

Chapter 6
Factorization
Exercise 6.1

1. Find the remainder (without division) on dividing $f(x)$ by $(x - 2)$ where

(i) $f(x) = 5x^2 - 7x + 4$

Solution

Let us assume $x - 2 = 0$

Then, $x = 2$

Given, $f(x) = 5x^2 - 7x + 4$

Now, substitute the value of x in $f(x)$,

$$f(2) = (5 \times 2^2) - (7 \times 2) + 4$$

$$= (5 \times 4) - 14 + 4$$

$$= 20 - 14 + 4$$

$$= 24 - 14$$

$$= 10$$

Therefore, the remainder is 10.

(ii) $f(x) = 2x^3 - 7x^2 + 3$

Solution

Let us assume $x - 2 = 0$

Then, $x = 2$

Given, $f(x) = 2x^3 - 7x^2 + 3$

Now, substitute the value of x in $f(x)$

$$f(2) = (2 \times 2^3) - (7 \times 2^2) + 3$$

$$= (2 \times 8) - (7 \times 4) + 3$$

$$= 16 - 28 + 3$$

$$= 19 - 28$$

$$= -9$$

Therefore, the remainder is -9 .

2. using the remainder theorem, find the remainder on dividing $f(x)$ by $(x + 3)$ where

(i) $f(x) = 2x^2 - 5x + 1$

Solution

Let us assume $x + 3 = 0$

Then, $x = -3$

Given, $f(x) = 2x^2 - 5x + 1$

Now, substitute the value of x in $f(x)$

$$f(-3) = (2 \times -3^2) - (5 \times (-3)) + 1$$

$$= (2 \times 9) - (-15) + 1$$

$$= 18 + 15 + 1$$

$$= 34$$

Therefore, the remainder is 34.

(ii) $f(x) = 3x^3 + 7x^2 - 5x + 1$

Solution

Let us assume $x + 3 = 0$

Then, $x = -3$

Given, $f(x) = 3x^3 + 7x^2 - 5x + 1$

Now, substitute the value of x in $f(x)$

$$f(-3) = (3 \times -3^3) + (7 \times -3^2) - (5 \times -3) + 1$$

$$= (3 \times -27) + (7 \times 9) - (-15) + 1$$

$$= -81 + 63 + 15 + 1$$

$$= -81 + 79$$

$$= -2$$

Therefore, the remainder is -2.

3. Find the remainder (without division) on dividing $f(x)$ by $(2x + 1)$ where,
(i) $f(x) = 4x^2 + 5x + 3$

Solution

Let us assume $2x + 1 = 0$

Then, $2x = -1$

$$x = -\frac{1}{2}$$

Given, $f(x) = 4x^2 + 5x + 3$

Now, substitute the value of x in $f(x)$,

$$f\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^2 + 5\left(-\frac{1}{2}\right) + 3$$

$$= \left(4 \times \frac{1}{4}\right) + \left(-\frac{5}{2}\right) + 3$$

$$= 1 - \frac{5}{2} + 3$$

$$= 4 - \frac{5}{2}$$

$$= \frac{8-5}{2}$$

$$= \frac{3}{2} = 1\frac{1}{2}$$

Therefore, the remainder is $1\frac{1}{2}$

(ii) $f(x) = 3x^3 - 7x^2 + 4x + 11$

Solution

Let us assume $2x + 1 = 0$

Then, $2x = -1$

$$x = -\frac{1}{2}$$

Given, $f(x) = 3x^3 - 7x^2 + 4x + 11$

Now, substitute the value of x in $f(x)$,

$$f\left(-\frac{1}{2}\right) = \left(3 \times \left(-\frac{1}{2}\right)^3\right) - \left(7 \times \left(-\frac{1}{2}\right)^2\right) + \left(4 \times -\frac{1}{2}\right) + 11$$

$$= 3 \times \left(-\frac{1}{8}\right) - \left(7 \times \frac{1}{4}\right) + (-2) + 11$$

$$= -\frac{3}{8} - \frac{7}{4} - 2 + 11$$

$$= -\frac{3}{8} - \frac{7}{4} + 9$$

$$= \frac{-3 - 14 + 72}{8}$$

$$= \frac{55}{8}$$

$$= 6\frac{7}{8}$$

4. Using remainder theorem, find the value of k if on dividing $2x^3 + 3x^2 - kx + 5$ by $x - 2$ leaves a remainder 7.

Solution

Let us assume, $x - 2 = 0$

Then, $x = 2$

Given, $2x^3 + 3x^2 - kx + 5$

Now, substitute the value of x in f(x)

$$\begin{aligned} f(2) &= (2 \times 2^3) + (3 \times 2^2) - (k \times 2) + 5 \\ &= (2 \times 8) + (3 \times 4) - 2k + 5 \\ &= 16 + 12 - 2k + 5 \\ &= 33 - 2k \end{aligned}$$

Form the question it is given that, remainder is 7.

$$\text{So, } 7 = 33 - 2k$$

$$2k = 33 - 7$$

$$2k = 26$$

$$K = \frac{26}{2}$$

$$K = 13$$

Therefore, the value of k is 13.

5. Using remainder theorem, find the value of 'a' if the division of $x^3 + 5x^2 - ax + 6$ by $(x - 1)$ leaves the remainder 2a.

Solution

Let us assume $x - 1 = 0$

Then, $x = 1$

Given, $f(x) = x^3 + 5x^2 - ax + 6$

Now, substitute the value of x in f(x),

$$f(1) = 1^3 + (5 \times 1^2) - (a \times 1) + 6$$

$$= 1 + 5 - a + 6$$

$$= 12 - a$$

From the question it is given that, remainder is 2a

$$\text{So, } 2a = 12 - a$$

$$2a + a = 12$$

$$3a = 12$$

$$A = \frac{12}{3}$$

$$A = 4$$

Therefore, the value of a is 4.

6. (i) what number must be divided be subtracted from $2x^2 - 5x$ so that the result polynomial leaves the remainder 2, when divided by $2x + 1$?

Solution

Let u assume 'p' be subtracted from $2x^2 - 5x$

So, dividing $2x^2 - 5x$ by $2x + 1$

$$\begin{array}{r|l}
 2x+1 & 2x^2 - 5x - p \\
 & 2x^2 + x \\
 \hline
 & -6x - p \\
 & -6x - 3 \\
 & + \quad + \\
 \hline
 & -p + 3
 \end{array}
 \quad x-3$$

Hence, remainder is $3 - p$

From the question it is given that, remainder is 2.

$$3 - p = 2$$

$$P = 3 - 2$$

$$P = 1$$

Therefore, 1 is to be subtracted.

(ii) What number must be added to $2x^3 - 7x^2 + 2x$ so that the resulting polynomial leaves the remainder -2 when divided by $2x - 3$?

Solution

Let us assume 'P' be subtracted from $2x^3 - 7x^2 + 2x$,

So, dividing it by $2x - 3$,

$2x-3$	$ \begin{array}{r} 2x^3 - 7x^2 + 2x + p \\ 2x^3 - 3x^2 \\ \hline -4x^2 + 2x \\ -4x^2 + 6x \\ \hline + \quad - \\ \hline -4x + p \\ -4x + 6 \\ \hline + \quad - \\ \hline P - 6 \end{array} $	$X^2 - 2x - 2$
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Hence, remainder is $P - 6$

From the question it is given that, remainder is -2

$$P - 6 = -2$$

$$P = -2 + 6$$

$$P = 4$$

Therefore, 4 is to be added.

7. (i) When divided by $x - 3$ the polynomials $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p + 3)x - 6$ leave the same remainder. Find the value of 'p'

Solution

From the question it is given that, by dividing $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p + 3)x - 6$ by $x - 3 = 0$, then $x = 3$

Let us assume $p(x) = x^3 - px^2 + x + 6$

Now, substitute the value of x in $p(x)$,

$$P(3) = 3^3 - (p \times 3^2) + 3 + 6$$

$$= 27 - 9p + 9$$

$$= 36 - 9p$$

Then, $q(x) = 2x^3 - x^2 - (p + 3)x - 6$

Now, substitute the value of x in $q(x)$,

$$q(3) = (2 \times 3^3) - (3)^2 - (p + 3) \times 3 - 6$$

$$= (2 \times 27) - 9 - 3p - 9 - 6$$

$$= 54 - 24 - 3P$$

$$= 30 - 3p$$

Given, the remainder in each case is same,

$$\text{So, } 36 - 9p = 30 - 3p$$

$$36 - 30 = 9p - 3p$$

$$6 = 6p$$

$$p = \frac{6}{6}$$

$$p = 1$$

therefore, value of p is 1.

(ii) Find 'a' if the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$, leaves the same remainder when divided by $x + 3$.

Solution

Let us assume $p(x) = ax^3 + 3x^2 - 9$ and $q(x) = 2x^3 + 4x + a$

From the question it is given that, both $p(x)$ and $q(x)$ leaves the same remainder when divided by $x + 3$.

Let us assume that, $x + 3 = 0$

Then, $x = -3$

Now, substitute the value of x in $p(x)$ and in $q(x)$,

So, $p(-3) = q(-3)$

$$a(-3)^3 + 3(-3)^2 - 9 = 2(-3)^3 + 4(-3) + a$$

$$- 27a + 27 - 9 = - 54 - 12 + a$$

$$- 27a - a = -66 - 18$$

$$- 28a = - 84$$

$$a = \frac{84}{28}$$

therefore, $a = 3$

(iii) The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ when divided by $x - 4$ leaves the remainder r_1 and r_2 respectively. If $2r_1 = r_2$, then find the value of a .

Solution

Let us assume $p(x) = ax^3 + 3x^2 - 3$ and $q(x) = 2x^3 - 5x + a$

From the question it is given that, both $p(x)$ and $q(x)$ leaves the remainder r_1 and r_2 respectively when divide by $x - 4$.

Also, given relation $2r_1 = r_2$

Let us assume that, $x - 4 = 0$

Then , $x = 4$

Now, substituted the value of x in $p(x)$ and in $q(x)$,

By factor theorem $r_1 = p(x)$ and $r_2 = q(x)$

So, $2 \times p(4) = q(4)$

$$2[a(4)^3 + 3(4)^2 - 3] = 128 - 20 + a$$

$$128a + 96 - 6 = 128 - 20 + a$$

$$128a + 90 = 108 + a$$

$$128a - a = 108 - 90$$

$$127a = 18$$

$$a = \frac{18}{127}$$

therefore, the value of $a = \frac{18}{127}$

8. using remainder thermo, find the remainders obtained when $x^3 + (kx + 8)x + k$ is divided by $x + 1$ and $x - 2$. Hence, find k if the sum of two remainders is 1.

Solution

Let us assume $p(x) = x^3 + (kx + 8)x + k$

From the question it is given that, the sum of the remainders when $p(x)$ is divided by $(x + 1)$ and $(x - 2)$ is 1.

Let us assume that, $x + 1 = 0$

Then, $x = -1$

Also, when $x - 2 = 0$

Then, $x = 2$

Now, by remainder theorem we have

$$P(-1) + p(2) = 1$$

$$(-1)^3 + [k(-1) + 8](-1) + k + (2)^3 + [k(2) + 8](2) + k = 1$$

$$-1 + k - 8 + k + 8 + 4k + 16 + k = 1$$

$$7k + 15 = 1$$

$$7k = 1 - 15$$

$$K = -\frac{14}{7}$$

$$K = -2$$

Therefore, $k = -2$

9. By factor theorem, show that $(x + 3)$ and $(2x - 1)$ are factors of $2x^2 + 5x - 3$.

Solution

Let us assume, $x + 3 = 0$

Then, $x = -3$

Given , $f(x) = 2x^2 + 5x - 3$

Now, substitute the value of x in $f(x)$,

$$f(-3) = (2 \times (-3)^2) + (5 \times -3) - 3$$

$$= (2 \times 9) + (-15) - 3$$

$$= 18 - 15 - 3$$

$$= 18 - 18$$

$$= 0$$

Now, $2x - 1 = 0$

Then, $2x = 1$

$$X = \frac{1}{2}$$

Given , $f(x) = 2x^2 + 5x - 3$

Now, substitute the value of x in $f(x)$

$$\begin{aligned}
f\left(\frac{1}{2}\right) &= \left(2 \times \left(\frac{1}{2}\right)^2\right) + \left(5 \times \frac{1}{2}\right) - 3 \\
&= \left(2 \times \left(\frac{1}{4}\right)\right) + \frac{5}{2} - 3 \\
&= \frac{1}{2} + \frac{5}{2} - 3 \\
&= \frac{1+5}{2} - 3 \\
&= \frac{6}{2} - 3 \\
&= 3 - 3 \\
&= 0
\end{aligned}$$

Hence, it is proved that $(x + 3)$ and $(2x - 1)$ are factors of $2x^2 + 5x - 3$.

10. Without actual division, prove that $x^4 + 2x^3 - 2x^2 + 2x + 3$ is exactly divisible by $x^2 + 2x - 3$.

Solution

Consider $x^2 + 2x - 3$

By factor method, $x^2 + 3x - x - 3$

$$= x(x + 3) - 1(x + 3)$$

$$= (x - 1)(x + 3)$$

$$\text{So, } f(x) = x^4 + 2x^3 - 2x^2 + 2x + 3$$

Now take, $x + 3 = 0$

$$X = -3$$

$$\text{Then, } f(-3) = (-3)^4 + 2 \times -(3^3) - (2 \times (-3)^2) + (2 \times -3) + 3$$

$$= 81 - 54 - 18 - 6 - 3$$

$$= 0$$

Therefore, $(x + 3)$ is a factor of $f(x)$

And also, take $x - 1 = 0$

$$X = 1$$

$$\text{Then, } f(1) = 1^4 + 2(1)^3 - 2(1)^2 + 2(1) - 3$$

$$= 0$$

Therefore, $(x - 1)$ is a factor of $f(x)$

By comparing both results, $p(x)$ is exactly divisible by $x^2 + 2x - 3$.

11. show that $(x - 2)$ is a factor of $3x^2 - x - 10$. Hence factories $3x^2 - x - 10$.

Solution

Let us assume $x - 2 = 0$

Then, $x = 2$

$$\text{Given, } f(x) = 3x^2 - x - 10$$

Now, substitute the value of x in $f(x)$,

$$f(2) = (3 \times 2^2) - 2 - 10$$

$$= (3 \times 4) - 2 - 10$$

$$= 12 - 12$$

$$= 0$$

Therefore, $(x - 2)$ is a factor of $f(x)$

Then, dividing $(3x^2 - x - 10)$ by $(x - 2)$ we get

$x-2$	$3x^2 - x - 10$ $3x^2 + x$ $- \quad +$	$3x + 5$
	$5x - 10$ $5x - 10$ $- \quad +$	
	0	

$$\text{Therefore, } 3x^2 - x - 10 = (x - 2)(3x + 5)$$

12. Using the factor theorem, show that $(x - 2)$ is a factor of $x^3 + x^2 - 4x - 4$. Hence factorize the polynomial completely.

Solution

Let us assume, $x - 2 = 0$

Then, $x = 2$

$$\text{Given, } f(x) = x^3 + x^2 - 4x - 4$$

Now, substitute the value of x in $f(x)$

$$f(2) = (2)^3 + (2)^2 - 4(2) - 4$$

$$= 8 - 4 - 8 - 4$$

$$= 0$$

Therefore, by factor theorem $(x - 2)$ is a factor of $x^3 + x^2 - 4x - 4$

Then, dividing $f(x)$ by $(x - 2)$, we get

	$x^2 + 3x + 2$
$x-2$	$x^3 + x^2 - 4x - 4$ $x^3 - 2x^2$ $-$
	$3x^2 - 4x - 4$ $-$ $3x^2 - 6x$
	$2x - 4$ $-$ $2x - 4$
	0

$$\text{Therefore, } x^3 + x^2 - 4x - 4 = (x - 2)(x^2 + 3x + 2)$$

$$= (x - 2)(x^2 + 2x + x + 2)$$

$$= (x - 2)(x(x + 2) + 1(x + 2))$$

$$= (x - 2)(x + 2)(x + 1)$$

13. Show that $2x + 7$ is a factor of $2x^3 + 5x^2 - 11x - 14$.hence factorize the given expression completely, using the factor theorem.

Solution

Let us assume $2x + 7 = 0$

Then, $2x = -7$

$$x = -\frac{7}{2}$$

Given, $f(x) = 2x^3 + 5x^2 - 11x - 14$

Now, substitute the value of x in $f(x)$,

$$f\left(-\frac{7}{2}\right) = 2\left(-\frac{7}{2}\right)^3 + 5\left(-\frac{7}{2}\right)^2 + 11\left(-\frac{7}{2}\right) - 14$$

$$= 2\left(-\frac{343}{8}\right) + 5\left(\frac{49}{4}\right) + \left(-\frac{77}{2}\right) - 14$$

$$= -\frac{343}{4} + \frac{245}{4} - \frac{77}{2} - 14$$

$$= \frac{-343+245+154-56}{4}$$

$$= -399 + \frac{399}{4}$$

$$= 0$$

Therefore, $(2x + 7)$ is a factor of $2x^3 + 5x^2 - 11x - 14$

Then, dividing $f(x)$ by $(2x + 1)$ we get

	$X^2 - x - 2$
$2x+7$	$2x^3 + 5x^2 - 11x - 14$
	$-$
	$2x^3 + 7x^2$
	$-2x^2 - 11x - 14$
	$-$
	$-2x^2 - 7x$
	$-4x - 14$
	$-$
	$-4x - 14$
	0

$$\begin{aligned}
 \text{Therefore, } 2x^3 + 5x^2 - 11x - 14 &= (2x + 7)(x^2 - x - 2) \\
 &= (2x + 7)(x^2 - 2x + x - 2) \\
 &= (2x + 7)(x(x - 2) + 1(x - 2)) \\
 &= (x + 1)(x - 2)(2x + 7)
 \end{aligned}$$

14. use factor theorem to factorize the following polynomials completely.

(i) $x^3 + 2x^2 - 5x - 6$

Solution

Let us assume $x = -1$

Given, $f(x) = x^3 + 2x^2 - 5x - 6$

Now, substitute the value of x in f(x)

$$f(-1) = (-1)^3 + 2(-1)^2 - 5(-1) - 6$$

$$= -1 + 2(1) + 5 - 6$$

$$= -1 + 2 + 5 - 6$$

$$= -7 + 7$$

$$= 0$$

Then, dividing f(x) by (x + 1) we get

X + 1	$\begin{array}{r} X^2 + x - 6 \\ X^3 + 2x^2 - 5x - 6 \\ - \\ X^3 + x^2 \\ \hline X^2 - 5x - 6 \\ - \\ X^2 + x \\ \hline -6x - 6 \\ - \\ -6x - 6 \\ \hline 0 \end{array}$
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$$\text{Therefore, } x^3 + 2x^2 - 5x - 6 = (x + 1)(x^2 + 3x - 2x - 6)$$

$$= (x + 1)(x(x + 3) - 2(x + 3))$$

$$= (x + 1)(x - 2)(x + 3)$$

(ii) $x^3 - 13x - 12$

Solution

Let us assume $x = -1$,

Given, $f(x) = x^3 - 13x - 12$

Now, substitute the value of x in $f(x)$

$$f(-1) = (-1)^3 - 13(-1) - 12$$

$$= -1 + 13 - 12$$

$$= -13 + 13$$

$$= 0$$

Then, dividing $f(x)$ by $(x + 1)$, we get

	$X^2 - x - 12$
$X + 1$	$X^3 + 0x^2 - 13x - 12$
	$-$
	$X^3 + x^2$
	$-x^2 - 13x - 12$
	$-$
	$-x^2 - x$
	$-12x - 12$
	$-$
	$-12x - 12$
	0

Therefore $x^3 - 13x - 12 = (x + 1)(x^2 - x - 12)$

$$= (x + 1)(x^2 - 4x + 3x - 12)$$

$$= (x + 1)(x(x - 4)) + 3(x - 4)$$

$$= (x + 1)(x + 3)(x - 4)$$

15. Use the remainder theorem to factorize the following expression

(i) $2x^3 + x^2 - 13x + 6$

Solution

Let us assume $x = 2$

Then, $f(x) = 2x^3 + x^2 - 13x + 6$

Now, substitute the value of x in $f(x)$

$$f(2) = (2 \times 2^3) + 2^2 - 13 \times 2 + 6$$

$$= (2 \times 8) + 4 - 26 + 6$$

$$= 16 + 4 - 26 + 6$$

$$= 26 - 26$$

$$= 0$$

Then , dividing $f(x)$ by $(x - 2)$ we get

	$2x^2 + 5x - 3$
$x - 2$	$2x^2 + x^2 - 13x + 6$
	$-$
	$2x^3 - 4x^2$
	$5x^2 - 13x + 6$
	$-$
	$5x^2 - 10x$
	$-3x + 6$
	$-$
	$-3x + 6$
	0

$$\begin{aligned}
\text{Therefore, } 2x^3 + x^2 - 13x + 6 &= (x - 2) (2x^2 + 5x - 3) \\
&= (x - 2) (2x^2 + 6x - x - 3) \\
&= (x - 2) (2x(x + 3) - 1(x + 3)) \\
&= (x - 2) (x + 3) (2x - 1)
\end{aligned}$$

(ii) $3x^3 + 2x^2 - 19x + 6$

Solution

Given $f(x) = 3x^3 + 2x^2 - 19x + 6$

Let us assume $x = 1$

$$\begin{aligned}
\text{Then } f(1) &= 3(1)^3 + 2(1)^2 - (19 \times 1) + 6 \\
&= 3 + 2 - 19 + 6 \\
&= 11 - 19 \\
&= -8
\end{aligned}$$

So, $-8 \neq 0$

Let us assume $x = -1$

$$\begin{aligned}
\text{Then, } f(-1) &= 3(-1)^3 + 2(-1)^2 - (19 \times (-1)) + 6 \\
&= -3 + 2 + 19 + 6 \\
&= -3 + 27 \\
&= 24
\end{aligned}$$

So, $24 \neq 0$

Now, assume $x = 2$

$$\begin{aligned}
 \text{Then, } f(2) &= 3(2)^3 + 2(2)^2 - (19 \times (2)) + 6 \\
 &= 24 + 8 - 38 + 6 \\
 &= 38 - 38
 \end{aligned}$$

$$\text{So, } 0 = 0$$

Therefore, $(x - 2)$ is a factor of $f(x)$

$$\begin{aligned}
 f(x) &= 3x^3 + 2x^2 - 19x + 6 \\
 &= 3x^3 - 6x^2 + 8x^2 - 16x - 3x + 6 \\
 &= 3x^2(x - 2) + 8x(x - 2) - 3(x - 2) \\
 &= (x - 2)(3x^2 + 8x - 3) \\
 &= (x - 2)(3x^2 + 9x - x - 3) \\
 &= (x - 2)(3x(x + 3) - 1(x + 3)) \\
 &= (x - 2)(3x(x + 3) - 1(x + 3)) \\
 &= (x - 2)(x + 3)(3x - 1)
 \end{aligned}$$

$$\text{(iii) } 2x^3 + 3x^2 - 9x - 10$$

Solution

$$\text{Given, } f(x) = 2x^3 + 3x^2 - 9x - 10$$

Let us assume, $x = -1$

$$\begin{aligned}
 &= 2(-1)^3 + 3(-1)^2 - 9(-1) - 10 \\
 &= -2 + 3 + 9 - 10
 \end{aligned}$$

$$= 12 - 12$$

$$= 0$$

Therefore, $(x + 1)$ is the factor of $2x^3 + 3x^2 - 9x - 10$

Then, dividing $f(x)$ by $(x + 1)$, we get

	$2x^2 + x - 10$
$X + 1$	$2x^3 + 3x^2 - 9x - 10$
	-
	$2x^3 + 2x^2$
	$X^2 - 9x - 10$
	-
	$X^2 + x$
	$-10x - 10$
	-
	$-10x - 10$
	0

$$\text{Therefore, } 2x^3 + 3x^2 - 9x - 10 = 2x^2 + 5x - 4x - 10$$

$$= x(2x + 5) - 2(2x + 5) - (2x + 5)(x - 2)$$

Hence the factors are $(x + 1)(x - 2)(2x + 5)$

(iv) $x^3 + 10x^2 - 37x + 26$

Solution

Given, $f(x) = x^3 + 10x^2 - 37x + 26$

Let us assume, $x = 1$

Then, $f(1) = 1^3 + 10(1)^2 - 37(1) + 26$

$= 1 + 10 - 37 + 26$

$= 37 - 37$

$= 0$

Therefore $x - 1$ is a factor of $x^3 + 10x^2 - 37x + 26$

Then, dividing $f(x)$ by $(x - 1)$ we get

	$X^2 + 11x - 26$
$X - 1$	$X^3 + 10x^2 - 37x + 26$
	-
	$X^3 - x^2$
	$11x^2 - 37x + 26$
	-
	$11x^2 - 11x$
	$-26x + 26$
	-
	$-26x + 26$
	0

$$\begin{aligned}
\text{Therefore, } x^3 + 10x^2 - 37x + 26 &= (x - 1)(x^2 + 11x - 26) \\
&= (x - 1)(x^2 + 13x - 2x - 26) \\
&= (x - 1)(x(x + 13) - 2(x + 13)) \\
&= (x - 1)((x - 2)(x + 13))
\end{aligned}$$

16. if $(2x + 1)$ is a factor of $6x^3 + 5x^2 + ax - 2$ find the value of a .

Solution

Let us assume $2x + 1 = 0$

Then, $2x = -1$

$$x = -\frac{1}{2}$$

Given, $f(x) = 6x^3 + 5x^2 + ax - 2$

Now substitute the value of x in $f(x)$

$$\begin{aligned}
f\left(-\frac{1}{2}\right) &= 6\left(-\frac{1}{2}\right)^3 + 5\left(-\frac{1}{2}\right)^2 + a\left(-\frac{1}{2}\right) - 2 \\
&= 6\left(-\frac{1}{8}\right) + 5\left(\frac{1}{4}\right) - \frac{1}{2}a - 2 \\
&= -\frac{3}{4} + \frac{5}{4} - \frac{a}{2} - 2 \\
&= \frac{-3+5-a-8}{4} \\
&= \frac{-6-a}{4}
\end{aligned}$$

From the question, $(2x + 1)$ is a factor of $6x^3 + 5x^2 + ax - 2$

Then remainder is 0.

$$\text{So, } \frac{-6-2}{4} = 0$$

$$-6 - 2a = 4 \times 0$$

$$-6 - 2a = 0$$

$$-2a = 6$$

$$a = -\frac{6}{2}$$

$$a = -3$$

Therefore, the value of a is -3 .

17. if $(3x - 2)$ is a factor of $3x^3 - kx^2 + 21x - 10$, find the value of k

Solution

Let us assume $3x - 2 = 0$

Then, $3x = 2$

$$x = \frac{2}{3}$$

Given, $f(x) = 3x^3 - kx^2 + 21x - 10$

Now, substitute the value of x in $f(x)$

$$f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^3 - k\left(\frac{2}{3}\right)^2 + 21\left(\frac{2}{3}\right) - 10$$

$$= 3\left(\frac{8}{27}\right) - k\left(\frac{4}{9}\right) + 14 - 10$$

$$= \frac{8}{9} - \frac{4k}{9} + 14 - 10$$

$$\begin{aligned}
&= \frac{8}{9} - \frac{4k}{9} + 4 \\
&= \frac{8-4k+36}{9} \\
&= \frac{44-4k}{9}
\end{aligned}$$

From the question, $(3x - 2)$ is a factor of $3x^3 - kx^2 + 21x - 10$

Then, remainder is 0

$$\text{So, } \frac{44-4k}{9} = 0$$

$$44 - 4k = 0 \times 9$$

$$44 = 4k$$

$$K = \frac{44}{4}$$

$$K = 11$$

18. if $(x - 2)$ is a factor of $2x^3 - x^2 + px - 2$, then (i) find the value of p (ii) with this value of p, factorize the above expression completely.

Solution

Let us assume $x - 2 = 0$

Then, $x = 2$

Given, $f(x) = 2x^3 - x^2 + px - 2$

Now, substitute the value of x in f(x)

$$f(2) = (2 \times 2^3) - 2^2 + (p \times 2) - 2$$

$$= (2 \times 8) - 4 + 2p - 2$$

$$= 16 - 4 + 2p - 2$$

$$= 16 - 6 + 2p$$

$$= 10 + 2p$$

From the question, $(x - 2)$ is a factor of $2x^3 - x^2 + px - 2$

Then, remainder is 0

$$10 + 2p = 0$$

$$2p = -10$$

$$P = -\frac{10}{2}$$

$$P = -5$$

So $(x - 2)$ is a factor of $2x^3 - x^2 + 5x - 2$

	$2x^2 + 3x + 11$
$X - 2$	$2x^3 - x^2 + 5x - 2$
	-
	$2x^3 - 4x^2$
	$3x^2 + 5x - 2$
	-
	$3x^2 - 6x$
	$11x - 2$
	-
	$11x - 22$
	20

$$\begin{aligned}
\text{Therefore, } 2x^3 - x^2 + 5x - 2 &= (x - 2)(2x^2 + 3x + 1) \\
&= (x - 2)(2x^2 + 2x + x + 1) \\
&= (x - 2)(2x(x + 1) + 1(x + 1)) \\
&= (x + 1)(x - 2)(2x + 1)
\end{aligned}$$

19. What number should be subtracted from $2x^3 - 5x^2 + 5x$ so that the resulting polynomial has $2x - 3$ as a factor?

Solution

Let us assume the number to be subtracted from $2x^3 - 5x^2 + 5x$ be p .

$$\text{Then } f(x) = 2x^3 - 5x^2 + 5x - p$$

$$\text{Given, } 2x - 3 = 0$$

$$x = \frac{3}{2}$$

$$f\left(\frac{3}{2}\right) = 0$$

$$\text{so, } f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 5\left(\frac{3}{2}\right)^2 + 5\left(\frac{3}{2}\right) - p = 0$$

$$2\left(\frac{27}{8}\right) - 5\left(\frac{9}{4}\right) + \frac{15}{2} - p = 0$$

$$\frac{27}{4} - \frac{45}{4} + \frac{15}{2} - p = 0 \quad [\text{multiply by 4 for all numerator}]$$

$$27 - 45 + 30 - 4p = 0$$

$$57 - 45 - 4p = 0$$

$$12 - 4p = 0$$

$$P = \frac{12}{4}$$

$$P = 3$$

Therefore, 3 is the number should be subtracted from $2x^3 - 5x^2 + 5x$

20. (i) Find the value of the constants a and b, if $(x - 2)$ and $(x + 3)$ are both factors of the expression $x^3 + ax^2 + bx - 12$

Solution

Let us assume $x - 2 = 0$

Then, $x = 2$

Given, $f(x) = x^3 + ax^2 + bx - 12$

Now, substitute the value of x in f(x)

$$f(2) = 2^3 + a(2)^2 + b(2) - 12$$

$$= 8 + 4a + 2b - 12$$

$$= 4a + 2b - 4$$

From the question, $(x - 2)$ is a factor of $x^3 + ax^2 + bx - 12$

$$\text{So, } 4a + 2b - 4 = 0$$

$$4a + 2b = 4$$

By dividing both the side by 2 we get,

$$2a + b = 2.... [\text{equation (i)}]$$

Now, assume $x + 3 = 0$

$$\text{Then , } x = - 3$$

$$\text{Given, } f(x) = x^3 + ax^2 + bx - 12$$

Now, substitute the value of x in f(x)

$$f(-3) = (-3)^3 + a(-3)^2 + b(-3) - 12$$

$$= - 27 + 9a - 3b - 12$$

$$= 9a - 3b - 39$$

From the question, (x- 3) is a factor of $x^3 + ax^2 + bx - 12$

$$\text{So, } 9a - 3b - 39 = 0$$

$$9a - 3b = 39$$

By dividing both the side by 3 we get,

$$3a - b = 13.....[\text{equation (ii)}]$$

Now, adding both equation (i) and equation (ii) we get,

$$(2a + b) + (3a - b) = 2 + 13$$

$$2a + 3b + b - b = 15$$

$$5a = 15$$

$$a = \frac{15}{5}$$

$$a = 3$$

consider the equation (i) to find out 'b'

$$2a + b = 2$$

$$2(3) + b = 2$$

$$6 + b = 2$$

$$B = 2 - 6$$

$$B = -4$$

(ii) if $(x + 2)$ and $(x + 3)$ are factors of $x^3 + ax + b$, find the values of a and b .

Solution

Let us assume $x + 2 = 0$

Then, $x = -2$

Given, $f(x) = x^3 + ax + b$

Now, substitute the value of x in $f(x)$

$$f(-2) = (-2)^3 + a(-2) + b$$

$$= -8 - 2a + b$$

From the question $(x + 2)$ is a factor of $x^3 + ax + b$

Therefore, remainder is 0

$$f(x) = 0$$

$$-8 - 2a + b = 0$$

$$-2a - b = -8 \dots [\text{equation (i)}]$$

Let us assume $x + 3 = 0$

Then, $x = -3$

Given, $f(x) = x^3 + ax + b$

Now, substitute the value of x in $f(x)$

$$f(-3) = (-3)^3 + a(-3) + b$$

$$= -27 - 3a + b$$

From the question, $(x + 3)$ is a factor of $x^3 + ax + b$

Therefore, remainder is 0

$$f(x) = 0$$

$$-27 - 3a + b = 0$$

$$3a - b = -27 \dots[\text{equation(i)}]$$

Now, subtracting both equation (i) and equation(ii) we get,

$$(2a - b) - (3a - b) = -8 - (-27)$$

$$2a - 3a - b + b = -8 + 27$$

$$-a = 19$$

$$a = -19$$

consider the equation (i) to find out 'b'.

$$2a - b = -8$$

$$2(-19) - b = -8$$

$$-38 - b = -8$$

$$b = -38 + 8$$

$$b = -30$$

21. if $(x + 2)$ and $(x - 3)$ are factors of $x^3 + ax + b$, find the value of a and b . With these values of a and b , factorize the given expression.

Solution

Let us assume $x + 2 = 0$

Then, $x = -2$

Given, $f(x) = x^3 + ax + b$

Now, substitute the value of x in $f(x)$,

$$f(-2) = (-2)^3 + a(-2) + b$$

$$= -8 - 2a + b$$

From the question, $(x + 2)$ is a factor of $x^3 + ax + b$

Therefore, remainder is 0.

$$f(x) = 0$$

$$-8 - 2a + b = 0$$

$$2a - b = -8 \dots [\text{equation (i)}]$$

Now, assume $x - 3 = 0$

Then, $x = 3$

Given, $f(x) = x^3 + ax + b$

Now, substitute the value of x in $f(x)$

$$f(3) = (3)^3 + a(3) + b$$

now, substitute the value of x in f(x)

$$f(3) = (3)^3 + a(3) + b$$

$$= 27 + 3a + b$$

From the question , $(x - 3)$ is a factor of $x^3 + ax + b$

Therefore, remainder is 0

$$f(x) = 0$$

$$27 + 3a + b = 0$$

$$3a + b = - 27 \text{[equation (ii)]}$$

Now, adding both equation (i) and equation (ii) we get,

$$(2a - b) + (3a + b) = - 8 - 27$$

$$2a - b + 3a + b = - 35$$

$$5a = - 35$$

$$a = -\frac{35}{5}$$

$$a = -7$$

consider the equation(i) to find out 'b'

$$2a - b = - 8$$

$$2(-7) - b = -8$$

$$-14 - b = - 8$$

$$b = - 14 + 8$$

$$b = - 6$$

therefore, value of $a = - 7$ and $b = - 6$

then, $f(x) = x^3 - 7x - 6$

$$(x + 2)(x - 3)$$

$$= x(x - 3) + 2(x - 3)$$

$$= x^2 - 3x + 2x - 6$$

$$= x^2 - x - 6$$

Dividing $f(x)$ by $x^2 - x - 6$ we get,

	$X + 1$
$X^2 - x - 6$	$X^3 + 0x^2 - 7x - 6$
	-
	$X^3 - x^2 - 6x$
	$X^2 - x - 6$
	-
	$X^2 - x - 6$
	0

Therefore, $x^3 - 7x - 6 = (x + 1)(x + 2)(x - 3)$

22. $(x - 2)$ is a factor of expression $x^3 + ax^2 + bx + 6$. When this expression is divided by $(x - 3)$, it leaves the remainder 3. Find the value of a and b .

Solution

From the question it is given that, $(x - 2)$ is a factor of the expression $x^3 + ax^2 + bx + 6$

Then, $f(x) = x^3 + ax^2 + bx + 6$ [equation (i)]

Let assume $x - 2 = 0$

Then, $x = 2$

Now, substitute the value of x in $f(x)$,

$$f(2) = 2^3 + a(2)^2 + 2b + 6$$

$$= 8 + 4a + 2b + 6$$

$$= 14 + 4a + 2b$$

By dividing the numbers by 2 we get,

$$= 7 + 2a + b$$

From the question, $(x - 2)$ is a factor of the expression $x^3 + ax^2 + bx + 6$.

So, remainder is 0.

$$f(x) = 0$$

$$7 + 2a + b = 0$$

$$2a + b = -7 \text{....[equation(ii)]}$$

Now, expression is divided by $(x - 3)$ it leaves the remainder 3.

	$X^2 + x(a + 3) + 3a + b + 9$
$X - 3$	$X^3 + ax^2 + bx + 6$
	-
	$X^3 \quad -3x^2$
	$X^2(a + 3) + bx + 6$
	-
	$X^2(a + 3) + x(-3a - 9)$
	$X(3a + b + 9) + 6$
	-
	$X(3a + b + 9) + -9a - 3b - 27$
	$9a + 3b + 33$

So, remainder = $33 + 9a + 3b = 3$

$$9a + 3b = 3 - 33$$

$$9a + 3b = -30$$

By dividing the numbers by 3 we get,

$$= 3a + b = -10 \dots [\text{equation(iii)}]$$

Now, subtracting equation(iii) from equation (ii) we get,

$$(3a + b) - (2a + b) = -10 - (-7)$$

$$3a - 2a + b - b = -10 + 7$$

$$a = -3$$

consider the equation (ii) to find out 'b'

$$2a + b = -7$$

$$2(-3) + b = -7$$

$$-6 + b = -7$$

$$b = -7 + 6$$

$$b = -1$$

23. If $(x - 2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by $(x - 3)$, it leaves a remainder 52, find the values of a and b .

Solution

From the question it is given that $(x - 2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$

Then, $f(x) = 2x^3 + ax^2 + bx - 14$[equation(i)]

Let assume $x - 2 = 0$

Then , $x = 2$

Now, substitute the value of x in $f(x)$,

$$f(2) = 2(2)^3 + a(2)^2 + 2b - 14$$

$$= 16 + 4a + 2b - 14$$

$$= 2 + 4a + 2b$$

By dividing the numbers by 2 we get,

$$= 1 + 2a + b$$

From the question, $(x - 2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$

So, remainder is 0.

$$f(x) = 0$$

$$1 + 2a + b = 0$$

$$2a + b = -1 \dots [\text{equation(ii)}]$$

Now, expression is divided by $(x - 3)$ it leaves the remainder 52.

$X - 3$	$2x^2 + x(a + 6) + 3a + b + 18$
	$2x^3 + ax^2 + bx - 14$
	-
	$2x^3 \qquad -6x^2$
	$X^2(a + 6) \qquad +bx \qquad -14$
	-
	$X^2(a + 6) + x(-3a - 18)$
	$X(3a + b + 18) \qquad -14$
	-
	$X(3a + b + 18) + -9a - 3b - 54$
	$9a + 3b + 40$

$$\text{So, remainder} = 9a + 3b + 40 = 52$$

$$9a + 3b = 52 - 40$$

$$9a + 3b = 12$$

By dividing the number by 3 we get,

$$= 3a + b = 4 \dots [\text{equation(iii)}]$$

Now, subtracting equation(iii) from equation(ii) we get,

$$(3a + b) - (2a + b) = 4 - (-1)$$

$$3a - 2a + b - b = 4 + 1$$

$$a = 5$$

$$a = 5$$

consider the equation (ii) to find out 'b'.

$$2a + b = -1$$

$$2(5) + b = -1$$

$$10 + b = -1$$

$$b = -1 - 10$$

$$b = -11$$

24. If $ax^3 + 3x^2 + bx - 3$ has a factor $(2x + 3)$ and leaves remainder -3 when divided by $(x + 2)$, find the value of a and b . With these values of a and b , factorize the given expression.

Solution

Let us assume, $2x + 3 = 0$

Then, $2x = -3$

$$x = -\frac{3}{2}$$

Given, $f(x) = ax^3 + 3x^2 + bx - 3$

Now, substitute the value of x in $f(x)$,

$$f\left(-\frac{3}{2}\right) = a\left(-\frac{3}{2}\right)^3 + 3\left(-\frac{3}{2}\right)^2 + b\left(-\frac{3}{2}\right) - 3$$

$$= a\left(-\frac{27}{8}\right) + 3\left(\frac{9}{4}\right) - \frac{3b}{2} - 3$$

$$= -\frac{27a}{8} + \frac{27}{4} - \frac{3b}{2} - 3$$

From the question it is given that, $ax^3 + 3x^2 + bx - 3$ has a factor $(2x + 3)$.

So, remainder is 0.

$$-\frac{27a}{8} + \frac{27}{4} - \frac{3b}{2} - 3 = 0$$

$$-27a + 54 - 12b - 24 = 0$$

$$-27a - 12b = -30$$

By dividing the numbers by -3 we get,

$$9a + 4b = 10 \text{ [equation(i)]}$$

Now, let us assume $x + 2 = 0$

$$\text{Then } x = -2$$

$$\text{Given, } f(x) = ax^3 + 3x^2 + bx - 3$$

Now, substitute the value of x in $f(x)$

$$f(-2) = a(-2)^3 + 3(-2)^2 + b(-2) - 3$$

$$= -8a + 12 - 2b - 3$$

$$= -8a - 2b + 9$$

Leaves the remainder -3

$$\text{So, } -8a - 2b + 9 = -3$$

$$-8a - 2b = -3 - 9$$

$$-8a - 2b = -12$$

By dividing both sides by -2 we get,

$$4a + b = 6 \text{ [equation(ii)]}$$

By multiplying equation(ii) by 4

$$16a + 4b = 24$$

Now, subtracting equation(ii) from equation (i) we get.

$$(16a + 4b) - (9a + 4b) = 24 - 10$$

$$16a - 9a + 4b - 4b = 14$$

$$7a = 14$$

$$a = \frac{14}{7}$$

$$a = 2$$

consider the equation (i) to find out 'b'

$$9a + 4b = 10$$

$$9(2) + 4b = 10$$

$$18 + 4b = 10$$

$$4b = 10 - 18$$

$$4b = -8$$

$$b = -\frac{8}{4}$$

$$b = -2$$

therefore, $f(x) = ax^3 + 3x^2 + bx - 3$

$$= 2x^3 + 3x^2 - 2x - 3$$

Given, $2x + 3$ is a factor of $f(x)$

So, divide $f(x)$ by $2x + 3$

	$X^2 - 1$
$2x + 3$	$2x^3 + 3x^2 - 2x - 3$
	$-$
	$2x^3 + 3x^2$
	$0 - 2x - 3$
	$-$
	$- 2x - 3x$
	0

Therefore, $2x^3 + 3x^2 - 2x - 3 = (2x + 3) (x^2 - 1)$

$= (2x + 3) (x + 1) (x - 1)$

25. Given $f(x) = ax^2 + bx + 2$ and $g(x) = bx^2 + ax + 1$. If $x - 2$ is a factor of $f(x)$ but leaves the remainder -15 when it divides $g(x)$, find the value of a and b . With these values of a and b , factorize the expression. $f(x) + g(x) + 4x^2 + 7x$

Solution

From the equation it is given that, $f(x) = ax^2 + bx + 2$ and $g(x) = bx^2 + ax + 1$ and $x - 2$ is a factor of $f(x)$

So, $x = 2$

Now, substitute the value of x in $f(x)$

$$f(2) = 0$$

$$a(2)^2 + b(2) + 2 = 0$$

$$4a + 2b + 2 = 0$$

By dividing both sides by 2 we get,

$$2a + b + 1 = 0 \dots [\text{equation (i)}]$$

Given, $g(x)$ divide by $(x - 2)$, leaves remainder -15

$$g(x) = bx^2 + ax + 1$$

$$\text{so, } g(2) = -15$$

$$b(2)^2 + 2a + 1 = -15$$

$$4b + 2a + 1 + 15 = 0$$

$$4b + 2a + 16 = 0$$

By dividing both sides by 2 we get,

$$2b + a + 8 = 0 \dots [\text{equation (ii)}]$$

Now, subtracting equation (ii) from equation (i) multiplied by 2,

$$(4a + 2b + 2) - (a + 2b + 8) = 0 - 0$$

$$4a - a + 2b - 2b + 2 - 8 = 0$$

$$3a - 6 = 0$$

$$3a = 6$$

$$a = \frac{6}{3}$$

$$a = 2$$

consider the equation (i) to find out 'b'

$$2a + b + 1 = 0$$

$$2(2) + b = -1$$

$$4 + b = -1$$

$$b = -1 - 4$$

$$b = -5$$

$$\text{now } f(x) = ax^2 + bx + 2 = 2x^2 - 5x + 2$$

$$g(x) = bx^2 + ax + 1 = -5x^2 + 2x + 1$$

$$\text{then, } f(x) + g(x) + 4x^2 + 7x$$

$$= 2x^2 - 5x + 2 - 5x^2 + 2x + 1 + 4x^2 + 7x$$

$$= x^2 + 4x + 3$$

$$= x^2 + 3x + x + 3$$

$$= x(x + 3) + 1(x + 3)$$

$$= (x + 1)(x + 3)$$

Chapter test

1. find the remainder when $2x^3 - 3x^2 + 4x + 7$ is divided by

(i) $x - 2$

(ii) $x + 3$

(iii) $2x + 1$

Solution

From the question it is given that, $f(x) = 2x^3 - 3x^2 + 4x + 7$

(i) consider $x - 2$

Let us assume $x - 2 = 0$

Then, $x = 2$

Now, substitute the value of x in $f(x)$

$$f(2) = 2(2)^3 - 3(2)^2 + 4(2) + 7$$

$$= 16 - 12 + 8 + 7$$

$$= 31 - 12$$

$$= 19$$

Therefore, the remainder is 19

(ii) consider $x + 3$

Let us assume $x + 3 = 0$

Then, $x = -3$

Now, substitute the value of x in $f(x)$

$$f(2) = 2(-3)^3 - 3(-3)^2 + 4(-3) + 7$$

$$= 2(-27) - 3(9) - 12 + 7$$

$$= -54 - 27 - 12 + 7$$

$$= -93 + 7$$

$$= -86$$

Therefore, remainder is -86

(iii) consider $2x + 1$

Let us assume $2x + 1 = 0$

Then, $2x = -1$

$$X = -\frac{1}{2}$$

Now, substitute the value of x in $f(x)$

$$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 7$$

$$= 2\left(-\frac{1}{8}\right) - 3\left(\frac{1}{4}\right) + 4\left(-\frac{1}{2}\right) + 7$$

$$= -\frac{1}{4} - \frac{3}{4} - 2 + 7$$

$$= 4$$

Therefore, remainder is 4

2. When $2x^3 - 9x^2 + 10x - p$ is divided by $(x + 1)$ the remainder is -24 find the value of p.

Solution

Let us assume $x + 1 = 0$

Then $x = -1$

Given $f(x) = 2x^3 - 9x^2 + 10x - p$

Now, substitute the value of x in f(x)

$$f(-1) = 2(-1)^3 - 9(-1)^2 + 10(-1) - p$$

$$= -2 - 9 - 10 + p$$

$$= -21 + p$$

From the question it is given that the remainder is -2 ,

$$\text{So, } -21 + p = -24$$

$$P = -24 + 21$$

$$P = -3$$

$$\text{So, } f(x) = 2x^3 - 9x^2 + 10x - (-3)$$

$$= 2x^3 - 9x^2 + 10x + 3$$

Therefore, the value of p is 3

3. If $(2x - 3)$ is a factor of $6x^2 + x + a$, find the value of a . With this value of a , factorise the given expression.

Solution

Let us assume $2x - 3 = 0$

Then, $2x = 3$

$$x = \frac{3}{2}$$

Given $f(x) = 6x^2 + x + a$

Now, substitute the value of x in $f(x)$

$$f\left(\frac{3}{2}\right) = 6\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right) + a$$

$$= 6\left(\frac{9}{4}\right) + \left(\frac{3}{2}\right) + a$$

$$= \frac{27}{2} + \frac{3}{2} + a$$

$$= \frac{30}{2} + a$$

$$= 15 + a$$

From the question $(2x - 3)$ is a factor of $6x^2 + x + a$

So remainder is 0

Then $15 + a = 0$

$$a = -15$$

therefore $f(x) = 6x^2 + x - 15$

dividing $f(x)$ by $2x - 3$ we get

	$3x + 5$
$X - 3$	$6x^2 + x - 15$
	-
	$6x^2 \quad -9x$
	$10x - 15$
	-
	$10x - 15$
	0

Therefore, $6x^2 + x - 15 = (2x - 3)(3x + 5)$

4. when $3x^2 - 5x + p$ is divided by $(x - 2)$ the remainder is 3.

Find the value of p. Also factorize the polynomial $3x^2 - 5x + p - 3$

Solution

Let us assume $x - 2 = 0$

Then, $x = 2$

Given, $f(x) = 3x^2 - 5x + p$

Now, substitute the value of x in f(x),

So, $f(2) = 3(2)^2 - 5(2) + p$

$-3(4) - 10 + p$

$= 12 - 10 + p$

$= 2 + p$

From the question it is given that, remainder is 3.

$$\text{So, } 2 + p = 3$$

$$P = 3 - 2$$

$$P = 1$$

$$\text{Therefore, } f(x) = 3x^2 - 5x + 1$$

$$\text{Consider the polynomial, } 3x^2 - 5x + p - 3$$

Now, substitute the value of p in polynomial,

$$= 3x^2 - 5x + 1 - 3$$

$$= 3x^2 - 5x - 2$$

Now by factorizing the polynomial $3x^2 - 5x - 2$.

Dividing $3x^2 - 5x - 2$ by $x - 2$ we get

	$3x + 1$
$X - 2$	$3x^2 - 5x - 2$
	$-$
	$3x^2 - 6x$
	$X - 2$
	$-$
	$X - 2$
	0

$$\text{Therefore, } 3x^2 - 5x - 2 = (x - 2)(3x + 1)$$

5. Prove that $(5x + 4)$ is a factor of $5x^3 + 4x^2 - 5x - 4$. Hence factorize the given polynomial completely.

Solution

Let us assume $(5x + 4) = 0$

Then, $5x = -4$

$$x = -\frac{4}{5}$$

Given, $f(x) = 5x^3 + 4x^2 - 5x - 4$

Now, substitute the value of x in $f(x)$

$$\begin{aligned}\text{So, } f\left(-\frac{4}{5}\right) &= 5\left(-\frac{4}{5}\right)^3 + 4\left(-\frac{4}{5}\right)^2 - 5\left(-\frac{4}{5}\right) - 4 \\&= 5\left(-\frac{64}{125}\right) + 4\left(\frac{16}{25}\right) + 4 - 4 \\&= -\frac{64}{25} + \frac{64}{25} \\&= \frac{-64+64}{25} \\&= \frac{0}{25} \\&= 0\end{aligned}$$

Hence, $(5x + 4)$ is a factor of $5x^3 + 4x^2 - 5x - 4$

So, dividing $5x^3 + 4x^2 - 5x - 4$ by $5x + 4$ we get,

	$X^2 - 1$
$5x + 4$	$5x^3 + 4x^2 - 5x - 4$
	$-$
	$5x^3 + 4x^2$
	$0 - 5x - 4$
	$-$
	$-5x - 4x$
	0

$$\begin{aligned}
 \text{Therefore, } 5x^3 + 4x^2 - 5x - 4 &= (5x + 4)(x^2 - 1) \\
 &= (5x + 4)(x^2 - 1^2) \\
 &= (5x + 4)(x + 1)(x - 1)
 \end{aligned}$$

6. use factor theorem to factorize the following polynomials completely:

(i) $4x^3 + 4x^2 - 9x - 9$

Solution

Let us assume $x = -1$

Given, $f(x) = 4x^3 + 4x^2 - 9x - 9$

Now, substitute the value of x in $f(x)$

$$\begin{aligned}
 f(-1) &= 4(-1)^3 + 4(-1)^2 - 9(-1) - 9 \\
 &= -4 + 4 + 9 - 9 \\
 &= 0
 \end{aligned}$$

Therefore, $x + 1$ is the factor of $4x^3 + 4x^2 - 9x - 9$

Now, dividing $4x^3 + 4x^2 - 9x - 9$ by $x + 1$ we get,

	$4x^2 - 9$
$X + 1$	$4x^3 + 4x^2 - 9x - 9$
	-
	$4x^3 + 4x^2$
	$0 - 9x - 9$
	-
	$-9x - 9x$
	0

Therefore , $4x^3 + 4x^2 - 9x - 9 = (x + 1) (4x^2 - 9)$

$$= (x + 1) ((2x)^2 - (3)^2)$$

$$= (x + 1) (2x + 3) (2x - 3)$$

(ii) $x^3 - 19x - 30$

Solution

Let us assume $x = -2$

$$\text{Given } f(x) = x^3 - 19x - 30$$

Now, substitute the value of x in $f(x)$

$$f(-1) = (-2)^3 - 19(-2) - 30$$

$$= -8 + 38 - 30$$

$$= -38 + 38$$

$$= 0$$

Therefore, $x + 2$ is the factor of $x^3 - 19x - 30$

Now, dividing $x^3 - 19x - 30$ by $x + 2$ we get,

	$X^2 - 2x - 15$
$X + 2$	$X^3 + 0x^2 - 19x - 30$
	-
	$X^3 + 2x^2$
	- $2x^2 - 19x - 30$
	-
	- $2x^2 - 4x$
	-15x - 30
	-
	-15x - 30
	0

$$\text{Therefore, } x^3 - 19x - 30 = (x + 2)(x^2 - 2x - 15)$$

$$= (x + 2)(x^2 - 5x + 3x - 15)$$

$$= (x + 2)(x - 5)(x + 3)$$

7. if $x^3 - 2x^2 + px + q$ has a factor $(x + 2)$ and leaves a remainder 9, when divided by $(x + 1)$, find the value of p and q with these values of p and q factorize the given polynomial completely.

Solution

From the question it is given that, $(x + 2)$ is a factor of the expression $x^3 - 2x^2 + px + q$

$$\text{Then, } f(x) = x^3 - 2x^2 + px + q$$

$$\text{Let assume } x + 2 = 0$$

$$\text{Then } x = -2$$

Now, substitute the value of x in $f(x)$

$$F(-2) = (-2)^3 - 2(-2)^2 + p(-2) + q$$

$$= -8 - 8 - 2p + q$$

$$= -16 - 2p + q$$

$$2p - q = -16 \dots\dots[\text{equation (i)}]$$

Now, consider $(x + 1)$

$$\text{Then, } f(x) = x^3 - 2x^2 + px + q$$

$$\text{Let assume } x + 1 = 0$$

$$\text{Then, } x = -1$$

Now, substitute the value of x in $f(x)$

$$\begin{aligned}
 F(-1) &= (-1)^3 - 2(-1)^2 + p(-1) + q \\
 &= -1 - 2 - p + q \\
 &= -3 - p + q
 \end{aligned}$$

Given, remainder is 9

$$\text{So, } -3 - p + q = 9$$

$$-p + q = 9 + 3$$

$$-p + q = 12 \dots [\text{equation(ii)}]$$

Now adding equation (i) and equation (ii) we get,

$$(2p - q) + (-p + q) = -16 + 12$$

$$2p - q - p + q = -4$$

$$P = -4$$

Consider the equation (ii) to find out 'b'

$$-p + q = 12$$

$$-(4) + q = 12$$

$$4 + q = 12$$

$$Q = 12 - 4$$

$$Q = 8$$

Therefore, by substituting the value of p and q $f(x) = x^3 - 2x^2 - 4x + 8$

Dividing $f(x)$ by $(x + 2)$ we get,

	$X^2 - 4x + 4$
$X + 2$	$X^3 - 2x^2 - 4x + 8$
	-
	$X^3 + 2x^2$
	$-4x^2 - 4x + 8$
	-
	$-4x^2 - 8x$
	$4x + 8$
	-
	$4x + 8$
	0

$$\begin{aligned}
 X^3 - 2x^2 - 4x + 8 &= (x + 2)(x^2 - 4x + 4) \\
 &= (x + 2)(x^2 - 2 \times x(-2) + 2^2) \\
 &= (x + 2)(x - 2)^2
 \end{aligned}$$

8. if $(x + 3)$ and $(x - 4)$ are factors of $x^3 + ax^2 - bx + 24$ find the values of a and b : with these values of a and b, factorize the given expression.

Solution

Let us assume $x + 3 = 0$

Then, $x = -3$

Given $f(x) = x^3 + ax^2 - bx + 24$

Now, substitute the value of x in f(x)

$$f(-3) = (-3)^3 + a(-3)^2 - b(-3) + 24$$

$$= -27 + 9a + 3b + 24$$

$$= 9a + 3b - 3$$

Dividing all terms by 3 we get,

$$= 3a + b - 1$$

From the question $(x + 3)$ is a factor of $x^3 + ax^2 - bx + 24$

Therefore, remainder is 0

$$f(x) = 0$$

$$3a + b - 1 = 0$$

$$3a + b = 1 \dots [\text{equation(i)}]$$

Now, assume $x - 4 = 0$

Then, $x = 4$

Given $f(x) = x^3 + ax^2 - bx + 24$

Now substitute the value of x in f(x)

$$F(4) = 4^3 + a(4)^2 - b(4) + 24$$

$$= 64 + 16a - 4b + 24$$

$$= 88 + 16a - 4b$$

Dividing all terms by 4 we get,

$$= 22 + 4a - b$$

From the question $(x - 4)$ is a factor of $x^3 + ax^2 - bx + 24$

Therefore remainder is 0

$$f(x) = 0$$

$$22 + 4a - b = 0$$

$$4a - b = -22 \dots [\text{equation(ii)}]$$

Now, adding both equation(i) and equation(ii) we get,

$$(3a + b) + (4a - b) = 1 - 22$$

$$3a + b + 4a - b = -21$$

$$7a = -21$$

$$a = -\frac{21}{7}$$

$$a = -3$$

consider the equation (i) to find out 'b'.

$$3a + b = 1$$

$$3(-3) + b = 1$$

$$-9 + b = 1$$

$$b = 1 + 9$$

$$b = 10$$

therefore value of $a = -3$ and $b = 10$

then, by substituting the value of a and b $f(x) = x^3 - 3x^2 - 10x + 24$

$$(x + 3)(x - 4)$$

$$= x(x - 4) + 3(x - 4)$$

$$= x^2 - 4x + 3x - 12$$

$$= x^2 - x - 12$$

Dividing $f(x)$ by $x^2 - x - 12$ we get,

	$X - 2$
$X^2 - x - 12$	$X^3 - 3x^2 - 10x + 24$
	-
	$X^3 - x^2 - 12x$
	$-2x^2 + 2x + 24$
	-
	$-2x^2 + 2x + 24$
	0

Therefore, $x^3 - 3x^2 - 10x + 24 = (x^2 - x - 12)(x - 2)$
 $= (x + 3)(x - 4)(x - 2)$

9. if $(2x + 1)$ is a factor of both the expressions $2x^2 - 5x + p$ and $2x^2 + 5x + q$, find the value of p and q hence find the other factors of both the polynomials.

Solution

Let us assume $2x + 1 = 0$

Then, $2x = -1$

$$x = -\frac{1}{2}$$

Given $p(x) = 2x^2 - 5x + p$

Now, substituted the value of x in p(x)

$$p\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^2 - 5\left(-\frac{1}{2}\right) + p$$

$$= 2\left(\frac{1}{4}\right) + \frac{5}{2} + p$$

$$= \frac{6}{2} + p$$

$$= 3 + p$$

From the question it is given that, $(2x + 1)$ is a factor of both the expressions $2x^2 - 5x + p$

So, remainder is 0

$$\text{Then, } 3 + p = 0$$

$$P = -3$$

Now consider $q(x) = 2x^2 + 5x + q$

Substitute the value of x in q(x)

$$q\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^2 + 5\left(-\frac{1}{2}\right) + q$$

$$= 2\left(\frac{1}{4}\right) - \frac{5}{2} + q$$

$$= \frac{1-5}{2} + q$$

$$= -\frac{4}{2} + q$$

$$= q - 2$$

From the question it is given that, $(2x + 1)$ is a factor of both the expressions $2x^2 + 5x + q$

So, remainder is 0

$$Q - 2 = 0$$

$$q = 2$$

therefore, $p = -3$ and $q = 2$

$$p(x) = 2x^2 - 5x - 3$$

$$q(x) = 2x^2 + 5x + 2$$

then, divide $p(x)$ by $2x + 1$

	$X - 3$
$2x + 1$	$2x^2 - 5x - 3$
	$-$
	$2x^2 + x$
	$-6x - 3$
	$-$
	$-6x - 3$
	0

$$\text{Therefore } 2x^2 - 5x - 3 = (2x + 1)(x - 3)$$

Now, divide $q(x)$ by $2x + 1$

	$X + 2$
$2x + 1$	$2x^2 + 5x + 2$
	$-$
	$2x^2 + x$
	$4x + 2$
	$-$
	$4x + 2$
	0

Therefore, $2x^2 + 5x + 2 = (2x + 1)(x + 2)$

10. if a polynomial $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ leaves remainder 5 and 19 when divided by $(x - 1)$ and $(x + 1)$ respectively. Find the value of a and b . Hence determined the remainder when $f(x)$ is divided by $(x - 2)$

Solution

From the question it is given that

$$f(x) = x^4 - 2x^3 + 3x^2 - ax + b$$

factor $(x - 1)$ leaves remainder 5,

factor $(x + 1)$ leaves remainder 19,

where $x = 1$ and $x = -1$

$$f(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b = 19$$

$$1 - 2(-1) + 3(1) - a(-1) + b = 19$$

$$1 + 2 + 3 + a + b = 19$$

$$6 + a + b = 19$$

$$A + b = 19 - 6$$

$$A + b = 13 \dots \dots [\text{equation (i)}]$$

$$f(1) = (1)^4 - 2(1)^3 + 3(1)^2 - a(1) + b = 5$$

$$1 - 2(1) + 3(1) - a(1) + b = 5$$

$$1 - 2 + 3 - a + b = 5$$

$$2 - a + b = 5$$

$$-a + b = 5 - 2$$

$$-a + b = 3 \dots \dots [\text{equation(ii)}]$$

Now, subtracting equation(ii) from equation (i) we get,

$$(a + b) - (-a + b) = 13 - 3$$

$$A + b + a - b = 10$$

$$2a = 10$$

$$a = \frac{10}{2}$$

$$a = 5$$

to find out the value of b, substitute the value of a in equation (i)

we get,

$$a + b = 13$$

$$5 + b = 13$$

$$B = 13 - 5$$

$$b = 8$$

therefore, value of $a = 5$ and $b = 8$

11. when a polynomial $f(x)$ is divided by $(x - 1)$ the remainder is 5 and when it is, divided by $(x - 2)$ the remainder is 7. Find the remainder when it is divided by $(x - 1)(x - 2)$

Solution

From the question it is given that,

Polynomial $f(x)$ is divided by $(x - 1)$

Remainder = 5

Let us assume $x - 1 = 0$

$$x = 1$$

$$f(1) = 5$$

and the divide be $(x - 2)$ remainder = 7

let us assume $x - 2 = 0$

$$x = 2$$

therefore, $f(2) = 7$

so, $f(x) = (x - 1)(x - 2)q(x) + ax + b$

where $q(x)$ is the quotient and $ax + b$ is remainder

now put $x = 1$ we get

$$f(1) = (1-1) (1 -2)q(1) + (a \times 1) + b$$

$$a + b = 5 \dots\dots[\text{equation (i)}]$$

$$x = 2$$

$$f(2) = (2 -1) (2-2) q(2) + (a \times 2) + b$$

$$2a + b = 7\dots\dots[\text{equation(ii)}]$$

Now subtracting equation (i) from equation (ii) we get,

$$(2a + b) - (a + b) = 7 - 5$$

$$2a + b - a - b = 2$$

$$A = 2$$

To find out the value of b , substitute the value of a in equation(i)

we get

$$a + b = 5$$

$$2 + b = 5$$

$$b = 5 - 2$$

$$b = 3$$

therefore, the remainder $= ax + b = 2x + 3$