

Chapter 8

Matrices

Exercise 8.1

Question 1.

Classify the following matrices:

i) $\begin{bmatrix} 2 & -1 \\ 5 & 1 \end{bmatrix}$

Solution:

It is square matrix of order 2

ii) $[2 \ 3 \ -7]$

Solution:

It is row matrix of order 1×3

iii) $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$

Solution:

It is column matrix of order 3×1

iv) $\begin{bmatrix} 2 & -4 \\ 0 & 0 \\ 1 & 7 \end{bmatrix}$

Solution:

It is a matrix of order 3×2

$$(v) \begin{bmatrix} 2 & 7 & 8 \\ -1 & \sqrt{2} & 0 \end{bmatrix}$$

Solution:

It is a matrix of order 2×3

Question 2.

i) If a matrix has 4 elements, what are the possible order it can have?

Solution:

It can have $1 \times 4, 4 \times 1, \text{ or } 2 \times 2$

ii) If a matrix has 4 elements, what are the possible orders it can have?

Solution:

It can have $1 \times 8, 8 \times 1, 2 \times 4, \text{ or } 4 \times 2$ order,

Question 3 . construct a 2×2 matrix whose elements a_{ij} are given by

i) $a_{ij} = 2i - j$

ii) $a_{ij} = i \cdot j$

Solution:

i) Given $a_{ij} = 2i - j$

Therefore matrix of order 2×2 is

$$\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$$

ii) Given $a_{ij} = i \cdot j$

Therefore matrix of order 2×2 is

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

Question 4. Find the values of x and y if:

$$\begin{bmatrix} 2x + y \\ 3x - 2y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} 2x + y \\ 3x - 2y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

Now by comparing the corresponding elements,

$$2x + y = 5 \dots\dots\dots i$$

$$3x - 2y = 4 \dots\dots\dots ii$$

Multiply (i) by 2 and (ii) 1 we get

$$4x + 2y = 10 \text{ and } 3x - 2y = 4$$

By adding we get

$$7x = 14$$

$$X = \frac{14}{7}$$

$$X = 2$$

Substituting the value of x in (i)

$$4 + y = 5$$

$$Y = 5 - 4$$

$$Y = 1$$

Hence $x = 2$ and $y = 1$

Question 5. Find the value of x if

$$\begin{bmatrix} 3x + y & -y \\ 2y - x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} 3x + y & -y \\ 2y - x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$$

Comparing the corresponding terms of given matrix we get

$$-y = 2$$

Therefore $y = -2$

Again we have

$$3x + y = 1$$

$$3x = 1 - y$$

Substituting the value of y we get

$$3x = 1 - (-2)$$

$$3x = 1 + 2$$

$$3x = 3$$

$$X = \frac{3}{3}$$

$$X = 1$$

Hence $x = 1$ and $y = -2$

Question 6. If

$$\begin{bmatrix} x + 3 & 4 \\ y - 4 & x + y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$$

Find the values of x and y.

Solution:

Given

$$\begin{bmatrix} x + 3 & 4 \\ y - 4 & x + y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$$

Comparing the corresponding terms, we get

$$X + 3 = 5$$

$$X = 5 - 3$$

$$X = 2$$

Again we have

$$Y - 4 = 3$$

$$Y = 3 + 4$$

$$Y = 7$$

Hence $x = 2$ and $y = 7$

Question 7. Find the values of x, y and Z if

$$\begin{bmatrix} x + 2 & 6 \\ 3 & 5z \end{bmatrix} = \begin{bmatrix} -5 & y^2 + y \\ 3 & -20 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} x + 2 & 6 \\ 3 & 5z \end{bmatrix} = \begin{bmatrix} -5 & y^2 + y \\ 3 & -20 \end{bmatrix}$$

Comparing the corresponding elements of given matrix, then we get

$$X + 2 = -5$$

$$X = -5 - 2$$

Also we have $5z = -20$

$$Z = -\frac{20}{5}$$

$$Z = -4$$

Again from given matrix we have

$$Y^2 + y - 6 = 0$$

The above equation can be written as

$$Y^2 + 3y - 2y - 6 = 0$$

$$Y(y + 3) - 2(y + 3) = 0$$

$$Y + 3 = 0 \text{ or } y - 2 = 0$$

$$Y = -3 \text{ or } y = 2$$

Hence $x = -7$, $y = -3$ and $Z = -4$

Question 8. Find the values of x, y, a and b if

$$\begin{bmatrix} x - 2 & y \\ a + 2b & 3a - b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} x - 2 & y \\ a + 2b & 3a - b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$$

Comparing the corresponding elements

$$X - 2 = 3 \text{ and } y = 1$$

$$X = 2 + 3$$

$$X = 5$$

Again we have

$$A + 2b = 5 \text{ i}$$

$$3a - b = 1 \text{ ii}$$

Multiply (i) by 1 and (ii) by 2

$$A + 2b = 5$$

$$6a - 2b = 2$$

Now by adding above equations we get

$$7a = 7$$

$$A = \frac{7}{7}$$

$$A = 1$$

Substituting the value of a in (i) we get

$$1 + 2b = 5$$

$$2b = 5 - 1$$

$$2b = 4$$

$$B = \frac{4}{2}$$

$$B = 2$$

Question 9: Find the values of a, b, c and d if

$$\begin{bmatrix} a + b & 3 \\ 5 + c & ab \end{bmatrix} \begin{bmatrix} 6 & d \\ -1 & 8 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} a + b & 3 \\ 5 + c & ab \end{bmatrix} \begin{bmatrix} 6 & d \\ -1 & 8 \end{bmatrix}$$

Comparing the corresponding terms, we get

$$3 = d$$

$$D = 3$$

Also we have

$$5 + c = -1$$

$$C = -1 - 5$$

$$C = -6$$

Also we have,

$$A + b = 6 \text{ and } a b = 8$$

We know that,

$$(a - b)^2 = (a + b)^2 - 4 ab$$

$$(6)^2 - 32 = 36 - 32 = 4 = (\pm 2)^2$$

$$a - b = \pm 2$$

$$\text{if } a - b = 2$$

$$a + b = 6$$

Adding the above two equations we get

$$2a = 4$$

$$A = \frac{4}{2}$$

$$A = 2$$

$$B = 6 - 4$$

$$B = 2$$

Again we have $a - b = -2$

And $a + b = 6$

Adding above equations we get

$$2a = 4$$

$$A = \frac{4}{2}$$

$$A = 2$$

Also, $b = 6 - 2 = 4$

$A = 2$ and $b = 4$

Exercise 8.2

Question 1. Given that $M = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ and $N = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$ find, $M + 2N$

Solution:

Given

$$M = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$$

$$N = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$$

Now we have to find $M + 2N$

$$M + 2N = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} + 2 \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$$

On simplifying we get,

$$= \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 + 4 & 0 + 0 \\ 1 - 2 & 2 + 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ -1 & 6 \end{bmatrix}$$

Question 2: If $A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$

Find $2A - 3B$

Solution:

$$A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

Now we have to find,

$$2A - 3B = 2 \begin{bmatrix} 0 & 0 \\ -3 & 1 \end{bmatrix} - 3 \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

On simplifying we get

$$= \begin{bmatrix} 4 & 0 \\ -6 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -6 & 9 \end{bmatrix} = \begin{bmatrix} 4 - 0 & 0 - 3 \\ -6 + 6 & 2 - 9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 \\ 0 & -7 \end{bmatrix}$$

Question 3. $\sin A \begin{bmatrix} \sin A & -\cos A \\ \cos A & \sin A \end{bmatrix} + \cos A \begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$

Solution:

Given,

$$\sin A \begin{bmatrix} \sin A & -\cos A \\ \cos A & \sin A \end{bmatrix} + \cos A \begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$$

On simplifications, we get

$$= \begin{bmatrix} \sin^2 A & -\sin A \cdot \cos A \\ \sin A \cdot \cos A & \sin^2 A \end{bmatrix} + \cos A \begin{bmatrix} \cos^2 A & \cos A \cdot \sin A \\ -\cos A \cdot \sin A & \cos^2 A \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2 A + \cos^2 A & -\sin A \cdot \cos A + \cos A \cdot \sin A \\ \sin A \cdot \cos A - \cos A \cdot \sin A & \sin^2 A + \cos^2 A \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ (Since, } \sin^2 A + \cos^2 A = 1 \text{)}$$

Question 4: $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$

Find $A + 2B - 3C$

Solution:

$$A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix}, C = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$$

Now we have to find $A + 2B - 3C$

$$= \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + 2 \begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix} - 3 \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} -4 & -2 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 6 & -3 \end{bmatrix}$$

= On simplifying we get

$$\begin{bmatrix} 1 - 4 - 0 & 2 - 2 - 9 \\ -2 + 2 - 6 & 3 + 4 + 3 \end{bmatrix} = \begin{bmatrix} -3 & -9 \\ -6 & 10 \end{bmatrix}$$

Question 5: If $A = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

Find the matrix X if :

i) $3A + X = B$

ii) $x - 3B = 2A$

Solution:

Given

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

Now we have to find

i) $3A + X = B$

$$x = B - 3A$$

Substituting the values we get

$$\begin{aligned} X &= \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} - 3 \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -3 \\ 3 & 6 \end{bmatrix} \\ &= \begin{bmatrix} 1-0 & 2+3 \\ -1-3 & 1-6 \end{bmatrix} \end{aligned}$$

$$\text{ii) } X - 3B = 2A$$

$$X = 2A + 3B$$

Now substituting the values A and B we get

$$\begin{aligned} X &= 2 \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} + 3 \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -3 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 0+3 & -2+6 \\ 2-3 & 4+3 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -1 & 7 \end{bmatrix} \end{aligned}$$

Question 6. Solve the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 5 & 0 \end{bmatrix} - 3X = \begin{bmatrix} -7 & 4 \\ 2 & 6 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} 2 & 1 \\ 5 & 0 \end{bmatrix} - 3X = \begin{bmatrix} -7 & 4 \\ 2 & 6 \end{bmatrix}$$

On simplification we get

$$X = \frac{1}{3} \begin{bmatrix} 9 & -3 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$$

Question 7: If $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3 \begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$ find the matrix M

Solution

Given,

$$\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3 \begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$$

$$2M = \begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 6 \\ 0 & -9 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} \quad (\text{After further simplification})$$

$$= \begin{bmatrix} 9 - 1 & 6 - 4 \\ 0 - (-2) & -9 - 3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 2 \\ 2 & -12 \end{bmatrix} \quad (\text{After subtraction of matrices})$$

$$M = \frac{1}{2} \begin{bmatrix} 8 & 2 \\ 2 & -12 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & -6 \end{bmatrix} \quad (\text{Dividing By 2})$$

Question 8: Given $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

Find the matrix X such that $A + 2x = 2B + C$

Solution:

$$A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}, C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\text{Let } x = \begin{bmatrix} x & y \\ z & t \end{bmatrix}$$

$$A + 2x = 2B + C \quad \text{(Given Condition)}$$

$$2x = 2B + C - A$$

$$2 \begin{bmatrix} x & y \\ z & t \end{bmatrix} = 2 \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} -6 & 4 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} \quad \text{(On further calculation)}$$

$$= \begin{bmatrix} -6 + 4 - 2 & 4 + 0 + 6 \\ 8 + 0 - 2 & 0 + 2 - 0 \end{bmatrix} = \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

(Addition and subtraction of matrices)

$$\therefore 2 \begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x & y \\ z & t \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix} \quad \text{(dividing by 2)}$$

Question 9: Find x and y if $x + y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $x - y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

Solution:

Given,

$$X + y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \quad \text{..... (i)}$$

$$X - y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad \text{.....(ii)}$$

Adding (i) and (ii) we get,

$$2x = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 7 + 3 & 0 + 0 \\ 2 + 0 & 5 + 3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

$$X = \frac{1}{2} \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

Now,

Subtracting (ii) from (i), we get

$$\begin{aligned} 2y &= \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \\ &= 2y = \begin{bmatrix} 7-3 & 0-0 \\ 2-0 & 5-3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix} \\ &= y = \frac{1}{2} \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \end{aligned}$$

Question 10: If $2 \begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$ find the values of **x** and **y**

Solution:

Given,

$$\begin{aligned} 2 \begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} &= \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix} \\ \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} &= \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix} \\ = \begin{bmatrix} 6+1 & 8+y \\ 10+0 & 2x+1 \end{bmatrix} &= \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix} \quad (\text{addition of matrices}) \end{aligned}$$

On comparing the corresponding elements, we have

$$8 + y = 0$$

$$\text{Then, } y = -8$$

$$\text{And, } 2x + 1 = 5$$

$$2x = 5 - 1 = 4$$

$$X = \frac{4}{2} = 2$$

Therefore $x = 2$, and $y = -8$

Question 11: If $2 \begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix}$ find the values of x and y

Solution:

Given,

$$2 \begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 6 + 1 & 8 + y \\ 10 + 0 & 2x + 1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix} \quad (\text{Addition of matrices})$$

$$= \begin{bmatrix} 7 & 8 + y \\ 10 & 2x + 1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix}$$

On comparing the corresponding terms, we have

$$2x + 1 = 5$$

$$2x = 5 - 1$$

$$X = \frac{4}{2} = 2$$

And,

$$8 + y = 0$$

$$Y = -8$$

And, $z = 7$

Therefore, $x = 2$, $y = -8$ and $Z = 7$

12: If $\begin{bmatrix} 5 & 2 \\ -1 & y+1 \end{bmatrix} - 2\begin{bmatrix} 1 & 2x-1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$ find the values of x and y

Solution:

Given,

$$\begin{bmatrix} 5 & 2 \\ -1 & y+1 \end{bmatrix} - 2\begin{bmatrix} 1 & 2x-1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ -1 & y+1 \end{bmatrix} - \begin{bmatrix} 2 & 4x-2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 5-2 & 2-4x+2 \\ -1-6 & y+1+4 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix} \quad (\text{Subtraction of matrices})$$

$$\begin{bmatrix} 3 & 4-4x \\ -7 & y+5 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$$

Now, comparing the corresponding terms, we get

$$4 - 4x = -8$$

$$4 + 8 = 4x$$

$$12 = 4x$$

$$X = \frac{12}{4}$$

$$X = 3$$

$$\text{And, } y + 5 = 2$$

$$Y = 2 - 5$$

$$Y = -3$$

Therefore, $x = 3$ and $y = -3$

Question 13: If $\begin{bmatrix} a & 3 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$

Find the value of a, b and c.

Solution:

Given

$$\begin{bmatrix} a & 3 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} a + 2 - 1 & 3 + b - 1 \\ 4 + 1 + 2 & 2 - 2 - c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} a + 1 & b + 2 \\ 7 & -c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix} \quad (\text{After further calculations})$$

Next, on comparing the corresponding terms, we have

$$A + 1 = 5$$

$$A = 4$$

$$B + 2 = 0$$

$$B = -2$$

$$-c = 3$$

$$C = 3$$

Therefore, the value of a, b and c are 4, -2 and -3 respectively.

Question 14: If $A = \begin{bmatrix} 2 & a \\ -3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 7 & b \end{bmatrix}$, $C = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$

And $5A + 2B = C$, find the values of a, b and C.

Solution:

Given,

$$A = \begin{bmatrix} 2 & a \\ -3 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 \\ 7 & b \end{bmatrix}, c = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix} \text{ and } 5A + 2B = C$$

So, we have

$$5 \begin{bmatrix} 2 & a \\ -3 & 5 \end{bmatrix} + 2 \begin{bmatrix} -2 & 3 \\ 7 & b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 5a \\ -15 & 25 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ 14 & 2b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$$

$$\begin{bmatrix} 10 - 4 & 5a + 6 \\ -15 + 14 & 25 + 2a \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 5a + 6 \\ -1 & 25 + 2a \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$$

On comparing the corresponding terms, we get

$$5a + 6 = 9$$

$$5a = 9 - 6$$

$$5a = 3 = \frac{3}{5}$$

And,

$$25 + 2b = -11$$

$$2b = -11 - 25$$

$$2b = -11 - 25$$

$$2b = -36$$

$$B = -\frac{36}{2}$$

$$B = -18$$

$$\text{And } C = 6$$

Therefore, the value of a, b and C are $\frac{3}{5}$, -18 and 6 respectively.

Exercise 8.3

1. If $A = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, is the product AB possible? Give a reason. If yes, find AB .

Solution:

Yes, the product is possible because of number of column in $A =$ number of row in B

That is order of matrix is 2×1

$$AB = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \times 2 + 5 \times 4 \\ 4 \times 2 + (-2) \times 4 \end{bmatrix} = \begin{bmatrix} 6 + 20 \\ 8 - 8 \end{bmatrix} = \begin{bmatrix} 26 \\ 0 \end{bmatrix}$$

2. If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$, find AB and BA , is $AB = BA$?

Solution:

Given

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$$

Now we have to find $A \times B$

$$\therefore A \times B = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 2 - 15 & -2 + 10 \\ 1 - 9 & -1 + 6 \end{bmatrix} = \begin{bmatrix} -13 & 8 \\ -8 & 5 \end{bmatrix}$$

Again have to find $B \times A$

$$B \times A = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 - 1 & 5 - 3 \\ -6 + 2 & -15 + 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -4 & -9 \end{bmatrix}$$

Hence AB is not equal to BA

3. If $A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$ Find $AB - 5C$

Solution:

Given

$$A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$$

On simplification we get

$$= \begin{bmatrix} 3 \times 0 + 7 \times 5 & 3 \times 2 + 7 \times 3 \\ 2 \times 0 + 4 \times 5 & 4 \times 2 + 4 \times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + 35 & 6 + 21 \\ 0 + 20 & 4 + 12 \end{bmatrix} = \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix}$$

$$5C = 5 \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$$

$$AB - 5C = \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix} = \begin{bmatrix} 30 & 52 \\ 40 & -14 \end{bmatrix}$$

4. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, find $A(BA)$

Solution:

Given

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 + 2 & 4 + 1 \\ 1 + 4 & 2 + 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$A(BA) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 10 & 5 + 8 \\ 8 + 5 & 10 + 4 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix}$$

5. Give matrices:

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}, C = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

Find the products of

(i) ABC

(ii) ACB and state whether they are equal.

Solution:

Given

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix},$$

$$C = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

Now consider,

$$\begin{aligned}ABC &= \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix} \times \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 6-1 & 8-2 \\ 12-2 & 16-4 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 5 & 6 \\ 10 & 12 \end{bmatrix} \times \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} \\ &= \begin{bmatrix} -15+0 & 5-12 \\ -30+0 & 10-24 \end{bmatrix} \\ &= \begin{bmatrix} -15 & -7 \\ -30 & -14 \end{bmatrix}\end{aligned}$$

$$\begin{aligned}ACB &= \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \times \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix} \\ &= \begin{bmatrix} -6+0 & 2-2 \\ -12+0 & 4-4 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix} \\ &= \begin{bmatrix} -6 & 0 \\ -12 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix} \\ &= \begin{bmatrix} -18+0 & -24+0 \\ -36+0 & -48+0 \end{bmatrix} \\ &= \begin{bmatrix} -18 & -24 \\ -36 & -48 \end{bmatrix}\end{aligned}$$

$\therefore ABC \neq ACB.$

6. Evaluate: $\begin{bmatrix} 4 \sin 30^\circ & 2 \cos 60^\circ \\ \sin 90^\circ & 2 \cos 0^\circ \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

Solution:

Given

$$\begin{bmatrix} 4 \sin 30^\circ & 2 \cos 60^\circ \\ \sin 90^\circ & 2 \cos 0^\circ \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$\sin 30^\circ = \frac{1}{2}, \cos 60^\circ = \frac{1}{2}$$

$$\sin 90^\circ = 1 \text{ and } \cos 0^\circ = 1$$

$$\begin{aligned} \therefore & \begin{bmatrix} 4 \times \frac{1}{2} & 2 \times \frac{1}{2} \\ 1 & 2 \times 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 2 \times 4 + 1 \times 5 & 2 \times 5 + 1 \times 4 \\ 1 \times 4 + 1 \times 5 & 1 \times 5 + 2 \times 4 \end{bmatrix} \\ &= \begin{bmatrix} 8 + 5 & 10 + 4 \\ 4 + 10 & 5 + 8 \end{bmatrix} \\ &= \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix} \end{aligned}$$

7. If $\mathbf{A} = \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix}$ find the matrix $\mathbf{AB} + \mathbf{BA}$

Solution:

Given

$$\mathbf{A} = \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix}$$

$$\mathbf{AB} = \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} -2 - 12 & 3 - 18 \\ 4 - 16 & -6 - 24 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & -15 \\ -12 & -30 \end{bmatrix}$$

$$\begin{aligned}
 BA &= \begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix} \times \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix} \\
 &= \begin{bmatrix} -2 - 6 & 6 - 12 \\ 4 - 12 & -12 - 24 \end{bmatrix} \\
 &= \begin{bmatrix} -8 & -6 \\ -8 & -36 \end{bmatrix}
 \end{aligned}$$

$\therefore AB + BA$

$$\begin{aligned}
 &= \begin{bmatrix} -14 & -15 \\ -12 & -30 \end{bmatrix} + \begin{bmatrix} -8 & -6 \\ -8 & -36 \end{bmatrix} \\
 &= \begin{bmatrix} -14 - 8 & -15 - 6 \\ -12 - 8 & -30 - 36 \end{bmatrix} \\
 &= \begin{bmatrix} -22 & -21 \\ -20 & -66 \end{bmatrix}
 \end{aligned}$$

8. If $A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$

Solution:

Given,

$$A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$$

$$2B = 2 \begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 4 \\ -4 & 2 \end{bmatrix}$$

Now,

$$A^2 = A \times A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4 & -2+2 \\ 2-2 & -4+1 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

$$\begin{aligned} \therefore 2B - A^2 &= \begin{bmatrix} 6 & 4 \\ -4 & 2 \end{bmatrix} - \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 6 - (-3) & 4 - 0 \\ -4 - 0 & 2 - (-3) \end{bmatrix} = \begin{bmatrix} 6+3 & 4 \\ -4 & 2+3 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 4 \\ -4 & 5 \end{bmatrix} \end{aligned}$$

9. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 5 & 1 \\ 7 & 4 \end{bmatrix}$, compute

- (i) $A(B + C)$
(ii) $(B + C)A$

Solution:

Given,

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 5 & 1 \\ 7 & 4 \end{bmatrix}$$

- (i) $A(B + C)$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \left(\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 1 \\ 7 & 4 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2+5 & 1+1 \\ 4+7 & 2+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 7 & 2 \\ 11 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 7+22 & 2+12 \\ 21+44 & 6+24 \end{bmatrix} = \begin{bmatrix} 29 & 14 \\ 65 & 30 \end{bmatrix}$$

$$\begin{aligned}
 & \text{(ii)} \quad (\mathbf{B} + \mathbf{C}) \mathbf{A} \\
 &= \begin{bmatrix} 7 & 2 \\ 11 & 6 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \\
 &= \begin{bmatrix} 7 + 6 & 14 + 8 \\ 11 + 18 & 22 + 24 \end{bmatrix} \\
 &= \begin{bmatrix} 13 & 22 \\ 29 & 46 \end{bmatrix}
 \end{aligned}$$

10. If $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$, Find the matrix $\mathbf{C}(\mathbf{B} - \mathbf{A})$.

Solution:

Given,

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}, \text{ and } \mathbf{C} = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$$

Now,

$$\mathbf{B} - \mathbf{A} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$\begin{aligned}
 \mathbf{C}(\mathbf{B} - \mathbf{A}) &= \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 + 3 & -1 - 3 \\ 3 + 1 & -3 - 1 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 4 & -4 \end{bmatrix}
 \end{aligned}$$

11. Let $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$ Find $\mathbf{A}^2 + \mathbf{AB} + \mathbf{B}^2$.

Solution:

Given,

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

Now,

$$\begin{aligned} A^2 &= A \times A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 \times 1 + 0 \times 2 & 1 \times 0 + 0 \times 1 \\ 2 \times 1 + 1 \times 2 & 2 \times 0 + 1 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 + 0 & 0 + 0 \\ 2 + 2 & 0 + 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} A \times B &= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 \times 2 + 0 \times 1 & 1 \times 3 + 0 \times 0 \\ 2 \times 2 + 1 \times (-1) & 2 \times 3 + 1 \times 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} B^2 &= B \times B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 2 \times 2 + 3 \times (-1) & 2 \times 3 + 3 \times 0 \\ -1 \times 2 + 0 \times (-1) & -1 \times 3 + 0 \times 0 \end{bmatrix} \\ &= \begin{bmatrix} 4 - 3 & 6 + 0 \\ -2 + 0 & -3 + 0 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} \end{aligned}$$

Hence,

$$\begin{aligned} A^2 + AB + B^2 &= \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 1 + 2 + 1 & 0 + 3 + 6 \\ 4 + 3 - 2 & 1 + 6 + (-3) \end{bmatrix} = \begin{bmatrix} 4 & 9 \\ 5 & 4 \end{bmatrix} \end{aligned}$$

12. Let $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$ $C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$ Find $A^2 + AC - 5B$.

Solution:

Given,

$$A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} \text{ and } C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$$

Now,

$$\begin{aligned} A^2 + AC - 5B &= \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix} - 5 \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 4 + 0 & 2 - 2 \\ 0 + 0 & 0 + 4 \end{bmatrix} + \begin{bmatrix} -6 - 1 & 4 + 4 \\ 0 + 2 & 0 - 8 \end{bmatrix} - 5 \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} \end{aligned}$$

(Substituting the value from given)

$$\begin{aligned} &= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix} - \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix} \\ &= \begin{bmatrix} 4 - 7 - 20 & 0 + 8 - 5 \\ 0 + 2 + 15 & 4 - 8 + 10 \end{bmatrix} = \begin{bmatrix} -23 & 3 \\ 17 & 6 \end{bmatrix} \end{aligned}$$

13. If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$ find $AC + B^2 - 10C$.

Solution:

Given,

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$

Now,

$$\begin{aligned} AC &= \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 2 - 3 & 0 + 12 \\ 5 - 7 & 0 + 28 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix} \end{aligned}$$

$$\begin{aligned}
 B^2 &= B \times B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} \\
 &= \begin{bmatrix} 0 - 4 & 0 + 28 \\ 0 - 7 & -4 + 49 \end{bmatrix} \\
 &= \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 10C &= 10 \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix} \\
 &= \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}
 \end{aligned}$$

Hence,

$$\begin{aligned}
 AC + B^2 - 10C &= \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix} + \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix} \\
 &= \begin{bmatrix} -1 - 4 - 10 & 12 + 28 - 0 \\ -2 - 7 + 10 & 28 + 45 - 40 \end{bmatrix} \\
 &= \begin{bmatrix} -15 & 40 \\ -1 & 33 \end{bmatrix}
 \end{aligned}$$

14. If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ find A^2 and A^3 . Also state that which of these is equal to A .

Solution:

Given,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{aligned}
 A^2 &= A \times A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 + 0 & 0 + 0 \\ 0 + 0 & 0 + 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
 \end{aligned}$$

Next,

$$\begin{aligned} A^3 &= A^2 \times A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0-1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \end{aligned}$$

From above, it's clearly seen that $A^3 = A$.

15. If $x = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$ show that $6x - x^2 = 9l$ where l is the unit matrix.

Solution:

Given,

$$x = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$$

Now,

$$\begin{aligned} x^2 &= x \times x = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 16-1 & 4+2 \\ -4-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix} \end{aligned}$$

Taking L.H.S. we have

$$\begin{aligned} 6x - x^2 &= 6 \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 24 & 6 \\ -6 & 12 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 24-15 & 6-6 \\ -6-6 & 12-3 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} \\ &= 9 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 9l = \text{R.H.S.} \end{aligned}$$

Hence proved.

16. $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ Show that is a solution of the matrix equation $x^2 - 2x - 3l = 0$, where l is the unit matrix of order 2.

Solution:

Given,

$$x^2 - 2x - 3l = 0$$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\Rightarrow x = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\begin{aligned} \therefore x^2 &= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} \end{aligned}$$

Now,

$$x^2 - 2x - 3l$$

$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - 2 \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5-2-3 & 4-4+0 \\ 4-4-0 & 5-2-3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore x^2 - 2x - 3l = 0$$

Hence proved.

17. Find the matrix 2×2 which satisfies the equation.

$$\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

Solution:

Given,

$$\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 + 35 & 6 + 21 \\ 0 + 20 & 4 + 12 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$2X = - \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} + \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} -34 & -32 \\ -24 & -10 \end{bmatrix}$$

$$X = \frac{1}{2} \begin{bmatrix} -34 & -32 \\ -24 & -10 \end{bmatrix} = \begin{bmatrix} -17 & -16 \\ -12 & -5 \end{bmatrix}$$

18. If $A = \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix}$, Find the value of x , so that $A^2 - 0$

Solution:

Given,

$$A = \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix},$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix} \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix}$$

$$= \begin{bmatrix} 1 + x & 1 + x \\ x + x^2 & x + x^2 \end{bmatrix} = 0$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

On comparing.

$$1 + x = 0$$

$$\therefore x = -1$$

19.

(i) Find x and y if $\begin{bmatrix} -3 & 2 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$

(ii) Find x and y if $\begin{bmatrix} 2x & x \\ y & 3y \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$

Solution:

(i) $\begin{bmatrix} -3 & 2 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} -3x & 4 \\ 0 & -10 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -3x & +4 \\ 0 & -10 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$$

Comparing the corresponding elements,

$$-3x + 4 = -5$$

$$-3x = -5 - 4 = -9$$

$$x = \frac{-9}{-3} = 3$$

Therefore, $x = 3$ and $y = -10$.

(ii) $\begin{bmatrix} 2x & x \\ y & 3y \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2x \times 3 + x \times 2 \\ y \times 3 + 3y \times 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6x + 2x \\ 3y + 6y \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8x \\ 9y \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

Comparing, we get

$$8x = 16$$

$$\Rightarrow x = \frac{16}{8} = 2$$

And, $9y = 9$

$$y = \frac{9}{9} = 1$$

20. Find the values of x and y if $\begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

Solution:

Given,

$$\begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+2y & -y \\ 4x & -x+y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & +y \\ 3x & +y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

On comparing the corresponding elements, we have

$$2x + y = 3 \quad (\text{i})$$

$$3x + y = 2 \quad (\text{ii})$$

Subtracting, we get

$$-x = 1 \Rightarrow x = -1$$

Substituting the values of x in (i).

$$2(-1) + y = 3$$

$$-2 + y = 3$$

$$y = 3 + 2 = 5$$

Therefore, $x = -1$ and $y = 5$.

CHAPTER TEST

1. Find the values of a and b if

$$\begin{bmatrix} a + 3 & b^2 + 2 \\ 0 & -6 \end{bmatrix} = \begin{bmatrix} 2a + 1 & 3b \\ 0 & b^2 - 5b \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} a + 3 & b^2 + 2 \\ 0 & -6 \end{bmatrix} = \begin{bmatrix} 2a + 1 & 3b \\ 0 & b^2 - 5b \end{bmatrix}$$

Comparing the corresponding elements

$$a + 3 = 2a + 1$$

$$\Rightarrow 2a - a = 3 - 1$$

$$\Rightarrow a = 2b^2 + 2 = 3b$$

$$\Rightarrow b^2 - 3b + 2 = 0$$

$$\Rightarrow b^2 - b - 2b + 2 = 0$$

$$\Rightarrow b(b - 1) - 2(b - 1) = 0$$

$$\Rightarrow (b - 1)(b - 2) = 0.$$

Either $b - 1 = 0$

Then $b = 2$

Hence $a = 2, b = 2$ or 1

2. Find a, b, c and d if $3 \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 4 & a + b \\ c + d & 3 \end{bmatrix} + \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix}$

Solution:

Given,

$$3 \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 4 & a + b \\ c + d & 3 \end{bmatrix} + \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix}$$

Now comparing the corresponding elements

$$3a = 4 + a$$

$$a - a = 4$$

$$2a = 4$$

Therefore, $a = 2$

$$3b = a + b + 6$$

$$3b - b = 2 + 6$$

$$2b = 8$$

Therefore, $b = 4$

$$3d = 3 + 2d$$

$$3d - 2d = 3$$

Therefore, $d = 3$

$$3c = c + d - 1$$

$$3c - c = 3 - 1$$

$$2c = 2$$

Therefore, $c = 1$

Hence, $a = 2, b = 4, c = 1$ and $d = 3$

3. Determine the matrices A and B when

$$\mathbf{A} + 2\mathbf{B} = \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix} \text{ and } 2\mathbf{A} - \mathbf{B} = \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix}$$

Solution:

Given,

$$\mathbf{A} + 2\mathbf{B} = \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix} \quad \dots \text{ (i)}$$

$$2\mathbf{A} - \mathbf{B} = \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \quad \dots \text{ (ii)}$$

Multiplying (i) by 1 and (ii) by 2

$$\mathbf{A} + 2\mathbf{B} = \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix}$$

$$4\mathbf{A} - 2\mathbf{B} = 2 \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 4 & -2 \end{bmatrix}$$

Now, adding we get

$$5\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 10 & -5 \end{bmatrix}$$

$$\mathbf{A} = \frac{1}{5} \begin{bmatrix} 5 & 0 \\ 10 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$$

$$\begin{aligned} \text{From (i) } \mathbf{A} + 2\mathbf{B} &= \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} + 2\mathbf{B} = \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix} \end{aligned}$$

$$2\mathbf{B} = \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 4 & -2 \end{bmatrix}$$

$$\therefore \mathbf{B} = \frac{1}{2} \begin{bmatrix} 0 & 2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$$

$$\text{Thus, } \mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$$

4.

(i) Find the matrix B is $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ and $A^2 = A + 2B$

(ii) If $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ -2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 0 \\ -1 & 1 \end{bmatrix}$ find A (4B - 3C)

Solution:

(i) Given,

$$A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$$

$$\text{Let } B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 16 + 2 & 4 + 3 \\ 8 + 6 & 2 + 9 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 7 \\ 14 & 11 \end{bmatrix}$$

$$A + 2B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} + 2 \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix} = \begin{bmatrix} 4 + 2a & 1 + 2b \\ 2 + 2c & 3 + 2d \end{bmatrix}$$

As, $A^2 = A + 2B$

$$\Rightarrow \begin{bmatrix} 18 & 7 \\ 14 & 11 \end{bmatrix} = \begin{bmatrix} 4 + 2a & 1 + 2b \\ 2 + 2c & 3 + 2d \end{bmatrix}$$

Comparing the corresponding elements, we have

$$4 + 2a = 18$$

$$2a = 18 - 4 = 14$$

$$a = \frac{14}{2}$$

$$\Rightarrow a = 7$$

$$1 + 2b = 7$$

$$2b = 7 - 1 = 6$$

$$b = \frac{6}{2}$$

$$\Rightarrow b = 3$$

$$2 + 2c = 14$$

$$2c = 14 - 2 = 12$$

$$2c = 12$$

$$c = \frac{12}{2}$$

$$\Rightarrow c = 6$$

$$3 + 2d = 11$$

$$2d = 11 - 3$$

$$d = \frac{8}{2}$$

$$\Rightarrow d = 4$$

Therefore, $a = 7, b = 3, c = 6$ and $d = 4$

$$\therefore B = \begin{bmatrix} 7 & 3 \\ 6 & 4 \end{bmatrix}$$

$$(ii) \quad A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 1 \\ -2 & 5 \end{bmatrix},$$

$$C = \begin{bmatrix} -2 & 0 \\ -1 & 1 \end{bmatrix}$$

$$4B - 3C = 4 \begin{bmatrix} 0 & 1 \\ -2 & 5 \end{bmatrix} - 3 \begin{bmatrix} -2 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 4 \\ -8 & 20 \end{bmatrix} - \begin{bmatrix} -6 & 0 \\ -3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 - (-6) & 4 - 0 \\ -8 - (-3) & 20 - 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + 6 & 4 - 0 \\ -8 + 3 & 20 - 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 4 \\ -5 & 17 \end{bmatrix}$$

Now,

$$A(4B - 3C) = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 6 & 4 \\ -5 & 17 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 6 + 2(-5) & 1 \times 4 + 2 \times 17 \\ -3 \times 6 + 4 \times (-5) & -3 \times 4 + 4 \times 17 \end{bmatrix}$$

$$= \begin{bmatrix} 6 - 10 & 4 + 34 \\ -18 - 20 & -12 + 68 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 38 \\ -38 & 56 \end{bmatrix}$$

5. If $A = \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ find the each of the following and state it they are equal:

(i) $(A + B)(A - B)$

(ii) $A^2 - B^2$

Solution:

Given,

$$A = \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$$

(i) $(A + B)(A - B)$

$$= \left\{ \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \right\} \times \left\{ \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 3+1 & 2+0 \\ 0+1 & 5+2 \end{bmatrix} \times \begin{bmatrix} 3-1 & 2-0 \\ 0-1 & 5-2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 \\ 1 & 7 \end{bmatrix} \times \begin{bmatrix} 2 & 2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 8-2 & 8+6 \\ 2-7 & 2+21 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 14 \\ -5 & 23 \end{bmatrix}$$

(ii) $A^2 - B^2$

$$= \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9+0 & 6+10 \\ 0+0 & 0+25 \end{bmatrix} - \begin{bmatrix} 1+0 & 0+0 \\ 1+2 & 0+4 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 16 \\ 0 & 25 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 9-1 & 16-0 \\ 0-3 & 25-4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 16 \\ -3 & 21 \end{bmatrix}$$

Hence, it's clearly seen that $(A + B)(A - B) \neq A^2 - B^2$

6. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ find $A^2 - 5A - 14I$, where I is unit matrix of order 2×2 .

Solution:

Given,

$$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{bmatrix} = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$$

$$5A = 5 \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix}$$

$$\begin{aligned} \therefore A^2 - 5A - 14I &= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - 14 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix} \\ &= \begin{bmatrix} 29 - 15 - 14 & -25 + 25 - 0 \\ -20 + 20 + 0 & 24 - 10 - 14 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

7. If $A = \begin{bmatrix} 3 & 3 \\ p & q \end{bmatrix}$ and $A^2 = 0$, find p and q .

Solution:

Given

$$A = \begin{bmatrix} 3 & 3 \\ p & q \end{bmatrix}$$

$$\begin{aligned} A^2 &= A \times A = \begin{bmatrix} 3 & 3 \\ p & q \end{bmatrix} \begin{bmatrix} 3 & 3 \\ p & q \end{bmatrix} \\ &= \begin{bmatrix} 9 + 3p & 9 + 3q \\ 3p + pq & 3p + q^2 \end{bmatrix} \end{aligned}$$

But $A^2 = 0$

$$\therefore \begin{bmatrix} 9 + 3p & 9 + 3q \\ 3p + pq & 3p + q^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

On comparing the corresponding elements, we have

$$9 + 3p = 0$$

$$3p = -9$$

$$p = \frac{-9}{3}$$

$$p = -3$$

And,

$$9 + 3q = 0$$

$$3q = -9$$

$$q = \frac{-9}{3}$$

$$q = -3$$

Therefore, $p = -3$ and $q = -3$

8. If $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and a, b, c and d .

Solution:

Given,

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -a + 0 & -b + 0 \\ 0 + c & 0 + d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -a & -b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

On comparing the corresponding elements, we have

$$-a = 1 \Rightarrow a = -1$$

$$-b = 0 \Rightarrow b = 0$$

$$c = 0 \text{ And } d = -1$$

Therefore, $a = -1, b = 0, c = 0$ and $d = -1$

9. $\begin{bmatrix} a - b & b - 4 \\ b + 4 & a - 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 14 & 0 \end{bmatrix}$ Find a and b if

Solution:

Given,

$$\begin{bmatrix} a - b & b - 4 \\ b + 4 & a - 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 14 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a - 2b + 0 & 0 + 2a - 2b \\ 2b + 8 + 0 & 0 + 2a - 4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 14 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a - 2b & 2a - 2b \\ 2b + 8 & 2a - 4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 14 & 0 \end{bmatrix}$$

On comparing the corresponding terms, we have

$$2a - 4 = 0$$

$$2a = 4$$

$$a = \frac{4}{2}$$

$$a = 2$$

And, $2a - 2b = -2$

$$2(2) - 2b = -2$$

$$4 - 2b = -2$$

$$2b = 4 + 2$$

$$b = \frac{6}{2}$$

$$b = 3$$

Therefore, $a = 2$ and $b = 3$.

10. If $A = \begin{bmatrix} \sec 60^\circ & \cos 90^\circ \\ -3 \tan 45^\circ & \sin 90^\circ \end{bmatrix}$ and $B = \begin{bmatrix} 0 & \cos 45^\circ \\ -2 & 3 \sin 90^\circ \end{bmatrix}$

Find

(i) $2A - 3B$

(ii) A^2

(iii) BA

Solution:

$$A = \begin{bmatrix} \sec 60^\circ & \cos 90^\circ \\ -3 \tan 45^\circ & \sin 90^\circ \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & \cos 45^\circ \\ -2 & 3 \sin 90^\circ \end{bmatrix}$$

$$A = \begin{bmatrix} \sec 60^\circ & \cos 90^\circ \\ -3 \tan 45^\circ & \sin 90^\circ \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -2 & 3 \end{bmatrix}$$

$$(\because \sec 60^\circ = 2, \cos 90^\circ = 0, \tan 45^\circ = 1, \sin 90^\circ = 1)$$

$$B = \begin{bmatrix} 0 & \cos 45^\circ \\ -2 & 3 \sin 90^\circ \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} \quad (\because \cot 45^\circ = 1)$$

(i) $2A - 3B$

$$= 2 \begin{bmatrix} 2 & 0 \\ -2 & 3 \end{bmatrix} - 3 \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -6 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 - 0 & 0 - 3 \\ -6 + 6 & 2 - 9 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 0 & -7 \end{bmatrix}$$

(ii) $A^2 = A \times A$

$$= \begin{bmatrix} 2 & 0 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 0 & 0 + 0 \\ -6 - 3 & 0 + 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -9 & 1 \end{bmatrix}$$

(iii) $BA = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -2 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 0 - 3 & 0 + 1 \\ -4 - 9 & 0 + 3 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -13 & 3 \end{bmatrix}$$