

Chapter 4

Linear inequations

Exercise 4

1. Solve the inequation, $3x - 11 < 3$ where $x \in \{1, 2, 3, \dots, 10\}$.

Also, represent its solution on a number line

Solution

Given inequation, $3x - 11 < 3$

$$3x < 3 + 11$$

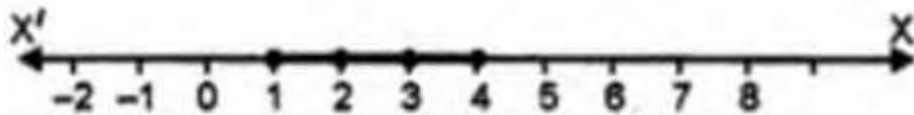
$$3x < 14$$

$$= x < \frac{14}{3}$$

But, $x \in \{1, 2, 3, \dots, 10\}$

Hence the solution set is $\{1, 2, 3, 4\}$

Representing the solution on a number line:



2. solve $2(x - 3) < 1$, $x \in \{1, 2, 3, \dots, 10\}$

Solution

Given inequation $2(x - 3) < 1$

$$2x - 6 < 1$$

$$2x < 7$$

$$= x < \frac{7}{2}$$

But, $x \in \{1, 2, 3, \dots, 10\}$

Hence the solution set is $\{1, 2, 3\}$

3. solve $5 - 4x > 2 - 3x$, $x \in \mathbb{w}$. Also represent its solution on the number line

Solution

Given inequation, $5 - 4x > 2 - 3x$

$$-4x + 3x > 2 - 5$$

$$-x > -3$$

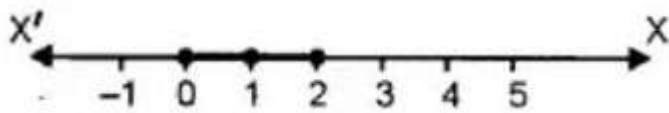
On multiplying both sides by -1 , the inequality reverses

$$= x < 3$$

Since, $x \in \mathbb{w}$

The solution set is $\{0, 1, 2\}$

Representing the solution on a number line:



4. List the solution set of $30 - 4(2x - 1) < 30$, given that x is a positive integer

Solution

Given inequation, $30 - 4(2x - 1) < 30$

$$30 - 8x + 4 < 30$$

$$34 - 8x < 30$$

$$-8x < 30 - 34$$

$$-8x < -4 \text{ [on multiplying both sides by } -1, \text{ the inequality reverse]}$$

$$8x > 4$$

$$x > \frac{4}{8}$$

$$= x > \frac{1}{2}$$

As x is a positive integer

The solution set is $\{ 1, 2, 3... \}$

5. solve: $2(x - 2) < 3x - 2$, $x \in \{-3, -2, -1, 0, 1, 2, 3\}$

Solution

Given inequation $2(x - 2) < 3x - 2$

$$2x - 4 < 3x - 2$$

$$2x - 3x < -2 + 4$$

$$-x < 2$$

$$= x > -2$$

But, $x \in \{-3, -2, -1, 0, 1, 2, 3\}$

Hence the solution set is $\{-1, 0, 1, 2, 3\}$

6. if x is a negative integer, find the solution set of

$$\frac{2}{3} + \frac{1}{3}(x + 1) > 0$$

solution

given inequation, $\frac{2}{3} + \frac{1}{3}(x + 1) > 0$

$$\frac{2}{3} + \frac{x}{3} + \frac{1}{3} > 0$$

$$\frac{x}{3} + 1 > 0$$

$$\frac{x}{3} > -1$$

$$= x > -3$$

As x is a negative integer

The solution set is $\{-1, -2\}$

7. Solve $x - 3(2 + x) > 2(3x - 1)$, $x \in \{-3, -2, -1, 0, 1, 2, 3\}$ also represent its solution on the number line

Solution

Given inequation $x - 3(2 + x) > 2(3x - 1)$

$$X - 6 - 3x > 6x - 2$$

$$-2x - 6 > 6x - 2$$

$$-6x - 2x > -2 + 6$$

$$-8x > 4$$

$$X < -\frac{4}{8}$$

$$= x < -\frac{1}{2}$$

But $x \in \{-3, -2, -1, 0, 1, 2, 3\}$

Hence the solution set is $\{-3, -2, -1\}$

8. given $x \in \{1, 2, 3, 4, 5, 6, 7, 9\}$ solve $x - 3 < 2x - 1$

Solution

Given inequation $x - 3 < 2x - 1$

$$X - 2x < -1 + 3$$

$$-x < 2$$

$$= x > -2$$

But, $x \in \{1, 2, 3, 4, 5, 6, 7, 9\}$

Hence the solution set is $\{1, 2, 3, 4, 5, 6, 7, 9\}$

9. List the solution set of the inequation $\frac{1}{2} + 8x > 5x - \frac{3}{2}$, $x \in \mathbb{Z}$

Solution

Given inequation, $\frac{1}{2} + 8x > 5x - \frac{3}{2}$

$$8x - 5x > -\frac{3}{2} - \frac{1}{2}$$

$$3x > -\frac{4}{2}$$

$$= x > -\frac{2}{3}$$

As $x \in \mathbb{Z}$

The solution set is $\{0, 1, 2, 3, 4, 5, \dots\}$

10. List the solution set of $\frac{11-2x}{5} \geq \frac{9-3x}{8} + \frac{3}{4}, x \in N$

Solution

Given inequation $\frac{11-2x}{5} \geq \frac{9-3x}{8} + \frac{3}{4}$

$$\frac{11-2x}{5} \geq \frac{9-3x+6}{8}$$

$$8(11-2x) \geq 5(15-3x)$$

$$88 - 16x \geq 75 - 15x$$

$$15x - 16x \geq 75 - 88$$

$$-x \geq -13$$

$$= x \leq 13$$

As $x \in N$

Hence the solution set is $\{1, 2, 3, 4, \dots, 13\}$

11. Find the values of x, which satisfy the inequation:

$-2 \leq \frac{1}{2} - \frac{2x}{3} \leq 1\frac{5}{6}, x \in N$. Graph the solution set on the number line.

Solution

Given inequation

$$-2 \leq \frac{1}{2} - \frac{2x}{3} \leq 1\frac{5}{6}$$

$$-2 \leq \frac{3-4x}{6} \leq \frac{11}{6}$$

$$-12 \leq 3 - 4x \leq 11$$

$$-12 - 3 \leq -4x \leq 11 - 3$$

$$-15 \leq -4x \leq 8$$

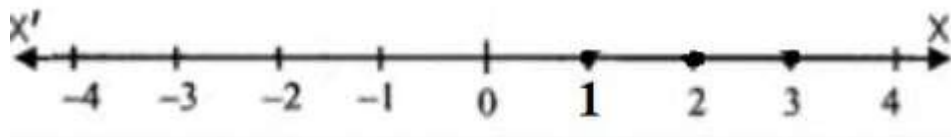
$$-\frac{15}{4} \leq -x \leq \frac{8}{4}$$

$$= \frac{15}{4} \geq x \geq -2$$

As $x \in \mathbb{N}$,

The solution set is $\{1, 2, 3\}$

Represent the solution on a number line:



12. if $x \in \mathbb{W}$, find the solution set of $\frac{3}{5}x - \frac{2x-1}{3} > 1$. Also graph the solution set on the number line, if possible

Solution

Given inequation, $\frac{3}{5}x - \frac{2x-1}{3} > 1$

$$\frac{9}{15}x - \frac{5(2x-1)}{15} > 1 \text{ [taking L.C.M]}$$

$$9x - 5(2x - 1) > 15 \text{ [multiplying by 15 on both sides]}$$

$$9x - 10x + 5 > 15$$

$$-x > 15 - 5$$

$$-x > 10$$

$$= x < -10$$

But, $x \in W$

Hence the solution set is a null set.

Thus, it can't be represent on number line.

13. Solve:

(i) $\frac{x}{2} + 5 \leq \frac{x}{3} + 6$, where x is a positive odd integer.

(ii) $\frac{2x+3}{3} \geq \frac{3x-1}{4}$, where x is positive even integer.

Solution

(i) given inequation $\frac{x}{2} + 5 \leq \frac{x}{3} + 6$

$$\frac{x+10}{2} \leq \frac{x+18}{3} \text{ [taking L.C.M on both sides]}$$

$$3(x + 10) \leq 2(x + 18) \text{ [on cross multiplying]}$$

$$3x + 30 \leq 2x + 36$$

$$3x - 2x \leq 36 - 30$$

$$= x \leq 6$$

As x is a positive odd integer.

Hence the solution set is $\{1, 3, 5\}$

(ii) given inequation $\frac{2x+3}{3} \geq \frac{3x-1}{4}$

$$4(2x + 3) \geq 3(3x - 1) \text{ [on cross multiplying]}$$

$$8x + 12 \geq 9x - 3$$

$$-9x + 8x \geq -12 - 3$$

$$-x \geq -15$$

$$= x \leq 15$$

As x is positive even integer.

Hence the solution set is $\{2, 4, 6, 8, 10, 12, 14\}$

14. given that $X \in 1$, solve the inequation and graph the solution on the number line:

$$3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2$$

Solution

Given inequation $3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2$

Now, let's take

$$3 \geq \frac{x-4}{2} + \frac{x}{3} \text{ we have}$$

$$3 \geq \frac{3x-12+2x}{6} \text{ [taking L.C.M]}$$

$$18 \geq 5x - 12$$

$$30 \geq 5x$$

$$= x \leq 6 \dots (i)$$

Next

$$\frac{x-4}{2} + \frac{x}{3} \geq 2$$

$$\frac{3x-12+2x}{6} \geq 2$$

$$5x - 12 \geq 12$$

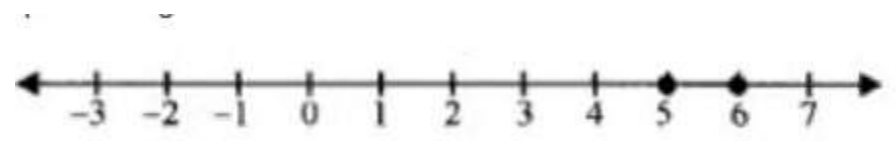
$$5x \geq 24$$

$$x \geq \frac{24}{5} = x \geq 4.8 \dots (ii)$$

Hence from (i) and (ii) we have

Solution of $x = \{ 5, 6 \}$

Representing the solution on a number line:



15. solve: $1 \geq 15 - 7x > 2x - 27$, $x \in \mathbb{N}$

Solution

Given inequation $1 \geq 15 - 7x > 2x - 27$

So we have

$$1 \geq 15 - 7x \text{ and } 15 - 7x > 2x - 27$$

$$7x \geq 15 - 1 \text{ and } -2x - 7x > -27 - 15$$

$$7x \geq 14 \text{ and } -9x > -42$$

$$x \geq 2 \text{ and } -x > -\frac{42}{9}$$

$$x \geq 2 \text{ and } x < \frac{14}{3}$$

$$= 2 \leq x < \frac{14}{3}$$

But as $x \in \mathbb{N}$

The solution set is $\{2, 3, 4\}$

16. If $x \in \mathbb{Z}$, solve $2 + 4x < 2x - 5 \leq 3x$. Also represent its solution on the number line.

Solution

Given inequation $2 + 4x < 2x - 5 \leq 3x$

So we have

$$2 + 4x < 2x - 5 \text{ and } 2x - 5 \leq 3x$$

$$4x - 2x < -5 - 2 \text{ and } 2x - 3x \leq 5$$

$$2x < -7 \text{ and } -x \leq 5$$

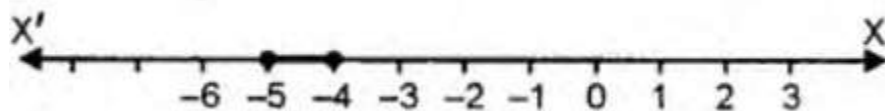
$$x < -\frac{7}{2} \text{ and } x \geq -5$$

$$= -5 \leq x < -\frac{7}{2}$$

As $x \in \mathbb{Z}$

The solution set is $\{-5, -4\}$

Representing the solution on a number line:



17. Solve: $\frac{4x-10}{3} \leq \frac{5x-7}{2}$, $x \in \mathbb{R}$ and represent the solution set on the number line.

Solution

Given inequation $\frac{4x-10}{3} \leq \frac{5x-7}{2}$

$$2(4x - 10) \leq 3(5x - 7) \text{ [on cross multiplying]}$$

$$8x - 20 \leq 15x - 21$$

$$8x - 15x \leq -21 + 20$$

$$-7x \leq -1$$

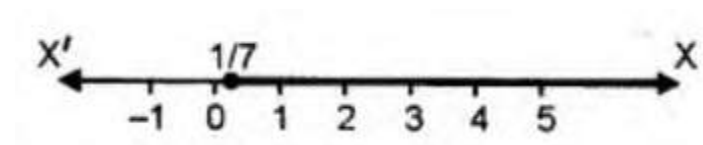
$$-x \leq -\frac{1}{7}$$

$$x \geq \frac{1}{7}$$

As $x \in \mathbb{R}$

Hence the solution set is $\{x : x \in \mathbb{R}, x \geq \frac{1}{7}\}$

Representing the solution on a number line:



18. Solve $\frac{3x}{5} - \frac{2x-1}{3} > 1$, $x \in \mathbb{R}$ and represent the solution set on the number line.

Solution

Given inequation $\frac{3x}{5} - \frac{2x-1}{3} > 1$

$$\frac{9x-10x+5}{15} > 1 \text{ [taking L.C.M]}$$

$$-x + 5 > 15$$

$$-x > 15 - 5$$

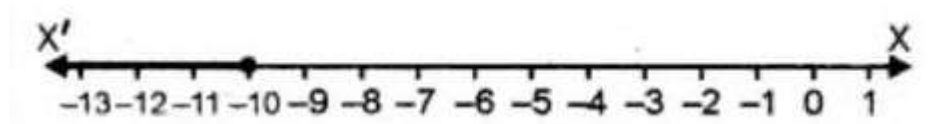
$$-x > 10$$

$$x < -10$$

As $x \in \mathbb{R}$

Hence, the solution set is $\{x : x \in \mathbb{R}, x < -10\}$

Representing the solution on a number line:



19. Given that $x \in \mathbb{R}$, solve the following inequation and graph the solution on the number line: $-1 \leq 3 + 4x < 23$

Solution

Given inequation $-1 \leq 3 + 4x < 23$

$$-1 - 3 \leq 4x < 23 - 3$$

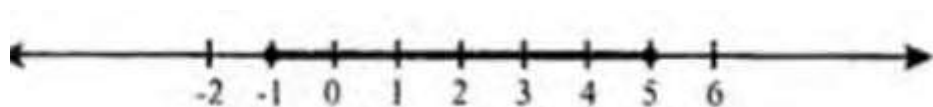
$$-4 \leq 4x < 20$$

$$-\frac{4}{4} \leq x < \frac{20}{4}$$

$$-1 \leq x < 5$$

Hence the solution set is $\{-1, \leq x < 5; x \in \mathbb{R}\}$

Representing the solution on a number line:



20. Solve the following inequation and graph the solution on the number line.

$$-2\frac{2}{3} \leq x + \frac{1}{3} < 3 + \frac{1}{3} \quad x \in \mathbf{R}$$

Solution

Given inequation

$$-2\frac{2}{3} \leq x + \frac{1}{3} < 3 + \frac{1}{3}$$

$$-\frac{8}{3} \leq \frac{3x+1}{3} < \frac{10}{3}$$

$$-8 \leq 3x + 1 < 10 \quad [\text{multiplying by 3}]$$

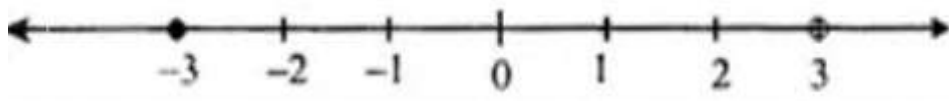
$$-8 - 1 \leq 3x < 10 - 1$$

$$-9 \leq 3x < 9$$

$$-3 \leq x < 3 \quad [\text{dividing by 3}]$$

Thus, the solution set is $\{x : x \in \mathbf{R}, -3 \leq x < 3\}$

Representing the solution on a number line:



21. solve the following inequation and represent the solution set on the number line:

$$-3 < -\frac{1}{2} - \frac{2}{3}x \leq \frac{5}{6}, x \in R$$

Solution

Given in equation,

$$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}, x \in R$$

$$-3 < -\frac{3+4x}{6} \leq \frac{5}{6} \text{ [taking L.C.M]}$$

$$-18 < -3 - 4x \leq 5 \text{ [multiplying by 6]}$$

$$-18 + 3 < -4x \leq 5 + 3$$

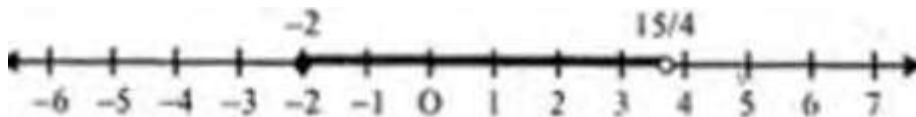
$$-15 < -4x \leq 8$$

$$-\frac{15}{4} < -x \leq \frac{8}{4}$$

$$-2 \leq x < \frac{15}{4}$$

Hence, the solution set is $\{ x : x \in R, -2 \leq x < \frac{15}{4} \}$

Represent the solution on a number line:



22. Solving the following inequation, write the solution set and represent it on the number line

$$-3(x - 7) \geq 15 - 7x > x - \frac{1}{3}, x \in R$$

Solution

Given inequation , $-3(x - 7) \geq 15 - 7x > x + \frac{1}{3}$

$$-3x + 21 \geq 15 - 7x > \frac{x+1}{3}$$

So,

$$-3x + 21 \geq 15 - 7x$$

$$7x - 3x \geq 15 - 21$$

$$4x > -6$$

$$X \geq -\frac{6}{4}$$

$$X \geq -\frac{3}{2}$$

And

$$15 - 7x > \frac{x+1}{3}$$

$$3(15 - 7x) > x + 1$$

$$45 - 21x > x + 1$$

$$-21x - x > 1 - 45$$

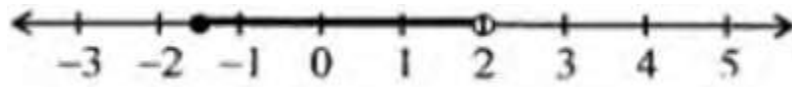
$$-22x > -44$$

$$-x > -\frac{44}{22}$$

$$X < 2$$

Hence, the solution set is $\{x : x \in R, -\frac{3}{2} \leq x < 2\}$

Represent the solution on a number line:



23. Solve the following inequation, write down the solution set and represent it in the real number line:

$$-2 + 10x \leq 13x + 10 \leq 24 + 10x, x \in Z$$

Solution

Given inequation, $-2 + 10x \leq 13x + 10 \leq 24 + 10x$

So, we have

$$-2 + 10x \leq 13x + 10 \text{ and } 13x + 10 \leq 24 + 10x$$

$$10x - 13x \leq 10 + 2 \text{ and } 13x - 10x \leq 24 - 10$$

$$-3x \leq 12 \text{ and } 3x \leq 14$$

$$X \geq -\frac{12}{3} \text{ and } x \leq \frac{14}{3}$$

$$X \geq -4 \text{ and } x \leq \frac{14}{3}$$

$$\text{So, } -4 \leq x \leq \frac{14}{3}$$

As $x \in Z$

Thus, the solution set is $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

Representing the solution on a number line:



24. Solve the inequation $2x - 5 \leq 5x + 4 < 11$ where $x \in \mathbb{I}$.

Also represent the solution set on the number line.

Solution

Given inequation, $2x - 5 \leq 5x + 4 < 11$

So, we have

$$2x - 5 \leq 5x + 4 \text{ and } 5x + 4 < 11$$

$$2x - 5x \leq 4 + 5 \text{ and } 5x < 11 - 4$$

$$-3x \leq 9 \text{ and } 5x < 7$$

$$-x < \frac{9}{3} \text{ and } x < \frac{7}{5}$$

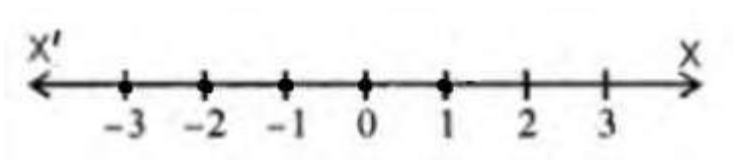
$$-3 \geq -3 \text{ and } x < \frac{7}{5}$$

$$-3 \leq x < \frac{7}{5}$$

As $x \in \mathbb{I}$

Thus, the solution set is $\{-3, -2, -1, 0, 1\}$

Representing the solution on a number line:



25. If $x \in \mathbb{I}$, A is the solution set of $2(x - 1) < 3x - 1$ and B is the solution set of $4x - 3 \leq 8 + x$, find $A \cap B$.

Solution

Given inequations,

$$2(x - 1) < 3x - 1 \text{ and } 4x - 3 \leq 8 + x \text{ for } x \in \mathbb{I}.$$

Solving for both, we have

$$2x - 3x < 2 - 1 \text{ and } 4x - x \leq 8 + 3$$

$$-x < 1 \text{ and } 3x \leq 11$$

$$x > -1 \text{ and } x \leq \frac{11}{3}$$

Hence,

$$\text{Solution set } A = \{0, 1, 2, 3, \dots\}$$

$$\text{Solution set } B = \{3, 2, 1, 0, -1, \dots\}$$

$$\text{Thus, } A \cap B = \{0, 1, 2, 3\}$$

26. If P is the solution set of $-3x + 4 < 2x - 3$, $x \in \mathbb{N}$ and Q is the solution set of $4x - 5 < 12$, $x \in \mathbb{W}$, find

(i) $P \cap Q$

(ii) $Q - P$.

Solution

Given inequations,

$-3x + 4 < 2x - 3$ where $x \in \mathbb{N}$ and $4x - 5 < 12$ where $x \in \mathbb{W}$.

So, solving

$-3x + 4 < 2x - 3$ where $x \in \mathbb{N}$

$-3x - 2x < -3 - 4$

$-5x < -7$

$x > \frac{7}{5}$

Hence, the solution set p is $\{2, 3, 4, 5\}$

And solving

$4x - 5 < 12$ where $x \in \mathbb{W}$

$4x < 12 + 5$

$4x < 17$

$x < \frac{17}{4}$

Hence the solution set Q is $\{0, 1, 2, 3, 4\}$

Therefore,

(i) $P \cap Q = \{2,3,4\}$

(ii) $Q - P = \{0,1\}$

27. $A = \{x : 11x - 5 > 7x + 3, x \in \mathbb{R}\}$ and

$B = \{x : 18x - 9 \geq 15 + 12x, x \in \mathbb{R}\}$

Find the range of set $A \cap B$ and represent it on number line

Solution

Given $A = \{x : 11x - 5 > 7x + 3, x \in \mathbb{R}\}$ and $B = \{x : 18x - 9 \geq 15 + 12x, x \in \mathbb{R}\}$

Solving for A

$$11x - 5 > 7x + 3$$

$$11x - 7x > 3 + 5$$

$$4x > 8$$

$$x > 2$$

Hence $A = \{x : x > 2, x \in \mathbb{R}\}$

Next, solving for B

$$18x - 9 \geq 15 + 12x$$

$$18x - 12x \geq 15 + 9$$

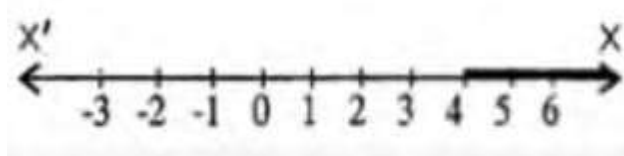
$$6x \geq 24$$

$$x \geq 4$$

Hence, $B = \{x : x \geq 4, x \in \mathbb{R}\}$

Thus, $A \cap B = x \geq 4$

Represent the solution on a number line:



28. Given: $P\{x : 5 < 2x - 1 \leq 11, x \in \mathbb{R}\}$

$Q\{x : -1 \leq 3 + 4x < 23, x \in \mathbb{I}\}$ where

$\mathbb{R}$ = (real numbers), $\mathbb{I}$ = (integers)

Represent P and Q on number line. Write down the elements of $P \cap Q$.

Solution

Given, $P\{x : 5 < 2x - 1 \leq 11, x \in \mathbb{R}\}$ and $Q\{x : -1 < 3 + 4x < 23, x \in \mathbb{I}\}$

Solving for p

$$5 < 2x - 1 \leq 11$$

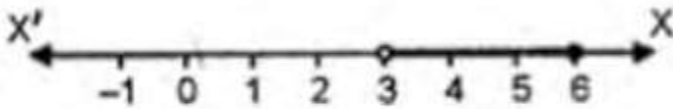
$$5 + 1 < 2x \leq 11 + 1$$

$$6 < 2x \leq 12$$

$$3 < x \leq 6$$

Hence, $P = \{x : 3 < x \leq 6, x \in R\}$

Representing the solution on a number line:



next, solving for Q

$$-1 \leq 3 + 4x < 23$$

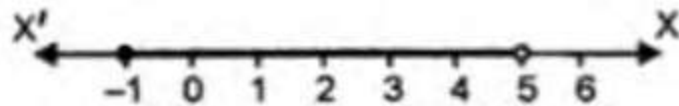
$$-1 - 3 \leq 4x < 23 - 3$$

$$-4 \leq 4x < 20$$

$$-1 \leq x < 5$$

Hence, solution $Q = \{-1, 0, 1, 2, 3, 4\}$

Representing the solution on a number line:



Therefore, $P \cap Q = \{4\}$

29. If $x \in 1$, find the smallest value of x which satisfies the inequation

$$2x + \frac{5}{2} > \frac{5x}{3} + 2$$

Solution

Given inequation, $2x + \frac{5}{2} > \frac{5x}{3} + 2$

$$\frac{4x+5}{2} > \frac{5x+6}{3} \text{ [taking L.C.M]}$$

$$3(4x + 5) > 2(5x + 6) \text{ [on cross- multiplication]}$$

$$12x + 15 > 10x + 12$$

$$12x - 10x > 12 - 15$$

$$2x > -3$$

$$X > -\frac{3}{2}$$

Hence, for $x \in \mathbb{R}$ the smallest value of x is -1

30. Given $20 - 5x < 5(x + 8)$, find the smallest value of x , when

(i) $x \in \mathbb{I}$

(ii) $x \in \mathbb{W}$

(iii) $x \in \mathbb{N}$.

Solution

Given inequation $20 - 5x < 5(x + 8)$

$$20 - 5x < 5x + 40$$

$$-5x - 5x < 40 - 20$$

$$-10x < 20$$

$$-x < \frac{20}{10}$$

$$X > -2$$

Thus,

(i) For $x \in 1$, the smallest value = -1

(ii) for $x \in W$, the smallest value = 0

(iii) For $x \in N$, the smallest value = 1

31. solve the following inequation and represent the solution set on the number line:

$$4x - 19 < \frac{3x}{5} - 2 \leq -\frac{2}{5} + x, x \in R$$

Solution

Given inequation,

$$4x - 19 < \frac{3x}{5} - 2 \leq -\frac{2}{5} + x, x \in R$$

So, we have

$$4x - 19 < \frac{3x}{5} - 2 \text{ and } \frac{3x}{5} - 2 \leq -\frac{2}{5} + x$$

$$4x - \frac{3x}{5} < 19 - 2 \text{ and } \frac{3x}{5} - x \leq 2 - \frac{2}{5}$$

$$\frac{20x - 3x}{5} < 17 \text{ and } \frac{3x - 5x}{5} < \frac{10 - 2}{5}$$

$$17x < 35 \text{ and } -2x \leq 8 \text{ [multiplying by 5]}$$

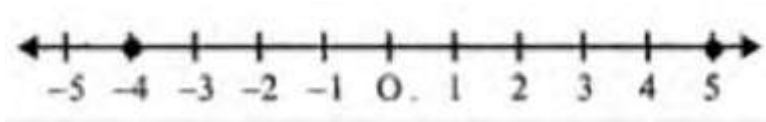
$$X < 5 \text{ and } -x \leq 4$$

$$X < 5 \text{ and } x \geq 4$$

$$-4 \leq x < 5, x \in \mathbb{R}$$

Hence, the solution set is $\{x : -4 \leq x < 5, x \in \mathbb{R}\}$

Representing the solution on a number line:



32. Solving the give inequation and graph the solution on the number line:

$$2y - 3 < y + 1 \leq 4y + 7;$$

$$Y \in \mathbb{R}$$

Solution

$$\text{Given inequation } 2y - 3 < y + 1 \leq 4y + 7$$

So, we have

$$2y - 3 < y + 1 \text{ and } y + 1 \leq 4y + 7$$

$$2y - y < 1 + 3 \text{ and } y - 4y \leq 7 - 1$$

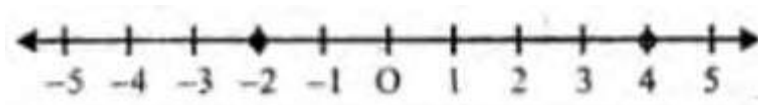
$$Y < 4 \text{ and } -3y \leq 6$$

$$Y < 4 \text{ and } -y \leq 2 = y \geq -2$$

$$\text{Thus, } -2 \leq y < 4$$

$$\text{The solution set is } \{y : -2 \leq y < 4, y \in \mathbb{R}\}$$

Representing the solution on a number line:



33. Solve the inequation and represent the solution set on the number line.

$$-3 + x \leq \frac{8x}{3} + 2 \leq \frac{14}{3} + 2x, \text{ where } x \in I$$

Solution

Given inequation,

$$-3 + x \leq \frac{8x}{3} + 2 \leq \frac{14}{3} + 2x, \text{ where } x \in I$$

So, we have

$$-3 + x \leq \frac{8x}{3} + 2 \text{ and } \frac{8x}{3} + 2 \leq \frac{14}{3} + 2x$$

$$x - \frac{8x}{3} \leq 2 + 3 \text{ and } \frac{8x}{3} - 2x \leq \frac{14}{3} - 2$$

$$\frac{3x - 8x}{3} \leq 5 \text{ and } \frac{8x - 6x}{3} \leq \frac{14 - 6}{3} \text{ [taking L.C.M]}$$

$$-\frac{5x}{3} \leq 5 \text{ and } 2x \leq 8$$

$$-5x \leq 15 \text{ and } x \leq \frac{8}{2}$$

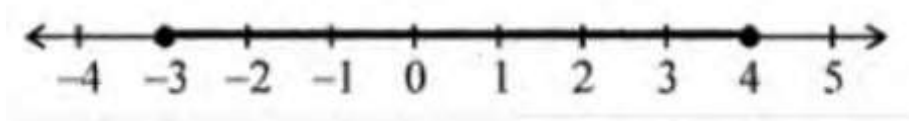
$$-x \leq 3 \text{ and } x \leq 4$$

$$x \geq -3 \text{ and } x \leq 4$$

$$= -3 \leq x \leq 4$$

Thus, the solution set is $\{-3, -2, -1, 0, 1, 2, 3, 4\}$

Representing the solution on a number line:



34. Find the greatest integer which is such that if 7 is added to its double, the resulting number becomes greater than three times the integer.

Solution

Lets consider the greatest integer to be x

Then according to the given condition, we have

$$2x + 7 > 3x$$

$$2x - 3x > -7$$

$$-x > -7$$

$$x < 7, x \in R$$

Hence, the greatest integer value is 6.

35. One – third of a bamboo pole is buried in mud, one – sixth of it is in water and the part above the water is greater than or equal to 3 meters. Find the length of the shortest pole.

Solution

Let's assume the length of the shortest pole = x meter

Now,

Length of the pole which is buried in mud = $\frac{x}{3}$

Length of the pole which is in the water = $\frac{x}{6}$

Then according to the given condition, we have

$$X - \left[\frac{x}{3} + \frac{x}{6} \right] \geq 3$$

$$X - \left[\frac{2x+x}{6} \right] \geq 3$$

$$X - \frac{3x}{6} \geq 3$$

$$X - \frac{x}{2} \geq 3$$

$$\frac{x}{2} \geq 3$$

$$X \geq 6[\text{multiplying by 6}]$$

Therefore, the length of the shortest pole is 6 meters.

Chapter test

1. Solve the inequation: $5x - 2 \leq 3(3 - x)$ where $x \in \{-2, -1, 0, 1, 2, 3, 4\}$. Also represent its solution on the number line.

Solution

Given inequation $5x - 2 \leq 3(3 - x)$

$$5x - 2 \leq 9 - 3x$$

$$5x + 3x \leq 9 + 2$$

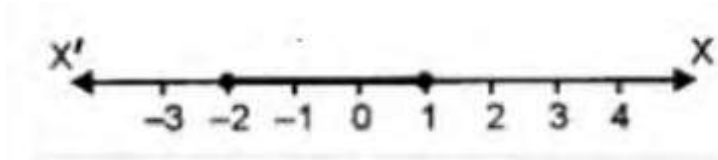
$$8x \leq 11$$

$$x \leq \frac{11}{8}$$

As $x \in \{-2, -1, 0, 1, 2, 3, 4\}$

The solution set is $\{-2, -1, 0, 1\}$

Representing the solution on a number line:



2. solve the inequation: $6x - 5 < 3x + 4$, $x \in \mathbb{I}$

Solution

Given inequation $6x - 5 < 3x + 4$

$$6x - 3x < 4 + 5$$

$$3x < 9$$

$$X < \frac{9}{3}$$

$$X < 3$$

As $x \in \mathbb{I}$

The solution set is $\{2, 1, 0, -1, -2, \dots\}$

3. find the solution set of the inequation $x + 5 \leq 2x + 3$; $x \in \mathbb{R}$

Graph the solution set on the number line

Solution

Given inequation $x + 5 \leq 2x + 3$

$$X - 2x \leq 3 - 5$$

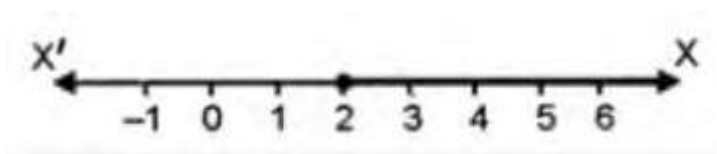
$$-x \leq -2$$

$$X \geq 2$$

As $x \in \mathbb{R}$

Thus, the solution set is $\{2, 3, 4, 5, \dots\}$

Representing the solution on a number line:



4. If $x \in R$ (real numbers) and $-1 < 3 - 2x \leq 7$, find solution set and present it on a number line.

Solution:

Given inequation $-1 < 3 - 2x \leq 7$

$$-1 - 3 < -2x \leq 7 - 3$$

$$-4 < -2x \leq 4$$

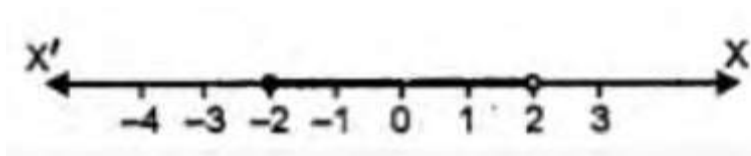
$$-\frac{4}{2} < -x \leq \frac{4}{2}$$

$$-2 < -x \leq 2$$

Thus $-2 \leq x < 2$

The solution set is $\{x : x \in R, -2 \leq x < 2\}$

Representing the solution on a number line:



5. Solve the inequation:

$$5x + \frac{1}{7} - 4\left(\frac{x}{7} + \frac{2}{5}\right) \leq 1\frac{3}{5} + \frac{3x-1}{7}, x \in R$$

Solution

Given inequation

$$\frac{5x+1}{7} - 4\left(\frac{x}{7} + \frac{2}{5}\right) \leq 1\frac{3}{5} + \frac{3x-1}{7}, x \in R$$

$$\frac{5x+1}{7} - \frac{4(5x+14)}{35} \leq \frac{8}{5} + \frac{3x-1}{7}$$

$$\frac{[5(5x+1)-4(5x+14)]}{35} \leq \frac{[56+5(3x-1)]}{35} \quad [\text{Taking L.C.M}]$$

$$(25x + 5 - 20x - 56) \leq 56 + 15x - 5$$

$$5x - 51 \leq 51 + 51$$

$$-10x \leq 102$$

$$-x \leq \frac{102}{10}$$

$$X \geq -\frac{51}{5}$$

Hence the solution set is $\{ x : x \in R, X \geq -\frac{51}{5} \}$

6. Find the range of values of a, which satisfy $7 \leq -4x + 2 < 12$, $x \in \mathbb{R}$. graph these values of a on the real number line.

$$7 < -4x + 2 < 12$$

$$7 < -4x + 2 \text{ and } -4x + 2 < 12$$

$$7. \text{ if } x \in \mathbb{R}, \text{ solve } 2x - 3 \geq x + \frac{1-x}{3} > \frac{2x}{5}$$

Solution

$$\text{Given inequation } 2x - 3 \geq x + \frac{1-x}{3} > \frac{2x}{5}$$

So, we have

$$2x - 3 \geq x + \frac{1-x}{3} \text{ and } x + \frac{1-x}{3} > \frac{2x}{5}$$

$$2x - 3 \geq \frac{3x+1-x}{3} \text{ and } \frac{3x+1-x}{3} > \frac{2x}{5} \text{ [on taking L.C.M]}$$

$$3(2x - 3) \geq 2x + 1 \text{ and } (2x + 1) \times 5 > 2x \times 3 \text{ [upon cross multiplication]}$$

$$6x - 9 \geq 2x + 1 \text{ and } 10x + 5 > 6x$$

$$6x - 2x \geq 1 + 9 \text{ and } 10x - 6x > -5$$

$$4x \geq 10 \text{ and } 4x > -5$$

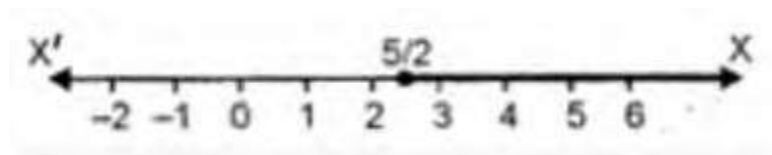
$$x \geq \frac{10}{4} \text{ and } x > -\frac{5}{4}$$

$$x \geq \frac{5}{2}$$

As $x \in \mathbb{R}$

Thus, the solution set is $\{ x : x \in \mathbb{R}, x \geq \frac{5}{2} \}$

Representing the solution on a number line:



7. If $x \in \mathbf{R}$, solve $2x - 3 \geq x + \frac{1-x}{3} > \frac{2x}{5}$. Also represent the solution on the number line.

Solution

Given inequation, $2x - 3 \geq x + \frac{1-x}{3} > \frac{2x}{5}$

So, we have

$$2x - 3 \geq x + \frac{1-x}{3} \text{ and } x + \frac{1-x}{3} > \frac{2x}{5}$$

$$2x - 3 \geq \frac{3x+1-x}{3} \text{ and } \frac{3x+1-x}{3} > \frac{2x}{5} \text{ [on taking L.C.M]}$$

$$3(2x - 3) \geq 2x + 1 \text{ and } 5 \times (2x + 1) > 3 \times 2x$$

$$6x - 9 \geq 2x + 1 \text{ and } 10x + 5 > 6x$$

$$6x - 2x \geq 1 + 9 \text{ and } 10x - 6x > -5$$

$$4x \geq 10 \text{ and } 4x > -5$$

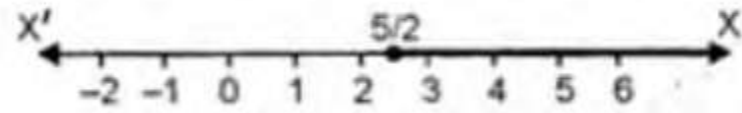
$$x \geq \frac{10}{4} \text{ and } x > -\frac{5}{4}$$

$$x \geq \frac{5}{2}$$

As $x \in \mathbb{R}$

The solution set = $\{ x : x \in \mathbb{R}, x \geq \frac{5}{2} \}$

Representing the solution on a number line:



8. Find positive integers which are such that if 6 is subtract from five times the integer than the resulting number cannot be greater than four times the integer.

Solution

Let's consider the positive integer be x

Then according to the problem, we have

$$5a - 6 < 4x$$

$$5a - 4x < 6$$

$$= x < 6$$

Hence the solution set = $\{ x : x < 6 \}$

$$= \{1, 2, 3, 4, 5, 6\}$$

9. Find three smallest consecutive natural numbers such that the difference between one –third of the largest and one – fifth of the smallest is at least 3.

Solution

Let's consider the first least natural number as x

Then, second number $= x + 1$

And third number $= x + 2$

So, according the conditions given in the problem, we have

$$\frac{1}{3} \times (x + 2) - \frac{x}{5} \geq 3$$

$$5x + 10 - 3x \geq 3 \times 15$$

[multiplying by 15 the L.C.M of 3 and 5]

$$2x \geq 45 - 10$$

$$2x \geq 35$$

$$X \geq \frac{35}{2}$$

$$X \geq 17.5$$

As x is a natural least number

Thus, first least natural number $= 18$

Second number $= 18 + 1 = 19$

And, third number $= 18 + 2 = 20$

Hence the least natural numbers are 18, 19 and 20