CHAPTER – 9

ARITHMETIC AND GEOMETRIC PROGRESSION

Exercise 9.1

1. For the following A.P.s, write the first term 'a' and the common difference 'd'.

(i) 3, 1, – 1, – 3, …

Solution:

From the question,

The first term a = 3

Then, difference d = 1 – 3 = -2

– 1 – 1 = – 2

– 3 – (– 1) = – 3 + 1 = – 2

Therefore, common difference d = – 2

(ii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, ...$

Solution:

From the question,

The first term a = $\frac{1}{3}$

Then, difference d = $\frac{5}{3} - \frac{1}{3} = \frac{(5-1)}{3} = \frac{4}{3}$

$\frac{9}{3} - \frac{5}{3} = \frac{(9-5)}{3} = \frac{4}{3}$

$\frac{13}{3} - \frac{9}{3} = \frac{(13-9)}{3} = \frac{4}{3}$

Therefore, common difference d = $\frac{4}{3}$

(iii) $\mathbf{-3.2, -3, -2.8, -2.6, \ldots}$

Solution:

From the question,

The first term a = -3.2

Then, difference d = $-3 - (-3.2) = -3 + 3.2 = 0.2$

$-2.8 - (-3) = -2.8 + 3 = 0.2$

$-2.6 - (-2.8) = -2.6 + 2.8 = 0.2$

Therefore, common difference d = 0.2

2. Write first four terms of the A.P., when the first term a and the common difference d are given as follows:

(i) a = 10, d = 10

Solution:

From the question it is given that,

First term a = 10

Common difference d = 10

Then the first four terms are = 10 + 10 = 20

20 + 10 = 30

30 + 10 = 40

Therefore, first four terms are 10, 20, 30, and 40.

(ii) a = - 2, d = 0

Solution:

From the question it is given that,

First term a = -2

Common difference d = 0

Then the first four terms are = -2 + 0 = -2

$-2 + 0 = -2$

$-2 + 0 = -2$

Therefore, first four terms are -2, $-2, -2$, and -2.

(iii) a = 4, d = -3

Solution:

From the question it is given that,

First term a = 4

Common difference d = -3

Then the first four terms are = $4 + (-3) = 4 - 3 = 1$

$1 + (-3) = 1 - 3 = -2$

$-2 + (-3) = -2 - 3 = -5$

Therefore, first four terms are 4, 1, -2, and -5.

(iv) a = $\frac{1}{2}$, d = $\frac{-1}{6}$

Solution:

From the question it is given that,

First term a = $\frac{1}{2}$

Common difference d = $\frac{-1}{6}$

Then the first four terms are = $\frac{1}{2} + \left(\frac{-1}{6}\right) = \frac{1}{2} - \frac{1}{6} = \frac{(3-1)}{6} = \frac{2}{6} = \frac{1}{3}$

$\frac{1}{3} + \left(\frac{-1}{6}\right) = \frac{1}{3} - \frac{1}{6} = \frac{(2-1)}{6} = \frac{1}{6}$

$\frac{1}{6} + \left(\frac{-1}{6}\right) = \frac{1}{6} - \frac{1}{6} = 0$

Therefore, first four terms are $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$ and 0.

3. Which of the following lists of numbers form an A.P.? If they form an A.P., Find the common difference d and write the next three terms:

(i) 4, 10, 16, 22 ….

Solution:

From the question it is given that,

First term a = 4

Common difference d = $10 - 4 = 6$

$16 - 10 = 6$

$22 - 16 = 6$

Therefore, common difference d = 6

Hence, the numbers are form A.P.

(ii) $\mathbf{-2, 2, -2, 2, \ldots}$

Solution:

From the question it is given that,

First term a = -2

Common difference d = $-2 - 2 = -4$

$-2 - 2 = -4$

$2 - (-2) = 2 + 2 = 4$

Therefore, common difference d is not same in the given numbers.

Hence, the numbers are not form A.P.

(iii) **2, 4, 8, 16 ….**

Solution:

From the question it is given that,

First term a = 2

Common difference d = $4 - 2 = 2$

$8 - 4 = 4$

$16 - 8 = 8$

Therefore, common difference d is not same in the given numbers.

Hence, the numbers are not form A.P.

(iv) $\mathbf{2, \frac{5}{2}, 3, \frac{7}{2}}$ ….

Solution:

From the question it is given that,

First term a = 2

Common difference d = $\frac{5}{2} - 2 = \frac{(5-4)}{2} = \frac{1}{2}$

$3 - \frac{5}{2} = \frac{(6-5)}{2} = \frac{1}{2}$

$\frac{7}{2} - 3 = \frac{(7-6)}{2} = \frac{1}{2}$

Therefore, common difference d = $\frac{1}{2}$

Hence, the numbers are form A.P.

(v) $\mathbf{-10, -6, -2, 2}$ ….

Solution:

From the question it is given that,

First term a = -10

Common difference d = $-6-(-10) = -6+10 = 4$

$-2-(-6) = -2+6 = 4$

$2-(-2) = 2+2 = 4$

Therefore, common difference d = 4

Hence, the numbers are form A.P.

(vi) $\mathbf{1^2, 3^2, 5^2, 7^2}$, ….

Solution:

From the question it is given that,

First term a = $1^2 = 1$

Common difference d = $3^2 - 1^2 = 9 - 1 = 8$

$5^2 - 3^2 = 25 - 9 = 16$

$7^2 - 5^2 = 49 - 25 = 24$

Therefore, common difference d is not same in the given numbers.

Hence, the numbers are not form A.P.

Exercise – 9.2

1. Find the A.P. whose n^{th} term is 7 – 3K. Also find the 20^{th} term.

Solution:

From the question it is given that

n^{th} term is 7 – 3K

So, $T_n = 7 - 3n$

Now, we start giving values, 1, 2, 3 …. In the place of n, we get,

$T_1 = 7 - (3 \times 1) = 7 - 3 = 4$

$T_2 = 7 - (3 \times 2) = 7 - 6 = 1$

$T_3 = 7 - (3 \times 3) = 7 - 9 = -2$

$T_4 = 7 - (3 \times 4) = 7 - 12 = -5$

$T_{20} = 7 - (3 \times 20) = 7 - 60 = -53$

Therefore, A.P. is $4, 1, -2, -5$ ...

So, 20^{th} term is -53.

2. Find the indicated terms in each of following A.P.s:

(i) 1, 6, 11, 16 …, a_{20}

Solution:

From the question,

The first term a = 1

Then, difference d = $6 - 1 = 5$

$11 - 6 = 5$

$16 - 11 = 5$

Therefore, common difference d = 5

From the formula, $a_n = a + (n-1)d$

So, $a_{20} = a + (20-1)d$

$= 1 + (20-1)5$

$= 1 + (19)5$

$= 1 + 95$

$= 96$

Therefore, $a_{20} = 96$

(ii) $\boldsymbol{-4, -7, -10, -13, \dots, a_{25}, a_n}$

Solution:

From the question,

The first term a = -4

Then, difference d = $-7 - (-4) = -7 + 4 = -3$

$-10 - (-7) = -10 + 7 = -3$

$-13 - (-10) = -13 + 10 = -3$

Therefore, common difference d = - 3

From the formula, $a_n = a + (n-1)d$

So, $a_{25} = a + (25-1)d$

$= -4 + (25-1)(-3)$

$= -4 + (24) - 3$

$= -4 - 72$

$= -76$

Therefore, $a_{25} = -76$

Now, $a_n = a + (n-1)d$

$a_n = -4 + (n-1) - 3$

$= -4 - 3n + 3$

$= -1 - 3n$

3. Find the n^th^ term and the 12^th^ term of the list of numbers: $\mathbf{5, 2, -1, -4, \ldots}$

Solution:

From the question,

The first term a = 5

Then, difference d = 2 – 5 = – 3

$-1 - 3 = -3$

$-4 - (-1) = -4 + 1 = -3$

Therefore, common difference d = - 3

From the formula, $a_n = a + (n-1)d$

$T_n = a + (n-1)d$

$= 5 + (n-1) - 3$

$= 5 - 3n + 3$

$= 8 - 3n$

So, $T_{12} = a + (12-1)d$

$= 5 + (12-1)(-3)$

$= 5 + (11) - 3$

$= 5 - 33$

$= -28$

4.

(i) If the common difference of an A.P. is – 3 and the 18th term is – 5, then find its first term.

Solution:

From the question,

The 18th term = – 5

Then, difference d = – 3

$T_n = a + (n - 1)d$

So, $T_{18} = a + (18 - 1)d$

$-5 = a + (18 - 1) - 3$

$-5 = a + (17)(-3)$

$-5 = a - 51$

$a = 51 - 5$

$a = 46$

Therefore, first term $a = 46$

(ii) If the first term of an A.P. is – 18 and its 10th term is zero, then find its common difference.

Solution:

From the question,

The 10th term = 0

Then, first term a = – 18

$T_n = a + (n-1)d$

So, $T_{10} = a + (10-1)d$

$0 = -18 + (10-1)d$

$0 = -18 + 9d$

$9d = 18$

$d = \frac{18}{9}$

$d = 2$

Therefore, common difference d = 2

5. Which term of the A.P.

(i) $3, 8, 13, 18, \ldots$ is 78?

Solution:

Let us assume 78 as nth term.

From the question,

The first term a = 3

Then, difference d = $8 - 3 = 5$

$13 - 8 = 5$

$18 - 13 = 5$

Therefore, common difference d = 5

$T_n = a + (n-1)d$

So, $78 = a + (n-1)d$

$78 = 3 + (n - 1)5$

$78 = 3 + 5n - 5$

$78 = -2 + 5\,n$

$5n = 78 + 2$

$5n = 80$

$n = \frac{80}{5}$

$n = 16$

Therefore, 78 is 16^{th} term.

(ii) $\mathbf{18, 15\frac{1}{2}, 13, \ldots}$ **is** $\mathbf{- 47}$?

Solution:

Convert mixed fraction into improper fraction = $15\frac{1}{2} = \frac{31}{2}$

Let us assume – 47 as n^{th} term.

From the question,

The first term $a = 18$

Then, difference d = $\frac{31}{2} - 18 = \frac{(31-36)}{2} = \frac{-5}{2}$

$13 - \frac{31}{2} = \frac{(26-31)}{2} = \frac{-5}{2}$

Therefore, common difference d = $\frac{-5}{2}$

$T_n = a + (n - 1)d$

So, $-47 = a + (n - 1)d$

$-47 = 18 + (n - 1)\left(\frac{-5}{2}\right)$

$-47 = 18 - \frac{5}{2}n + \frac{5}{2}$

$-47 - 18 = \frac{-5}{2}n + \frac{5}{2}$

$-65 = \frac{-5}{2}n + \frac{5}{2}$

$-65 - \frac{5}{2} = \frac{-5}{2}n$

$\frac{(-130-5)}{2} = \frac{-5}{2}n$

$\frac{-135}{2} = \frac{-5}{2}n$

$n = \left(\frac{-135}{2}\right) \times \left(\frac{-2}{5}\right)$

$n = \frac{-135}{-5}$

$n = 27$

Therefore, – 47 is 27^{th} term.

6.

(i) Check whether – 150 is a term of the A.P. 11, 8, 5, 2 ….

Solution:

From the question it is given that,

The first term a = 11

Then, difference d = 8 – 11 = - 3

5 – 8 = - 3

2 – 5 = - 3

Then, common difference d = -3

Let us assume – 150 as n^{th} term,

$T_n = a + (n-1)d$

So, $-150 = 11 + (n-1)(-3)$

$-150 = 11 - 3n + 3$

$-150 = 14 - 3n$

$3n = 150 + 14$

$3n = 164$

$n = \frac{164}{3}$

$n = 54\frac{2}{3}$

Therefore, – 150 is not a term of the A.P. 11, 8, 5, 2…

(ii) Find whether 55 is a term of the A.P. 7, 10, 13 … or not. If yes, find which term is it.

Solution:

From the question it is given that,

The first term a = 7

Then, difference d = 10 – 7 = 3

13 – 10 = 3

Then, common difference d = 3

Let us assume 55 as n^{th} term,

$T_n = a + (n-1)d$

So, $55 = 7 + (n-1)3$

$55 = 7 + 3n - 3$

$55 = 4 + 3n$

$3n = 55 - 4$

$3n = 51$

$n = \frac{51}{3}$

$n = 17$

Therefore, 55 is 17th term of the A.P. 7, 10, 13….

7.

(i) Find the 20th term from the last term of the A.P. 3, 8, 13 …, 253.

Solution:

Let us assume 255 as nth term,

From the question,

The first term a = 3

Then, difference d = 8 – 3 = 5

13 – 8 = 5

Therefore, common difference d = 5

$T_n = a + (n - 1)d$

So, $253 = a + (n - 1)d$

$253 = 3 + (n - 1)5$

$253 = 3 + 5n - 5$

$253 = -2 + 5n$

$5n = 253 + 2$

$5n = 255$

$n = \frac{255}{5}$

$n = 51$

Therefore, 255 is 51st term.

(ii) Find the 12th from the end of the A.P. – 2, – 4, – 6 …, – 100.

Solution:

Let us assume – 100 as nth term,

From the question,

The first term a = – 2

Then, difference d = $-4-(-2) = -4+2 = -2$

$-6-(-4) = -6+4 = -2$

Therefore, common difference d = - 2

$T_n = a + (n-1)d$

So, $-100 = a + (n-1)d$

$-100 = -2 + (n-1)(-2)$

$-100 = -2 - 2n + 2$

$-100 = -2n$

$n = \frac{-100}{-2}$

$n = 50$

Therefore, - 100 is 50th term.

Now, assume 'P' be the 12th term from the last.

Then, $P = L - (n-1)d$

$= -100 - (12 - 1)(-2)$

$= -100 - (11)(-2)$

$= -100 + 22$

$P = -78$

Therefore, - 78 is the 12^{th} term from the last of the A.P. $-2, -4, -6, \ldots$

8. Find the sum of the two middle most terms of the A.P.

$\boldsymbol{\frac{-4}{3}, -1, \frac{-2}{3}, \ldots, 4\frac{1}{3}}$

Solution:

From the question,

Last term (n^{th}) $= 4\frac{1}{3} = \frac{13}{3}$

First term $a = \frac{-4}{3}$

Then, difference $d = -1 - \left(\frac{-4}{3}\right) = -1 + \frac{4}{3} = \frac{(-3+4)}{3} = \frac{1}{3}$

$= \frac{-2}{3} - (-1) = \frac{-2}{3} + 1 = \frac{(-2+3)}{3} = \frac{1}{3}$

Therefore, common difference $d = \frac{1}{3}$

We know that,

$T_n = a + (n - 1)d$

So, $\frac{13}{3} = \frac{-4}{3} + (n - 1)\left(\frac{1}{3}\right)$

$\frac{13}{3} + \frac{4}{3} = \frac{1}{3}n - \frac{1}{3}$

$\frac{13}{3} + \frac{4}{3} + \frac{1}{3} = \frac{1}{3}n$

$\frac{(13+4+1)}{3} = \frac{1}{3}n$

$\frac{18}{3} = \frac{1}{3}n$

$6 = \frac{1}{3}n$

$n = 6 \times 3$

$n = 18$

So, middle term is $\frac{18}{2}$ and $\left(\frac{18}{2}\right) + 1 = 9^{th}$ and 10^{th} term.

Then, $a_9 + a_{10} = a + 8d + a + 9d$

$= 2a + 17d$

Now substitute the value of 'a' and 'd'.

$= 2\left(\frac{-4}{3}\right) + 17\left(\frac{1}{3}\right)$

$= \frac{-8}{3} + \frac{17}{3}$

$= \frac{(-8+17)}{3}$

$= \frac{9}{3}$

$= 3$

Therefore, the sum of the two middle most terms of the A.P. is 3.

9. Which term of the A.P. 53, 48, 43 …. is the first negative term?

Solution:

From the question,

The first term a = 53

Then, difference d = 48 – 53 = – 5

= 43 – 48 = -5

Therefore, common difference d = -5

$T_n = a + (n - 1)d$

= $53 + (n - 1)(-5)$

= $53 - 5n + 5$

= $58 - 5n$

$5n = 58$

$n = \frac{58}{5}$

$n = 11.6 \approx 12$

Therefore, 12th term is the first negative term of the A.P. 53, 48, 43 …

10. Determine the A.P. whose third term is 16 and the 7th term exceeds the 5th term by 12.

Solution:

From the question it is given that,

$T_3 = 16$

The 7th term exceeds the 5th term by 12 = $T_7 - T_5 = 12$

We know that, $T_n = a + (n - 1)d$

So, $T_3 = a + 2d = 16$ … [equation (ii)]

$T_7 - T_5 = (a + 6d) - (a + 4d) = 12$ … [Equation (ii)]

$12 = a + 6d - a - 4d$

$12 = 2d$

$d = \frac{12}{2}$

$d = 6$

Now, substitute value of d in equation (i) we get,

Then, $T_3 = a + 2d$

$16 = a + 2(6)$

$a = 16 - 12$

$a = 4$

Therefore, A.P. is $4 + 6 = 10, 10 + 6 = 16, 16 + 6 = 22$

Hence, the four term of A.P. is 4, 10, 16, 22….

11. Find the 20th term of the A.P. whose 7th term is 24 less than the 11th term, first term being 12.

Solution:

From the question it is given that,

First term a = 12

7th term is 24 less than the 11th term = $T_{11} - T_7 = 24$

$T_{11} - T_7 = (a + 10d) - (a + 6d) = 24$

$24 = a + 10d - a - 6d$

$24 = 4d$

$d = \frac{24}{4}$

$d = 6$

Now, $T_{20} = a + 19d$

Substitute the values of a and d,

$T_{20} = 12 + 19(6)$

$T_{20} = 12 + 114$

$T_{20} = 126$

12. Find the 31st term of an A.P. whose 11th term is 38 and 6th term is 73.

Solution:

From the question it is given that,

$T_{11} = 38$

$T_6 = 73$

Let us assume 'a' be the first term and 'd' be the common difference.

So, $T_{11} = a + 10d = 38$ (i)

$T_6 = a + 5d = 73$ (ii)

Subtracting both equation (i) and equation (ii)

$(a + 10d) - (a + 5d) = 73 - 38$

$a + 10d - a - 5d = 35$

$5d = 35$

$d = \frac{35}{5}$

$d = 7$

Now, substitute the value of d in equation (i) to find out a, we get

$a + 10d = 38$

$a + 10(7) = 38$

$a + 70 = 38$

$a = 38 - 70$

$a = -32$

Therefore, $T_{31} = a + 30d$

$= -32 + 30(7)$

$= -32 + 210$

$= 178$

13. If the seventh term of an A.P. is $\frac{1}{9}$ and its ninth term is $\frac{1}{7}$, find its 63rd term.

Solution:

From the question it is given that,

$T_9 = \frac{1}{7}$

$T_7 = \frac{1}{9}$

Let us assume 'a' be the first term and 'd' be the common difference.

So, $T_9 = a + 8d = \frac{1}{7}$ (i)

$T_7 = a + 6d = \frac{1}{9}$ (ii)

Subtracting both equation (i) and equation (ii)

$(a + 6d) - (a + 8d) = \frac{1}{9} - \frac{1}{7}$

$a + 6d - a - 8d = \frac{(7-9)}{63}$

$-2d = \frac{-2}{63}$

$d = \left(\frac{-2}{63}\right) \times \left(\frac{-1}{2}\right)$

$d = \frac{1}{63}$

Now, substitute the value of d in equation (ii) to find out a, we get

$a + 6\left(\frac{1}{63}\right) = \frac{1}{9}$

$a = \frac{1}{9} - \frac{6}{63}$

$a = \frac{(7-6)}{63}$

$a = \frac{1}{63}$

Therefore, $T_{63} = a + 62d$

$= \frac{1}{63} + 62\left(\frac{1}{63}\right)$

$= \frac{1}{63} + \frac{62}{63}$

$= \frac{1+62}{63}$

$= \frac{63}{63}$

$= 1$

14.

(i) The 15th term of an A.P. is 3 more than twice its 7th term. If the 10th term of the A.P. is 41, find its nth term.

Solution:

From the question it is given that,

$T_{10} = 41$

$T_{10} = a + 9d = 41$ (i)

$T_{15} = a + 14d = 2T_7 + 3$

$= a + 14d = 2(2 + 6d) + 3$

$= a + 14d = 2a + 12d + 3$

$-3 = 2a - a + 12d - 14d$

$a - 2d = -3$ (ii)

Now, subtracting equation (ii) from (i), we get,

$(a + 9d) - (a - 2d) = 41 - (-3)$

$a + 9d - a + 2d = 41 + 3$

$11d = 44$

$d = \frac{44}{11}$

$d = 4$

Then, substitute the value of d is equation (i) to find a.

$a + 9(4) = 41$

$a + 36 = 41$

$a = 41 - 36$

$a = 5$

Therefore, n[th] term = $T_n = a + (n - 1)d$

$= 5 + (n - 1)4$

$= 5 + 4n - 4$

$= 1 + 4n$

(ii) The sum of 5^{th} and 7^{th} terms of an A.P. is 52 and the 10^{th} term is 46. Find the A.P.

Solution:

From the question it is given that,

$a_5 + a_7 = 52$

$(a + 4d) + (a + 6d) = 52$

$a + 4d + a + 6d = 52$

$2a + 10d = 52$

Divide both the side by 2 we get,

$a + 5d = 26$ (i)

Given, $a_{10} = a + 9d = 46$

$a + 9d = 46$ (ii)

Now subtracting equation (i) from equation (ii),

$(a + 9d) - (a + 5d) = 46 - 26$

$a + 9d - a - 5d = 20$

$4d = 20$

$d = \frac{20}{4}$

$d = 5$

Substitute the value of d in equation (i) to find out a,

$a + 5d = 26$

$a + 5(5) = 26$

$a + 25 = 26$

$a = 26 - 25$

$a = 1$

Then, $a_2 = a + d$

$= 1 + 5 = 6$

$a_3 = a_2 + d$

= 6 + 5

= 11

$a_4 = a_3 + d$

= 11 + 5

= 16

Therefore, 1, 6, 11, 16 … are A.P.

15. If 8th term of an A.P. is zero, prove that its 38th term is triple of its 18th term.

Solution:

From the question it is given that,

$T_8 = 0$

We have to prove that, 38th term is triple of its 18th term = $T_{38} = 3T_{18}$

$T_8 = a + 7d = 0$

$T_8 = a = -7d$

$T_{38} = a + 37d$

$= -7d + 37d$

= $30d$

Take, $T_{18} = a + 17d$

Substitute the value of a and d,

$T_{18} = -7d + 17d$

$T_{18} = 10d$

By comparing results of T_{38} and T_{18}, 38th term is triple of its 18th term.

16. Which term of the A.P. 3, 10, 17 …. Will be 84 more than its 13th term?

Solution:

From the question it is given that,

First term a = 3

Common difference d = $10 - 3 = 7$

Then, $T_{13} = a + 12d$

= $3 + 12(7)$

= $3 + 84$

= 87

Let us assume that, nth term is 84 more than its 13th term.

So, $T_n = 84 + 87$

= 171

We know that, $T_n = a + (n-1)d = 171$

$3 + (n-1)7 = 171$

$3 + 7n - 7 = 171$

$7n - 4 = 171$

$7n = 171 + 4$

$7n = 175$

$n = \frac{175}{7}$

$n = 25$

17.

(i) How many two digit numbers are divisible by 3?

Solution:

The two digits numbers divisible by 3 are 12, 15, 18, 21, 24 …, 99.

The above numbers are A.P.

So, first number $a = 12$

Common difference d = $15 - 12 = 3$

Then, last number is 99

We know that, T_n (last number) = $a + (n-1)d$

$99 = 12 + (n-1)3$

$99 = 12 + 3n - 3$

$99 = 9 + 3n$

$99 - 9 = 3n$

$3n = 90$

$n = \frac{90}{3}$

$n = 30$

Therefore, 30 two digits number are divisible by 3.

(ii) Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

Solution:

The natural numbers which are divisible by both 2 and 5 are 110, 120, 130, 140 …., 999

The above numbers are A.P.

So, first number a = 110

Common difference d = 120 – 110 = 10

Then, last number is 999

We know that, T_n (last number) = $a + (n - 1)d$

$999 = 110 + (n - 1)10$

$999 = 110 + 10n - 10$

$999 = 100 + 10n$

$10n = 888$

$n = \frac{888}{10}$

$n = 88$

The number of natural numbers which are divisible by both 2 and 5 are 88.

18. If the number n – 2, 4n – 1 and 5n + 2 are in A.P., find the value of n.

Solution:

From the question it is given that, $n - 2, 4n - 1$ and $5n + 2$ are in A.P/

Multiplying by 2 to 4n – 1 then it becomes = 8n – 2

So, 8n – 2 = n – 2 + 5n + 2

$8n - 2 = 6n$

$8n - 6n = 2$

$2n = 2$

$n = \frac{2}{2}$

$n = 1$

19. The sum of three numbers in A.P. is 3 and their product is – 35. Find the number.

Solution:

From the question it is given that,

The sum of three numbers in A.P. = 3

Given, their product = -35

Let us assume the 3 numbers which are in A.P. are, $a - d, a, a + d$

Now adding 3 numbers = $a - d + a + a + d = 3$

$3a = 3$

$a = \frac{3}{3}$

$a = 1$

From the question, product of 3 number is – 35

So, $(a - d) \times (a) \times (a + d) = -35$

$(1 - d) \times (1) \times (1 + d) = -35$

$1^2 - d^2 = -35$

$d^2 = 35 + 1$

$d^2 = 36$

$d = \sqrt{36}$

$d = \pm 6$

Therefore, the numbers are $(a - d) = 1 - 6 = -5$

$a = 1$

$(a + d) = 1 + 6 = 7$

If $d = -6$

The numbers are $(a - d) = 1 - (-6) = 1 + 6 = 7$

$a = 1$

$(a + d) = 1 + (-6) = 1 - 6 = -5$

Therefore, the number $-5, 1, 7, \ldots$ and $7, 1, -5, \ldots.$ are in A.P.

20. The sum of three numbers in A.P. is 30 and the ratio of first number to the third number is 3: 7. Find the numbers.

Solution:

From the question it is given that, sum of three numbers in A.P. = 30

The ratio of first number to the third number is 3: 7

Let us assume the 3 numbers which are in A.P. are, $a - d, a, a + d$

Now adding 3 numbers = $a - d + a + a + d = 30$

$3a = 30$

$a = \frac{30}{3}$

$a = 10$

Given ratio 3: 7 = $a - d : a + d$

$\frac{3}{7} = \frac{(a-d)}{(a+d)}$

$(a-d)7 = 3(a+d)$

$7a - 7d = 3a + 3d$

$7a - 3a = 7d + 3d$

$4a = 10d$

$4(10) = 10d$

$40 = 10d$

$d = \frac{40}{10}$

$d = 4$

Therefore, the numbers are $a - d = 10 - 4 = 6$

$a = 10$

$a + d = 10 + d = 14$

So, $6, 10, 14$ ... are in A.P.

21. The sum of the first three terms of an A.P. is 33. If the product of the first and the third terms exceeds the second term by 29, find the A.P.

Solution:

From the question it is given that, sum of the first three terms of an A.P. is 33.

Let us assume the 3 numbers which are in A.P. are, $a - d, a, a + d$

$3a = 33$

$a = \frac{33}{3}$

$a = 11$

Given, the product of the first and the third terms exceeds the second term by 29.

$(a - d)(a + d) = a + 29$

$a^2 - d^2 = 11 + 29$

$11^2 - d^2 = 40$

$121 - 40 = d^2$

$d^2 = 81$

$d = \sqrt{81}$

$d = \pm 9$

If d = 9

Therefore, the numbers are $(a - d) = 11 - 9 = 2$

$a = 11$

$(a + d) = 11 + 9 = 20$

If d = - 9

The number are $(a - d) = 1 - (-9) = 11 + 9 = 20$

$a = 11$

$(a + d) = 11 + (-9) = 11 - 9 = 2$

Therefore, the numbers 2, 11, 20 … and 20, 11, 2 …. are in A.P.

Exercise – 9.3

1. Find the sum of the following A.P.s:

(i) 2, 7, 12 … to 10 terms

Solution:

From the question,

First terms $a = 2$

Then, d = 7 – 2 = 5

12 – 7 = 5

So, common difference d = 5

n = 10

$S_{10} = \frac{n}{2}(2a + (n-1)d)$

$= \frac{10}{2}\left((2 \times 2) + (10-1)5\right)$

$= 5(4 + 45)$

$= 5(49)$

$= 245$

(ii) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$ … to 11 terms

Solution:

From the question,

First terms $a = \frac{1}{15}$

Then, $d = \frac{1}{12} - \frac{1}{15}$

$= \frac{(5-4)}{60}$

$= \frac{1}{60}$

So, common difference $d = \frac{1}{60}$

$n = 11$

$S_{11} = \frac{11}{2}(2a + (n-1)d)$

$= \frac{11}{2}\left(\left(2 \times \left(\frac{1}{5}\right)\right) + (11-1)\left(\frac{1}{60}\right)\right)$

$= \frac{11}{2}\left(\left(\frac{2}{15}\right) + \left(\frac{10}{60}\right)\right)$

$= \frac{11}{2}\left(\frac{2}{15} + \frac{1}{6}\right)$

$= \frac{11}{2}\frac{(4+5)}{30}$

$= \frac{11}{2}\left(\frac{9}{30}\right)$

$= \frac{11}{2}\left(\frac{3}{10}\right)$

$= \frac{33}{20}$

2. Find the sums given below:

(i) $\mathbf{34 + 32 + 30 + \cdots + 10}$

Solution:

From the question,

First term $a = 34$,

Difference d = $32 - 34 = -2$

So, common difference d = -2

Last term $T_n = 10$

We know that, $T_n = a + (n - 1)d$

$10 = 34 + (n - 1)(-2)$

$-24 = -2(n - 1)$

$-24 = -2n + 2$

$2n = 24 + 2$

$2n = 26$

$n = \frac{26}{2}$

$n = 13$

$S_n = \frac{n}{2}(a + 1)$

$= \frac{13}{2}(34 + 10)$

$= \frac{13}{2}(44)$

$= 13(22)$

= 286

(ii) $\boldsymbol{-5 + (-8) + (-11) + \cdots + (-230)}$

Solution:

From the question,

First term = $a = -5$

Difference d = $-8 - (-5) = -8 + 5 = -3$

So, common difference d = -3

Last term $T_n = -230$

We know that, $T_n = a + (n-1)d$

$-230 = -5 + (n-1)(-3)$

$-230 = -5 - 3n + 3$

$-230 = -2 - 3n$

$3n = 230 - 2$

$3n = 228$

$n = \frac{228}{3}$

$n = 76$

Therefore, $S_n = \frac{n}{2}(a + 1)$

$= \frac{76}{2}\left(-5 + (-230)\right)$

$= 38(-5 - 230)$

$= 38\ (235)$

$= -8930$

3. In an A.P. (with usual notations):

(i) Given a = 5, d = 3, $a_n = 50$ find n and S_n

Solution:

From the question,

First term = $a = 5$

Then common difference d = 3

$a_n = 50,$

We know that, $a_n = a + (n-1)d$

$50 = 5 + (n-1)3$

$50 = 5 + 3n - 3$

$50 = 2 + 3n$

$3n = 50 - 2$

$3n = 48$

$n = \frac{48}{3}$

$n = 16$

So, $S_n = \left(\frac{n}{2}\right)(2a + (n-1)d)$

$= \left(\frac{16}{2}\right)\left((2 \times 5) + (16-1) \times 3\right)$

$= 8(10 + 45)$

$= 8(55)$

$= 440$

(ii) Given a = 7, $a_{13} = 35$ find d and S_{13}

Solution:

From the question,

First term = $a = 7$

$a_{13} = 35,$

We know that, $a_n = a + (n-1)d$

$35 = 7 + (13-1)d$

$35 = 7 + 13d - d$

$35 = 7 + 12d$

$12d = 35 - 7$

$12d = 28$

$d = \frac{28}{12}$ [Divide by 4]

$d = \frac{7}{3}$

So, $S_{13} = \left(\frac{n}{2}\right)(2a + (n-1)d)$

$= \left(\frac{13}{2}\right)\left((2 \times 7) + (13-1) \times \left(\frac{7}{3}\right)\right)$

$= \left(\frac{13}{2}\right)\left(14 + \left(12 \times \frac{7}{3}\right)\right)$

$= \left(\frac{13}{2}\right)(14 + 28)$

$= \left(\frac{13}{2}\right)(42)$

$= 13 \times 21$

$= 273$

(iii) Given $d = 5, S_9 = 75$, find a and a_9.

Solution:

From the question it is given that,

Common difference d = 5

$S_9 = 75$

We know that, $a_n = a + (n-1)d$

$a_9 = a + (9-1)5$

$a_9 = a + 45 - 5$

$a_9 = a + 40$ (i)

Then, $S_9 = \left(\frac{n}{2}\right)(2a + (n-1)d)$

$75 = \left(\frac{9}{2}\right)(2a + (9-1)5)$

$75 = \left(\frac{9}{2}\right)(2a + (8)5)$

$\frac{(75 \times 2)}{9} = 2a + 40$

$\frac{150}{9} = 2a + 40$

$2a = \frac{150}{9} - 40$

$2a = \frac{50}{3} - 40$

$2a = \frac{(50-120)}{13}$

$2a = \frac{-70}{3}$

$a = \frac{-70}{(3 \times 2)}$

$a = \frac{-35}{3}$

Now, substitute the value of a in equation (i),

$a_9 = a + 40$

$= \frac{-35}{3} + 40$

$= \frac{(-35+120)}{3}$

$= \frac{85}{3}$

(iv) Given $\boldsymbol{a = 8, a_n = 62, S_n = 210}$, find n and d

Solution:

From the question it is give that,

First term $a = 8$,

$a_n = 62$ And $S_n = 210$

We know that, $a_n = a + (n-1)d$

$62 = 8 + (n-1)d$

$(n-1)d = 62 - 8$

$(n-1)d = 54$ (i)

Then, $S_n = \left(\frac{n}{2}\right)(2a + (n-1)d)$

$210 = \left(\frac{n}{2}\right)\left((2 \times 8) + 54\right)$ [From equation (i) $(n-1)d = 54$]

$210 = \left(\frac{n}{2}\right)(16 + 54)$

$420 = n(70)$

$n = \frac{420}{70}$

$n = 6$

Now, substitute the value of n is equation (i),

$(n-1)d = 54$

$(6-1)d = 54$

$5d = 54$

$d = \frac{54}{5}$

Therefore, $d = \frac{54}{5}$ and $n = 6$

(v) Given $a = 3, n = 8, S = 192$, find d.

Solution:

From the question it is give that,

First term $a = 3$,

$n = 8$

$S = 192$

We know that, $S_n = \left(\frac{n}{2}\right)(2a + (n-1)d)$

$$192 = \left(\frac{8}{2}\right)\left((2 \times 3) + (8-1)d\right)$$

$$192 = 4(6 + 7d)$$

$$48 = 6 + 7d$$

$$48 - 6 = 7d$$

$$42 = 7d$$

$$d = \frac{42}{7}$$

$$d = 6$$

Therefore, common difference d is 6.

4.

(i) The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Solution:

From the question it is give that,

First term $a = 5$

Last term = 45

Then, sum = 400

We know that, last term = $a + (n-1)d$

$45 = 5 + (n-1)d$

$(n-1)d = 45 - 5$

$(n-1)d = 40$ (i)

So, $S_n = \left(\frac{n}{2}\right)(2a + (n-1)d)$

$400 = \left(\frac{n}{2}\right)\left((2 \times 5) + 40\right)$ [From equation (i) $(n-1)d = 40$]

$800 = n(10 + 40)$

$800 = 50n$

$n = \frac{800}{50}$

$n = 16$

(ii) The sum of first 15 terms of an A.P. is 750 and its first term is 15. Find its 20th term.

Solution:

From the question it is give that,

First term $a = 15$

Therefore, sum of first n terms of an A.P. is given by,

$S_n = \left(\frac{n}{2}\right)(2a + (n-1)d)$

$S_{15} = \left(\frac{15}{2}\right)(2a + (15-1)d)$

$750 = \left(\frac{15}{2}\right)(2a + 14d)$

$\frac{(750\times2)}{15} = 2a + 14d$

$100 = 2a + 14d$

Dividing both the side by 2 we get,

$50 = a + 7d$

Now, substitute the value a,

$50 = 15 + 7d$

$7d = 50 - 15$

$7d = 35$

$d = \frac{35}{7}$

$d = 5$

So, 20th term $a_{20} = a + 19d$

$= 15 + 19(5)$

$= 15 + 95$

$= 110$

5. The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution:

From the question it is give that,

First term $a = 17$

Last term (l) = 350

Common difference d = 9

We know that, l = $T_n = a + (n-1)d$

$350 = 17 + (n-1) \times 9$

$350 - 17 = 9n - 9$

$333 + 9 = 9n$

$342 = 9n$

$n = \frac{342}{9}$

$n = 38$

So, $S_n = \left(\frac{n}{2}\right)(2a + (n-1)d)$

= $\left(\frac{38}{2}\right)\left((2 \times 17) + (38-1)d\right)$

= $19\left(34 + (37 \times 9)\right)$

= $19(34 + 333)$

= 19×367

= 6973

Therefore, $n = 38$ and $S_n = 6973$

6. Solve for x : $1 + 4 + 7 + 10 + \cdots + x = 287$.

Solution:

From the question it is give that,

First term $a = 1$

Difference d = $4 - 1 = 3$

$n = x$

$x = a = (n - 1)d$

$x - 1 = (n - 1)d$

$S_n = \left(\frac{n}{2}\right)(2a + (n - 1)d)$

$287 = \left(\frac{n}{2}\right)\left((2 \times 1) + (n - 1)3\right)$

$= n(2 + 3n - 3)$

$574 = n(2 + 3n - 3)$

$574 = 2n + 3n^2 - 3n$

$574 = -n + 3n^2$

$3n^2 - n - 574 = 0$

$3n^2 - 42n + 41 - 574 = 0$

$3n(n - 14) + 41(n - 14) = 0$

$(n - 14)(3n + 41) = 0$

If $n - 14 = 0$

$n = 14$

Or $3n + 41 = 0$

$3n = -41$

$n = \frac{-41}{3}$

We have to take positive number so n = 14

Then, $= a + (n - 1)d$

$= 1 + (14 - 1)3$

$= 1 + (13)3$

= $1 + 39$

= 40

Therefore, $x = 40$

7.

(i) How many terms of the A.P. 25, 22, 19 …. are needed to give the sum 116? Also find the last term.

Solution:

From the question it is give that,

First term $a = 25$

Common difference d = $22 - 25 = -3$

Sum = 116

$S_n = \left(\frac{n}{2}\right)(2a + (n-1)d)$

$116 = \left(\frac{n}{2}\right)(2a + (n-1)d)$

By cross multiplication,

$232 = n\big((2 \times 25) + (n-1)(-3)\big)$

$232 = n(50 - 3n + 3)$

$232 = n(53 - 3n)$

$232 = 53n - 3n^2$

$3n^2 - 53n + 232 = 0$

$3n^2 - 24n - 29n + 232 = 0$

$3n(n-8) - 29(n-8) = 0$

$(n-8)(3n-29) = 0$

If $n - 8 = 0$

$n = 8$

Or $3n - 29 = 0$

$3n = 29$

$n = \frac{29}{3}$

Not possible to take fraction,

So, $n = 8$

Then, $T = a + (n - 1)d$

$= 25 + (8 - 1)(-3)$

$= 25 + 7(-3)$

$= 25 - 21$

$= 4$

(ii) **How many terms of the A.P. 24, 21, 18 …. must be taken so that the sum is 78? Explain the double answer.**

Solution:

From the question it is give that,

First term $a = 24$

Common difference d = $21 - 24 = -3$

Sum = 78

$S_n = \left(\frac{n}{2}\right)(2a + (n - 1)d)$

$78 = \left(\frac{n}{2}\right)(2a + (n - 1)d)$

By cross multiplication,

$156 = n\big((2 \times 24) + (n-1)(-3)\big)$

$156 = n(48 - 3n + 3)$

$156 = n(51 - 3n)$

$156 = 51n - 3n^2$

$3n^2 - 51n + 156 = 0$

$3n^2 - 12n - 39n + 156 = 0$

$3n(n-1) - 39(n-1) = 0$

$(n-4)(3n-39) = 0$

If $n - 4 = 0$

$n = 4$

Or $3n - 39 = 0$

$3n = 39$

$n = \frac{39}{3}$

$n = 13$

Now we have to consider both values

So, $n = 4$

Then, $T = a + (n-1)d$

$= 24 + (4-1)(-3)$

$= 24 + 3(-3)$

$= 24 - 9$

$= 15$

$n = 13$

Then, $T = a + (n - 1)d$

$= 24 + (13 - 1)(-3)$

$= 24 + 12(-3)$

$= 24 - 36$

$= -12$

So, $\left(12 + 9 + 6 + 3 + 0 + (-3) + (-6) + (-9) + (-12)\right) = 0$

8. Find the sum of first 22 terms, of an A.P. in which $d = 7$ and a_{22} is 149.

Solution:

From the question it is given that,

Common difference d = 7

$a_{22} = 149$

$n = 22$

We know that,

$a_{22} = (n - 1)d$

$149 = a + (22 - 1)7$

$149 = a + (22)7$

$149 = a + 147$

$a = 149 - 147$

$a = 2$

So, $S_n = \left(\frac{n}{2}\right)(2a + (n - 1)d)$

$= \left(\frac{22}{2}\right)\left((2 \times 2) + (22 - 1)7\right)$

$= 11(4 + (21)7)$

$= 11(4 + 147)$

$= 11(151)$

$= 1661$

9. In an A.P. the fourth and sixth terms are 8 and 14 respectively. Find the:

(i) First term

(ii) Common difference

(iii) Sum of the first 20 terms.

Solution:

From the question it is given that,

$T_4 = 8$ and $T_6 = 14$

$\Rightarrow a + 3d = 8$ (i)

$\Rightarrow a + 5d = 14$ (ii)

(i) Substituting the value of d in (i), we have

$a + 3(3) = 8$

$a = 8 - 9$

$a = -1$

Hence, the first term = -1

(ii) Subtracting (i) from (ii), we get

$(a + 5d) - (a + 3d) = 14 - 8$

$5d - 3d = 6$

$2d = 6$

$d = \frac{6}{2}$

$d = 3$

Hence, common difference d = 3

(iii) The sum of first 20 terms

$n = 20$

$S_{20} = \frac{n}{2} \times [2a + (n - 1)d]$

$= \frac{20}{2} \times [2(-1) + (20 - 1)(3)]$

$= 10 \times [-2 + (19)(3)]$

$= 10 \times [-2 + 57]$

$= 10 \times 55$

$= 550$

10.

(i) Find the sum of first 51 terms of the A.P. whose second and third terms are 14 and 18 respectively.

Solution:

From the question it is given that,

$T_2 = 14, T_3 = 18$

So, common difference d = $T_3 - T_2$

$= 18 - 14$

$= 4$

Where, $a = T_1 = 14 - 4 = 10$

$n = 51$

We know that,

$S_{51} = \left(\frac{n}{2}\right)(2a + (n-1)d)$

$= \left(\frac{51}{2}\right)\left((2 \times 10) + (51-1)4\right)$

$= \left(\frac{51}{2}\right)(20 + (50 \times 4))$

$= \left(\frac{51}{2}\right)(20 + 200)$

$= \left(\frac{51}{2}\right) \times 220$

$= 5610$

(ii) The 4th term of A.P. is 22 and 15th term is 66. Find the first term and the common difference. Hence, find the sum of first 8 terms of the A.P.

Solution:

From the question it is given that,

$T_4 = 22, T_{15} = 66$

$\Rightarrow a + 3d = 22$ (i)

$\Rightarrow a + 14d = 66$ (ii)

Subtracting (i) from (ii), we get

$(a + 14d) - (a + 3d) = 66 - 22$

$14d - 3d = 44$

$11d = 44$

$d = \frac{44}{11}$

$d = 4$

So, common difference d = 4

Substituting the value of d in (i), we have

$a + 3(4) = 22$

$a = 22 - 12$

$a = 10$

Hence, the first term $a = 10$

Now, the sum of first 8 terms of the A.P. is

$n = 8$

$S_n = \frac{n}{2} \times [2a + (n - 1)d]$

$S_8 = \frac{8}{2} \times [2(10) + (8 - 1)(4)]$

$= 10 \times [20 + (7)(4)]$

$= 10 \times [20 + 28]$

= 480

11. If the sum of first 6 terms of an A.P. is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.

Solution:

From the question it is given that,

$S_6 = 36$

$S_{16} = 256$

We know that,

$S_n = \left(\frac{n}{2}\right)(2a + (n-1)d)$

$S_6 = \left(\frac{6}{2}\right)(2a + (6-1)d) = 36$

$3(2a + 5d) = 36$

Divide both the side by 3,

$2a + 5d = 12$ (i)

Now, $S_{16} = \left(\frac{16}{2}\right)(2a + (16-1)d) = 256$

$8(2a + 15d) = 256$

Divide both the side by 8,

$2a + 15d = 32$ (ii)

Then, subtract equation (ii) from equation (i) we get,

$(2a + 5d) - (2a + 15d) = 12 - 32$

$2a + 5d - 2a - 15d = -20$

$-10d = -20$

$d = \frac{-20}{-10}$

$d = 2$

Substitute the value of d in equation (i) to find a,

$2a + 5d = 12$

$2a + 5(2) = 12$

$2a + 10 = 12$

$2a = 12 - 10$

$2a = 2$

$a = \frac{2}{2}$

$a = 1$

So, $S_{10} = \left(\frac{n}{2}\right)(2a + (n-1)d)$

$= \left(\frac{10}{2}\right)\left((2 \times 1) + (10 - 1)2\right)$

$= 5(2 + 18)$

$= 5(20)$

$= 100$

Therefore, the sum of first 10 terms is 100.

12. Show that $a_1, a_2, a_3, \ldots$ from an A.P. where a_n is defined as $a_n = 3 + 4n$. Also find the sum of first 15 terms.

Solution:

From the question it is given that,

n^{th} term is 3 + 4n

So, $a_n = 3 + 4n$

Now, we start giving values, 1, 2, 3 …. in the place of n, we get,

$a_1 = 3 + (4 \times 1) = 3 + 4 = 7$

$a_2 = 3 + (4 \times 2) = 3 + 8 = 11$

$a_3 = 3 + (4 \times 3) = 3 + 12 = 15$

$a_4 = 3 + (4 \times 4) = 3 + 16 = 19$

So. The numbers are 7, 11, 15, 19 ….

Then, first term $a = 7$, common difference d = $11 - 7 = 4$

We know that,

$S_{15} = \left(\frac{n}{2}\right)(2a + (n-1)d)$

= $\left(\frac{15}{2}\right)\left((2 \times 7) + (15 - 1) \times 4\right)$

= $\left(\frac{15}{2}\right)\left(14 + (14 \times 4)\right)$

= $\left(\frac{15}{2}\right)(14 + 56)$

= $\left(\frac{15}{2}\right) \times 70$

= 525

Therefore, the sum of first 15 terms is 525.

13. The sum of first six terms of an arithmetic progression is 42. The ratio of the 10th term to the 30th term is 1: 3. Calculate the first and the thirteenth term.

Solution:

$S_6 = 42$ and $\frac{T_{10}}{T_{30}} = \frac{1}{3}$

We know that,

$S_n = \left(\frac{n}{2}\right)(2a + (n-1)d)$

$T_n = a + (n-1)d$

So, we have

$S_6 = \left(\frac{6}{2}\right) \times [2a + (6-1)d] = 42$

$3 \times (2a + 5d) = 42$

$2a + 5d = 14$ (i) [Dividing by 3]

And,

$\frac{T_{10}}{T_{30}} = \frac{[a+(10-1)d]}{[a+(30-1)d]} = \frac{1}{3}$

$\frac{(a+9d)}{(a+29d)} = \frac{1}{3}$

On cross – multiplication, we have

$3(a + 9d) = a + 29d$

$3a + 27d = a + 29d$

$3a - a + 27d - 29d = 0$

$2a - 2d = 0$

$a - d = 0$

$a = d$

Using the above the relation in (i), we get

$2a + 5a = 14$

$7a = 14$

$a = \frac{14}{7}$

$a = 2$

Hence, the first term of the A.P. is 2.

Now, the thirteenth term is given by

$T_{13} = 2 + (13 - 1)(2)$

$= 2 + 12 \times 2$

$= 2 + 24$

$= 26$

14. In an A.P., the sum of its first n terms is $6n - n^2$. find its 25th term.

Solution:

Given,

$S_n = 6n - n^2$

Now,

$S_1 = 6(1) - (1)^2 = 6 - 1 = 5$

So, the first term $a = 5$

And,

$S_2 = 6(2) - (2)^2 = 12 - 4 = 8$

So, $T_1 + T_2 = 8$

$5 + T_2 = 8$

$T_2 = 8 - 5 = 3$

So, the common difference d = $T_2 - T_1$

$d = 3 - 5 = -2$

Thus, 25th term in the A.P. is given by

$T_{25} = 5 + (25 - 1)(-2)$

$= 5 + 25(-2)$

$= 5 - 48$

$= -43$

15. If S_n denotes the sum of first n terms of an A.P., Prove that $S_{30} = 3(S_{20} - S_{10})$.

Solution:

We know that,

$S_n = \frac{n}{2} \times [2a + (n - 1)d]$

So,

$S_{10} = \frac{10}{2} \times [2a + (10 - 1)d]$

$= 5 \times (2a + 9d)$

$= 10a + 45d$

$S_{20} = \frac{20}{2} \times [2a + (20 - 1)d]$

$= 10 \times (2a + 19d)$

$= 20a + 190d$

$S_{30} = \frac{30}{2} \times [2a + (30 - 1)d]$

$= 15 \times (2a + 29d)$

$= 30a + 435d$

Now, taking L.H.S. we have

$3(S_{20} - S_{10}) - 3[20a + 190d - (10a + 45d)]$

$= 3(20a + 190d - 10a - 45d)$

$= 3(10a + 145d)$

$= 30a + 435d$

$= S_{30}$

= R.H.S.

Hence Proved.

16.

(i) Find the sum of first 1000 positive integers.

(ii) Find the sum of first 15 multiples of 8.

Solution:

(i) First 1000 positive integers are:

1, 2, 3, 4 …... 1000

This is an A.P. with first term $a = 1$ and common difference $d = 1$

We know that,

$S_n = \frac{n}{2} \times [2a + (n-1)d]$

Thus,

$S_{1000} = \frac{1000}{2} \times [2(1) + (1000-1)(1)]$

$= 500 \times (2 + 99)$

$= 500 \times 101$

$= 50500$

Therefore, the sum of first 1000 positive integers is 50500.

(ii) The first 15 multiples of 8 are:

8, 16, 24 …. Where n = 15

This forms an A.P. with first term $a = 8$ and common difference $d = 16 - 8 = 8$

We know that,

$S_n = \frac{n}{2} \times [2a + (n-1)d]$

Thus,

$S_{15} = \frac{15}{2} \times [2(8) + (14 - 1)(8)]$

$= \frac{15}{2} \times (16 + 13 \times 8)$

$= \frac{15}{2} \times (16 + 104)$

$= \frac{15}{2} \times 120$

$= 15 \times 60$

$= 900$

Therefore, the sum of first 15 multiples of 8 is 900.

17.

(i) Find the sum of all two digit natural numbers which are divisible by 4.

(ii) Find the sum of all natural numbers between 100 and 200 which are divisible by 4.

(iii) Find the sum of all multiples of 9 lying between 300 and 700.

(iv) Find the sum of all natural numbers less than 100 which are divisible by 4.

Solution:

(i) The two – digit natural numbers which are divisible by 4 are:

4, 8, 12, 16 ….

This form an A.P.

The last term in this series is found out by dividing 100 by 4

$\frac{100}{4}$ = 25 and remainder is zero

So, the last two – digit number which is divisible by 4 is 96

And, it is the 24th term

So, we have

$a = 4, d = 4$ Last term l = 96 and n = 24

Thus, sum of these numbers is

$S_{24} = \frac{24}{2}(4 + 96)$

$= 12 \times 100$

= 1200

(ii) The natural numbers between 100 and 200 which are divisible by 4 are:

104, 108, 112…

This form an A.P.

The last term in this series is found out by dividing 200 by 4

$\frac{200}{4}$ = 50 and remainder is zero

So, the last natural number between 100 and 200 which are divisible by 4 is 196

So, we have

$a = 104, d$ = 4 and last term l = 96

The number of terms is found out by

$T_n = 196 = 104 + (n - 1)(4)$

$196 = 104 + 4n - 4$

$4n = 196 - 104 + 4$

$n = \frac{96}{4}$

$n = 24$

Thus, sum of these numbers is

$S_{24} = \frac{24}{2}(104 + 196)$

$= 12 \times 200$

$= 2400$

(iii) The multiples of 9 lying between 300 and 700 are:

306, 315, 324 ….

This form an A.P. where $a = 306$ and $d = 9$

The last term in this series is found out by dividing 700 by 9

$\frac{700}{9}$ Gives 77 as quotient and 7 as remainder.

So, the last number between 300 and 700 which is a multiple is 700 – 7 = 693

Now, we have

$a = 306, d = 9$ And l = 693

The number of terms is found out by

$T_n = 693 = 306 + (n - 1)(9)$

$693 = 306 + 9n - 9$

$9n = 693 - 306 + 9$

$9n = 369$

$n = \frac{396}{9}$

$n = 44$

Thus, sum of these numbers is

(iv) The natural numbers less than 100 which are divisible by 4 are:

4, 8, 12…., 96

This forms an A.P. where a = 4, d = 4 and l = 96

Now, for calculating *n*

$T_n = 96 = 4 + (n - 1)4$

$96 = 4 + 4n - 4$

$4n = 96$

$n = \frac{96}{4}$

$n = 24$

Thus, sum of these numbers is

$S_{24} = \frac{24}{2}(4 + 96)$

$= 12 \times 100$

$= 1200$

Exercise 9.4

1.

(i) **Find the next term of the list of numbers $\frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \ldots.$**

Solution:

From the question,

First term $a = \frac{1}{6}$

Then, $r = \left(\frac{1}{3}\right) \div \left(\frac{1}{6}\right)$

$r = \left(\frac{1}{3}\right) \times \left(\frac{6}{1}\right)$

$r = \frac{6}{3}$

$r = 2$

Therefore, next term = $\frac{2}{3} \times 2 = \frac{4}{3}$

(ii) **Find the next term of the list of numbers $\frac{3}{16}, \frac{-3}{8}, \frac{3}{4}, \frac{-3}{2} \ldots.$**

Solution:

From the question,

First term $a = \frac{3}{16}$

Then, $r = \left(\frac{-3}{8}\right) \div \left(\frac{3}{16}\right)$

$r = \left(\frac{-3}{8}\right) \times \left(\frac{16}{3}\right)$

$r = \frac{(-3 \times 16)}{(8 \times 3)}$

$r = \frac{(-1\times2)}{(1\times1)}$

$r = -2$

Therefore, next term = $\frac{-3}{2} \times (-2) = \frac{6}{2} = 3$

(iii) Find the 15th term of the series $\frac{\sqrt{3}+1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \cdots$

Solution:

From the question,

First term $a = \sqrt{3}$

Then, $r = \left(\frac{1}{\sqrt{3}}\right) \div \left(\sqrt{3}\right)$

$r = \left(\frac{1}{\sqrt{3}}\right) \times \left(\frac{1}{\sqrt{3}}\right)$

$r = \frac{(1\times1)}{(\sqrt{3}\times\sqrt{3})}$

$r = \frac{1}{\left(\sqrt{3}\right)^2}$

$r = \frac{1}{3}$

So, $a_{15} = ar^{n-1}$

$= \sqrt{3}\left(\frac{1}{3}\right)^{15-1}$

$= \sqrt{3}\left(\frac{1}{3}\right)^{14}$

$= \sqrt{3} \times \left(\frac{1}{3^{14}}\right)$

Therefore, $a_{15} = \sqrt{3} \times \left(\frac{1}{3^{14}}\right)$

(iv) **Find the 4th term of the list of number $\frac{1}{\sqrt{2}}, -2, 4\sqrt{2}, -16 \ldots$**

Solution:

From the question it is given that,

First term $a = \frac{1}{\sqrt{2}}$

Then, $r = -2 \div \left(\frac{1}{\sqrt{2}}\right)$

$r = \left(\frac{-2}{1}\right) \times \left(\frac{\sqrt{2}}{1}\right)$

$r = \frac{(-2\times\sqrt{2})}{(1\times1)}$

$r = -2\sqrt{2}$

So, $a_n = ar^{n-1}$

$= \left(\frac{1}{\sqrt{2}}\right)\left(-2\sqrt{2}\right)^{n-1}$

$= \left(\frac{1}{\sqrt{2}}\right)(-1)^{n-1} \times \left[\left(\sqrt{2}\right)^2 \times \sqrt{2}\right]^{n-1}$

$= (-1)^{n-1} \times \frac{1}{\sqrt{2}} \times \left[\left(\sqrt{2}\right)^3\right]^{n-1}$

$= (-1)^{n-1} \times \frac{1}{\sqrt{2}} \times \left(\sqrt{2}\right)^{3n-3}$

$= (-1)^{n-1} \times \left(\sqrt{2}\right)^{3n-3-1}$

$= (-1)^{n-1} \times \left(\sqrt{2}\right)^{3n-4}$

$= (-1)^{n-1} \times 2^{\frac{(3n-4)}{2}}$

Therefore, $a_n = (-1)^{n-1} \times 2^{\frac{(3n-4)}{2}}$

(v) Find the 10^{th} and n^{th} terms of the list of numbers 5, 25, 125, …

Solution:

From the question it is given that,

First term $a = 5$

Then, $r = (25) \div (5)$

$r = (25) \times \left(\frac{1}{5}\right)$

$r = 5$

So, $a_{10} = ar^{n-1}$

$= 5 \times (5)^{10-1}$

$= 5 \times 5^9$

$= 5^{9+1}$ [By $a^m \times a^n = a^{m+n}$]

$= 5^{10}$

Therefore, $a_{10} = ar^{n-1}$

$= 5 \times 5^{n-1}$

$= 5^{n-1+1}$

$= 5^n$

(vi) Find the 6^{th} and the n^{th} terms of th list of numbers $\frac{3}{2}, \frac{3}{4}, \frac{3}{8}$ …

Solution:

From the question it is given that,

First term $a = \frac{3}{2}$

Then, $r = \left(\frac{3}{4}\right) \div \left(\frac{3}{2}\right)$

$r = \left(\frac{3}{4}\right) \times \left(\frac{2}{3}\right)$

$r = \frac{(3\times2)}{(4\times3)}$

$r = \frac{(1\times1)}{(2\times1)}$

$r = \frac{1}{2}$

So, $a_n = ar^{n-1}$

$= \left(\frac{3}{2}\right) \times \left(\frac{1}{2}\right)^{n-1}$

$= 3 \times \frac{1}{2} \times \left(\frac{1}{2}\right)^{n-1}$

$= 3 \times \left(\frac{1}{2}\right)^{n-1+1}$

$= 3 \times \left(\frac{1}{2}\right)^{n}$

$= \frac{3}{2^n}$

Therefore, $a_6 = \frac{3}{2^n}$

$= \frac{3}{2^6}$

$= \frac{3}{64}$

(vii) **Find the 6th term from the end of the list of numbers 3, -6, 12, -24 …, 12288.**

Solution:

From the question it is given that,

Last term = 12288

First term a = 3,

Then, $r = (-6) \div (3)$

$r = (-6) \times \left(\frac{1}{3}\right)$

$r = \frac{(-6\times1)}{(1\times3)}$

$r = \frac{(-2\times1)}{(1\times1)}$

$r = -2$

Then, 6th term from the end,

$a_6 = l \times \left(\frac{1}{r}\right)^{n-1}$

$= 12288 \times \left(\frac{1}{-2}\right)^{6-1}$

$= 12288 \times \left(\frac{1}{-2^5}\right)$

$= \frac{12288}{-32}$

$= -384$

2. Which term of the G.P.

(i) $2, 2\sqrt{2}, 4$ ... is 128?

Solution:

From the question it is given that,

Last term = 128

First term $a = 2$

Then, $r = \left(-2\sqrt{2}\right) \div (2)$

$r = \frac{(-2\sqrt{2})}{(2)}$

$r = \sqrt{2}$

Then, $a_n = ar^{n-1}$

So, 128, = $2\left(\sqrt{2}\right)^{n-1}$

$2^7 = 2\left(\sqrt{2}\right)^{n-1}$

$\frac{2^7}{2} = \left(\sqrt{2}\right)^{n-1}$

$2^{7-1} = \left(\sqrt{2}\right)^{n-1}$

$2^6 = \left(\sqrt{2}\right)^{n-1}$

$\left(\sqrt{2}\right)^{n-1} = \left(\sqrt{2}\right)^{12}$

Now, comparing the powers

$n - 1 = 12$

$n = 12 + 1$

$n = 13$

Therefore, 128 is the 13th term.

(ii) $\mathbf{1, \frac{1}{3}, \frac{1}{9}, \ldots}$ **is** $\mathbf{\frac{1}{243}}$

Solution:

From the question it is given that,

Last term $(a_n) = \frac{1}{243}$

First term $a = 1$

Then, $r = \left(\frac{1}{3}\right) \div (1)$

$r = \left(\frac{1}{3}\right) \times \left(\frac{1}{1}\right)$

$r = \frac{1}{3}$

Then, $a_n = ar^{n-1}$

$\frac{1}{243} = 1 \times \left(\frac{1}{3}\right)^{n-1}$

$\left(\frac{1}{3}\right)^5 = \left(\frac{1}{3}\right)^{n-1}$

By comparing both left hand side and right hand side,

$5 = n - 1$

$n = 5 + 1$

$n = 6$

Therefore, $\frac{1}{243}$ is 6^{th} term

3. Determine the 12^{th} term of a G.P. whose 8^{th} term is 192 and common ratio is 2.

Solution:

From the question it is given that,

$a_8 = 192$ and $r = 2$

Then, by the formula $a_n = ar^{n-1}$

$a_8 = ar^{8-1}$

$192 = a(2)^{8-1}$

$192 = a(2)^7$

$a = \frac{192}{2^7}$

$a = \frac{192}{128}$

$a = \frac{3}{2}$

Now, $a_{12} = \left(\frac{3}{2}\right)(2)^{12-1}$

$= \left(\frac{3}{2}\right) \times (2)^{11}$

$= \left(\frac{3}{2}\right) \times 2048$

$= 3072$

$a_8 = 3072$

4. In a G.P., the third is 24 and 6th term is 192. Find the 10th term.

Solution:

From the question it is given that,

$a_3 = 24$

$a_6 = 192$

Then, by the formula $a_n = ar^{n-1}$

$a_6 = ar^{6-1}$

$192 = ar^{6-1}$

$192 = ar^5$ (i)

Now, $a_3 = ar^{3-1}$

$24 = ar^{3-1}$

$24 = ar^2$ (ii)

By dividing equation (i) by equation (ii)

$\frac{ar^5}{ar^2} = \frac{192}{24}$

$r^{5-2} = 8$

$r^3 = 8$

$r^3 = 2^3$

$r = 2$

Now, substitute the value r in equation (i),

$192 = ar^5$

$192 = a(2)^5$

$a = \frac{192}{32}$

$a = 6$

So, $a_{10} = ar^{10-1}$

$= ar^9$

$= 6(2)^9$

$= 6(512)$

$= 3072$

5. Find the number of terms of a G.P. whose first term is $\frac{3}{4}$, common ratio is 2 and the last term is 384.

Solution:

From the question it is given that,

First term of G.P. $a = \frac{3}{4}$

Common ratio $(r) = 2$

Last term = 384

Then, by the formula $a_n = ar^{n-1}$

$384 = \left(\frac{3}{4}\right)(2)^{n-1}$

$\frac{(384\times4)}{3} = (2)^{n-1}$

$\frac{(1536)}{3} = (2)^{n-1}$

$512 = 2^{n-1}$

$2^9 = 2^{n-1}$

By comparing both left had side and right hand side,

$9 = n - 1$

$n = 9 + 1$

$n = 10$

The number of term of a G.P. is 10.

6. Find the value of x such that,

(i) $\frac{-2}{7}, x, \frac{-7}{2}$ are three consecutive terms of a G.P.?

Solution:

From the question,

$x^2 = \frac{-2}{7} \times \frac{-7}{2}$

$x^2 = 1$

$x = \pm 1$

Therefore, $x = 1$ or $x = -1$

(ii) x + 9, x – 6 and 4 are three consecutive terms of a G.P.

Solution:

From the question,

$(x-6)^2 = (x+9) \times 4$

$x^2 - 12x + 36 = 4x + 36$

$x^2 - 12x - 4x + 36 - 36 = 0$

$x^2 - 16x = 0$

$x(x-16) = 0$

Either let us take $x - 16 = 16$

Or $x = 0$

So, $x = 0, 16$

(iii) x, x + 3, x + 9 are first three terms of a G.P. find the value of x.

Solution:

From the question,

$(x+3)^2 = x(x+9)$

$x^2 + 6x + 9 = x^2 + 9x$

$9 = 9x - 6x$

$9 = 3x$

$x = \frac{9}{3}$

$x = 3$

7. If the fourth, seventh the tenth terms of a G.P. are x, y, z respectively, Prove that x, y, z are in G.P.

Solution:

From the question it is given that,

$a_4 = x$

$a_7 = y$

$a_{10} = z$

Now we have to prove that, x, y, z are in G.P.

Then, by the formula $a_n = ar^{n-1}$

$a_4 = ar^{4-1}$

$a_4 = a^3$

$a_4 = x$

So, $a_7 = a^{7-1}$

$a_7 = a^6$

$a_7 = y$

$a_{10} = a^{10-1}$

$a_{10} = a^9$

$a_{10} = z$

x, y, z are in G.P. then,

$y^2 = xz$

Then, $xz = ar^3 \times ar^9$

$= a^2r^3 + 9$

$= a^2r^{12}$

$y^2 = (ar^6)^2$

$y^2 = a^2r^{12}$

By comparing left hand side and right hand side

LHS = RHS

Therefore, *x, y* and *z* are in G.P.

8. The 5th, 8th and 11th term of a G.P. and p, q and s respectively. Show that $q^2 = ps$.

Solution:

From the question it is given that,

$a_5 = p$

$a_5 = q$

$a_{11} = s$

Now we have to prove that, $q^2 = ps$

Then, by the formula $a_n = ar^{n-1}$

$a_5 = ar^{5-1}$

$a_5 = a^4$

$a_5 = p$

So, $a_8 = a^{8-1}$

$a_8 = a^7$

$a_8 = q$

$a_{11} = a^{11-1}$

$a_{11} = a^{10}$

$a_{11} = s$

p, q, s are in G.P. then,

$q^2 = (ar^7)^2$

$= ar^{14}$

Then, $px = ar^4 \times ar^{10}$

= $a^2 r^4 + 10$

= $a^2 r^{14}$

Therefore, $q^2 = ps$

9. If $\boldsymbol{a}, \boldsymbol{a^2 + 2}$ and $\boldsymbol{a^3 + 10}$ are in G.P., then find the values (s) of $\boldsymbol{a}$.

Solution:

From the question

$(a^2 + 2)^2 = a(a^3 + 10)$

$a^4 + 4 = a^4 + 10a$

$4a^2 - 10a + 4 = 0$

$2a^2 - 5a + 2 = 0$

$2a^2 - a - 4a + 2 = 0$

$a(2a - 1) - 2(2a - 1) = 0$

$(2a - 1)(a - 2) = 0$

Then, $2a - 1 = 0$

$a = \frac{1}{2}$

$a - 2 = 0$

$a = 2$

Therefore, $a = 2$ or $a = \frac{1}{2}$.

10. Find the geometric progression whose 4th term is 54 and the 7th term is 1458.

Solution:

From the question it is given that,

4th term $a_4 = 54$

7th term $a_7 = 1458$

$ar^3 = 54$

$ar^6 = 1458$

Now dividing we get,

$\frac{ar^6}{ar^3} = \left(\frac{1458}{54}\right)$

$r^{6-3} = 27$

$r^3 = 3^3$

$r = 3$

Then, $ar^3 = 54$

$a \times 27 = 54$

$a = \frac{54}{57}$

$a = 2$

Therefore G.P. is 2, 6, 18, 54 …

11. The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

Solution:

From the question it is give that,

The sum of first three terms of a G.P. is $\frac{39}{10}$

The product of first three terms of a G.P. is 1

Let us assume that a be the first term and 'r' be the common ratio,

And also assume that, three terms of the G.P. is $\frac{a}{r}, a, ar$

The sum of three terms = $\left(\frac{a}{r}\right) + a + r = \frac{39}{10}$

Take out 'a' as common then, we get

$a\left(\frac{1}{r} + 1 + r\right) = \frac{39}{10}$ (i)

Now, product of three terms = $\left(\frac{a}{r}\right) \times a \times ar = 1$

$\frac{a^3 r}{r} = 1$

$a^3 = 1$

$a^3 = 1^3$

$a = 1$

Substitute the value of 'a' in equation (i).

$1\left(\frac{1}{r} + 1 + r\right) = \frac{39}{10}$

$\frac{(1+r+r^2)}{r} = \frac{39}{10}$

By cross multiplication we get,

$\frac{10(1+r+r^2)}{r} = 39r$

$10 + 10r + 10r^2 = 39r$

Transposing $39r$ from right hand side to left hand side it becomes $-39r$

$10 + 10r + 10r^2 - 39r = 0$

$10r^2 - 29r + 10 = 0$

$10r^2 - 25r - 4r + 10 = 0$

$5r(2r - 5) - 2(2r - 5) = 0$

$(2r - 5)(5r - 2) = 0$

So,

$2r - 5 = 0$

$r = \frac{5}{2}$

$5r - 2 = 0$

$r = \frac{2}{5}$

Therefore, $r = \frac{5}{2}$ or $\frac{2}{5}$

Then the terms if $r = \frac{5}{2}$ are $1, \frac{5}{2}, \frac{25}{4}$

The terms if $r = \frac{2}{5}$ are $1, \frac{2}{5}, \frac{4}{25}$

12. Three numbers are in A.P. and their sum is 15. If 1, 4, and 19 are added to these numbers respectively, the resulting numbers are in G.P. Find the numbers.

Solution:

From the question it is given that,

The sum of first three terms of a A.P. is 15

Let us assume three numbers are $a-d, a, a+d$.

The sum of three terms = $a-d+a+a+d=15$

$a=\frac{15}{3}$

$a=5$

Then, adding 1, 4, 19 in the terms

The numbers becomes, $a-d+1, a+4, a+d+19$

Therefore, $b^2=ac$

$(a+4)^2=(a-d+1)(a+d+19)$

Simplify the above terms,

$a^2+8a+16=a^2+ad+19a-ad-a^2-19d+a+d+19$

$a^2+8a+16=a^2-d^2-18d+20a+19$

$8a+16=2a-18d-d^2+19$

$8a+16-20a-18d-d^2+19$

$8a+16-20a+18d+d^2-19=0$

$d^2+18d-12a-3=0$

$d^2+18d-(12\times 5)-3=0$

$d^2+18d-60-3=0$

$d^2+18d-63=0$

$d^2+21d-3d-63=0$

$d(d+21)-3(d+21)=0$

$(d + 21)(d - 3) = 0$

So, $d + 21 = 0$

$d = -21$

$d - 3 = 0$

$d = 3$

Then the terms if d = 3 and a = 5,

Then G.P. $5 - 3 = 2, 5, 5 + 3 = 8$

The terms if $d = -21$ are $5 - (-21) = 5 + 21 = 26, 5, 5 - 21 = -16$

Exercise 9.5

1. Find the sum of:

(i) 20 terms of the series 2 + 6 + 18 + …

Solution:

From the question,

First term $a = 2$

Common ratio $r = \frac{6}{2} = 3$

Number of terms $n = 20$

So, $S_{20} = \frac{a(r^n - 1)}{r - 1}$

$= \frac{2(3^{20} - 1)}{3 - 1}$

$= \frac{2(3^{20} - 1)}{2}$

$= 3^{20} - 1$

Therefore, $S_{20} = 3^{20} - 1$

(ii) 10 terms of series $\mathbf{1 + \sqrt{3} + 3 + \cdots}$

Solution:

From the question,

First term $a = 1$

Common ratio $r = \frac{\sqrt{3}}{1} = \sqrt{3}$

Number of terms $n = 10$

So, $S_{10} = \frac{a(r^n-1)}{r-1}$

$= \frac{1\left(\left(\sqrt{3}\right)^{10}-1\right)}{\sqrt{3}-1}$

Multiplying $\left(\sqrt{3}+1\right)$ for both numerator and denominator we get,

$= \frac{\left(\left(\sqrt{3}\right)^{10}-1\right)\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}$

$= \frac{(3^5-1)\left(\sqrt{3}+1\right)}{3-1}$ [By rationalizing the denominator]

$= \frac{(243-1)\left(\sqrt{3}+1\right)}{2}$

$= \frac{242\left(\sqrt{3}+1\right)}{2}$

$= 121\left(\sqrt{3}+1\right)$

Therefore, $S_{10} = 121\left(\sqrt{3}+1\right)$

(iii) 6 terms of the G.P. $1, \frac{-2}{3}, \frac{4}{9}, \ldots$

Solution:

From the question,

First term $a = 1$

Common ratio $r = \frac{-2}{3} \times 1 = \frac{-2}{3}$

Number of terms $n = 6$

So, $S_6 = \frac{a(r^n-1)}{r-1}$

$= \frac{1\left[1-\left(\frac{-2}{3}\right)^6\right]}{\left(1+\left(\frac{2}{3}\right)\right)}$

$= \left(\frac{3}{5}\right)\left(1 - \left(\frac{-2^6}{3^6}\right)\right)$

$= \left(\frac{3}{5}\right)\left(1 - \left(\frac{64}{729}\right)\right)$

$= \left(\frac{3}{5}\right)\left(\frac{(729-64)}{729}\right)$

$= \frac{3}{5} \times \left(\frac{665}{729}\right)$

$= \frac{133}{243}$

(iv) 5 terms and n terms of the series $1 + \frac{2}{3} + \frac{4}{9} + \cdots$

Solution:

From the question,

First term $a = 1$

Common ratio $r = \frac{2}{3} \times 1 = \frac{2}{3}$

Number of terms $n = 5$

So, $S_n = \frac{a(r^n-1)}{r-1}$

$= \frac{1\left[1-\left(\frac{2}{3}\right)^n\right]}{\left(1-\frac{2}{3}\right)}$

$S_n = 3\left[1 - \left(\frac{2}{3}\right)^n\right]$

Then, $S_5 = 3\left[1 - \left(\frac{2}{3}\right)^5\right]$

$= 3\left[1 - \left(\frac{32}{243}\right)\right]$

$= 3\left(\frac{(243-32)}{243}\right)$

$= \frac{211}{81}$

2. Find the sum of the series $81 - 27 + 9 \ldots \frac{-1}{27}$

Solution:

From the question it is given that,

First term $a = 81$

$r = \frac{-27}{81}$

$= \frac{-1}{3}$

Last term l = $\frac{-1}{27}$

$S_n = \frac{(a-lr)}{(l-r)}$

$= \frac{\left[81+\left(\left(\frac{1}{27}\right)\times\left(\frac{-1}{3}\right)\right)\right]}{\left[1+\left(\frac{1}{3}\right)\right]}$

$= \frac{\left[\left(81-\left(\frac{1}{81}\right)\right)\right]}{\left(\frac{4}{3}\right)}$

$= \frac{(6561-1)}{\left[81\times\left(\frac{4}{3}\right)\right]}$

$= \frac{(6560\times3)}{(81\times4)}$

$= \frac{1640}{27}$

3. The n^{th} term of a G.P. is 128 and the sum of its n terms is 255. If its common ratio is 2, then find its first term.

Solution:

From the question it is given that,

The n^{th} term of a G.P. $T_n = 128$

The sum of its n terms $S_n = 255$

Common ratio r = 2

We know that, $T_n = ar^{n-1}$

$128 = a2^{n-1}$

$a = \frac{128}{2^{n-1}}$ (i)

Also we know that, $S_n = \frac{a(r^n-1)}{(r-1)}$

$255 = \frac{a(2^n-1)}{(2-1)}$

By cross multiplication we get,

$255 = a(2^n - 1)$

$a = \frac{255}{(2^n-1)}$ (ii)

By cross multiplication we get,

$255 \times 2^n - 1 = 128(2^n - 1)$

$255 \times 2^n - 1 = 125 \times 2^n - 128$

$\frac{(255\times2^n)}{2} = 128 \times 2^n - 128$

$255 \times 2^n = 256 \times 2^n - 256$

$256 \times 2^n - 255 \times 2^n = 256$

By simplification,

$2^n = 256$

$2^n = 2^8$

By comparing both LHS and RHS, we get,

Then, $128 = a2^7$

$128 = a \times 128$

$a = \frac{128}{128}$

$a = 1$

Therefore, the value of a is 1.

4.

(i) How many terms of the G.P. $3, 3^2, 3^3, \ldots$ are needed to give the sum 120?

Solution:

From the question it is given that,

Terms of the G.P. $3, 3^2, 3^3, \ldots$

Sum of the terms = 120

The first terms $a = 3$

$r = \frac{3^2}{3}$

$= \frac{9}{3}$

= 3

We know that, $S_n = \frac{a(r^n - 1)}{r - 1} = 120$

$\frac{3(3^n-1)}{3-1} = 120$

$\frac{3(3^n-1)}{2} = 120$

By cross multiplication we get,

$3^n - 1 = \frac{(120\times2)}{3}$

$3^n - 1 = \frac{240}{3}$

$3^n - 1 = 80$

$3^n = 80 + 1$

$3^n = 81$

$3^n = 3^4$

Therefore, $n = 4$

(ii) How many terms of the G.P. 1, 4, 16 … must be taken to have their sum equal to 341?

Solution:

From the question it is given that,

Terms of the G.P. $1, 4, 16$, ...

Sum of the terms = 341

The first terms $a = 1$

$r = \frac{4}{1}$

= 4

We know that, $S_n = \frac{a(r^n-1)}{r-1} = 341$

$= \frac{1(4^n - 1)}{4 - 1} = 341$

$= \frac{1(4^n - 1)}{3} = 341$

By cross multiplication we get,

$4^n - 1 = (341 \times 3)$

$4^n - 1 = 1023$

$4^n = 1023 + 1$

$4^n = 1024$

4	1024
4	256
4	64
4	16
4	4
	1

$4^n = 4^5$

Therefore, $n = 5$

5. How many terms of the $\frac{2}{9} - \frac{1}{3} + \frac{1}{2} + \cdots$ will make the sum $\frac{55}{72}$?

Solution:

From the question it is given that,

Terms of G.P. is $\frac{2}{9} - \frac{1}{3} + \frac{1}{2} + \cdots$

Sum of the terms $= \frac{55}{72}$

The first term $a = \frac{2}{9}$

$r = \frac{-1}{3} \div \frac{2}{9} = \left(\frac{-1}{3}\right) \times \left(\frac{9}{2}\right) = \frac{-3}{2}$

We know that, $S_n = \frac{a(r^n-1)}{r-1} = \frac{55}{72}$

$$\frac{\left[\left(\frac{2}{9}\right)\left(1-\left(\frac{-3}{2}\right)^n\right)\right]}{\left(1+\left(\frac{3}{2}\right)\right)} = \frac{55}{72}$$

$$1 - \left(\frac{-3}{2}\right)^n = \left(\frac{55}{72}\right) \times \left(\frac{5}{2}\right) \times \left(\frac{9}{2}\right)$$

$$\left(1 - (-1)\right)^n \left(\frac{3}{2}\right)^n = \frac{275}{32}$$

$$1 + 1\left(\frac{3}{2}\right)^n = \frac{275}{32}$$

$$\left(\frac{3}{2}\right)^n = \frac{275}{32} - 1$$

$$\left(\frac{3}{2}\right)^n = \frac{(275-32)}{32}$$

$$\left(\frac{3}{2}\right)^n = \frac{243}{32}$$

$$\left(\frac{3}{2}\right)^n = \left(\frac{3}{2}\right)^5$$

Therefore, $n = 5$

6. The 2nd and 5th terms of a geometric series are $-\frac{1}{2}$ and sum $\frac{1}{16}$ respectively. Find the sum of the series up to 8 terms.

Solution:

From the question it is given that,

$a_2 = \frac{-1}{2}$

$a_5 = \frac{1}{16}$

We know that, $a_2 = ar^{n-1}$

$= ar^{2-1}$

$a_2 = ar = \frac{-1}{2}$ (i)

$a_5 = ar^{5-1}$

$= ar^4$

$a_5 = ar^4 = \frac{1}{16}$ (ii)

Now, dividing equation (ii) by (i) we get,

$r^3 = \frac{1}{16} + \left(\frac{-1}{2}\right)$

$= \left(\frac{1}{16}\right) \times (-2)$

$= \frac{-1}{8}$

$r^3 = \left(\frac{-1}{2}\right)^3$

So, $r = \frac{-1}{2}$

$ar = \frac{-1}{2}$

$a \times \left(\frac{-1}{2}\right) = \frac{-1}{2}$

$a = \frac{-1}{2} \times \left(\frac{-2}{1}\right)$

$a = 1$

Therefore, $a = 1$ and $r = \frac{-1}{2}$

Then, $S_8 = \frac{a(r^n-1)}{r-1}$

$= \frac{1\left[1-\left(\frac{-1}{2}\right)^8\right]}{\left(1+\frac{1}{2}\right)}$

$= \frac{\left[1-\left(\frac{1}{256}\right)\right]}{\left(\frac{3}{2}\right)}$

$= \left(\frac{255}{256}\right) \times \left(\frac{2}{3}\right)$

$= \left(\frac{510}{768}\right)$

$= \frac{85}{128}$

7. The first term of G.P. is 27 and 8^th^ term is $\frac{1}{81}$. Find the sum of its first 10 terms.

Solution:

From the question it is given that,

First term $a = 27$

8th term $a_8 = \frac{1}{81}$

Then, $a_n = ar^{n-1}$

$a_8 = ar^{8-1} = \frac{1}{81}$

$a_8 = ar^7 = \frac{1}{81}$

$ar^7 = \frac{1}{81}$

$27r^7 = \frac{1}{81}$

$r^7 = \frac{1}{(81 \times 27)}$

$r^7 = \frac{1}{2187}$

$r^7 = \frac{1}{3^7}$

$r = \frac{1}{3}$

So, $S_{10} = \frac{a(1-r^n)}{r-1}$

$= \frac{27\left[1-\left(\frac{1}{3}\right)^{10}\right]}{\left(1-\frac{1}{3}\right)}$

$= \frac{27\left[1-\left(\frac{1}{3^{10}}\right)\right]}{\left(\frac{(3-1)}{3}\right)}$

$= \left(\frac{27 \times 3}{2}\right)\left[1 - \frac{1}{3^{10}}\right]$

$= \left(\frac{81}{2}\right)\left[1 - \frac{1}{3^{10}}\right]$

8. Find the first term of the G.P. whose common ratio is 3, last term is 486 and the sum of whose terms is 728.

Solution:

From the question it is given that,

Common ratio $r = 3$

Last term = 486

Sum of the terms = 728

We know that, $S_n = \frac{a(r^n-1)}{r-1}$

$= \frac{a(3^n-1)}{3-1} = 728$

$= \frac{a(3^n-1)}{2} = 728$

$= a(3^n - 1) = 728 \times 2$

$= a(3^n - 1) = 1456$ (i)

Then, last term = ar^{n-1}

$486 = a \times 3^{n-1}$

$486 = a\left(\frac{3^n}{3}\right)$

$1458 = a3^n$ (ii)

Consider equation (i), $a(3^n - 1) = 1456$

$a3^n - a = 1456$

Substitute the value of $a3^n$ in equation (i),

$1458 - a = 1456$

$a = 1458 - 1456$

$a = 2$

Therefore, the first term a is 2.

9. In a G.P. the first term is 7, the last term is 448, and the sum is 889. Find the common ratio.

Solution:

From the question it is given that,

First term a is = 7

Then, last terms is = 448

Sum = 889

We know that, last term = ar^{n-1}

$7r^{n-1} = 448$

$r^{n-1} = \frac{448}{7}$

$r^{n-1} = 64$ (i)

So, sum = $\frac{a(r^{n-1})}{(r-1)} = 889$

$\frac{7(r^{n-1})}{(r-1)} = 889$

$\frac{(r^{n-1})}{(r-1)} = \frac{889}{7}$

$\frac{(r^{n-1})}{(r-1)} = 127$ (ii)

Consider the equation (i),

$\frac{r^n}{r} = 64$

$r^n = 64r$

Now substitute the value of r^n in equation (ii),

$\frac{(64r-1)}{(r-1)} = 127$

$64r - 1 = 127r - 127$

$127r - 64r = -1 + 127$

$63r = 126$

$r = \frac{126}{63}$

$r = 2$

Therefore, common ratio = 2

10. Find the third term of a G.P. whose common ratio is 3 and the sum of whose first seven terms is 2186.

Solution:

From the question it is given that,

Common ratio $r = 3$

$S_7 = 2186$

We know that, $S_n = \frac{a(r^n-1)}{r-1}$

$S_7 = \frac{a(3^n-1)}{3-1}$

$2186 = \frac{a(3^n-1)}{2}$

By cross multiplication,

$(2186 \times 2) = a(3^7 - 1)$

$(4372) = a(2187 - 1)$

$4372 = a2186$

$a = \frac{4372}{2186}$

$a = 2$

Then, $a_3 = ar^{3-1}$

$= ar^2$

$= 2 \times 3^2$

$= 2 \times 9$

$a_3 = 18$

11. If the first term of a G.P. is 5 and the sum of first three terms is $\frac{31}{5}$, find the common ratio.

Solution:

From the question it is given that,

First term of a G.P. is $a = 5$

The sum of first three terms is $S_3 = \frac{31}{5}$

We know that, $S_n = \frac{a(r^n-1)}{r-1}$

$$S_3 = \frac{a(r^3-1)}{r-1}$$

$$\frac{31}{5} = \frac{5(r^3-1)}{(r-1)}$$

$$\frac{31}{(5\times5)} = \frac{(r^3-1)}{(r-1)}$$

$$\frac{31}{25} = \frac{(r^3-1)}{(r-1)}$$

$$\frac{(r-1)(r^2+r+1)}{(r-1)} = \frac{31}{25}$$

$$r^2 + r + 1 = \frac{31}{25}$$

By cross multiplication we get,

$$25(r^2 + r + 1) = 31$$

$$25r^2 + 25r + 25 = 31$$

Transposing 31 from right hand side to left hand side it becomes – 31.

$$25r^2 + 25r + 25 - 31 = 0$$

$$25r^2 + 25r - 6 = 0$$

$25r^2 + 30r - 5r - 6 = 0$

$5r(5r + 6) - 1(5r + 6) = 0$

$(5r - 1)(5r + 6) = 0$

Take $5r - 1 = 0$

$r = \frac{1}{5}$

Or $5r + 6 = 0$

$r = \frac{-6}{5}$

Therefore, Common ratio $r = \frac{1}{5}$ or $\frac{-6}{5}$.

CHAPTER TEST

1. Write the first four terms of the A.P. when its first term is – 5 and term is – 5 and the common difference is – 3.

Solution:

From the question it is given that,

First term $a = -5$

Common difference $d = -3$

Then the first four terms are = $-5 + (-3) = -5 - 3 = -8$

$-8 + (-3) = -8 - 3 = -11$

$-11 + (-3) = -11 - 3 = -14$

Therefore, first four terms are $-5, -8, -11$ and -14.

2. Verify that each of the following lists of numbers is an A.P., and the write its next three terms:

(i) $\mathbf{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \ldots}$

Solution:

From term question it is given that,

First term $a = 0$

Common difference = $\frac{1}{4} - 0 = \frac{1}{4}$

So, next three numbers are $\frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

$1 + \frac{1}{4} = \frac{(4+1)}{4} = \frac{5}{4}$

$\frac{5}{4} + \frac{1}{4} = \frac{6}{4} = \frac{3}{2}$

Therefore, the next three term are $1, \frac{5}{4}$ and $\frac{3}{2}$.

(ii) $\mathbf{5, \frac{14}{3}, \frac{13}{3}, 4} \ldots$

Solution:

From term question it is given that,

First term $a = 5$

Common difference = $\frac{14}{3} - 5 = \frac{(14-15)}{3} = \frac{-1}{3}$

So, next three numbers are $4 + \left(\frac{-1}{3}\right) = \frac{(12-1)}{3} = \frac{11}{3}$

$\frac{11}{3} + \left(\frac{-1}{3}\right) = \frac{(11-1)}{3} = \frac{10}{3}$

$\frac{10}{3} + \left(\frac{-1}{3}\right) = \frac{(10-1)}{3} = \frac{9}{3} = 3$

Therefore, the next three term are $\frac{11}{3}, \frac{10}{3}$ and 3.

3. The nth term of an A.P. is $6n + 2$. find the common difference

Solution:

From the question it is given that,

nth term is $6n + 2$

So, $T_n = 6n + 2$

Now, we start giving values, 1, 2, 3 … in the place of n, we get,

$T_1 = (6 \times 1) + 2 = 6 + 2 = 8$

$T_2 = (6 \times 2) + 2 = 12 + 2 = 14$

$T_3 = (6 \times 3) + 2 = 18 + 2 = 20$

$T_4 = (6 \times 4) + 2 = 24 + 2 = 26$

Therefore, A.P. is $8, 14, 20, 26$ ...

So, common difference $d = 14 - 8 = 6$

4. Show that the list of numbers 9, 12, 15, 18 … from an A.P. find its 16th term and the nth.

Solution:

From the question,

The first term $a = 9$

Then, difference $d = 12 - 9 = 3$

$15 - 12 = 3$

$18 - 15 = 3$

Therefore, common difference $d = 3$

From the formula, $a_n = a + (n - 1)d$

$T_n = a + (n - 1)d$

$= 9 + (n - 1)3$

$= 9 + 3n - 3$

$= 6 + 3n$

So, $T_{16} = a + (n - 1)d$

$= 9 + (16 - 1)3$

$= 9 + (15)(3)$

$= 9 + 45 = 54$

5. Find the 6th term from the end of the A.P. 17, 14, 11 …., - 40.

Solution:

From the question it is given that,

First term $a = 17$

Common difference = $14 - 17 = -3$

Last term l = -40

$L = a + (n - 1)d$

$-40 = 17 + (n - 1)(-3)$

$-40 - 17 = -3n + 3$

$-57 - 3 = -3n$

$n = \frac{-60}{-3}$

$n = 20$

Therefore, 6th term form the end = $l - (n - 1)d$

= $-40 - (6 - 1)(-3)$

= $-40 - (5)(-3)$

= $-40 + 15$

= -25

6. If the 8th term of an A.P. is 31 and the 15th term is 16 more than its 11th term, then find the A.P.

Solution:

From the question it is given that,

$a_8 = 31$

a_{15} = the 15th term is 16 more than its 11th term = $a_{11} + 16$

We know that, $a_n = a + (n-1)d$

So, $a_8 = a + 7d = 31$ (i)

$a_{15} = a + 14d = a + 10d + 16$

$14d - 10d = 16$

$4d = 16$

$d = \frac{16}{4}$

$d = 4$

Now substitute the value of d in equation (i) we get,

$a + (7 \times 4) = 31$

$a + 28 = 31$

$a = 31 - 28$

$a = 3$

So, $3 + 4 = 7, 7 + 4 = 11, 11 + 4 = 15$

Therefore, A.P. is $3, 7, 11, 15$ ....

7. The 17th term of an A.P. is 5 more than twice its 8th term. If the 11th term of the A.P. is 43, then find the wth term.

Solution:

From the question it is given that,

a_{17} = 5 More than twice its 8th term = $2a_8 + 5$

$a_{11} = 43$

a_n = ?

We know that, $a_{11} = a + 10d = 43$ (i)

$a_{17} = 2a_8 + 5$

$a + 16d = 2(a + 7d) + 5$

$a + 16d = 2a + 14d + 5$

$2a - a = 16d - 14d - 5$

$a = 2d - 5$ (ii)

Now substitute the value of a in equation (i) we get,

$2d - 5 + 10d = 43$

$12d = 43 + 5$

$12d = 48$

$d = \frac{48}{12}$

$d = 4$

To find out the value of a substitute the value of d in equation (i)

$a + (10 \times 4) = 43$

$a + 40 = 43$

$a = 43 - 40$

$a = 3$

Then, $a_n = a + (n - 1)d$

$= 3 + 4(n - 1)$

$= 3 + 4n - 4$

$= 4n - 1$

8. The 19th term of an A.P. is equal to three times its 6th term. If its 9th term is 19, find the A.P.

Solution:

From the question it is given that,

a_{19} = 19th term of an A.P. is equal to three times its 6th term = $3a_6$

$a_9 = 19$

As we know, $a_n = a + (n-1)d$

$a_9 = a + 8d = 19$ (i)

Then, $a_{19} = 3(a + 5d)$

$a + 18d = 3a + 15d$

$3a - a = 18d - 15d$

$2a = 3d$

$a = \left(\frac{3}{2}\right)d$

Now substitute the value of a in equation (i) we get

$\left(\frac{3}{2}\right)d + 8d = 19$

$\frac{(3d+16d)}{2} = 19$

$\left(\frac{19}{2}\right)d = 19$

$d = \frac{(19\times2)}{19}$

$d = 2$

To find out the value of a substitute the value of d in equation (i)

$a + 8d = 19$

$a + (8 \times 2) = 19$

$a + 16 = 19$

$a = 19 - 16$

$a = 3$

Therefore, A.P. is $3, 5, 7, 9$ ...

9. If the 3rd and the 9th terms of an A.P. are 4 and – 8 respectively, then which term of this A.P. is zero?

Solution:

From the equation it is given that,

$a_3 = 4$

$a_9 = -8$

We know that, $a_3 = a + 2d = 4$ (i)

$a_9 = a + 8d = -8$ (ii)

Now, subtracting equation (i) from equation (ii)

$(a + 8d) - (a + 2d) = -8 - 4$

$a + 8d - a - 2d = -12$

$6d = -12$

$d = \frac{-12}{6}$

$d = -2$

To find out the value of a substitute the value of d in equation (i)

$a + 2d = 4$

$a + \left(2 \times (-2)\right) = 4$

$a - 4 = 4$

$a = 4 + 4$

$a = 8$

Let us assume n^{th} term be zero, then

$a + (n - 1)d = 0$

$8 + (n - 1)(-2) = 0$

$-2n + 2 = -8$

$-2n = -8 - 2$

$-2n = -10$

$n = \frac{-10}{-2}$

$n = 5$

Therefore, 0 will be the fifth term.

10. Which term of the list of number 5, 2, -1, - 4 … is -55?

Solution:

From the question it is given that,

First term $a = 5$

n^{th} term = 55

Common difference $d = 2 - 5 = -3$

We know that, $a_n = a + (n - 1)d$

$-55 = 5 + (n - 1)(-3)$

$-55 - 5 = -3n + 3$

$-60 - 3 = -3n$

$-63 = -3n$

$n = \frac{-63}{-3}$

$n = 21$

Therefore, -55 is the 21^{st} term.

11. The 24^{th} term of an A.P. is twice its 10^{th} term. Show that its 72^{nd} term is four times its 15^{th} term.

Solution:

From the question it is given that,

The 24^{th} term of an A.P. is twice its 10^{th} term = $a_{24} = 2a_{10}$

We have to show that, 72^{nd} term is four times its 15^{th} term = $a_{72} = 4a_{15}$

We know that, $a_{24} = a + 23d = 2a_{10}$

$a + 23d = 2(a + 9d)$

$a + 23d = 2a + 18d$

$2a - a = 23d - 18d$

$a = 5d$ (i)

$a_{72} = 4a_{15}$

$a + 71d = 4(a + 14d)$

Substitute the value of we get,

$5d + 71d = 4(5d + 14d)$

$76d = 4(19d)$

Therefore, it is proved that 72^{nd} term is four times its 15^{th} term.

12. Which term of the list of numbers 20, $19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$ ... is the first negative term?

Solution:

From the question it is given that,

First term $a = 20$

Common difference $d = 19\frac{1}{4} - 20 = \frac{77}{4} - 20 = \frac{(77-80)}{4} = \frac{-3}{4}$

We know that, $a_n = a + (n-1)d$

$a_n = 20 + (n-1)\left(\frac{-3}{4}\right)$

$a_n = 20 - \frac{3}{4}n + \frac{3}{4}$

$a_n = 20 + \frac{3}{4} - \frac{3}{4}n$

$a_n = \frac{(80+3)}{4} - \frac{3}{4}n$

$a_n = \frac{83}{4} - \frac{3}{4}n$

$a_n = \frac{83}{4} - \frac{3}{4}n < 0$

$\frac{83}{4} < \frac{3}{4}n$

$83 < 3n$

$\frac{83}{3} < n$

$28 < n$

Therefore, 28^{th} is the first negative term.

13. How many three digit numbers are divisible by 9?

Solution:

The three digits numbers which are divisible by 9 are 108, 117, 126 …, and 999

Then, first term $a = 108$

Common difference = 9

Last term = 999

We know that, l = $a_n = a + (n - 1)d$

$999 = 108 + (n - 1)9$

$999 - 108 = 9n - 9$

$891 + 9 = 9n$

$900 = 9n$

$n = \frac{900}{9}$

$n = 100$

Therefore, there are 100 three digit numbers.

14. The sum of three number in A.P. is – 3 and the product is 8. Find the number.

Solution:

From the question it is given that,

The sum of three numbers in A.P. = - 3

The product of three numbers in A.P. = 8

Let us assume the 3 numbers which are in A.P. are, $a - d, a, a + d$

Now adding 3 numbers = $a - d + a + a + d = -3$

$3a = -3$

$a = \frac{-3}{3}$

$a = -1$

From the question, product of 3 numbers is – 35

So, $(a - d) \times (a) \times (a + d) = 8$

$a(a^2 - d^2) = 8$

$-1((-1)^2 - d^2) = 8$

$1 - d^2 = \frac{8}{-1}$

$1 - d^2 = -8$

$d^2 = 8 + 1$

$d^2 = 9$

$d = \sqrt{9}$

$d = \pm 3$

Therefore, the numbers are if $d = 3(a - d) = -1 - 3 = -4$

$a = -1$

$(a + d) = -1 + 3 = 2$

If $d = -6$

The numbers are $(a - d) = -1 - (-3) = -1 + 3 = 2$

$a = -1$

Therefore, the numbers $-4, -1, 2,$ .... and $2, -1, -4, \dots$ are in A.P.

15. The angles of a quadrilateral are in A.P. If the greatest angle is double of the smallest angle, find all the four angles.

Solution:

From the question it is given that,

The angles of a quadrilateral are in A.P.

Greatest angle is double of the smallest angle

Let us assume the greatest angle of the quadrilateral is $a + 3d$.

Then, the other angles are $a + d, a - d, a - 3d$

So, $a - 3d$ is the smallest

Therefore, $a + 3d = 2(a - 3d)$

$a + 3d = 2a - 6d$

$6d + 3d = 2a - a$

$9d = a$ (i)

We know that the sum all angles of quadrilateral is 360°

$a - 3d + a - d + a + d + a + 3d = 360°$

$4a = 360°$

$a = \frac{360}{4}$

$a = 90°$

Now, substitute the value of a in equation (i) we get,

$9d = 90$

$d = \frac{90}{9}$

$d = 10$

Substitute the value of a and d in assumed angles, Greatest angle = $a + 3d = 90 + (3 \times 10) = 90 + 30 = 120°$

Then, other angles are = $a + d = 90° + 10° = 100°$

$a - d = 90° - 10° = 80°$

$a - 3d = 90° - (3 \times 10) = 90 - 30 = 60°$

Therefore, the angles of quadrilateral are 120°, 100°, 80° and 60°.

16. Find the sum of first 20 term of an A.P. whose n^{th} term is 15 – 4n.

Solution:

From the question it is given that,

n^{th} term is 15 – 4n

So, $a_n = 15 - 4n$

Now, we start giving values, 1, 2, 3 … in the place of *n* we get,

$a_1 = 15 - (4 \times 1) = 15 - 4 = 11$

$a_2 = 15 - (4 \times 2) = 15 - 8 = 7$

$a_3 = 15 - (4 \times 3) = 15 - 12 = 3$

$a_4 = 15 - (4 \times 4) = 15 - 16 = -1$

Then, $a_{20} = 15 - (4 \times 20) = 15 - 80 = -65$

So, 11, 7, 3, -1 … -65 are in A.P.

Therefore, first term $a = 11$

Common difference = -4

$n = 20$

$S_{20} = \left(\frac{n}{2}\right)[2a + (n - 1)d]$

$= \left(\frac{20}{2}\right)[(2 \times 11) + (20 - 1)(-4)]$

$= 10[22 - (19)(-4)]$

$= 10[22 - 76]$

$= 10(-54)$

$= -540$

Therefore, the sum of first 20 terms of an A.P. is -540.

17. Find the sum: $\mathbf{18 + 15\frac{1}{2} + 13 + \cdots + \left(-49\frac{1}{2}\right)}$

Solution:

From the question it is given that,

First term $a = 18$

Common difference $d = 15\frac{1}{2} - 18$

$= \frac{31}{2} - 18$

$= \frac{(31-36)}{2}$

$= \frac{-5}{2}$

Last term $= -49\frac{1}{2} = \frac{-99}{2}$

We know that, $a_n = a + (n-1)d$

$\frac{-99}{2} = 18 + (n-1)\left(\frac{-5}{2}\right)$

$\left(\frac{-99}{2}\right) - \left(\frac{18}{1}\right) = (n-1)\left(\frac{-5}{2}\right)$

$\frac{(-99-36)}{2} = \left(\frac{-5}{2}\right)(n-1)$

$\left(\frac{-135}{2}\right) = \left(\frac{-5}{2}\right)(n-1)$

$\left(\frac{-135}{2}\right) \times \left(\frac{-2}{5}\right) = n-1$

$\frac{-135}{-5} = n-1$

$27 = n-1$

$n = 27+1$

$n = 28$

Then, $S_n = \left(\frac{n}{2}\right)[2a+(n-1)d]$

$S_{28} = \left(\frac{28}{2}\right)\left[(2 \times 18) + (28-1)\left(\frac{-5}{2}\right)\right]$

$S_{28} = 14\left[36 + \left(27 \times \left(\frac{-5}{2}\right)\right)\right]$

$S_{28} = 14\left[36 - \left(\frac{135}{2}\right)\right]$

$S_{28} = 14\left[\frac{(72-135)}{2}\right]$

$S_{28} = 14\left(\frac{-63}{2}\right)$

$S_{28} = -441$

18.

(i) **How many terms of the A.P. – 6, $\left(\frac{-11}{2}\right)$, -5, ... make the sum – 25?**

Solution:

From the question it is given that,

Terms of the A.P. is $-6, \left(\frac{-11}{2}\right), -5, \ldots$

The first term $a = -6$

Common difference $d = \left(\frac{-11}{2}\right) - (-6)$

$= \left(\frac{-11}{2}\right) + 6$

$= \frac{(-11+12)}{2}$

$= \frac{1}{2}$

The terms are make the sum – 25

Then, $S_n = \left(\frac{n}{2}\right) [2a + (n-1)d]$

$-25 = \left(\frac{n}{2}\right) \left[\left(2 \times (-6)\right) + (n-1) \left(\frac{1}{2}\right)\right]$

$-25 \times 2 = n \left[-12 + \frac{1}{2}n - \frac{1}{2}\right]$

$-50 = n \left[\left(\frac{-25}{2}\right) + \frac{1}{2}n\right]$

$\frac{1}{2}n^2 - \left(\frac{25}{2}\right)n + 50 = 0$

$n^2 - 25n + 100 = 0$

$n^2 - 5n - 20 + 100 = 0$

$n(n-5) - 20(n-5) = 0$

$(n-5)(n-20) = 0$

So, $n - 5 = 0$

$n = 5$

Or $n - 20 = 0$

$n = 20$

Therefore, number of terms are 5 or 20.

(ii) Solve the equation $2 + 5 + 8 + \cdots + x = 155$

Solution:

From the question it is given that,

The first term $a = 2$

Last term = x

Common difference $d = 5 - 2 = 3$

Then, sum of the terms = 155

$L = a + (n - 1)d$

$x = 2 + (n - 1)3$

$x = 2 + 3n - 3$

$x = 3n - 1$ (i)

We know that, $S_n = \left(\frac{n}{2}\right)[2a + (n - 1)d]$

$155 = \left(\frac{n}{2}\right)[(2 \times 2) + (n - 1) \times 3]$

$155 \times 2 = n[4 + 3n - 3]$

$310 = n(3n + 1)$

$3n^2 + n - 310 = 0$

$3n^2 - 30n + 31n - 310 = 0$

$3n(n - 10) + 31(n - 10) = 0$

$(n - 10)(3n - 31) = 0$

So, $n - 10 = 0$

$n = 10$

Or $3n + 31 = 0$

$n = \frac{-31}{3}$

Negative is not possible

Therefore, $n = 10$

Now, substitute the value of n in equation (i),

$x = 3n - 1$

$= (3 \times 10) - 1$

$= 30 - 1$

$= 29$

19. If the third term of an A.P. is 5 and the ratio of its 6th term to the 10th term is 7: 13, then find the sum of first 20 term of this A.P.

Solution:

From the question it is given that,

The third term of an A.P. $a_3 = 5$

The ratio of its 6th term to the 10th term $a_6 : a_{10} = 7 : 13$

We know that, $a_n = a + (n - 1)d$

$a_n = a + (3 - 1)d = 5$

$= a + 2d = 5$ (i)

Then, $\frac{a_6}{a_{10}} = \frac{7}{13}$

$\frac{(a+5d)}{(a+9d)} = \frac{7}{13}$

By cross multiplication we get,

$13(a + 5d) = 7(a + 9d)$

$13a + 75d = 7a + 63d$

$13a - 7a + 65d - 63d = 0$

$6a + 2d = 0$

Divide by 2 on both side we get,

$3a + d = 0$

$d = -3a$ (i)

Substitute the value of d in equation (i),

$a + 2(-3a) = 5$

$a - 6a = 5$

$-5a = 5$

$a = \frac{-5}{5}$

$a = -1$

Now substitute the value of a in equation (ii).

$d = -3(-1)$

$d = 3$

Then, sum of first 20 terms,

$= \left(\frac{n}{2}\right)[2a + (n - 1)d]$

$= \left(\frac{20}{2}\right)\left[\left(2 \times (-3)\right) + \left(2 - (-1)\right)3\right]$

$= 10[-2 + 57]$

$= 10 \times 55$

$= 550$

20. The sum of first 14 terms of an A.P. is 1505 and its first term is 10. Find its 25th term.

Solution:

From the question it is given that,

First term $a = 10$

The sum of first 14 terms of an A.P. = 1505

25th term =?

We know that, $S_n = \left(\frac{n}{2}\right)[2a + (n-1)d]$

$S_{14} = \left(\frac{n}{2}\right)[2a + (n-1)d]$

$1505 = \left(\frac{14}{2}\right)[(2 \times 10) + (14 + 1)d]$

$1505 = 7[20 + 13d]$

$\frac{1505}{7} = 20 + 13d$

$215 = 20 + 13d$

$13d = 195$

$d = \frac{195}{13}$

$d = 15$

Then, $a_n = a + (n-1)d$

$a_{25} = a + (25 - 1)(15)$

$= 10 + (24)15$

$= 10 + 360$

$= 370$

21. Find the geometric progression whose 4th term is 54 and 7th term is 1458.

Solution:

From the question it is given that,

The geometric progression whose 4th term $a_4 = 54$

The geometric progression whose 7th term $a_7 = 1458$

We know that, $a_n = ar^{n-1}$

$a_4 = ar^{4-1}$

$a_4 = ar^3 = 54$

$a_7 = ar^6 = 1458$

By dividing both we get,

$\frac{ar^6}{ar^3} = \frac{1458}{54}$

$r^{6-3} = 27$

$r^3 = 3^3$

$r = 3$

To find out a, consider $ar^3 = 54$

$a(3)^3 = 54$

$a = \frac{54}{27}$

$a = 2$

Therefore, $a = 2, r = 3$

So, G.P. is 2, 6, 18, 54 ….

22. The fourth term of a G.P. is the square of its second term and the first term is -3. Find its 7th term.

Solution:

From the question it is given that,

The fourth term of a G.P. is the square of its second term = $a_4 = (a_2)^2$

The first term $a_1 = -3$

We know that, $a_n = ar^{n-1}$

$a_4 = ar^{4-1}$

$a_4 = ar^3$

$a_2 = ar$

Now, $ar^3 = (ar)^2$

$ar^3 = a^2r^2$

$\frac{r^3}{r^2} = \frac{a^2}{a}$

$r^{3-2} = a^{2-1}$ $\left[\text{From } \frac{a^m}{a^n} = a^{m-n}\right]$

$r = a$

$a_1 = -3$

$a_7 = ar^{7-1}$

$a_7 = ar^6$

$= -3 \times (-3)^6$

$= -3 \times 729$

$= -2187$

Therefore, the 7th term $a_7 = -2187$

23. If the 4th, 10th and 16th terms of a G.P. are x, y and z respectively, prove that x, y and z are in G.P.

Solution:

From the question it is given that,

$a_4 = x$

$a_{10} = y$

$a_{16} = z$

Now, we have to show that x, y and z are in G.P.

We know that,

$a_n = ar^{n-1}$

$a_4 = ar^{4-1}$

$a_4 = ar^3 = x$

$a_{10} = ar^9 = y$

$a_{16} = ar^{15} = z$

x, y, z are in G.P.

If $y^2 = xy$

Substitute the value of x and y.

$y^2 = (ar^9)^2$

$y^2 = a^2r^{18}$

Then, $xz = ar^2 \times ar^{15}$

= $a^{1+1}r^{3+15}$ [From $a^m \times a^n = a^{m+n}$]

= a^2r^{18}

So, $y^2 = xy$

Therefore, it is proved that x, y, z are in G.P.

24. How many terms of the G.P. $3, \frac{3}{2}, \frac{3}{4}$ are needed to give the sum $\frac{3069}{512}$?

Solution:

From the question it is given that,

Sum of the terms $S_n = \frac{3069}{512}$

First term $a = 3$

Common ratio r = $\frac{\left(\frac{3}{2}\right)}{3}$

$= \left(\frac{3}{2}\right) \times \left(\frac{1}{3}\right)$

= $\frac{1}{2}$

We know that, $S_n = \frac{a(1-r^n)}{(1-r)}$

$\left(\frac{3069}{512}\right) = \frac{3\left[1-\left(\frac{1}{2}\right)^n\right]}{\left(1-\frac{1}{2}\right)}$

$$\left(\frac{3069}{512}\right) = (2 \times 3)\left[1 - \left(\frac{1}{2}\right)^n\right]$$

$$1 - \left(\frac{1}{2}\right)^n = \frac{3069}{(512\times6)}$$

$$\left(\frac{1}{2}\right)^n = \frac{1023}{1024}$$

$$\left(\frac{1}{2}\right)^n = \frac{(1024-1023)}{1024}$$

$$\left(\frac{1}{2}\right)^n = \frac{1}{1024}$$

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$$\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{10}$$

By comparing both LHS and RHS,

$n = 10$

Therefore, there are 10 terms are in the G.P.