Ex 17.1

Q1(i)

$^{14}C_3$

$= \frac{14!}{3!(14-3)!}$ $\left(\because\ ^nC_r = \frac{n!}{r!(n-r)!}\right)$

$= \frac{14!}{3!11!}$

$= \frac{14 \times 13 \times 12 \times 11!}{3 \times 2 \times 1 \times 11!}$

$= \frac{14 \times 13 \times 12}{6}$

$= 364$

Q1(ii)

$^{12}C_{10}$

$= \frac{12!}{10!(12-10)!}$ $\left(\because\ ^nC_r = \frac{n!}{r!(n-r)!}\right)$

$= \frac{12 \times 11 \times 10!}{10! \times 2 \times 1}$

$= 66$

Q1(iii)

$^{35}C_{35}$

$= \frac{35!}{35!(35-35)!}$ $\left(\because\ ^nC_r = \frac{n!}{r!(n-r)!}\right)$

$= 1$

Q1(iv)

${}^{n+1}C_n$

$= \frac{(n+1)!}{(n!)(n+1-n)} \qquad \left(\because {}^nC_r = \frac{n}{r!(n-r)!}\right)$

$= \frac{(n+1)\times n!}{n!\times 1!}$

$= n+1$

Q1(v)

$\sum_{r=1}^{5} {}^5C_r$

$= {}^5C_1 + {}^5C_2 + {}^5C_3 - {}^5C_4 + {}^5C_5$

$- \frac{5!}{1!4!} + \frac{5!}{2!3!} + \frac{5!}{3!2} + \frac{5!}{4!1!} + \frac{5!}{5!0!} \qquad \left(\because {}^nC_r - \frac{n!}{r!(n-r)!}\right)$

$= 5 - \frac{5\times 4}{2} + \frac{5\times 4}{2} + 5 + 1$

$= 5 \quad 10 + 10 + 5 + 1$

$= 31$

Q2

${}^nC_r = \frac{n!}{r!(n-r)!}$

Hence $n = n$

$r = 12$ and 5

Applying formula

${}^nC_p = {}^nC_q = n$

Then $p + q = n$

$\Rightarrow {}^nC_{12} = {}^nC_5$

$12 + 5 = n$

$\Rightarrow \quad n = 17$

Q3

If ${}^nC_p = {}^nC_q$

Then $p + q = n$

Also ${}^nC_r = \frac{n!}{r!(n-r)}$ ....(i)

$\Rightarrow \quad {}^nC_4 = {}^nC_6$

$4 + 6 = n$

$\Rightarrow \quad n = 10$

then ${}^{12}C_n = {}^{12}C_{10}$

Applying (i)

$${}^{12}C_{10} = \frac{12!}{10!\,2!}$$

$$= \frac{12 \times 11 \times 10!}{10! \times 2 \times 1}$$

$$= \frac{12 \times 11}{2 \times 1} = 66$$

Q4

If ${}^nC_p = {}^nC_q$

Then $p + q = n$

$\Rightarrow \quad {}^nC_{10} = {}^nC_{12}$

$10 + 12 = n$

$\Rightarrow \quad n = 22$

Find ${}^{23}C_n$

$\Rightarrow \quad {}^{23}C_{22}$

$$= \frac{23!}{22!\,1!}$$

$$= \frac{23 \times 22!}{22!}$$

$$= 23$$

Q5

If ${}^nC_p = {}^nC_r$ then $p + r = n$

$\therefore \quad x + 2x + 3 = 24$

$3x = 21$

$x = 7$

Q6

If ${}^nC_p = {}^nC_q$

$\Rightarrow \quad p + q = n$

also $C_x = {}^{18}C_{x+2}$

$\Rightarrow \quad x + x + 2 = 18$

$2x + 2 = 18$

$2x = 18 - 2 = 16$

$2x = 16$

$x = 8$

Q7

If ${}^nC_p = {}^nC_q$

Then $p + q = n$

$\Rightarrow \quad {}^{15}C_{3r} = {}^{15}C_{r+3}$

$\Rightarrow \quad 3r + r + 3 = 15$

$4r + 3 = 15$

$4r = 15 - 3 = 12$

$r = 3$

Q8

$^8C_r = ^7C_2 + ^7C_3$

Applying formula $^nC_r = \dfrac{n!}{r!(n-r)!}$

$$\frac{8!}{r!(8-r)!} = \frac{7!}{2!\ 5!} + \frac{7!}{3!\ 4!}$$

$$\frac{8 \times 7!}{r!\ (8-r)!} = \frac{7!}{2 \times 5 \times 4!} + \frac{7!}{3 \times 2 \times 4!}$$

$$\frac{8 \times 7!}{r!(8-r)!} = \frac{7!}{2 \times 4!}\left(\frac{1}{5} + \frac{1}{3}\right)$$

Cancelling 7! from both sides

$$\frac{8}{r!\ (8-r)!} = \frac{8}{2 \times 15 \times 4!}$$

Cancelling 8 on both sides

$$2 \times 5 \times 3 \times 4 \times 3 \times 2 \times 1 = r!(8-r)!$$

$$(3 \times 2)(5 \times 4 \times 3 \times 2 \times 1) = r!(8-r)!$$

$$\Rightarrow r! = 3!$$

$$r = 3$$

or $\quad r! = 5!$

$$r = 5$$

Q9

$$\frac{\frac{15!}{(15-r)!\ r!}}{\frac{15!}{(15-r+1)!(r-1)!}} = \frac{11}{5}$$

$$\frac{\frac{15!}{(15-r)(16-r)!\ r(r-1)!}}{\frac{15!}{(16-r)!(r-1)!}} = \frac{11}{5}$$

$$\Rightarrow \quad \frac{16-r}{r} = \frac{11}{5}$$

$$80 - 5r = 11r$$
$$80 = 16r$$
$$r = \frac{80}{16}$$
$$= 5$$
$$r = 5$$

Q10

${}^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$

$$\frac{\frac{(n+2)!}{8!(n-6)!}}{\frac{(n-2)!}{(n-6)!}} = \frac{57}{16}$$

$$\Rightarrow \quad \frac{(n+2)(n+1)(n)(n-1)(n-2)!}{8!(n-2)!} = \frac{57}{16}$$

Cancelling $(n-2)!$ from numerator and denominator

$$\Rightarrow \quad (n+2)(n+1)(n)(n-1) = \frac{57 \times 7 \times 6 \times 5 \times 4 \times 3 \times 1 \times 16}{16}$$

$$\Rightarrow \quad (n+2)(n+1)(n)(n-1) = 21 \times 20 \times 19 \times 10$$

comparing both sides $n = 19$

Q11

$$\frac{\dfrac{28!}{(2r)!\,(28-2r)!}}{\dfrac{24!}{(2r-4)!\,\left(24-(2r-4)\right)!}}=\frac{225}{11}$$

$$\Rightarrow \quad \frac{28\times27\times26\times25\times24!\,(2r-4)!\,(28-2r)!}{(2r)!\,(28-2r)!\,24!}=\frac{225}{11}$$

$$\Rightarrow \quad \frac{28\times27\times26\times25}{2r\times(2r-1)\times(2r-2)(2r-3)}=\frac{225}{11}$$

$$\Rightarrow \quad \frac{28\times27\times26\times25\times11}{15\times15}=2r(2r-1)(2r-2)(2r-3)$$

$$\Rightarrow \quad 11\times12\times13\times14=2r(2r-1)(2r-2)(2r-3)$$

Comparing both sides $r=7$

Q12

$$\frac{\dfrac{4n!}{(2n)!(2n)!}}{\dfrac{2n!}{n!\,n!}} \qquad \left(\because\ {}^nC_r=\frac{n!}{r!(n-r)!}\right)$$

$$=\frac{(4n)!}{(2n)!(2n)!}\frac{\times(n!)^2}{\times(2n)!^2}$$

$$=\frac{\left[1.2.3.4\ldots(4n-1)(4n)\right](n!^2)}{(2n)!\left[1.2.3.4\ldots(2n-2)(2n-1)(2n)\right]^2}$$

$$=\frac{\left[1.3.5\ldots(4n-1)\right]\times\left[2.4.6\ldots4n\right]\times(n!)^2}{(2n)!\left[1.3.5\ldots(2n-1)\right]^2\times\left[2.4.6\ldots(2n-2)(2n)\right]^2}$$

$$=\frac{\left[1.3.5\ldots(4n-1)\right]\times2^{2n}\times\left[1.2.3\ldots2n\right]\times n!^2}{(2n)!\times\left[1.3.5\ldots(2n-1)^2\times2^{2n}\times n!^2\right]}$$

$$=\frac{\left[1.3.5\ldots(4n-1)\right]}{\left[1.3.5\ldots(2n-1)\right]^2}$$

Hence Proved

Q13

$$\frac{\frac{2n!}{3!\,(2n-3)!}}{\frac{n!}{2!\,(n-2)!}} = \frac{44}{3}$$

$$\Rightarrow \quad \frac{2n!\,2!\,(n-2)!}{3!\,(2n-3)!\,n!} = \frac{44}{3}$$

$$\Rightarrow \quad \frac{2n!}{3n!\,(n-1)(2n-3)!} = \frac{44}{3}$$

$$\Rightarrow \quad 2n(2n-1)(2n-2) = 44n(n-1)$$

$$\Rightarrow \quad (2n-1)(n-1) = 11(n-1)$$

$Q \quad n = 6$

$\therefore \quad n = 6$

Q14

If ${}^{n}C_r = {}^{n}C_p$

then $r + p = n$

$\therefore \quad 16 = r + r + 2$

$r = 7$

then ${}^{r}C_4 = {}^{7}C_4 \qquad (\because r = 7)$

$$\Rightarrow \quad \frac{7!}{4!(7-4)!} \qquad \left(\because {}^{n}C_r = \frac{n!}{r!(n-r)!}\right)$$

$$\Rightarrow \quad \frac{7 \times 5 \times 6}{3 \times 2}$$

$= 35$

Q15

$${}^{20}C_5 + \sum_{r=2}^{5} {}^{25-r}C_4$$

$$\Rightarrow \left({}^{20}C_5 + {}^{20}C_4\right) + {}^{21}C_4 + {}^{22}C_4 + {}^{23}C_4$$

$$\Rightarrow \left({}^{21}C_5 + {}^{21}C_4\right) + {}^{22}C_4 + {}^{23}C_4 \qquad \left(\because {}^{n}C_{r-1} + {}^{n}C_r = {}^{n+1}C_r\right)$$

$$\Rightarrow \left({}^{22}C_5 + {}^{22}C_4\right) + {}^{23}C_4 \qquad \left(\because {}^{n}C_{r-1} + {}^{n}C_r = {}^{n+1}C_r\right)$$

$$\Rightarrow {}^{23}C_5 + {}^{23}C_4 \qquad \left(\because {}^{n}C_{r-1} + {}^{n}C_r = {}^{n+1}C_r\right)$$

$$\Rightarrow {}^{24}C_5$$

$$\Rightarrow 42504$$

Q16

$$\text{Product} = \left[(2n+1)(2n+3)(2n+5)\ldots(2n+r)\right]$$

$$= \frac{(2n)!\left[(2n+1)(2n+3)\ldots(2n+r)\right]}{(2n)!}$$

$$= \frac{(2n)\left[(2n-1)(2n-2)\ldots4.2(2n+1)(2n+3)\right]}{(2n)!}$$

$$= \frac{(2n+r)!}{(2n)!}$$

Hence $r = 2n$

$$= \frac{(2n+2n)!}{2n}$$

$$= \frac{(4n)!}{(2n)!}$$

$$= (2n)!$$

Q17

L.H.S, $= {}^{2n}C_n + {}^{2n}C_{n-1}$

$$\frac{2n!}{n!\,n!} + \frac{2n!}{(n-1)!(n-1)!}$$

$$= (2n)!\left[\frac{1}{n(n-1)!(n)(n-1)!} + \frac{1}{(n-1)!(n-1)!}\right]$$

$$= \frac{(2n)!}{(n-1)!(n-1)!}\left[\frac{1+n^2}{n^2}\right] \ldots\ldots\ldots\ldots (i)$$

$${}^{2n+2}C_{n+1} = \frac{(2n+2)!}{(n+1)!(n+1)!}$$

$$= \frac{(2n+2)(2n+1)(2n)!}{n(n+1)(n-1)!(n+1)n(n-1)!} \ldots\ldots\ldots (ii)$$

$$\Rightarrow \quad \frac{(2n)!}{(n-1)!(n-1)!} \times \frac{(n+1)^2(n)^2(n-1)!(n-1)!}{(2n+2)(2n+1)(2n)!} \times \left(\frac{1+n^2}{n^2}\right)$$

$$= \frac{(n+1)^2\left(1+n^2\right)}{(2n+2)(2n+1)} = \frac{(n+1)(n+1)\left(n^2+1\right)}{2(n+1)(2n+1)}$$

$$= \frac{(n+1)\left(n^2+1\right)}{(2n+1)} \times \frac{1}{2}$$

Q18

${}^nC_4, {}^nC_5$, and ${}^nC_6$ are in A.P

$\therefore \quad {}^nC_5 - {}^nC_4 = {}^nC_6 - {}^nC_5$

$$\frac{n!}{5!(n-5)!} - \frac{n!}{4!(n-4)!} = \frac{n!}{6!(n-6)!} - \frac{n!}{6!(n-5)!}$$

$$\Rightarrow \quad \frac{n!}{4!(n-5)!}\left[\frac{1}{5} - \frac{1}{n-4}\right] = \frac{n!}{5!(n-6)!}\left[\frac{1}{6} - \frac{1}{n-5}\right]$$

$$\Rightarrow \quad \frac{1}{n-5}\left[\frac{n-4-5}{5(n-4)}\right] = \frac{1}{5}\left[\frac{n-5-6}{6(n-5)}\right]$$

$$\Rightarrow \quad \frac{n-9}{n-4} = \frac{n-11}{6}$$

$$\Rightarrow \quad 6n - 54 = n^2 - 15n + 44$$

$$\Rightarrow \quad n^2 - 21n + 98 = 0$$

$$n = 7, 14$$

n is 7 or 14

Q19

We have $\alpha = {}^mC_2 = \dfrac{m(m-1)}{2}$ $\left({}^nC_r = \dfrac{n!}{r!(n-r)!} \right)$

Now ${}^{\alpha}C_2 = \dfrac{\alpha(\alpha-1)}{2}$

$$= \frac{\left(\frac{m(m-1)}{2}\right)\left(\frac{m(m-1)}{2}-1\right)}{2}$$

$$= \frac{m(m-1)(m^2-m-2)}{2\times2\times2} = \frac{m(m-1)(m+1)(m-2)}{8}$$

$$= \frac{m(m-1)(m+1)(m-2)}{4\times2}$$

multiplying with 3, numerator and denominator to make 4:

Or $$= \frac{m(m+1)m(m-1)(m-2)}{4.3.2.1}$$

$$= \frac{3(m+1)m(m-1)(m-2)}{4!}$$

$$= 3.\ {}^{m+1}C_4 \qquad \left(\because\ {}^nC_r = \frac{n!}{r!(n-r)!}\right)$$

Q20(i)

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$${}^nC_{r-1} = \frac{n!}{(r-1)!(n-r+1)!}$$

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n!(r-1)!(n-r+1)!}{r!(n-r)!\,n!}$$

$$= \frac{(r-1)!(n-r+1)\times(n-r)!}{r\times(r-1)!(n-r)!}$$

$$= \frac{n-r+1}{r}$$

Hence Proved.

Q20(ii)

$$n \times {}^{n-1}C_{r-1}$$

$$= n \times \frac{(n-1)!}{(r-1)!\left[(n-1)-(r-1)\right]!}$$

$$= \frac{n! \times (n-r+1)}{(r-1)!(n-r)!(n-r+1)}$$

multiplying numerator and denominator by $(n-r+1)]$

$$= \frac{(n-r+1) \times n!}{(r-1)!(n-r+1)!}$$

$$= (n-r+1)^{n}C_{r-1}$$

Hence Proved

Q20(iii)

$${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$${}^{n-1}C_{r-1} = \frac{(n-1)!}{(r-1)!(n-1)-(r-1)!}$$

Or $\frac{{}^{n}C_{r}}{{}^{n-1}C_{r-1}} = \frac{n!(r-1)!(n-r)!}{r!(n-r)!(n-1)!}$

$$= \frac{n \times (n-1)!(r-1)! \times (n-r)!}{r \times (n-1)! \times (r-1)! \times (n-r)!}$$

$$= \frac{n}{r}$$

Hence Proved

Q20(iv)

$$L.H.S \Rightarrow {}^{n}C_{r} + 2\,{}^{n}C_{r-1} + {}^{n}C_{r-2}$$

$$= \left({}^{n}C_{r} + {}^{n}C_{r-1}\right) + \left({}^{n}C_{r-2} + {}^{n}C_{r-1}\right)$$

$$= {}^{n+1}C_{r} + {}^{n+1}C_{r-1} \qquad \left[\because\ {}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}\right]$$

$$= \left(n+1\right) + {}^{1}C_{r}$$

$$= {}^{n+2}C_{r}$$

Ex 17.2

Q1

No of players = 15
No of players to be selected = 11

Number of combinations

$= {}^{15}C_{11}$

$= \frac{15!}{11!\ 4!} = \frac{15\times14\times13\times12}{4\times3\times2}$

$= 1365$ ways

Q2

Total boy = 25
Total girls = 10

Party of 8 to be made from 25 boy and 10 girls, selecting 5 boy and 3 girls

$\Rightarrow \quad {}^{25}C_5$ and ${}^{10}C_3$

$= {}^{25}C_5 \times {}^{10}C_3$

Now, ${}^{25}C_5 = \frac{n!}{r!(n-r)!}$

$\Rightarrow \quad \frac{25!}{5!\ 20!}\times\frac{10}{3!\ 7!} = \frac{25\times24\times23\times22\times21\times10\times9\times8}{5\times4\times3\times2\times3\times2}$

$= 6375600$

Q3

Out of 9 courses 2 are compulsory. So students can choose from 7 courses only. Also out of 5 courses that students need to choose, 2are compulsory.
So they have to choose 3 courses out of 7 courses. This can be done ${}^7C_3=35$ ways.

Q4

No of players = 16

No of players to be selected = 11

$\therefore$ No of combination = ${}^{16}C_{11}$

$$= \frac{16!}{11!\ 5!} = \frac{16 \times 15 \times 14 \times 13 \times 12}{5 \times 4 \times 3 \times 2} = 4368$$

(i) *I*nclude 2 particular players

$\rightarrow$ Now we have to select 9 more out of remaining 14

$\therefore$ *R*equired number of ways

$$= {}^{14}C_9$$

$$= \frac{14!}{9!\ 5!} = \frac{14 \times 13 \times 12 \times 11 \times 10}{5 \times 4 \times 3 \times 2}$$

$$= 2002$$

(ii) *E*xclude 2 particular players $\rightarrow$ now we have to select 11 players out of 14 players

$$= {}^{14}C_{11} = \frac{14!}{11!\ 3!} = \frac{14 \times 13 \times 12}{3 \times 2}$$

$$= 364$$

Q5

Total number of professor = 10
Total number of student = 20

Committee of 2 professor and 3 student can be selected in $^{10}C_2 \times ^{20}C_3$ ways.

$$= \frac{10!}{2!\ 8!} \times \frac{20!}{3!\ 17!}$$

$$= \frac{10 \times 9}{2} \times \frac{20 \times 19 \times 18}{3 \times 2}$$

= 51300 ways

(i) a particular professor is included

$\therefore$ committee is $^{9}C_1 \times ^{20}C_3$

$$= \frac{9!}{8!} \times \frac{20}{3! \times 17!} = \frac{9 \times 20 \times 19 \times 18}{3 \times 2}$$

= 10260

(ii) a particular student is included

$\therefore$ committee is $^{10}C_2 \times ^{19}C_2$

$$= \frac{10!}{2 \times 8!} \times \frac{19}{2! \times 17!} = \frac{10 \times 9 \times 19 \times 18}{2 \times 2 \times 1} = 7695$$

(iii) a particular student is excluded $\rightarrow$ now total student are 19

$\therefore$ committee is $^{10}C_2 \times ^{19}C_3$

$$= \frac{10!}{2 \times 8!} \times \frac{19}{3! \times 16!} = \frac{10 \times 9 \times 19 \times 18 \times 17}{2 \times 3 \times 2} = 43605$$

Q6

The we can multiplying 2 or 3 or 4 digits.
Then number of ways of multiplying 4 digits at a time

$= {}^4C_4 \dots\dots\dots (i)$

The number of ways of multiplying 3 digits at a time

$= {}^4C_3 \dots\dots\dots (ii)$

The number of ways of multiplying 2 digits at a time

$= {}^4C_2 \dots\dots\dots (iii)$

∴ Total number of ways

$= {}^4C_4 + {}^4C_2 + {}^4C_3$

$\Rightarrow \quad = 1 + \frac{4 \times 3}{2} + 4$

$\Rightarrow \quad = 11$

= There are 11 ways

Q7

Total number of boys = 12
Total number of girls = 10
Total number of girls for the competition

$= 10 + 2 = 12$

Total students chosen for competition

$= 10 - 2$ (at least 4 boys and 4 girls)

∴ Selection can be made in

${}^{12}C_4 \times {}^8C_4 + {}^{12}C_5 \times {}^8C_3 + {}^{12}C_6 \times {}^8C_2$

$= \frac{12!}{4!\,8!} \times \frac{8!}{4!\,4!} + \frac{12!}{5!\,7!} \times \frac{8!}{3!\,5!} + \frac{12!}{6!\,6!} \times \frac{8!}{2!\,6!}$

$= \left(\frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 4 \times 3 \times 2}\right) + \left(\frac{12 \times 11 \times 10 \times 9 \times 8 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 3 \times 2}\right) + \left(\frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2 \times 2}\right)$

$= 55440 + 44352 + 181104$
$= 280896$

∴ Total number of ways = 385770 − 280896 = 104874
(385770 = from 10 girls 4 are chosen)

Q8

Total number of books = 10
total books to be selected = 4

(i) there is no restriction

$$= {}^{10}C_4 = \frac{10!}{4!\,6!} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2}$$

$$= 210$$

(ii) two particular books are always selected

these the total books = 10 − 2 = 8

So out of rem aining 8 books selection od 2 books canbe done in ${}^8C_2$ way

$$= \frac{8!}{2!\,6!} = \frac{8 \times 7}{2 \times 1} = 28 \text{ ways}$$

(iii) two particular books are never selected

these the total number of books = 10 − 2 = 8

so out of rem aining 8 books, 4 books can be selected in ${}^8C_4$ way

$$= \frac{8!}{4!\,4!} = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2}$$

$$= 14 \times 5$$

$$= 70 \text{ ways}$$

Q9

Total number of officer = 4
Total number of jawans = 8
Total number of selection to be made = 6

(i) to include exactly one officer

This can be done is ${}^4C_1 \times {}^8C_5$ ways

$$= \frac{4!}{1!\,3!} \times \frac{8!}{5!\,3!}$$

$$= \frac{4 \times 8 \times 7 \times 6}{3 \times 2} = 224 \text{ ways}$$

(ii) to induce at least one officer

This can be done is following ways

$${}^4C_1 \times {}^8C_5 + {}^4C_2 \times {}^8C_4 + {}^4C_3 \times {}^8C_3 + {}^4C_4 \times {}^8C_2$$

$$= \frac{4 \times 8!}{5!\,3!} + \frac{4!}{2!\,2!} \times \frac{8!}{4!\,4!} + \frac{4!}{3!\,1!} \times \frac{8!}{3!\,5!} + \frac{1 \times 8!}{2!\,5!}$$

$$= \left(\frac{4 \times 8 \times 7 \times 6}{3 \times 2}\right) + \left(\frac{4 \times 3 \times 8 \times 7 \times 6 \times 5}{2 \times 4 \times 3 \times 2}\right) + \left(\frac{4 \times 8 \times 7 \times 6}{3 \times 2}\right) + \left(\frac{8 \times 7}{2 \times 1}\right)$$

$$= (4 \times 8 \times 7) + (4 \times 3 \times 7 \times 5) + (4 \times 8 \times 7) + (4 \times 7)$$

$$= 224 + 420 + 224 + 28$$

$$= 896 \text{ ways}$$

Q10

Total number of students is XI = 20
Total number of students is XII = 20

Total number of students to be selected is a team = 11
(at least 5 from XI and 5 from XII)
this can be done is following ways

$${}^{20}C_5 \times {}^{20}C_6 + {}^{20}C_6 \times {}^{20}C_5$$

$$= 2\left({}^{20}C_6 \times {}^{20}C_5\right)$$

$$= 2\left(\frac{20!}{6!\,14!} \times \frac{20!}{5!\,15!}\right)$$

or $$= \frac{2 \times 20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 20 \times 19 \times 18 \times 17 \times 16}{6 \times 5 \times 4 \times 3 \times 2 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= 19 \times 17 \times 16 \times 15 \times 2 \times 19 \times 3 \times 17 \times 8$$

$$= 1201870080 \text{ ways}$$

Q11

Total number of questions = 10

Question in part A = 6

Question in part B = 7

Selecting to questions with at least 4 from each part A and part B.
can from done in following way.

$${}^6C_4 \times {}^7C_6 + {}^6C_5 \times {}^7C_5 + {}^6C_6 \times {}^7C_4$$

$$= \left(\frac{6!}{4!\ 2!} \times \frac{7!}{6!\ 1!}\right) + \left(\frac{6!}{5!\ 1!} \times \frac{7!}{5!\ 2!}\right) + \left(\frac{1 \times 7!}{4!\ 3!}\right) \qquad \left(\because {}^nC_r = \frac{n!}{r!(n-r)!}\right)$$

$$= \left(\frac{6 \times 5 \times 7}{2}\right) + \left(\frac{6 \times 7 \times 6}{2}\right) + \left(\frac{7 \times 6 \times 5}{3 \times 2}\right)$$

$$= (105) + (126) + (35)$$

= 266 ways

Q12

Total number of question = 5

Total number of question to be answered = 4

Given that 1 and 2 question are compulsory, the number of ways in which a student can choose the questions will follow the following way.

Total question = 5 − 2 = 3

Out of 3 remaining questions a student has to select any 2 for answering

$\Rightarrow {}^3C_2 = 3$ ways

Q13

Total number of questions = 12
Total number of questions to be answered = 7

Each group has 6 questions (6 + 6) more than 5 question from either group is not permitted, therefore the number of ways a student can choose questions can be done in following ways.

$$^6C_2 \times {^6C_5} + {^6C_3} \times {^6C_4} + {^6C_4} \times {^6C_3} + {^6C_5} \times {^6C_2}$$

$$= 2\left({^6C_2} \times {^6C_5} + {^6C_3} \times {^6C_4}\right)$$

$$= 2\left(\frac{6!}{2!\ 4!} \times \frac{6!}{5!\ 1!} + \frac{6!}{3!\ 3!} \times \frac{6!}{4!\ 2!}\right)$$

$$= 2\left(\frac{6 \times 5 \times 6}{2} + \frac{5 \times 5 \times 4 \times 6 \times 5}{3 \times 2 \times 2}\right)$$

$$= \frac{2 \times 6 \times 5 \times 6}{2}\left(1 + \frac{20}{6}\right)$$

$$= 180\left(\frac{26}{6}\right)$$

$$= 30 \times 26 = 780$$

= 780 ways

Q14

Number of point = 10
Number of collinear points = 4

Since 4 out of 10 points are collinear, so the number of lines will be $\left({}^4C_2 - 1\right)$ lie from ${}^{10}C_2$

$\left\{\text{one is subtracted from } {}^4C_2 \text{ to count for the Line on which 4 collinear points lie}\right\}$

$\therefore$ number of lines $= {}^{10}C_2 - \left({}^4C_2 - 1\right)$

$= {}^{10}C_2 - {}^4C_2 + 1$

$= \frac{10!}{2!\,8!} - \frac{4!}{2!\,2!} + 1$

$= \frac{10 \times 9}{2} - \frac{4 \times 3}{2} + 1$

$= 45 - 6 + 1$

$= 40$

Q15

(i) hexagon $\rightarrow$ A hexagon has 6 angular points. By joining any two angular points we get a line which is either a side or a diagonal.

$\therefore$ Number of lines $= {}^6C_2 = \frac{6!}{2!\,4!}$

$= \frac{6 \times 5}{2} = 15$

Number of sides = 6

$\therefore$ Number of diagonals $= 15 - 6 = 9$

(ii) Polygon of 16 sides will have 16 angular points. By joining any 2 points we get a line which is either a side or a diagonal.

$\therefore$ number of lines $= {}^{16}C_2 = \frac{16!}{2!\,14!}$

$= \frac{16 \times 15}{2} = 120$

$\Rightarrow$ number of sides = 16

$\therefore$ number of diagonals $= 120 - 16 = 104$

Q16

Since 5 out of 12 points are collinear, so the number of triangles will be ${}^5C_3$ less from ${}^{12}C_3$

$$= {}^{12}C_3 - {}^5C_3$$

$$= \frac{12!}{3!\,9!} - \frac{5!}{3!\,2!}$$

$$= \frac{12\times11\times10}{3\times2} - \frac{5\times4}{2}$$

$$= 220 - 10$$

$$= 210$$

Q17

Total men = 6
Total women = 4

Total persons in committee = 5

(where at least are women has to be selected)
This can be done in

$${}^4C_1 \times {}^6C_4 + {}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1$$

$$\left({}^nC_r = \frac{n!}{r!(n-r)!}\right) \left({}^nC_r = 1,\ {}^nC_1 = n\right)$$

$$= \left(\frac{4\times6!}{4!\times2!}\right) + \left(\frac{4!}{2!\,2!}\times\frac{6!}{3!\,3!}\right) + \left(\frac{4!}{3!\,1!}\times\frac{6!}{2!\,4!}\right) + (1\times6)$$

$$= \left(\frac{4\times6\times5}{2}\right) + \left(\frac{4\times3}{2}\times\frac{6\times5\times4}{3\times2}\right) + \left(\frac{4\times6\times5}{2}\right) + (6)$$

$$= (60) + (120) + 60 + 6$$

$$= 246 \text{ ways}$$

Q18

52 families have at most 2 children, while 35 families have 2 children.
The selection of 20 families of which at least 18 families must have at most 2 children can be made as under

i) 18 families out of 52 and 2 families out of 35

ii) 19 families out of 52 and 1 family out of 35

iii) 20 families out of 52

Therefore the number of ways are $= {}^{52}C_{18} \times {}^{35}C_{2} + {}^{52}C_{19} \times {}^{35}C_{1} + {}^{52}C_{20} \times {}^{35}C_{0}$

Q19

i) Since, the team does not include any girl therefore, only boys are to be selected.
5 boys out of 7 boys can be selected in ${}^7C_5$ ways.

$$= {}^7C_5 = \frac{7!}{5!\,2!} = \frac{6 \times 7}{2} = 21$$

ii) Since, at least one boy and one girl are to be there in every team. The team consist of

a) 1 boy and 4 girls i.e. ${}^7C_1 \times {}^4C_4$
b) 2 boys and 3 girls i.e. ${}^7C_2 \times {}^4C_3$
c) 3 boys and 2 girls i.e. ${}^7C_3 \times {}^4C_2$
d) 4 boys and 1 girls i.e. ${}^7C_4 \times {}^4C_1$

∴ The required number of ways

$$= {}^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$$

$$= 7 + 84 + 210 + 140$$

$$= 441$$

iii) Since, the team has to consist of at least 3 girls, the team can consist of
a) 3 girls and 2 boys $= {}^7C_2 \times {}^4C_3$ ways
b) 4 girls and 1 boy $= {}^4C_4 \times {}^7C_1$, ways

∴ The required number of ways

$$= {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1$$
$$= 84 + 7$$
$$= 91$$

Q20

The number of ways selecting of 3 people out of 5

$$= {}^5C_3 = \frac{5!}{3!\,2!} = \frac{5\times 4}{2} = 10.$$

1 man can be selected from 2 men in ${}^2C_1$ ways and 2 women can be selected from 3 women in ${}^3C_2$ ways.

$\therefore$ The required number of committees

$$= {}^2C_1 \times {}^3C_2$$

$$= \frac{2!}{1!\,1!} \times \frac{3!}{2!\,1!}$$

$$= 6$$

Q21

A decagon has 10 sides
By joining any two angular points

we get a line which is either a side or a diagonal

$\therefore$ number of lines $= {}^{10}C_2 = \frac{10!}{2!\,8!} = \frac{10\times 9}{2} = 45$

$\therefore$ number of sides = 10

$\therefore$ number of diagonals = 45 – 10 = 35

Also,by joining 3 angular points a triangle in formed

$$= {}^{10}C_3$$

$$= \frac{10!}{3!\,7!} = \frac{10\times 9\times 8}{3\times 2} = \frac{720}{6} = 120$$

$$= 120$$

Q22

Out of the 52 cards 4 are kings and 48 are Non-kings.
Five cards with at least one king

= (one king and 4 non-kings) or (two kings and 3 non kings) or
(3 kings and 2 non kings) or (4 kings and 1 non kings)

$$= \left({}^{4}C_1 \times {}^{48}C_4\right) + \left({}^{4}C_2 \times {}^{48}C_3\right) + \left({}^{4}C_3 \times {}^{48}C_2\right) + \left({}^{4}C_4 \times {}^{48}C_1\right)$$

$$= 4 \times \frac{48 \times 47 \times 46 \times 45}{4 \times 3 \times 2} + \frac{4 \times 3}{2} \times \frac{48 \times 47 \times 46}{3 \times 2} + 4 \times \frac{48 \times 47}{2} + 1 \times 48$$

$= 778320 + 103776 + 4512 + 48$
$= 886656$

Required Number of ways = 886656

Q23

Total persons = 8 Selection to be made = 6 person.

If A is chosen then B must be chosen.
$\Rightarrow$ A and B are chosen together

$\therefore$ Selection can be made in

$${}^{6}C_4 = \frac{6!}{4!\,2!} = \frac{6 \times 5}{2} = 15 \text{ ways}$$

Also the number of selections in which A and B are not chosen are

$${}^{7}C_6 = \frac{7!}{6!\,1!} = 7 \text{ ways}$$

Total number of ways in which selection is made = 15 + 7
= 22 ways

Q24

There are 5 boys and 4 girls.
The team consists of 3 boys and 3 girls.

Number of ways to from the leom

$$= {}^5C_3 \times {}^4C_3$$

$$= \frac{5!}{3!2!} \times \frac{4!}{3!}$$

$$= \frac{5 \times 4}{2} \times 4$$

$$= 40$$

Number of ways = 40

Q25

There are 6 red balls, 5 white balls and 5 blue balls.

Number of ways to select 9 balls consisting of 3 balls of each colour.

= (3 red out of 6 red) and
(3 white out of 5 white) and
(3 blue out of 5 blue balls)

$$= {}^6C_3 \times {}^5C_3 \times {}^5C_3$$

$$= \frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times \frac{5 \times 4}{2} \times \frac{5 \times 4}{2}$$

$$= 2000$$

Required Number of ways = 2000

Q26

Out of 52 cards 4 are ace and
and 48 are Non ace.

Number of ways to select 5 cards with exactly one ace.

= (one ace out of 4 ace) and
(4 non-ace out of 48 Non-ace)

$= {}^{4}C_{1} \times {}^{48}C_{4}$

$= 4 \times \frac{48 \times 47 \times 46 \times 45}{4 \times 3 \times 2 \times 1}$

= 778320

Required Number of ways = 778320

Q27

There are total 5 bowlers and 12 batsman are available to select from.

Number of ways to select a team of 11 that includes exactly 4 bowlers.

= (7 batsman out of 12 batsman) and
(4 bowlers out of 5 bowlers)

$= {}^{12}C_{7} \times {}^{5}C_{4}$

$= \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} \times 5$

= 3960

Required number of ways = 3960

Q28

Bag contains 5 black and 6 red balls.

Number of ways to select 2 black balls out of 5 black and 3 red balls out of 6 red balls.

$= {}^{5}C_{2} \times {}^{6}C_{3}$

$= \frac{5 \times 4}{2} \times \frac{6 \times 5 \times 4}{3 \times 2}$

= 200

Required number of ways = 200

Q29

There are total 9 courses are available and out of these 2 subjects are compulsory. So,

Number of ways to select 2 compulsory and 3 option out of 9 − 2 = 7 subjects

$= {}^{2}C_{2} \times {}^{7}C_{3}$

$= 1 \times \frac{7 \times 6 \times 5}{3 \times 2}$

= 35

Required number of ways = 35

Q30

i) The committee consists of exactly 3 girls.

∴ We have to select 4 boys from 9 boys.

This can be done in ${}^9C_4$ ways and 3 girls out of 4 girls can be selected in ${}^4C_3$ ways.

∴ The required number ways $= {}^9C_4 \times {}^4C_3$

$$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times 4$$

$$= 504$$

ii) At least 3 girls are there.

∴ There are 3 or more than i.e. 3 or 4 girls

∴ *a*) 3 girls and 4 boys i.e. ${}^4C_3 \times {}^9C_3$ ways

b) 4 girls and 3 boys i.e. ${}^4C_4 \times {}^9C_3$ ways

∴ The required number of ways

$= {}^4C_3 \times {}^9C_4 - {}^4C_4 \times {}^9C_3$

$= 504 + 84$

$= 588$

iii) For at most 3 girls there are 3,2,1 or 0 girls

i.e *a*) 0 girls and 7 boys $= {}^4C_0 \times {}^9C_7$

b) 1 girls and 6 boys $= {}^4C_1 \times {}^9C_6$

c) 2 girls and 5 boys $= {}^4C_2 \times {}^9C_5$

d) 3 girls and 4 boys $= {}^4C_3 \times {}^9C_4$,

∴ Total number of required ways

$\Rightarrow \quad {}^4C_0 \times {}^9C_7 + {}^4C_1 \times {}^9C_6 + {}^9C_2 \times {}^9C_5 + {}^4C_3 \times {}^9C_4$

$\Rightarrow \quad 0 \times \frac{9 \times 8}{2} + 4 \times \frac{9 \times 8 \times 7}{3 \times 2} + \frac{4 \times 3}{2} \times \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} + 504$

$\Rightarrow \quad 36 + 48 \times 7 + 18 \times 42 + 504$

$\Rightarrow \quad 1632$

Q31

Here, part I has 5 questions and part II has 7 questions.

Student has to attempt 8 questions selecting at least 3 from each section.
So,
Number of ways to select at least 3 from each section and a total of 8 questions.

$$= \left(3 \text{ from part I and } 5 \text{ from part II}\right) \text{ or}$$
$$\left(4 \text{ from part I and } 4 \text{ from part II}\right) \text{ or}$$
$$\left(5 \text{ from part I and } 3 \text{ from part II}\right)$$

$$= \left({}^5C_3 \times {}^7C_5\right) + \left({}^5C_4 \times {}^7C_5\right) + \left({}^5C_3 \times {}^7C_3\right)$$

$$= \left(\frac{5 \times 4}{2 \times 1} \times \frac{7 \times 6}{2 \times 1}\right) + \left(5 \times \frac{7 \times 6 \times 5}{3 \times 2}\right) + \left(1 \times 7 \frac{5 \times 6 \times 5}{3 \times 2}\right)$$

$$= 210 + 175 + 35$$
$$= 420$$

Required number of ways = 420

Q32

In a parallelgram, there are 2 sets of parallel lines. Each set of parallel lines consists of $(m+2)$ lines and, each parallelogram is formed by choosing two lines from the first set and two straight lines from the second set.

Hence, the total number of parallelogram $= {}^{m+2}C_2 \times {}^{m+2}C_2$

$$= \left({}^{m+2}C_2\right)^2$$

Q33

There are 18 points in a plane out of which 5 points are collinear.

Then number of striaght lines joining these points are

$\Rightarrow \quad {}^{n}C_2 - \left({}^{p}C_2 - 1\right)$

$\Rightarrow \quad {}^{n}C_2 - {}^{p}C_2 + 1 \begin{pmatrix} \text{where } n = 18 \\ p = 5 \end{pmatrix}$

$\Rightarrow \quad {}^{18}C_2 - {}^{5}C_2 + 1$

$\Rightarrow \quad \frac{18 \times 17}{2} - \frac{5 \times 4}{2} + 1$

$\Rightarrow \quad 144$

number of triangle = ${}^{13}C_3$

$= \frac{13!}{3!\,10!} = \frac{13 \times 12 \times 11}{3 \times 2}$

$= 13 \times 2 \times 11$

$= 13 \times 22$

$= 806$

Ex 17.3

Q1

Total vowels are 5
Total consonants are 17

Vowels formed from 5 vowels and 17 consonants by selecting 2 vowels and 3 consonants are.

$$= {}^5C_2 \times {}^{17}C_3 \times 5!$$

$$= \frac{5!}{2!\ 3!} \times \frac{17!}{3!\ 4!} \times 120$$

$$= \frac{5 \times 4}{2} \times \frac{17 \times 16 \times 15}{3 \times 2} \times 120$$

$$= 10 \times 17 \times 8 \times 5 \times 120$$

$$= 400 \times 17 \times 120$$
$$= 6800 \times 120$$

$$= 816000$$

Q2

Total persons=10

Number of persons to be selected=5

Condition $= p_1$ must and p_4, p_5 must not be there

Remaining number of persons required is 4 out of 10-3=7

$${}^7C_4 \times 5!$$

Q3

(i) Total number of 4 letter words formed from the letters of the word 'MONDAY' is $= {}^6C_4 \times 4! = 360$

(ii) Total number of words formed by using all letters of the word 'MONDAY' is $= 6! = 720$

(iii)
There are two vowels A and O. So, first place can be filled in 2 ways and the remaining 5 places can be filled in 5! ways.
So, total number of words beginning with a vowel $= 2 \times 5! = 240$

Q4

First separate the 3 and then arrange the remaining things

$^{n-3}C_{r-3}\,(r-2)! \times 3!$

Q5

I N V O L U T E

Number of letters = 8

Wovels = I,O,U,E

Consonents = N,V,L,T,

Number of ways to select 3 wovels = $^4C_3$

Number of ways to select 2 consonents = $^4C_2$

Number of ways to arrange these five letters

$= {}^4C_3 \times {}^4C_2 \times 5!$

$= 4 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 2880$

Required number of ways = 2880

Q6

There are x things

Two specific things are to occur together, so remaining things are $(r-2)$.

Now, number of ways to arrange $(r-2)$ things out of $(n-2)$ = $^{(n-2)}P_{(r-2)}$

Two things can be arranged is $(r-1)$ ways.

and these two can be placed in 2 ways.

Therefore,

Required number of ways = $2(r-1)\,{}^{(n-2)}P_{(r-2)}$

Q7

The given word is P R O P O R T I O N.
Total letters = 10

Number of P = 2, Number of R = 2
Number of O = 3, Number of T = 1
Number of I = 1, Number of N = 1

(i) Case I: There are 6 different letters is which all the four are distirct to selected.
Number of ways to select therefour = $^6C_4$
= 15

Case II: Two same and two distirct letters are selected there are three pairs which more than, letters.
Number of ways to select therefour
$= {}^3C_1 \times {}^5C_2$
$= 3 \times 10$
$= 30$

Case III: Two alike of one kind and two alike of other kind.
There are 3 pairs of letters is the more than one letters. Any 2 of these 3 letters.
Number of ways to select these letters
$= {}^3C_2$
$= 3$

Case IV: Three alike and one dofferent.
Number of ways to select these letters
$= 1 \times {}^5C_1$
$= 5$
Therefore,
Number of ways to select four letters
$= 15 + 30 + 3 + 5$
$= 53$
Required number of ways to select = 53

(ii) For case I:

Number of arrangements of four letters all distirct = $^6C_4 \times 4!$

$= 15 \times 24$

$= 360$

For case II:

Number of arrangements of four letters two same kind and two of different kind

$= {}^3C_1 \times {}^5C_2 \times \frac{4!}{2!1!1!}$

$= 3 \times 10 \times 12$

$= 360$

For case III:

Number of arrangements of four letters two alike of one kind and two of other kind

$= {}^3C_2 \times \frac{4!}{2!2!}$

$= 3 \times 6$

$= 18$

Case IV:

Number of arrangements of four letters 3 alike and 1 other kind

$= 1 \times {}^5C_1 \times \frac{4!}{3!1!}$

$= 20$

Therefore,

Total number of arrangements of four letters selected = $360 + 360 + 18 + 20$

Required number of arrangement = 758

Q8

M O R A D A B A D

Number of M = 1, Number of O = 1

Number of R = 1, Number of A = 3

Number of D = 2, Number of B = 1

(i) $\dfrac{\text{Four distirct letters}}{\text{There are 6 letters}}$

Number of arrangement of 4 letters

selected from these 6 = $^6C_4 \times 4!$

$= 15 \times 24$

$= 360$

(ii) Two alike and two different letters

There are 2 pairs with more than one

So, one pair from these and 2 from letters from rest 5 letters.

Number of ways to arrange therefour

$= {}^2C_1 \times {}^5C_2 \times \dfrac{4!}{2!}$

$= 2 \times 10 \times 12$

$= 240$

(iii) Two alike and two alike of other kinds.

Number of ways to arrange therefour

$= {}^2C_2 \times {}^5C_2 \times \dfrac{4!}{2!2!}$

$= 6$

(iv) There alike and one different number of ways to arrange therefour

$= 1 \times {}^5C_1$

$= 5 \times \dfrac{4!}{3!1!}$

$= 20$

Therefore,

Required number of ways $= 240 + 360 + 6 + 20$

Required number ways $= 626$

Q9

In one round table the business man can accommodate the guests in ${}^{21}C_{15}$ ways. In the second round table he can accommodate the guests in ${}^{6}C_{6}$ ways. Keeping one guest as fixed in the first round table, the other 14 guests can be arrange in 14! ways. Keeping one guest as fixed in the second round table, the other 5 guests can be arrange in 5! ways.
Therefore the total number of ways in which the guests can be arrange is
$= {}^{21}C_{15} \times {}^{6}C_{6} \times 14! \times 5!$ ways

Q10

The word EXAMINATION has letters E,X,A,M,I,N,T,O where A,I,N repeat twice.

$\therefore$ The total number of letter = 11

The number of ways of selecting 4 letters.

$$= {}^{11}C_4 = \frac{11!}{4!\,7!} = \frac{11 \times 10 \times 9 \times 8}{4 \times 3 \times 2}$$

= 330.

The number of arranging 4 letters

a) All different ${}^{8}C_4 \times 4! = {}^{8}P_4 = \frac{8!}{4!}$
$= 8 \times 7 \times 6 \times 5$
$= 56 \times 30$
$= 1680$

b) 2 distinct and 2 alike

$$= {}^{3}C_1 \times {}^{7}C_2 = \frac{3 \times 7 \times 6}{2} = 63 \times \frac{4!}{2!}$$
$= 378$

c) 2 alike of one kind and 2 alike of other kind

$$ {}^{3}C_2 \times \frac{4!}{2\;2} = 3 \times 6 = 18$$

d) 3 alike and 1 distinct letter

$$ {}^{3}C_1 \times {}^{7}C_2 = \frac{3 \times 7 \times 6}{2} = 378$$

$\therefore$ Total number of ways in which 4 letter words are formed = 1680 + 378 + 18 + 378
= 2454 ways

Q11

$No\,of\ persons = 16$

$Condition\,on\,specific\ persons = 4\,and\ 2 = 6$

Remaining people=16-6=10

So lets fill 8 people on both sides first from these 10.

First side, we can select 4 out of 10.

$^{10}C_4 \times {}^{6}C_6$

Now we can arrange these 8 people on both sides in $8! \times 8!$ ways

Answer= $^{10}C_4 \times {}^{6}C_6 \times 8! \times 8!$