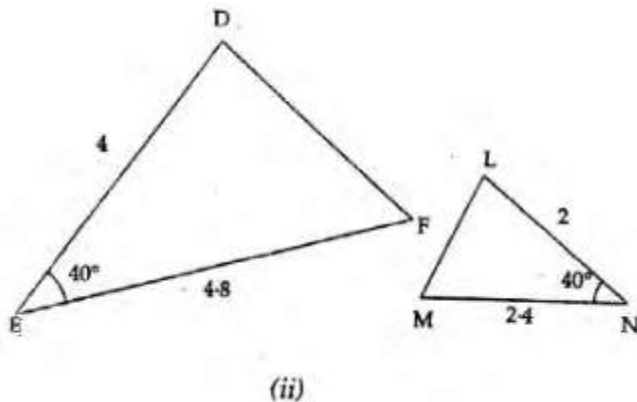
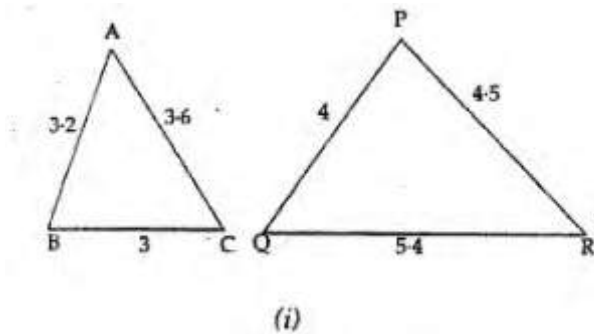


Chapter 13

Similarity

Exercise 13.1

1. State which pairs of triangles in the figure given below are similar. Write the similarity rule used and also write the pairs of similar triangles in symbolic form (all lengths of sides are in cm):



Solution : -

(i) From the $\triangle ABC$ and $\triangle PQR$

$$\frac{AB}{PQ} = \frac{3.2}{4}$$

$$= \frac{32}{40}$$

Divide both numerator and denominator by 8 we get,

$$= \frac{4}{5}$$

$$\frac{AC}{PR} = \frac{3.6}{4.5}$$

$$= \frac{36}{45}$$

Divide both numerator and denominator by 9 we get,

$$= \frac{4}{5}$$

$$\frac{BC}{QR} = \frac{3}{5.4}$$

$$= \frac{30}{54}$$

Divide both numerator and denominator by 6 we get,

$$= \frac{5}{9}$$

By comparing all the results, the sides are not equal,

Therefore, the triangles are not equal.

(ii) By comparing all the results, the sides are not equal.

Therefore, the triangles are not equal.

(iii) From the $\triangle DEF$ and $\triangle LMN$

$$\angle E = \angle N = 40^\circ$$

$$\text{Then, } \frac{DE}{LN} = \frac{4}{2}$$

Divide both numerator and denominator by 2 we get, $= 2$

$$\frac{EF}{MN} = \frac{4.8}{2.4}$$

$$= \frac{48}{24}$$

Divide both numerator and denominator by 24 we get, $= 2$

Therefore,

$$\triangle DEF \sim \triangle LMN$$

2. It is given that $\triangle DEF \sim \triangle RPQ$. Is it true to say that $\angle D = \angle R$ and $\angle F = \angle P$? Why?

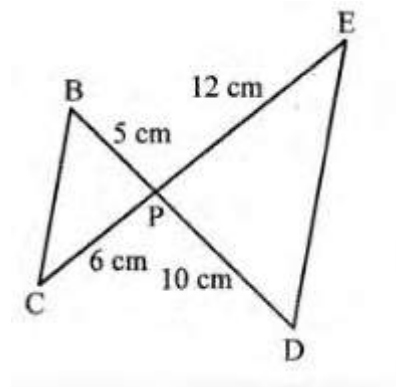
Solution:

From the question it is given that, $\triangle DEF \sim \triangle RPQ$

$$\angle D = \angle R \text{ and } \angle F = \angle Q \text{ not } \angle P$$

No, $\angle F \neq \angle P$

3. If in two right triangles, one of the acute angles of one triangle is equal to an acute angle of the other triangle, can you say that the two triangles are similar? Why?



Solution:

From the figure, two line segments are intersecting each other at P.

In $\triangle BCP$ and $\triangle DPE$

$$\frac{5}{10} = \frac{6}{12}$$

Dividing LHS and RHS by 2 we get,

$$\frac{1}{2} = \frac{1}{2}$$

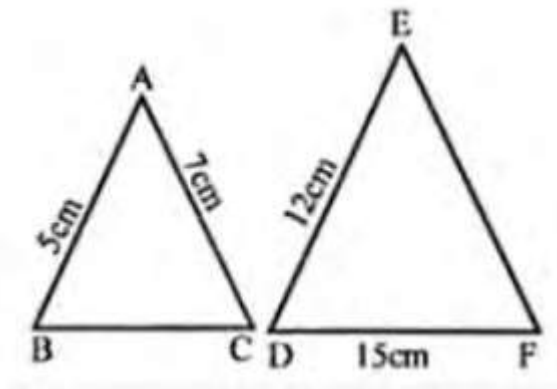
Therefore, $\triangle BCD \sim \triangle DEP$

5. It is given that $\triangle ABC \sim \triangle EDF$ such that $AB = 5\text{cm}$, $AC = 7\text{cm}$, $DF = 15\text{cm}$ and $DE = 12\text{cm}$.

Find the lengths of the remaining sides of the triangles.

Solution:

As per the dimensions give in the question,



From the question it is given that,

$$\triangle DEF \sim \triangle LMN$$

$$\text{So, } \frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF}$$

$$\text{Consider } \frac{AB}{ED} = \frac{AC}{EF}$$

$$\frac{5}{12} = \frac{7}{EF}$$

By cross multiplication,

$$EF = \frac{(7 \times 12)}{5}$$

$$EF = 16.8 \text{ cm}$$

$$\text{Now, consider } \frac{AB}{ED} = \frac{BC}{DF}$$

$$\frac{5}{12} = \frac{BC}{15}$$

$$BC = \frac{(5 \times 15)}{12}$$

$$BC = \frac{75}{12}$$

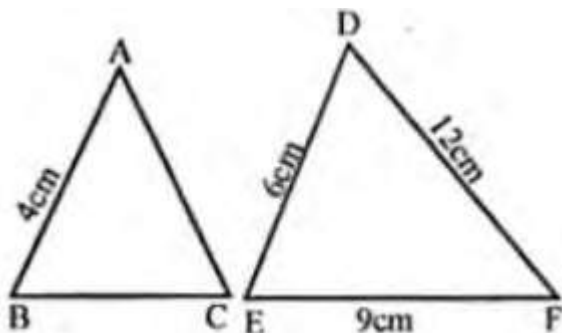
$$BC = 6.25$$

$$BC = 6.25$$

6. If $\triangle ABC \sim \triangle DEF$, $AB = 4\text{cm}$, $DE = 6\text{cm}$, $EF = 9\text{cm}$ and $FD = 12\text{cm}$, then find the perimeter of $\triangle ABC$.

Solution:

As per the dimensions given in the questions,



Now, we have to find out the perimeter of $\triangle ABC$

Let $\triangle ABC \sim \triangle DEF$

$$\text{So, } \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

$$\text{Consider, } \frac{AB}{DE} = \frac{AC}{DF}$$

$$\frac{4}{6} = \frac{AC}{12}$$

By cross multiplication we get,

$$AC = \frac{(4 \times 12)}{6}$$

$$AC = \frac{48}{6}$$

$$AC = 8 \text{ cm}$$

$$\text{Then, consider } \frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{4}{6} = \frac{BC}{9}$$

$$BC = \frac{(4 \times 9)}{6}$$

$$BC = \frac{36}{6}$$

$$BC = 6\text{cm}$$

Therefore, the perimeter $\Delta ABC = AB + BC + AC$

$$= 4 + 6 + 8$$

$$= 18 \text{ cm}$$

(b) If $\Delta ABC \sim \Delta PQR$, perimeter of $\Delta ABC = 32\text{cm}$, perimeter of $\Delta PQR = 48 \text{ cm}$ and $PR = 6 \text{ cm}$, then find the length of AC .

Solution:

From the question it is given that,

$$\Delta ABC \sim \Delta PQR$$

$$\text{Perimeter of } \Delta ABC = 32\text{cm}$$

$$\text{Perimeter of } \Delta PQR = 48\text{cm}$$

$$\text{So, } \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$$

$$\text{Then, perimeter of } \frac{\Delta ABC}{\text{perimeter of } \Delta PQR} = \frac{AC}{PR}$$

$$\frac{32}{48} = \frac{AC}{6}$$

$$AC = \frac{(32 \times 6)}{48}$$

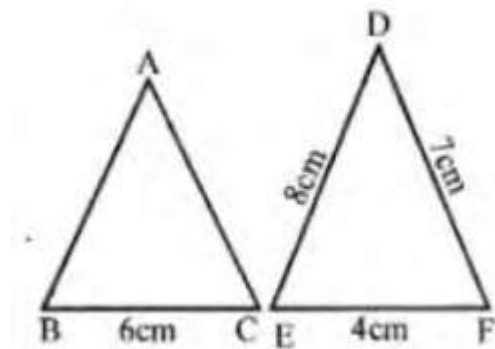
$$AC = 4$$

Therefore, the length of $AC = 4 \text{ cm}$.

7. Calculate the Other sides of a triangle whose shortest side is 6cm and which is similar to a triangle whose sides are 4cm, 7cm and 8cm.

Solution:

Let us assume that, $\Delta ABC \sim \Delta DEF$



ΔABC is $BC = 6\text{cm}$

$\Delta ABC \sim \Delta DEF$

$$\text{So, } \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\text{Consider } \frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{AB}{8} = \frac{6}{4}$$

$$AB = \frac{(6 \times 8)}{4}$$

$$AB = \frac{48}{4}$$

$$AB = 12$$

$$\text{Now, consider } \frac{BC}{EF} = \frac{AC}{DF}$$

$$\frac{6}{4} = \frac{AC}{7}$$

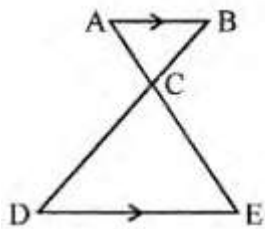
$$AC = \frac{(6 \times 7)}{4}$$

$$AC = \frac{42}{4}$$

$$AC = \frac{21}{2}$$

$$AC = 10.5 \text{ cm}$$

8. (a) In the figure given below, $AB \parallel DE$, $AC = 3 \text{ cm}$, $CE = 7.5 \text{ cm}$ and $BD = 14 \text{ cm}$. Calculate CB and DC .



Solution:

From the question it is given that,

$$AB \parallel DE$$

$$AC = 3 \text{ cm}$$

$$CE = 7.5 \text{ cm}$$

$$BD = 14 \text{ cm}$$

From the figure,

$$\angle ACB = \angle DCE \quad [\text{because vertically opposite angles}]$$

$$\angle BAC = \angle CED \quad [\text{alternate angles}]$$

Then, $\Delta ABC \sim \Delta CDE$

$$\text{So, } \frac{AC}{CE} = \frac{BC}{CD}$$

$$\frac{3}{7.5} = \frac{BC}{CD}$$

By cross multiplication we get,

$$7.5BC = 3CD$$

Let us assume $BC = x$ and $CD = 14 - x$

$$7.5 \times x = 3 \times (14 - x)$$

$$7.5x = 42 - 3x$$

$$7.5x + 3x = 42$$

$$10.5x = 42$$

$$x = \frac{42}{10.5}$$

$$x = 4$$

Therefore, $BC = x = 4\text{cm}$

$$CD = 14 - x$$

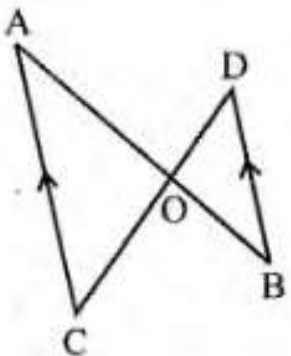
$$= 14 - 4$$

$$= 10 \text{ cm}$$

(b) In the figure (2) given below, $CA \parallel BD$, the lines AB and CD meet at G .

(i) Prove that $\Delta ACO \sim \Delta BDO$.

(ii) If $BD = 2.4$ cm, $OD = 4$ cm, $OB = 3.2$ cm and $AC = 3.6$ cm, calculate OA and OC .



Solution:

(i) We have to prove that, $\Delta ACO \sim \Delta BDO$.

So, from the figure

Consider ΔACO and ΔBDO

Then,

$\angle AOC = \angle BOD$ [from vertically opposite angles]

$\angle A = \angle B$

Therefore, $\Delta ACO \sim \Delta BDO$

Given, $BD = 2.4$ cm, $OD = 4$ cm, $OB = 3.2$ cm, $AC = 3.6$ cm,

$\Delta ACO \sim \Delta BOD$

So, $\frac{AO}{OB} = \frac{CO}{OD} = \frac{AC}{BD}$

Consider $\frac{AC}{BD} = \frac{AO}{OB}$

$$\frac{3.6}{2.4} = \frac{AO}{3.2}$$

$$AO = \frac{(3.6 \times 3.2)}{2.4}$$

$$AO = 4.8 \text{ cm}$$

$$\text{Now, consider } \frac{AC}{BD} = \frac{CO}{OD}$$

$$\frac{3.6}{2.4} = \frac{CO}{4}$$

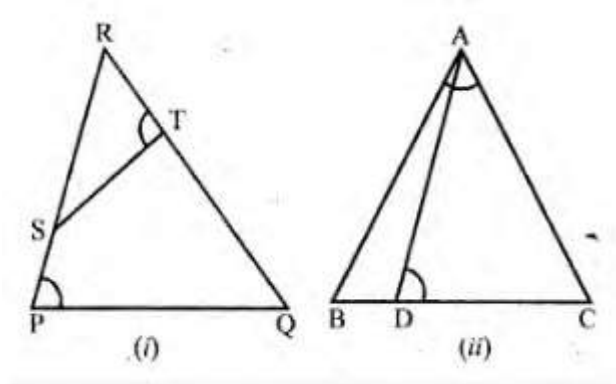
$$CO = \frac{(3.6 \times 4)}{2.4}$$

$$CO = 6 \text{ cm}$$

9. (a) In the figure

(i) given below, $\angle P = \angle RTS$.

Prove that $\triangle RPQ \sim \triangle RTS$.



Solution:

From the given figure, $\angle P = \angle RTS$

So we have to prove that $\triangle RPQ \sim \triangle RTS$

So we have to prove that $\Delta RPQ \sim \Delta RTS$

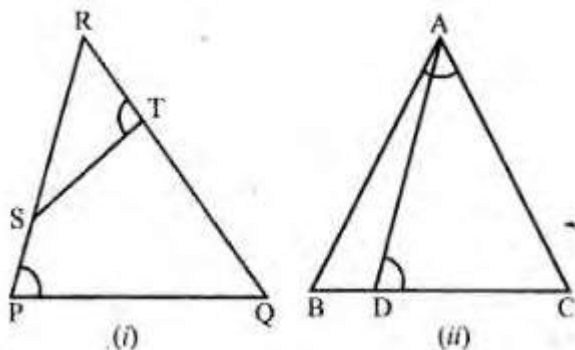
In ΔRPQ and ΔRTS

$\angle R = \angle R$ (common angle for both triangle)

$\angle P = \angle RTS$ (from the question)

$\Delta RPQ \sim \Delta RTS$

(b) In the figure (ii) given below, $\angle ADC = \angle BAC$. Prove that $CA^2 = DC \times BC$



Solution:

From the figure, $\angle ADC = \angle BAC$

So, we have to prove that $CA^2 = DC \times BC$

In ΔABC and ΔADC

$\angle C = \angle C$ (common angle for both triangle)

$\angle BAC = \angle ADC$ (from the question)

$\Delta ABC \sim \Delta ADC$

Therefore, $\frac{CA}{DC} = \frac{BC}{CA}$

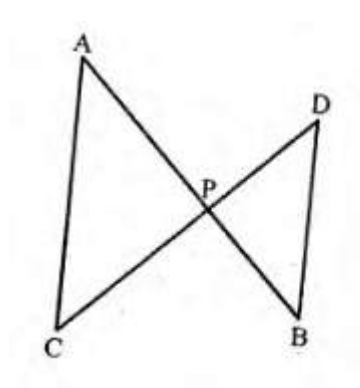
We know that, corresponding sides are proportional,

Therefore, $CA^2 = DC \times BC$

10. (a) In the figure(1) given below, $AP = 2PB$ and $CP = 2PD$.

(i) Prove that $\triangle ACP$ is similar to $\triangle BDP$ and $AC \parallel BD$.

(ii) If $AC = 4.5$ cm, Calculate the length of BD .



Solution:

From the question it is given that,

$$AP = 2PB, CP = 2PD$$

(i) We have to prove that, $\triangle ACP$ is similar to $\triangle BDP$ and $AC \parallel BD$

$$AP = 2PB, CP = 2PD$$

(i) We have to prove that, $\triangle ACP$ is similar to $\triangle BDP$ and $AC \parallel BD$ $AP = 2PB$

$$\frac{AP}{PB} = \frac{2}{1}$$

Then, $CP = 2PD$

$$\frac{CP}{PD} = \frac{2}{1}$$

$\angle APC = \angle BPD$ [from vertically opposite angles]

So, $\triangle ACP \sim \triangle BDP$

Therefore, $\angle CAP = \angle PBD$ [alternate angles]

Hence, $AC \parallel BD$

$$(ii) \frac{AP}{PB} = \frac{AC}{BD} = \frac{2}{1}$$

$$AC = 2BD$$

$$2BD = 4.5 \text{ cm}$$

$$BD = \frac{4.5}{2}$$

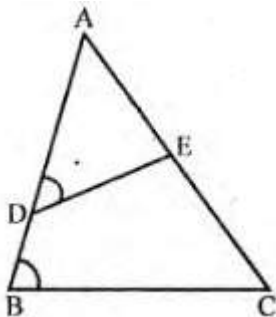
$$BD = 2.25 \text{ cm}$$

(b) In the figure (2) given below,

$$\angle ADE = \angle ACB.$$

(i) Prove that $\triangle ABC$ and $\triangle AED$ are similar.

(ii) If $AE = 3\text{cm}$, $BD = 1 \text{ cm}$ and $AB = 6\text{cm}$, calculate AC .



Solution:

From the given figure,

(i) $\angle A = \angle A$ (common angle for both triangles)

$\angle ACB = \angle ADE$ [given]

Therefore, $\Delta ABC \sim \Delta AED$

(ii) from (i) proved that, $\Delta ABC \sim \Delta AED$

$$\text{So, } \frac{BC}{DE} = \frac{AB}{AE} = \frac{AC}{AD}$$

$$AD = AB - BD$$

$$= 6 - 1 = 5$$

$$\text{Consider, } \frac{AB}{AE} = \frac{AC}{AD}$$

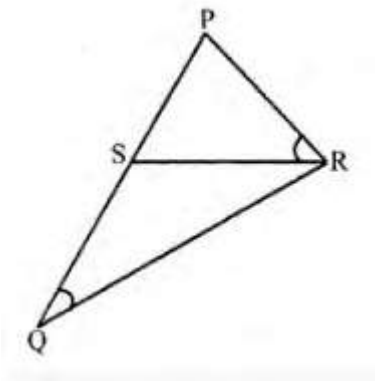
$$\frac{6}{3} = \frac{AC}{5}$$

$$AC = \frac{(6 \times 5)}{3}$$

$$AC = \frac{30}{3}$$

$$AC = 10 \text{ cm}$$

(c) In the figure (3) given below, $\angle PQR = \angle PRS$. Prove that triangles PQR and PRS are similar. If $PR = 8\text{cm}$, $PS = 4\text{cm}$, calculate PQ.



Solution:-

From the figure,

$\angle P = \angle P$ (common angle for both triangles)

$\angle PQR = \angle PRS$ [from the question]

So, $\Delta PQR \sim \Delta PRS$

Then, $\frac{PQ}{PR} = \frac{PR}{PS} = \frac{QR}{SR}$

Consider $\frac{PQ}{PR} = \frac{PR}{PS}$

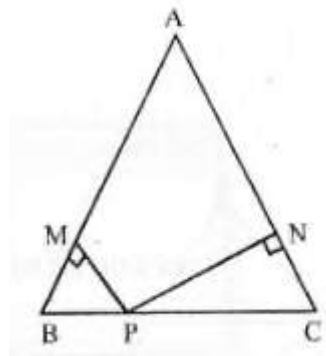
$$\frac{PQ}{8} = \frac{8}{4}$$

$$PQ = \frac{(8 \times 8)}{4}$$

$$PQ = \frac{64}{8}$$

$$PQ = 16 \text{ cm}$$

11. In the given figure, ABC is a triangle in which $AB = AC$. P is a point on the side BC such that $PM \perp AB$ and $PN \perp AC$. Prove that $BM \times NP = CN \times MP$.



Solution:

From the question it is given that, ABC is a triangle in which $AB = AC$.

P is a point on the side BC such that $PM \perp AB$ and $PN \perp AC$.

We have to prove that, $BM \times NP = CN \times MP$

Consider the $\triangle ABC$

$AB = AC$ [from the question]

$\angle B = \angle C$... [angles opposite to equal sides]

Then, consider $\triangle BMP$ and $\triangle CNP$

$\angle M = \angle N$

Therefore, $\triangle BMP \sim \triangle CNP$

So, $\frac{BM}{CN} = \frac{MP}{NP}$

By cross multiplication we get,

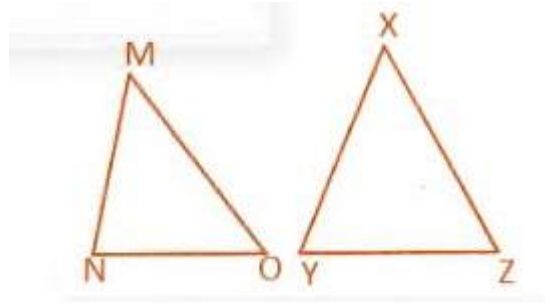
$BM \times NP = CN \times MP$

Hence it is proved.

12. Prove that the ratio of the perimeters of two similar triangles is the same as the ratio of their corresponding sides.

Solution:

Consider the two triangles, $\triangle MNO$ and $\triangle XYZ$



From the question it is given that, two triangles are similar triangles

So, $\triangle MNO \sim \triangle XYZ$

If two triangles are similar, the corresponding angles are equal and their corresponding sides are proportional.

$$\frac{MN}{XY} = \frac{NO}{YZ} = \frac{MO}{XZ}$$

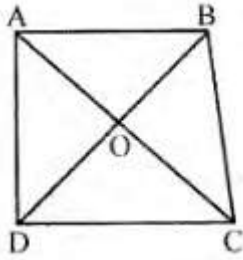
Perimeter of $\triangle MNO = MN + NO + MO$

$$\text{Therefore, } \left(\frac{MN}{XY} = \frac{NO}{YZ} = \frac{MO}{XZ} \right) = \left(\frac{MN}{XY} = \frac{NO}{YZ} = \frac{MO}{XZ} \right)$$

$$= \text{Perimeter of } \frac{\triangle MNO}{\text{perimeter of } \triangle XYZ}$$

13. In the adjoining figure, ABCD is a trapezium in which $AB \parallel DC$.

The diagonals AC and BD intersect at O. Prove that $\frac{AO}{OC} = \frac{BO}{OD}$



Using the above result, find the values of x if $OA = 3x - 19$, $OB = x - 4$, $OC = x - 3$ and $OD = 4$.

Solution:

From the given figure, ABCD is a trapezium in which $AB \parallel DC$,

The diagonals AC and BD intersect at O.

So we have to prove that, $\frac{AO}{OC} = \frac{BO}{OD}$

Consider the $\triangle AOB$ and $\triangle COD$,

$\angle AOB = \angle COD$ [vertically opposite angles]

$\angle OAB = \angle OCD$

Therefore, $\triangle AOB \sim \triangle COD$

So, $\frac{OA}{OC} = \frac{OB}{OD}$

Now by using above result we have to find the value of x if $OA = 3x - 19$, $OB = x - 4$, $OC = x - 3$ and $OD = 4$.

$$\frac{OA}{OC} = \frac{OB}{OD}$$

$$\frac{(3x-19)}{(x-3)} = \frac{(x-4)}{4}$$

By cross multiplication we get,

$$(x - 3)(x - 4) = 4(3x - 19)$$

$$X^2 - 4x - 3x + 12 = 12x - 76$$

$$X^2 - 7x + 12 - 12x + 76 = 0$$

$$X^2 - 19x + 88 = 0$$

$$X^2 - 8x - 11x + 88 = 0$$

$$X(x - 8) - 11(x - 8) = 0$$

$$(x - 8)(x - 11) = 0$$

$$\text{Take } x - 8 = 0$$

$$X = 8$$

$$\text{Then, } x - 11 = 0$$

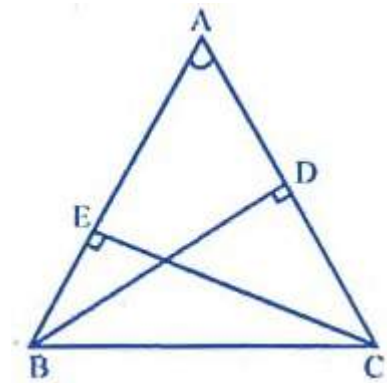
$$X = 11$$

Therefore, the value of x is 8 and 11.

14. In $\triangle ABC$, $\angle A$ is acute. BD and CE are perpendicular on AC and AB respectively. Prove that $AB \times AE = AC \times AD$.

Solution:

Consider the $\triangle ABC$



So, we have to prove that, $AB \times AE = AC \times AD$

Now, consider the $\triangle ADB$ and $\triangle AEC$,

$\angle A = \angle A$ [common angle for both triangles]

$\angle ADB = \angle AEC$ [both angles are equal to 90°]

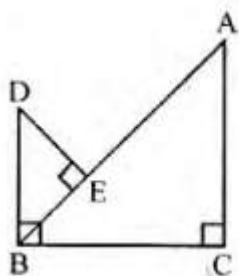
$\triangle ADB \sim \triangle AEC$

So, $\frac{AB}{AC} = \frac{AD}{AE}$

By cross multiplication we get,

$$AB \times AE = AC \times AD$$

15. In the given figure, $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$



Solution :

From the figure, $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$

We have to prove that, $\frac{BE}{DE} = \frac{AC}{BC}$

Consider the $\triangle ABC$ and $\triangle DEB$,

$$\angle C = 90^\circ$$

$$\angle A + \angle ABC = 90^\circ \text{ [from the figure equation (i)]}$$

Now in $\triangle DEB$

$$\angle DBE + \angle ABC = 90^\circ \text{ [from the figure equation (ii)]}$$

From equation (i), we get

$$\angle A = \angle DBE$$

Then, in $\triangle ABC$ and $\triangle DBE$

$$\angle C = \angle E \text{ [both angles are equal to } 90^\circ \text{]}$$

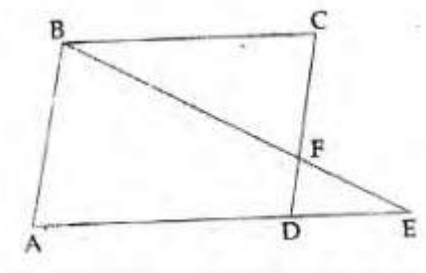
So, $\triangle ABC \sim \triangle DBE$

$$\text{Therefore, } \frac{AC}{BE} = \frac{BC}{DE}$$

By cross multiplication, we get

$$\frac{AC}{BC} = \frac{BE}{DE}$$

16. (a) In the figure (1) given below, E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. show that $\triangle ABE \sim \triangle CFB$.



Solution:

From the figure, ABCD is a parallelogram,

Then, E is a point on AD and produced and BE intersects CD at F.

We have to prove that $\Delta ABE \sim \Delta CFB$

Consider ΔABE and ΔCFB

$\angle A = \angle C$ [Opposite angles of a parallelogram]

$\angle ABE = \angle BFC$ [O

$\angle ABE = \angle BFC$ [Alternate angles are equal]

$\Delta ABE \sim \Delta CFB$

(b) In the figure (2) given below, PQRS is a parallelogram; PQ = 16cm, QR = 10cm. L is a point on PR such that RL : LP = 2 : 3. QL produced meets RS at M and PS produced at N.

(i) Prove that triangle RLQ is similar to triangle PLN. Hence, find PN.

Solution:

From the question it is give that,

$\angle RLQ = \angle NLP$ [vertically opposite angles are equal]

$\angle RQL = \angle LNP$ [alternate angle are equal]

Therefore, $\Delta RLQ \sim \Delta PLN$

$$\text{So, } \frac{QR}{PN} = \frac{RL}{LP} = \frac{2}{3}$$

$$\frac{QR}{PN} = \frac{2}{3}$$

$$\frac{10}{PN} = \frac{2}{3}$$

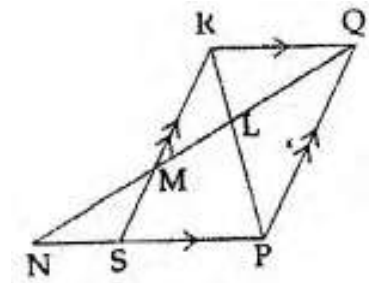
$$PN = \frac{(10 \times 3)}{2}$$

$$PN = \frac{30}{2}$$

$$PN = 15\text{cm}$$

Therefore, $PN = 15\text{cm}$

(ii) Name a triangle similar to triangle RLM. Evaluate RM.



Solution:

From the figure,

Consider $\triangle RLM$ and $\triangle QLP$

Then, $\angle RLM = \angle QLP$ [verticallu opposite angles are equal]

$\angle LRM = \angle LPQ$ [alternate angles are equal]

Therefore, $\triangle RLM \sim \triangle QLP$

$$\text{Then, } \frac{RM}{PQ} = \frac{RL}{LP} = \frac{2}{3}$$

$$\text{So, } \frac{RM}{16} = \frac{2}{3}$$

$$RM = \frac{(16 \times 2)}{3}$$

$$RM = \frac{32}{3}$$

$$RM = 10\frac{2}{3}$$

17. The altitude BN and CM of $\triangle ABC$ meet at H. Prove that

(i) $CN \times HM = BM \times HN$

(ii) $\frac{HC}{HB} = \sqrt{[(CN \times HN)/(BM \times HM)]}$

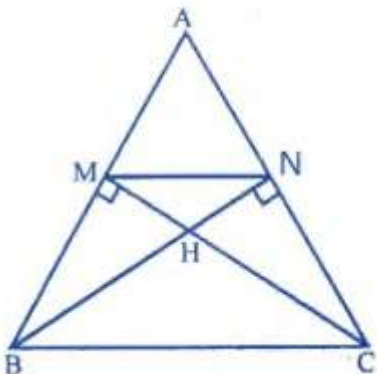
(iii) $\triangle MHN \sim \triangle BHC$

Solution:

Consider the $\triangle ABC$,

Where, the altitude BN and CM of $\triangle ABC$ meet at H.

and construction : Join MN



(i) We have to prove that, $CN \times HM = BM \times HN$

In $\triangle BHM$ and $\triangle CHN$

$\angle BHM = \angle CHN$ [because vertically opposite angles are equal]

$\angle M = \angle N$ [both angles are equal to 90°]

Therefore, $\triangle BHM \sim \triangle CHN$

So, $\frac{HM}{HN} = \frac{BM}{CN} = \frac{HB}{HC}$

Then, by cross multiplication we get

$$CN \times HM = BM \times HN$$

$$\begin{aligned}
 \text{(ii) Now, } \frac{HC}{HB} &= \sqrt{\frac{(HN \times CN)}{(HM \times BM)}} \\
 &= \sqrt{\frac{(CN \times HN)}{(BM \times HM)}}
 \end{aligned}$$

Because, M and N divide AB and AC in the same ratio.

(iii) Now consider $\triangle MHN$ and $\triangle BHC$

$\angle MHN = \angle BHC$ [because vertically opposite angles are equal]

$\angle MNH = \angle HBC$ [because alternate angles are equal]

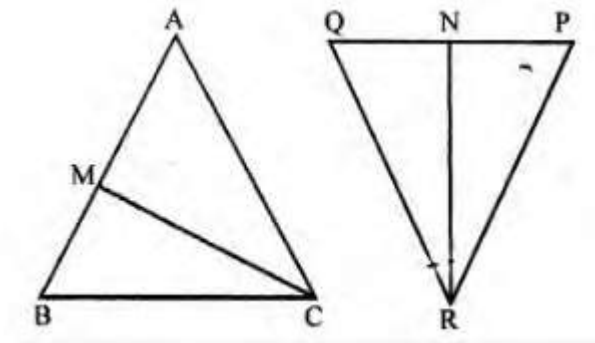
Therefore, $\triangle MHN \sim \triangle BHC$

18. In the given figure, CM and RN are respectively the medians of $\triangle ABC$ and $\triangle PQR$. If $\triangle ABC \sim \triangle PQR$, prove that :

(i) $\triangle AMC \sim \triangle PQR$

(ii) $\frac{CM}{RN} = \frac{AB}{PQ}$

(iii) $\triangle CMB \sim \triangle RNQ$



Solution :-

From the given figure it is given that, CM and RN are respectively the medians of ΔABC and ΔPQR .

(i) We have to prove that, $\Delta AMC \sim \Delta PQR$

Consider the ΔABC and ΔPQR

As $\Delta ABC \sim \Delta PQR$

$$\angle A = \angle P, \angle B = \angle Q \text{ and } \angle C = \angle R$$

And also corresponding sides are proportional

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

Then, consider the ΔAMC and ΔPNR ,

$$\angle A = \angle P$$

$$\frac{AC}{PR} = \frac{AM}{PN}$$

$$\text{Because, } \frac{AB}{PQ} = \frac{\frac{1}{2}AB}{\frac{1}{2}PQ}$$

$$\frac{AB}{PQ} = \frac{AM}{PN}$$

Therefore, $\Delta AMC \sim \Delta PNR$

(ii) From solution

$$(i) \frac{CM}{RN} = \frac{AM}{PN}$$

$$\frac{CM}{RN} = \frac{2AM}{2PN}$$

$$\frac{CM}{RN} = \frac{AB}{PQ}$$

(iii) Now consider the ΔCMB and ΔRNQ

$$\angle B = \angle Q$$

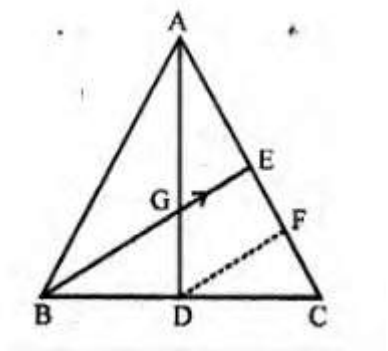
$$\frac{BC}{QP} = \frac{BM}{QN}$$

Therefore, $\Delta CMB \sim \Delta RNQ$

19. In the adjoining figure, medians AD and BE of ΔABC meet at the point G, and DF is drawn parallel to BE. Prove that

(i) $EF = FC$

(ii) $AG : GD = 2 : 1$



Solution :

From the figure it is given that, medians AD and BE of ΔABC meet at the point G, and DF is drawn parallel to BE.

(i) We have to prove that, $EF = FC$

From the figure, D is the midpoint of BC and also DF parallel to BE.

So, F is the midpoint of EC

Therefore, $EF = FC$

$$= \frac{1}{2} EC$$

Therefore, $EF = FC$

$$= \frac{1}{2} EC$$

$$EF = \frac{1}{2} AE$$

(ii) Now consider the $\triangle AGE$ and $\triangle ADF$

Then, $(BG \text{ or } GE) \parallel DF$

Therefore, $\triangle AGE \sim \triangle ADF$

$$\text{So, } \frac{AG}{GD} = \frac{AE}{EF}$$

$$\frac{AG}{GD} = \frac{1}{1/2}$$

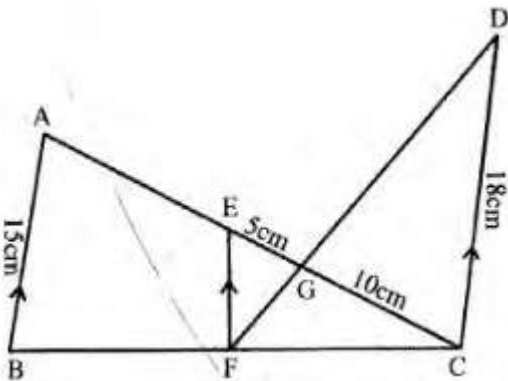
$$\frac{AG}{GD} = 1 \times \left(\frac{2}{1}\right)$$

Therefore, $AG : GD = 2 : 1$

20. In the figure given below, AB , EF and CD are parallel lines. Given that $AB = 15\text{cm}$, $EG = 5\text{cm}$, $GC = 10\text{cm}$ and $DC = 18\text{ cm}$. Calculate

(i) EF

(ii) AC



Solution:-

From the figure it is given that, AB, EF and CD are parallel lines.

(i) Consider the $\triangle EFG$ and $\triangle CGD$

$$\angle EGF = \angle CGD \quad [\text{Because vertically opposite angles are equal}]$$

$$\angle FEG = \angle GCD \quad [\text{alternate angles are equal}]$$

Therefore, $\triangle EFG \sim \triangle CGD$

$$\text{Then, } \frac{EG}{GC} = \frac{EF}{CD}$$

$$\frac{5}{10} = \frac{EF}{18}$$

$$EF = \frac{(5 \times 18)}{10}$$

Therefore, $EF = 9 \text{ cm}$

(ii) Now, consider the $\triangle ABC$ and $\triangle EFC$

$$EF \parallel AB$$

So, $\triangle ABC \sim \triangle EFC$

$$\text{Then, } \frac{AC}{EC} = \frac{AB}{EF}$$

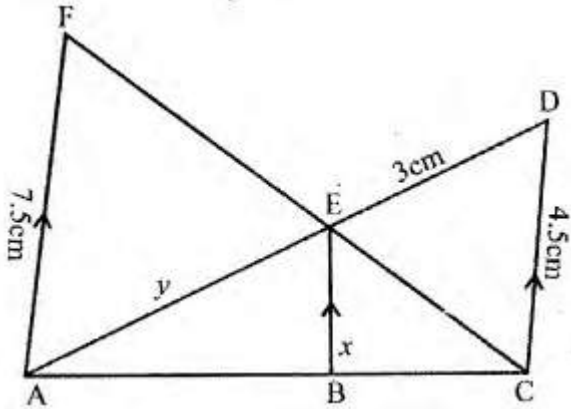
$$\frac{AC}{(5+10)} = \frac{15}{9}$$

$$\frac{AC}{(15)} = \frac{15}{9}$$

$$AC = \frac{(15 \times 5)}{9}$$

Therefore, $AC = 25 \text{ cm}$

(b) In the figure given below, AF, BE and CD are parallel line. Given that AF = 7.5 cm, CD = 4.5cm, ED = 3cm, BE = x and AE = y. Find the values of x and y.



Solution:

From the figure, AF, BE and CD are parallel lines.

Consider the $\triangle AEF$ and $\triangle CED$

$\angle AEF$ and $\angle CED$ [because vertically opposite angles are equal]

$\angle F = \angle C$ [altinate angles are equal]

Therefore, $\triangle AEF \sim \triangle CED$

$$\text{So, } \frac{AF}{CD} = \frac{AE}{ED}$$

$$= \frac{7.5}{4.5} = \frac{y}{3}$$

By cross multiplications,

$$y = \frac{(7.5 \times 3)}{4.5}$$

$$y = 5 \text{ cm}$$

So, similarly in $\triangle ACD$, $BE \parallel CD$

Therefore, $\triangle ABE \sim \triangle ACD$

$$\frac{EB}{CD} = \frac{AE}{AD}$$

$$\frac{x}{CD} = \frac{y}{y} + 3$$

$$\frac{x}{4.5} = \frac{5}{(5+3)}$$

$$\frac{x}{4.5} = \frac{5}{8}$$

$$x = \frac{(4.5 \times 5)}{8}$$

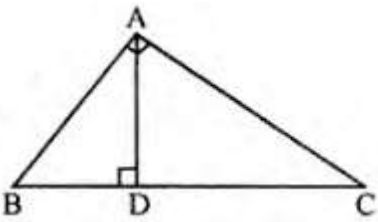
$$x = \frac{22.5}{8}$$

$$x = \frac{225}{80}$$

$$x = \frac{45}{16}$$

$$x = 2\frac{13}{16}$$

21. In the given figure, $\angle A = 90^\circ$ and $AD \perp BC$ If $BD = 2$ cm and $CD = 8$ cm, find AD .



Solution:

From the figure, consider $\triangle ABC$,

So, $\angle A = 90^\circ$

And $AD \perp BC$

$\angle BAC = 90^\circ$

Then, $\angle BAD + \angle DAC = 90^\circ \dots$ [equation (i)]

Now, consider $\triangle ADC$

$$\angle ADC = 90^\circ$$

So, $\angle DCA + \angle DAC = 90^\circ \dots$ [equation (ii)]

From equation (i) and equation (ii)

we have

$$\angle BAD + \angle DAC = \angle DCA + \angle DAC$$

$$\angle BAD = \angle DCA \dots$$
 [equation (iii)]

So, from $\triangle BDA$ and $\triangle ADC$

$$\angle BDA = \angle ADC \dots$$
 [both the angles are equal to 90°]

$$\angle BAD = \angle DCA \dots$$
 [from equation (iii)]

Therefore, $\triangle BDA \sim \triangle ADC$

$$\frac{BD}{AD} = \frac{AD}{DC} = \frac{AB}{AC}$$

Because, corresponding sides of similar triangles are proportional

$$\frac{BD}{AD} = \frac{AD}{DC}$$

By cross multiplication we get,

$$AD^2 = BD \times CD$$

$$AD^2 = 2 \times 8 = 16$$

$$AD = \sqrt{16}$$

$$AD = 4$$

22. A 15 metres high tower casts a shadow of 24 metres long at a certain time and at the same time, a telephone pole casts a shadow 16 metres long. Find the height of the telephone pole.

Solution:

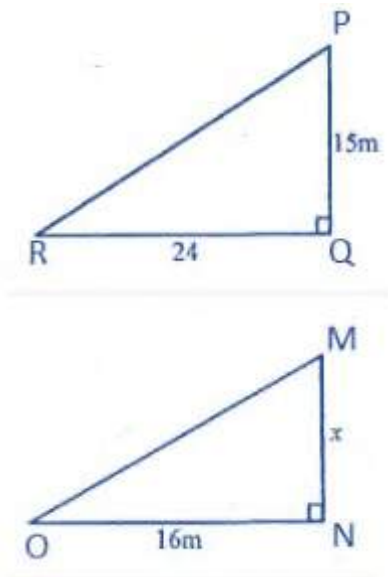
From the question it is given that,

Height of a tower $PQ = 15\text{m}$

Its shadow $QR = 24\text{m}$

Let us assume the height of a telephone pole $MN = x$

Its shadow $NO = 16\text{ m}$



Given at the same time,

$$\triangle PQR \sim \triangle MNO$$

$$\text{Therefore, } \frac{PQ}{MN} = \frac{OQ}{RQ}$$

$$\frac{15}{x} = \frac{24}{16}$$

By cross multiplication we get,

$$x = \frac{(15 \times 16)}{24}$$

$$x = \frac{240}{24}$$

$$x = 10$$

Therefore, height of pole = 10m.

23. A street light bulb is fixed on a pole 6m above the level of street. If a woman of height casts a shadow of 3m, find how far she is away from the base of the pole ?

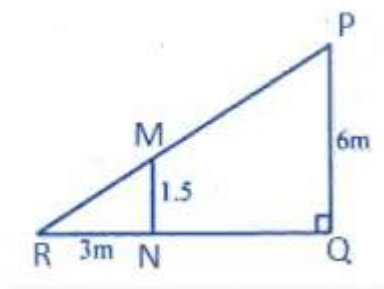
Solution:

From the question it is given that,

Height of pole (PQ) = 6m

Height of a woman = 1.5m

So, shadow NR = 3m



Therefore, pole and woman are standing in the same line

PM || MR

$\Delta PRQ \sim \Delta MNR$

$$\text{So, } \frac{RQ}{RN} = \frac{PQ}{MN}$$

$$\frac{(3+x)}{3} = \frac{6}{1.5}$$

$$\frac{(3+x)}{3} = \frac{60}{15}$$

$$\frac{(3+x)}{3} = \frac{4}{1}$$

$$(3 + x) = 12$$

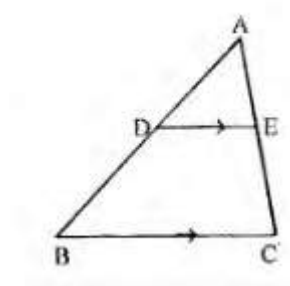
$$X = 12 - 3$$

$$X = 9m$$

Therefore, women is 9m away from the pole.

Exercise 13.2

1. (a) In the figure (i) given below if $DE \parallel BG$, $AD = 3\text{cm}$, $BD = 4\text{cm}$ and $BC = 5\text{cm}$. Find (i) $AE : EC$ (ii) DE .



Solution:

From the figure,

$DE \parallel BG$, $AD = 3\text{cm}$, $BD = 4\text{cm}$ and $BC = 5\text{cm}$

(i) $AE : EC$

$$\text{So, } \frac{AD}{BD} = \frac{AE}{EC}$$

$$\frac{AE}{EC} = \frac{AD}{BD}$$

$$\frac{AE}{EC} = \frac{3}{4}$$

(ii) Consider $\triangle ADE$ and $\triangle ABC$

$$\angle D = \angle B$$

$$\angle E = \angle C$$

Therefore, $\triangle ADE \sim \triangle ABC$

$$\text{Then, } \frac{DE}{BC} = \frac{AD}{AB}$$

$$\frac{DE}{5} = \frac{3}{(3+4)}$$

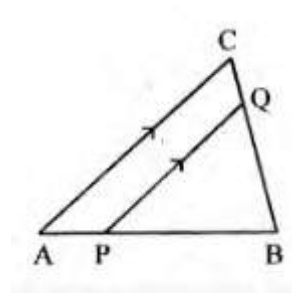
$$\frac{DE}{5} = \frac{3}{7}$$

$$DE = \frac{(3 \times 5)}{7}$$

$$DE = \frac{15}{7}$$

$$DE = 2\frac{1}{7}$$

(b) In the figure (ii) given below, $PQ \parallel AC$, $AP = 4\text{cm}$, $PB = 6\text{ cm}$ and $BC = 8\text{cm}$, Find CQ and BQ .



Solution :

From the figure,

$PQ \parallel AC$, $AP = 4\text{cm}$, $PB = 6\text{cm}$ and $BC = 8\text{cm}$

$\angle BQP = \angle BCA$ [because alternate angles are equal]

Also, $\angle B = \angle B$ [common for both the triangles]

Therefore, $\triangle ABC \sim \triangle BPQ$

$$\text{Then, } \frac{BQ}{BC} = \frac{BP}{AB} = \frac{PQ}{AC}$$

$$\frac{BQ}{BC} = \frac{6}{(6+4)} = \frac{PQ}{AC}$$

$$\frac{BQ}{BC} = \frac{6}{(10)} = \frac{PQ}{AC}$$

$$\frac{BQ}{8} = \frac{6}{(10)} = \frac{PQ}{AC} \dots\dots[\text{because } BC = 8\text{cm given}]$$

$$\text{Now, } \frac{BQ}{8} = \frac{6}{(10)}$$

$$BQ = \frac{(6 \times 10)}{8}$$

$$BQ = \frac{48}{10}$$

$$BQ = 4.8 \text{ cm}$$

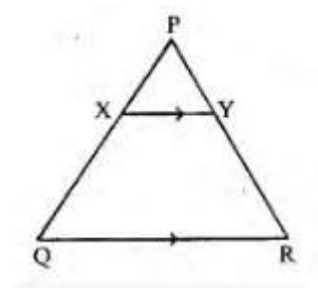
$$\text{Also, } CQ = BC - BQ$$

$$CQ = (8 - 4.8)\text{cm}$$

$$CQ = 3.2 \text{ cm}$$

Therefore, $CQ = 3.2 \text{ cm}$ and $BQ = 4.8 \text{ cm}$

(c) In the figure (iii) given below, if $XY \parallel QR$, $PX = 1 \text{ cm}$, $QX = 3\text{cm}$, $YR = 4.5 \text{ cm}$ and $QR = 9\text{cm}$, find PY and XY .



Solution :

From the figure,

$XY \parallel QR$, $PX = 1\text{cm}$, $QX = 3\text{cm}$, $YR = 4.5 \text{ cm}$ and $QR = 9\text{cm}$,

$$\text{So, } \frac{PX}{QX} = \frac{PY}{YR}$$

$$= \frac{1}{3} = \frac{PY}{4.5}$$

By cross multiplication we get,

$$\frac{(4.5 \times 1)}{3} = PY$$

$$PY = \frac{45}{30}$$

$$PY = 1.5$$

Then, $\angle X = \angle Q$

$$\angle Y = \angle R$$

So, $\Delta PXY \sim \Delta PQR$

$$\text{Therefore, } \frac{XY}{QR} = \frac{PX}{PQ}$$

$$\frac{XY}{9} = \frac{1}{(1+3)}$$

$$\frac{XY}{9} = \frac{1}{(4)}$$

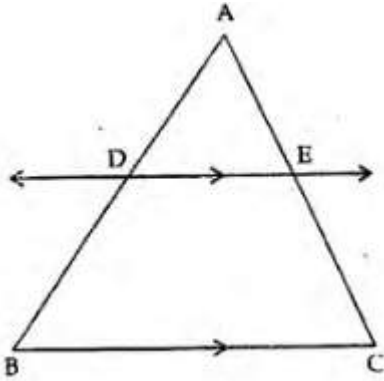
$$XY = \frac{9}{4}$$

$$XY = 2.25$$

2. In the given figure, $DE \parallel BC$.

(i) If $AD = x$, $DB = x - 2$, $AE = x + 2$ and $EC = x - 1$, find the value of x .

(ii) If $DB = x - 3$, $AB = 2x$, $EC = x - 2$ and $AC = 2x + 3$, find the value of x .



Solution:

(i) From the figure, it is given that,

Consider the ΔABC ,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{(x-2)} = \frac{(x+2)}{(x-1)}$$

By cross multiplication we get,

$$X(x - 1) = (x - 2)(x + 2)$$

$$x^2 - x = x^2 - 4$$

$$-x = -4$$

$$x = 4$$

(ii) From the question it is given that,

$$DB = x - 3, AB = 2x, EC = x - 2 \text{ and } AC = 2x + 3$$

Consider the ΔABC ,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{2x}{(x-2)} = \frac{(2x+3)}{(x-3)}$$

By cross multiplication we get,

$$2x(x-2) = (2x+3)(x-3)$$

$$2x^2 - 4x = 2x^2 - 6x + 3x - 9$$

$$2x^2 - 4x - 2x^2 + 6x - 3x = -9$$

$$-7x + 6x = -9$$

$$-x = -9$$

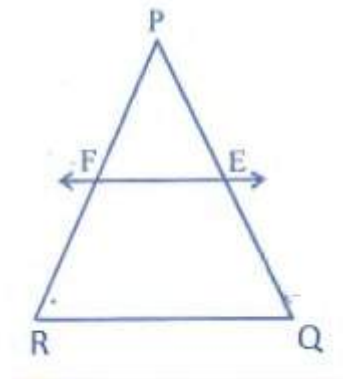
$$x = 9$$

3. E and F are points on the sides PQ and PR respectively of a ΔPQR . For each of the following cases, state whether $EF \parallel QR$:
(i) $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 8$ cm and $RF = 9$ cm.

Solution:

From the given dimensions,

Consider the ΔPQR



$$\text{So, } \frac{PE}{EQ} = \frac{3.9}{3}$$

$$= \frac{39}{30}$$

$$= \frac{13}{10}$$

$$\text{Then, } \frac{PF}{FR} = \frac{8}{9}$$

By comparing both the results,

$$\frac{13}{10} \neq \frac{8}{9}$$

$$\text{Therefore, } \frac{PE}{EQ} \neq \frac{PF}{FR}$$

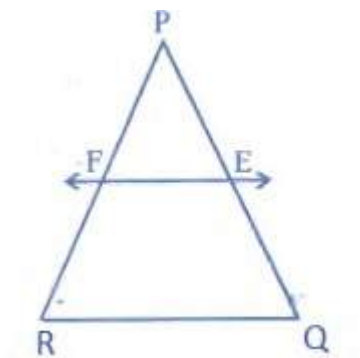
So, EF is not parallel to QR

(ii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18cm and PF = 0.36 cm.

Solution:

From the dimensions given in the question,

Consider the ΔPQR



$$\text{So, } \frac{PQ}{PE} = \frac{1.28}{0.18}$$

$$= \frac{128}{18}$$

$$= \frac{64}{9}$$

$$\text{Then } \frac{PR}{RF} = \frac{2.56}{0.36}$$

$$= \frac{256}{36}$$

$$= \frac{64}{9}$$

By comparing both the results,

$$\frac{64}{9} = \frac{64}{9}$$

$$\text{Therefore, } \frac{PQ}{PE} = \frac{PR}{PF}$$

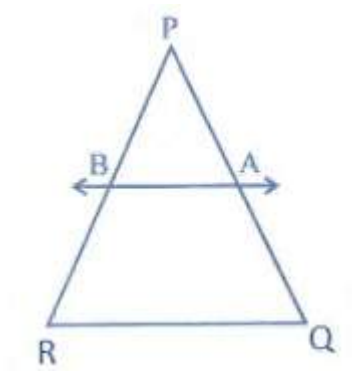
So, EF is parallel to QR.

4. A and B are respectively the points on the sides PQ and PR of a triangle PQR such that PQ = 12.5cm, PA= 5cm, BR = 6cm and PB = 4cm. Is AB || QR ? Give reasons for your answer.

Solution:

From the dimensions given in the question,

Consider the ΔPQR



$$\text{So, } \frac{PQ}{PA} = \frac{12.5}{5}$$

$$= \frac{2.5}{1}$$

$$\frac{PR}{PB} = \frac{(PB+BR)}{PB}$$

$$= \frac{(4+6)}{4}$$

$$= \frac{10}{4}$$

$$= 2.5$$

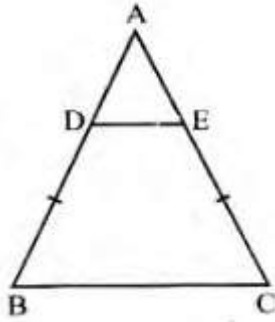
by comparing both the results,

$$2.5 = 2.5$$

$$\text{Therefore, } \frac{PQ}{PA} = \frac{PR}{PB}$$

So, AB is parallel to QR.

5. (a) In figure (i) given below, $DE \parallel BC$ and $BD = CE$. Prove that $\triangle ABC$ is an isosceles triangle.



Solution :

From the question it is given that,

$DE \parallel BC$ and $BD = CE$

So, we have to prove that $\triangle ABC$ is an isosceles triangle.

Consider the triangle ABC,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Given, $DB = EC$ [equation (i)]

Then $AD = AE$ [equation (ii)]

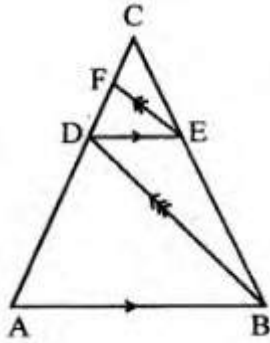
By adding equation (i) and equation (ii) we get,

$$AD + DB = AE + EC$$

So, $AB = AC$

Therefore, $\triangle ABC$ is an isosceles triangle.

(b) In figure (ii) given below, $AB \parallel DE$ and $BD \parallel EF$. Prove that $DC^2 = CF \times AC$.



Solution:

From the figure it is given that, $AB \parallel DE$ and $BD \parallel EF$.

We have to prove that, $DC^2 = CF \times AC$

Consider the $\triangle ABC$,

$$\frac{DC}{CA} = \frac{CE}{CB} \dots[\text{equation (i)}]$$

Now, consider $\triangle CDE$

$$\frac{CF}{CD} = \frac{CE}{CB} \dots[\text{equation (ii)}]$$

From equation (i) and equation (ii),

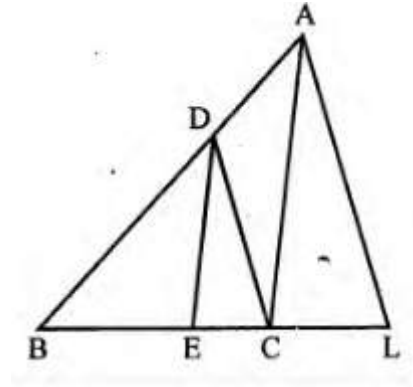
$$\frac{DC}{CA} = \frac{CF}{CD}$$

$$\frac{DC}{AC} = \frac{CF}{DC}$$

By cross multiplication we get,

$$DC^2 = CF \times AC$$

6. (a) In the figure (i) given below, $CD \parallel LA$ and $DE \parallel AC$. Find the length of CL if $BE = 4\text{cm}$ and $EC = 2\text{cm}$.



Solution:

From the given figure, $CD \parallel LA$ and $DE \parallel AC$,

Consider the $\triangle BCA$,

$$\frac{BE}{BC} = \frac{BD}{BA}$$

By using the corollary of basic proportionality theorem,

$$\frac{BE}{(BE+EC)} = \frac{BD}{AB}$$

$$\frac{4}{(4+2)} = \frac{BD}{AB} \dots \text{[equation (i)]}$$

Then, consider the $\triangle BLA$

$$\frac{BC}{BL} = \frac{BD}{AB}$$

By using the corollary of basic proportionality theorem,

$$\frac{6}{(6+CL)} = \frac{BD}{AB} \dots \text{[equation (ii)]}$$

Now, combining the equation (i) and equation (ii), we get

$$\frac{6}{(6+CL)} = \frac{4}{6}$$

By cross multiplication we get,

$$6 \times 6 = 4 \times (6 + CL)$$

$$24 + 4CL = 36$$

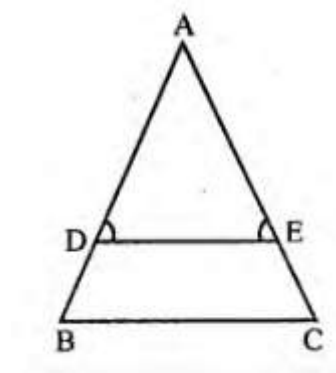
$$4CL = 36 - 24$$

$$CL = \frac{12}{4}$$

$$CL = 3 \text{ cm}$$

Therefore, the length of CL is 3 cm.

(b) In the give figure, $\angle D = \angle E$ and $\frac{AD}{BD} = \frac{AE}{EC}$. Prove that BAC is an isosceles triangle.



Solution:

From the given figure, $\angle D = \angle E$ and $\frac{AD}{BD} = \frac{AE}{EC}$.

We have to prove that, BAC is an isosceles triangle

So, consider the $\triangle ADE$

$\angle D = \angle E$...[from the question]

$AD = DE$ [sides opposite to equal angles]

Consider the $\triangle ABC$,

Then, $\frac{AD}{DB} = \frac{AE}{EC}$ [equation (i)]

Therefore, DE parallel to BC

Because AD = DE

DB = EC[equation (ii)]

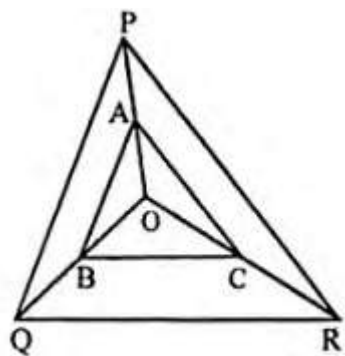
By adding equation (i) and equation (ii) we get,

$$AD + DB = AE + EC$$

$$AB = AC$$

Therefore, $\triangle ABC$ is an isosceles triangle.

7. In the adjoining given below, A, B and C are points on OP, OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. show that $BC \parallel QR$.



Solution:

Consider the $\triangle POQ$

$AB \parallel PQ$ [given]

So, $\frac{OA}{AP} = \frac{OB}{BQ}$ [equation (i)]

Then, consider the ΔOPR

$AC \parallel PR$

$$\frac{OA}{AP} = \frac{OC}{CR} \dots[\text{equation (ii)}]$$

Now by comparing both equation (i) and equation (ii),

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

Then, in ΔOQR

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

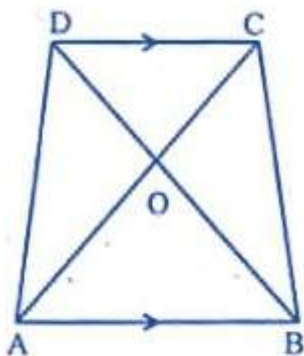
Therefore, $BC \parallel QR$

8. ABCD is a trapezium in which $AB \parallel DC$ and its diagonals intersect each other at O. Using Basic Proportionality theorem, Prove that $\frac{AO}{BO} = \frac{CO}{DO}$

Solution:

From the question It is given that,

ABCD is a trapezium in which $AB \parallel DC$ and its diagonal intersect each other at O



Now consider the ΔOAB and ΔOCD ,

$\angle AOB = \angle COD$ [because vertically opposite angles are equal]

$\angle OBA = \angle ODC$ [because alternate angles are equal]

Therefore, $\Delta OAB \sim \Delta OCD$

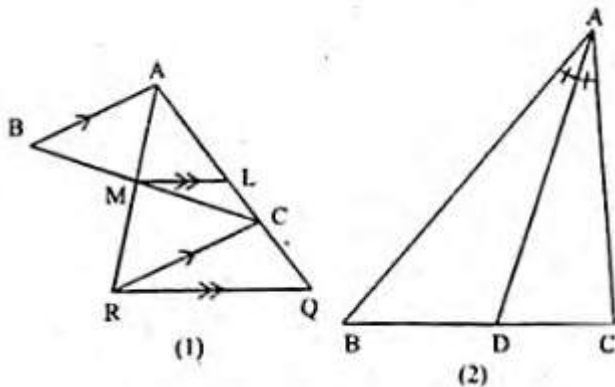
Then, $\frac{OA}{OC} = \frac{OB}{OD}$

$\frac{AO}{OB} = \frac{CO}{DO}$.. [by alternate angles]

9. (a) In the figure (i) given below, $AB \parallel CR$ and $LM \parallel QR$.

(i) Prove that $\frac{BM}{MC} = \frac{AL}{LQ}$

(ii) Calculate $LM : QR$, given that $BM : MC = 1 : 2$.



Solution:

From the question it is given that, $AB \parallel CR$ and $LM \parallel QR$

(i) We have to prove that, $\frac{BM}{MC} = \frac{AL}{LQ}$

Consider the ΔARQ

$LM \parallel QR$[from the question]

So, $\frac{AM}{MR} = \frac{AL}{LQ}$ [equation (i)]

Now, consider the ΔAMB and ΔMCR

$\angle AMB = \angle CMR$ [because vertically opposite angles are equal]

$\angle MBA = \angle MCR$ [because alternate angles are equal]

Therefore, $\frac{AM}{MR} = \frac{BM}{MC}$ [equation (ii)]

From equation (i) and equation (ii) we get,

$$\frac{BM}{MR} = \frac{AL}{LQ}$$

(ii) Given, $BM : MC = 1 : 2$

$$\frac{AM}{MR} = \frac{BM}{MC}$$

$$\frac{AM}{MR} = \frac{1}{2} \dots[\text{equation (iii)}]$$

$LM \parallel QR$ [given from equation]

$$\frac{AM}{MR} = \frac{LM}{QR} \dots[\text{equation (iv)}]$$

$$\frac{AR}{AM} = \frac{QR}{LM}$$

$$\frac{(AM+MR)}{AM} = \frac{QR}{LM}$$

$$1 + \frac{MR}{AM} = \frac{QR}{LM}$$

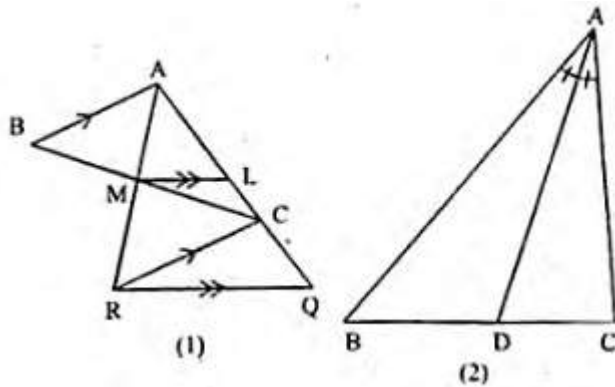
$$1 + \frac{2}{1} = \frac{QR}{LM}$$

$$\frac{3}{1} = \frac{QR}{LM}$$

$$\frac{LM}{QR} = \frac{1}{3}$$

Therefore, the ratio of LM : QR is 1 : 3.

(b) In the figure (2) given below AD is bisector of $\angle BAC$. If AB = 6cm, AC = 4cm and BD = 3cm, find BC



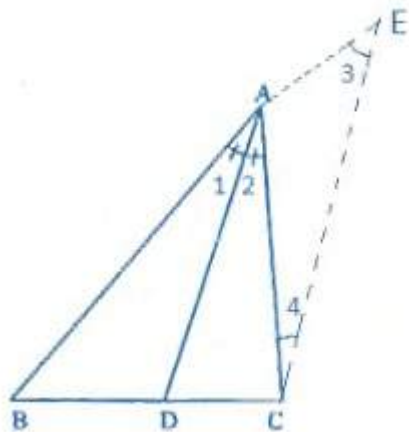
Solution:

From the question it is given that,

AD is bisector of $\angle BAC$

AB = 6cm, AC = 4cm and BD = 3cm

Construction, from C draw a straight line CE parallel to DA and join AE



$$\angle 1 = \angle 2 \dots [\text{equation (i)}]$$

By construction $CE \parallel DE$

$$\text{So, } \angle 2 = \angle 4 \dots [\text{because alternate angles are equal}] \quad [\text{equation (ii)}]$$

Again by construction $CE \parallel DE$

$$\angle 1 = \angle 3 \dots [\text{because corresponding angles are equal}] \quad [\text{equation (iii)}]$$

By comparing equation(i), equation (ii) and equation (iii) we get,

$$\angle 3 = \angle 4$$

$$\text{So, } AC = AE \dots [\text{equation (iv)}]$$

Now, consider the $\triangle BCE$,

$CE \parallel DE$

$$\frac{BD}{DC} = \frac{AB}{AE}$$

$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\frac{3}{DC} = \frac{6}{4}$$

by cross multiplication we get,

$$3 \times 4 = 6 \times DC$$

$$DC = \frac{(3 \times 4)}{6}$$

$$DC = \frac{(12)}{6}$$

$$DC = 2$$

$$\text{Therefore, } BC = BD + DC$$

$$= 3 + 2$$

$$= 5\text{cm}$$

Exercise 13.3

1. Given that Δ s ABC and PQR are similar :

Find :

- (i) The ratio of the area of ΔABC to the area of ΔPQR if their corresponding sides are in the ratio 1 : 3.
- (ii) The ratio of their corresponding sides if area of ΔABC : area of $\Delta PQR = 25 : 36$.

Solution:-

From the question it is given that,

- (i) The ratio of the area of ΔABC to the area of ΔPQR if their corresponding sides are in the ratio 1 : 3.

Then, $\Delta ABC \sim \Delta PQR$

$$\text{area of } \frac{\Delta ABC}{\text{area of } \Delta PQR} = \frac{BC^2}{QR^2}$$

So, $BC : QR = 1 : 3$

$$\text{Therefore, } \frac{\Delta ABC}{\text{area of } \Delta PQR} = \frac{1^2}{3^2}$$

$$= \frac{1}{9}$$

Hence the ratio of the area of ΔABC to the area of ΔPQR is 1 : 9.

(ii) The area of ΔABC to the area of ΔPQR if their corresponding sides are in the ratio 25 : 36.

Then, $\Delta ABC \sim \Delta PQR$

$$\text{Area of } \frac{\Delta ABC}{\text{area of } \Delta PQR} = \frac{BC^2}{QR^2}$$

$$\text{area of } \frac{\Delta ABC}{\text{area of } \Delta PQR} = \frac{25}{36}$$

$$= \left(\frac{BC}{QR}\right)^2 = \left(\frac{5}{6}\right)^2$$

$$= \frac{BC}{QR} = \frac{5}{6}$$

Hence the ratio of their corresponding sides is 5 : 6.

2. $\Delta ABC \sim \Delta DEF$. If area of $\Delta ABC = 9$ sq. cm., area of $\Delta DEF = 16$ sq. cm and $BC = 2.1$ cm., find the length of EF.

Solution:

From the question it is given that,

$$\Delta ABC \sim \Delta DEF$$

$$\text{Area of } \Delta ABC = 9 \text{ sq. cm}$$

$$\text{Area of } \Delta DEF = 16 \text{ sq. cm}$$

We know that,

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \frac{BC^2}{EF^2}$$

$$\frac{9}{16} = \frac{BC^2}{EF^2}$$

$$\frac{9}{16} = \frac{(2.1)^2}{x^2}$$

$$\frac{2.1}{x} = \frac{\sqrt{9}}{\sqrt{16}}$$

$$\frac{2.1}{x} = \frac{3}{4}$$

By cross multiplication we get,

$$2.1 \times 4 = 3 \times x$$

$$8.4 = 3x$$

$$x = \frac{8.4}{3}$$

$$x = 2.8$$

Therefore, EF = 2.8 cm

3. $\triangle ABC \sim \triangle DEF$. If BC = 3cm, EF = 4cm and area of $\triangle ABC = 54$ sq. cm. Determine the area of $\triangle DEF$.

Solution:

From the question it is given that,

$$\triangle ABC \sim \triangle DEF$$

$$BC = 3\text{cm}, EF = 4\text{cm}$$

$$\text{Area of } \triangle ABC = 54 \text{ sq. cm.}$$

We know that,

$$\frac{\text{Area of } \triangle ABC}{\text{area of } \triangle DEF} = \frac{BC^2}{EF^2}$$

$$\frac{54}{\text{area of } \triangle DEF} = \frac{3^2}{4^2}$$

$$\frac{54}{\text{area of } \triangle DEF} = \frac{9}{16}$$

By cross multiplication we get,

$$\text{Area of } \triangle DEF = \frac{(54 \times 16)}{9}$$

$$= 6 \times 16$$

$$= 96 \text{ cm}$$

4. The area of two similar triangles are 36cm^2 and 25cm^2 . If an altitude of the first triangle is 2.4cm, find the corresponding altitude of the other triangle.

Solution :

From the question it is given that,

The area of two similar triangles are 36cm^2 and 25cm^2

Let us assume $\Delta PQR \sim \Delta XYZ$, PM and XN are their altitudes.

So, area of $\Delta PQR = 36\text{cm}^2$

Area of $\Delta XYZ = 25\text{cm}^2$

PM = 2.4 cm

Assume XN = a

we know that,

$$\frac{\text{area of } \Delta PQR}{\text{area of } \Delta XYZ} = \frac{PM^2}{XN^2}$$

$$\frac{36}{25} = \frac{(2.4)^2}{a^2}$$

By cross multiplication we get,

$$36a^2 = 25(2.4)^2$$

$$a^2 = 5.76 \times \frac{25}{36}$$

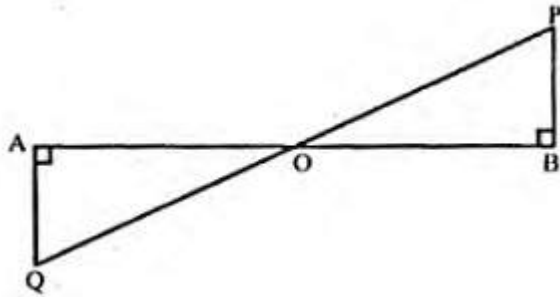
$$a^2 = \frac{144}{36}$$

$$a^2 = \sqrt{4}$$

$$a = 2 \text{ cm}$$

Therefore, altitude of the other triangle XN = 2 cm.

5. (a) In the figure, (i) given below, PB and QA perpendiculars to the line segment AB. If PO = 6cm, QO = 9cm and the area of $\Delta POB = 120\text{cm}^2$, find the area of ΔQOA .



Solution:

From the question it is given that, PO = 6cm, QO = 9 cm and the area of $\Delta POB = 120\text{cm}^2$

From the figure,

Consider the ΔAOQ and ΔBOP ,

$\angle OAQ = \angle OBP$...[both angles are equal to 90°]

$\angle AOQ = \angle BOP$..[because vertically opposite angles are equal]

Therefore, $\Delta AOQ \sim \Delta BOP$

Then, $\frac{\text{area of } \Delta AOQ}{\text{area of } \Delta BOP} = \frac{OQ^2}{PO^2}$

$$\frac{\text{Area of } \Delta AOQ}{120} = \frac{9^2}{6^2}$$

$$\frac{\text{Area of } \Delta AOQ}{120} = \frac{81}{36}$$

$$\text{Area of } \Delta AOQ = \frac{(81 \times 120)}{36}$$

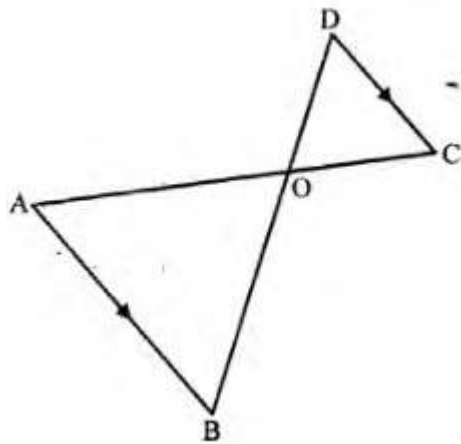
$$\text{Area of } \Delta AOQ = 270 \text{ cm}^2$$

b) In the figure (ii) given below, $AB \parallel DC$. $AO = 10\text{cm}$, $OC = 5\text{cm}$, $AB = 6.5\text{ cm}$ and $OD = 2.8\text{ cm}$.

(i) Prove that $\Delta OAB \sim \Delta OCD$.

(ii) Find CD and OB .

(iii) find the ratio of areas of ΔOAB and ΔOCD



Solution:

from the question it is given that,

$AB \parallel DC$. $AO = 10\text{cm}$, $OC = 5\text{ cm}$, $AB = 6.5\text{cm}$ and $OD = 2.8\text{ cm}$

(i) We have to prove that, $\Delta OAB \sim \Delta OCD$

So, consider the ΔOAB and ΔOCD

$\angle AOB = \angle COD \dots$ [because vertically opposite angles are equal]

$\angle OBA = \angle OCD \dots$ [because alternate angles are equal]

Therefore, $\Delta OAB \sim \Delta OCD \dots$ [from AAA axiom]

(ii) Consider the ΔOAB and ΔOCD

$$\frac{OA}{OC} = \frac{OB}{OD} = \frac{AB}{CD}$$

Now consider $\frac{OA}{OC} = \frac{OB}{OD}$

$$\frac{10}{5} = \frac{OB}{2.8}$$

$$OB = \frac{(10 \times 2.8)}{5}$$

$$OB = 2 \times 2.8$$

$$OB = 5.6 \text{ cm}$$

$$\text{Then, consider } \frac{OA}{OC} = \frac{AB}{CD}$$

$$\frac{10}{5} = \frac{6.5}{CD}$$

$$CD = \frac{(6.5 \times 5)}{10}$$

$$CD = \frac{32.5}{10}$$

$$CD = 3.25 \text{ cm}$$

(iii) We have to find the ratio of areas of ΔOAB and ΔOCD .

From (i) we proved that, $\Delta OAB \sim \Delta OCD$

$$\text{Then, } \frac{\text{area of } (\Delta OAB)}{\text{area of } \Delta OCD}$$

$$\frac{AB^2}{CD^2} = \frac{(6.5)^2}{(3.25)^2}$$

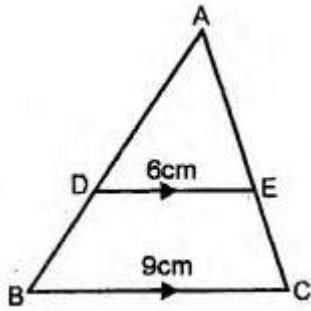
$$= \frac{(6.5 \times 6.5)}{(3.25 \times 3.25)}$$

$$= 2 \times \frac{2}{1}$$

$$= \frac{4}{1}$$

Therefore, the ratio of areas of ΔOAB and $\Delta OCD = 4:1$

6. (a) In the figure (i) given below, $DE \parallel BC$. If $DE = 6\text{cm}$, $BC = 9\text{cm}$ and area of $\Delta ADE = 28\text{ sq. cm}$, find the area of ΔABC .



Solution:

From the question it is given that,

$DE \parallel BC$, $DE = 6\text{cm}$, $BC = 9\text{ cm}$ and area of $\Delta ADE = 28\text{ sq. cm}$

From the fig. $\angle D = \angle B$ and $\angle E = \angle C \dots$ [corresponding angles are equal]

Now, consider the ΔADE and ΔABC ,

$\angle A = \angle A \dots$ [common angles for both triangles]

Therefore, $\Delta ADE \sim \Delta ABC$

$$\text{Then, } \frac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC} = \frac{(DE)^2}{(BC)^2}$$

$$\frac{28}{\text{area of } \Delta ABC} = \frac{(6)^2}{(9)^2}$$

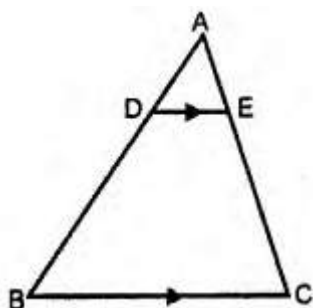
$$\frac{28}{\text{area of } \Delta ABC} = \frac{36}{81}$$

$$\text{area of } \Delta ABC = \frac{(28 \times 81)}{36}$$

$$\text{area of } \Delta ABC = \frac{2268}{36}$$

$$\text{area of } \Delta ABC = 63\text{cm}^2$$

(b) In the figure (ii) given below, $DE \parallel BC$ and $AD : DB = 1 : 2$, find the ratio of the areas of $\triangle ADE$ and trapezium DBCE.



Solution :

From the question it is given that, $DE \parallel BC$ and $AD : DB = 1 : 2$,
 $\angle D = \angle B, \angle E = \angle C$ [corresponding angles are equal]

Consider the $\triangle ADE$ and $\triangle ABC$,

$\angle A = \angle A$ [common angles for both triangles]

Therefore, $\triangle ADE \sim \triangle ABC$

But, $\frac{AD}{DB} = \frac{1}{2}$

Then, $\frac{DB}{AD} = \frac{2}{1}$

Now, adding 1 for both side LHS and RHS,

$$\left(\frac{DB}{AD} + 1\right) = \left(\frac{2}{1} + 1\right)$$

$$\left(\frac{DB+AD}{AD}\right) = (2 + 1)$$

Therefore, $\triangle ADE \sim \triangle ABC$

Then, $\frac{\text{area of } \triangle ADE}{\text{area of } \triangle ABC} = \frac{AD^2}{AB^2}$

$$\frac{\text{area of } \triangle ADE}{\text{area of } \triangle ABC} = \left(\frac{1}{3}\right)^2$$

$$\frac{\text{area of } \triangle ADE}{\text{area of } \triangle ABC} = \frac{1}{9}$$

$$\text{area of } \triangle ABC = 9 \text{ area of } \triangle ADE$$

Area of trapezium DBCE

Area of $\triangle ABC$ - Area of $\triangle ADE$

9 area of $\triangle ADE$ - area of $\triangle ADE$

8 area of $\triangle ADE$

$$\text{Therefore, } \frac{\text{area of } \triangle ADE}{\text{area of trapezium}} = \frac{1}{8}$$

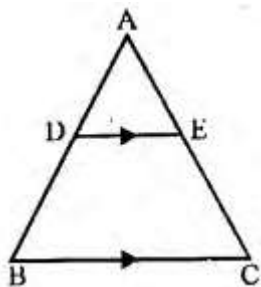
Then area of $\triangle ADE$: area of trapezium DBCE = 1 : 8

7. In the given figure, $DE \parallel BC$.

(i) Prove that $\triangle ADE$ and $\triangle ABC$ are similar.

(ii) Given that $AD = \frac{1}{2} BD$, calculate DE if $BC = 4.5$ cm.

(iii) If area of $\triangle ABC = 18\text{cm}^2$, find the area of trapezium DBCE



Solution:

(i) From the question it is given that, $DE \parallel BC$

We have to prove that, $\triangle ADE$ and $\triangle ABC$ are similar

$\angle A = \angle A \dots \dots$ [Common angle for both triangles]

$\angle ADE = \angle ABC \dots$ [because corresponding angles are equal]

Therefore, $\Delta ADE \sim \Delta ABC \dots$ [AA axiom]

(ii) From (i) we proved that, $\Delta ADE \sim \Delta ABC$

$$\text{Then, } \frac{AD}{AB} = \frac{AB}{AC} = \frac{DE}{BC}$$

$$\text{So, } \frac{AD}{(AD+BD)} = \frac{DE}{BC}$$

$$\frac{\left(\frac{1}{2}BD\right)}{\left(\left(\frac{1}{2}BD\right)+BD\right)} = \frac{DE}{4.5}$$

$$\frac{\left(\frac{1}{2}BD\right)}{\left(\frac{3}{2}BD\right)} = \frac{DE}{4.5}$$

$$\frac{1}{2} \times \left(\frac{2}{3}\right) = \frac{DE}{4.5}$$

$$\frac{1}{3} = \frac{DE}{4.5}$$

$$\text{Therefore, } DE = \frac{4.5}{3}$$

$$DE = 1.5 \text{ cm}$$

(iii) From the question it is given that, area of $\Delta ABC = 18\text{cm}^2$

$$\text{Then, } \frac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC} = \frac{DE^2}{BC^2}$$

$$\frac{\text{area of } \Delta ADE}{18} = \left(\frac{DE}{BC}\right)^2$$

$$\frac{\text{area of } \Delta ADE}{18} = \left(\frac{AD}{AB}\right)^2$$

$$\frac{\text{area of } \triangle ADE}{18} = \left(\frac{1}{3}\right)^2$$

$$\frac{\text{area of } \triangle ADE}{18} = \frac{1}{9}$$

$$\text{area of } \triangle ADE = 18 \times \frac{1}{9}$$

$$\text{area of } \triangle ADE = 2$$

$$\text{So, area of trapezium DBCE} = \text{area of } \triangle ABC - \text{area of } \triangle ADE$$

$$= 18 - 2$$

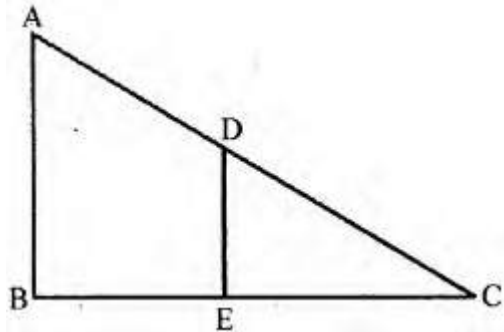
$$= 16 \text{ cm}^2$$

8. In the given figure, AB and DE are perpendicular to BC.

(i) Prove that $\triangle ABC \sim \triangle DEC$

(ii) If $AB = 6\text{cm}$, $DE = 4\text{cm}$ and $AC = 15\text{ cm}$, calculate CD.

(iii) Find the ratio of the area of $\triangle ABC$: area of $\triangle DEC$.



Solution:

(i) Consider the $\triangle ABC$ and $\triangle DEC$.

$$\angle ABC = \angle DEC \dots [\text{both angles are equal to } 90^\circ]$$

$$\angle C = \angle C \dots [\text{common angle for both triangles}]$$

Therefore, $\triangle ABC \sim \triangle DEC$ [by AA axiom]

$$(ii) \frac{AC}{CD} = \frac{AB}{DE}$$

Corresponding sides of similar triangles are proportional

$$\frac{15}{CD} = \frac{6}{4}$$

$$CD = \frac{(15 \times 4)}{6}$$

$$CD = \frac{60}{6}$$

$$CD = 10 \text{ cm}$$

$$(iii) \text{ we know that, } \frac{\text{area of } \triangle ABC}{\text{area of } \triangle DEC} = \frac{AB^2}{DE^2}$$

$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle DEC} = \frac{6^2}{4^2}$$

$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle DEC} = \frac{36}{16}$$

$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle DEC} = \frac{9}{4}$$

Therefore, the ratio of the area of $\triangle ABC$: area of $\triangle DEC$ is 9 : 4.

$$\text{Area of } \triangle ABC = 9 \text{ area of } \triangle ADE$$

$$\text{Area of trapezium DBCE}$$

$$\text{Area of } \triangle ABC - \text{Area of } \triangle ADE$$

$$9 \text{ area of } \triangle ADE - \text{area of } \triangle ADE$$

$$8 \text{ area of } \triangle ADE$$

$$\text{Therefore, } \frac{\text{area of } \triangle ADE}{\text{Area of trapezium}} = \frac{1}{8}$$

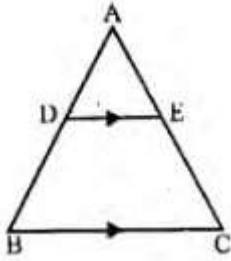
$$\text{Then area of } \triangle ADE : \text{Area of trapezium DBCE} = 1 : 8$$

7. In the given figure, $DE \parallel BC$.

(i) Prove that $\triangle ADE$ and $\triangle ABC$ are similar.

(ii) Given that $AD = \frac{1}{2}BD$, calculate DE if $BC = 4.5$ cm.

(iii) If area of $\triangle ABC = 18\text{cm}^2$, find the area of trapezium $DBCE$



Solution:

(i) From the question it is given that, $DE \parallel BC$

We have to prove that, $\triangle ADE$ and $\triangle ABC$ are similar

$\angle A = \angle A$...[common angle for both triangles]

$\angle ADE = \angle ABC$..[because corresponding angles are equal]

Therefore, $\triangle ADE \sim \triangle ABC$[AA axiom]

(ii) From (i) we proved that, $\triangle ADE \sim \triangle ABC$

$$\text{Then, } \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

$$\text{So, } \frac{AD}{(AD+BD)} = \frac{DE}{BC}$$

$$\frac{\left(\frac{1}{2}BD\right)}{\left(\left(\frac{1}{2}BD\right)+BD\right)} = \frac{DE}{4.5}$$

$$\frac{\left(\frac{1}{2}BD\right)}{\left(\left(\frac{3}{2}BD\right)\right)} = \frac{DE}{4.5}$$

$$\frac{1}{2} \times \left(\frac{2}{3}\right) = \frac{DE}{4.5}$$

$$\frac{1}{3} = \frac{DE}{4.5}$$

$$\text{Therefore, } DE = \frac{4.5}{3}$$

$$DE = 1.5 \text{ cm}$$

(iii) From the question it is given that, area of $\triangle ABC = 18\text{cm}^2$

$$\text{Then, } \frac{\text{area of } \triangle ADE}{\text{area of } \triangle ABC} = \frac{DE^2}{BC^2}$$

$$\frac{\text{area of } \triangle ADE}{18} = \left(\frac{DE}{BC}\right)^2$$

$$\frac{\text{area of } \triangle ADE}{18} = \left(\frac{AD}{AB}\right)^2$$

$$\frac{\text{area of } \triangle ADE}{18} = \left(\frac{1}{3}\right)^2$$

$$\frac{\text{area of } \triangle ADE}{18} = \frac{1}{9}$$

$$\text{area of } \triangle ADE = 18 \times \frac{1}{9}$$

$$\text{area of } \triangle ADE = 2$$

$$\text{So, area of trapezium DBCE} = \text{area of } \triangle ABC - \text{area of } \triangle ADE$$

$$= 18 - 2$$

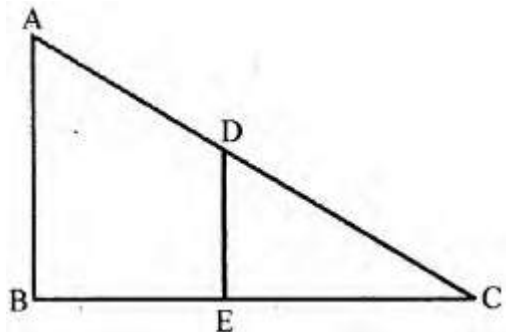
$$= 16\text{cm}^2$$

8. In the given figure, AB and DE are perpendicular to BC.

(i) Prove that $\triangle ABC \sim \triangle DEC$

(ii) If $AB = 6\text{cm}$, $DE = 4\text{cm}$ and $AC = 15\text{ cm}$, calculate CD .

(iii) Find the ratio of the area of $\triangle ABC$: *area of* $\triangle DEC$.



Solution:

(i) Consider the $\triangle ABC$ and $\triangle DEC$,

$\angle ABC = \angle DEC$ [both angles are equal to 90°]

$\angle C = \angle C$...[common angle for both triangles]

Therefore, $\triangle ABC \sim \triangle DEC$... [by AA axiom]

$$(ii) \frac{AC}{CD} = \frac{AB}{DE}$$

Corresponding sides of similar triangles are proportional

$$\frac{15}{CD} = \frac{6}{4}$$

$$CD = \frac{(15 \times 4)}{6}$$

$$CD = \frac{60}{6}$$

$$CD = 10\text{ cm}$$

(iii) we know that, $\frac{\text{area of } \triangle ABC}{\text{area of } \triangle DEC} = \frac{AB^2}{DE^2}$

$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle DEC} = \frac{6^2}{4^2}$$

$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle DEC} = \frac{36}{16}$$

$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle DEC} = \frac{9}{4}$$

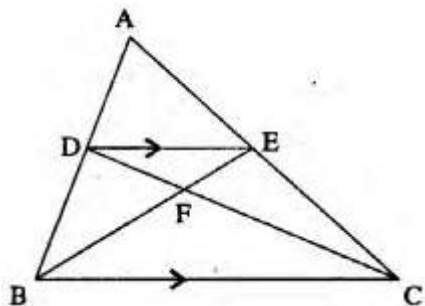
Therefore, the ratio of the area of $\triangle ABC$: area of $\triangle DEC$ is 9 : 4.

9. In the adjoining figure, ABC is a triangle. DE is parallel to BC and $\frac{AD}{DB} = \frac{3}{2}$.

(i) Determine the ratios $\frac{AD}{DB}$, $\frac{DE}{BC}$

(ii) Prove that $\triangle DEF$ is similar to $\triangle CBF$. Hence, find $\frac{EF}{FB}$.

(iii) What is the ratio of the areas of $\triangle DEF$ and $\triangle CBF$?



Solution:

(i) We have to find the ratios $\frac{AD}{AB}$, $\frac{DE}{BC}$,

From the question it is given that, $\frac{AD}{DB} = \frac{3}{2}$,

$$\text{Then, } \frac{DB}{AD} = \frac{2}{3}$$

Now add 1 for both LHS and RHS we get,

$$\left(\frac{DB}{AD} + 1\right) = \left(\frac{2}{3} + 1\right)$$

$$\frac{(DB+AD)}{AD} = \frac{(2+3)}{3}$$

From the figure $(DB + AD) = AB$

$$\text{So, } \frac{AB}{AD} = \frac{5}{3}$$

Now, consider the $\triangle ADE$ and $\triangle ABC$,

$\angle ADE = \angle B$...[corresponding angles are equal]

$\angle AED = \angle C$...[corresponding angles are equal]

Therefore, $\triangle ADE \sim \triangle ABC$..[by AA similarity]

$$\text{Then, } \frac{AD}{DB} = \frac{DE}{BC} = \frac{3}{5}$$

(ii) Now consider the $\triangle DEF$ and $\triangle CBF$

$\angle EDF = \angle BCF$...[because alternate angles are equal]

$\angle DEF = \angle FBC$ [because alternate angles are equal]

$\angle DFE = \angle ABFC$ [because vertically opposite angles are equal]

Therefore, $\triangle DEF \sim \triangle CBF$

$$\text{So, } \frac{EF}{FB} = \frac{DE}{BC} = \frac{3}{5}$$

(iii) we have to find the ratio of the areas of $\triangle DEF$ and $\triangle CBF$,

We know that, $\frac{\text{Area of } \triangle DEF}{\text{Area of } \triangle BFC} = \frac{DE^2}{BC^2}$

$$\frac{\text{Area of } \triangle DEF}{\text{Area of } \triangle BFC} = \left(\frac{DE}{BC}\right)^2$$

$$\frac{\text{Area of } \triangle DEF}{\text{Area of } \triangle BFC} = \left(\frac{3}{5}\right)^2$$

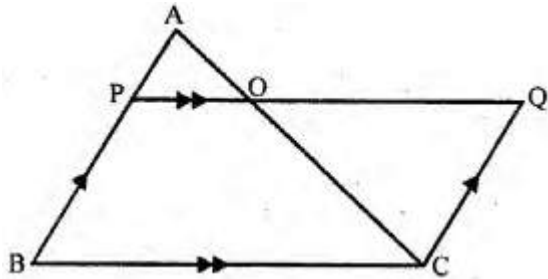
$$\frac{\text{Area of } \triangle DEF}{\text{Area of } \triangle BFC} = \frac{9}{25}$$

Therefore, the ratio of the areas of $\triangle DEF$ and $\triangle CBF$ is 9 : 25.

10. In $\triangle ABC$, $AP : PB = 2 : 3$. PO is parallel to BC and is extended to Q so that CQ is parallel to BA . find:

(i) Area $\triangle APO$: area $\triangle ABC$.

(ii) Area $\triangle APO$: Area $\triangle CQO$.



Solution:

From the question it is given that,

$$PB = 2 : 3$$

PO is parallel to BC and is extended to Q so that CQ is parallel to BA .

(i) we have to find the area ΔAPO : *area* ΔABC ,

Then,

$\angle A = \angle A \dots$ [common angles for both triangles]

$\angle APO = \angle ABC \dots$ [because corresponding angles are equal]

Then, $\Delta APO \sim \Delta ABC \dots$ [AA axiom]

We know that, $\frac{\text{area of } \Delta APO}{\text{area of } \Delta ABC} = \frac{AP^2}{AB^2}$

$$= \frac{AP^2}{(AP+PB)^2}$$

$$= \frac{2^2}{(2+3)^2}$$

$$= \frac{4}{5^2}$$

$$= \frac{4}{25}$$

Therefore, *area* ΔAPO : *Area* ΔABC is 4 : 25

(ii) we have to find the area ΔAPO : *area* ΔCQO

Then, $\angle AOP = \angle COQ \dots$ [because vertically opposite angles are equal]

$\angle APQ = \angle OQC \dots$ [because alternate angles are equal]

Therefore, $\frac{\text{area of } \Delta APO}{\text{area of } \Delta CQO} = \frac{AP^2}{CQ^2}$

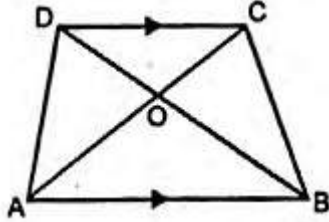
$$\frac{\text{area of } \Delta APO}{\text{area of } \Delta CQO} = \frac{AP^2}{PB^2}$$

$$\frac{\text{area of } \Delta APO}{\text{area of } \Delta CQO} = \frac{2^2}{3^2}$$

$$\frac{\text{area of } \Delta APO}{\text{area of } \Delta CQO} = \frac{4}{9}$$

Therefore, *area* ΔAPO : *area* ΔCQO is 4 : 9.

11. (a) In the figure (i) given below, ABCD is a trapezium in which $AB \parallel DC$ and $AB = 2CD$. Determine the ratio of the areas of $\triangle AOB$ and $\triangle COD$.



Solution:

From the question it is given that,

ABCD is a trapezium in which $AB \parallel DC$ and $AB = 2CD$,

Then, $\angle OAB = \angle OCD$... [*because alternate angles are equal*]

$\angle OBA = \angle ODC$

Then, $\triangle AOB \sim \triangle COD$

$$\text{So, } \frac{\text{area of } \triangle AOB}{\text{area of } \triangle COD} = \frac{AB^2}{CD^2}$$

$$= \frac{(2CD)^2}{CD^2} \dots \text{ [because } AB = 2CD \text{]}$$

$$= \frac{4CD^2}{CD^2}$$

$$= \frac{4}{1}$$

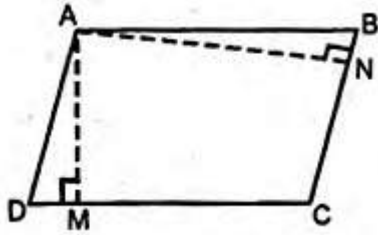
Therefore, the ratio of the areas of $\triangle AOB$ and $\triangle COD$ is 4 : 1.

(b) In the figure (ii) given below, ABCD is a parallelogram. $AM \perp DC$ and $AN \perp CB$. If $AM = 6\text{ cm}$, $AN = 10\text{ cm}$ and the area of parallelogram ABCD is 45 cm^2 , find

(i) AB

(ii) BC

(iii) area of $\triangle ADM$: area of $\triangle ANB$.



Solution:

From the question it is given that,

ABCD is a parallelogram, $AM \perp DC$ and $AN \perp CB$

$AM = 6\text{ cm}$

$AN = 10\text{ cm}$

The area of parallelogram ABCD is 45 cm^2

Then, area of parallelogram ABCD = $AC \times AM = BC \times AN$

$45 = DC \times 6 = BC \times 10$

(i) $DC = \frac{45}{6}$

Divide both numerator and denominator by 3 we get,

$$= \frac{15}{2}$$

$$= 7.5\text{ cm}$$

Therefore, $AB = DC = 7.5 \text{ cm}$

(ii) $BC \times 10 = 45$

$$BC = \frac{45}{10}$$

$$BC = 4.5 \text{ cm}$$

(iii) Now, consider $\triangle ADM$ and $\triangle ABN$

$\angle D = \angle B \dots$ [because opposite angles of a parallelogram]

$\angle M = \angle N \dots$ [both angles are equal to 90°]

Therefore, $\triangle ADM \sim \triangle ABN$

$$\text{Therefore, } \frac{\text{area of } \triangle ADM}{\text{area of } \triangle ABN} = \frac{AD^2}{AB^2}$$

$$= \frac{BC^2}{AB^2}$$

$$= \frac{4.5^2}{7.5^2}$$

$$= \frac{20.25}{56.25}$$

$$= \frac{2025}{5625}$$

$$= \frac{81}{225}$$

$$= \frac{9}{25}$$

Therefore, area of $\triangle ADM$: area of $\triangle ANB$ is 9:25

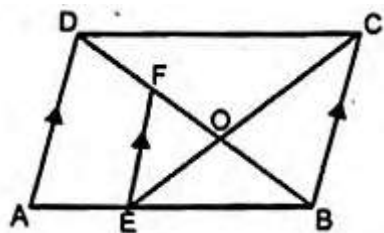
(c) In the figure (iii) given below, ABCD is a parallelogram. E is a point on AB, CE intersects the diagonal BD at O and $EF \parallel BC$. If $AE : EB = 2 : 3$, find

(i) $EF : AD$

(ii) Area of $\triangle BEF : \text{area of } \triangle ABD$

(iii) area of $\triangle ABD : \text{area of trapezium AFED}$

(iv) area of $\triangle FEO : \text{area of } \triangle OBC$.



Solution:

From the question it is given that, ABCD is a parallelogram.

E is a point on AB, CE intersects the diagonal BD at O.

$AE : EB = 2 : 3$

(i) We have to find $EF : AD$

$$\text{So, } \frac{AB}{BE} = \frac{AD}{EF}$$

$$\frac{EF}{AD} = \frac{BE}{AB}$$

$$\frac{AE}{EB} = \frac{2}{3} \dots [\text{given}]$$

Now add 1 to both LHS and RHS we get,

$$\frac{(AE+EB)}{EB} = \left(\frac{2}{3}\right) + 1$$

$$\frac{(AE+EB)}{EB} = \left(\frac{2+3}{3}\right)$$

$$\frac{AB}{EB} = \frac{5}{3}$$

$$\frac{EB}{AB} = \frac{3}{5}$$

Therefore, EF : AD is 3 : 5

(ii) we have to find area of $\triangle BEF$: *area of* $\triangle ABD$,

$$\text{Then, } \frac{\text{area of } \triangle BEF}{\text{area of } \triangle ABD} = \frac{(EF)^2}{(AD)^2}$$

$$\begin{aligned} \frac{\text{area of } \triangle BEF}{\text{area of } \triangle ABD} &= \frac{(3)^2}{(5)^2} \\ &= \frac{9}{25} \end{aligned}$$

Therefore, area of $\triangle BEF$: *area of* $\triangle ABD$ is 9 : 25

$$\text{(iii) from(ii) } \frac{\text{area of } \triangle ABD}{\text{area of } \triangle BEF} = \frac{25}{9}$$

$$25 \text{ area of } \triangle BEF = 9 \text{ area of } \triangle ABD$$

$$25 (\text{ area of } \triangle ABD - \text{ area of trapezium AEFD}) = 9 \text{ area of } \triangle ABD$$

$$25 \text{ area of } \triangle ABD - 25 \text{ area of trapezium AEFD} = 9 \text{ area of } \triangle ABD$$

$$25 \text{ area of trapezium AEFD} = 25 \text{ area of } \triangle ABD - 9 \text{ area of } \triangle ABD$$

$$25 \text{ area of trapezium AEFD} = 16 \text{ area of } \triangle ABD$$

$$\frac{\text{area of } \triangle ABD}{\text{area of trapezium AEFD}} = \frac{25}{16}$$

Therefore, area of $\triangle ABD$: area of trapezium AFED = 25 : 16

(iv) Now we have to find area of $\triangle FEO$: area of $\triangle OBC$

So, consider $\triangle FEO$ and $\triangle OBC$,

$\angle EOF = \angle BOC$.. [because vertically opposite angles are equal]

$\angle F = \angle OBC$.. [because alternate angles are equal]

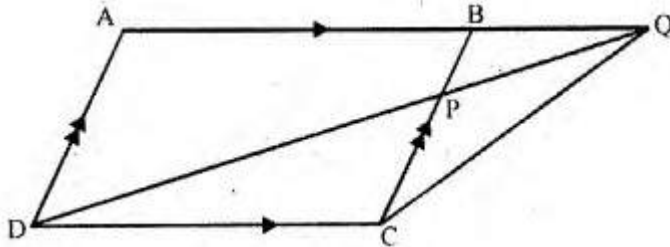
$\triangle FEO \sim \triangle OBC$

Then, $\frac{\text{area of } FEO}{\text{area of } \triangle OBC} = \frac{EF^2}{BC^2}$

$$\frac{EF^2}{AD^2} = \frac{9}{25}$$

Therefore, area of $\triangle FEO$: area of $\triangle OBC = 9 : 25$

12. In the adjoining figure, ABCD is a parallelogram. P is a point on BC such that $BP : PC = 1 : 2$ and DP produced meets AB produced at Q. If area of $\triangle CPQ = 20 \text{ cm}^2$, find



(i) area of $\triangle BPQ$.

(ii) area $\triangle CDP$.

(iii) area of parallelogram ABCD.

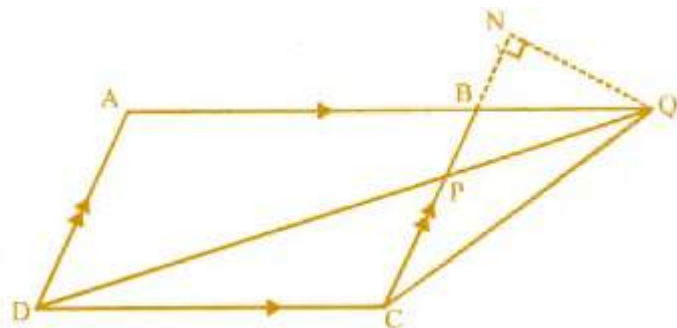
Solution:

From the question it is given that, ABCD is a parallelogram.

$$BP : PC = 1 : 2$$

$$\text{area of } \triangle CPQ = 20 \text{ cm}^2$$

Construction : draw QN perpendicular CB and Join BN.



$$\text{Then, } \frac{\text{area of } \triangle BPQ}{\text{area of } \triangle CPQ} = \frac{\left(\frac{1}{2}BP\right) \times QN}{\left(\frac{1}{2}PC\right) \times QN}$$

$$= \frac{BP}{PC} = \frac{1}{2}$$

(i) So, area $\triangle BPQ = \frac{1}{2}$ area of $\triangle CPQ$

$$= \frac{1}{2} \times 20$$

Therefore, area of $\triangle BPQ = 10\text{cm}^2$

(ii) Now we have to find area of $\triangle CDP$,

Consider the $\triangle CDP$ and $\triangle BQP$,

Then, $\angle CPD = \angle QPD$.. [because vertically opposite angles are equal]

$\angle PDC = \angle PQB$... [because alternate angles are equal]

Therefore, $\triangle CDP \sim \triangle BQP$...[AA axiom]

$$\frac{\text{area of } \triangle CDP}{\text{area of } \triangle BQP} = \frac{PC^2}{BP^2}$$

$$\frac{\text{area of } \triangle CDP}{\text{area of } \triangle BQP} = \frac{2^2}{1^2}$$

$$\frac{\text{area of } \triangle CDP}{\text{area of } \triangle BQP} = \frac{4}{1}$$

$$\text{area of } \triangle CDP = 4 \times \text{area } \triangle BQP$$

$$\begin{aligned}\text{Therefore, area of } \triangle CDP &= 4 \times 10 \\ &= 40 \text{ cm}^2\end{aligned}$$

(iii) We have to find the area of parallelogram ABCD,

$$\text{Area of parallelogram ABCD} = 2 \text{ area of } \triangle DCQ$$

$$= 2 \text{ area } (\triangle DCP + \triangle CPQ)$$

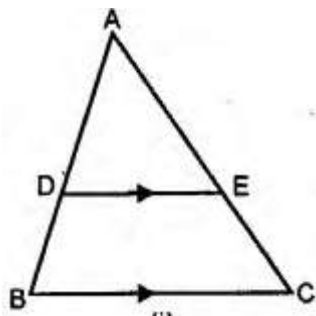
$$= 2 (40 + 20) \text{ cm}^2$$

$$= 2 \times 60 \text{ cm}^2$$

$$= 120 \text{ cm}^2$$

Therefore, the area of parallelogram ABCD is 120 cm^2 .

13. (a) In the figure (i) given below, $DE \parallel BC$ and the ratio of the areas of $\triangle ADE$ and trapezium DBCE is $4 : 5$. Find the ratio of $DE : BC$.



Solution:

From the question it is given that,

$$DE \parallel BC$$

The ratio of the areas of $\triangle ADE$ and trapezium DBCE is 4 : 5

Now, consider the $\triangle ABC$ and $\triangle ADE$

$$\angle A = \angle A \dots [\text{common angle for both triangles}]$$

$$\angle D = \angle B \text{ and}$$

$$\angle E = \angle C \dots [\text{because corresponding angles are equal}]$$

Therefore, $\triangle ADE \sim \triangle ABC$

$$\text{So, } \frac{\text{area of } \triangle ADE}{\text{area of } \triangle ABC} = \frac{(DE)^2}{(BC)^2} \dots [\text{equation (i)}]$$

$$\text{Then, } \frac{\text{area of } \triangle ADE}{\text{area of trapezium DBCE}} = \frac{4}{5}$$

$$\frac{\text{area of trapezium DBCE}}{\text{area of } \triangle ADE} = \frac{5}{4}$$

Add 1 for both LHS and RHS we get,

$$\left(\frac{\text{area of trapezium DBCE}}{\text{area of } \triangle ADE} \right) + 1 = \left(\frac{5}{4} \right) + 1$$

$$\frac{(\text{area of trapezium DBCE} + \text{area of } \triangle ADE)}{\text{area of } \triangle ADE} = \frac{(5+4)}{4}$$

$$\frac{(\text{area of } \triangle ABC)}{\text{area of } \triangle ADE} = \frac{9}{4}$$

$$\frac{\text{area of } \triangle ADE}{\text{area of } \triangle ABC} = \frac{4}{9}$$

From equation (i),

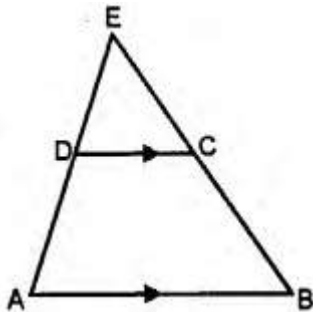
$$\frac{\text{area of } \triangle ADE}{\text{area of } \triangle ABC} = \frac{(DE)^2}{(BC)^2}$$

$$\frac{\text{area of } \triangle ADE}{\text{area of } \triangle ABC} = \frac{(4)^2}{(9)^2}$$

$$\frac{\text{area of } \triangle ADE}{\text{area of } \triangle ABC} = \frac{2}{3}$$

Therefore, $DE : BC = 2 : 3$

- (b) In the figure (ii) given below, $AB \parallel DC$ and $AB = 2 DC$. If $AD = 3\text{cm}$, $BC = 4\text{cm}$ and AD , BC produced meet at E , find (i) ED (ii) BE (iii) area of $\triangle EDC$: area of trapezium $ABCD$.



Solution:

From the question it is given that,

$$AB \parallel DC$$

$$AB = 2DC, \quad AD = 3\text{cm}, \quad BC = 4\text{cm}$$

Now consider $\triangle EAB$,

$$\frac{EA}{DA} = \frac{EB}{CB} = \frac{AB}{DC} = \frac{2DC}{DC} = \frac{2}{1}$$

$$(i) EA = 2, DA = 2 \times 3 = 6cm$$

$$\text{Then, } ED = EA - DA$$

$$= 6 - 3$$

$$= 3 \text{ cm}$$

$$(ii) \frac{EB}{CB} = \frac{2}{1}$$

$$EB = 2 \text{ CB}$$

$$EB = 2 \times 4$$

$$EB = 8 \text{ cm}$$

(iii) Now, consider the $\triangle EAB, DC \parallel AB$

So, $\triangle EDC \sim \triangle EAB$

$$\text{Therefore, } \frac{\text{area of } \triangle EDC}{\text{area of } \triangle ABE} = \frac{DC^2}{AB^2}$$

$$\frac{\text{area of } \triangle EDC}{\text{area of } \triangle ABE} = \frac{DC^2}{(2DC)^2}$$

$$\frac{\text{area of } \triangle EDC}{\text{area of } \triangle ABE} = \frac{DC^2}{4DC^2}$$

$$\frac{\text{area of } \triangle EDC}{\text{area of } \triangle ABE} = \frac{1}{4}$$

Therefore, area of ABE = 4 area of $\triangle EDC$

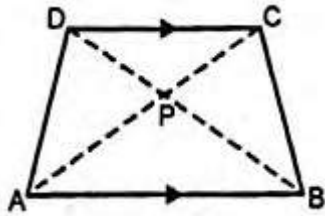
Then, area of $\triangle EDC$ + area of trapezium ABCD = 4 area of $\triangle EDC$

Area of trapezium ABCD = 3 area of $\triangle EDC$

$$\text{So, } \frac{\text{area of } \triangle EDC}{\text{area of trapezium } ABCD} = \frac{1}{3}$$

Therefore, area of ΔEDC : area of trapezium ABCD = 1 : 3

14. (a) In the figure given below, ABCD is a trapezium in which DC is parallel to AB. If AB = 9 cm, DC = 6cm and BB = 12 cm. find (i) BP
(ii) the ratio of areas of ΔAPB and ΔDPC .



Solution:

From the question it is given that,

DC is parallel to AB

AB = 9 cm, DC = 6cm and BB = 12cm

(i) Consider the ΔAPB and ΔCPD

$\angle APB = \angle CPD$..[because vertically opposite angles are equal]

$\angle PAB = \angle PCD$... [because alternate angles are equal]

So, $\Delta APB \sim \Delta CPD$

Then, $\frac{BP}{PD} = \frac{AB}{CD}$

$$\frac{BP}{(12-BP)} = \frac{9}{6}$$

$$6BP = 108 - 9BP$$

$$6BP + 9BP = 108$$

$$15BP = 108$$

$$BP = \frac{108}{15}$$

Therefore, $BP = 7.2$ cm

(ii) We know that, $\frac{\text{area of } \triangle APB}{\text{area of } \triangle CPD} = \frac{AB^2}{CD^2}$

$$\frac{\text{area of } \triangle APB}{\text{area of } \triangle CPD} = \frac{9^2}{6^2}$$

$$\frac{\text{area of } \triangle APB}{\text{area of } \triangle CPD} = \frac{81}{36}$$

By dividing both numerator and denominator by 9, we get,

$$\frac{\text{area of } \triangle APB}{\text{area of } \triangle CPD} = \frac{9}{4}$$

Therefore, the ratio of areas of $\triangle APB$ and $\triangle DPC$ is 9 :4.

(b) In the figure given below, $\angle ABC = \angle DAC$ and $AB = 8$ cm, $AC = 4$ cm, $AD = 5$ cm.

(i) Prove that $\triangle ACD$ is similar to $\triangle BCA$

(ii) Find BC and CD

(iii) Find the area of $\triangle ACD$: *area of* $\triangle ABC$.

Solution:

From the question it is given that,

$$\angle ABC = \angle DAC$$

$$AB = 8\text{cm}, AC = 4\text{cm}, AD = 5\text{cm}$$

(i) Now, consider $\triangle ACD$ and $\triangle BCA$

$\angle C = \angle C$ [common angle for both triangles]

$\angle ABC = \angle CAD$..[common angle for both triangles]

So, $\triangle ACD \sim \triangle BCA$..[by AA axiom]

$$(ii) \frac{AC}{BC} = \frac{CD}{CA} = \frac{AD}{AB}$$

$$\text{Consider } \frac{AC}{BC} = \frac{AD}{AB}$$

$$\frac{4}{BC} = \frac{5}{8}$$

$$BC = \frac{(4 \times 8)}{5}$$

$$BC = \frac{32}{5}$$

$$BC = 6.4 \text{ cm}$$

$$\text{Then, consider } \frac{CD}{CA} = \frac{AD}{AB}$$

$$\frac{CD}{4} = \frac{5}{8}$$

$$CD = \frac{(4 \times 5)}{8}$$

$$CD = \frac{20}{8}$$

$$CD = 2.5 \text{ cm}$$

(iii) from (i) we proved that, $\triangle ACD \sim \triangle BCA$

$$\frac{\text{area of } \triangle ACB}{\text{area of } \triangle BCA} = \frac{AC^2}{AB^2}$$

$$= \frac{4^2}{8^2}$$

$$= \frac{16}{64}$$

By dividing both numerator and denominator by 16, we get, $= \frac{1}{4}$

Therefore, the area of $\triangle ACD$: *area of $\triangle ABC$* is 1 : 4.

15. ABC is a right angles triangle with $\angle ABC = 90^\circ$. D is any point on AB and DE is perpendicular to AC. Prove that :

(i) $\triangle ADE \sim \triangle ACB$.

(ii) If $AC = 13$ cm, $BC = 5$ cm and $AE = 4$ cm. Find DE and AD.

(iii) Find, area of $\triangle ADE$: *area of quadrilateral BCED*.

Solution:

From the question it is given that,

$$\angle ABC = 90^\circ$$

AB and DE is perpendicular to AC

(i) Consider the $\triangle ADE$ and $\triangle ACB$.

$$\angle A = \angle A \dots [\text{common angle for both triangle}]$$

$$\angle B = \angle E \dots [\text{both angles are equal to } 90^\circ]$$

Therefore, $\triangle ADE \sim \triangle ACB$

(ii) from (i) we proved that, $\Delta ADE \sim \Delta ACB$

So, $\frac{AE}{AB} = \frac{AD}{AC} = \frac{DE}{BC} \dots$ [equation (i)]

Consider the ΔABC , is a right angle triangle

From Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$13^2 = AB^2 + 5^2$$

$$169 = AB^2 + 25$$

$$169 - 25 = AB^2$$

$$144 = AB^2$$

$$AB = \sqrt{144}$$

$$AB = 12 \text{ cm}$$

Consider the equation (i),

$$\frac{AE}{AB} = \frac{AD}{AC} = \frac{DE}{BC}$$

$$\text{Take, } \frac{AE}{AB} = \frac{AD}{AC}$$

$$\frac{4}{12} = \frac{AD}{13}$$

$$\frac{1}{3} = \frac{AD}{13}$$

$$\frac{(1 \times 13)}{3} = AD$$

$$AD = 4.33 \text{ cm}$$

$$\text{Now, take } \frac{AE}{AB} = \frac{DE}{BC}$$

$$\frac{4}{12} = \frac{DE}{5}$$

$$\frac{1}{3} = \frac{DE}{5}$$

$$DE = \frac{(5 \times 1)}{3}$$

$$DE = \frac{5}{3}$$

$$DE = 1.67 \text{ cm}$$

(iii) Now, we have to find area of $\triangle ADE$: area of quadrilateral BCED,

$$\text{We know that, Area of } \triangle ADE = \frac{1}{2} \times AE \times DE$$

$$= \frac{1}{2} \times 4 \times \left(\frac{5}{3}\right)$$

$$= \frac{10}{3} \text{ cm}^2$$

Then, area of quadrilateral BCED = area of $\triangle ABC$ – area of $\triangle ADE$.

$$= \frac{1}{2} \times BC \times AB - \frac{10}{3}$$

$$= \frac{1}{2} \times BC \times AB - \frac{10}{3}$$

$$= 1 \times 5 - 6 = \frac{10}{3}$$

$$= 30 - \frac{10}{3}$$

$$= \frac{(90-10)}{3}$$

$$= \frac{80}{3} \text{ cm}^2$$

So, the ratio of area of $\triangle ADE$: *area of quadrilateral BCED*

$$= \frac{\left(\frac{10}{3}\right)}{\left(\frac{80}{3}\right)}$$

$$= \left(\frac{10}{3}\right) \times \left(\frac{3}{80}\right)$$

$$= \frac{(10 \times 3)}{(3 \times 80)}$$

$$= \frac{(1 \times 1)}{(1 \times 8)}$$

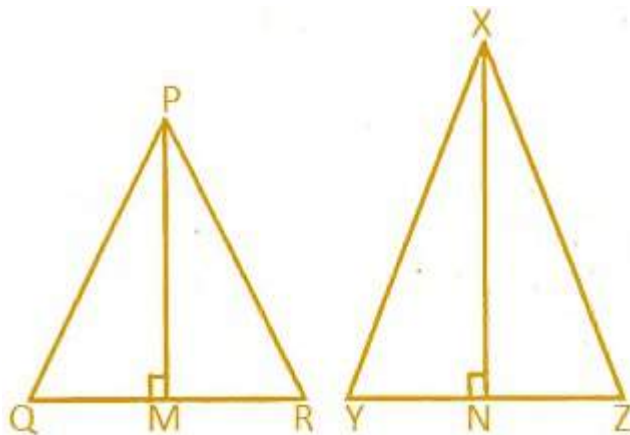
$$= \frac{1}{8}$$

Therefore, area of $\triangle ADE$: *area of quadrilateral BCED* is 1 : 8.

16. Two isosceles triangles have equal vertical angles and their areas are in the ratio 7 : 16. Find the ratio of their corresponding height.

Solution:

Consider the two isosceles triangle PQR and XYZ.



$$\angle P = \angle X \dots\dots [\text{from the question}]$$

$$\text{So, } \angle Q + \angle R = \angle Y + \angle Z$$

$$\angle Q = \angle R \text{ and } \angle Y = \angle Z \text{ [because opposite angles of equal sides]}$$

$$\text{Therefore, } \angle Q = \angle Y \text{ and } \angle R = \angle Z$$

$$\Delta PQR \sim \Delta XYZ$$

$$\text{Then, } \frac{\text{area of } \Delta PQR}{\text{area of } \Delta XYZ} = \frac{PM^2}{XN^2} \dots\dots [\text{from corollary of theorem}]$$

$$\frac{PM^2}{XN^2} = \frac{7}{16}$$

$$\frac{PM}{XN} = \frac{\sqrt{7}}{\sqrt{16}}$$

$$\frac{PM}{XN} = \frac{\sqrt{7}}{4}$$

$$\text{Therefore, ratio of PM : DM} = \sqrt{7} : 4$$

17. On a map drawn to a scale of 1 : 250000, a triangular plot of land has the following measurements : AB = 3cm, BC = 4 cm and $\angle ABC = 90^\circ$. Calculate

(i) the actual length of AB in km.

(ii) the area of the plot in sq. km :

Solution:

From the question it is given that,

Map drawn to a scale of 1 : 250000

AB = 3cm, BC = 4cm and $\angle ABC = 90^\circ$

(i) We have to find the actual length of AB in km.

Let us assume scale factor $K = 1 : 250000$

$$K = \frac{1}{250000}$$

Then, length of AB of actual plot $= \frac{1}{k} \times \text{length of AB on the map}$

$$= \left(\frac{1}{(250000)} \times 3 \right)$$

$$= 250000 \times 3$$

To convert cm into km divide by 100000

$$= \frac{(25000 \times 3)}{(100 \times 1000)}$$

$$= \frac{15}{2}$$

length of AB of actual plot = 7.5 km

(ii) We have to find the area of the plot in sq. km

Area of plot on the map $= \frac{1}{2} \times AB \times BC$

$$= \frac{1}{2} \times 3 \times 4$$

$$= \frac{1}{2} \times 12$$

$$= 1 \times 6$$

$$= 6 \text{ cm}^2$$

Then, area of actual plot $= \frac{1}{k^2} \times 6 \text{ cm}^2$

$$= 250000^2 \times 6$$

To convert cm into km divide by $(100000)^2$

$$= \frac{(25000 \times 25000 \times 6)}{100000 \times 100000}$$

$$\begin{aligned}
 &= \left(\frac{25}{4}\right) \times 6 \\
 &= \frac{75}{2} \\
 &= 3.75 \text{ km}^2
 \end{aligned}$$

18. On a map drawn to a scale of 1 : 25000, a rectangular plot of land, ABCD has the following measurements AB = 12cm and BG = 16 cm. Calculate :

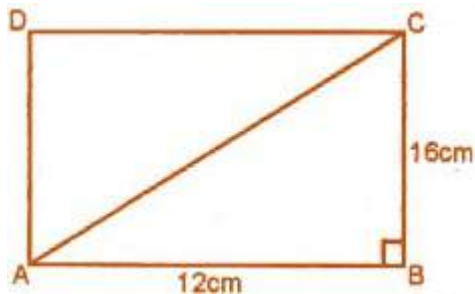
- (i) The distance of a diagonal of the plot in km.
- (ii) The area of the plot in sq. km.

Solution :

From the question it is given that,

Map drawn to a scale of 1 : 25000

AB = 12cm, BG = 16cm



Consider the ΔABC ,

From the Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$AC = \sqrt{(AB^2 + BC^2)}$$

$$AC = \sqrt{((12)^2 + (16)^2)}$$

$$= \sqrt{144 + 256}$$

$$= \sqrt{400}$$

$$= 20 \text{ cm}$$

Then, area of rectangular plot $ABCD = AB \times BC$

$$= 12 \times 16$$

$$= 192 \text{ cm}^2$$

(i) We have to find the distance of a diagonal of the plot in km.

Let us assume scale factor $K = 1 : 25000$

$$K = \frac{1}{250000}$$

Then, length of AB of actual plot $= \frac{1}{k} \times$
length of diagonal of rectangular plot

$$= \left(\frac{1}{\left(\frac{1}{25000} \right)} \right) \times 3$$

$$= 25000 \times 20$$

To convert cm into km divide by 100000

$$= \frac{(25000 \times 20)}{(100 \times 1000)}$$

$$= 5 \text{ km}$$

(ii) We have to find the area of the plot in sq. km.

Then, area of actual plot $= \frac{1}{k^2} \times 192 \text{ cm}^2$

$$= 25000^2 \times 192$$

To covert cm into divide by $(100000)^2$

$$= \frac{(25000 \times 25000 \times 192)}{(100000 \times 100000)}$$

$$= 12 \text{ km}^2$$

19. The model of a building is constructed with she scale factor 1 : 30.

- (i) If the height of the model is 80 cm, find the actual height of the building in metres.
- (ii) If the actual volume of a tank at the top of the building is 27m^3 , find the volume of the tank on the top of the model.

Solution:

From the question it is given that,

The model of a building is constructed with the scale factor 1 : 30

So,

$$\frac{\text{Height of the model}}{\text{Height of actual building}} = \frac{1}{30}$$

(i) Given, the height of the model is 80 cm

$$\text{Then, } \frac{80}{H} = \frac{1}{30}$$

$$H = (80 \times 30)$$

$$H = 2400 \text{ cm}$$

$$H = \frac{2400}{100}$$

$$H = 24 \text{ m}$$

(ii) Given, the actual volume of a tank at the top of the building is $27m^3$

$$\frac{\text{Volume of model}}{\text{Volume of tank}} = \left(\frac{1}{30}\right)^3$$

$$\frac{V}{27} = \frac{1}{27000}$$

$$V = \frac{27}{27000}$$

$$V = \frac{1}{1000} m^3$$

Therefore, Volume of model = 1000 cm^3

20. A model of a ship is made to a scale of 1 : 200.

(i) If the length of the model is 4m, find the length of the ship.

(ii) If the area of the deck of the ship is 16000 m^2 , find the area of the deck of the model.

(iii) If the volume of the model is 200 litres, find the volume of the ship in m^3 . (100 litres = $1m^3$)

Solution:

From the question it is given that, a model of a ship is made to a scale of 1 : 200

(i) Given, the length of the model is 4 m

$$\text{Then, length of the ship} = \frac{(4 \times 200)}{1}$$

$$= 800 \text{ m}$$

(ii) Given, the area of the deck of the ship is 160000 m^2

$$\text{Then, area of deck of the model} = 160000 \times \left(\frac{1}{200}\right)^2$$

$$= 160000 \times \left(\frac{1}{40000}\right)$$

$$= 4 \text{ m}^2$$

(iii) Given, the volume of the model is 200 liters

$$\text{Then, Volume of ship} = 200 \times \left(\frac{200}{1}\right)^3$$

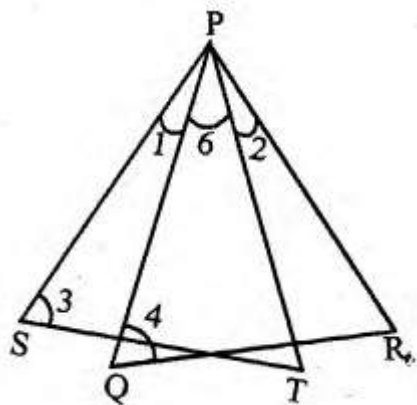
$$= 200 \times 8000000$$

$$= \frac{(200 \times 8000000)}{100}$$

$$= 1600000 \text{ m}^3$$

Chapter Test

1. In the adjoining figure, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$. Show that $PT \times QR = PR \times ST$.



Solution :

From the question it is given that,

$$\angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4$$

We have to prove that, $PT \times QR = PR \times ST$

Given, $\angle 1 = \angle 2$

Adding $\angle 6$ to both LHS and RHS we get,

$$\angle 1 + \angle 6 = \angle 2 + \angle 6$$

$$\angle SPT = \angle QPR$$

Consider the ΔPQR and ΔPST ,

From above $\angle SPT = \angle QPR$

$$\angle 3 = \angle 4$$

Therefore, $\Delta PQR \sim \Delta PST$

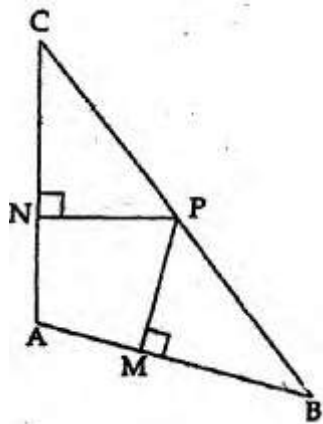
So, $\frac{PT}{PR} = \frac{ST}{QR}$

By cross multiplication we get,

$$PT \times QR = PR \times ST$$

Hence, it is proved that $PT \times QR = PR \times ST$

2. In the adjoining figure, $AB = AC$. If $PM \perp AB$ and $PN \perp AC$, show that $PM \times PC = PN \times PB$.



Solution:

From the given figure,

$AB = AC$, If $PM \perp AB$ and $PN \perp AC$

We have to show that, $PM \times PC = PN \times PB$

Consider the $\triangle ABC$,

$AB = AC$...[given]

$\angle B = \angle C$

Then, consider $\triangle CPN$ and $\triangle BPM$

$\angle N = \angle M$ [both angles are equal to 90°]

$\angle C = \angle B$ [from above]

Therefore, $\triangle CPN \sim \triangle BPM$...[from AA axiom]

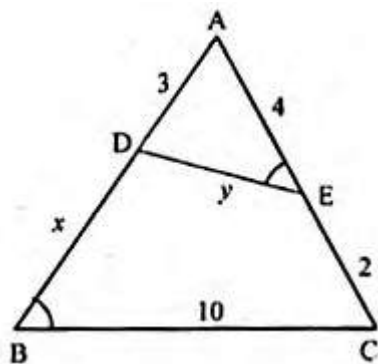
$$\text{So, } \frac{PC}{PB} = \frac{PN}{PM}$$

By cross multiplication we get,

$$PC \times PM = PN \times PB$$

Therefore, it is proved that, $PM \times PC = PN \times PB$

3. (a) In the figure given below. $\angle AED = \angle ABC$. Find the values of x and y.



Solution:

From the figure it is given that,

$$\angle AED = \angle ABC$$

Consider the $\triangle ABC$ and $\triangle ADE$

$$\angle AED = \angle ABC \dots [\text{from the figure}]$$

$$\angle A = \angle A \dots [\text{common angle for both triangles}]$$

Therefore, $\triangle ABC \sim \triangle ADE \dots$ [by AA axiom]

$$\text{Then, } \frac{AD}{AC} = \frac{DE}{BC}$$

$$\frac{3}{(4+2)} = \frac{y}{10}$$

$$\frac{3}{(6)} = \frac{y}{10}$$

By cross multiplication we get,

$$y = \frac{(3 \times 10)}{6}$$

$$y = \frac{30}{6}$$

$$y = 5$$

Now, consider $\frac{AB}{AE} = \frac{BC}{DE}$

$$\frac{(3+x)}{4} = \frac{10}{y}$$

Substitute the value of y,

$$\frac{(3+x)}{4} = \frac{10}{5}$$

By cross multiplication,

$$5(3+x) = 10 \times 4$$

$$15 + 5x = 40$$

$$5x = 40 - 15$$

$$5x = 25$$

$$x = \frac{25}{5}$$

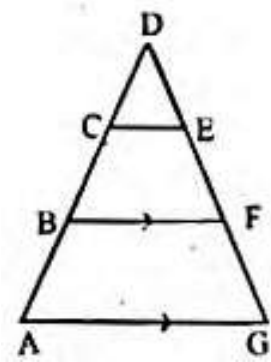
$$x = 5$$

Therefore, the value of $x = 5$ cm and $y = 5$ cm

(b) In the figure given below, $CD = \frac{1}{2}AC$, B is mid-point of AC and E is mid-point of DF. If $BF \parallel AG$, Prove that :

(i) $CE \parallel AG$

(ii) $3ED = GD$



Solution:

From the question it is given that,

$$CD = \frac{1}{2}AC$$

$$BF \parallel AG$$

(i) We have to prove that $CE \parallel AG$

$$\text{Consider, } CD = \frac{1}{2}AC$$

$$AC = 2BC \dots [\text{because from the figure B is mid-point of AC}]$$

$$\text{So, } CD = \frac{1}{2}(2BC)$$

$$CD = BC$$

$$\text{Hence, } CE \parallel BF \dots [\text{equation (i)}]$$

$$\text{Given, } BF \parallel AG \dots [\text{equation (ii)}]$$

By comparing the results of equation (i) and equation (ii) we get,

$$CE \parallel AG$$

(ii) We have to prove that, $3ED = GD$

Consider the $\triangle AGD$,

$CE \parallel AG$ above it is proved]

$$\text{So, } \frac{ED}{GD} = \frac{DC}{AD}$$

$$AD = AB + BC + DC$$

$$= DC + DC + DC$$

$$= 3DC$$

$$\text{So, } \frac{ED}{GD} = \frac{DC}{(3DC)}$$

$$\frac{ED}{GD} = \frac{1}{(3(1))}$$

$$\frac{ED}{GD} = \frac{1}{3}$$

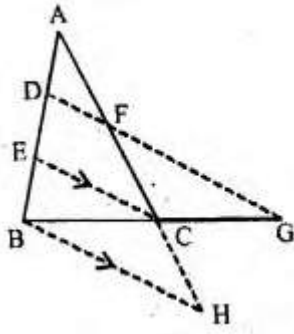
$$3ED = GD$$

Hence it is proved that, $3ED = GD$

4. In the adjoining figure, $2AD = BD$, E is mid-point of BD and F is mid-point of AC and $EC \parallel BH$. Prove that :

(i) $DF \parallel BH$

(ii) $AH = 3 AF$.



Solution:

From the question it is given that, $2AD = BD$, $ED \parallel BH$

(i) given, E is mid-point of BD

$$2DE = BD \dots [\text{equation (i)}]$$

$$2AD = BD \dots [\text{equation (ii)}]$$

from equation (i) and equation (ii) we get,

$$2DE = AD$$

$$DE = AD$$

Also given that, F is mid-point of AC

$$DF \parallel EC \dots [\text{equation (iii)}]$$

$$\text{Given, } EC \parallel BH \dots [\text{equation (iv)}]$$

By comparing equation (iii) and equation (iv) we get,

$$DF \parallel BH$$

(ii) We have to prove that, $AH = 3 AF$.

Given, E is mid- point of BD and $EC \parallel BH$.

And c is midpoint of AH,

$$\text{Thenm } FC = CH \dots [\text{equation (v)}]$$

Also given F is mid-point of AC

$$AF = FC \dots [\text{equation (vi)}]$$

By comparing both equation (v) and equation (vi) we get,

$$FC = AF = CH$$

$$AF = \left(\frac{1}{3}\right)AH$$

By cross multiplication we get,

$$3AF = AH$$

Therefore, it is proved that $3AF = AH$

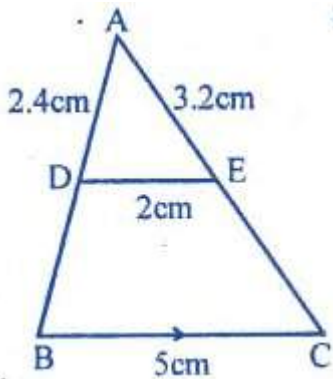
5. In a $\triangle ABC$, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$. If $AD = 2.4$ cm, $AE = 3.2$ cm, $DE = 2$ cm and $BC = 5$ cm, find BD and CE.

Solution:

From the question it is given that, In a $\triangle ABC$, D and E are points on the sides AB and AC respectively.

$$DE \parallel BC$$

$$AD = 2.4 \text{ cm}, AE = 3.2 \text{ cm}, DE = 2 \text{ cm and } BC = 5 \text{ cm}$$



Consider the $\triangle ABC$,

Given, $DE \parallel BC$

$$\text{So, } \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

$$\text{Now, consider } \frac{AD}{AB} = \frac{DE}{BC}$$

$$\frac{2.4}{AB} = \frac{2}{5}$$

$$AB = \frac{(2.4 \times 5)}{2}$$

$$AB = \frac{12}{2}$$

$$AB = 6 \text{ cm}$$

$$\text{Then, consider } \frac{AE}{AC} = \frac{DE}{BC}$$

$$\frac{3.2}{AC} = \frac{2}{5}$$

$$AC = \frac{(3.2 \times 5)}{2}$$

$$AC = \frac{16}{2}$$

$$AC = 8 \text{ cm}$$

$$\text{Hence, } BD = AB - AD$$

$$= 6 - 2.4$$

$$= 3.6 \text{ cm}$$

$$CE = AC - AE$$

$$= 8 - 3.2$$

$$= 4.8 \text{ cm}$$

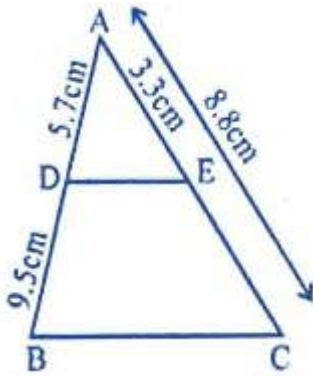
6. In a $\triangle ABC$, D and E are points on the sides AB and AC respectively such that $AD = 5.7$ cm, $BD = 9.5$ cm, $AE = 3.3$ cm and $AC = 8.8$ cm. Is $DE \parallel BC$? Justify your answer.

Solution:

From the question it is given that,

In a $\triangle ABC$, D and E are points on the sides AB and AC respectively.

$AD = 5.7$ cm, $BD = 9.5$ cm, $AE = 3.3$ cm and $AC = 8.8$ cm



Consider the $\triangle ABC$,

$$EC = AC - AE$$

$$= 8.8 - 3.3$$

$$= 5.5 \text{ cm}$$

$$\text{Then, } \frac{AD}{DB} = \frac{5.7}{9.5}$$

$$= \frac{57}{95}$$

By dividing both numerator and denominator by 19 we get,

$$= \frac{3}{5}$$

$$\frac{AE}{EC} = \frac{3.3}{5.5}$$

$$= \frac{33}{55}$$

By dividing both numerator and denominator by 11 we get,

$$= \frac{3}{5}$$

$$\text{So, } \frac{AD}{DB} = \frac{AE}{EC}$$

$$\text{Therefore, } \frac{DE}{BC}$$

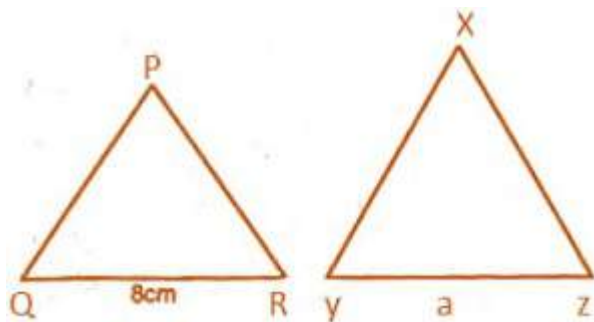
7. If the areas of two similar triangles are 360 cm^2 and 250 cm^2 and if one side of the first triangle is 8cm, find the length of the corresponding side of the second triangle.

Solution:

From the question it is given that, the areas of two similar triangles are 360 cm^2 and 250 cm^2

One side of the first triangle is 8 cm

So, PQR and XYZ are two similar triangles,



So, let us assume area of $\Delta PQR = 360 \text{ cm}^2$, $QR = 8 \text{ cm}$

And area of $\Delta XYZ = 250 \text{ cm}^2$

Assume $YZ = a$

We know that, $\frac{\text{area of } \Delta PQR}{\text{area of } \Delta XYZ} = \frac{QR^2}{yz^2}$

$$\frac{360}{250} = \frac{(8)^2}{a^2}$$

$$\frac{360}{250} = \frac{64}{a^2}$$

By cross multiplication we get,

$$a^2 = \frac{(250 \times 64)}{360}$$

$$a^2 = \frac{400}{9}$$

$$a = \sqrt{\frac{400}{9}}$$

$$a = \frac{20}{3}$$

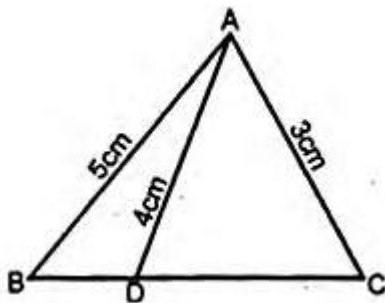
$$a = 6\frac{2}{3}$$

8. In the adjoining figure, D is a point on BC such that $\angle ABD = \angle CAD$.
If AB = 5cm, AC = 3cm and AD = 4cm, find

(i) BC

(ii) DC

(iii) area of $\triangle ACD$: area of $\triangle BCA$.



Solution:

From the question it is given that,

$$\angle ABD = \angle CAD$$

$$AB = 5\text{cm}, AC = 3\text{cm and } AD = 4\text{ cm}$$

Now, consider the $\triangle ABC$ and $\triangle ACD$

$$\angle C = \angle C \dots [\text{common angle for both triangles}]$$

$$\angle ABC = \angle CAD \dots [\text{from the question}]$$

$$\text{So, } \triangle ABC \sim \triangle ACD$$

$$\text{Then, } \frac{AB}{AD} = \frac{BC}{AC} = \frac{AC}{DC}$$

$$(i) \text{ Consider } \frac{AB}{AD} = \frac{BC}{AC}$$

$$\frac{5}{4} = \frac{BC}{3}$$

$$BC = \frac{(5 \times 3)}{4}$$

$$BC = \frac{15}{4}$$

$$BC = 3.75 \text{ cm}$$

$$(ii) \text{ Consider } \frac{AB}{AD} = \frac{AC}{DC}$$

$$\frac{5}{4} = \frac{3}{DC}$$

$$DC = \frac{(3 \times 4)}{5}$$

$$DC = \frac{12}{5}$$

$$DC = 2.4 \text{ cm}$$

(iii) Consider the $\triangle ABC$ and $\triangle ACD$

$\angle CAD = \angle ABC$ [from the question]

$\angle ACD = \angle ACB$...[common angle for both triangle]

Therefore, $\triangle ACD \sim \triangle ABC$

Then, $\frac{\text{area of } \triangle ACD}{\text{area of } \triangle ABC} = \frac{AD^2}{AB^2}$

$$= \frac{4^2}{5^2}$$

$$= \frac{16}{25}$$

Therefore, area of $\triangle ACD$: area of $\triangle BCA$ is 16 : 25.

9. In the adjoining figure, the diagonals of a parallelogram intersect at O. OE is drawn parallel to CB to meet AB at E, find area of $\triangle AOE$: area of parallelogram ABCD.

Solution:

From the given figure,

The diagonals of a parallelogram intersect at O.

OE is drawn parallel to CB to meet AB at E.

In the figure four triangles have equal area.

So, area of $\triangle OAB = \frac{1}{4}$ area of parallelogram ABCD

Then, O is midpoint of AC of $\triangle ABC$ and $DE \parallel CB$

E is also midpoint of AB

Therefore, OE is the median of $\triangle OAB$

Area of $\triangle AOE = \frac{1}{2}$ area of $\triangle AOB$

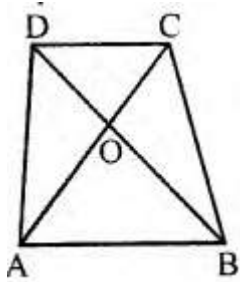
$$= \frac{1}{2} \times \frac{1}{4} \text{ area of parallelogram ABCD}$$

$$= \frac{1}{8} \text{ area of parallelogram } ABCD$$

$$\text{So, } \frac{\text{area of } \triangle AOE}{\text{area of parallelogram } ABCD} = \frac{1}{8}$$

Therefore, area of $\triangle AOE$: area of parallelogram $ABCD$ is 1 : 8.

10. In the given figure, $ABCD$ is a trapezium in which $AB \parallel DC$. If $2AB = 3DC$, find the ratio of the areas of $\triangle AOB$ and $\triangle COD$.



Solution:

From the question it is given that, $ABCD$ is a trapezium in which $AB \parallel DC$. If $2AB = 3DC$.

$$\text{So, } 2AB = 3DC$$

$$\frac{AB}{DC} = \frac{3}{2}$$

Now, consider $\triangle AOB$ and $\triangle COD$

$$\angle AOB = \angle COD \text{ .. [because vertically opposite angles are equal]}$$

$$\angle OAB = \angle OCD \text{ [because alternate angles are equal]}$$

Therefore, $\triangle AOB \sim \triangle COD$...[from AA axiom]

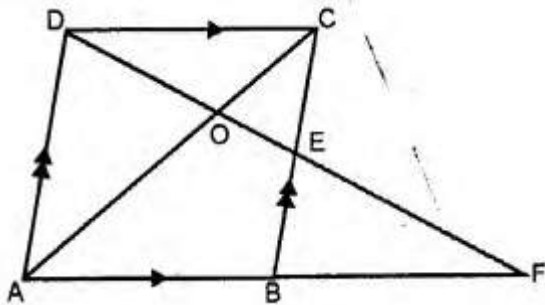
$$\text{Then, } \frac{\text{area of } \triangle AOB}{\text{area of } \triangle COD} = \frac{AB^2}{DC^2}$$

$$= \frac{3^2}{2^2}$$

$$= \frac{9}{4}$$

Therefore, the ratio of the areas of $\triangle AOB$ and $\triangle COD$ is 9:4

11. In the adjoining figure, ABCD is a parallelogram. E is mid-point of BC. DE meets the diagonal AC at O and meet AB (produced) at F. Prove that



Solution:

From the question it is given that,

ABCD is a parallelogram. E is mid-point of BC.

DE meets the diagonal AC at O.

(i) Now consider the $\triangle AOD$ and $\triangle EOC$

$\angle AOD = \angle EOC$ [because Vertically opposite angles are equal]

$\angle OAD = \angle OCB$ [because alternate angles are equal]

Therefore, $\triangle AOD \sim \triangle EOC$

$$\text{Then, } \frac{OA}{OC} = \frac{DO}{OE} = \frac{AD}{EC} = \frac{2EC}{EC}$$

$$\frac{OA}{OC} = \frac{DO}{OE} = \frac{2}{1}$$

Therefore, $OA : OC = 2 : 1$

(ii) From (i) we proved that $\Delta AOD \sim \Delta EOC$

$$\text{So, } \frac{\text{area of } \Delta OEC}{\text{area of } \Delta AOD} = \frac{OE^2}{DO^2}$$

$$\frac{\text{area of } \Delta OEC}{\text{area of } \Delta AOD} = \frac{1^2}{2^2}$$

$$\frac{\text{area of } \Delta OEC}{\text{area of } \Delta AOD} = \frac{1}{4}$$

Therefore, area of ΔOEC : area of ΔAOD is 1 : 4.

13. A model of a ship is made to a scale of 1 : 250 calculate :

(i) The length of the ship, if the length of model is 1.6m.

(ii) The area of the deck of the ship, if the area of the deck of model is $2.4m^2$.

(iii) The volume of the model, if the volume of the ship is $1km^3$.

Solution;

From the question it is given that, a model of a ship is made to a scale of 1 : 250

(i) Given, the length of the model is 1.6m

$$\text{Then, length of the ship} = \frac{(1.6 \times 250)}{1}$$

$$= 400 \text{ m}$$

(ii) Given, the area of the deck of the ship is $2.4m^2$

$$\text{Then, area of deck of the model} = 2.4 \times \left(\frac{1}{250}\right)^2$$

$$= 1,50,000 m^2 = 4m^2$$

(iii) Given, the volume of the model is 1 km^3

$$\text{Then, Volume of ship} = \left(\frac{1}{250^3}\right) \times 1\text{ km}^3$$

$$= \left(\frac{1}{250^3}\right) \times 1000^3$$

$$= 4^3$$

$$= 64\text{ m}^3$$

Therefore, volume of ship is 64 m^3 .