

Chapter 3 Expansions

Exercise – 3.1

1.

Solution

(i) $(2x + 7y)^2$

It is in the form of $(a + b)^2 = a^2 + 2ab + b^2$

$$\therefore a = 2x ; b = 7y$$

$$\begin{aligned}\therefore (2x + 7y)^2 &= (2x)^2 + 2 \cdot 2x \cdot 7y + (7y)^2 \\ &= 4x^2 + 28xy + 49y^2\end{aligned}$$

(ii) $\left(\frac{1}{2}x + \frac{2}{3}y\right)^2$

$$\Rightarrow \left(\frac{1}{2}x\right)^2 + 2 \cdot \frac{1}{2} \cdot x \cdot \frac{2}{3} \cdot y + \left(\frac{2}{3}y\right)^2$$

$$\Rightarrow \frac{x^2}{4} + \frac{2}{3}xy + \frac{4}{9}y^2$$

2.

Solution

(i) $\left(3x + \frac{1}{2}x\right)^2$

It is in the form of $(a+b)^2 = a^2 + 2ab + b^2$

$$\therefore (3x)^2 + 2 \cdot 3x \cdot \frac{1}{2x} + \left(\frac{1}{2x}\right)^2$$

$$\Rightarrow 9x^2 + 3 + \frac{1}{4x^2}$$

(ii) $(3x^2y + 5z)^2$

It is in the form of $(a+b)^2 = a^2 + 2ab + b^2$

Here $a = 3x^2y$; $b = 5z$

$$\Rightarrow (3x^2y)^2 + 2 \cdot 3x^2y \cdot 5z + (5z)^2$$

$$\Rightarrow 9x^4y^2 + 30x^2yz + 25z^2$$

3.

Solution

(i) $\left(3x - \frac{1}{2x}\right)^2$

It is in the form of $(a-b)^2 = a^2 - 2ab + b^2$

Here $a = 3x$; $b = \frac{1}{2x}$

$$\Rightarrow (3x)^2 - 2 \cdot 3x \cdot \frac{1}{2x} + \left(\frac{1}{2x}\right)^2$$

$$\Rightarrow 3^2 - x^2 - 3 + \frac{1}{2^2x^2}$$

$$\Rightarrow 9x^2 - 3 + \frac{1}{4x^2}$$

$$(ii) \left(\frac{1}{2}x - \frac{3}{2}y\right)^2$$

It is in the form of $(a - b)^2 = a^2 - 2ab + b^2$

$$\text{Here } a = \frac{1}{2}x \quad ; \quad b = \frac{3}{2}y$$

$$\therefore \left(\frac{1}{2}x\right)^2 - 2 \cdot \frac{1x}{2} \cdot \frac{3}{2}y + \left(\frac{3}{2}y\right)^2$$

$$\Rightarrow \frac{x^2}{4} - \frac{3}{2}xy + \frac{9y^2}{4}$$

4.

Solution

$$(i) (x + 3)(x + 5)$$

$$= x(x + 5) + 3(x + 5)$$

$$= x^2 + 5x + 3x + 15$$

$$= x^2 + 8x + 15$$

$$(ii) (x + 3)(x - 5)$$

$$= x(x - 5) + 3(x - 5)$$

$$= x \cdot x - x \cdot 5 + 3 \cdot x - 3 \cdot 5$$

$$= x^2 - 5x + 3x - 15$$

$$= x^2 - 2x - 15$$

$$\text{(iii) } (x - 7)(x + 9)$$

$$= x(x + 9) - 7(x + 9)$$

$$= x \cdot x + 9 \cdot x - 7 \cdot x - 7 \cdot 9$$

$$= x^2 + 9x - 7x - 63$$

$$= x^2 + 2x - 63$$

$$\text{(iv) } (x - 2y)(x - 3y)$$

$$= x(x - 3y) - 2y(x - 3y)$$

$$= x \cdot x - x \cdot 3y - 2y \cdot x + 2y \cdot 3y$$

$$= x^2 - 3xy - 2xy + 6y^2$$

$$= x^2 - 5xy + 6y^2$$

5.

Solution

$$\text{(i) } (x - 2y - z)^2$$

It is in the form of $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

There $a = x$; $b = -2y$; $c = -z$

$$\therefore \Rightarrow x^2 + (-2y)^2 + (-z)^2 + 2(x(-2y) + (-2y)(-z) + (-z) \cdot x)$$

$$\Rightarrow x^2 + 4y^2 + z^2 + 2(-2xy + 2yz - zx)$$

$$\Rightarrow x^2 + 4y^2 + z^2 + 4yz - 4xy - 2zx$$

(ii) $(2x - 3y + 4z)^2$

It is in the form of $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Here $a = 2x$; $b = -3y$; $c = 4z$

$$\therefore \Rightarrow (2x)^2 + (-3y)^2 + (4z)^2 + 2(2x \cdot (-3y) + (-3y)(4z) + 4z \cdot 2x)$$

$$\Rightarrow 4x^2 + 9y^2 + 16z^2 + 2(-6xy - 12yz + 8xz)$$

$$\Rightarrow 4x^2 + 9y^2 + 16z^2 - 12xy - 24yz + 16xz$$

6.

Solution

(i) $\left[2x + \frac{3}{x} - 1\right]^2$

It is in the form of $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Here $a = 2x$; $b = \frac{3}{x}$; $c = -1$

$$\therefore \Rightarrow (2x)^2 + \left(\frac{3}{x}\right)^2 + (-1)^2 + 2\left(2x \cdot \frac{3}{x} + \frac{3}{x}(-1) + (-1) \cdot 2x\right)$$

$$\Rightarrow 4x^2 + \frac{9}{x^2} + 1 + 2\left(6 - \frac{3}{x} - 2x\right)$$

$$\Rightarrow 4x^2 + \frac{9}{x^2} + 1 + 12 - \frac{6}{x} - 4x$$

$$\Rightarrow 4x^2 + \frac{9}{x^2} - \frac{6}{x} - 4x + 13$$

$$(ii) \left(\frac{2}{3}x - \frac{3}{2x} - 1 \right)^2$$

It is in the form of $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$$\text{Here } a = \frac{2}{3}x ; b = -\frac{3}{2x} ; c = -1$$

$$\begin{aligned} \therefore &\Rightarrow \left(\frac{2}{3}x \right)^2 + \left(-\frac{3}{2x} \right)^2 + (-1)^2 + 2 \left[\frac{2}{3}x \left(-\frac{3}{2x} \right) + \right. \\ &\left. \left(-\frac{3}{2x} \right) \cdot (-1)(-1) \left(\frac{2}{3}x \right) \right] \\ &\Rightarrow \frac{4}{9}x^2 - \frac{9}{4x^2} + 1 - 2 + \frac{6}{2x} - \frac{4x}{3} \end{aligned}$$

$$\Rightarrow \frac{4}{9}x^2 - \frac{9}{4x^2} + \frac{3}{x} - \frac{4x}{3} - 1$$

7.

Solution

$$(i) (x + 3)^3$$

It is in the form of $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$$\text{Here } a = x ; b = 3$$

$$\therefore \Rightarrow x^3 + 3.x^2 . 3 + 3.x.3^2 + 3^3$$

$$\Rightarrow x^3 + 9x^2 + 27x + 27$$

$$\Rightarrow x^3 + 9x^2 + 27x + 27$$

$$\text{(ii) } (2a + b)^3$$

$$\Rightarrow (2a)^3 + 3.(2a)^2.b + 3.2a.b^2 + b^3$$

$$\Rightarrow 8a^3 + 3.4a^2.b + 6ab^2 + b^3$$

$$\Rightarrow 8a^3 + 12a^2b + 6ab^2 + b^3$$

8.

Solution

$$\text{(i) } \left(3x + \frac{1}{x}\right)^3$$

It is in the form of $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$$a = 3x ; b = \frac{1}{x}$$

$$\therefore \Rightarrow (3x)^3 + 3.(3x)^2 \cdot \frac{1}{x} + 3.3x \cdot \left(\frac{1}{x}\right)^2 + \left(\frac{1}{x}\right)^3$$

$$\Rightarrow 27x^3 + 3.9x^2 \cdot \frac{1}{x} + 9x \cdot \frac{1}{x^2} + 9x \cdot \frac{1}{x^2} + \frac{1}{x^3}$$

$$\Rightarrow 27x^3 + 27x + \frac{9}{x} + \frac{1}{x^3}$$

$$\text{(ii) } (2x-1)^3$$

It is in the form of $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Here $a = 2x ; b = 1$

$$\therefore \Rightarrow (2x)^3 - 3.(2x)^2 \cdot 1 + 3(2x)(1)^2 - (1)^3$$

$$\Rightarrow 8x^3 - 3.4x^2 + 6x - 1$$

$$\Rightarrow 8x^3 - 12x^2 + 6x - 1$$

9.

Solution

(i) $(5x - 3y)^3$

It is in the form of $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$$a = 5x ; b = 3y$$

$$\therefore \Rightarrow (5x)^3 - 3.(5x)^2 . 3y + 3.5x . (3y)^2 - (3y)^3$$

$$\Rightarrow 125x^3 - 3.25x^2 . 3y + 3.5x . 9y^2 - 27y^3$$

$$\Rightarrow 125x^3 - 225x^2y + 135y^2 . X - 27y^3$$

(ii) $\left(2x - \frac{1}{3y}\right)^3$

$$\therefore \Rightarrow (2x)^3 - 3.(2x)^2 . \frac{1}{3y} + 3.2x . \left(\frac{1}{3y}\right)^2 - \left(\frac{1}{3y}\right)^3$$

$$\Rightarrow 8x^3 - 3.4x^2 . \frac{1}{3y} + 3.2x \frac{1}{9y^2} - \frac{1}{27y^3}$$

$$\Rightarrow 8x^3 - \frac{4x^2}{y} + \frac{2x}{3y^2} - \frac{1}{27y^3}$$

10.

Solution

(i) $(a + b)^2 + (a - b)^2$

$$\Rightarrow a^2 + 2ab + b^2 + a^2 - 2ab + b^2$$

$$\Rightarrow 2a^2 + 2b^2$$

$$\Rightarrow 2(a^2 + b^2)$$

$$\text{(ii)} \quad (\mathbf{a+b})^2 - (\mathbf{a-b})^2$$

$$\Rightarrow (a^2 + 2ab + b^2) - (a^2 - 2ab + b^2)$$

$$\Rightarrow a^2 + 2ab + b^2 - a^2 + 2ab - b^2$$

$$\Rightarrow 2ab + 2ab$$

$$\Rightarrow 4ab$$

11.

Solution

$$\text{(i)} \quad \left(a + \frac{1}{a}\right)^2 + \left(a - \frac{1}{a}\right)^2$$

$$\Rightarrow \left(a^2 + 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2}\right) + \left(a^2 + 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2}\right)$$

$$\Rightarrow a^2 + 2 + \frac{1}{a^2} + a^2 - 2 + \frac{1}{a^2}$$

$$\Rightarrow 2a^2 + \frac{2}{a^2}$$

$$\Rightarrow 2\left(a^2 + \frac{2}{a^2}\right)$$

$$\text{(ii)} \quad \left(a + \frac{1}{a}\right)^2 - \left(a - \frac{1}{a}\right)^2$$

$$\Rightarrow \left(a^2 + 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2}\right) - \left(a^2 - 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2}\right)$$

$$\Rightarrow a^2 + 2 + \frac{1}{a^2} - a^2 + 2 - \frac{1}{a^2}$$

$$\Rightarrow 2 + 2$$

$$\Rightarrow 4$$

12.

Solution

(i) $(3x - 1)^2 - (3x - 2)(3x + 1)$

$$\Rightarrow (3x)^2 - 2 \cdot 3x \cdot 1 + 1^2 - 3x(3x + 1) + 2(3x + 1)$$

$$\Rightarrow 9x^2 - 6x + 1 - 9x^2 - 3x + 6x + 2$$

$$\Rightarrow 3x + 3$$

$$\Rightarrow 3(x + 1)$$

(ii) $(4x + 3y)^2 - (4x - 3y)^2 - 48$

$$\Rightarrow (4x)^2 + 2 \cdot 3y \cdot 4x + (3y)^2 - ((4x)^2 - 2 \cdot 4x \cdot 3y + (3y)^2) - 48$$

$$\Rightarrow 16x^2 + 24xy + 9y^2 - 16yx^2 + 24xy - 9y^2 - 48$$

$$\Rightarrow 48xy - 48$$

$$\Rightarrow 48(xy - 1)$$

13.

Solution

(i) $(7p + 9q)(7p - 9q)$

$$\Rightarrow 7p(7p - 9q) + 9q(7p - 9q)$$

$$\Rightarrow 49p^2 - 63pq + 63pq - 81q^2$$

$$\Rightarrow 49p^2 - 81q^2$$

$$(ii) \left(2x - \frac{3}{x}\right) \left(2x + \frac{3}{x}\right)$$

$$\Rightarrow (2x)^2 - \left(\frac{3}{x}\right)^2$$

Since it is in the form of $(a+b)(a-b) = a^2 - b^2$

$$\Rightarrow 4x^2 - \frac{9}{x^2}$$

14.

Solution

$$(i) (2x - y + 3) (2x - y - 3)$$

$$((2x - y) + 3) ((2x - y) - 3)$$

It is in the form of $(a + b)(a - b) = a^2 - b^2$

$$\Rightarrow (2x - y)^2 - 3^2$$

$$\Rightarrow (2x)^2 - 2 \cdot 2x \cdot y + y^2 - 9$$

$$\Rightarrow 4x^2 - 4xy + y^2 - 9$$

$$(ii) (3x + y - 5) (3x - y - 5)$$

$$\Rightarrow (3x + (y - 5)) (3x - (y + 5))$$

$$\Rightarrow [(3x - 5) + y] [(3x - 5) - y]$$

It is in the form of $(a + b)(a - b) = a^2 - b^2$

$$\because a = 3x - 5 ; b = y$$

$$\therefore (3x - 5)^2 - y^2$$

$$\Rightarrow (3x)^2 - 2 \cdot 3x \cdot 5 + 5^2 - y^2$$

$$\Rightarrow 9x^2 - 30x + 25 - y^2$$

15.

Solution

$$(i) \left(x + \frac{2}{x} - 3\right) \left(x - \frac{2}{x} - 3\right)$$

$$\Rightarrow \left((x - 3) + \frac{2}{x}\right) \left((x - 3) - \frac{2}{x}\right)$$

It is in the form of $(a + b)(a - b) = a^2 - b^2$

$$\because a = x - 3 ; b = \frac{2}{x}$$

$$\Rightarrow (x-3)^2 - \left(\frac{2}{x}\right)^2$$

$$\Rightarrow x^2 - 2 \cdot x \cdot 3 + 3^2 - \frac{4}{x^2}$$

$$\Rightarrow x^2 - 6x + 9 - \frac{4}{x^2}$$

$$(ii) (5 - 2x)(5 + 2x) (25 + 4x^2)$$

It is in the form of $(a + b)(a - b) = a^2 - b^2$

$$\because (5^2 - (2x)^2) (25 + 4x^2)$$

$$\Rightarrow (25 - 4x^2) (25 + 4x^2)$$

$$\Rightarrow (25)^2 - (4x^2)^2$$

$$\Rightarrow 625 - 16x^4$$

16.

Solution

(i) $(x + 2y + 3) (2y + x + 7)$

$$\Rightarrow x(2y + x + 7) + 2y(2y + x + 7) + 3(2y + x + 7)$$

$$\Rightarrow 2xy + x^2 + 7x + 4y^2 + 2xy + 14y + 6y + 3x + 21$$

$$\Rightarrow x^2 + 4y^2 + 4xy + 10x + 20y + 21$$

(ii) $(2x + y + 5) (2x + y - 9)$

$$\Rightarrow 2x(2x + y - 9) + y(2x + y - 9) + 5(2x + y - 9)$$

$$\Rightarrow 4x^2 + 2xy - 18x + 2xy + y^2 - 9y + 10x + 5y - 45$$

$$\Rightarrow 4x^2 + y^2 + 4xy - 8x - 4y - 45$$

(iii) $(x - 2y - 5) (x - 2y + 3)$

$$\Rightarrow x(x - 2y + 3) - 2y(x - 2y + 3) - 5(x - 2y + 3)$$

$$\Rightarrow x^2 - 2xy + 3x - 2xy + 4y^2 - 6y - 5x + 10y - 15$$

$$\Rightarrow x^2 + 4y^2 - 4xy - 2x - 4y - 15$$

(iv) $(3x - 4y - 2) (3x - 4y - 6)$

$$\Rightarrow 3x(3x - 4y - 6) - 4y(3x - 4y - 6) - 2(3x - 4y - 6)$$

$$\Rightarrow 9x^2 - 12xy - 18x - 12xy + 16y^2 + 24y - 6x + 8y + 12$$

$$\Rightarrow 9x^2 + 16y^2 - 24xy - 24x + 32y + 12$$

17.

Solution

(i) $(2p + 3q) (4p^2 - 6pq + 9q^2)$

$$(2p + 3q) ((2p)^2 - 2p \cdot 3q + (3q)^2)$$

It is in the form of $(a+b) (a^2 - ab + b^2)$ is $a^3 + b^3$

$$\therefore \text{ here } a = 2p \text{ ; } b = 3q$$

$$\therefore (2p)^3 + (3q)^3$$

$$\Rightarrow 8p^3 + 27q^3$$

(ii) $\left(x + \frac{1}{x}\right) \left(x^2 - 1 + \frac{1}{x^2}\right)$

It is in the form of $(a + b) (a^2 - ab + b^2)$ is $a^3 + b^3$

$$\therefore \text{ here } a = x \text{ ; } b = \frac{1}{x}$$

$$\therefore x^3 + \frac{1}{x^3}$$

18.

Solution

(i) $(3p - 4q) (9p^2 + 12pq + 16q^2)$

$$(3p - 4q) ((3p)^2 + 3p \cdot 4q + (4q)^2)$$

It is in the form of $(a - b)(a^2 + ab + b^2)$ is $a^3 - b^3$

$$\therefore \text{ here } 3p = a ; b = 4q$$

$$\therefore (3p)^3 - (4q)^3$$

$$\Rightarrow 27p^3 - 64q^3$$

$$(ii) \left(x - \frac{3}{x}\right) \left(x^2 + 3 + \frac{9}{x^2}\right)$$

$$\therefore \left(x - \frac{3}{x}\right) \left(x^2 + x\frac{3}{x} + \left(\frac{3}{x}\right)^2\right)$$

It is in the form of $(a - b)(a^2 + ab + b^2)$ is $a^3 - b^3$

$$\therefore \text{ here } a = x ; b = \frac{3}{x}$$

$$\therefore x^3 - \left(\frac{3}{x}\right)^3$$

$$\Rightarrow x^3 - \frac{27}{x^3}$$

19.

Solution

$$\text{Given } (2x + 3y + 4z) (4x^2 + 9y^2 + 16z^2 - 6xy - 12yz - 8zx)$$

$$\Rightarrow (2x + 3y + 4z) ((2x)^2 + (3y)^2 + (4z)^2 - 2x \cdot 3y - 3y \cdot 4z - 4z \cdot 2x)$$

$\therefore$ it is in the form of

$$(a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc$$

$$\therefore \text{ here } a = 2x ; b = 3y ; c = 4z$$

$$\therefore (2x)^3 + (3y)^3 + (4z)^3 - 3 \cdot 2x \cdot 3y \cdot 4z$$

$$\Rightarrow 8x^3 + 27y^3 + 64z^3 - 72xyz$$

20.

Solution

(i) $(x + 1)(x + 2)(x + 3)$

$$[x(x+2) + 1(x+2)](x+3)$$

$$\Rightarrow (x^2 + 2x + x + 2)(x + 3)$$

$$\Rightarrow (x^2 + 3x + 2)(x + 3)$$

$$\Rightarrow (x^2 + 3x + 2)(x) + (x^2 + 3x + 2)3$$

$$\Rightarrow x^3 + 5x^2 + 2x + 3x^2 + 9x + 6$$

$$\Rightarrow x^3 + 6x^2 + 11x + 6$$

(ii) $(x - 2)(x - 3)(x + 4)$

$$\Rightarrow [x(x - 3) - 2(x - 3)](x + 4)$$

$$\Rightarrow (x^2 - 3x - 2x + 6)(x + 4)$$

$$\Rightarrow (x^2 - 5x + 6)(x + 4)$$

$$\Rightarrow (x^2 - 5x + 6)x + (x^2 - 5x + 6)4$$

$$\Rightarrow x^3 - 5x^2 + 6x + 4x^2 - 20x + 24$$

$$\Rightarrow x^3 - x^2 - 14x + 24$$

21.

Solution

Given $(x - 3)(x + 7)(x - 4)$

$$(x(x + 7) - 3(x + 7))(x - 4)$$

$$(x^2 + 7x - 3x - 21)(x - 4)$$

$$(x^2 + 4x - 21)(x - 4)$$

$$(x^2 + 4x - 21)x - 4(x^2 + 4x - 21)$$

$$x^3 + 4x^2 - 21x - 4x^2 - 16x + 84$$

$$x^3 - 37x + 84$$

$\therefore$ the coefficient of x^2 is 0

The coefficient of x is -37

22.

Solution

$$\text{Given } a^2 + 4a + x = (a + 2)^2$$

$$\therefore a^2 + 4a + x = a^2 + 2.a.2 + 2^2$$

$$a^2 + 4a + x = a^2 + 4a + 4$$

$$\Rightarrow x = a^2 + 4a + 4 - a^2 - 4a$$

$$X = 4$$

23.

Solution

$$(i) (101)^2$$

$$\Rightarrow (100 + 1)^2$$

$$\Rightarrow (100)^2 + 2.100.1 + 1^2$$

$$\Rightarrow 10000 + 200 + 1$$

$$\Rightarrow 10201$$

$$\textbf{(ii) } (1003)^2$$

$$\Rightarrow (1000 + 3)^2$$

$$\Rightarrow (1000)^2 + 2.1000.3 + 3^2$$

$$\Rightarrow 1000000 + 6000 + 9$$

$$\Rightarrow 1006009$$

$$\textbf{(iii) } (10.2)^2$$

$$(10 + 0.2)^2$$

$$(10)^2 + 2 \times 10 \times 0.2 + (0.2)^2$$

$$100 + 4 + 0.04$$

$$104.04$$

24.

Solution

$$\textbf{(i) } 99^2$$

$$\Rightarrow (100 - 1)^2$$

$$\Rightarrow (100)^2 - 2.100.1 + 1^2$$

$$\Rightarrow 10000 - 200 + 1$$

$$\Rightarrow 9801$$

$$\text{(ii) } (997)^2$$

$$\Rightarrow (1000 - 3)^2$$

$$\Rightarrow 1000^2 - 2 \cdot 1000 \cdot 3 + 3^2$$

$$\Rightarrow 1000000 - 6000 + 9$$

$$\Rightarrow 994009$$

In this we used the $(a - b)^2$ formulae i.e, $a^2 - 2ab + b^2$

$$\text{(iii) } (9.8)^2$$

$$\Rightarrow (10 - 0.2)^2$$

$$\Rightarrow 10^2 - 2 \times 10 \times 0.2 + (0.2)^2$$

$$\Rightarrow 100 - 4 + 0.4$$

$$\Rightarrow 96.04$$

25.

Solution

$$\text{(i) } (103)^2$$

$$\Rightarrow (100 + 3)^2$$

$\therefore$ it is in the form of

$$(a + b)^2 = a^2 + 2ab + b^2$$

$\therefore$ here $a = 100$; $b = 3$

$$\Rightarrow (100)^3 + 3.(100)^2.3 + 3100 . 3^2 + 3^3$$

$$\Rightarrow 1000000 + 90000 + 2700 + 27$$

$$\Rightarrow 1092727$$

$$\textbf{(ii) } 99^3$$

$$\Rightarrow (100 - 1)^3$$

$$\Rightarrow 100^3 - 3.100^2 . 1 + 3.100.1^2 - 1^3$$

$$\Rightarrow 1000000 - 30000 + 300 - 1$$

$$\Rightarrow 970299$$

$$\textbf{(iii) } (10.1)^3$$

$$\Rightarrow (10 + 0.1)^3$$

$$\Rightarrow 10^3 + 3.10^2.(0.1) + 3.10(0.1)^2 + (0.1)^3$$

$$\Rightarrow 1000 + 30 + 3 + 0.01$$

$$\Rightarrow 1030.301$$

26.

Solution

$$\text{Given } 2a - b + c = 0$$

$$\Rightarrow (2a + c) = b$$

Squaring on both sides

$$\Rightarrow (2a + c)^2 = b^2$$

$$\Rightarrow (2a)^2 + 2 \cdot 2a \cdot c + c^2 = b^2$$

$$\Rightarrow 4a^2 + 4ac + c^2 = b^2$$

$$\Rightarrow 4a^2 - b^2 + c^2 + 4ac = 0$$

Hence proved

27.

Solution

Given $a + b + 2c = 0$

$$a + b = -2c \dots(i)$$

cubing on both sides

$$(a+b)^3 = (-2c)^3$$

$$a^3 + b^3 + 3a^2b + 3ab^2 = -8c^3$$

$$a^3 + b^3 + 3ab(a + b) = -8c^3 \dots \text{from (i)}$$

$$a^3 + b^3 + 3ab(-2c) = -8c^3$$

$$a^3 + b^3 - 6abc = -8c^3$$

$$a^3 + b^3 + 8c^3 = 6abc$$

hence proved

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Solution

Given $a + b + c = 0$

$$a + b = -c \dots(i)$$

cubing on both sides

$$(a + b)^3 = (-c)^3$$

$$\Rightarrow a^3 + b^3 + 3a^2b + 3ab^2 = -c^3$$

$$\Rightarrow a^3 + b^3 + 3ab(a + b) = -c^3$$

$$\Rightarrow a^3 + b^3 + 3ab(-c) = -c^3$$

$$\Rightarrow a^3 + b^3 - 3abc = -c^3$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

$$\Rightarrow \frac{a^3 + b^3 + c^3}{abc} = 3$$

$$\Rightarrow \frac{a^3}{abc} + \frac{b^3}{abc} + \frac{c^3}{abc} = 3$$

$$\Rightarrow \frac{a^2}{bc} + \frac{b^2}{ac} + \frac{c^2}{ab} = 3$$

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Solution

Given $x + y = 4$

Cubing on both sides

$$(x + y)^3 = 4^3$$

$$\Rightarrow x^3 + 3x^2y + 3xy^2 + y^3 = 64$$

$$\Rightarrow x^3 + 3xy(x + y) + y^3 = 64$$

$$\Rightarrow x^3 + 3xy(4) + y^3 = 64$$

$$\Rightarrow x^3 + 12xy + y^3 = 64$$

$$\Rightarrow x^3 + y^3 + 12xy - 64 = 0$$

30

Solution

(i) $(27)^3 + (-17)^3 + (-10)^3$

$\therefore$ if $a + b + c = 0$; then $a^3 + b^3 + c^3 = 3abc$

$\therefore$ here $a = 27$

$$b = -17$$

$$c = -10$$

$\therefore 27 - 17 - 10 = 0$

$\therefore a^3 + b^3 + c^3 = 3abc$

$$= 3 \cdot 27 \cdot (-17) \cdot (-10)$$

$$= 13770$$

(ii) $(-28)^3 + 15^3 + 13^3$

$\therefore$ if $a + b + c = 0$; then $a^3 + b^3 + c^3 = 3abc$

$\Rightarrow -28 + 15 + 13 = 0$

$\therefore a^3 + b^3 + c^3 = 3abc$

$$= 3(-28)(15)(13)$$

$$= - 16380$$

31.

Solution

Given $\frac{86 \times 86 \times 86 + 14 \times 14 \times 14}{86 \times 86 - 86 \times 14 + 14 \times 14}$

$\therefore$ It is in the form of $\frac{a^3+b^3}{a^2-ab+b^2} = (a+b)$

$$\therefore \frac{86^3+14^3}{86^2-86 \cdot 14+14^2} = 86+14$$

$$= 100$$

Exercise 3.2

1.

Solution

Given $x - y = 8 \dots (i)$

$$xy = 5 \dots (ii)$$

squaring on both sides in equal (i)

$$\therefore (x - y)^2 = 8^2$$

$$X^2 - 2.xy + y^2 = 64$$

$$X^2 - 2(5) + y^2 = 64 \quad \therefore \text{from eg (ii)}$$

$$X^2 - 10 + y^2 = 64$$

$$X^2 + y^2 = 64 + 10$$

$$X^2 + y^2 = 74$$

2.

Solution

Given $x + y = 10 \dots (i)$

$$xy = 21 \dots (ii)$$

squaring on both sides in equal(i)

$$(x + y)^2 = 10^2$$

$$x^2 + 2xy + y^2 = 100$$

$$x^2 + 2(21) + y^2 = 100$$

$$x^2 + 4^2 + y^2 = 100$$

$$x^2 + y^2 = 100 - 42$$

$$x^2 + y^2 = 58$$

$$2(x^2 + y^2) = 2 \times 58 = 116$$

3.

Solution

Given $2a + 3b = 7$ (i)

$$ab = 2$$
(ii)

squaring on both sides in equal(i)

$$(2a + 3b)^2 = 7^2$$

$$(2a)^2 + 2 \cdot 2a \cdot 3b + (3b)^2 = 49$$

$$4a^2 + 12ab + 9b^2 = 49$$

$$4a^2 + 12(2) + 9b^2 = 49$$

$$4a^2 + 24 + 9b^2 = 49$$

$$4a^2 + 9b^2 = 49 - 24$$

$$4a^2 + 9b^2 = 25$$

4.

Solution

Given $3x - 4y = 16$ (i)

$$xy = 4$$
(ii)

squaring on both sides in equal(i)

$$(3x - 4y)^2 = 16^2$$

$$(3x)^2 - 2 \cdot 3x \cdot 4y + (4y)^2 = 256$$

$$9x^2 - 24xy + 16y^2 = 256$$

$$9x^2 - 24(4) + 16y^2 = 256$$

$$9x^2 - 96 + 16y^2 = 256$$

$$9x^2 + 16y^2 = 256 + 96$$

$$9x^2 + 16y^2 = 352$$

5.

Solution

Given $x + y = 8 \dots (i)$

$$x - y = 2 \dots (ii)$$

since we know that

$$2(x^2 + y^2) = (x + y)^2 + (x - y)^2$$

From (i) squaring on both sides

$$(x + y)^2 = 8^2$$

$$= 64$$

From (ii) squaring on both sides

$$(x - y)^2 = 2^2$$

$$= 4$$

$$\therefore 2(x^2 + y^2) = (x + y)^2 + (x - y)^2$$

$$= 64 + 4$$

$$= 68$$

6.

Solution

Given $a^2 + b^2 = 13 \dots (i)$

$$ab = 6 \dots (ii)$$

(i) $a + b$

$$\therefore (a + b)^2 = a^2 + 2ab + b^2$$

$$= a^2 + b^2 + 2ab$$

$$= 13 + 2(6)$$

$$= 13 + 12$$

$$(a + b)^2 = 25$$

$$a + b = \sqrt{25}$$

$$a + b = 5$$

(ii) $a - b$

$$\therefore (a - b)^2 = a^2 - 2ab + b^2$$

$$= a^2 + b^2 - 2ab$$

$$= 13 - 2(6)$$

$$= 13 - 12$$

$$(a - b)^2 = 1$$

$$(a - b) = \sqrt{1}$$

$$a - b = 1$$

7.

Solution

$$\text{Given } a + b = 4 \dots\dots(i)$$

$$ab = -12\dots\dots(ii)$$

(i) a- b

From (i) and (2)

$$ab = -12$$

$$a(4 - a) = -12$$

$$4a - a^2 = -12$$

$$a^2 - 12 - 4a = 0$$

$$a^2 - 4a - 12 = 0$$

$$a^2 - 6a + 2a - 12 = 0$$

$$a(a-6) + 2(a -b) = 0$$

$$(a - 6) (a+2) = 0$$

$$a - 6 = 0 \quad ; \quad a + 2 = 0$$

$$a = 6 \quad \quad \quad a = -6$$

$$a + b = 4$$

$$a + b = 4$$

$$6 + b = 4$$

$$-2 + b = 4$$

$$b = 4 - 6$$

$$b = 4 + 2$$

$$b = -2$$

$$b = 6$$

$$(a, b) = (6, -2)$$

$$(i) \ a - b = 6 - (-2) = 8$$

$$(ii) \ a^2 - b^2 = (a + b)(a - b)$$

$$= (4)(8)$$

$$= 32$$

8.

Solution

$$\text{Given } p - q = 9 \dots (i)$$

$$pq = 36 \dots (ii)$$

from (i) and (ii)

$$p = q + 9$$

$$\therefore pq = 36$$

$$(q + 9)q = 36$$

$$q^2 + 9q = 36$$

$$q^2 + 9q - 36 = 0$$

$$q^2 + 19q - 3q - 36 = 0$$

$$q(q + 12) - 3(q + 12) = 0$$

$$(q + 12)(q - 3) = 0$$

$$q + 12 = 0$$

$$q = -12$$

$$q - 3 = 0$$

$$q = 3$$

$$p - q = 9$$

$$p - (-12) = 9$$

$$p = 9 - 12$$

$$p = -3$$

$$p = 12 ; q = 3$$

$$p - q = 9$$

$$p - 3 = 9$$

$$p = 9 + 3$$

$$p = 12$$

$$(i) p + q$$

$$12 + 3$$

$$= 15$$

$$(ii) p^2 - q^2$$

$$= (p + q) (p - q)$$

$$= 15 \times 9$$

$$= 135$$

9

Solution

$$\text{Given } x + y = 6 \dots\dots(i)$$

$$x - y = 4 \dots\dots(ii)$$

from (i) squaring on both sides

$$(x + y)^2 = 6^2$$

$$x^2 + y^2 + 2xy = 36$$

$$x^2 + y^2 = 36 - 2xy \dots\dots(\text{iii})$$

from (i) squaring on both sides

$$(x - y)^2 = 4^2$$

$$x^2 - 2xy + y^2 = 16$$

$$x^2 + y^2 = 16 + 2xy \dots\dots(\text{iv})$$

$\therefore$ equate equation (iii) and (iv)

$$36 - 2xy = 16 + 2xy$$

$$36 - 16 = 2xy + 2xy$$

$$20 = 4xy$$

$$4xy = 20$$

$$xy = \frac{20}{4}$$

$$xy = 5$$

(i) $x^2 + y^2$

From equation (iii) $x^2 + y^2 = 36 - 2xy$

$$= 36 - 2(5)$$

$$= 36 - 10 = 26$$

(ii) $xy = 5$

10.

Solution

Given $x - 3 = \frac{1}{x}$

$$x - \frac{1}{x} = 3$$

squaring on both sides

$$\left(x - \frac{1}{x}\right)^2 = 3^2$$

$$x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 9$$

$$x^2 - 2 + \frac{1}{x^2} = 9$$

$$x^2 + \frac{1}{x^2} = 9 + 2$$

$$x^2 + \frac{1}{x^2} = 11$$

11.

Solution

Given $x + y = 8 \dots\dots(i)$

$$xy = 3\frac{3}{4} = \frac{15}{4} \dots\dots(ii)$$

from equation (i) $= y = 8 - x$

from equation (ii) $= xy = \frac{15}{4}$

$$x(8 - x) = \frac{15}{4}$$

$$8x - x^2 = \frac{15}{4}$$

$$4(8x - x^2) = 15$$

$$32x - 4x^2 = 15$$

$$4x^2 - 32x + 15 = 0$$

$$4x^2 - 30x - 2x + 15 = 0$$

$$2x(2x - 15) - 1(2x - 15) = 0$$

$$(2x - 15)(2x - 1) = 0$$

$$2x - 15 = 0 \quad ; \quad 2x - 1 = 0$$

$$2x = 15 \quad \quad \quad 2x = 1$$

$$x = \frac{15}{2} \quad \quad \quad x = \frac{1}{2}$$

$$x + y = 8 \quad \quad \quad x + y = 8$$

$$\frac{15}{2} + y = 8 \quad \quad \quad \frac{1}{2} + y = 8$$

$$y = 8 - \frac{15}{2} \quad \quad \quad y = 8 - \frac{1}{2}$$

$$y = \frac{16-15}{2} \quad \quad \quad y = \frac{16-1}{2}$$

$$y = \frac{1}{2} \quad \quad \quad y = \frac{15}{2}$$

$$x = \frac{15}{2} \quad ; \quad y = \frac{1}{2}$$

$$\textbf{(i) } \mathbf{x - y}$$

$$= \frac{15}{2} - \frac{1}{2}$$

$$= 15 - \frac{1}{2}$$

$$= \frac{14}{2} = 7$$

$$\textbf{(ii) } \mathbf{3(x^2 + y^2)}$$

$$= 3 \left[\left(\frac{15}{2} \right)^2 + \left(\frac{1}{2} \right)^2 \right]$$

$$= 3 \left[\frac{225}{4} + \frac{1}{4} \right]$$

$$= 3 \left[\frac{225+1}{4} \right]$$

$$= 3 \left(\frac{226}{4} \right)$$

$$= \frac{3 \times 113}{2}$$

$$= \frac{339}{2}$$

$$\textbf{(iii) } \mathbf{5(x^2 + y^2) + 4(x - y)}$$

$$5 \left(\left(\frac{15}{2} \right)^2 + \left(\frac{1}{2} \right)^2 \right) + 4(7)$$

$$= 5 \left(\frac{225}{4} + \frac{1}{4} \right) + 4(7)$$

$$= 5 \left(\frac{113}{2} \right) + 28$$

$$= \frac{565}{2} + 28$$

$$= \frac{565+56}{2}$$

$$= \frac{621}{2}$$

12.

Solution

$$\text{Given } x^2 + y^2 = 34$$

$$xy = 10\frac{1}{2} = \frac{21}{2}$$

$$(x + y)^2 = x^2 + y^2 + 2xy$$

$$= 34 + 2 \cdot \frac{21}{2}$$

$$= 34 + 21$$

$$= 55$$

$$(x - y)^2 = x^2 + y^2 - 2xy$$

$$= 34 - 2 \cdot \frac{21}{2}$$

$$= 34 - 21$$

$$= 13$$

$$2(x + y)^2 + (x - y)^2$$

$$= 2(55) + 13$$

$$= 110 + 13$$

$$= 123$$

13.

Solution

Given $a - b = 3$ (i)

$$ab = 4$$
(ii)

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

from (i) squaring on both sides

$$(a - b)^2 = 3^2$$

$$a^2 + b^2 - 2ab = 9$$

$$a^2 + b^2 - 2(4) = 9$$

$$a^2 + b^2 - 8 = 9$$

$$a^2 + b^2 = 9 + 8$$

$$a^2 + b^2 = 17$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$= 3 \cdot (a^2 + b^2 + ab)$$

$$= 3(17 + 4)$$

$$= 3(21)$$

$$= 63$$

14.

Solution

Given $2a - 3b = 3 \dots (i)$

$$ab = 2 \dots (2)$$

from (i) equation on both sides

$$(2a - 3b)^2 = 3^2$$

$$(2a)^2 - 2 \cdot 2a \cdot 3b + (3b)^2 = 9$$

$$4a^2 - 12ab + 9b^2 = 9$$

$$4a^2 + 9b^2 - 12(2) = 9$$

$$4a^2 + 9b^2 - 24 = 9$$

$$4a^2 + 9b^2 - 24 = 9$$

$$4a^2 + 9b^2 = 9 + 24$$

$$= 33$$

$$(2a)^3 - (3b)^3 = 8a^3 - 27b^3$$

$$= (2a - 3b)((2a)^2 + 2a \cdot 3b + (3b)^2)$$

$$= 3(4a^2 + 6ab + 9b^2)$$

$$= 3(33 + 6(2))$$

$$= 3(33 + 12)$$

$$= 3(45)$$

$$= 135$$

15.

Solution

given $x + \frac{1}{x} = 4$

(i) squaring on both sides

$$\left(x + \frac{1}{x}\right)^2 = 4^2$$

$$x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 16$$

$$x^2 + 2 + \frac{1}{x^2} = 16$$

$$x^2 + \frac{1}{x^2} = 16 - 2$$

$$x^2 + \frac{1}{x^2} = 14$$

(ii) $x^4 + \frac{1}{x^4}$

We know that $x^2 + \frac{1}{x^2} = 14$

Squaring on both sides

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 14^2$$

$$(x^2)^2 + 2 \cdot x^2 \cdot \frac{1}{x^2} + \left(\frac{1}{x^2}\right)^2 = 196$$

$$x^4 + 2 + \frac{1}{x^4} = 196$$

$$x^4 + \frac{1}{x^4} = 196 - 2$$

$$x^4 + \frac{1}{x^4} = 194$$

$$\text{(iii) } x^3 + \frac{1}{x^3}$$

$$\text{We know } x + \frac{1}{x} = 4$$

Cubing on both sides

$$\left(x + \frac{1}{x}\right)^3 = 4^3$$

$$x^3 + 3 \cdot x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) + \frac{1}{x^3} = 64$$

$$x^3 + 3\left(x + \frac{1}{x}\right) + \frac{1}{x^3} = 64$$

$$x^3 + \frac{1}{x^3} + 3(4) = 64$$

$$\begin{aligned} x^3 + \frac{1}{x^3} &= 64 - 12 \\ &= 52 \end{aligned}$$

$$\text{(iv) } x - \frac{1}{x}$$

$$\begin{aligned} \left(x - \frac{1}{x}\right)^2 &= x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} \\ &= x^2 + \frac{1}{x^2} - 2 \end{aligned}$$

$$\begin{aligned} \text{from (i) } x^2 + \frac{1}{x^2} &= 14 \\ &= 14 - 2 \end{aligned}$$

$$\left(x - \frac{1}{x}\right)^2 = 12$$

$$\begin{aligned}
 x - \frac{1}{x} &= \sqrt{12} \\
 &= \sqrt{4 \times 3} \\
 &= 2\sqrt{3}
 \end{aligned}$$

16.

Solution

Given $x - \frac{1}{x} = 5$

Squaring on both sides

$$\left(x - \frac{1}{x}\right)^2 = 5^2$$

$$x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 25$$

$$x^2 + \frac{1}{x^2} = 25 + 2$$

$$x^2 + \frac{1}{x^2} = 23$$

squaring on both sides

$$\left(x^2 - \frac{1}{x^2}\right)^2 = 23^2$$

$$(x^2)^2 + 2 \cdot x^2 \cdot \frac{1}{7^2} + \left(\frac{1}{x^2}\right)^2 = 529$$

$$x^4 + 2 + \frac{1}{x^4} = 529$$

$$x^4 + \frac{1}{x^4} = 529 - 2$$

$$x^4 + \frac{1}{x^4} = 527$$

17.

Solution

Given

$$x - \frac{1}{x} = \sqrt{5}$$

(i) squaring on both sides

$$\left(x - \frac{1}{x}\right)^2 = (\sqrt{5})^2$$

$$x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 5$$

$$x^2 - 2 + \frac{1}{x^2} = 5$$

$$x^2 + \frac{1}{x^2} = 5 + 2$$

$$x^2 + \frac{1}{x^2} = 7$$

$$\text{(ii) } x + \frac{1}{x}$$

$$\begin{aligned} &= \left(x + \frac{1}{x}\right)^2 = x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} \\ &= x^2 + \frac{1}{x^2} + 2 \quad [\because \text{from (i) }] \\ &= 7 + 2 \\ &= 9 \end{aligned}$$

$$\left(x + \frac{1}{x}\right)^2 = 9$$

$$x + \frac{1}{x} = \sqrt{9}$$

$$x + \frac{1}{x} = 3$$

$$\text{(iii) } x^3 + \frac{1}{x^3}$$

$$\begin{aligned} &= \left(x + \frac{1}{x}\right) \left(x^2 - x \cdot \frac{1}{x} + \frac{1}{x^2}\right) \\ &= \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2} - 1\right) \\ &= 3 \cdot (7-1) \\ &= 3 \times 6 \\ &= 18 \end{aligned}$$

18.

Solution

Given $x + \frac{1}{x} = 6$

Squaring on both sides

$$\left(x + \frac{1}{x}\right)^2 = 6^2$$

$$x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 36$$

$$x^2 + 2 + \frac{1}{x^2} = 36$$

$$\begin{aligned} x^2 + \frac{1}{x^2} &= 36 - 2 \\ &= 34 \end{aligned}$$

(i) $x - \frac{1}{x}$

$$= \left(x - \frac{1}{x}\right)^2 = x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2}$$

$$= x^2 + \frac{1}{x^2} - 2$$

$$= 34 - 2$$

$$\left(x - \frac{1}{x}\right)^2 = 32$$

$$x - \frac{1}{x} = \sqrt{32}$$

$$x - \frac{1}{x} = \sqrt{16 \times 2}$$

$$x - \frac{1}{x} = 4\sqrt{2}$$

$$(ii) \quad x^2 - \frac{1}{x^2}$$

$$= x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right)$$

$$= 6 \cdot 4\sqrt{2}$$

$$= 24\sqrt{2}$$

19.

Solution

$$\text{Given } x + \frac{1}{x} = 2$$

Squaring on both sides

$$\left(x + \frac{1}{x}\right)^2 = 2^2$$

$$x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 4$$

$$x^2 + 2 + \frac{1}{x^2} = 4$$

$$x^2 + \frac{1}{x^2} = 4 - 2$$

$$x^2 + \frac{1}{x^2} = 2 \dots\dots(i)$$

$$\therefore x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right) \left(x^2 - x \cdot \frac{1}{x} + \frac{1}{x^2}\right)$$

$$= 2 \cdot \left(x^2 + \frac{1}{x^2} - 1\right)$$

$$= 2 (2 -1)$$

$$= 2(1)$$

$$= 2.....(ii)$$

$$\text{From (1) } = x^2 + \frac{1}{x^2} = 2$$

Squaring on both sides

$$= \left(x^2 + \frac{1}{x^2} \right)^2 = 2^2$$

$$= (x^2)^2 + 2 \cdot x^2 \cdot \frac{1}{x^2} + \left(\frac{1}{x^2} \right)^2 = 4$$

$$= x^4 + 2 + \frac{1}{x^4} = 4$$

$$= x^4 + \frac{1}{x^4} = 4 - 2$$

$$= x^4 + \frac{1}{x^4} = 4 - 2$$

$$= 2(iii)$$

From equation (i) , (ii) and (iii)

$$x^2 + \frac{1}{x^2} = 2$$

$$x^3 + \frac{1}{x^3} = 2$$

$$x^4 + \frac{1}{x^4} = 2$$

$$\therefore x^2 + \frac{1}{x^2} = x^3 + \frac{1}{x^3} = x^4 + \frac{1}{x^4}$$

Hence proved

20.

Solution

$$\text{Given } x - \frac{2}{x} = 3$$

Cubing on both sides

$$\left(x - \frac{2}{x}\right)^3 = 3^3$$

$$x^3 - 3 \cdot x \cdot \frac{2}{x} \left(x - \frac{2}{x}\right) - \left(\frac{2}{x}\right)^3 = 27$$

$$x^3 - 6 \left(x - \frac{2}{x}\right) - \frac{8}{x^3} = 27$$

$$x^3 - \frac{8}{x^3} - 6(3) = 27$$

$$x^3 - \frac{8}{x^3} = 27 + 18$$
$$= 45$$

21.

Solution

$$\text{Given } a + 2b = 5$$

Cubing on both sides

$$(a + 2b)^3 = 5^3$$

$$a^3 + 3a \cdot 2b(a + 2b) + (2b)^3 = 125$$

$$a^3 + 6ab(5) + 8b^3 = 125$$

$$a^3 + 30ab + 8b^3 = 125$$

$$\therefore a^3 + 8b^3 + 30ab = 125$$

22.

Solution

$$\text{Given } a + \frac{1}{a} = p$$

Cubing on both sides

$$\left(a + \frac{1}{a}\right)^3 = p^3$$

$$a^3 + 3a \cdot \frac{1}{a} \left(a + \frac{1}{a}\right) + \frac{1}{a^3} = p^3$$

$$a^3 + 3(p) + \frac{1}{a^3} = p^3$$

$$a^3 + \frac{1}{a^3} = p^3 - 3p$$

$$a^3 + \frac{1}{a^3} = p(p^2 - 3)$$

23.

Solution

$$\text{Given } x^2 + \frac{1}{x^2} = 27$$

$$\begin{aligned} \therefore \left(x - \frac{1}{x}\right)^2 &= x^2 - 2x \cdot \frac{1}{x} + \frac{1}{x^2} \\ &= x^2 - 2 + \frac{1}{x^2} \end{aligned}$$

$$= x^2 + \frac{1}{x^2} - 2$$

$$= 27 - 2$$

$$\left(x - \frac{1}{x}\right)^2 = 25$$

$$x - \frac{1}{x} = \sqrt{25}$$

$$x - \frac{1}{x} = 5$$

24.

Solution

Given $x^2 + \frac{1}{x^2} = 27$

Take $\left(x - \frac{1}{x}\right)^2 = x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2}$

$$= x^2 - 2 + \frac{1}{x^2}$$

$$= x^2 + \frac{1}{x^2} - 2$$

$$= 27 - 2$$

$$\left(x - \frac{1}{x}\right)^2 = 25$$

$$x - \frac{1}{x} = \sqrt{25}$$

$$x - \frac{1}{x} = 5$$

$$\begin{aligned}
& \therefore 3x^3 + 5x - \frac{3}{x^3} - \frac{5}{x} \\
& = 3x^3 - \frac{3}{x^3} + 5x - \frac{5}{x} \\
& = 3\left(x^3 - \frac{1}{x^3}\right) + 5\left(x - \frac{1}{x}\right) \\
& = 3\left(x - \frac{1}{x}\right)\left(x^2 + x \cdot \frac{1}{x} + \frac{1}{x^2}\right) + 5\left(x - \frac{1}{x}\right) \\
& = 3\left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 1\right) + 5\left(x - \frac{1}{x}\right) \\
& = 3(5)(27+1) + 5(5) \\
& = 15(28) + 25 \\
& = 420 + 25 = 445
\end{aligned}$$

25.

Solution

$$\begin{aligned}
\text{Given } x^2 + \frac{1}{25x^2} &= 8\frac{3}{5} \\
x^2 + \left(\frac{1}{5x}\right)^2 &= \frac{43}{5}
\end{aligned}$$

$\therefore$ let us consider

$$\begin{aligned}
\left(x + \frac{1}{5x}\right)^2 &= x^2 + 2 \cdot x \cdot \frac{1}{5x} + \left(\frac{1}{5x}\right)^2 \\
&= x^2 + \frac{2}{5} + \frac{1}{25x^2} \\
&= x^2 + \frac{1}{25x^2} + \frac{2}{5} \\
&= \frac{43}{5} + \frac{2}{5}
\end{aligned}$$

$$\left(x + \frac{1}{5x}\right)^2 = \frac{43+2}{5}$$

$$\left(x + \frac{1}{5x}\right)^2 = \frac{47}{5}$$

$$x + \frac{1}{5x} = \sqrt{\frac{47}{5}}$$

26.

Solution

$$\text{Given } x^2 + \frac{1}{4x^2} = 8$$

$$x^2 + \left(\frac{1}{2x}\right)^2 = 8$$

let us consider

$$\left(x + \frac{1}{2x}\right)^2 = x^2 + 2 \cdot x \cdot \frac{1}{2x} + \left(\frac{1}{2x}\right)^2$$

$$= x^2 + 1 + \frac{1}{4x^2}$$

$$= x^2 + \frac{1}{4x^2} + 1$$

$$= 8 + 1$$

$$\left(x + \frac{1}{2x}\right)^2 = 9$$

$$x + \frac{1}{2x} = \sqrt{9}$$

$$\therefore x^3 + \left(\frac{1}{2x}\right)^3$$

$$= x^3 + \frac{1}{8x^3} = \left(x + \frac{1}{2x}\right) \left(x^2 - x \cdot \frac{1}{2x} + \left(\frac{1}{2x}\right)^2\right)$$

$$\begin{aligned}
&= x^3 + \frac{1}{8x^3} = \left(x + \frac{1}{2x}\right) \left(x^2 + \frac{1}{4x^2} - 1\right) \\
&= 3(8-1) \\
&= 3(7) \\
&= 21
\end{aligned}$$

27.

Solution

Given $a^2 - 3a + 1 = 0$

Dividing each term by a , we get

$$\frac{a^2}{a} - \frac{3a}{a} + \frac{1}{a} = 0$$

$$a - 3 + \frac{1}{a} = 0$$

$$a + \frac{1}{a} = 3$$

now

$$(i) \left(a + \frac{1}{a}\right)^2 = a^2 + 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2}$$

$$\left(a + \frac{1}{a}\right)^2 = a^2 + 2 + \frac{1}{a^2}$$

$$a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$$

$$= 3^2 - 2$$

$$= 9 - 2$$

$$= 7$$

$$\text{(ii) } a^3 + \frac{1}{a^3}$$

$$= \left(a + \frac{1}{a}\right) \left(a^2 - a \cdot \frac{1}{a} + \frac{1}{a^2}\right)$$

$$= \left(a + \frac{1}{a}\right) \left(a^2 + \frac{1}{a^2} - 1\right)$$

$$= (3)(7-1)$$

$$= 3 \times 6$$

$$= 18$$

28.

Solution

$$\text{Given } a = \frac{1}{a-5}$$

$$a(a-5) = 1$$

$$a^2 - 5a = 1$$

$$a^2 - 5a - 1 = 0$$

(i)

Dividing each term by a , we get

$$\frac{a^2}{a} - \frac{5a}{a} - \frac{1}{a} = 0$$

$$a - 5 - \frac{1}{a} = 0$$

$$\therefore a - \frac{1}{a} = 5$$

(ii) now $\left(a + \frac{1}{a}\right)$

$$\therefore a - \frac{1}{a} = 5$$

Squaring on both sides

$$\left(a - \frac{1}{a}\right)^2 = 5^2$$

$$a^2 - 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2} = 25$$

$$a^2 + \frac{1}{a^2} = 25 - 2$$

$$a^2 + \frac{1}{a^2} = 23$$

$$\left(a + \frac{1}{a}\right)^2 = a^2 + 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2}$$

$$= a^2 + \frac{1}{a^2} + 2$$

$$= 23 + 2$$

$$= 25$$

$$\left(a + \frac{1}{a}\right)^2 = 25$$

$$a + \frac{1}{a} = \sqrt{25}$$

$$= 5$$

(iii) $a^2 - \frac{1}{a^2} = \left(a + \frac{1}{a}\right)\left(a - \frac{1}{a}\right)$

$$= 5 \times 5 = 25$$

29.

Solution

$$\text{Given } \left(x + \frac{1}{x}\right)^2 = 3$$

$$x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 3$$

$$x^2 + \frac{1}{x^2} = 3 - 2$$

$$x^2 + \frac{1}{x^2} = 1$$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right) \left(x^2 - x \cdot \frac{1}{x} + \frac{1}{x^2}\right)$$

$$= \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2} - 1\right)$$

$$= \sqrt{3}(1 - 1)$$

$$= \sqrt{3}(0)$$

$$= 0$$

30.

Solution

$$\text{Given } x = 5 - 2\sqrt{6}$$

Squaring on both sides

$$x^2 = 5$$

31.

Solution

Given $a + b + c = 12$

Squaring on both sides

$$(a + b + c)^2 = 12^2$$

$$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 144$$

$$\therefore \text{from given } ab + bc + ca = 22$$

$$a^2 + b^2 + c^2 + 2(22) = 144$$

$$a^2 + b^2 + c^2 + 44 = 144$$

$$a^2 + b^2 + c^2 = 144 - 44$$

$$a^2 + b^2 + c^2 = 100$$

32.

Solution

Given $a + b + c = 12$

Squaring on both sides

$$(a + b + c)^2 = 12^2$$

$$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 144$$

$$\therefore a^2 + b^2 + c^2 = 100$$

$$\therefore 100 + 2(ab + bc + ca) = 144$$

$$2(ab + bc + ca) = 144 - 100$$

$$2(ab + bc + ca) = 44$$

$$ab + bc + ca = \frac{44}{2} = 22$$

33.

Solution

$$\text{Given } a^2 + b^2 + c^2 = 125$$

$$\therefore ab + bc + ca = 50$$

$$\begin{aligned} \because (a + b + c)^2 &= a^2 + b^2 + c^2 + 2(ab + bc + ca) \\ &= 125 + 2(50) \\ &= 125 + 100 \\ &= 225 \end{aligned}$$

$$(a + b + c)^2 = 225$$

$$a + b + c = \sqrt{225}$$

$$a + b + c = 15$$

34.

Solution

$$\text{Given } a + b - c = 5$$

$$a^2 + b^2 + c^2 = 29$$

$$(a + b - c)^2 = a^2 + b^2 + c^2 + 2(ab - bc - ca)$$

$$5^2 = 29 + 2(ab - bc - ca)$$

$$25 = 29 + 2(ab - bc - ca)$$

$$25 - 29 = 2(ab - bc - ca)$$

$$-4 = 2(ab - bc - ca)$$

$$ab - bc - ca = -\frac{4}{2} = -2$$

35.

Solution

$$\text{Given } a - b = 7$$

$$a^2 + b^2 = 85$$

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$7^2 = 85 - 2ab$$

$$49 = 85 - 2ab$$

$$2ab = 85 - 49$$

$$2ab = 36$$

$$ab = \frac{36}{2}$$

$$ab = 18$$

$$\therefore a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$= 7(a^2 + b^2 + ab)$$

$$= 7(85 + 18)$$

$$= 7(103)$$

$$= 721$$

36.

Solution

Given the number is x

$$\therefore x = y - 3$$

$$x - y = -3$$

$$y - x = 3$$

$$\text{and } x^2 + y^2 = 29$$

$$\therefore y - x = 3$$

Squaring on both sides

$$(y - x)^2 = 3^2$$

$$y^2 + x^2 - 2xy = 9$$

$$29 - 2xy = 9$$

$$29 - 9 = 2xy$$

$$2xy = 20$$

$$xy = \frac{20}{2}$$

$$xy = 10$$

37

Solution

Given sum of two numbers = 8

Product of two numbers = 15

Let number be x and y

$$\therefore x + y = 8$$

$$xy = 15$$

$$x + y = 8$$

squaring on both sides

$$(x+y)^2 = 8^2$$

$$x^2 + y^2 + 2xy = 64$$

$$x^2 + y^2 + 2xy = 64$$

$$x^2 + y^2 + 2(15) = 64$$

$$x^2 + y^2 = 64 - 30$$

$$x^2 + y^2 = 34$$

$$\therefore x^3 + y^3$$

$$= (x + y) (x^2 - xy + y^2)$$

$$= 8(x^2 + y^2 - xy)$$

$$= 8(34 - 15)$$

$$= 8(19)$$

$$= 152$$

Chapter Test

1. Find the expansions of the following :

(i) $(2x + 3y + 5)(2x + 3y - 5)$

(ii) $(6 - 4a - 7b)^2$

(iii) $(7 - 3xy)^3$

(iv) $(x + y + 2)^3$

Solution:

(i) $(2x + 3y + 5)(2x + 3y - 5)$

Let us simplify the expression, we get

$$(2x + 3y + 5)(2x + 3y - 5) = [(2x + 3y) + 5][(2x + 3y) - 5]$$

By using the formula, $(a)^2 - (b)^2 = [(a + b)(a - b)]$

$$= (2x + 3y)^2 - (5)^2$$

$$= (2x)^2 + (3y)^2 + 2 \times 2x \times 3y - 5 \times 5$$

$$= 4x^2 + 9y^2 + 12xy - 25$$

(ii) $(6 - 4a - 7b)^2$

Let us simplify the expression, we get

$$\begin{aligned}(6 - 4a - 7b)^2 &= [6 + (-4a) + (-7b)]^2 \\&= (6)^2 + (-4a)^2 + (-7b)^2 + 2(6)(-4a) + 2(-4a)(-7b) + 2(-7b)(6) \\&= 36 + 16a^2 + 49b^2 - 48a + 56ab - 84b\end{aligned}$$

(iii) $(7 - 3xy)^3$

Let us simplify the expression

By using the formula, we get

$$\begin{aligned}(7 - 3xy)^3 &= (7)^3 - (3xy)^3 - 3(7)(3xy)(7 - 3xy) \\&= 343 - 27x^3y^3 - 63xy(7 - 3xy) \\&= 343 - 27x^3y^3 - 441xy + 189x^2y^2\end{aligned}$$

(iv) $(x + y + 2)^3$

Let us simplify the expression

By using the formula, we get

$$\begin{aligned}(x + y + 2)^3 &= [(x + y) + 2]^3 \\&= (x + y)^3 + (2)^3 + 3(x + y)(2)(x + y + 2) \\&= x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6(x + y)[(x + y) + 2] \\&= x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6(x + y)^2 + 12(x + y) \\&= x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6(x^2 + y^2 + 2xy) + 12x + 12y = x^3 +\end{aligned}$$

$$y^3 + 3x^2y + 3xy^2 + 8 + 6x^2 + 6y^2 + 12xy + 12x + 12y$$

$$= x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6x^2 + 6y^2 + 12x + 12y + 12xy$$

2. Simplify: $(x - 2)(x + 2)(x^2 + 4)(x^4 + 16)$

Solution:

Let us simplify the expression, we get

$$(x - 2)(x + 2)(x^4 + 4)(x^4 + 16) = (x^2 - 4)(x^4 + 4)(x^4 + 16)$$

$$= [(x^2)^2 - (4)^2](x^4 + 16)$$

$$= (x^4 - 16)(x^4 + 16)$$

$$= (x^4)^2 - (16)^2$$

$$= x^8 - 256$$

3. Evaluate 1002×998 by using a special product.

Solution:

Let us simplify the expression, we get

$$1002 \times 998 = (1000 + 2)(1000 - 2)$$

$$= (1000)^2 - (2)^2$$

$$= 1000000 - 4$$

$$= 999996$$

4. If $a + 2b + 3c = 0$, Prove that $a^3 + 8b^3 + 27c^3 = 18abc$

Solution:

Given:

$$a + 2b + 3c = 0, a + 2b = -3c$$

Let us cube on both the sides, we get

$$(a + 2b)^3 = (-3c)^3$$

$$a^3 + (2b)^3 + 3(a)(2b)(a + 2b) = -27c^3$$

$$a^3 + 8b^3 + 6ab(-3c) = -27c^3$$

$$a^3 + 8b^3 - 18abc = -27c^3$$

$$a^3 + 8b^3 + 27c^3 = 18abc$$

Hence proved.

5. If $2x = 3y - 5$, then find the value of $8x^3 - 27y^3 + 90xy + 125$.

Solution:

Given:

$$2x = 3y - 5$$

$$2x - 3y = -5$$

Now, let us cube on both sides, we get

$$(2x - 3y)^3 = (-5)^3$$

$$(2x)^3 - (3y)^3 - 3 \times 2x \times 3y (2x - 3y) = -125$$

$$8x^3 - 27y^3 - 18xy (2x - 3y) = -125$$

Now, substitute the value of $2x - 3y = -5$

$$8x^3 - 27y^3 - 18xy (-5) = -125$$

$$8x^3 - 27y^3 + 90xy = -125$$

$$8x^3 - 27y^3 + 90xy + 125 = 0$$

6. If $a^2 - \left(\frac{1}{a^2}\right) = 5$, evaluate $a^4 + \left(\frac{1}{a^4}\right)$

Solution:

It is given that,

$$a^2 - \left(\frac{1}{a^2}\right) = 5$$

So,

By using the formula, $(a + b)^2$

$$\left[a^2 - \frac{1}{a^2}\right]^2 = a^4 + \left(\frac{1}{a^4}\right) - 2$$

$$\left[a^2 - \frac{1}{a^2}\right]^2 + 2 = a^4 + \frac{1}{a^4}$$

Substitute the value of $a^2 - \frac{1}{a^2} = 5$, we get

$$5^2 + 2 = a^4 + \frac{1}{a^4}$$

$$a^4 + \frac{1}{a^4} = 25 + 2$$

$$= 27$$

7. If $a + \frac{1}{a} = p$ and $a - \frac{1}{a} = q$, Find the relation between p and q.

Solution:

It is given that,

$$a + \frac{1}{a} = p \text{ and } a - \frac{1}{a} = q$$

so,

$$\left(a + \frac{1}{a}\right)^2 - \left(a - \frac{1}{a}\right)^2 = 4(a) \left(\frac{1}{a}\right)$$

$$= 4$$

By substituting the values, we get

$$p^2 - q^2 = 4$$

Hence the relation between p and q is that $p^2 - q^2 = 4$.

8. If $\frac{a^2 + 1}{a} = 4$, find the value of $\frac{2a^3 + 2}{a^3}$

Solution:

It is given that,

$$\frac{a^2 + 1}{a} = 4$$

$$\frac{a^2}{a} + \frac{1}{a} = 4$$

$$a + \frac{1}{a} = 4$$

So by multiplying the expression by $2a$, we get

$$\begin{aligned} 2a^3 + \frac{2}{a^3} &= 2 \left[a^3 + \frac{1}{a^3} \right] \\ &= 2 \left[\left(a + \frac{1}{a} \right)^3 - 3(a) \left(\frac{1}{a} \right) \left(a + \frac{1}{a} \right) \right] \\ &= 2 [(4)^3 - 3(4)] \\ &= 2 [64 - 12] \\ &= 2 (52) \\ &= 104 \end{aligned}$$

9. If $x = \frac{1}{4-x}$, find the value of

(i) $x + \frac{1}{x}$

(ii) $x^3 + \frac{1}{x^3}$

(iii) $x^6 + \frac{1}{x^6}$

Solution:

It is given that,

$$x = \frac{1}{4-x}$$

So,

$$(i) \ x(4 - x) = 1$$

$$4x - x^2 = 1$$

Now let us divide both sides by x, we get

$$4 - x = \frac{1}{x}$$

$$4 = \frac{1}{x} + x$$

$$\frac{1}{x} + x = 4$$

$$\frac{1}{x} + x = 4$$

$$(ii) \ x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^2 - 3\left(x + \frac{1}{x}\right)$$

By substituting the values, we get

$$= (4)^3 - 3(4)$$

$$= 64 - 12$$

$$= 52$$

$$(iii) \ x^6 + \frac{1}{x^6} = \left(x^3 + \frac{1}{x^3}\right)^2 - 2$$

$$= (52)^2 - 2$$

$$= 2704 - 2$$

$$= 2702$$

10. If $x - \frac{1}{x} = 3 + 2\sqrt{2}$, find the value of $\frac{1}{4} \left(x^3 - \frac{1}{x^3} \right)$

Solution:

It is given that,

$$x - \frac{1}{x} = 3 + 2\sqrt{2}$$

So,

$$x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x} \right)^3 + 3 \left(x - \frac{1}{x} \right)$$

$$= (3 + 2\sqrt{2})^3 + 3(3 + 2\sqrt{2})$$

By using the formula, $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$$= (3)^3 + (2\sqrt{2})^3 + 3(3)(2\sqrt{2})(3 + 2\sqrt{2}) + 3(3 + 2\sqrt{2})$$

$$= 27 + 16\sqrt{2} + 54\sqrt{2} + 72 + 9 + 6\sqrt{2}$$

$$= 108 + 76\sqrt{2}$$

Hence,

$$\frac{1}{4} \left(x^3 - \frac{1}{x^3} \right) = \frac{1}{4} (108 + 76\sqrt{2})$$

$$= 27 + 19\sqrt{2}$$

11. If $x + \frac{1}{x} = 3\frac{1}{3}$, find the value of $x^3 - \frac{1}{x^3}$

Solution:

It is given that,

$$x + \frac{1}{x} = 3\frac{1}{3}$$

we know that,

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$= x^2 + \frac{1}{x^2} + 2 - 4$$

$$= \left(x + \frac{1}{x}\right)^2 - 4$$

$$\text{But } x + \frac{1}{x} = 3\frac{1}{3} = \frac{10}{3}$$

So,

$$\left(x - \frac{1}{x}\right)^2 = \left(\frac{10}{3}\right)^2 - 4$$

$$= \frac{100}{9} - 4$$

$$= \frac{100 - 36}{9}$$

$$= \frac{64}{9}$$

$$x - \frac{1}{x} = \sqrt{\frac{64}{9}}$$

$$= \frac{8}{3}$$

Now,

$$\begin{aligned}x^3 - \frac{1}{x^3} &= \left(x - \frac{1}{x}\right)^3 + 3 \left(x\right) \left(\frac{1}{x}\right) \left(x - \frac{1}{x}\right) \\&= \left(\frac{8}{3}\right)^3 + 3 \left(\frac{8}{3}\right) \\&= \left(\frac{512}{27} + 8\right) \\&= \frac{728}{27} \\&= 26 \frac{26}{27}\end{aligned}$$

12. If $x = 2 - \sqrt{3}$, then find the value of $x^3 - \frac{1}{x^3}$

Solution:

It is given that,

$$x = 2 - \sqrt{3}$$

so,

$$\frac{1}{x} = \frac{1}{2 - \sqrt{3}}$$

By rationalizing the denominator, we get

$$\begin{aligned}&= \left[\frac{1(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})} \right] \\&= \left[\frac{(2 + \sqrt{3})}{2^2 - \sqrt{3}^2} \right] \\&= \left[\frac{2 + \sqrt{3}}{4 - 3} \right]\end{aligned}$$

$$= 2 + \sqrt{3}$$

Now,

$$x - \frac{1}{x} = 2 - \sqrt{3} - 2 - \sqrt{3}$$

$$= -2\sqrt{3}$$

Let us cube on both sides, we get

$$\left(x - \frac{1}{x}\right)^3 = (-2\sqrt{3})^3$$

$$x^3 - \frac{1}{x^3} - 3(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right) = 24\sqrt{3}$$

$$x^3 - \frac{1}{x^3} - 3(-2\sqrt{3}) = -24\sqrt{3}$$

$$x^3 - \frac{1}{x^3} + 6\sqrt{3} = -24\sqrt{3}$$

$$x^3 - \frac{1}{x^3} = -24\sqrt{3} - 6\sqrt{3}$$

$$= -30\sqrt{3}$$

Hence,

$$x^3 - \frac{1}{x^3} = -30\sqrt{3}$$

13. If the sum of two numbers is 11 and sum of their cubes is 737, find the sum of their squares.

Solution:

Let us consider x and y be two numbers

Then,

$$x + y = 11$$

$$x^3 + y^3 = 735 \text{ and } x^2 + y^2 = ?$$

Now,

$$x + y = 11$$

Let us cube on both the sides,

$$(x + y)^3 = (11)^3$$

$$x^3 + y^3 + 3xy(x + y) = 1331$$

$$735 + 3x \times 11 = 1331$$

$$33xy = 1331 - 735$$

$$= 596$$

$$xy = \frac{596}{33}$$

$$xy = 8$$

We know that, $x + y = 11$

By squaring on both sides, we get

$$(x + y)^2 = (11)^2$$

$$x^2 + y^2 + 2xy = 121 \quad x^2 + y^2 + 2 \times 18 = 121$$

$$x^2 + y^2 + 36 = 121$$

$$x^2 + y^2 = 121 - 36$$

$$= 85$$

Hence sum of the squares = 85

14. If $a - b = 7$ and $a^3 - b^3 = 133$, find:

(i) ab

(ii) $a^2 + b^2$

Solution:

It is given that,

$$a - b = 7$$

let us cube on both sides, we get

$$(i) (a - b)^3 = (7)^3$$

$$a^3 + b^3 - 3ab(a - b) = 343$$

$$133 - 3ab \times 7 = 343$$

$$133 - 21ab = 343$$

$$- 21ab = 343 - 133$$

$$= 210$$

$$ab = - \frac{210}{21}$$

$$ab = -10$$

$$(ii) a^2 + b^2$$

$$\text{Again } a - b = 7$$

Let us square on both sides, we get

$$(a - b)^2 = (7)^2$$

$$a^2 + b^2 - 2ab = 49$$

$$a^2 + b^2 - 2 \times (-10) = 49$$

$$a^2 + b^2 + 20 = 49$$

$$a^2 + b^2 = 49 - 20$$

$$= 29$$

$$\text{Hence, } a^2 + b^2 = 29$$

15. Find the coefficient of x^2 expansion of $(x^2 + x + 1)^2 + (x^2 - x + 1)^2$

Solution:

Given:

The expression, $(x^2 + x + 1)^2 + (x^2 - x + 1)^2$

$$\begin{aligned}
(x^2 + x + 1)^2 + (x^2 - x + 1)^2 &= [((x^2 + 1) + x)^2 + ((x^2 + 1) - x)^2] \\
&= (x^2 + 1)^2 + x^2 + 2(x^2 + 1)(x) + (x^2 + 1)^2 + x^2 - 2(x^2 + 1)(x) \\
&= (x^2)^2 + (1)^2 + 2 \times x^2 \times 1 + x^2 + (x^2)^2 + 1 + 2 \times x^2 + 1 + x^2 \\
&= x^4 + 1 + 2x^2 + x^2 + x^4 + 1 + 2x^2 + x^2 \\
&= 2x^4 + 6x^2 + 2
\end{aligned}$$

$\therefore$ Co-efficient of x^2 is 6.