

CHAPTER – 11

SECTION FORMULA

Exercise – 11.1

1. Find the co-ordinates of the mid – point of the line segments joining the following pairs of points:

- (i) $(2, -3), (-6, 7)$
- (ii) $(5, -11), (4, 3)$
- (iii) $(a + 3, 5b), (2a - 1, 3b + 4)$

Solution:

Co-ordinates of midpoint of line joining the points (x_1, y_1) and $(x_2, y_2) = \left\{ \frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2} \right\}$

(i) Co-ordinates of midpoint of line joining the points $(2, -3)$ and $(-6, 7) = \left\{ \frac{(2 + (-6))}{2}, \frac{((-3) + 7)}{2} \right\}$

$$= \left(\frac{-4}{2}, \frac{4}{2} \right)$$
$$= (-2, 2)$$

Hence the co-ordinates of midpoint of line joining the points $(2, -3)$ and $(-6, 7)$ is $(-2, 2)$.

(ii) Co-ordinates of midpoint of line joining the points (x_1, y_1) and $(x_2, y_2) = \left\{ \frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2} \right\}$

Co-ordinates of midpoint of line joining the points $(5, -11)$ and $(4, 3) = \left\{ \frac{(5 + 4)}{2}, \frac{((-11) + 3)}{2} \right\}$

$$= \left(\frac{9}{2}, \frac{-8}{2} \right)$$

$$= \left(\frac{9}{2}, -4 \right)$$

Hence the co-ordinates of midpoint of line joining the points $(5, -11)$ and $(4, 3)$ is $\left(\frac{9}{2}, -4 \right)$.

(iii) Co-ordinates of midpoint of line joining the points (x_1, y_1) and $(x_2, y_2) = \left\{ \frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2} \right\}$

Co-ordinates of midpoint of line joining the points $(a + 3, 5b)$ and $(2a - 1, 3b + 4) = \left\{ \frac{(a+3+2a-1)}{2}, \frac{(5b+3b+4)}{2} \right\}$

$$= \left\{ \frac{(3a+2)}{2}, \frac{(8b+4)}{2} \right\}$$

$$= \left\{ \frac{(3a+2)}{2}, (4b + 2) \right\}$$

Hence the co-ordinates of midpoint of line joining the points $(a + 3, 5b)$ and $(2a - 1, 3b + 4)$ is $\left\{ \frac{(3a+2)}{2}, (4b + 2) \right\}$.

2. The co-ordinates of two points A and B are $(-3, 3)$ and $(12, -7)$ respectively. P is a point on the line segment AB such that AP: PB = 2: 3. Find the co-ordinates of P.

Solution:

Let the co-ordinates of P (x, y) divides AB in the ratio m: n.

A $(-3, 3)$ and B $(12, -7)$ are the given points.

Given m: n = 2: 3

$$x_1 = -3, y_1 = 3, x_2 = 12, y_2 = -7, m = 2 \text{ and } n = 3$$

By Section formula $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$$x = \frac{(2 \times 12 + 3 \times -3)}{(2+3)}$$

$$x = \frac{(24-9)}{5}$$

$$x = \frac{15}{5}$$

$$x = 3$$

By section formula $y = \frac{(my_2 + ny_1)}{(m+n)}$

$$y = \frac{(2 \times -7 + 3 \times 3)}{5}$$

$$y = \frac{(-14+9)}{5}$$

$$y = \frac{-5}{5}$$

$$y = -1$$

Hence the co-ordinate of point P are (3, -1).

3. P divides the distance between A (– 2, 1) and B (1, 4) in the ratio of 2: 1. Calculate the co-ordinates of the point P.

Solution:

Let the co-ordinates of P (x, y) divides AB in the ratio m: n.

A (– 2, 1) and B (1, – 4) are the given points.

Given m: n = 2: 1

$$x_1 = -2, y_1 = 1, x_2 = 1, y_2 = 4, m = 2 \text{ and } n = 1$$

By Section formula $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$$x = \frac{(2 \times 1 + 1 \times -2)}{(2+1)}$$

$$x = \frac{(2-2)}{3}$$

$$x = \frac{0}{3}$$

$$x = 0$$

By section formula $y = \frac{(my_2 + ny_1)}{(m+n)}$

$$y = \frac{(2 \times 4 + 1 \times 1)}{(2+1)}$$

$$y = \frac{(8+1)}{3}$$

$$y = \frac{9}{3}$$

$$y = 3$$

Hence the co-ordinate of point P are (0, 3).

4.

- (i) Find the co-ordinates of the points of trisection of the line segment joining the point (3, -3) and (6, 9).
- (ii) The line segment joining the points (3, -4) and (1, 2) is trisected at the points P and Q. If the coordinates of P and Q are (p, -2) and $(\frac{5}{3}, q)$ respectively, find the values of p and q.

Solution:



(i) Let P and Q be the points of trisection of AB i.e. $AP = PQ = QB$

Given A (3, -3) and B (6, 9)

$$x_1 = 3, y_1 = -3, x_2 = 6, y_2 = 9$$

P (x, y) divides AB internally in the ratio 1: 2.

$$m : n = 1 : 2$$

By applying the section formula, the coordination of P are as follow.

$$\text{By Section formula } x = \frac{(mx_2 + nx_1)}{(m+n)}$$

$$x = \frac{(1 \times 6 + 2 \times 3)}{(1+2)}$$

$$x = \frac{(6+6)}{3}$$

$$x = \frac{12}{3}$$

$$x = 4$$

$$\text{By section formula } y = \frac{(my_2 + ny_1)}{(m+n)}$$

$$y = \frac{(1 \times 9 + 2 \times -3)}{(2+1)}$$

$$y = \frac{(9-6)}{3}$$

$$y = \frac{3}{3}$$

$$y = 1$$

Hence the co-ordinate of point P are (4, 1).

Now, Q also divides AB internally in the ratio 2: 1.

$$m : n = 1 : 2$$

By applying the section formula, the coordination of P are as follow.

By Section formula $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$$x = \frac{(2 \times 6 + 1 \times 3)}{(1+2)}$$

$$x = \frac{(12+3)}{3}$$

$$x = \frac{15}{3}$$

$$x = 5$$

By section formula $y = \frac{(my_2 + ny_1)}{(m+n)}$

$$y = \frac{(2 \times 9 + 1 \times -3)}{(2+1)}$$

$$y = \frac{(18-3)}{3}$$

$$y = \frac{15}{3}$$

$$y = 5$$

Hence the co-ordinate of point Q are (5, 5).

- (ii) Let P (p, -2) and Q $\left(\frac{5}{3}, q\right)$ be the points of trisection of AB i.e.,
AP = PQ = QB



Given A (3, -4) and B (1, 2)

$$x_1 = 3, y_1 = -4, x_2 = 1, y_2 = 2$$

P (p, -2) divides AB internally in the ratio 1: 2.

By Section formula $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$$p = \frac{(1 \times 1 + 2 \times 3)}{(1+2)}$$

$$p = \frac{(1+6)}{3}$$

$$p = \frac{7}{3}$$

Now, Q also divides AB internally in the ratio 2: 1.

$$m : n = 1 : 2$$

Q $\left(\frac{5}{3}, q\right)$ divides AB internally in the ratio 2: 1.

By Section formula $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$$q = \frac{(2 \times 2 + 1 \times 4)}{(2+1)}$$

$$q = \frac{(4+4)}{3}$$

$$q = \frac{0}{3}$$

$$q = 0$$

Hence the value of p and q are $\frac{7}{3}$ and 0 respectively.

5.

- (i) The line segment joining the points A (3, 2) and B (5, 1) is divided at the point P in the ratio 1: 2 and it lies on the line $3x - 18y + k = 0$. Find the value of k .
- (ii) A point P divides the line segment joining the points A (3, -5) and B (-4, 8) such that $\frac{AP}{PB} = \frac{k}{1}$ if P lies on the line $x + y = 0$, then find the value of k .

Solution:

(i) Let the co-ordinates of P (x, y) divides AB in the ratio m: n.

A (3, 2) and B (5, 1) are the given points.

Given m: n = 1: 2

$$x_1 = 3, y_1 = 2, x_2 = 5, y_2 = 1, m = 1 \text{ and } n = 2$$

$$\text{By Section formula } x = \frac{(mx_2 + nx_1)}{(m+n)}$$

$$x = \frac{(1 \times 5 + 2 \times 3)}{(1+2)}$$

$$x = \frac{(5+6)}{3}$$

$$x = \frac{11}{3}$$

$$\text{By section formula } y = \frac{(my_2 + ny_1)}{(m+n)}$$

$$y = \frac{(1 \times 1 + 2 \times 2)}{(1+2)}$$

$$y = \frac{(1+4)}{3}$$

$$y = \frac{5}{3}$$

Given P lies on the line $3x - 18y + k = 0$

Substitute x and y in above equation

$$3 \times \left(\frac{11}{3}\right) - 18 \times \left(\frac{5}{3}\right) + k = 0$$

$$11 - 30 + k = 0$$

$$-19 + k = 0$$

$$k = 19$$

Hence the value of k is 19.

(ii) Let the co-ordinates of P (x, y) divides AB in the ratio m: n.

A (3, -5) and B (-4, 8) are the given points.

$$\text{Given } \frac{AP}{PB} = \frac{k}{1}$$

$$m : n = 1 : 2$$

$$x_1 = 3, y_1 = -5, x_2 = -4, y_2 = 8, m = k \text{ and } n = 1$$

$$\text{By Section formula } x = \frac{(mx_2 + nx_1)}{(m+n)}$$

$$x = \frac{(k \times -4 + 1 \times 3)}{(k+1)}$$

$$x = \frac{(-4k+3)}{(k+1)}$$

$$\text{By section formula } y = \frac{(my_2 + ny_1)}{(m+n)}$$

$$y = \frac{(k \times 8 + 1 \times -5)}{(k+1)}$$

$$y = \frac{(8k+3)}{(k+1)}$$

$$\text{Co-ordinate of P is } \left(\frac{(-4k+3)}{(k+1)}, \frac{(8k+3)}{(k+1)} \right)$$

Given P lies on the line $x + y = 0$

Substitute x and y in above equation

$$\frac{(-4k+3)}{(k+1)} + \frac{(8k+3)}{(k+1)} = 0$$

$$(-4k + 3) + (8k - 5) = 0$$

$$4k - 2 = 0$$

$$4k = 2$$

$$k = \frac{2}{4} = \frac{1}{2}$$

Hence the value of k is $\frac{1}{2}$.

6. Find the coordinates of the point which is three-fourth of the way from A (3, 1) to B (-2, 5).

Solution:



Let P be the point which is three-fourth of the way from A (3, 1) to B (-2, 5).

$$\frac{AP}{AB} = \frac{3}{4}$$

$$AB = AP + PB$$

$$\frac{AP}{AB} = \frac{AP}{AP+PB} = \frac{3}{4}$$

$$4AP = 3AP + 3PB$$

$$AP = 3PB$$

$$\frac{AP}{AB} = \frac{3}{4}$$

The ratio m: n = 3: 1

$$x_1 = 3, y_1 = 1, x_2 = -2, y_2 = 5$$

By Section formula $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$$x = \frac{(3 \times 5 + 1 \times 1)}{(3+1)}$$

$$x = \frac{(-6+3)}{4}$$

$$x = \frac{-3}{4}$$

By section formula $y = \frac{(my_2 + ny_1)}{(m+n)}$

$$y = \frac{(3 \times 5 + 1 \times 1)}{(3+1)}$$

$$y = \frac{(15+1)}{4}$$

$$y = \frac{16}{4}$$

$$y = 4$$

Hence the co-ordinates of P are $\left(\frac{-3}{4}, 4\right)$.

7. Point P (3, – 5) is reflected P' in the x-axis. Also P on reflection in the y-axis is mapped as P''.

- (i) Find the co-ordinates to P' and P''**
- (ii) Compute the distance P'P''.**
- (iii) Find the middle point of the line segment P'P''.**
- (iv) On which co-ordinate axis does the middle point of the line segment P'P'' lie?**

Solution:

(i) The image of P (3, – 5) when reflected in x-axis will be (3, 5).

When you reflect a point across the x-axis, the x-coordinate remains the same, but the y-coordinate is transformed into its opposite (its sign is changed).

Co-ordinates of P' = (3, 5)

Image of P (3, – 5) when reflected in y-axis will be (-3, – 5).

When you reflect a point across the y-axis, the y-coordinate remains the same, but the x-coordinate is transformed into the opposite (its sign is changed).

Co-ordinates of $P'' = (-3, -5)$

(ii) Let $P'(x_1, y_1)$ and $P''(x_2, y_2)$ be the given points

By distance formula $d(P'P'') = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Co-ordinates of $P' = (3, 5)$

Co-ordinates of $P'' = (-3, -5)$

Here $x_1 = 3, y_1 = 5, x_2 = -3, y_2 = -5$

$D(P'P'') = \sqrt{[(-3 - 3)^2 + (-5 - 5)^2]}$

$$= \sqrt{[(-6)^2 + (-10)^2]}$$

$$= \sqrt{36 + 100}$$

$$= \sqrt{136}$$

$$= \sqrt{(4 \times 34)}$$

$$= 2\sqrt{34}$$

Hence the distance between P' and P'' is $2\sqrt{34}$ units.

(iii) **Co-ordinates of $P' = (3, 5)$**

Co-ordinates of $P'' = (-3, -5)$

Here $x_1 = 3, y_1 = 5, x_2 = -3, y_2 = -5$

Let $Q(x, y)$ be the midpoint of $P'P''$

By midpoint formula,

$$x = \frac{(x_1 + x_2)}{2}$$

$$y = \frac{(y_1 + y_2)}{2}$$

$$x = \frac{(3 + (-3))}{2} = \frac{0}{2} = 0$$

$$x = \frac{(5 + (-5))}{2} = \frac{0}{2} = 0$$

Hence the co-ordinate of midpoint of P'P'' is (0, 0).

(iv) Co-ordinates of P' = (3, -5)

Co-ordinates of P'' = (-3, -5)

Here $x_1 = 3, y_1 = -5, x_2 = -3, y_2 = -5$

Let R (x, y) be the midpoint of P'P''

By midpoint formula,

$$x = \frac{(x_1 + x_2)}{2}$$

$$y = \frac{(y_1 + y_2)}{2}$$

$$x = \frac{(3 + (-3))}{2} = \frac{0}{2} = 0$$

$$x = \frac{(-5 + (-5))}{2} = \frac{-10}{2} = -5$$

So the co-ordinate of midpoint of P'P'' is (0, -5).

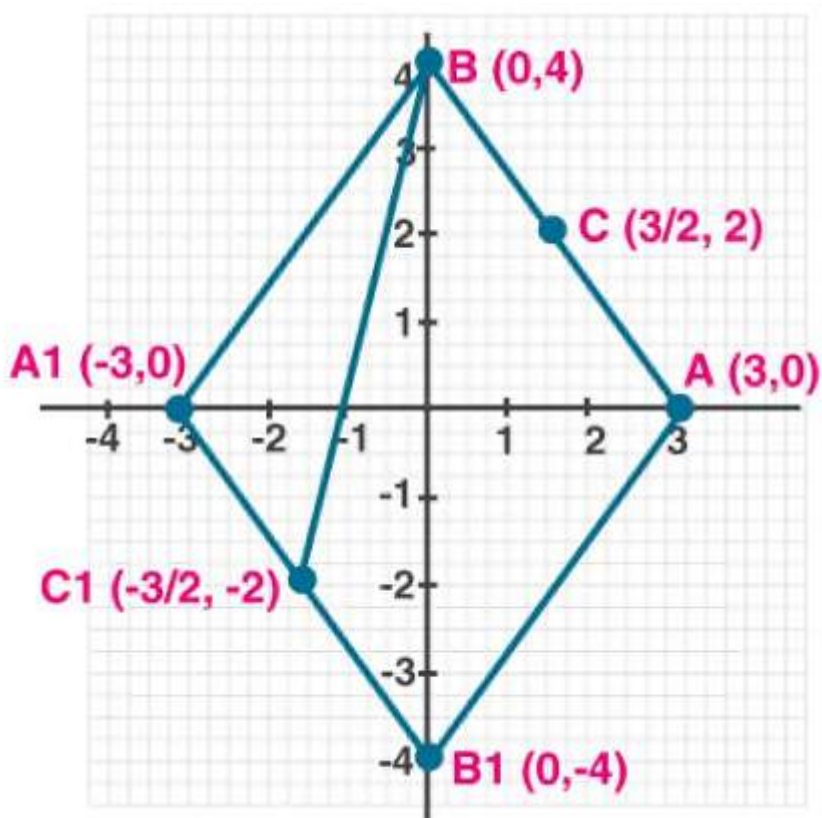
Here x co-ordinate is zero.

Hence the point lies on y-axis.

**8. Use graph paper for this question. Take 1 cm = 1 unit on both axes.
Plot the points A (3, 0) and B (0, 4).**

- (i) Write down the co-ordinates of A_1 , the reflection of A in the y -axis.
- (ii) Write down the co-ordinate of B_1 , the reflection of B in the x -axis.
- (iii) Assign the special name to the quadrilateral ABA_1B_1 .
- (iv) If C is the midpoint of AB . Write down the co-ordinates of the point C_1 , the reflection of C in the origin.
- (v) Assign the special name to quadrilateral ABC_1B_1 .

Solution:



- (i) Co-ordinate of point A are $(3, 0)$.

When you reflect a point across the y -axis, the y -coordinate remains the same, but the x -coordinate is transformed into its opposite (its sign is changed)

Hence the reflection of A in the y -axis is $(-3, 0)$.

(ii) Co-ordinate of point B are (0, 4).

When you reflect a point across the x-axis, the x-coordinate remains the same, but the y-coordinate is transformed into its opposite (its sign is changed)

Hence the reflection of B in the x-axis is (0, -4).

(iii) The quadrilateral ABA₁B₁ will be a rhombus.

(iv) Let C be midpoint of AB

Co-ordinate of C = $\left(\frac{(3+0)}{2}, \frac{(0+4)}{2}\right) = \left(\frac{3}{2}, 2\right)$ (midpoint formula)

In a point reflection in the origin, the image of the point (x, y) is the point (-x, -y).

Hence the reflection of C in the origin is $\left(\frac{-3}{2}, -2\right)$

(v) In quadrilateral ABC₁B₁, ABB₁C₁

Hence the quadrilateral ABC₁B₁ is a trapezium.

9. The line segment joining A (-3, 1) and B (5, -4) is a diameter of a circle whose centre is C. Find the co-ordinates of the point C. (1990)

Solution:

Given Co-ordinates of A = (-3, 1)

Co-ordinates of B = (5, -4)

Here $x_1 = -3, y_1 = 1, x_2 = 5, y_2 = -4$

Let C (x, y) be the midpoint of AB

By midpoint formula,

$$x = \frac{(x_1 + x_2)}{2}$$

$$y = \frac{(y_1+y_2)}{2}$$

$$x = \frac{(-3+5)}{2} = \frac{2}{2} = 1$$

$$y = \frac{(1+(-4))}{2} = \frac{-3}{2}$$

Hence the co-ordinate of midpoint of AB is C $\left(1, \frac{-3}{2}\right)$.

10. The mid-point of the line segment joining the points (3m, 6) and (-4, 3n) is (1, 2m – 1). Find the values of m and n.

Solution:

Let the midpoint of line joining the points (3m, 6) and (-4, 3n) is (1, 2m – 1).

$$\text{Here } x_1 = 3m, y_1 = 6, x_2 = -4, y_2 = 3n$$

$$x = 1, y = 2m - 1$$

By midpoint formula,

$$x = \frac{(x_1+x_2)}{2}$$

$$1 = \frac{(3m+4)}{2}$$

$$3m - 4 = 2$$

$$3m = 2 + 4$$

$$m = \frac{6}{3} = 2$$

By midpoint formula,

$$y = \frac{(y_1+y_2)}{2}$$

$$2m - 1 = \frac{(6+3n)}{2}$$

$$4m - 2 = 6 + 3n$$

Put $m = 2$ in above equation

$$4 \times 2 - 2 = 6 + 3n$$

$$8 - 2 - 6 = 3n$$

$$3n = 0$$

$$n = 0$$

Hence the value of m and n are 2 and 0 respectively.

11. The co-ordinates of the mid-point of the line segment PQ are (1, -2). The co-ordinates of P are (-3, 2). Find the co-ordinates of Q. (1992).

Solution:

Let the co-ordinates of Q be (x_2, y_2)

Given co-ordinate of P = $(-3, 2)$

Co-ordinates of midpoint = $(1, -2)$

Here $x_1 = -3, y_1 = 2, x_2 = 1, y_2 = -2$

By midpoint formula,

$$x = \frac{(x_1 + x_2)}{2}$$

$$1 = \frac{(-3 + x_2)}{2}$$

$$2 = -3 + x_2$$

$$x_2 = 2 + 3 = 5$$

By midpoint formula,

$$y = \frac{(y_1 + y_2)}{2}$$

$$-2 = \frac{(2 + y_2)}{2}$$

$$-4 = 2 + y_2$$

$$y_2 = -4 - 2$$

$$y_2 = -6$$

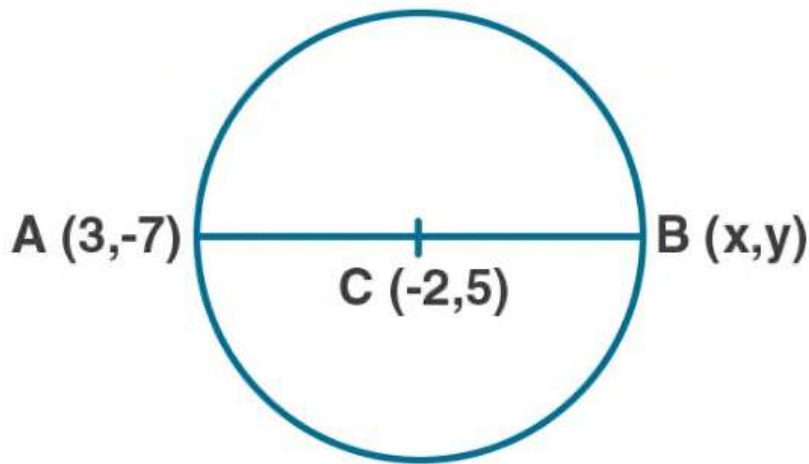
Hence the co-ordinates of Q are (5, -6).

12. AB is a diameter of a circle with centre C (-2, 5). If point A is (3, -7). Find:

(i) The length of radius AC.

(ii) The coordinates of B.

Solution:



(i) Length of radius AC = d(A, C)

Co-ordinates of A = (3, -7)

Co-ordinates of C = (-2, 5)

Here $x_1 = 3, y_1 = -7, x_2 = -2, y_2 = 5$

By distance formula $d(A, C) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

$$= \sqrt{[((-2) - 3)^2 + (5 - (-7))^2]}$$

$$= \sqrt{[(-5)^2 + (12)^2]}$$

$$= \sqrt{[25 + 144]}$$

$$= \sqrt{169}$$

$$= 13$$

Hence the radius is 13 units.

(ii) Given AB is the diameter and C is the centre of the circle.

By midpoint formula, $-2 = \frac{(x+3)}{2}$

$$-4 = x + 3$$

$$x = -4 - 3 = -7$$

By midpoint formula, $5 = \frac{(-7+y)}{2}$

$$10 = -7 + y$$

$$y = 10 + 7 = 17$$

Hence the co-ordinates of B are $(-7, 17)$.

13. Find the reflection (image) of the point (5, -3) in the point (-1, 3).

Solution:

Let the co-ordinates of the image of the point P (5, -3) be

P1 (x, y) in the point (-1, 3) then the point (-1, 3) will be the midpoint of PP1.

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$$-1 = \frac{(5+x_2)}{2} \quad [x = -1, x_1 = 5]$$

$$-2 = 5 + x_2$$

$$x_2 = -2 - 5 = -7$$

By midpoint formula, $y = \frac{(y_1+y_2)}{2}$

$$3 = \frac{(-3+y_2)}{2} \quad [y = 3, y_1 = -3]$$

$$6 = -3 + y_2$$

$$y_2 = 6 + 3 = 9$$

Hence the co-ordinates of the image of P is (-7, -9).

14. The line segment joining A $\left(-1, \frac{5}{3}\right)$ the points B (a, 5) is divided in the ratio 1: 3 at P, the point where the line segment AB intersects y-axis. Calculate

- (i) The value of a**
- (ii) The co-ordinates fo P. (1994)**

Solution:

- (i) Let P (x, y) divides the line segment joining the points A $\left(-1, \frac{5}{3}\right)$, B (a, 5) in the ratio 1: 3,**

Here m: n = 1: 3

$$x_1 = -1, y_1 = \frac{5}{3}, x_2 = a, y_2 = 5$$

By Section formula $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$$x = \frac{(1 \times a + 3 \times -1)}{(1+3)}$$

$$x = \frac{(a-3)}{4} \quad \dots \text{ (i)}$$

By section formula $y = \frac{(my_2 + ny_1)}{(m+n)}$

$$y = \frac{\left(\frac{1 \times 5 + 3 \times 5}{3}\right)}{(3+1)}$$

$$y = \frac{(5+5)}{4}$$

$$y = \frac{10}{4}$$

$$y = \frac{5}{2} \quad \dots \text{ (ii)}$$

Given P meets y-axis. So its x co-ordinate will be zero.

$$\text{i.e. } \frac{(a-3)}{4} = 0$$

$$a - 3 = 0$$

$$a = 3$$

$$\text{(ii)} \quad x = \frac{(a-3)}{4} \quad [\text{From (i)}]$$

Substitute $a = 3$ in above equation.

$$x = \frac{(3-3)}{4} = 0$$

$$y = \frac{5}{2} \quad [\text{From (ii)}]$$

Hence the co-ordinates of P are $\left(0, \frac{5}{2}\right)$.

15. The point P (-4, 1) divides the line segment joining the points A (2, -2) and B in the ratio of 3: 5. Find the point B.

Solution:

Let the co-ordinates of B be (x_2, y_2)

Given co-ordinates of A = (2, -2)

Co-ordinates of P = (-4, 1)

Ratio m: n = 3: 5

$x_1 = 2, y_1 = -2, x_2 = -4, y_2 = 1$

P divides AB in the ratio 3: 5

By section formula, $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$$-4 = \frac{(3 \times x_2 + 5 \times 2)}{(3+5)}$$

$$-4 = \frac{(3x_2 + 10)}{8}$$

$$-32 = 3x_2 + 10$$

$$3x_2 = -32 - 10 = -42$$

$$x_2 = \frac{-42}{3} = -14$$

By section formula $y = \frac{(my_2 + ny_1)}{(m+n)}$

$$1 = \frac{(3 \times y_2 + 5 \times (-2))}{(3+5)}$$

$$1 = \frac{(3y_2 - 10)}{8}$$

$$8 = 3y_2 - 10$$

$$3y_2 = 8 + 10 = 18$$

$$y = \frac{18}{3} = 6$$

Hence the co-ordinates of B are (-14, 6).

16.

- (i) In what ratio does the point (5, 4) divide the line segment joining the points (2, 1) and (7, 6)?
- (ii) In what ratio does the point (-4, b) divide the line segment joining the point P (2, -2), Q (-14, 6)?

Solution:

- (i) Let the ratio that the point (5, 4) divide the line segment joining the points (2, 1) and (7, 6) be m: n.

Here $x_1 = 2, y_1 = 1, x_2 = 7, y_2 = 6, x = 5, y = 4$

By section formula, $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$$5 = \frac{(m \times 7 + n \times 2)}{(m+n)}$$

$$5 = \frac{(7m + 2n)}{(m+n)}$$

$$5(m + n) = 7m + 2n$$

$$5m + 5n = 7m + 2n$$

$$5m - 7m = 2n - 5n$$

$$-2m = -3n$$

$$\frac{m}{n} = \frac{-3}{-2} = \frac{3}{2}$$

Hence the ratio m: n is 3: 2.

- (ii) Let the ratio that the point $(-4, b)$ divide the line segment joining the point $(2, -2)$ and $(-14, 6)$ be $m: n$.

Here $x_1 = 2, y_1 = -2, x_2 = -14, y_2 = 6, x = -4, y = b$

By section formula, $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$$-4 = \frac{(m \times -14 + n \times 2)}{(m+n)}$$

$$-4 = \frac{(-14m + 2n)}{(m+n)}$$

$$-4(m+n) = -14m + 2n$$

$$-4m - 4n = -14m + 2n$$

$$-4m + 14m = 2n + 4n$$

$$10m = 6n$$

$$\frac{m}{n} = \frac{6}{10} = \frac{3}{5}$$

Hence the ratio $m: n$ is $3: 5$.

By section formula $y = \frac{(my_2 + ny_1)}{(m+n)}$

$$b = \frac{(3 \times 6 + 5 \times -2)}{(3+5)}$$

$$b = \frac{(18 - 10)}{8}$$

$$b = \frac{8}{8}$$

$$b = 1$$

Hence the value of b is 1 and the ratio $m: n$ is $3: 5$.

17. The line segment joining A $(2, 3)$ and B $(6, -5)$ is intercepted by the x-axis at the point K. Write the ordinate of the point k. Hence,

find the ratio in which K divides AB. Also, find the coordinates of the point K.

Solution:

Since the point K is on x-axis, its y co-ordinate is zero.

Let the point K be (x, 0).

Let the point K divides the line segment joining A (2, 3) and B (6, -5) in the ratio m: n.

Here $x_1 = 2, y_1 = 3, x_2 = 6, y_2 = -5, y = 0$

By section formula $y = \frac{(my_2 + ny_1)}{(m+n)}$

$$0 = \frac{(m \times -5 + n \times 3)}{(m+n)}$$

$$0 = \frac{(-5m + 3n)}{m+n}$$

$$-5m + 3n = 0$$

$$-5m = -3n$$

$$\frac{m}{n} = \frac{-3}{-5} = \frac{3}{5}$$

Hence the point K divides the line segment in the ratio 3: 5.

By section formula, $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$$x = \frac{(3 \times 6 + 5 \times 2)}{(3+5)}$$

$$x = \frac{(18+10)}{8}$$

$$x = \frac{28}{8} = \frac{7}{2}$$

Hence the co-ordinate of K are $\left(\frac{7}{2}, 0\right)$

18. If A (-4, 3) and B (8, -6),

(i) Find the length of AB.

**(ii) In what ratio is the line joining AB, divided by the x-axis?
(2008)**

Solution:

(i) Given points are A (-4, 3) and B (8, -6).

Here $x_1 = -4, y_1 = 3$

$x_2 = 8, y_2 = -6$

By distance formula $d(AB) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

$$D(AB) = \sqrt{[(8 - (-4))^2 + ((-6) - 3)^2]}$$

$$D(AB) = \sqrt{[(12)^2 + (-9)^2]}$$

$$D(AB) = \sqrt{[144 + 81]}$$

$$D(AB) = \sqrt{225}$$

$$D(AB) = 15$$

Hence the length of AB is 15 units.

(ii) Let $m : n$ be the ratio in which the line AB is divided by the x-axis.

Since the line meets x-axis, its y co-ordinate is zero.

By section formula, $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$$0 = \frac{(m \times -6 + n \times 3)}{(m+n)}$$

$$0 = \frac{(-6m+3n)}{(m+n)}$$

$$-6m + 3n = 0$$

$$-6m = -3n$$

$$\frac{m}{n} = \frac{-3}{-6} = \frac{3}{6} = \frac{1}{2}$$

Hence the ratio is 1: 2.

19.

- (i) Calculate the ratio in which the line segment joining (3, 4) and (-2, 1) is divided by the y-axis.**
- (ii) In what ratio does the line $x - y - 2 = 0$ divide the line segment joining the points (3, -1) and (8, 9)? Also, find the coordinates of the point of division.**

Solution:

- (i) Let $m : n$ be the ratio in which the line segment joining (3, 4) and (-2, 1) is divided by the y-axis.**

Since the line meets y-axis, its x co-ordinate is zero.

$$\text{Here } x_1 = 3, y_1 = 4$$

$$x_2 = -2, y_2 = 1$$

$$\text{By section formula, } x = \frac{(mx_2 + nx_1)}{(m+n)}$$

$$0 = \frac{(m \times -2 + n \times 3)}{(m+n)}$$

$$0 = \frac{(-2m + 3n)}{(m+n)}$$

$$0 = -2m + 3n$$

$$2m = 3n$$

$$\frac{m}{n} = \frac{3}{2}$$

Hence the ratio m: n is 3: 2.

(ii) Let the line $x - y - 2 = 0$ divide the line segment joining the points (3, -1) and (8, 9) in the ratio m: n at the point P (x, y).

Here $x_1 = 3, y_1 = -1$

$x_2 = 8, y_2 = 9$

By section formula, $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$$x = \frac{(m \times 8 + n \times 3)}{(m+n)}$$

$$x = \frac{(8m + 3n)}{(m+n)} \quad \dots \text{ (i)}$$

By section formula $y = \frac{(my_2 + ny_1)}{(m+n)}$

$$y = \frac{(m \times 9 + n \times -1)}{(m+n)}$$

$$y = \frac{(9m - n)}{(m+n)} \quad \dots \text{ (ii)}$$

Since the point P (x, y) lies on the line $x - y - 2 = 0$,

Equation (i) and (ii) will satisfy the equation $x - y - 2 = 0 \quad \dots \text{ (iii)}$

Substitute (i) and (ii) in (iii)

$$\left[\frac{(8m + 3n)}{(m+n)} \right] - \left[\frac{(9m - n)}{(m+n)} \right] - 2 = 0$$

$$\left[\frac{(8m + 3n)}{(m+n)} \right] - \left[\frac{(9m - n)}{(m+n)} \right] - \left[\frac{2(m+n)}{(m+n)} \right] = 0$$

$$8m + 3n - (9m - n) - 2(m + n) = 0$$

$$8m + 3n - 9m + n - 2m - 2n = 0$$

$$-3m + 2n = 0$$

$$-3m = -2n$$

$$\frac{m}{n} = \frac{-2}{-3} = \frac{2}{3}$$

Hence the ratio m: n is 2: 3.

Substitute m and n in (i)

$$x = \frac{(8m+3m)}{(m+n)}$$

$$x = \frac{(8 \times 2 + 3 \times 3)}{(2+3)}$$

$$x = \frac{(16+9)}{5}$$

$$x = \frac{25}{5} = 5$$

Substitute m and n in (ii)

$$y = \frac{(9m-n)}{(m+n)}$$

$$y = \frac{(9 \times 2 - 3)}{(2+3)}$$

$$y = \frac{(18-3)}{5}$$

$$y = \frac{15}{5} = 3$$

Hence the co-ordinates of P are (5, 3).

20. Given a line segment AB joining the points A (-4, 6) and B (8, -3). Find:

- (i) The ratio in which AB is divided by the y-axis.**
- (ii) Find the coordinates of the point of intersection.**
- (iii) The length of AB.**

Solution:

- (i) Let $m : n$ be the ratio in which the line segment joining A (-4, 6) and B (8, -3) is divided by the y-axis.

Since the line meets y-axis, its x co-ordinate is zero.

$$\text{Here } x_1 = -4, y_1 = 6$$

$$x_2 = 8, y_2 = -3$$

$$\text{By section formula, } x = \frac{(mx_2 + nx_1)}{(m+n)}$$

$$0 = \frac{(m \times 8 + n \times -4)}{(m+n)}$$

$$0 = \frac{(8m + (-4n))}{(m+n)}$$

$$0 = 8m + (-4n)$$

$$8m = 4n$$

$$\frac{m}{n} = \frac{4}{8} = \frac{1}{2}$$

Hence the ratio m: n is 1: 2.

- (ii) By section formula $y = \frac{(my_2 + ny_1)}{(m+n)}$

Substitute m and n in above equation

$$y = \frac{(1 \times (-3) + 2 \times 6)}{(1+2)}$$

$$y = \frac{(-3+12)}{3}$$

$$y = \frac{9}{3} = 3$$

So the co-ordinates of the point of intersection are (0, 3).

- (iii) By distance formula $d(AB) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

$$D(AB) = \sqrt{[(8 - (-4))^2 + ((-3) - 6)^2]}$$

$$D(AB) = \sqrt{[(12)^2 + (-9)^2]}$$

$$D(AB) = \sqrt{[144 + 81]}$$

$$D(AB) = \sqrt{225}$$

$$D(AB) = 15$$

Hence the length of AB is 15 units.

21.

- (i) Write down the co-ordinates of the point P that divides the line joining A (-4, 1) and B (17, 10) in the ratio 1: 2.**
- (ii) Calculate the distance OP where O is the origin.**
- (iii) In what ratio does the y-axis divide the line AB?**

Solution:

- (i) Let P (x, y) divides the line segment joining the points A (-4, 1), B (17, 10) in the ratio 1: 2,**

$$\text{Here } x_1 = -4, y_1 = 1$$

$$x_2 = 17, y_2 = 10$$

$$m : n = 1 : 2$$

$$\text{By section formula, } x = \frac{(mx_2 + nx_1)}{(m+n)}$$

$$x = \frac{(1 \times 17 + 2 \times -4)}{(1+2)}$$

$$x = \frac{(17+8)}{3}$$

$$x = \frac{9}{3}$$

$$x = 3$$

$$\text{By section formula } y = \frac{(my_2 + ny_1)}{(m+n)}$$

$$y = \frac{(1 \times 10 + 2 \times 1)}{(1+2)}$$

$$y = \frac{(10+2)}{3}$$

$$y = \frac{12}{3} = 4$$

Hence the co-ordinates of the point P are (3, 4).

(ii) Since O is the origin, the co-ordinates of O are (0, 0).

$$\text{By distance formula } d(OP) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

$$D(OP) = \sqrt{[(0 - 3)^2 + (0 - 4)^2]}$$

$$D(OP) = \sqrt{[(3)^2 + (4)^2]}$$

$$D(OP) = \sqrt{(9 + 16)}$$

$$D(OP) = \sqrt{25}$$

$$D(OP) = 5$$

Hence the distance OP is 5 units.

(iii) Let $m : n$ be the ratio in which y-axis divide line AB.

Since AB touches y-axis, its x co-ordinate will be zero.

$$\text{Here } x_1 = -4, y_1 = 1$$

$$x_2 = 17, y_2 = 10$$

$$\text{By section formula, } x = \frac{(mx_2 + nx_1)}{(m+n)}$$

$$0 = \frac{(m \times 17 + n \times -4)}{(m+n)}$$

$$0 = \frac{(17m+4n)}{(m+n)}$$

$$0 = 17m - 4n$$

$$17m = 4n$$

$$\frac{m}{n} = \frac{4}{17}$$

$$m : n = 4 : 17$$

Hence the ratio in which y-axis divide line AB is 4: 17.

22. Calculate the length of the median through the vertex A of the triangle ABC with vertices A (7, - 3), B (5, 3) and C (3, - 1).

Solution:

Let M (x, y) be the median of ΔABC through A to BC.

M will be the midpoint of BC.

$$x_1 = 5, y_1 = 3$$

$$x_2 = 3, y_2 = -1$$

$$\text{By midpoint formula, } x = \frac{(x_1+x_2)}{2}$$

$$x = \frac{(5+3)}{2} = \frac{8}{2} = 4$$

$$\text{By midpoint formula, } y = \frac{(y_1+y_2)}{2}$$

$$y = \frac{(3+(-1))}{2} = \frac{2}{2} = 1$$

Hence the co-ordinates of M are (4, 1).

$$\text{By distance formula } d(AM) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

$$x_1 = 7, y_1 = -3$$

$$x_2 = 4, y_2 = 1$$

$$D(AM) = \sqrt{[(4 - 7)^2 + (1 - (-3))^2]}$$

$$D(AM) = \sqrt{[(-3)^2 + (4)^2]}$$

$$D(AM) = \sqrt{(9 + 16)}$$

$$D(AM) = \sqrt{25} = 5$$

Hence the distance AM is 5 units.

23. Three consecutive vertices of a parallelogram ABCD are A (1, 2), B (1, 0) and C (4, 0). Find the fourth vertex D.

Solution:

Let M be the midpoint of the diagonals of the parallelogram ABCD.

Co-ordinate of M will be the midpoint of diagonal AC.

Given point are A (1, 2), B (1, 0) and C (4, 0).

Consider the AC.

$$x_1 = 1, y_1 = 2$$

$$x_2 = 4, y_2 = 0$$

$$\text{By midpoint formula, } x = \frac{(x_1 + x_2)}{2}$$

$$x = \frac{(1+4)}{2} = \frac{5}{2}$$

$$\text{By midpoint formula, } y = \frac{(y_1 + y_2)}{2}$$

$$y = \frac{(2+0)}{2} = \frac{2}{2} = 1$$

Hence the co-ordinates of M are $\left(\frac{5}{2}, 1\right)$.

M is also the midpoint of diagonal BD.

Consider line BD and M as midpoint.

$$x_1 = 1, y_1 = 0$$

$$x = \frac{5}{2}, y = 1$$

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$$\frac{5}{2} = \frac{(1+x_2)}{2}$$

$$5 = 1 + x_2$$

$$x_2 = 5 - 1 = 4$$

By midpoint formula, $y = \frac{(y_1+y_2)}{2}$

$$1 = \frac{(0+y_2)}{2}$$

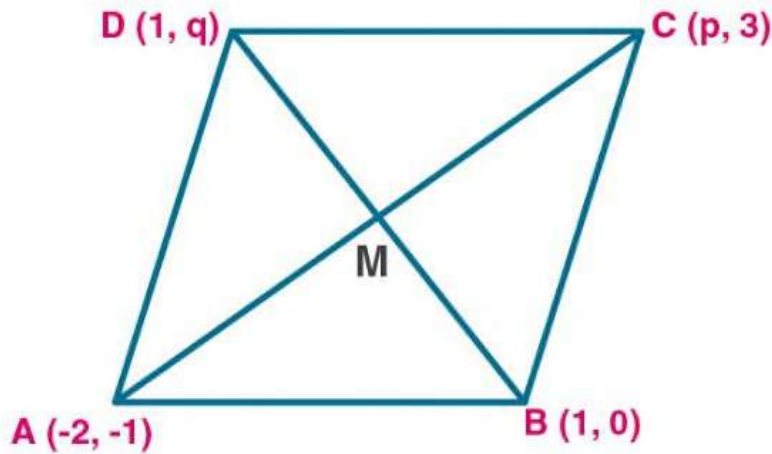
$$1 = \frac{y_2}{2}$$

$$y_2 = 2$$

Hence the co-ordinates of D are (4, 2).

24. If the points A (-2, -1), B (1, 0), C (p, 3) and D (1, q) form a parallelogram ABCD, find the values of p and q.

Solution:



Given vertices of the parallelogram are A (-2, -1), B (1, 0), C (p, 3) and D (1, q).

Let M (x, y) be the midpoint of the diagonals of the parallelogram ABCD.

Diagonals AC and BD bisect each other at M.

When M is the midpoint of AC

By midpoint formula,

$$x = \frac{(-2+p)}{2} = \frac{(p-2)}{2} \quad \dots (i)$$

$$y = \frac{(-1+3)}{2} = \frac{2}{2} = 1 \quad \dots (ii)$$

When M is the midpoint of BD

By midpoint formula,

$$x = \frac{(1+1)}{2} = \frac{2}{2} = 1 \quad \dots (iii)$$

$$y = \frac{(q+0)}{2} = \frac{q}{2} \quad \dots (iv)$$

Equating (i) and (ii), we get

$$\frac{(p-2)}{2} = 1$$

$$p - 2 = 2$$

$$p = 2 + 2 = 4$$

Equating (iii) and (iv), we get

$$\frac{q}{2} = 1$$

$$q = 2$$

Hence the value of p and q are 4 and 2 respectively.

25. If two vertices of a parallelogram are (3, 2) (-1, 0) and its diagonals meet at (2, -5), find the other two vertices of the parallelogram.

Solution:

Let A (3, 2) and B (-1, 0) be the two vertices of the parallelogram ABCD.

Let M (2, -5) be the point where diagonals meet.

Since the diagonals of the parallelogram bisect each other, M is the midpoint of AC and BD.

Consider A – M – C.

Let Co-ordinate of C be (x_2, y_2)

$$x_1 = 3, y_1 = 2$$

$$x = 2, y = -5$$

By midpoint formula, $x = \frac{(x_1 + x_2)}{2}$

$$2 = \frac{(3 + x_2)}{2}$$

$$3 + x_2 = 4$$

$$x_2 = 4 - 3 = 1$$

By midpoint formula, $y = \frac{(y_1+y_2)}{2}$

$$-5 = \frac{(2+y_2)}{2}$$

$$-10 = 2 + y_2$$

$$y_2 = -10 - 2 = -12$$

Hence the co-ordinates of the point C are (1, -12).

Consider B – M – D

Let co-ordinate of D be (x_2, y_2)

$$x_1 = -1, y_1 = 0$$

$$x = 2, y = -5$$

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$$2 = \frac{(-1+x_2)}{2}$$

$$-1 + x_2 = 4$$

$$x_2 = 4 + 1 = 5$$

By midpoint formula, $y = \frac{(y_1+y_2)}{2}$

$$-5 = \frac{(0+y_2)}{2}$$

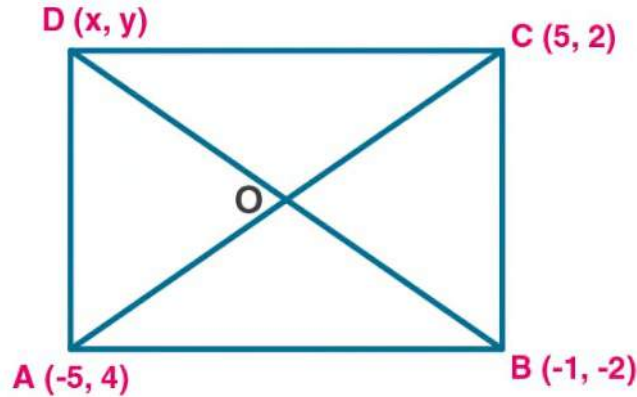
$$-10 = 0 + y_2$$

$$y_2 = -10$$

Hence the co-ordinate of the point D are (5, -10).

26. Prove that the points A (-5, 4), B (-1, -2) and C (5, 2) are the vertices of an isosceles right angled triangle. Find the co-ordinates of D so that ABCD is a square.

Solution:



Given points are A (-5, 4), B (-1, -2) and C (5, 2) are given.

Since these are vertices of an isosceles triangle ABC then $AB = BC$.

By distance formula $d(AB) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Here $x_1 = -5, y_1 = 4$

$x_2 = -1, y_2 = -2$

$$D(AB) = \sqrt{[(-1 - (-5))^2 + (-2 - 4)^2]}$$

$$D(AB) = \sqrt{[(4)^2 + (6)^2]}$$

$$D(AB) = \sqrt{(16 + 36)}$$

$$D(AB) = \sqrt{52} \quad \dots (i)$$

By distance formula $d(BC) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Here $x_1 = -1, y_1 = -2$

$x_2 = 5, y_2 = 2$

$$D(BC) = \sqrt{[(5 - (-1))^2 + (-2 - (-2))^2]}$$

$$D(BC) = \sqrt{[(6)^2 + (4)^2]}$$

$$D(BC) = \sqrt{(36 + 16)}$$

$$D(BC) = \sqrt{52} \quad \dots (ii)$$

From (i) and (ii) $AB = BC$

So given points are the vertices of isosceles triangle.

$$\text{By distance formula } d(AC) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

$$\text{Here } x_1 = -5, y_1 = 4$$

$$x_2 = 5, y_2 = 2$$

$$D(AC) = \sqrt{[(5 - (-5))^2 + (2 - 4)^2]}$$

$$D(AC) = \sqrt{[(10)^2 + (-2)^2]}$$

$$D(AC) = \sqrt{(100 + 4)}$$

$$D(AC) = \sqrt{104} \quad \dots (iii)$$

Apply Pythagoras theorem to triangle ABC

$$AB^2 + BC^2 = (\sqrt{52})^2 + (\sqrt{52})^2$$

$$= 52 + 52$$

$$= 104 \quad \dots (iv)$$

$$AC^2 = (\sqrt{104})^2 = 104 \quad \dots (v)$$

From (iv) and (v) we got

$$AB^2 + BC^2 = AC^2$$

So Pythagoras theorem is satisfied.

So the triangle is an isosceles right angled triangle.

Hence proved.

If ABCD is a square, let the diagonals meet at O.

Diagonals of a square bisect each other. So, O is the midpoint of AC and BD.

Consider A-O-C

$$x_1 = -5, y_1 = 4$$

$$x_2 = 5, y_2 = 2$$

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$$x = \frac{(-5+5)}{2} = \frac{0}{2} = 0$$

By midpoint formula, $y = \frac{(y_1+y_2)}{2}$

$$y = \frac{(4+2)}{2} = \frac{6}{2} = 3$$

So co-ordinate of O is (0, 3)

Consider B-O-D

Let co-ordinate of D be (x_2, y_2)

$$x_1 = -1, y_1 = -2$$

$$x = 0, y = 3$$

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$$0 = \frac{(-1+x_2)}{2}$$

$$-1 + x_2 = 0$$

$$x_2 = 1$$

By midpoint formula, $y = \frac{(x_1 + x_2)}{2}$

$$3 = \frac{(-2 + y_2)}{2}$$

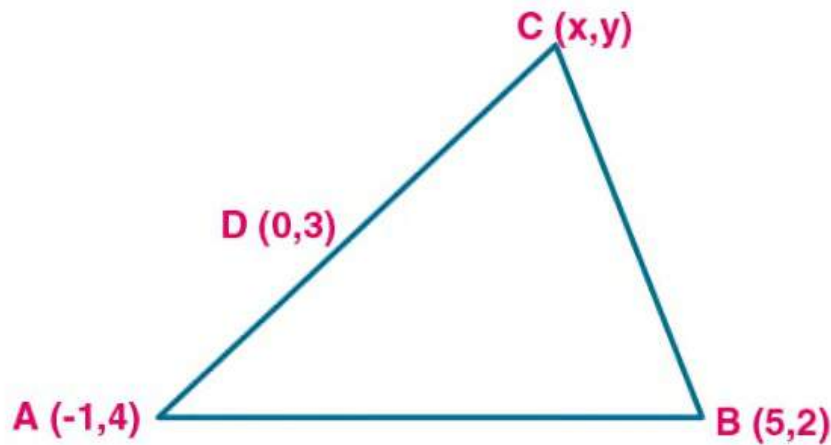
$$6 = -2 + y_2$$

$$y_2 = 6 + 2 = 8$$

Hence the co-ordinates of the point D are (1, 8).

27. Find the third vertex of a triangle if its two vertices are (-1, 4) and (5, 2) and midpoint of one sides is (0, 3).

Solution:



Let A (-1, 4) and B (5, 2) are the vertices of the triangle and let D (0, 3) is the midpoint of side AC.

Let co-ordinate of C be (x, y).

Consider D (0, 3) as midpoint of AC

By midpoint formula,

$$\frac{(-1 + x)}{2} = 0$$

$$-1 + x = 0$$

$$x = 1$$

By midpoint formula,

$$\frac{(4+y)}{2} = 3$$

$$4 + y = 6$$

$$y = 6 - 4 = 2$$

So the co-ordinates of C are (1, 2).

Consider D (0, 3) as midpoint of BC

By midpoint formula,

$$\frac{(5+x)}{2} = 0$$

$$5 + x = 0$$

$$x = -5$$

By midpoint formula,

$$\frac{(2+y)}{2} = 3$$

$$2 + y = 6$$

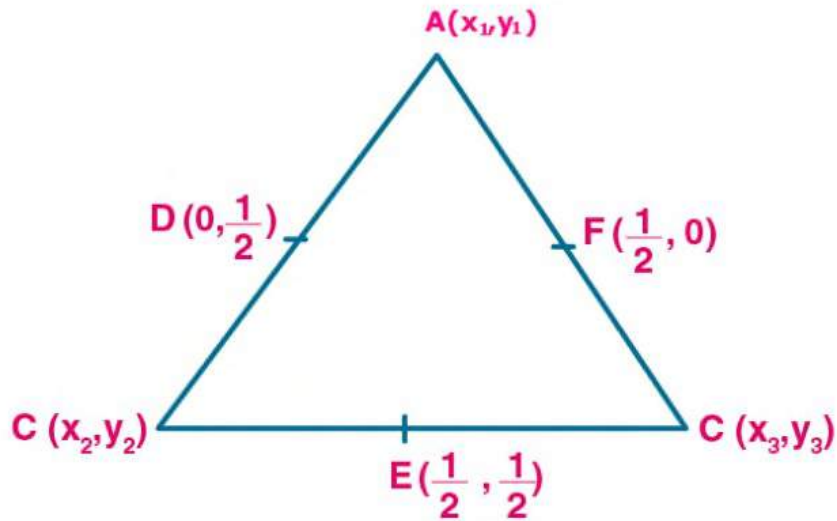
$$y = 6 - 2 = 4$$

So the co-ordinates of C are (-5, 4).

Hence the co-ordinates of the point C will be (1, 2) or (-5, 4).

28. Find the coordinates of the vertices of the triangle the middle points of whose sides are $\left(0, \frac{1}{2}\right)$, $\left(\frac{1}{2}, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, 0\right)$.

Solution:



Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of the triangle ABC.

Consider AB

By midpoint formula, $\frac{(x_1+x_2)}{2} = 0$

$$x_1 + x_2 = 0$$

$$x_1 = -x_2 \quad \dots (i)$$

By midpoint formula, $\frac{(y_1+y_2)}{2} = \frac{1}{2}$

$$y_1 + y_2 = 1$$

$$y_1 = 1 - y_2 \quad \dots (ii)$$

Consider AC

By midpoint formula, $\frac{(x_1+x_3)}{2} = \frac{1}{2}$

$$x_1 + x_3 = 1 \quad \dots (iii)$$

By midpoint formula, $\frac{(y_1+y_3)}{2} = 0$

$$y_1 + y_3 = 0$$

$$y_1 = -y_3 \quad \dots (iv)$$

Consider BC

By midpoint formula, $\frac{(x_2+x_3)}{2} = \frac{1}{2}$

$$x_2 + x_3 = 1 \quad \dots (v)$$

By midpoint formula, $\frac{(y_2+y_3)}{2} = \frac{1}{2}$

$$y_2 + y_3 = 1 \quad \dots (vi)$$

Substitute (i) in (iii)

Then (iii) becomes $-x_2 + x_3 = 1$

Equation (v) $x_2 + x_3 = 1$

Adding above two equation, we get

$$2x_3 = 2$$

$$x_3 = \frac{2}{2} = 1$$

Substitute $x_3 = 1$ in (iii), we get $x_1 = 0$

$$x_2 = 0 \quad [\text{From (i)}]$$

So $x_1 = 0, x_2 = 0, x_3 = 1$

Substitute (iv) in (ii)

Then (ii) becomes $-y_3 + y_2 = 1$

Equation (vi) $y_2 + y_3 = 1$

Adding above two equation, we get

$$2y_2 = 2$$

$$y_2 = \frac{2}{2} = 1$$

Substitute $y_2 = 1$ in (i), we get $y_1 = 0$

$$y_3 = 0$$

$$\text{So } y_1 = 0, y_2 = 1, y_3 = 0$$

Hence the co-ordinates of vertices are A (0, 0), B (0, 1) and C (1, 0).

29. Show by section formula that the points (3, -2), (5, 2) and (8, 8) are collinear.

Solution:

Let the point B (5, 2) divides the line joining A (3, -2) and C (8, 8) in the ratio m: n.

$$\text{Then by section formula, } x = \frac{(mx_2 + nx_1)}{(m+n)}$$

$$5 = \frac{(m \times 8 + n \times 3)}{(m+n)}$$

$$5 = \frac{(8m + 3n)}{(m+n)}$$

$$5m + 5n = 8m + 3n$$

$$2n = 3m$$

$$\frac{m}{n} = \frac{2}{3} \quad \dots \text{ (i)}$$

$$\text{By section formula } y = \frac{(my_2 + ny_1)}{(m+n)}$$

$$2 = \frac{(m \times 8 + n \times -2)}{(m+n)}$$

$$2 = \frac{(8m - 2n)}{(m+n)}$$

$$2m + 2n = 8m - 2n$$

$$6m = 4n$$

$$\frac{m}{n} = \frac{4}{6} = \frac{2}{3} \quad \dots \text{ (ii)}$$

Here ratios are same.

So the points are collinear.

30. Find the value of p for which the points (-5, 1), (1, p) and (4, -2) are collinear.

Solution:

Let A (-5, 1) divides the line joining (1, p) and (4, -2) in the m: n.

Then by section formula, $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$$-5 = \frac{(m \times 4 + n \times 1)}{(m+n)}$$

$$-5 = \frac{(4m+n)}{(m+n)}$$

$$-5m + 5n = 4m + n$$

$$-9m = -4n$$

$$\frac{m}{n} = \frac{-4}{-9} = \frac{4}{9} \quad \dots (i)$$

By section formula $y = \frac{(my_2 + ny_1)}{(m+n)}$

$$1 = \frac{(m \times -2 + n \times p)}{(m+n)}$$

$$2 = \frac{(-2m - pn)}{(m+n)}$$

$$m + n = -2m - pn$$

$$3m = -(p+1)n$$

$$\frac{m}{n} = \frac{-(p+1)}{3} \quad \dots (ii)$$

Equating (i) and (ii)

$$\frac{(p-1)}{3} = \frac{-2}{3}$$

$$p - 1 = -2$$

$$p = -2 + 1 = -1$$

Hence the value of p is -1 .

31. A (10, 5), B (6, -3) and C (2, 1) are the vertices of triangle ABC. L is the midpoint of AB, M is the mid-point of AC. Write down the co-ordinates of L and M. Show that $LM = \frac{1}{2} BC$.

Solution:

Given points are A (10, 5), B (6, -3) and C (2, 1)

Let L (x, y) be the midpoint of AB.

$$\text{Here } x_1 = 10, y_1 = 5$$

$$x_2 = 6, y_2 = -3$$

$$\text{By midpoint formula, } x = \frac{(x_1+x_2)}{2}$$

$$x = \frac{(10+6)}{2} = \frac{16}{2} = 8$$

$$\text{By midpoint formula, } y = \frac{(y_1+y_2)}{2}$$

$$y = \frac{(5-3)}{2} = \frac{2}{2} = 1$$

So co-ordinates of L are (8, 1).

Let M (x, y) be the midpoint of AC.

$$\text{Here } x_1 = 10, y_1 = 5$$

$$x_2 = 2, y_2 = 1$$

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$$x = \frac{(10+2)}{2} = \frac{12}{2} = 6$$

By midpoint formula, $y = \frac{(y_1+y_2)}{2}$

$$y = \frac{(5+1)}{2} = \frac{6}{2} = 3$$

So co-ordinates of M are (6, 3).

By distance formula $d(LM) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Here $x_1 = 8, y_1 = 1$

$x_2 = 6, y_2 = 3$

$$D(LM) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

$$D(LM) = \sqrt{[(6 - 8)^2 + (3 - 1)^2]}$$

$$D(LM) = \sqrt{[(-2)^2 + (2)^2]}$$

$$D(LM) = \sqrt{(4 + 4)}$$

$$D(LM) = \sqrt{8} = 2\sqrt{2} \quad \dots (i)$$

By distance formula $d(BC) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

The points are B (6, -3) and C (2, 1)

So $x_1 = 6, y_1 = -3$

$x_2 = 2, y_2 = 1$

$$D(BC) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

$$D(BC) = \sqrt{[(2 - 6)^2 + (1 - (-3))^2]}$$

$$D(BC) = \sqrt{[(-4)^2 + (4)^2]}$$

$$D(BC) = \sqrt{(16 + 16)}$$

$$D(BC) = \sqrt{32} = 4\sqrt{2} \quad \dots (ii)$$

From (i) and (ii), $LM = \frac{1}{2} BC$.

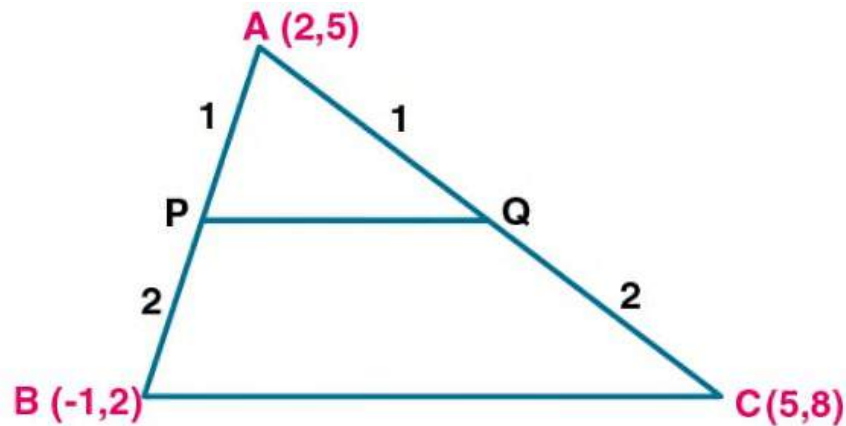
32. A (2, 5), B (-1, 2) and C (5, 8) are the vertices of a triangle ABC.

P and Q are points on AB and AC respectively such that AP: PB = AQ: QC = 1: 2.

(i) Find the co-ordinates of P and Q.

(ii) Show that $PQ = \frac{1}{3} BC$

Solution:



(i) Given vertices of the ABC are A (2, 5), B(-1, 2) and C (5, 8)

P and Q are points on AB and AC respectively such that AP: PB = AQ: QC = 1: 2.

P (x, y) divides AB in the ratio 1: 2.

$$x_1 = 2, y_1 = 5$$

$$x_2 = -1, y_2 = 2$$

$$m : n = 1 : 2$$

$$\text{By section formula, } x = \frac{(mx_2 + nx_1)}{(m+n)}$$

$$x = \frac{(1 \times -1 + 2 \times 2)}{(1+2)}$$

$$x = \frac{(-1+4)}{3}$$

$$x = \frac{3}{3} = 1$$

$$\text{By section formula } y = \frac{(my_2 + ny_1)}{(m+n)}$$

$$y = \frac{(1 \times 2 + 2 \times 5)}{(1+2)}$$

$$y = \frac{(2+10)}{3}$$

$$y = \frac{12}{3} = 4$$

Co-ordinates of P are (1, 4).

Q (x, y) divides AC in the ratio 1: 2.

$$x_1 = 2, y_1 = 5$$

$$x_2 = 5, y_2 = 8$$

$$m : n = 1 : 2$$

$$\text{By section formula, } x = \frac{(mx_2 + nx_1)}{(m+n)}$$

$$x = \frac{(1 \times 5 + 2 \times 2)}{(1+2)}$$

$$x = \frac{(5+4)}{3}$$

$$x = \frac{9}{3} = 3$$

By section formula $y = \frac{(my_2 + ny_1)}{(m+n)}$

$$y = \frac{(1 \times 8 + 2 \times 5)}{(1+2)}$$

$$y = \frac{(8+10)}{3}$$

$$y = \frac{18}{3} = 6$$

Co-ordinates of Q are (3, 6).

(ii) By distance formula $d(PQ) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

The points are P (1, 4) and Q (3, 6)

So $x_1 = 1, y_1 = 4$

$x_2 = 3, y_2 = 6$

$$D(PQ) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

$$D(PQ) = \sqrt{[(3 - 1)^2 + (6 - 4)^2]}$$

$$D(PQ) = \sqrt{[(2)^2 + (2)^2]}$$

$$D(PQ) = \sqrt{(4 + 4)}$$

$$D(PQ) = \sqrt{8} = 2\sqrt{2} \dots (i)$$

By distance formula $d(BC) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

The points are B (-1, 2) and C (5, 8)

Points are B (-1, 2) and C (5, 8).

So $x_1 = -1, y_1 = 2$

$x_2 = 5, y_2 = 8$

$$D(BC) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

$$D(BC) = \sqrt{[(5 - (-1))^2 + (8 - 2)^2]}$$

$$D(BC) = \sqrt{[(6)^2 + (6)^2]}$$

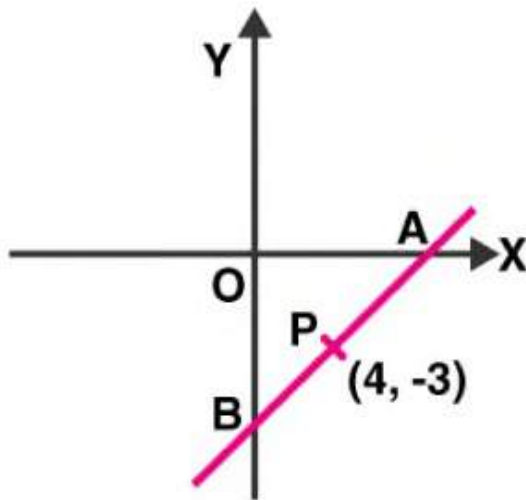
$$D(BC) = \sqrt{(36 + 36)}$$

$$D(BC) = \sqrt{72} = \sqrt{(36 \times 2)} = 6\sqrt{2} \quad \dots (ii)$$

$$\frac{BC}{3} = \frac{6\sqrt{2}}{3} = \frac{2}{\sqrt{2}} = PQ$$

Hence proved.

33. The mid-point of the line segment AB shown in the adjoining diagram is (4, -3). Write down the co-ordinates of A and B.



Solution:

Let P (4, -3) be the midpoint of line joining the points A and B.

Since A lies on x-axis, its co-ordinates are $(x_2, 0)$

Since B lies on y-axis, its co-ordinates are $(0, y_1)$

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$$4 = \frac{(0+x_2)}{2}$$

$$x_2 = 4 \times 2 = 8$$

By midpoint formula, $y = \frac{(y_1+y_2)}{2}$

$$-3 = \frac{(y_1+0)}{2}$$

$$y_1 = -3 \times 2 = -6$$

Hence the co-ordinates of A and B are (8, 0) and (0, -6) respectively.

34. Find the co-ordinates of the centroid of a triangle whose vertices are A (-1, 3), B (1, -1) and C (5, 1). (2006)

Solution:

Given vertices of the triangle are A (-1, 3), B (1, -1) and C (5, 1).

Co-ordinates of the centroid of a triangle, whose vertices are (x_1, y_1) , (x_2, y_2) and (x_3, y_3)

$$\left[\frac{(x_1+x_2+x_3)}{3}, \frac{(y_1+y_2+y_3)}{3} \right]$$

$$(x_1, y_1) = (-1, 3)$$

$$(x_2, y_2) = (1, -1)$$

$$(x_3, y_3) = (5, 1)$$

$$\frac{(x_1+x_2+x_3)}{3} = \frac{(-1+1+5)}{3} = \frac{5}{3}$$

$$\frac{(y_1+y_2+y_3)}{3} = \frac{(3-1+1)}{3} = \frac{3}{3} = 1$$

Hence the co-ordinates of centroid are $\left(\frac{5}{3}, 1\right)$.

35. Two vertices of a triangle are $(3, -5)$ and $(-7, 4)$. Find the third vertex given that the centroid is $(2, -1)$.

Solution:

Let third vertex be $C(x_3, y_3)$.

Given $(x_1, y_1) = (3, -5)$

$(x_2, y_2) = (-7, 4)$

Co-ordinates of centroid are $(2, -1)$

Co-ordinates of the centroid of a triangle, whose vertices are (x_1, y_1) , (x_2, y_2) and (x_3, y_3)

$$\left[\frac{(x_1+x_2+x_3)}{3}, \frac{(y_1+y_2+y_3)}{3} \right]$$

$$\frac{(x_1+x_2+x_3)}{3} = \frac{(3+7+x_3)}{3} = 2 \quad [x \text{ Co-ordinate of centroid}]$$

$$-4 + x_3 = 2 \times 3$$

$$-4 + x_3 = 6$$

$$x_3 = 6 + 4$$

$$x_3 = 10$$

$$\frac{(y_1+y_2+y_3)}{3} = -1 \quad [y \text{ Co-ordinate of centroid}]$$

$$-5 + 4 + y_3 = -1 \times 3$$

$$-1 + y_3 = -3$$

$$y_3 = -3 + 1$$

$$y_3 = -2$$

Hence the third vertex is $(10, -2)$.

36. The vertices of a triangle are A (-5, 3), B (p, -1) and C (6, q). Find the values of p and q if the centroid of the triangle ABC is the point (1, -1).

Solution:

Given vertices of the triangle are A (-5, 3), B (p, -1) and C (6, q).

Co-ordinates of centroid are (1, -1).

Co-ordinates of the centroid of a triangle, whose vertices are (x_1, y_1) , (x_2, y_2) and (x_3, y_3)

$$\left[\frac{(x_1+x_2+x_3)}{3}, \frac{(y_1+y_2+y_3)}{3} \right]$$

$$(x_1, y_1) = (-5, 3)$$

$$(x_2, y_2) = (p, -1)$$

$$(x_3, y_3) = (6, q)$$

$$\text{X co-ordinate of centroid, } \frac{(x_1+x_2+x_3)}{3} = \frac{(-5+p+6)}{3} = 1$$

$$p + 1 = 3$$

$$p = 3 - 1$$

$$p = 2$$

$$\text{Y co-ordinate of centroid, } \frac{(y_1+y_2+y_3)}{3} = \frac{(3-1+q)}{3} = -1$$

$$2 + q = 3 \times -1$$

$$2 + q = -3$$

$$q = -3 - 2$$

$$q = -5$$

Hence the value of p and q are 2 and -5 respectively.