

Chapter 1

Rational and Irrational Numbers

Exercise 1.1

Question 1.

Insert a rational number between and $\frac{2}{9}$ and $\frac{3}{8}$ arrange in descending order:

Solution :

Given :

Rational numbers: $\frac{2}{9}$ and $\frac{3}{8}$

Let us rationalize the numbers,

By taking LCM for denominators 9 and 8 which is 72.

$$\frac{2}{9} = \frac{(2 \times 8)}{9 \times 8} = \frac{16}{72}$$

$$\frac{3}{8} = \frac{3 \times 9}{8 \times 9} = \frac{27}{72}$$

Since ,

$$\frac{16}{72} < \frac{27}{72}$$

$$\text{So, } \frac{2}{9} < \frac{3}{8}$$

The rational number between $\frac{2}{9}$ and $\frac{3}{8}$ is

$$= \frac{\frac{2}{9} + \frac{3}{8}}{2}$$

$$= \frac{\frac{(2 \times 8) + (3 \times 9)}{72}}{2}$$

$$= \frac{16 + 27}{72 \times 2}$$

$$= \frac{43}{144}$$

$$\text{Hence, } \frac{3}{8} > \frac{43}{144} > \frac{2}{9}$$

The descending order of the numbers is $\frac{3}{8} > \frac{43}{144} > \frac{2}{9}$

Question 2.

Insert two rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$ and arrange in ascending order:

Solution:

Given

The rational numbers $\frac{1}{3}$ and $\frac{1}{4}$

By taking LCM and rationalizing, we get

$$= \frac{\frac{1}{3} + \frac{1}{4}}{2}$$

$$= \frac{\frac{4+3}{12}}{2}$$

$$= \frac{7}{12 \times 2}$$

$$= \frac{7}{24}$$

Now let us find the rational number between $\frac{1}{4}$ and $\frac{7}{24}$

By taking LCM and rationalizing, we get

$$= \frac{\frac{1}{4} + \frac{7}{24}}{2}$$

$$= \frac{\frac{6+7}{24}}{2}$$

$$= \frac{13}{24 \times 2}$$

$$= \frac{13}{48}$$

So,

The two rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$ are

$$\frac{7}{24} \text{ and } \frac{13}{48}$$

Hence we, know that, $\frac{1}{3} > \frac{7}{24} > \frac{13}{48} > \frac{1}{4}$

The ascending order is as following : $\frac{1}{4}, \frac{13}{48}, \frac{7}{24}, \frac{1}{3}$

Question 3.

Insert two rational numbers between $-\frac{1}{3}$ and $-\frac{1}{2}$ and arrange in ascending order.

Solution:

Given :

The rational numbers - $-\frac{1}{3}$ and $-\frac{1}{2}$

By taking LCM and rationalizing, we get

$$= \frac{\frac{-1}{3} + \frac{-1}{2}}{2}$$

$$= \frac{\frac{-2-3}{6}}{2}$$

$$= \frac{-5}{6 \times 2}$$

$$= \frac{-5}{12}$$

So, the rational number between $\frac{-1}{3}$ and $\frac{-1}{2}$ is $\frac{-5}{12}$

$$\frac{-1}{3} > \frac{-5}{12} > \frac{-1}{2}$$

Now, let us find the rational number between $\frac{-1}{3}$ and $\frac{-5}{12}$

By taking LCM and rationalizing , we get

$$= \frac{\frac{-1}{3} + \frac{-5}{12}}{2}$$

$$= \frac{\frac{-4-5}{12}}{2}$$

$$= \frac{-9}{12 \times 2}$$

$$= \frac{-9}{24}$$

$$= \frac{-3}{8}$$

So, the rational number between $\frac{1}{3}$ and $\frac{5}{12}$ is $\frac{3}{8}$

$$\frac{-1}{3} > \frac{-3}{8} > \frac{-5}{12}$$

Hence, the two rational numbers between $\frac{1}{3}$ and $-\frac{1}{2}$

$$\frac{-1}{3} > \frac{-3}{8} > \frac{-5}{12} > -\frac{1}{2}$$

The ascending is as follows:

$$-\frac{1}{2}, \frac{-5}{12}, \frac{-3}{8}, -\frac{1}{3}$$

Question 4.

Insert three rational numbers between $\frac{1}{3}$ and $\frac{4}{5}$ and arrange in descending order.

Solution

Given:

The rational numbers $\frac{1}{3}$ and $\frac{4}{5}$

By taking LCM and rationalizing, we get

$$= \frac{\frac{1}{3} + \frac{4}{5}}{2}$$

$$= \frac{\frac{5+12}{15}}{2}$$

$$= \frac{17}{15 \times 2}$$

$$= \frac{17}{30}$$

So, the rational number between $\frac{1}{3}$ and $\frac{4}{5}$ is $\frac{17}{30}$

$$\frac{1}{3} < \frac{17}{30} < \frac{4}{5}$$

Now , let us find the rational numbers between $\frac{1}{3}$ and $\frac{17}{30}$

By taking LCM and rationalizing, we get

$$= \frac{\frac{1}{3} + \frac{17}{30}}{2}$$

$$= \frac{\frac{10+17}{30}}{2}$$

$$= \frac{27}{30 \times 2}$$

$$= \frac{27}{60}$$

So, the rational number between $\frac{1}{3}$ and $\frac{17}{30}$ is $\frac{27}{60}$

$$\frac{1}{3} < \frac{27}{60} < \frac{17}{30}$$

Now, let us find the rational numbers between $\frac{17}{30}$ and $\frac{4}{5}$

By taking LCM and rationalizing , we get

$$= \frac{\frac{17}{30} + \frac{4}{5}}{2}$$

$$= \frac{\frac{17+24}{30}}{2}$$

$$= \frac{41}{30 \times 2}$$

$$= \frac{41}{60}$$

So, the rational number between $\frac{17}{30}$ and $\frac{4}{5}$ is $\frac{41}{60}$

$$\frac{17}{30} < \frac{41}{60} < \frac{4}{5}$$

Hence, the three rational numbers between $\frac{1}{3}$ and $\frac{4}{5}$ are

$$\frac{1}{3} < \frac{27}{60} < \frac{17}{30} < \frac{41}{60} < \frac{4}{5}$$

The descending order is as follows: $\frac{4}{5}, \frac{41}{60}, \frac{17}{30}, \frac{27}{60}, \frac{1}{3}$

Question 5.

Insert three rational numbers between 4 and 4.5

Solution:

Given :

The rational numbers 4 and 4.5

By rationalizing, we get

$$= \frac{(4 + 4.5)}{2}$$

$$= \frac{8.5}{2}$$

$$= 4.25$$

So, the rational number between 4 and 4.5 is 4.25

$$4 < 4.25 < 4.5$$

Now, let us find the rational number between 4 and 4.25

By rationalizing , we get

$$\begin{aligned} &= \frac{(4 + 4.5)}{2} \\ &= \frac{8.5}{2} \\ &= 4.125 \end{aligned}$$

So, the rational number between 4 and 4.25 is 4.125

$$4 < 4.125 < 4.25$$

Now, let us find the rational number between 4 and 4.125

By rationalizing , we get

$$\begin{aligned} &= \frac{(4 + 4.125)}{2} \\ &= \frac{8.125}{2} \\ &= 4.0625 \end{aligned}$$

So, the rational number between 4 and 4.125 is 4.0625

$$4 < 4.0625 < 4.125$$

Hence, the rational numbers between 4 and 4.5 are

$$4 < 4.0625 < 4.125 < 4.25 < 4.5$$

The three rational numbers between 4 and 4.5

$$4.0625, 4.125, 4.25$$

Question 6.

Find six rational numbers between 3 and 4.

Solution:

Given :

The rational number 3 and 4

So let us find the six rational numbers between 3 and 4,

First rational number between 3 and 4 is

$$= \frac{(3 + 4)}{2}$$

$$= \frac{7}{2}$$

Second rational number between 3 and $\frac{7}{2}$ is

$$= \frac{3 + \frac{7}{2}}{2}$$

$$= \frac{(6+7)}{(2 \times 2)} \text{ [By taking 2 LCM]}$$

$$= \frac{13}{4}$$

Third rational number between $\frac{7}{2}$ and 4 is

$$= \frac{\frac{7}{2} + 4}{2}$$

$$= \frac{(7+8)}{(2 \times 2)} \text{ [By taking 2 as LCM]}$$

$$= \frac{15}{4}$$

Fourth rational number between 3 and $\frac{13}{4}$ is

$$\begin{aligned}
&= \frac{3 + \frac{13}{4}}{2} \\
&= \frac{(12+13)}{(4 \times 2)} \text{ [By taking 4 as LCM]} \\
&= \frac{25}{8}
\end{aligned}$$

Find rational number between $\frac{13}{4}$ and $\frac{7}{2}$ is

$$\begin{aligned}
&= \frac{\left(\frac{13}{4} + \frac{7}{2}\right)}{2} \\
&= \frac{\left(\frac{13+14}{4}\right)}{2} \text{ [By taking 4 as LCM]} \\
&= \frac{(13+14)}{(4 \times 2)} \\
&= \frac{27}{8}
\end{aligned}$$

Sixth rational number between $\frac{7}{2}$ and $\frac{15}{4}$ is

$$\begin{aligned}
&= \frac{\left(\frac{7}{2} + \frac{15}{4}\right)}{2} \\
&= \frac{\left(\frac{14+15}{4}\right)}{2} \text{ [By taking 4 as LCM]} \\
&= \frac{(14+15)}{(4 \times 2)} \\
&= \frac{29}{8}
\end{aligned}$$

Hence, the six rational numbers between 3 and 4 are

$$\frac{25}{8}, \frac{13}{4}, \frac{27}{8}, \frac{7}{2}, \frac{29}{8}, \frac{15}{4}$$

7. find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$

Solution:

Given:

The rational numbers $\frac{3}{5}$ and $\frac{4}{5}$

Now, let us find the five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$

So we need to multiply both numerator and denominator with $5+1 = 6$ we get,

$$\frac{3}{5} = \frac{(3 \times 6)}{(5 \times 6)} = \frac{18}{30}$$

$$\frac{4}{5} = \frac{(4 \times 6)}{(5 \times 6)} = \frac{24}{30}$$

$$\text{Now, we have } \frac{18}{30} < \frac{19}{30} < \frac{20}{30} < \frac{21}{30} < \frac{22}{30} < \frac{23}{30} < \frac{24}{30}$$

Hence , the five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are

$$\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$$

Question 8.

Find ten rational numbers between $-\frac{2}{5}$ and $\frac{1}{7}$

Solution :

Given :

The rational numbers $-\frac{2}{5}$ and $\frac{1}{7}$

By taking LCM for 5 and 7 which is 35

$$\text{So, } \frac{-2}{5} = \frac{(-2 \times 7)}{(5 \times 7)} = \frac{-14}{35}$$

$$\frac{1}{7} = \frac{(1 \times 5)}{(7 \times 5)} = \frac{5}{35}$$

Now, we can insert any 10 numbers between $\frac{14}{35}$ and $\frac{5}{35}$

i.e. $\frac{-13}{35}, \frac{-12}{35}, \frac{-11}{35}, \frac{-10}{35}, \frac{-9}{35}, \frac{-8}{35}, \frac{-7}{35}, \frac{-6}{35}, \frac{-5}{35}, \frac{-4}{35}, \frac{-3}{35}, \frac{-2}{35}, \frac{-1}{35},$
 $\frac{1}{35}, \frac{2}{35}, \frac{3}{35}, \frac{4}{35}$

hence, the ten rational numbers between $\frac{2}{5}$ and $\frac{1}{7}$ are

$$\frac{-6}{35}, \frac{-5}{35}, \frac{-4}{35}, \frac{-3}{35}, \frac{-2}{35}, \frac{-1}{35}, \frac{1}{35}, \frac{2}{35}, \frac{3}{35}, \frac{4}{35}$$

Question 9.

Find six rational numbers between $\frac{1}{2}$ and $\frac{2}{3}$

Solution:

Given :

The rational numbers $\frac{1}{2}$ and $\frac{2}{3}$

To make the denominators similar let us take LCM for 2 and 3 which is 6

$$\frac{1}{2} = \frac{(1 \times 3)}{(2 \times 3)} = \frac{3}{6}$$

$$\frac{2}{3} = \frac{(2 \times 2)}{(3 \times 2)} = \frac{4}{6}$$

Now, we need to insert six rational numbers, so multiply both numerator and denominator by $6 + 1 = 7$

$$\frac{3}{6} = \frac{(3 \times 7)}{(6 \times 7)} = \frac{21}{42}$$

$$\frac{4}{6} = \frac{(4 \times 7)}{(6 \times 7)} = \frac{28}{42}$$

We know that, $\frac{21}{42} < \frac{22}{42} < \frac{23}{42} < \frac{24}{42} < \frac{25}{42} < \frac{26}{42} < \frac{27}{42} < \frac{28}{42}$

Hence, the six rational numbers between $\frac{1}{2}$ and $\frac{2}{3}$ are

$$\frac{22}{42}, \frac{23}{42}, \frac{24}{42}, \frac{25}{42}, \frac{26}{42}, \frac{27}{42}$$

Exercise 1.2

Question 1.

Prove that, $\sqrt{5}$ is an irrational number.

Solution:

Let us consider $\sqrt{5}$ be a rational, number, then

$\sqrt{5} = \frac{p}{q}$, where 'p' and 'q' are integer, $q \neq 0$ and p,q have no common factors (except 1) .

So,

$$5 = \frac{p^2}{q^2}$$

$$p^2 = 5q^2 \dots\dots (1)$$

As we know, '5' divides $5q^2$, so, '5' divides p^2 as well . hence, '5' is prime.

So 5 divides p

Now, let $p = 5k$ where k is an integer

Square on both sides, we get

$$p^2 = 25k^2$$

$$5q^2 = 25k^2 \text{ [since, } p^2 = 5q^2, \text{ from equation (1)]}$$

$$q^2 = 5k^2$$

As we know, '5' divides $5k^2$, so, '5' divides q^2 as well. But '5' is prime.

So 5 divides q

Thus, p and q have a common factor 5. This statement contradicts that 'p' and 'q' has no common factors (except 1) .

We can say that, $\sqrt{5}$ is not an rational number.

$\sqrt{5}$ is an irrational number.

Hence proved.

Question 2.

Prove that, $\sqrt{7}$ be a rational number, then

$\sqrt{7} = \frac{p}{q}$ where 'p' and 'q' are integers $q \neq 0$ and p,q have no common factors (except 1).

So,

$$7 = \frac{p^2}{q^2}$$

$$P^2 = 7q^2 \dots\dots(1)$$

As we know, '7' divides so, '7' divides p^2 as well. Hence, '7' is prime.

So 7 divides p

Now, let $p = 7k$, where 'k' is an integer 'square on both sides, we get

$$P^2 = 49k^2$$

$$7q^2 = 49k^2 \text{ [since , } p^2 = 7q^2 \text{ , from equation (1)]}$$

$$Q^2 = 7k^2$$

As we know , '7' divides $7k^2$, so '7' divides q^2 as well. But '7' is prime. So 7 divides q

Thus, p and q have a common factor 7 . this statement contradicts that 'p' and 'q' has no common factors (except 1).

We can say that, $\sqrt{7}$ is not a rational number.

$\sqrt{7}$ is an irrational number.

Hence proved.

Question 3.

Prove that $\sqrt{6}$ is an irrational number.

Solution:

Let us consider $\sqrt{6}$ be a rational number, then

$\sqrt{6} = \frac{p}{q}$ where 'p' and 'q' are integer $q \neq 0$ and p, q have no common factors (except 1).

So,

$$6 = \frac{p^2}{q^2}$$

$$P^2 = 6q^2 \dots(1)$$

As we know, '2' divides $6q^2$, so '2' divides p^2 as well. Hence , '2' is prime.

So 2 divides p

Now, let $p = 2k$, where 'k' is an integer

Square on both sides, we get

$$p^2 = 4k^2$$

$$6q^2 = 4k^2 \text{ [Since, } p^2 = 6q^2 \text{ , from equation (1)]}$$

$$3q^2 = 2k^2$$

As we know, '2' divides $2k^2$, so '2' divides $3q^2$ as well.

'2' should either 3 or divide q^2 .

but '2' does not divide 3. '2' divides q^2 so '2' is prime.

So 2 divides q

Thus, p and q have a common factor 2. This statement contradicts that 'p' and 'q' has no common factors (except 1) .

We can say that, $\sqrt{6}$ is not a rational number.

$\sqrt{6}$ is an irrational number.

Hence proved.

Question 4.

Prove that $\frac{1}{\sqrt{11}}$ is an irrational number.

Solution:

Let us consider $\frac{1}{\sqrt{11}}$ be a rational number, then

$\frac{1}{\sqrt{11}} = \frac{p}{q}$ where 'p' and 'q' are integers $q \neq 0$ and p,q have no common factors (except 1.)

So,

$$\frac{1}{11} = \frac{p^2}{q^2}$$

$$Q^2 = 11p^2 \dots (1)$$

As we know, '11' divides $11p^2$, so '11' divides q^2 as well. Hence, '11' is prime.

So 11 divides q

Now, let $q = 11k$, where 'k' is an integer

Square on both sides, we get

$$Q^2 = 121k^2$$

$$11p^2 = 121k^2 \text{ [since, } q^2 = 11p^2 \text{ , from equation (1)]}$$

$$p^2 = 11k^2$$

As we know. '11' divides $11k^2$, so '11' divides p^2 as well. But '11' is prime.

So 11 divides p

Thus, p and q have a common factor. 11. This statement contradicts that ' p ' and ' q ' has no common factor (except) .

We can say that, $\frac{1}{\sqrt{11}}$ is not a rational number.

$\frac{1}{\sqrt{11}}$ is an irrational number.

Hence proved.

Question 5.

Prove that $\sqrt{2}$ is an irrational number. Hence show that $3 - \sqrt{2}$ is an irrational .

Solution:

Let us consider $\sqrt{2}$ is an irrational number. Then

$\sqrt{2} = \frac{p}{q}$ where 'p' and 'q' are integers, $q \neq 0$ and p,q have no common

Factors (except 1)

So,

$$2 = \frac{p^2}{q^2}$$

$$p^2 = 2q^2 \dots\dots(1)$$

As we know, '2' divides $2q^2$, so '2' divides p^2 as well. Hence, '2' is prime.

So 2 divides p

Now, let $p = 2k$, where 'k' is an integer

Square on both sides, we get

$$p^2 = 4k^2$$

$$2q^2 = 4k^2 \text{ [Since, } p^2 = 2q^2 \text{, from equation (1)]}$$

$$q^2 = 2k^2$$

As we know, '2' divides $2k^2$, so '2' divides q^2 as well. but '2' is prime.

So 2 divides q

Thus, p and q have a common factor 2. This statement contradicts that

‘p’ and ‘q’ has no common factors (except 1.)

We can say that. $\sqrt{2}$ is not a rational number.

$\sqrt{2}$ is not a rational number.

Now, let us assume $3 - \sqrt{2}$ be a rational number, ‘r’

$$\text{So, } 3 - \sqrt{2} = r$$

$$3 - r = \sqrt{2}$$

We know that, ‘r’ is rational, ‘3 – r’ is rational, so $\sqrt{2}$ is also rational .this contradicts the statement that $\sqrt{2}$ is irrational .

So , $3 - \sqrt{2}$ is irrational number.

Hence proved.

Question 6.

Prove that, $\sqrt{3}$ is an irrational number . hence show that $\frac{2}{5} \times \sqrt{3}$

Is an irrational number.

Solution:

Let us consider $\sqrt{3}$ be a rational number, then

$\sqrt{3} = \frac{p}{q}$ where ‘p’ and ‘q’ are integers, $q \neq 0$ and p, q have no common factors (except 1).

So,

$$3 = \frac{p^2}{q^2}$$

$$P^2 = 3q^2 \dots (1)$$

As we know, '3' divides $3q^2$, so '3' divides p^2 as well. Hence, '3' is prime.

So 3 divides p

Now, let $p = 3k$, where 'k' is an integer

Square on both sides, we get

$$P^2 = 9k^2$$

$$3q^2 = 9k^2 \text{ [since, } p^2 = 3q^2 \text{ , from equation (1)]}$$

$$Q^2 = 3k^2$$

As we know, '3' divides $3k^2$, so '3' divides q^2 as well. But '3' is prime.

So 3 divides q

Thus, p and q have a common factor, 3 this statement contradicts that 'p' and 'q' has no common factors (except 1)

We can say that, $\sqrt{3}$ is not a rational number.

$\sqrt{3}$ is an irrational number.

Now, let us assume $\left(\frac{2}{5}\right)\sqrt{3}$ be a rational number, 'r'

$$\text{So, } \left(\frac{2}{5}\right)\sqrt{3}$$

$$\frac{5r}{2} = \sqrt{3}$$

We know that, 'r' is rational, $\frac{5r}{2}$ is rational, so ' $\sqrt{3}$ ' is also rational

This contradicts the statement that $\sqrt{3}$ is irrational.

So, $\left(\frac{2}{5}\right)\sqrt{3}$ is irrational number.

Hence proved.

Question 7.

Prove that $\sqrt{5}$ be a rational number, then $\sqrt{5}$ is an irrational number.

Hence, show that $-3 + 2\sqrt{5}$ is an irrational number.

Solution:

Let us consider $\sqrt{5}$ be a rational number, then

$\sqrt{5} = \frac{p}{q}$ where 'p' and 'q' are integers, $q \neq 0$ and p, q have no common factors (except 1).

So,

$$5 = \frac{p^2}{q^2}$$

$$P^2 = 5q^2 \dots (1)$$

As we know, '5' divides $5q^2$, so '5' divides p^2 as well. Hence, '5' is prime.

So 5 divides p

Now, let $p = 5k$, where 'k' is an integer

Square on both sides, we get

$$P^2 = 25k^2$$

$$5q^2 = 25k^2 \text{ [since , } p^2 = 5q^2, \text{ from equation (1)]}$$

$$Q^2 = 5k^2$$

As we know, '5' divides $5k^2$, so '5' divides q^2 as well. But '5' is prime. So 5 divides q

Thus, p and q have a common factor 5. This statement contradicts 'p' and 'q' has no common factors (except 1)

We can say that, $\sqrt{5}$ is not a rational number.

$\sqrt{5}$ is an irrational number.

Now, let us assume $-3 + 2\sqrt{5}$ be a rational number, 'r'

$$\text{So, } -3 + 2\sqrt{5} = r$$

$$-3 - r = 2\sqrt{5}$$

$$\frac{(-3 - r)}{2} = \sqrt{5}$$

We know that, 'r' rational $\frac{(-3-r)}{2}$ is rational, so ' $\sqrt{5}$ ' is also rational.

This contradicts the statement that $\sqrt{5}$ is irrational.

So, $-3 + 2\sqrt{5}$ is irrational number.

Hence proved.

Question 8.

(i) $5 + \sqrt{2}$

(ii) $3 - 5\sqrt{3}$

(iii) $2\sqrt{3} - 7$

(iv) $\sqrt{2} + \sqrt{5}$

Solution:

(i) $5 + \sqrt{2}$

Now, let us assume $5 + \sqrt{2}$ be a rational number, 'r'

So, $5 + \sqrt{2} = r$

$R - 5 = \sqrt{2}$

We know that, 'r' is rational, 'r - 5' is rational so ' $\sqrt{2}$ ' is also rational. This contradicts the statement that ' $\sqrt{2}$ ' is irrational.

So, $5 + \sqrt{2}$ is irrational number .

(ii) $3 - 5\sqrt{3}$

Now, let us assume $3 - 5\sqrt{3}$ be a rational number, 'r'

So, $3 - 5\sqrt{3} = r$

$3 - r = 5\sqrt{3}$

$\frac{(3-r)}{5} = \sqrt{3}$

We know that, 'r' is rational, $\frac{(3-r)}{5}$ is rational, so ' $\sqrt{3}$ ' is also rational.

This contradicts the statement that ' $\sqrt{3}$ ' is irrational.

So, $3 - 5\sqrt{3}$ is irrational number.

(iii) $2\sqrt{3} - 7$

Now, let us assume $2\sqrt{3} - 7$ be a rational number, 'r'

$$\text{So, } 2\sqrt{3} - 7 = r$$

$$2\sqrt{3} = r + 7$$

$$\sqrt{3} = \frac{r+7}{2}$$

We know that, 'r' is rational, $\frac{r+7}{2}$ is rational, so ' $\sqrt{3}$ ' is also rational.

This contradicts the statement that $\sqrt{3}$ is irrational.

So, $2\sqrt{3} - 7$ is irrational number.

$$\text{(iv) } \sqrt{2} + \sqrt{5}$$

Now, let us assume $\sqrt{2} + \sqrt{5}$ be a rational number, 'r'

$$\text{So } \sqrt{2} + \sqrt{5} = r$$

$$\sqrt{5} = r - \sqrt{2}$$

Square on both sides,

$$(\sqrt{5})^2 = (r - \sqrt{2})^2$$

$$5 = r^2 + (\sqrt{2})^2 - 2r\sqrt{2}$$

$$5 = r^2 + 2 - 2\sqrt{2}r$$

$$5 - 2 = r^2 - 2\sqrt{2}r$$

$$R^2 - 3 = 2\sqrt{2}r$$

$$\frac{r^2 - 3}{2r} = \sqrt{2}$$

We know that, 'r' is rational, $\frac{r^2 - 3}{2r}$ is rational, so ' $\sqrt{2}$ ' is also

rational. This contradicts the statement that $\sqrt{2}$ is irrational. So,

$\sqrt{2} + \sqrt{5}$ is irrational number.

Exercise 1.3

Question 1.

Locate $\sqrt{10}$ and $\sqrt{17}$ on the number line.

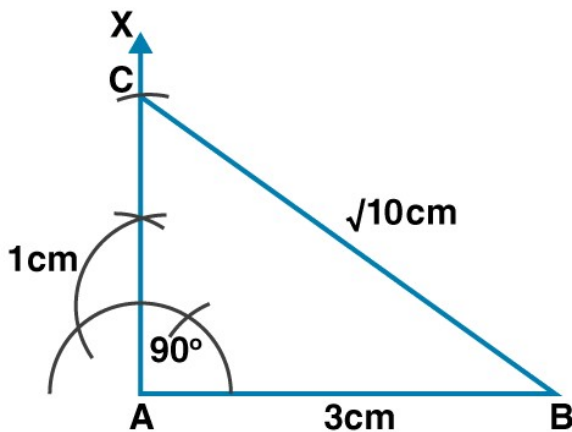
Solution:

$$\sqrt{10}$$
$$\sqrt{10} = \sqrt{(9 + 1)} = \sqrt{((3)^2 + 1^2)}$$

Now let us construct:

- Draw a line segment $AB = 3\text{cm}$.
- At point A, draw a perpendicular AX and cut off $AC = 1\text{cm}$.
- Join BC .

$$BC = \sqrt{10} \text{ cm}$$



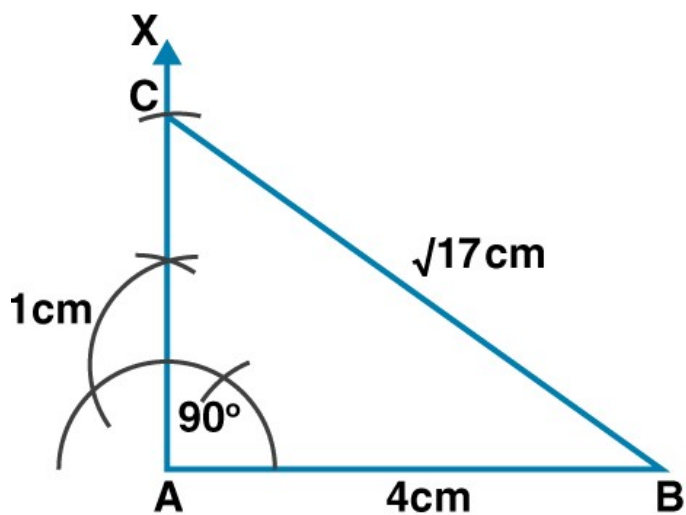
$$\sqrt{17}$$

$$\sqrt{17} = \sqrt{(16 + 1)} = \sqrt{((4)^2 + 1^2)}$$

Now let us construct:

- Draw a line segment $AB = 4\text{cm}$.
- At point A, draw a perpendicular AX and cut off $AC = 1\text{cm}$.
- Join BC .

$$BC = \sqrt{17} \text{ cm}$$



Question 2

Write the decimal expansion of each of the following numbers and say what kind of decimal expansion each has:

(i) $\frac{36}{100}$

(ii) $4\frac{1}{8}$

(iii) $\frac{2}{9}$

(iv) $\frac{2}{11}$

(v) $\frac{3}{13}$

(vi) $\frac{329}{400}$

Solution:

(i) $\frac{36}{100}$

[illegible]

$$\frac{36}{100} = 0.36$$

It is a terminating decimal.

(ii) $4\frac{1}{8}$

$$4\frac{1}{8} = \frac{(4 \times 8 + 1)}{8} = \frac{33}{8}$$

$$\begin{array}{r}
 \begin{array}{r}
 0 \quad 4. \quad 1 \quad 2 \quad 5 \\
 8 \overline{) 3 \quad 3. \quad 0 \quad 0 \quad 0} \\
 - \quad 0 \\
 \hline
 3 \quad 3 \\
 - \quad 3 \quad 2 \\
 \hline
 1 \quad 0 \\
 - \quad 8 \\
 \hline
 2 \quad 0 \\
 - \quad 1 \quad 6
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 40 \\
 - 40 \\
 \hline
 0
 \end{array}$$

$$\frac{33}{8} = 4.125$$

It is a terminating decimal.

(iii) $\frac{2}{9}$

$$\begin{array}{r}
 0.222 \\
 9 \overline{) 2.000} \\
 - 0 \\
 \hline
 20 \\
 - 18 \\
 \hline
 20 \\
 - 18 \\
 \hline
 20 \\
 - 18 \\
 \hline
 2
 \end{array}$$

$$\frac{2}{9} = 0.222$$

It is a non-terminating recurring decimal.

$$(iv) \frac{2}{11}$$

$$\begin{array}{r}
 0.181 \\
 11 \overline{) 2.000} \\
 - 0 \\
 \hline
 20 \\
 - 11 \\
 \hline
 90 \\
 - 88 \\
 \hline
 20 \\
 - 11 \\
 \hline
 9
 \end{array}$$

$$\frac{2}{11} = 0.181$$

It is a non-terminating recurring decimal.

$$(v) \frac{3}{13}$$

$$\begin{array}{r}
 0.2307692307 \\
 13 \overline{) 3.0000000000} \\
 - 0 \\
 \hline
 30 \\
 - 26 \\
 \hline
 40 \\
 - 39 \\
 \hline
 10 \\
 - 0 \\
 \hline
 100 \\
 - 91 \\
 \hline
 90 \\
 - 78 \\
 \hline
 120 \\
 - 117 \\
 \hline
 3
 \end{array}$$

$$\begin{array}{r}
 30 \\
 - 26 \\
 \hline
 40 \\
 - 39 \\
 \hline
 10 \\
 - 0 \\
 \hline
 100 \\
 - 91 \\
 \hline
 9
 \end{array}$$

$$\frac{3}{13} = 0.2317692307$$

It is a non-terminating recurring decimal.

$$(vi) \frac{329}{400}$$

$$\begin{array}{r}
 000.8225 \\
 400 \overline{) 329.0000} \\
 \underline{- 0} \\
 32 \\
 0 \\
 \underline{3290} \\
 - 3200 \\
 \hline
 900 \\
 - 800 \\
 \hline
 1000 \\
 800 \\
 \hline
 2000 \\
 - 2000 \\
 \hline
 0
 \end{array}$$

$$\frac{329}{400} = 0.8225$$

It is a terminating decimal

Question 3.

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non – terminating repeating decimal expansion:

(i) $\frac{13}{3125}$

(ii) $\frac{17}{8}$

(iii) $\frac{23}{75}$

(iv) $\frac{6}{15}$

(v) $\frac{1258}{625}$

(vi) $\frac{77}{210}$

Solution:

We know that, if the denominator of a fraction has only 2 or 5 or both factors, it is a terminating decimal otherwise it is non-terminating repeating decimals.

(i) $\frac{13}{3125}$

5		3125
5		625
5		125
5		25
5		5
		1

$$3125 = 5 \times 5 \times 5 \times 5 \times 5$$

Prime factor of 3125 = 5, 5, 5, 5, 5, 5 [i.e in the form of $2^n, 5^n$]
It is a terminating decimal.

(ii) $\frac{17}{8}$

$$\begin{array}{r|l} 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$$8 = 2 \times 2 \times 2$$

Prime factor of 8 = 2, 2, 2 [i.e in the form of $2^n, 5^n$] it is a terminating decimal.

(iii) $\frac{23}{75}$

$$\begin{array}{r|l} 3 & 75 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$75 = 3 \times 5 \times 5$$

Prime factor of 75 = 3, 5, 5
It is a non-terminating repeating decimal.

(iv) $\frac{6}{15}$

Let us divide both numerator and denominator by 3

$$\begin{aligned} \frac{6}{15} &= \frac{(6 \div 3)}{(15 \div 3)} \\ &= \frac{2}{5} \end{aligned}$$

Since the denominator is 5.

It is a terminating decimal.

$$\begin{array}{r}
 0.40 \\
 15 \overline{) 6.00} \\
 \underline{- 0} \\
 60 \\
 \underline{- 60} \\
 00 \\
 \underline{- 00} \\
 0
 \end{array}$$

$$\frac{6}{15} = 0.4$$

$$(v) \frac{1258}{625}$$

$$\begin{array}{r|l}
 5 & 625 \\
 \hline
 5 & 125 \\
 \hline
 5 & 25 \\
 \hline
 5 & 5 \\
 \hline
 & 1
 \end{array}$$

$$625 = 5 \times 5 \times 5 \times 5$$

Prime factor of 625 = 5, 5, 5, 5 [i.e in the form of 2^n , 5^n]
 It is a terminating decimal.

$$(vi) \frac{77}{210}$$

Let us divide both numerator and denominator by 7

$$\begin{aligned}
 \frac{77}{210} &= \frac{(77 \div 7)}{(210 \div 7)} \\
 &= \frac{11}{30}
 \end{aligned}$$

$$\begin{array}{r|l}
 2 & 30 \\
 \hline
 3 & 15 \\
 \hline
 5 & 5 \\
 \hline
 & 1
 \end{array}$$

$$30 = 2 \times 3 \times 5$$

Prime factor $30 = 2, 3, 5$

It is a non – terminating repeating decimal.

Question 4.

Without actually performing the long division, find if $\frac{987}{10500}$ will have terminating or non – terminating repeating decimal expansion.

Give reasons for your answer.

Solution:

Given:

The fraction $\frac{987}{10500}$

Let us divide numerator and denominator by 21, we get

$$\begin{aligned}
 \frac{987}{10500} &= \frac{(987 \div 21)}{(10500 \div 21)} \\
 &= \frac{47}{500}
 \end{aligned}$$

So,

The prime factors for denominator $500 = 2 \times 2 \times 5 \times 5 \times 5$

Since it is of the form : $2^n, 5^n$

Hence it is a terminating decimal.

Question 5.

Write the decimal expansions of the following numbers which have Terminating decimal expansions:

Solution :

(i) $\frac{17}{8}$

(ii) $\frac{13}{3125}$

(iii) $\frac{7}{80}$

(iv) $\frac{6}{15}$

(v) $2^2 \times \frac{7}{5^4}$

(vi) $\frac{237}{1500}$

Solution:

(i) $\frac{17}{8}$

2		8
2		4
2		2
		1

Denominator, $8 = 2 \times 2 \times 2$
 $= 2^3$

It is a terminating decimal.

When we divide $\frac{17}{8}$, we get

$$\begin{array}{r}
 \\
 8 \overline{) 17.00000} \\
 \underline{- 0} \\
 17 \\
 \underline{- 16} \\
 10 \\
 \underline{- 8} \\
 20 \\
 \underline{16} \\
 40 \\
 \underline{- 40} \\
 00 \\
 \underline{- 0} \\
 0
 \end{array}$$

$$\frac{17}{8} = 2.125$$

$$(ii) \frac{13}{3125}$$

$$\begin{array}{r|l}
 5 & 3125 \\
 \hline
 5 & 625 \\
 \hline
 5 & 125 \\
 \hline
 5 & 25 \\
 \hline
 5 & 5 \\
 \hline
 & 1
 \end{array}$$

$$3125 = 5 \times 5 \times 5 \times 5 \times 5$$

Prime factor of 3125 = 5, 5, 5, 5, 5 [i. e in the form of 2^n , 5^n]
It is a terminating decimal.

When we divide $\frac{13}{3125}$, we get

$$\begin{array}{r}
 \begin{array}{cccccccc}
 & & & 0 & 0. & 0 & 0 & 4 & 1 & 6 \\
 3 & 1 & 2 & 5 & | & 1 & 3. & 0 & 0 & 0 & 0 & 0
 \end{array} \\
 - & \frac{0}{1} & 3 & & & & & & & & \\
 - & & 0 & & & & & & & & \\
 & 1 & 3 & 0 & & & & & & & \\
 - & & & 0 & & & & & & & \\
 & 1 & 3 & 0 & 0 & & & & & & \\
 & & & & 0 & & & & & & \\
 & 1 & 3 & 0 & 0 & 0 & & & & & \\
 - & 1 & 2 & 5 & 0 & 0 & & & & & \\
 & & & 5 & 0 & 0 & 0 & & & & \\
 & & - & 3 & 1 & 2 & 5 & & & & \\
 & & & 1 & 8 & 7 & 5 & 0 & & & \\
 & & - & 1 & 8 & 7 & 5 & 0 & & & \\
 & & & & & & & 0 & & & \\
 & & & & & & & & & & 0
 \end{array}$$

$$\frac{13}{3125} = 0.00416$$

$$(iii) \frac{7}{80}$$

$$\begin{array}{r|l}
 2 & 80 \\
 \hline
 2 & 40 \\
 \hline
 2 & 20 \\
 \hline
 2 & 10 \\
 \hline
 5 & 5 \\
 \hline
 & 1
 \end{array}$$

$$80 = 2 \times 2 \times 2 \times 2 \times 2$$

Prime factor of 80 = $2^4, 5^1$ [i.e in the form of $2^n, 5^n$]

It is a terminating decimal.

When we divide $\frac{7}{80}$ we, get

$$\begin{array}{r}
 \begin{array}{r}
 0. \ 0 \ 8 \ 7 \ 5 \\
 8 \ 0 \ \overline{) 7. \ 0 \ 0 \ 0 \ 0} \\
 - \ 0 \\
 \hline
 7 \ 0 \\
 - \ 0 \\
 \hline
 7 \ 0 \ 0 \\
 - \ 6 \ 4 \ 0 \\
 \hline
 6 \ 0 \ 0 \\
 - \ 5 \ 6 \ 0 \\
 \hline
 4 \ 0 \ 0 \\
 - \ 4 \ 0 \ 0 \\
 \hline
 0
 \end{array}
 \end{array}$$

$$\frac{7}{80} = 0.0875$$

$$(iv) \frac{6}{15}$$

Let us divide both numerator and denominator by 3, we get

$$\begin{aligned}
 \frac{6}{15} &= \frac{(6 \div 3)}{(15 \div 3)} \\
 &= \frac{2}{5}
 \end{aligned}$$

Since the denominator is 5,
It is terminating decimal.

$$\begin{array}{r}
 \begin{array}{r}
 0. \ 4 \ 0 \\
 1 \ 5 \ \overline{) 6. \ 0 \ 0} \\
 - \ 0 \\
 \hline
 6 \ 0 \\
 - \ 6 \ 0 \\
 \hline
 0 \ 0 \\
 - \ 0 \\
 \hline
 0
 \end{array}
 \end{array}$$

$$\frac{6}{15} = 0.4$$

$$(v) \frac{2^2 \times 7}{5^4}$$

We know that the denominator is 5^4 it is a terminating decimal.

$$\begin{aligned} \frac{2^2 \times 7}{5^4} &= \frac{(2 \times 2 \times 7)}{(5 \times 5 \times 5 \times 5)} \\ &= \frac{28}{625} \end{aligned}$$

$$\begin{array}{r} \\ 625 \overline{) 28.000000} \\ \underline{- 0} \\ 28 \\ \underline{- 0} \\ 280 \\ \underline{- 0} \\ 2800 \\ \underline{- 2500} \\ 3000 \\ \underline{- 2500} \\ 5000 \\ \underline{- 5000} \\ 0 \\ \underline{ } \\ 0 \end{array}$$

$$\frac{28}{625} = 0.0448$$

It is a terminating decimal.

$$(vi) \frac{237}{1500}$$

Let us divide both numerator and denominator by 3, we get

$$\frac{237}{1500} = \frac{(237 \div 3)}{(1500 \div 3)}$$

$$= \frac{79}{500}$$

Since the denominator is 500,

Its factors are, $500 = 2 \times 2 \times 5 \times 5 \times 5$

$$= 2^2 \times 5^3$$

It is terminating decimal.

$$\begin{array}{r} 5 \quad 0 \quad 0 \quad \overline{) \begin{array}{r} 0 \quad 0. \quad 1 \quad 5 \quad 8 \\ 7 \quad 9. \quad 0 \quad 0 \quad 0 \\ - \quad 0 \\ \hline 7 \quad 9 \\ - \quad 0 \\ \hline 7 \quad 9 \quad 0 \\ - \quad 5 \quad 0 \quad 0 \\ \hline 2 \quad 9 \quad 0 \quad 0 \\ - \quad 2 \quad 5 \quad 0 \quad 0 \\ \hline 4 \quad 0 \quad 0 \quad 0 \\ - \quad 4 \quad 0 \quad 0 \quad 0 \\ \hline 0 \end{array}} \end{array}$$

$$\frac{237}{1500} = \frac{79}{500} = 0.1518$$

Write the denominator of the rational number $\frac{257}{5000}$ in the form $2^m \times 5^n$

Where m , n is non – negative integers. Hence, write its decimal Expansion on without actually division.

Solution:

Given :

The fraction = $\frac{257}{500}$

Since the denominator is 5000,
The factors for 5000 are:

2	5000
2	2500
2	1250
5	625
5	125
5	25
5	5
	1

$$\begin{aligned} 5000 &= 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 \\ &= 2^3 \times 5^4 \\ \frac{257}{5000} &= \frac{257}{2^3 \times 5^4} \end{aligned}$$

It is a terminating decimal.

So,

Let us multiply both numerator and denominator by 2, we get

$$\begin{aligned} \frac{257}{5000} &= \frac{(257 \times 2)}{(5000 \times 2)} \\ &= \frac{514}{10000} \\ &= 0.0514 \end{aligned}$$

Question 7

Write the decimal expansion of $\frac{1}{7}$. hence, write the decimal
Expression of ? $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$, and $\frac{6}{7}$

Solution:

Given :

The fraction : $\frac{1}{7}$

$$\frac{1}{7} = 0.142857142857$$

$$\frac{1}{7} = 0.142857142857$$

Since it is recurring ,

$$= 0.\overline{142857}$$

Now,

$$\begin{aligned}\frac{2}{7} &= 2 \times \left(\frac{1}{7}\right) \\ &= 2 \times 0.\overline{142857} \\ &= 0.\overline{285714}\end{aligned}$$

$$\begin{aligned}\frac{3}{7} &= 3 \times \left(\frac{1}{7}\right) \\ &= 3 \times 0.\overline{142857} \\ &= 0.\overline{428571}\end{aligned}$$

$$\begin{aligned}\frac{4}{7} &= 4 \times \left(\frac{1}{7}\right) \\ &= 4 \times 0.\overline{142857} \\ &= 0.\overline{714285}\end{aligned}$$

$$\begin{aligned}\frac{6}{7} &= 6 \times \left(\frac{1}{7}\right) \\ &= 6 \times 0.\overline{142857} \\ &= 0.\overline{857142}\end{aligned}$$

Question 8.

Express the following numbers in the form $\frac{p}{q}$. where p and q are both integers and $q \neq 0$;

(i) $0.\overline{3}$

(ii) $5.\overline{2}$

(iii) $0.404040.....$

(iv) $0.4\overline{7}$

(v) $0.1\overline{34}$

(vi) $0.\overline{001}$

Solution:

(i) $0.\overline{3}$

Let $x = 0.\overline{3} = 0.3333\dots$

Since there is one repeating digit after the decimal point,

Multiplying by 10 on both sides, we get

$$10x = 3.3333\dots$$

Now, subtract both the values,

$$9x = 3$$

$$x = \frac{3}{9}$$

$$= \frac{1}{3}$$

$$0.\overline{3} = \frac{1}{3}$$

(ii) $5.\overline{2}$

Let $x = 5.\overline{2} = 5.2222\dots$

Since there is one repeating digit after the decimal point,

Multiplying by 10 on both sides, we get

$$10x = 52.2222\dots$$

Now, subtract both the values,

$$9x = 52 - 5$$

$$9x = 47$$

$$x = \frac{47}{9}$$

$$5.\overline{2} = \frac{47}{9}$$

(iii) $0.404040\dots$

Let $x = 0.404040\dots$

Since there is two repeating digit after the decimal point,

Multiplying by 100 on both sides, we get

$$100x = 40.404040....$$

Now, subtract both the values,

$$99x = 40$$

$$X = \frac{40}{99} 0.404040.... = \frac{40}{99}$$

$$(iv) 0.4\overline{7}$$

$$\text{Let } x = 0.4\overline{7} = 0.47777....$$

Since there is one non – repeating digit after the decimal point,

Multiplying by 10 on both sides, we get

$$10x = 4.7777$$

Since there is one repeating digit after the decimal point,

Multiplying by 10 on both sides, we get

$$100x = 47.7777$$

Now, subtract both the values,

$$90x = 47 - 4$$

$$90x = 43$$

$$X = \frac{43}{90}$$

$$0.4\overline{7} = \frac{43}{90}$$

$$(v) 0.1\overline{34}$$

$$\text{Let } x = 0.1\overline{34} = 0.13434343....$$

Since there is one non- repeating digit after the decimal point,

Multiplying by 10 on both sides, we get

$$10x = 1.343434$$

Since there is two repeating digit after the decimal point,

Multiplying by 100 on both sides, we get

$$1000x = 134.343434$$

Now, subtract both the values,

$$990x = 133$$

$$X = \frac{133}{990}$$

$$0.1\overline{34} = \frac{133}{990}$$

$$(vi) 0.\overline{001}$$

$$\text{Let } x = 0.\overline{001} = 0.001001001...$$

Since There is three repeating digit after the decimal point,

Multiplying by 1000 on both sides, we get

$$1000x = 1.001001$$

Now, subtract both the values,

$$999x = 1$$

$$X = \frac{1}{999}$$

$$0.\overline{001} = \frac{1}{999}$$

Question 9 .

Classify the following numbers as rational or irrational:

$$(i) \sqrt{23}$$

$$(ii) \sqrt{225}$$

$$(iii) 0.3796$$

$$(iv) 7.478478$$

$$(v) 1.10101001000100001....$$

$$(vi) 345.\overline{0.456}$$

Solution:

$$(i) \sqrt{23}$$

Since , 23 is not a perfect square,

$\sqrt{23}$ is an irrational number.

$$(ii) \sqrt{225}$$

$\sqrt{225} = \sqrt{(15)^2} = 15$
Since , 225 is a perfect square,
 $\sqrt{225}$ = is a rational number.

(iii) 0.3796

$0.3796 = \frac{3796}{10000}$
Since , the decimal expansion is terminating decimal.
0.3796 is a rational number.

(iv) 7.478478

Let $x = 7.478478$

Since there is three repeating digit after the decimal point,

Multiplying by 1000 on both sides, we get

$$1000x = 7478.478478....$$

Now, subtract both the values,

$$999x = 7478 - 7$$

$$999x = 7471$$

$$X = \frac{7471}{999}$$

$$7.478478 = \frac{7471}{999}$$

Hence, it is neither terminating nor non – terminating or non – repeating decimal.

7.478478 is an irrational number.

(v) $1.101001000100001\dots$

Since number of zero's between two consecutive ones are increasing.
So it is non – terminating or non – repeating decimal.
 $1.101001000100001\dots$ is an irrational number.

(vi) $345.\overline{0456}$

Let $x = 345.0456456$

Multiplying by 10 on both sides, we get

$$10x = 3450.456456$$

Since there is three repeating digit after the decimal point,

Multiplying by 1000 on both sides, we get

$$1000x = 3450456.456456\dots$$

Now, subtract both the values,

$$10000x - 10x = 3450456 - 345$$

$$9990x = 3450111$$

$$X = \frac{3450111}{9990}$$

Since, it is non- terminating repeating decimal.

$345.\overline{0456}$ is a rational number.

Question 10.

Insert.... following .

- (i) One irrational number between $\frac{1}{3}$ and $\frac{1}{2}$
- (ii) One irrational number between $\frac{2}{5}$ and $\frac{1}{2}$
- (iii) One irrational number between 0 and 0.1

Solution:

- (i) One irrational number between $\frac{1}{3}$ and $\frac{1}{2}$

$$\begin{array}{r}
 0. \quad 3 \quad 3 \quad 3 \\
 3 \overline{) 1. \quad 0 \quad 0 \quad 0} \\
 - \quad 0 \\
 \hline
 1 \quad 0 \\
 - \quad 9 \\
 \hline
 1 \quad 0 \\
 - \quad 9 \\
 \hline
 1 \\
 \hline
 \hline
 \end{array}$$

$$\frac{1}{3} = 0.333$$

$$\begin{array}{r}
 0. \quad 5 \\
 2 \overline{) 1. \quad 0} \\
 - \quad 0 \\
 \hline
 1 \quad 0 \\
 - \quad 1 \quad 0 \\
 \hline
 0 \\
 \hline
 \hline
 \end{array}$$

$$\frac{1}{2} = 0.5$$

So there are infinite irrational numbers between $\frac{1}{3}$ and $\frac{1}{2}$

(ii) One irrational number between $\frac{2}{5}$ and $\frac{1}{2}$

$$+ \begin{array}{r} -0.4 \\ 5 \overline{-2.0} \\ -0 \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$$

$$\frac{-2}{5} = -0.4$$

$$\begin{array}{r} 0. 5 \\ 2 \overline{) 1. } 0 \\ - 0 \\ \hline 1 0 \\ - 1 0 \\ \hline 0 \end{array}$$

$$\frac{1}{2} = 0.5$$

(iii) One irrational number between 0 and 0.1

There are infinite irrational numbers between 0 and 1.
One irrational number among them can be 0.06006000600006....

Question 11.

Insert two irrational numbers between 2 and 3.

Solution:

2 is expressed as $\sqrt{4}$

And 3 is expressed as $\sqrt{9}$

So, two irrational numbers between 2 and 3 or $\sqrt{4}$ and $\sqrt{9}$ are , $\sqrt{5}$
 $\sqrt{6}$

Question 12.

Write two irrational numbers between $\frac{4}{9}$ and $\frac{7}{11}$.

Solution :

$\frac{4}{9}$ is expressed as 0.4444.....

$\frac{7}{11}$ is expressed as 0.636363....

So, two irrational numbers between $\frac{4}{9}$ and $\frac{7}{11}$ are 0.4040040004....and
0.6060060006...

Question 13.

Find one rational number between $\sqrt{2}$ and $\sqrt{3}$

Solution:

$\sqrt{2}$ and $\sqrt{3}$ is 1.5

Question 14.

Find two rational numbers between $\sqrt{12}$ and $\sqrt{15}$

Solution:

$$\sqrt{12} = \sqrt{(4 \times 3)} = 2\sqrt{3}$$

Since, $12 < 12.25 < 12.96 < 15$

$$\text{So, } \sqrt{12} < \sqrt{12.25} < \sqrt{12.96} < \sqrt{15}$$

Hence, two rational numbers between $\sqrt{12}$ and $\sqrt{15}$ are

$$[\sqrt{12.25}, \sqrt{12.96}] \text{ or } [\sqrt{3.5}, \sqrt{3.6}].$$

Question 15.

Insert irrational numbers between $\sqrt{5}$ and $\sqrt{7}$.

Solution:

Since, $5 < 6 < 7$

So, irrational number between $\sqrt{5}$ and $\sqrt{7}$ is $\sqrt{6}$

Question 16.

Insert two irrational numbers between $\sqrt{3}$ and $\sqrt{7}$

Solution:

Since, $3 < 4 < 5 < 6 < 7$

So,

$$\sqrt{3} < \sqrt{4} < \sqrt{5} < \sqrt{6} < \sqrt{7}$$

But $\sqrt{4} = 2$, which is a rational number .

So,

Two irrational numbers between $\sqrt{3}$ and $\sqrt{7}$ are $\sqrt{5}$ and $\sqrt{6}$

Exercise 1.4

Question 1.

Simplify the following :

(i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

(ii) $3\sqrt{3} + 2\sqrt{27} + \frac{7}{\sqrt{3}}$

(iii) $6\sqrt{5} \times 2\sqrt{5}$

(iv) $8\sqrt{15} \div 2\sqrt{3}$

(v) $\frac{\sqrt{24}}{8} + \frac{\sqrt{54}}{9}$

(vi) $\frac{3}{\sqrt{8}} + \frac{1}{\sqrt{2}}$

Solution:

(i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

Let us simplify the expression ,

$$\begin{aligned} & \sqrt{45} - 3\sqrt{20} + 4\sqrt{5} \\ &= \sqrt{9 \times 5} - 3\sqrt{4 \times 5} + 4\sqrt{5} \\ &= 3\sqrt{5} - 3 \times 2\sqrt{5} + 4\sqrt{5} \\ &= 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5} \\ &= \sqrt{5} \end{aligned}$$

$$(ii) 3\sqrt{3} + 2\sqrt{27} + \frac{7}{\sqrt{3}}$$

Let us simplify the expression,

$$3\sqrt{3} + 2\sqrt{27} + \frac{7}{\sqrt{3}}$$

$$3\sqrt{3} + 2\sqrt{(9 \times 3)} + \frac{7\sqrt{3}}{(\sqrt{3} \times \sqrt{3})} \text{ (By rationalizing)}$$

$$= 3\sqrt{3} + (2 \times 3) \sqrt{3} + \frac{7\sqrt{3}}{3}$$

$$= 3\sqrt{3} + 6\sqrt{3} + \left(\frac{7}{3}\right) \sqrt{3}$$

$$= \sqrt{3} \left(3 + 6 + \frac{7}{3} \right)$$

$$= \sqrt{3} \left(9 + \frac{7}{3} \right)$$

$$= \sqrt{3} \frac{(27+7)}{3}$$

$$= \frac{34}{3} \sqrt{3}$$

$$(iii) 6\sqrt{5} \times 2\sqrt{5}$$

Let us simplify the expression,

$$6\sqrt{5} \times 2\sqrt{5}$$

$$= 12 \times 5$$

$$= 60$$

$$(iv) 8\sqrt{15} \div 2\sqrt{3}$$

Let us simplify the expression,

$$8\sqrt{15} \div 2\sqrt{3}$$

$$= \frac{(8\sqrt{5}\sqrt{3})}{2\sqrt{3}}$$

$$= 4\sqrt{5}$$

$$(v) \frac{\sqrt{24}}{8} + \frac{\sqrt{54}}{9}$$

Let us simplify the expression,

$$\frac{\sqrt{24}}{8} + \frac{\sqrt{54}}{9}$$

$$= \frac{\sqrt{(4 \times 6)}}{8} + \frac{\sqrt{(9 \times 6)}}{9}$$

$$= \frac{2\sqrt{6}}{8} + \frac{3\sqrt{6}}{9}$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{6}}{3}$$

By taking LCM

$$= \frac{3\sqrt{6} + 4\sqrt{6}}{12}$$

$$= \frac{7\sqrt{6}}{12}$$

$$(vi) \frac{3}{\sqrt{8}} + \frac{1}{\sqrt{2}}$$

Let us simplify the expression,

$$\frac{3}{\sqrt{8}} + \frac{1}{\sqrt{2}}$$

$$= \frac{3}{2}\sqrt{2} + \frac{1}{\sqrt{2}}$$

By taking LCM

$$= \frac{(3+2)}{2\sqrt{2}}$$

$$= \frac{5}{2\sqrt{2}}$$

By rationalizing,

$$= \frac{5\sqrt{2}}{2\sqrt{2} \times 2\sqrt{2}}$$

$$= \frac{5\sqrt{2}}{2 \times 2}$$

$$= \frac{5\sqrt{2}}{4}$$

Question 2.

Simplify the following :

(i) $(5 + \sqrt{7})(2 + \sqrt{5})$

(ii) $(5 + \sqrt{5})(5 - \sqrt{5})$

(iii) $(\sqrt{5} + \sqrt{2})^2$

(iv) $(\sqrt{3} + \sqrt{7})^2$

(v) $(\sqrt{2} + \sqrt{3})(\sqrt{5} + \sqrt{7})$

(vi) $(4 + \sqrt{5})(\sqrt{3} - \sqrt{7})$

Solution:

(i) $(5 + \sqrt{7})(2 + \sqrt{5})$

Let us simplify the expression,

$$= 5(2 + \sqrt{5}) + \sqrt{7}(2 + \sqrt{5})$$

$$= 10 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{35}$$

$$(ii) (5 + \sqrt{5})(5 - \sqrt{5})$$

Let us simplify the expression,
By using the formula,

$$(a)^2 - (b)^2 = (a + b)(a - b)$$

So,

$$= (5)^2 - (\sqrt{5})^2$$

$$= 25 - 5$$

$$= 20$$

$$(iii) (\sqrt{5} + \sqrt{2})^2$$

Let us simplify the expression,
By using the formula,

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$(\sqrt{5} + \sqrt{2})^2 = (5)^2 + (\sqrt{2})^2 + 2\sqrt{5}\sqrt{2}$$

$$= 5 + 2 + 2\sqrt{10}$$

$$= 7 + 2\sqrt{10}$$

$$(iv) (\sqrt{3} - \sqrt{7})^2$$

Let us simplify the expression,
By using the formula,

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$(\sqrt{3} - \sqrt{7})^2 = (\sqrt{3})^2 + (\sqrt{7})^2 - 2\sqrt{3}\sqrt{7}$$

$$= 3 + 7 - 2\sqrt{21}$$

$$= 10 - 2\sqrt{21}$$

$$(v) (\sqrt{2} + \sqrt{3})(\sqrt{5} + \sqrt{7})$$

Let us simplify the expression,

$$= \sqrt{2} (\sqrt{5} + \sqrt{7}) \sqrt{3}(\sqrt{5} + \sqrt{7})$$

$$= \sqrt{2} \times \sqrt{5} \times \sqrt{2} \times \sqrt{7} \times \sqrt{3} \times \sqrt{5} \times \sqrt{3} \times \sqrt{7}$$

$$= \sqrt{10} + \sqrt{14} + \sqrt{15} + \sqrt{21}$$

$$(vi) (4 + \sqrt{5})(\sqrt{3} - \sqrt{7})$$

Let us simplify the expression,

$$= 4(\sqrt{3} - \sqrt{7}) + \sqrt{5}(\sqrt{3} - \sqrt{7})$$

$$= 4\sqrt{3} - 4\sqrt{7} + \sqrt{15} - \sqrt{35}$$

Question 3.

If $\sqrt{2} = 1.414$, then find the value of

(i) $\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$

(ii) $3\sqrt{32} - 2\sqrt{50} + 4\sqrt{128} - 20\sqrt{18}$

Solution:

(i) $\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$

Let us simplify the expression,

$$\begin{aligned} & \sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98} \\ &= \sqrt{2 \times 4} + \sqrt{2 \times 25} + \sqrt{2 \times 36} + \sqrt{2 \times 49} \\ &= \sqrt{2} \sqrt{4} + \sqrt{2} \sqrt{25} + \sqrt{2} \sqrt{36} + \sqrt{2} \sqrt{49} \\ &= 20\sqrt{2} \\ &= 20 \times 1.414 \\ &= 28.28 \end{aligned}$$

(ii) $3\sqrt{32} - 2\sqrt{50} + 4\sqrt{128} - 20\sqrt{18}$

Let us simplify the expression,

$$\begin{aligned} & 3\sqrt{32} - 2\sqrt{50} + 4\sqrt{128} - 20\sqrt{18} \\ & 3\sqrt{16 \times 2} - 2\sqrt{25 \times 2} + 4\sqrt{64 \times 2} - 20\sqrt{9 \times 2} \\ &= 3 \sqrt{16} \sqrt{2} - 2\sqrt{25}\sqrt{2} + 4\sqrt{64}\sqrt{2} - 20\sqrt{9} \sqrt{2} \end{aligned}$$

$$= 3.4\sqrt{2} - 2.5\sqrt{2} + 4.8\sqrt{2} - 20.3\sqrt{2}$$

$$= 12\sqrt{2} - 10\sqrt{2} + 32\sqrt{2} - 60\sqrt{2}$$

$$= (12 - 10 + 32 - 60) \sqrt{2}$$

$$= -26 \sqrt{2}$$

$$= -26 \times 1.414$$

$$= -36.764$$

Question 4.

If $\sqrt{3} = 1.732$, then find the value of

$$(i) \sqrt{27} + \sqrt{75} + \sqrt{108} - \sqrt{243}$$

$$(ii) 5\sqrt{12} - 3\sqrt{48} + 6\sqrt{75} + 7\sqrt{108}$$

Solution:

$$(i) \sqrt{27} + \sqrt{75} + \sqrt{108} - \sqrt{243}$$

Let us simplify the expression,

$$\sqrt{27} + \sqrt{75} + \sqrt{108} - \sqrt{243}$$

$$= \sqrt{9 \times 3} + \sqrt{25 \times 3} + \sqrt{36 \times 3} - \sqrt{81 \times 3}$$

$$= \sqrt{9} \sqrt{3} + \sqrt{25} \sqrt{3} + \sqrt{36} \sqrt{3} - \sqrt{81} \sqrt{3}$$

$$= 3\sqrt{3} + 5\sqrt{3} + 6\sqrt{3} - 9\sqrt{3}$$

$$= (3 + 5 + 6 - 9)$$

$$= 5\sqrt{3}$$

$$= 5 \times 1.732$$

$$= 8.660$$

$$(ii) 5\sqrt{12} - 3\sqrt{48} + 6\sqrt{75} + 7\sqrt{108}$$

Let us simplify the expression,

$$5\sqrt{12} - 3\sqrt{48} + 6\sqrt{75} + 7\sqrt{108}$$

$$= 5\sqrt{(4 \times 3)} - 3\sqrt{(16 \times 3)} + 6\sqrt{(25 \times 3)} + 7\sqrt{36 \times 3}$$

$$= 5\sqrt{4} \sqrt{3} - 3\sqrt{16}\sqrt{3} + 6\sqrt{25}\sqrt{3} + 7\sqrt{36}\sqrt{3}$$

$$= 5.2\sqrt{3} - 3.4\sqrt{3} + 6.5\sqrt{3} + 7.6\sqrt{3}$$

$$= 10\sqrt{3} - 12\sqrt{3} + 30\sqrt{3} + 42\sqrt{3}$$

$$= (10 - 12 + 30 + 42) \sqrt{3}$$

$$= 70\sqrt{3}$$

$$= 70 \times 1.732$$

$$= 121.24$$

Question 5.

State which of the following are rational or irrational decimals.

$$(i) \sqrt{\left(\frac{4}{9}\right)}, \frac{-3}{70}, \sqrt{\left(\frac{7}{25}\right)}, \sqrt{\left(\frac{16}{5}\right)}$$

$$(ii) -\sqrt{\left(\frac{2}{49}\right)}, \frac{3}{200}, \sqrt{\left(\frac{25}{3}\right)}, -\sqrt{\left(\frac{49}{16}\right)}$$

Solution:

$$(i) \sqrt{\left(\frac{4}{9}\right)}, \frac{-3}{70}, \sqrt{\left(\frac{7}{25}\right)}, \sqrt{\left(\frac{16}{5}\right)}$$

$$\sqrt{\left(\frac{4}{9}\right)} = \frac{2}{3}$$

$$\frac{-3}{70} = \frac{-3}{70}$$

$$\sqrt{\left(\frac{7}{25}\right)} = \frac{\sqrt{7}}{5}$$

$$\sqrt{\left(\frac{16}{5}\right)} = \frac{4}{\sqrt{5}}$$

So,

$\frac{\sqrt{7}}{5}$ and $\frac{4}{\sqrt{5}}$ are irrational decimals.

$\frac{2}{3}$ and $\frac{-3}{70}$ are rational decimals.

$$(ii) -\sqrt{\left(\frac{2}{49}\right)}, \frac{3}{200}, \sqrt{\left(\frac{25}{3}\right)}, -\sqrt{\left(\frac{49}{16}\right)}$$

$$-\sqrt{\left(\frac{2}{49}\right)} = -\frac{\sqrt{2}}{7}$$

$$\frac{3}{200} = \frac{3}{200}$$

$$\sqrt{\left(\frac{25}{3}\right)} = \frac{5}{\sqrt{3}}$$

$$-\sqrt{\left(\frac{49}{16}\right)} = \frac{-7}{4}$$

So,

- $\frac{\sqrt{2}}{7}$ and $\frac{5}{\sqrt{3}}$ are irrational decimals.

$\frac{3}{200}$ and $\frac{-7}{4}$ are rational decimals .

Question 6.

State which of the following are rational or irrational decimals.

(i) $-3\sqrt{2}$

(ii) $\sqrt{\frac{256}{81}}$

(iii) $\sqrt{\frac{27}{16}}$

(iv) $\sqrt{\frac{5}{36}}$

Solution:

(i) $-3\sqrt{2}$

We know that $\sqrt{2}$ is an irrational number.

So, $-3\sqrt{2}$ will also be irrational number.

$$(ii) \sqrt{\frac{256}{81}}$$

$$\sqrt{\frac{256}{81}} = \frac{16}{9} = \frac{4}{3}$$

It is a rational number.

$$(iii) \sqrt{\frac{27}{16}}$$

$$\sqrt{\frac{27}{16}} = \sqrt{9 \times 3 \times 16} = 3 \times 4 \sqrt{3} = 12\sqrt{3}$$

It is an irrational number.

$$(iv) \sqrt{\frac{5}{36}}$$

$$\sqrt{\frac{5}{36}} = \frac{\sqrt{5}}{6}$$

It is an irrational number.

Question 7.

State which of the following are irrational numbers.

$$(i) 3 - \sqrt{\frac{7}{25}}$$

$$(ii) -\frac{2}{3} + 3\sqrt{2}$$

$$(iii) \frac{3}{\sqrt{3}}$$

$$(iv) \frac{-2}{7} 3\sqrt{5}$$

$$(v) (2 - \sqrt{3})(2 + \sqrt{3})$$

$$(vi) (3 + \sqrt{5})^2$$

$$(vii) \left(\frac{2}{5} \sqrt{7}\right)^2$$

$$(viii) (3 - \sqrt{6})^2$$

Solution:

$$(i) 3 - \sqrt{\frac{7}{25}}$$

Let us simplify,

$$3 - \sqrt{\frac{7}{25}} = 3 - \sqrt{\frac{7}{25}}$$

$$3 - \frac{\sqrt{7}}{5}$$

Hence, $3 - \frac{\sqrt{7}}{5}$ is an irrational number.

$$(ii) -\frac{2}{3} + 3\sqrt{2}$$

Let us simplify,

$$-\frac{2}{3} + 3\sqrt{2} = -\frac{2}{3} + 2^{\frac{1}{3}}$$

Since, 2 is not a perfect cube.

Hence it is an irrational number.

(iii) $\frac{3}{\sqrt{3}}$

Let us simplify,

By rationalizing. We get

$$\frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$= \frac{3\sqrt{3}}{3}$$

$$= \sqrt{3}$$

Hence, $\frac{3}{\sqrt{3}}$ is an irrational number.

(iv) $\frac{-2}{7}3\sqrt{5}$

Let us simplify,

$$\frac{-2}{7}3\sqrt{5} = \frac{-2}{7}5^{\frac{1}{2}}$$

Since, 5 is not a perfect cube.

Hence it is an irrational number.

(v) $(2 - \sqrt{3}) (2 + \sqrt{3})$

Let us simplify,

By using the formula,

$$(a + b)(a - b) = (a)^2 - (b)^2$$

$$\begin{aligned}
 (2 - \sqrt{3}) (2 + \sqrt{3}) &= 3^2 + (\sqrt{5})^2 + 2.3 \sqrt{5} \\
 &= 9 + 5 + 6\sqrt{5} \\
 &= 14 + 6\sqrt{5}
 \end{aligned}$$

Hence, it is an irrational number.

$$\begin{aligned}
 \text{(vii)} \left(\frac{2}{5} \sqrt{7}\right)^2 &= \left(\frac{2}{5} \sqrt{7}\right)^2 \times \left(\frac{2}{5} \sqrt{7}\right)^2 \\
 &= \frac{4}{25} \times 7 \\
 &= \frac{28}{25}
 \end{aligned}$$

Hence it is a rational number.

$$\text{(viii)} (3 - \sqrt{6})^2$$

Let us simplify,

By using $(a - b)^2 = a^2 + b^2 - 2ab$

$$\begin{aligned}
 (3 - \sqrt{6})^2 &= 3^2 + (\sqrt{6})^2 - 2.3 \sqrt{6} \\
 &= 9 + 6 - 6\sqrt{6} \\
 &= 15 - 6\sqrt{6}
 \end{aligned}$$

Hence it is an irrational number.

Question 8.

Prove the following are irrational numbers.

(i) $3\sqrt{2}$

(ii) $\sqrt[3]{3}$

(iii) $\sqrt[4]{5}$

Solution:

(i) $3\sqrt{2}$

We know that $3\sqrt{2} = 2^{\frac{1}{3}}$

Let us consider $2^{\frac{1}{3}} = \frac{p}{q}$, where p, q are integers, $q > 0$.

P and q have no common factors (except 1).

So,

$$2^{\frac{1}{3}} = \frac{p}{q}$$

$$2 = \frac{p^3}{q^3}$$

$$P^3 = 2q^3 \dots\dots (1)$$

We know that, 2 divides $2q^3$ then 2 divides p^3

So, 2 divides p

Now, let us consider $p = 2k$, where k is an integer

Substitute the value of p in (1) , we get

$$P^3 = 2q^3$$

$$(2k)^3 = 2q^3$$

$$8k^3 = 2q^3$$

$$4k^3 = q^3$$

We know that, 2 divides $4k^3$ then 2 divides q^3

So, 2 divides q

Thus p and q have a common factor '2'

This contradicts the statement, p and q have no common factor (except 1) .

Hence, $3\sqrt{2}$ is an irrational number.

(ii) $\sqrt[3]{3}$

We know that $\sqrt[3]{3} = 3^{\frac{1}{3}}$

Let us consider $3^{\frac{1}{3}} = \frac{p}{q}$, where p, q are integers, $q > 0$

p and q have no common factors (except 1.)

So,

$$3^{\frac{1}{3}} = \frac{p}{q}$$

$$3 = \frac{p^3}{q^3}$$

$$p^3 = 3q^3 \dots (1)$$

We know that , 3 divides $3q^3$ then 3 divides p^3

So, 3 divides p

Now, let us consider $p = 3k$, where k is an integer

Substitute the value of p in (1) , we get

$$p^3 = 3q^3$$

$$(3k)^3 = 3q^3$$

$$9k^3 = 3q^3$$

$$3k^3 = q^3$$

We know that , 3 divides $9k^3$ then 3 divides q^3

So, 3 divides q

Thus p and q have a common factor '3'

This contradicts the statement, p and q have no common factor (except 1).

Hence, $\sqrt[3]{3}$ is an irrational number.

(iii) $\sqrt[4]{5}$

We know that $\sqrt[4]{5} = 5^{\frac{1}{4}}$

Let us consider $5^{\frac{1}{4}} = \frac{p}{q}$, where p, q are integers, $q > 0$.

P and q have no common factors (except 1)

So,

$$5^{\frac{1}{4}} = \frac{p}{q}$$

$$5 = \frac{p^4}{q^4}$$

$$P^4 = 5q^4 \dots (1)$$

We know that, 5 divides $5q^4$ then 5 divides p^4

So, 5 divides p

Now, let us consider $p = 5k$, where k is an integer

Substitute the value of p in (1) , we get

$$p^4 = 5q^4$$

$$(5k)^4 = 5q^4$$

$$625k^4 = 5q^4$$

$$125 k^4 = q^4$$

We know that, 5 divides $125k^4$ then 5 divides q^4

So, 5 divides q

Thus p and q have a common factor '5'

This contradicts the statement, p and q have no common factor (except 1).

Hence, $\sqrt[4]{5}$ is an irrational number.

Question 9.

Find the greatest and the smallest real numbers.

$$(i) 2\sqrt{3}, \frac{3}{\sqrt{2}}, -7, \sqrt{15}$$

$$(ii) -3\sqrt{2}, \frac{9}{\sqrt{5}}, -4, \frac{4}{3}\sqrt{5}, \frac{3}{2}\sqrt{3}$$

Solution:

$$(i) 2\sqrt{3}, \frac{3}{\sqrt{2}}, -7, \sqrt{15}$$

Let us simplify each fraction

$$2\sqrt{3} = \sqrt{(4 \times 3)} = \sqrt{12}$$

$$\frac{3}{\sqrt{2}} = \frac{3 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = 3 \frac{\sqrt{2}}{2} = \sqrt{\left(\frac{9}{4}\right)} \times 2 = \sqrt{\frac{9}{2}} = \sqrt{4.5}$$

$$- \sqrt{7} = - \sqrt{7}$$

$$\sqrt{15} = \sqrt{15}$$

So,

The greatest real number = $\sqrt{15}$

Smallest real number = $-\sqrt{7}$

$$(ii) -3\sqrt{2}, \frac{9}{\sqrt{5}}, -4, \frac{4}{3}\sqrt{5}, \frac{3}{2}\sqrt{3}$$

Let us simplify each fraction.

$$-3\sqrt{2} = -\sqrt{(9 \times 2)} = -\sqrt{18}$$

$$\frac{9}{\sqrt{5}} = \frac{(9 \times \sqrt{5})}{(\sqrt{5} \times \sqrt{5})} = \frac{9\sqrt{5}}{5} = \sqrt{\left(\frac{81}{25}\right) \times 5} = \sqrt{\frac{81}{5}} = \sqrt{16.2}$$

$$-4 = -\sqrt{16}$$

$$\frac{4}{3}\sqrt{5} = \sqrt{\left(\frac{16}{9}\right) \times 5} = \sqrt{\frac{80}{9}} = \sqrt{8.88} = \sqrt{8.8}$$

$$\frac{3}{2}\sqrt{3} = \sqrt{\frac{9}{4}} \times 3 = \sqrt{\frac{27}{4}} = \sqrt{6.25}$$

So,

The greatest real number = $9\sqrt{5}$

Smallest real number = $-3\sqrt{2}$

Question 10.

Write in ascending order.

$$(i) 3\sqrt{2} = \sqrt{9 \times 2} = \sqrt{18}$$

$$2\sqrt{3} = \sqrt{4 \times 3} = \sqrt{12}$$

$$\sqrt{15} = \sqrt{15}$$

$$4 = \sqrt{16}$$

Now, let us arrange in ascending order

$$\sqrt{12}, \sqrt{15}, \sqrt{16}, \sqrt{18}$$

$$\text{So, } 2\sqrt{3}, \sqrt{15}, 4, 3\sqrt{2}$$

$$(ii) 3\sqrt{2}, 2\sqrt{8}, 4, \sqrt{50}, 4\sqrt{3}$$

$$3\sqrt{2} = \sqrt{9 \times 2} = \sqrt{18}$$

$$2\sqrt{8} = \sqrt{4 \times 8} = \sqrt{32}$$

$$4 = \sqrt{16}$$

$$\sqrt{50} = \sqrt{50}$$

$$4\sqrt{3} = \sqrt{16 \times 3} = \sqrt{48}$$

Now, let us arrange in ascending order.

$$\sqrt{16} \sqrt{18} \sqrt{32} \sqrt{48} \sqrt{50}$$

$$\text{So, } 4, 3\sqrt{2}, 4\sqrt{3}, \sqrt{50}$$

Question 11.

Write in descending order:

$$(i) \frac{9}{\sqrt{2}}, \frac{3}{2} \sqrt{5}, 4\sqrt{3}, 3\sqrt{\frac{6}{5}}$$

$$(ii) \frac{5}{\sqrt{3}}, \frac{7}{3} \sqrt{2} - \sqrt{3}, 3\sqrt{5}, 2\sqrt{7}$$

Solution:

$$\frac{9}{\sqrt{2}}, \frac{3}{2}\sqrt{5}, 4\sqrt{3}, 3\sqrt{\frac{6}{5}}$$

$$\frac{9}{\sqrt{2}} = \frac{9 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{9\sqrt{2}}{2} = \sqrt{\frac{81}{4}} \times 2 = \sqrt{\frac{81}{2}} = \sqrt{40.5}$$

$$\frac{3}{2}\sqrt{5} = \sqrt{\frac{9}{4}} \times 5 = \sqrt{\frac{45}{4}} = \sqrt{11.25}$$

$$4\sqrt{3} = \sqrt{(16 \times 3)} = \sqrt{48}$$

$$3\sqrt{\frac{6}{5}} = \sqrt{\frac{(9 \times 6)}{5}} = \sqrt{\frac{54}{5}} = \sqrt{10.8}$$

Now, let us arrange in descending order

$$\sqrt{48}, \sqrt{40.5}, \sqrt{11.25}, \sqrt{10.8}$$

So,

$$4\sqrt{3}, \frac{9}{\sqrt{2}}, \frac{3}{2}\sqrt{5}, 3\sqrt{\frac{6}{5}}$$

$$(ii) \frac{5}{\sqrt{3}}, \frac{7}{3}\sqrt{2}, -\sqrt{3}, 3\sqrt{5}, 2\sqrt{7}$$

$$\frac{5}{\sqrt{3}} = \sqrt{\frac{25}{3}} = \sqrt{8.33}$$

$$\frac{7}{3}\sqrt{2} = \sqrt{\frac{49}{9}} \times 2 = \sqrt{\frac{98}{9}} = \sqrt{10.88}$$

$$-\sqrt{3} = -\sqrt{3}$$

$$3\sqrt{5} = \sqrt{(9 \times 5)} = \sqrt{45}$$

$$2\sqrt{7} = \sqrt{(4 \times 7)} = \sqrt{28}$$

Now, let us arrange in descending order

$$\sqrt{45}, \sqrt{28}, \sqrt{10.88}, \sqrt{8.33}, -\sqrt{3}$$

So,

$$3\sqrt{5}, 2\sqrt{7}, \frac{7}{3}\sqrt{2}, \frac{5}{\sqrt{3}}, -\sqrt{3}$$

Question 12.

Arrange in ascending order.

$$\sqrt[3]{2}, \sqrt{3}, \sqrt[6]{5}$$

Solution:

Here we can express the given expressions as:

$$\sqrt[3]{2} = 2^{\frac{1}{3}}$$

$$\sqrt{3} = 3^{\frac{1}{2}}$$

$$\sqrt[6]{5} = 5^{\frac{1}{6}}$$

Let us make the roots common so,

$$2^{\frac{1}{3}} = 2^{(2 \times \frac{1}{2} \times \frac{1}{3})} = 4^{\frac{1}{6}}$$

$$3^{\frac{1}{2}} = 3^{(3 \times \frac{1}{3} \times \frac{1}{2})} = 27^{\frac{1}{6}}$$

$$5^{\frac{1}{6}} = 5^{\frac{1}{6}}$$

Now, let us arrange in ascending order,

$$4^{\frac{1}{6}}, 5^{\frac{1}{6}}, 27^{\frac{1}{6}}$$

So,

$$2^{\frac{1}{3}}, 5^{\frac{1}{6}}, 3^{\frac{1}{2}}$$

So,

$$\sqrt[3]{2}, \sqrt[6]{5}, \sqrt{3}$$

Exercise 1.5

Question 1 .

Rationalize the following :

(i) $\frac{3}{4}\sqrt{5}$

(ii) $\frac{5\sqrt{7}}{\sqrt{3}}$

(iii) $\frac{3}{4-\sqrt{7}}$

(iv) $\frac{17}{3\sqrt{2}} + 1$

(v) $\frac{16}{\sqrt{41}-5}$

(vi) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(vii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

(viii) $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

Solution:

(i) $\frac{3}{4}\sqrt{5}$

Let us rationalize,

$$\begin{aligned}\frac{3}{4}\sqrt{5} &= \frac{3 \times \sqrt{5}}{4\sqrt{5} \times \sqrt{5}} \\ &= \frac{3\sqrt{5}}{4 \times 5} \\ &= \frac{3\sqrt{5}}{20}\end{aligned}$$

$$(ii) \frac{5\sqrt{7}}{\sqrt{3}}$$

Let us rationalize,

$$\begin{aligned} \frac{5\sqrt{7}}{\sqrt{3}} &= \frac{5\sqrt{7} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\ &= \frac{5\sqrt{21}}{3} \end{aligned}$$

$$(iii) \frac{3}{4-\sqrt{7}}$$

Let us rationalize,

$$\begin{aligned} \frac{3}{4-\sqrt{7}} &= \frac{3 \times 4 + \sqrt{7}}{(4-\sqrt{7}) \times (4+\sqrt{7})} \\ &= \frac{3(4+\sqrt{7})}{(4^2 - (\sqrt{7})^2)} \\ &= \frac{3(4+\sqrt{7})}{(16-7)} \\ &= \frac{3(4+\sqrt{7})}{9} \\ &= \frac{(4+\sqrt{7})}{3} \end{aligned}$$

$$(iv) \frac{17}{3\sqrt{2}} + 1$$

Let us rationalize,

$$\begin{aligned} \frac{17}{3\sqrt{2}} + 1 &= \frac{17(3\sqrt{2}-1)}{(3\sqrt{2}+1)(3\sqrt{2}-1)} \\ &= \frac{17(3\sqrt{2}-1)}{(3\sqrt{2})^2 - 1^2} \\ &= \frac{17(3\sqrt{2}-1)}{9 \cdot 2 - 1} \\ &= \frac{17(3\sqrt{2}-1)}{18-1} \\ &= \frac{17(3\sqrt{2}-1)}{17} \\ &= (3\sqrt{2} - 1) \end{aligned}$$

$$\text{(v)} \frac{16}{\sqrt{41}-5}$$

Let us rationalize,

$$\begin{aligned} \frac{16}{\sqrt{41}-5} &= \frac{16(\sqrt{41}+5)}{(\sqrt{41}-5)(\sqrt{41}+5)} \\ &= \frac{16(\sqrt{41}+5)}{(\sqrt{41})^2-5^2} \\ &= \frac{16(\sqrt{41}+5)}{41-25} \\ &= \frac{16(\sqrt{41}+5)}{16} \\ &= (\sqrt{41}+5) \end{aligned}$$

$$\text{(vi)} \frac{1}{\sqrt{7}-\sqrt{6}}$$

Let us rationalize,

$$\begin{aligned} \frac{1}{\sqrt{7}-\sqrt{6}} &= \frac{1(\sqrt{7}+\sqrt{6})}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})} \\ &= \frac{(\sqrt{7}+\sqrt{6})}{(\sqrt{7})^2-(\sqrt{6})^2} \\ &= \frac{\sqrt{7}+\sqrt{6}}{7-6} \\ &= \frac{\sqrt{7}+\sqrt{6}}{1} \\ &= (\sqrt{7}+\sqrt{6}) \end{aligned}$$

$$\text{(vii)} \frac{1}{\sqrt{5}+\sqrt{2}}$$

Let us rationalize,

$$\begin{aligned} \frac{1}{\sqrt{5}+\sqrt{2}} &= \frac{1(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} \\ &= \frac{(\sqrt{5}-\sqrt{2})}{[(\sqrt{5})^2-(\sqrt{2})^2]} \end{aligned}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{3}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{3}$$

$$\text{(viii)} \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

Let us rationalize,

$$\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{(\sqrt{2}+\sqrt{3})(\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})}$$

$$= \frac{(\sqrt{2}+\sqrt{3})^2}{(\sqrt{2})^2 - (\sqrt{3})^2}$$

$$= \frac{2+3+2\sqrt{2}\sqrt{3}}{2-3}$$

$$= \frac{5+2\sqrt{6}}{1}$$

$$= -(5 + 2\sqrt{6})$$

Question 2.

Simplify:

$$\text{(i)} \frac{(7 + 3\sqrt{5})}{(7 - 3\sqrt{5})}$$

$$\text{(ii)} \frac{(3 - 2\sqrt{2})}{(3 + 2\sqrt{2})}$$

$$\text{(iii)} \frac{(5 - 3\sqrt{14})}{(7 + 2\sqrt{14})}$$

Solution:

$$(i) \frac{(7 + 3\sqrt{5})}{(7 - 3\sqrt{5})}$$

Let us rationalize the denominator, we get

$$\begin{aligned} \frac{(7 + 3\sqrt{5})}{(7 - 3\sqrt{5})} &= \frac{[(7 + 3\sqrt{5}) (7 + 3\sqrt{5})]}{[(7 - 3\sqrt{5}) (7 + 3\sqrt{5})]} \\ &= \frac{(7+3\sqrt{5})^2}{7^2-(3\sqrt{5})^2} \\ &= \frac{7^2+3(\sqrt{5})^2+2.7.3\sqrt{5}}{49-9.5} \\ &= \frac{49+9.5+42\sqrt{5}}{49-45} \\ &= \frac{[49 + 45 + 42\sqrt{5}]}{[4]} \\ &= \frac{[94 + 42\sqrt{5}]}{4} \\ &= \frac{2[47 + 21\sqrt{5}]}{4} \\ &= \frac{[47 + 21\sqrt{5}]}{2} \end{aligned}$$

$$(ii) \frac{(3 - 2\sqrt{2})}{(3 + 2\sqrt{2})}$$

Let us rationalize the denominator, we get

$$\begin{aligned} \frac{(3 - 2\sqrt{2})}{(3 + 2\sqrt{2})} &= \frac{[(3 - 2\sqrt{2}) (3 - 2\sqrt{2})]}{[(3 + 2\sqrt{2}) (3 - 2\sqrt{2})]} \\ &= \frac{(3-2\sqrt{2})^2}{(3^2-2\sqrt{2})^2} \\ &= \frac{[3^2+(2\sqrt{2})^2-2.3.2\sqrt{2}]}{[9-4.2]} \\ &= \frac{[9 + 4.2 - 12\sqrt{2}]}{[9 - 8]} \\ &= \frac{[9 + 8 - 12\sqrt{2}]}{1} \end{aligned}$$

$$= 17 - 12\sqrt{2}$$

$$\text{(iii)} \frac{(5 - 3\sqrt{14})}{(7 + 2\sqrt{14})}$$

Let us rationalize the denominator, we get

$$\begin{aligned} \frac{(5 - 3\sqrt{14})}{(7 + 2\sqrt{14})} &= \frac{[(5 - 3\sqrt{14}) (7 - 2\sqrt{14})]}{[(7 + 2\sqrt{14}) (7 - 2\sqrt{14})]} \\ &= \frac{[5(7 - 2\sqrt{14}) - 3\sqrt{14} (7 - 2\sqrt{14})]}{[7^2 - (2\sqrt{14})^2]} \\ &= \frac{[35 - 10\sqrt{14} - 21\sqrt{14} + 6 \cdot 14]}{[49 - 4 \cdot 14]} \\ &= \frac{[35 - 31\sqrt{14} + 84]}{[49 - 56]} \\ &= \frac{[119 - 31\sqrt{14}]}{[-7]} \\ &= \frac{-[119 - 31\sqrt{14}]}{7} \\ &= \frac{[31\sqrt{14} - 119]}{7} \end{aligned}$$

Question 3.

Simplify:

$$\left(\frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}} \right) - \left(\frac{2\sqrt{5}}{(\sqrt{6} + \sqrt{5})} \right) - \left(\frac{3\sqrt{2}}{(\sqrt{15} + 3\sqrt{2})} \right)$$

Solution:

Let us simplify individually,

$$\left(\frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}} \right)$$

Let us rationalize the denominator,

$$\begin{aligned} \left(\frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}} \right) &= \frac{[7\sqrt{3} (\sqrt{10} - \sqrt{3})]}{[(\sqrt{10} + \sqrt{3}) (\sqrt{10} - \sqrt{3})]} \\ &= \frac{[7\sqrt{3} \cdot \sqrt{10} - 7\sqrt{3} \cdot \sqrt{3}]}{[(\sqrt{10})^2 - (\sqrt{3})^2]} \end{aligned}$$

$$= \frac{[7\sqrt{30} - 7.3]}{[10 - 3]}$$

$$= \frac{7[\sqrt{30} - 3]}{7}$$

$$= \sqrt{30} - 3$$

Now

$$\left(\frac{2\sqrt{5}}{(\sqrt{6} + \sqrt{5})} \right)$$

Let us rationalize the denominator, we get

$$\left(\frac{2\sqrt{5}}{(\sqrt{6} + \sqrt{5})} \right) = \frac{[2\sqrt{5}(\sqrt{6} - \sqrt{5})]}{[(\sqrt{6} + \sqrt{5})(\sqrt{6} - \sqrt{5})]}$$

$$= \frac{[2\sqrt{5}.\sqrt{6} - 2\sqrt{5}.\sqrt{5}]}{[(\sqrt{6})^2 - (\sqrt{5})^2]}$$

$$= \frac{[2\sqrt{30} - 2.5]}{[6 - 5]}$$

$$= \frac{[2\sqrt{30} - 10]}{1}$$

$$= 2\sqrt{30} - 10$$

Now,

$$\left(\frac{3\sqrt{2}}{(\sqrt{15} + 3\sqrt{2})} \right)$$

Let us rationalize the denominator, we get

$$\left(\frac{3\sqrt{2}}{(\sqrt{15} + 3\sqrt{2})} \right) = \frac{[3\sqrt{2}(\sqrt{15} - 3\sqrt{2})]}{[(\sqrt{15} + 3\sqrt{2})(\sqrt{15} - 3\sqrt{2})]}$$

$$= \frac{[3\sqrt{2}.\sqrt{15} - 3\sqrt{2}.3\sqrt{2}]}{[(\sqrt{15})^2 - (3\sqrt{2})^2]}$$

$$= \frac{[3\sqrt{30} - 9.2]}{[15 - 9.2]}$$

$$= \frac{[3\sqrt{30} - 18]}{[15 - 18]}$$

$$= \frac{3[\sqrt{30} - 6]}{[-3]}$$

$$= \frac{[\sqrt{30}-6]}{-1}$$

$$= 6 - \sqrt{30}$$

So, according to the question let us substitute the obtained values,

$$\left(\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}\right) - \left(\frac{2\sqrt{5}}{(\sqrt{6}+\sqrt{5})}\right) - \left(\frac{3\sqrt{2}}{(\sqrt{15}+3\sqrt{2})}\right)$$

$$= (\sqrt{30}-3) - (2\sqrt{30}-10) - (6-\sqrt{30})$$

$$= \sqrt{30}-3-2\sqrt{30}+10-6+\sqrt{30}$$

$$= 2\sqrt{30}-2\sqrt{30}-3+10-6$$

$$= 1$$

Question 4.

Simplify:

$$\left[\frac{1}{\sqrt{4}+\sqrt{5}}\right] + \left[\frac{1}{\sqrt{5}+\sqrt{6}}\right] + \left[\frac{1}{\sqrt{6}+\sqrt{7}}\right] + \left[\frac{1}{\sqrt{7}+\sqrt{8}}\right] + \left[\frac{1}{\sqrt{8}+\sqrt{9}}\right]$$

Solution:

let us simplify individually,

$$\left[\frac{1}{\sqrt{4}+\sqrt{5}}\right]$$

Rationalize the denominator, we get

$$\left[\frac{1}{\sqrt{4}+\sqrt{5}}\right] = \left[\frac{\frac{1}{\sqrt{4}-\sqrt{5}}}{(\sqrt{4}+\sqrt{5})(\sqrt{4}-\sqrt{5})}\right]$$

$$= \frac{(\sqrt{4}-\sqrt{5})}{(\sqrt{4})^2 - (\sqrt{5})^2}$$

$$= \left[\frac{(\sqrt{4}-\sqrt{5})}{(4-5)}\right]$$

$$= \left[\frac{(\sqrt{4}-\sqrt{5})}{-1} \right]$$

$$= -(\sqrt{4} - \sqrt{5})$$

Now,

$$\left[\frac{1}{\sqrt{5}+\sqrt{6}} \right]$$

Rationalize the denominator, we get

$$\left[\frac{1}{\sqrt{5}+\sqrt{6}} \right] = \left[\frac{\frac{1}{\sqrt{5}-\sqrt{6}}}{(\sqrt{5}+\sqrt{6})(\sqrt{5}-\sqrt{6})} \right]$$

$$= \left[\frac{\sqrt{5}-\sqrt{6}}{(\sqrt{5})^2(\sqrt{6})^2} \right]$$

$$= \left[\frac{(\sqrt{5}-\sqrt{6})}{(5-6)} \right]$$

$$= \left[\frac{(\sqrt{5}-\sqrt{6})}{-1} \right]$$

$$= -(\sqrt{5} - \sqrt{6})$$

Now,

$$\left[\frac{1}{\sqrt{6}+\sqrt{7}} \right]$$

Rationalize the denominator, we get

$$\left[\frac{1}{\sqrt{6}+\sqrt{7}} \right] = \left[\frac{\frac{1}{\sqrt{6}-\sqrt{7}}}{(\sqrt{6}+\sqrt{7})(\sqrt{6}-\sqrt{7})} \right]$$

$$= \left[\frac{(\sqrt{6}-\sqrt{7})}{(\sqrt{6})^2-(\sqrt{7})^2} \right]$$

$$= \left[\frac{(\sqrt{6}-\sqrt{7})}{(6-7)} \right]$$

$$= \left[\frac{(\sqrt{6}-\sqrt{7})}{-1} \right]$$

$$= -(\sqrt{6} - \sqrt{7})$$

Now ,

$$\left[\frac{1}{(\sqrt{7}-\sqrt{8})} \right]$$

Rationalize the denominator , we get

$$\begin{aligned}
\left[\frac{1}{(\sqrt{7}-\sqrt{8})} \right] &= \left[\frac{\frac{1}{(\sqrt{7}-\sqrt{8})}}{(\sqrt{7}+\sqrt{8})(\sqrt{7}-\sqrt{8})} \right] \\
&= \left[\frac{(\sqrt{7}-\sqrt{8})}{(\sqrt{7})^2-(\sqrt{8})^2} \right] \\
&= \left[\frac{(\sqrt{7}-\sqrt{8})}{(7-8)} \right] \\
&= \left[\frac{(\sqrt{7}-\sqrt{8})}{-1} \right] \\
&= -(\sqrt{7} - \sqrt{8})
\end{aligned}$$

Now,

$$\left[\frac{1}{\sqrt{8}+\sqrt{9}} \right]$$

Rationalize the denominator, we get

$$\begin{aligned}
\left[\frac{1}{\sqrt{8}+\sqrt{9}} \right] &= \left[\frac{\frac{1}{\sqrt{8}-\sqrt{9}}}{(\sqrt{8}+\sqrt{9})(\sqrt{8}-\sqrt{9})} \right] \\
&= \frac{(\sqrt{8}-\sqrt{9})}{(\sqrt{8})^2-(\sqrt{9})^2} \\
&= \left[\frac{\sqrt{8}-\sqrt{9}}{8-9} \right] \\
&= \left[\frac{(\sqrt{8}-\sqrt{9})}{-1} \right] \\
&= -(\sqrt{8} - \sqrt{9})
\end{aligned}$$

So, according to the question let us substitute the obtained values,

$$\begin{aligned}
&\left[\frac{1}{\sqrt{4}+\sqrt{5}} \right] + \left[\frac{1}{\sqrt{5}+\sqrt{6}} \right] + \left[\frac{1}{\sqrt{6}+\sqrt{7}} \right] + \left[\frac{1}{\sqrt{7}+\sqrt{8}} \right] + \left[\frac{1}{\sqrt{8}+\sqrt{9}} \right] \\
&= -(\sqrt{4} - \sqrt{5}) + -(\sqrt{5} - \sqrt{6}) + -(\sqrt{6} - \sqrt{7}) + -(\sqrt{7} - \sqrt{8}) + -(\sqrt{8} - \sqrt{9}) \\
&= -\sqrt{4} + \sqrt{5} - \sqrt{5} + \sqrt{6} - \sqrt{6} + \sqrt{7} - \sqrt{7} + \sqrt{8} - \sqrt{8} + \sqrt{9} \\
&= -\sqrt{4} + \sqrt{9} \\
&= -2 + 3 \\
&= 1
\end{aligned}$$

Question 5.

Given , Find the value of a and b, if

$$(i) \frac{[3-\sqrt{5}]}{[3+2\sqrt{5}]} = -1 \frac{19}{11} + a\sqrt{5}$$

$$(ii) \frac{[\sqrt{2}+\sqrt{3}]}{[3\sqrt{2}-2\sqrt{3}]} = a - b\sqrt{6}$$

$$(iii) \frac{[7+\sqrt{5}]}{[7-\sqrt{5}]} - \frac{[7-\sqrt{5}]}{[7+\sqrt{5}]} = a + \frac{7}{11} b\sqrt{5}$$

Solution:

$$(i) \frac{[3-\sqrt{5}]}{[3+2\sqrt{5}]} = -1 \frac{19}{11} + a\sqrt{5}$$

Let us consider LHS

$$\frac{[3-\sqrt{5}]}{[3+2\sqrt{5}]}$$

Rationalize the denominator,

$$\begin{aligned} \frac{[3-\sqrt{5}]}{[3+2\sqrt{5}]} &= \frac{\frac{[3-\sqrt{5}]}{[3-2\sqrt{5}]}}{[(3+2\sqrt{5})(3-2\sqrt{5})]} \\ &= \left[\frac{3(3-2\sqrt{5})-\sqrt{5}(3-2\sqrt{5})}{3^2-(2\sqrt{5})^2} \right] \\ &= \frac{[9-6\sqrt{5}-3\sqrt{5}+2.5]}{[9-4.5]} \\ &= \frac{[9-6\sqrt{5}-3\sqrt{5}+10]}{[9-20]} \\ &= \frac{[19-9\sqrt{5}]}{-11} \end{aligned}$$

$$= -\frac{19}{11} + \frac{9\sqrt{5}}{11}$$

So when comparing with RHS

$$-\frac{19}{11} + \frac{9\sqrt{5}}{11} = -\frac{19}{11} + a\sqrt{5}$$

$$\text{Hence, value of } a = \frac{9}{11}$$

$$(ii) \frac{[\sqrt{2}+\sqrt{3}]}{[3\sqrt{2}-2\sqrt{3}]} = \mathbf{a - b\sqrt{6}}$$

Let us consider LHS

$$\frac{[\sqrt{2}+\sqrt{3}]}{[3\sqrt{2}-2\sqrt{3}]}$$

Rationalize the denominator,

$$\frac{[\sqrt{2}+\sqrt{3}]}{[3\sqrt{2}-2\sqrt{3}]} = \left[\frac{(\sqrt{2}+\sqrt{3})(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}+2\sqrt{3})} \right]$$

$$= \frac{[\sqrt{2}(3\sqrt{2}+2\sqrt{3})+\sqrt{3}(3\sqrt{2}+2\sqrt{3})]}{[(3\sqrt{2})^2-(2\sqrt{3})^2]}$$

$$= \frac{[3.2+2\sqrt{2}\sqrt{3}+3\sqrt{2}\sqrt{3}+2.3]}{[9.2-4.3]}$$

$$= \frac{[6+2\sqrt{6}+3\sqrt{6}+6]}{[18-12]}$$

$$= \frac{[12+5\sqrt{6}]}{6}$$

$$= \frac{12}{6} + \frac{5\sqrt{6}}{6}$$

$$= 2 + \frac{5\sqrt{6}}{6}$$

$$= 2 - \left(- \frac{5\sqrt{6}}{6} \right)$$

So when comparing with RHS

$$2 - \left(- \frac{5\sqrt{6}}{6} \right) = a - b\sqrt{6}$$

Hence, value of $a = 2$ and $b = -\frac{5}{6}$

$$\text{(iii)} \left[\frac{7+\sqrt{5}}{7-\sqrt{5}} \right] - \left[\frac{7-\sqrt{5}}{7+\sqrt{5}} \right] = a + \frac{7}{11} b\sqrt{5}$$

$$\frac{[7+\sqrt{5}]}{[7-\sqrt{5}]} - \frac{[7-\sqrt{5}]}{[7+\sqrt{5}]} = a + \frac{7}{11} b\sqrt{5}$$

Let us consider LHS

Since there are two terms, let us solve individually

$$\frac{[7+\sqrt{5}]}{[7-\sqrt{5}]}$$

Rationalize the denominator,

$$\begin{aligned} \frac{[7+\sqrt{5}]}{[7-\sqrt{5}]} &= \frac{[(7+\sqrt{5})(7+\sqrt{5})]}{[(7-\sqrt{5})(7+\sqrt{5})]} \\ &= \frac{[(7+\sqrt{5})]^2}{[7^2-(\sqrt{5})^2]} \end{aligned}$$

$$= \frac{[7^2+(\sqrt{5})^2+2.7.\sqrt{5}]}{[49-5]}$$

$$= \frac{[49+5+14\sqrt{5}]}{[44]}$$

$$= \frac{[54+14\sqrt{5}]}{44}$$

Now,

$$\frac{[7-\sqrt{5}]}{[7+\sqrt{5}]}$$

Rationalize the denominator,

$$\frac{[7-\sqrt{5}]}{[7+\sqrt{5}]} = \frac{(7-\sqrt{5})(7-\sqrt{5})}{(7+\sqrt{5})(7-\sqrt{5})}$$

$$= \left[\frac{(7-\sqrt{5})^2}{(7^2-(\sqrt{5})^2)} \right]$$

$$= \frac{[7^2+(\sqrt{5})^2-2\cdot 7\cdot\sqrt{5}]}{[49-5]}$$

$$= \frac{[49+5-14\sqrt{5}]}{[44]}$$

$$= \frac{[54-14\sqrt{5}]}{44}$$

So, according to the question

$$\frac{[7+\sqrt{5}]}{[7-\sqrt{5}]} - \frac{[7-\sqrt{5}]}{[7+\sqrt{5}]}$$

By substituting the obtained values,

$$= \frac{[54+14\sqrt{5}]}{44} - \frac{[54-14\sqrt{5}]}{44}$$

$$= \frac{[54+14\sqrt{5}-54+14\sqrt{5}]}{44}$$

$$= \frac{28\sqrt{5}}{44}$$

$$= \frac{7\sqrt{5}}{11}$$

So when comparing with RHS

$$\frac{7\sqrt{5}}{11} = a + \frac{7}{11}b\sqrt{5}$$

Hence, value of $a = 0$ and $b = 1$

Question 6.

Simplify:

$$\frac{[7+3\sqrt{5}]}{[3+\sqrt{5}]} - \frac{[7-3\sqrt{5}]}{[3-\sqrt{5}]} = \mathbf{p + q\sqrt{5}}$$

Solution:

Let us consider LHS

Since there are two terms, let us solve individually

$$\frac{[7+3\sqrt{5}]}{[3+\sqrt{5}]}$$

Rationalize the denominator ,

$$\begin{aligned}\frac{[7+3\sqrt{5}]}{[3+\sqrt{5}]} &= \frac{(7+3\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})} \\&= \frac{[7(3-\sqrt{5})+3\sqrt{5}(3-\sqrt{5})]}{[3^2-(\sqrt{5})^2]} \\&= \frac{[21-7\sqrt{5}+9\sqrt{5}-3.5]}{[9-5]} \\&= \frac{[21+2\sqrt{5}-15]}{4} \\&= \frac{6+2\sqrt{5}}{4}\end{aligned}$$

$$= \frac{2[3+\sqrt{5}]}{4}$$

$$= \frac{[3+\sqrt{5}]}{2}$$

Rationalize the denominator,

$$\frac{[7-3\sqrt{5}]}{[3-\sqrt{5}]} = \frac{[(7-3\sqrt{5})(3+\sqrt{5})]}{[(3-\sqrt{5})(3+\sqrt{5})]}$$

$$= \frac{[7(3+\sqrt{5})-3\sqrt{5}(3+\sqrt{5})]}{[3^2-(\sqrt{5})^2]}$$

$$= \frac{[21+7\sqrt{5}-9\sqrt{5}-3.5]}{[9-5]}$$

$$= \frac{[21-2\sqrt{5}-15]}{4}$$

$$= \frac{[6-2\sqrt{5}]}{4}$$

$$= \frac{2[3-\sqrt{5}]}{4}$$

$$= \frac{[3-\sqrt{5}]}{2}$$

So, according to the question

$$\frac{[7+3\sqrt{5}]}{[3+\sqrt{5}]} - \frac{[7-3\sqrt{5}]}{[3-\sqrt{5}]}$$

By substituting the obtained values,

$$= \frac{[3+\sqrt{5}]}{2} - \frac{[3-\sqrt{5}]}{2}$$

$$= \frac{[3 + \sqrt{5} - 3 + \sqrt{5}]}{2}$$

$$= \frac{[2\sqrt{5}]}{2}$$

$$= \sqrt{5}$$

So when comparing with RHS

$$\sqrt{5} = p + q\sqrt{5}$$

Hence, value of $p = 0$ and $q = 1$

Question 7.

If $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$ find

(i) $\frac{\sqrt{2}}{(2 + \sqrt{2})}$

(ii) $\frac{1}{(\sqrt{3} + \sqrt{2})}$

Solution:

(i) $\frac{\sqrt{2}}{(2 + \sqrt{2})}$

By rationalizing the denominator,

$$\begin{aligned} \frac{\sqrt{2}}{(2 + \sqrt{2})} &= \frac{[\sqrt{2}(2 - \sqrt{2})]}{[(2 + \sqrt{2})(2 - \sqrt{2})]} \\ &= \frac{[2\sqrt{2} - 2]}{[2^2 - (\sqrt{2})^2]} \\ &= \frac{[2\sqrt{2} - 2]}{[4 - 2]} \end{aligned}$$

$$= \frac{2[\sqrt{2}-1]}{2}$$

$$= \sqrt{2} - 1$$

$$= 1.414 - 1$$

$$= 0.414$$

$$(ii) \frac{1}{(\sqrt{3} + \sqrt{2})}$$

By rationalizing the denominator,

$$\frac{1}{(\sqrt{3} + \sqrt{2})} = \frac{[1(\sqrt{3}-\sqrt{2})]}{[\sqrt{3}+\sqrt{2}][\sqrt{3}-\sqrt{2}]}$$

$$= \frac{[(\sqrt{3}-\sqrt{2})]}{[(\sqrt{3})^2-(\sqrt{2})^2]}$$

$$= \left[\frac{(\sqrt{3}-\sqrt{2})}{3-2} \right]$$

$$= \left(\frac{(\sqrt{3}-\sqrt{2})}{1} \right)$$

$$= 1.732 - 1.414$$

$$= 0.318$$

Question 8.

If $a = 2 + \sqrt{3}$, find $\frac{1}{a}$, $(a - \frac{1}{a})$

Solution :

Given:

$$a = 2 + \sqrt{3}$$

so,

$$\frac{1}{a} = \frac{1}{2+\sqrt{3}}$$

By rationalizing the denominator,

$$\frac{1}{(2+\sqrt{3})} = \frac{1(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$$

$$= \frac{[(2-\sqrt{3})]}{[2^2-(\sqrt{3})^2]}$$

$$= \frac{[(2-\sqrt{3})]}{[4-3]}$$

$$= (2 - \sqrt{3})$$

Then ,

$$A - \frac{1}{a} = 2 + \sqrt{3} - (2 - \sqrt{3})$$

$$= 2 + \sqrt{3} - 2 + \sqrt{3}$$

$$= 2\sqrt{3}$$

Question 9.

Solve:

$$\text{If } x = 1 - \sqrt{2}, \text{ find } \frac{1}{x}, (x - \frac{1}{x})^4$$

Solution:

Given:

$$X = 1 - \sqrt{2}$$

So,

$$\frac{1}{x} = \frac{1}{(1 - \sqrt{2})}$$

By rationalizing the denominator,

$$\frac{1}{(1 - \sqrt{2})} = \frac{[1(1 + \sqrt{2})]}{[(1 - \sqrt{2})(1 + \sqrt{2})]}$$

$$= \left[\frac{(1+\sqrt{2})}{1^2-(\sqrt{2})^2} \right]$$

$$= \left[\frac{(1+\sqrt{2})}{1-2} \right]$$

$$= \frac{(1+\sqrt{2})}{-1}$$

$$= -(1 + \sqrt{2})$$

Then ,

$$\left(x - \frac{1}{x}\right)^4 = [1 - \sqrt{2} - (-1 - \sqrt{2})]^4$$

$$= [1 - \sqrt{2} + 1 + \sqrt{2}]^2$$

$$= 2^4$$

$$= 16$$

Question : 10.

$$\text{If } x = 5 - 2\sqrt{6}, \text{ find } \frac{1}{x}, \left(x^2 - \frac{1}{x^2}\right)$$

Solution:

Given :

$$X = 5 - 2\sqrt{6}$$

So,

$$\frac{1}{x} = \frac{1}{(5 - 2\sqrt{6})}$$

By rationalizing the denominator,

$$\frac{1}{(5 - 2\sqrt{6})} = \frac{[1(5+2\sqrt{6})]}{[(5-2\sqrt{6})(5+2\sqrt{6})]}$$

$$= \frac{[(5+2\sqrt{6})]}{[5^2-(2\sqrt{6})^2]}$$

$$= \frac{[(5+2\sqrt{6})]}{[25-4.6]}$$

$$= \frac{[(5+2\sqrt{6})]}{[25-24]}$$

$$(5 + 2\sqrt{6})$$

Then ,

$$X + \frac{1}{x} = 5 - 2\sqrt{6} + (5 + 2\sqrt{6})$$

$$= 10$$

Square on both sides we get

$$\left(x + \frac{1}{x}\right)^2 = 10^2$$

$$x^2 + \frac{1^2}{x} + 2x \cdot \frac{1}{x} = 100$$

$$x^2 + \frac{1^2}{x} + 2 = 100$$

$$x^2 + \frac{1^2}{x} = 100 - 2$$

$$= 98$$

Question 11.

If $p = \frac{(2-\sqrt{5})}{(2+\sqrt{5})}$ **and** $q = \frac{(2+\sqrt{5})}{(2-\sqrt{5})}$, find the values of

(i) $p + q$

(ii) $p - q$

$$(iii) p^2 + q^2$$

$$(iv) p^2 - q^2$$

Solution:

Given :

$$P = \frac{(2 - \sqrt{5})}{(2 + \sqrt{5})} \text{ and } q = \frac{(2 + \sqrt{5})}{(2 - \sqrt{5})}$$

$$(i) p + q$$

$$\frac{(2 - \sqrt{5})}{(2 + \sqrt{5})} + \frac{(2 + \sqrt{5})}{(2 - \sqrt{5})}$$

So by rationalizing the denominator, we get

$$= \left[\frac{(2 - \sqrt{5})^2 + (2 + \sqrt{5})^2}{2^2 - (\sqrt{5})^2} \right]$$

$$= \frac{[4 + 5 - 4\sqrt{5} + 4 + 5 + 4\sqrt{5}]}{[4 - 5]}$$

$$= \frac{18}{-1}$$

$$= -18$$

$$(ii) p - q$$

$$\frac{(2 - \sqrt{5})}{(2 + \sqrt{5})} - \frac{(2 + \sqrt{5})}{(2 - \sqrt{5})}$$

So by rationalizing the denominator, we get

$$\begin{aligned}
&= \frac{(2-\sqrt{5})^2 - (2+\sqrt{5})^2}{(2^2 - (\sqrt{5})^2)} \\
&= \frac{[4+5-4\sqrt{5} - (4+5+4\sqrt{5})]}{4-5} \\
&= \frac{[9-4\sqrt{5} - 9-4\sqrt{5}]}{-1} \\
&= \frac{-8\sqrt{5}}{-1} \\
&= 8\sqrt{5}
\end{aligned}$$

(iii) $p^2 + q^2$

We know that $(p + q)^2 = p^2 + q^2 + 2pq$

So,

$$p^2 + q^2 = (p + q)^2 - 2pq$$

$$pq = \left[\frac{(2-\sqrt{5})}{(2+\sqrt{5})} \right] \times \left[\frac{(2+\sqrt{5})}{(2-\sqrt{5})} \right]$$

$$= 1$$

$$p + q = -18$$

So,

$$p^2 + q^2 = (p + q)^2 - 2pq$$

$$= (-18)^2 - 2(1)$$

$$= 324 - 2$$

$$= 322$$

$$(iv) p^2 - q^2$$

We know that, $p^2 - q^2 = (p + q)(p - q)$

So, by substituting the values

$$p^2 - q^2 = (p + q)(p - q)$$

$$= (-18)(8\sqrt{5})$$

$$= -144\sqrt{5}$$

Question 12.

If $x = \frac{(\sqrt{2} - 1)}{(\sqrt{2} + 1)}$ and $y = \frac{(\sqrt{2} + 1)}{(\sqrt{2} - 1)}$ find

(i) $x + y$

(ii) xy

Solution:

Given :

$$x = \frac{(\sqrt{2} - 1)}{(\sqrt{2} + 1)} \text{ and } y = \frac{(\sqrt{2} + 1)}{(\sqrt{2} - 1)}$$

(i) $x + y$

$$= \left[\frac{(\sqrt{2} - 1)}{(\sqrt{2} + 1)} \right] + \left[\frac{(\sqrt{2} + 1)}{(\sqrt{2} - 1)} \right]$$

By rationalizing the denominator

$$= \left[\frac{(\sqrt{2} - 1)^2 + (\sqrt{2} + 1)^2}{(\sqrt{2})^2 - 1^2} \right]$$

$$= \frac{[2+1-2\sqrt{2}+2+1+2\sqrt{2}]}{[2-1]}$$

$$= \frac{6}{1}$$

$$= 6$$

(ii) xy

$$\left[\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)} \right] \times \left[\frac{(\sqrt{2}+1)}{(\sqrt{2}-1)} \right]$$

$$= 1$$