

Chapter 4

Factorization

Exercise 4.1

Factorise the following (1 to 9)

1. (i) $8xy^3 + 12x^2y^2$

Solution:

$$8xy^3 + 12x^2y^2$$

Take out common in both terms,

$$\text{Then, } 4xy^2(2y + 3x)$$

Therefore, HCF of $8xy^3$ and $12x^2y^2$ is $4xy^2$

(ii) $15x^3 - 9ax^2$

Solution:

$$15x^3 - 9ax^2$$

Take out common in both terms,

$$\text{Then, } 3ax^2 (5x - 3)$$

Therefore, HCF of $15ax^3$ and $9a^2$ is $3ax^2$.

Question 2.

(i) $21py^2 - 56py$

Solution:

$$21py^2 - 56py$$

Take out common in both terms,

Then , $7py (3y - 8)$

Therefore , HCL of $21py^2$ and $56 py$ is $7py$.

(ii) $4x^3 - 6x^2$

Solution:

$$4x^3 - 6x^2$$

Take out common in both terms,

Then , $2x^2 (2x - 3)$

Therefore, HCF of $4x^3$ and $6x^2$ is $2x^2$.

Question 3.

(i) $2\pi r^2 - 4\pi r$

Solution:

Take out common in both terms,

Then , $2\pi r(r - 2)$

Therefore , HCL of $2\pi r^2$ and $4\pi r$ is $2\pi r$.

(ii) $18m + 16n$

Solution :

$$18m + 16n$$

Take out common in both terms ,

Then , $2 (9m - 8n)$

Therefore , HCF of $18m$ and $16n$ is 2 .

Question 4.

(i) $25abc^2 - 15a^2b^2c$

Solution:

$$25abc^2 - 15a^2b^2c$$

Take out common in both terms,

Then , $5abc (5c - 3ab)$

Therefore, HCF of $25abc^2$ and $15a^2b^2c$ is $5abc$

(ii) $28p^2q^2r - 42pq^2r^2$

Solution:

$$28p^2q^2r - 42pq^2r^2$$

Take out common in both terms,

Then , $14pq^2r (2p - 3r)$

Therefore , HCF or $28p^2q^2r$ and $42pq^2r^2$ is $14pq^2r$.

Question 5.

(i) $8x^3 - 6x^2 + 10x$

Solution:

$$8x^3 - 6x^2 + 10x$$

Take out common in both terms,

Then , $2x (4x^2 - 3x + 5)$

Therefore, HCF of $8x^3$, $6x^2$ and $10x$ is $2x$.

(ii) $14mn + 22m - 62p$

Solution:

$$14mn + 22m - 62p$$

Take out common in both terms,

Then , $2 (7mn + 11m - 31p)$

Therefore, HCF of $14mn$, $22m$ and $62p$ is 2 .

Question 6.

(i) $18p^2q^2 - 24pq^2 + 30p^2q$

Solution:

$$18p^2q^2 - 24pq^2 + 30p^2q$$

Take out common in both terms,

Then, $6pq (3pq - 4q + 5q)$

Therefore, HCL of $18p^2q^2$, $24pq^2$ and $30p^2q$ is $6pq$.

(ii) $27a^3b^3 - 18a^2b^3 + 75a^3b^2$

Solution:

$$27a^3b^3 - 18a^2b^3 + 75a^3b^2$$

Take out common in both terms,

Then , $3a^2b^2 (9a - 6b + 25a)$

Therefore , HCL of $27a^3b^3$, $18a^2b^3$ and $75a^3b^2$ is $3 a^2b^2$

Question 7.

(i) $15a (2p - 3q) - 10b (2p - 3q)$

Solution:

$$15a (2p - 3q) - 10b (2p - 3q)$$

Take out common in both terms,

Then $5 (2p - 3q) [3a - 2b]$

Therefore, HCF of $15a (2p - 3q)$ and $10b (2p - 3q)$ is $5 (2p - 3q) .$

Question 8.

(i) $6(x + 2y)^3 + 8 (x + 2y)^2$

Solution:

$$6(x + 2y)^3 + 8 (x + 2y)^2$$

Take out common in all terms,

Then, $2(x + 2y)^2 [3 (x + 2y) + 4]$

Therefore, HCF of $6(x + 2y)^3$ and $8 (x + 2y)^2$ is $2(x + 2y)^2$

(ii) $14 (a - 3b)^3 - 21p (a - 3b)$

Solution:

$$14 (a - 3b)^3 - 21p (a - 3b)$$

Take out common in all terms,

$$\text{Then , } 7(a - 3b) [2(a - 3b)^2 - 3p]$$

Therefore, HCF of $14 (a - 3b)^3$ and $21p (a - 3b)$ is $7 (a - 3b)$.

Question 9.

(i) $10a(2p + q)^3 - 15b (2p + q)^2 + 35 (2p + q)$

Solution:

$$10a(2p + q)^3 - 15b (2p + q)^2 + 35 (2p + q)$$

Take out common in all terms,

$$\text{Then , } 5 (2p + q) [2a (2p + q)^2 - 3b (2p + q) + 7]$$

Therefore , HCF of $10a(2p + q)^3$, $15b (2p + q)^2$ and $35 (2p + q)$ is $5(2p + q)$.

(ii) $x (x^2 + y^2 - z^2) + y(-x^2 - y^2 + z^2) - z(x^2 + y - z^2)$

Solution:

$$x (x^2 + y^2 - z^2) + y(-x^2 - y^2 + z^2) - z(x^2 + y - z^2)$$

take out common in all terms,

then , $(x^2 + y^2 - z^2) [x - y - z]$

Therefore HCL of $x (x^2 + y^2 - z^2)$, $y(-x^2 - y^2 + z^2)$ and $,z(x^2 + y - z^2)$
is $(x^2 + y^2 - z^2)$

Exercise 4.2

Factorise the following (1 to 13) :

Question 1.

(i) $x^2 + xy - x - y$

Solution :

$$x^2 + xy - x - y$$

take out common in all terms,

$$x(x + y) - 1(x + y)$$

$$(x + y)(x - 1)$$

(ii) $y^2 - yz - 5y + 5z$

Solution:

$$y^2 - yz - 5y + 5z$$

take out common in all terms,

$$y(y - z) - 5(y - z)$$

$$(y - z)(y - 5)$$

(ii) $5p^2 - 8pq - 10p + 16q$

Solution:

$$5p^2 - 8pq - 10p + 16q$$

take out common in all terms,

$$p(5p - 8q) - 2(5p - 8q)$$

$$(5p - 8q)(p - 2)$$

Question 3.

(i) $a^2b - ab^2 + 3a - 3b$

Solution:

$$a^2b - ab^2 + 3a - 3b$$

take out common in all terms,

$$ab(a - b) + 3(a - b)$$

$$(a - b)(ab + 3)$$

(ii) $x^3 - 3x^2 + x - 3$

Solution:

$$x^3 - 3x^2 + x - 3$$

take out common in all terms,

$$x^2(x - 3) + 1(x - 3)$$

$$(x - 3)(x^2 + 1)$$

Question 4.

(i) $6xy^2 - 3xy - 10y + 5$

Solution:

$$6xy^2 - 3xy - 10y + 5$$

take out common in all terms,

$$3xy (2y - 1) - 5 (2y - 1)$$

$$(2y - 1) (3xy - 5)$$

$$(ii) 3ax - 6ay - 8by + 4bx$$

Solution:

$$3ax - 6ay - 8by + 4bx$$

take out common in all terms,

$$3a (x - 2y) + 4b (x - 2y)$$

$$(x - 2y) (3a + 4b)$$

Question 5.

$$(i) 1 - a - b + ab$$

Solution:

$$1 - a - b + ab$$

Take out common in all terms,

$$1(1 - a) - b(1 - a)$$

$$(1 - a) (1 - b)$$

$$(ii) a (a - 2b - c) + 2bc$$

Solution:

$$a (a - 2b - c) + 2bc$$

above question can be written as,

$$a^2 - 2ab - ac + 2bc$$

Take out common in all terms,

$$a (a - 2b) - c (a + 2b)$$

$$(a - 2b) (a - c)$$

Question 6.

$$(i) x^2 + xy (1 + y) + y^3$$

Solution:

$$x^2 + xy (1 + y) + y^3$$

Above question can be written as,

$$X^2 + xy + xy^2 + y^3$$

Take out common in all terms,

$$X(x + y) + y^2(x + y)$$

$$(x + y) (x + y^2)$$

$$(ii) y^2 - xy (1 - x) - x^3$$

Solution:

$$y^2 - xy (1 - x) - x^3$$

Above question can be written as,

$$Y^2 - xy + x^2y - x^3$$

Take out common in all terms,

$$Y(y - x) + x^2 (y - x)$$

$$(y - x) (y + x^2)$$

Question 7.

$$(i) ab^2 + (a - 1)b - 1$$

Solution:

$$ab^2 + (a - 1)b - 1$$

Above question can be written as,

$$Ab^2 + ab - b - 1$$

Take out common in all terms,

$$Ab(b + 1) - 1(b + 1)$$

$$(b + 1)(ab - 1)$$

$$(ii) 2a - 4b - xa + 2bx$$

Solution:

$$2a - 4b - xa + 2bx$$

Take out common in all terms,

$$2(a - 2b) - x(a - 2b)$$

$$(a - 2b)(2 - x)$$

Question 8.

(i) $5ph - 10qk + 2rph - 4qrk$

Solution:

$$5ph - 10qk + 2rph - 4qrk$$

Re – arranging the given question we get,

$$5ph + 2rph - 10qk - 4qrk$$

Take out common in all terms,

$$Ph(5 + 2r) - 2qk(5 + 2r)$$

$$(5 + 2r)(ph - 2qk)$$

(ii) $x^2 - x(a + 2b) + 2ab$

Solution:

$$x^2 - x(a + 2b) + 2ab$$

above question can be written as,

$$x^2 - xa - 2xb + 2ab$$

take out common in all terms,

$$x(x - a) - 2b(x - a)$$

$$(x - a)(x - 2b)$$

Question 9.

(i) $ab (x^2 + y^2) - xy (a^2 + b^2)$

Solution:

$$ab (x^2 + y^2) - xy (a^2 + b^2)$$

above question can be written as,

$$abx^2 + aby^2 - xya^2 - xyb^2$$

Re – arranging the above we get,

$$Abx^2 - xyb^2 + aby^2 - xya^2$$

Take out common in all terms,

$$bx (ax - by) + ay (by - xy)$$

$$bx (ax - by) - ay (ax - by)$$

$$(ax - by) (bx - ay)$$

(ii) $(ax + by)^2 + (bx - ay)^2$

Solution:

By expanding the give question, we get,

$$(ax)^2 + (by)^2 + 2axby + (bx)^2 + (ay)^2 - 2bxay$$

$$A^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$$

Re – arranging the above we get,

$$A^2x^2 + a^2y^2 + b^2y^2 + b^2x^2$$

Take out common in all terms,

$$A^2 (x^2 + y^2) + b^2 (x^2 + y^2)$$

$$(x^2 + y^2) (a^2 + b^2)$$

Question 10.

(i) $a^3 + ab(1 - 2a) - 2b^2$

Solution:

$$a^3 + ab(1 - 2a) - 2b^2$$

above question can be written as ,

$$a^3 + ab - 2a^2b - 2b^2$$

Re- arranging the above we get,

$$a^3 - 2a^2b + ab - 2b^2$$

take out common in all terms,

$$a^2(a - 2b) + b(a - 2b)$$

$$(a - 2b)(a^2 + b)$$

(ii) $3x^2y - 3xy + 12x - 12$

Solution:

$$3x^2y - 3xy + 12x - 12$$

Take out common in all terms,

$$3xy(x - 1) + 12(x - 1)$$

$$(x - 1)(3xy + 12)$$

Question 11.

$$A^2b + ab^2 - abc - b^2c + axy + bxy$$

Solution:

$$A^2b + ab^2 - abc - b^2c + axy + bxy$$

Re – arranging the above we get,

$$A^2b - abc + axy + ab^2 - b^2c + bxy$$

Take out common in all terms.

$$A(ab - bc + xy) + b(ab - bc + xy)$$

$$(a + b)(ab - bc + xy)$$

Question 12.

$$Ax^2 - bx^2 + ay^2 - by^2 + az^2 - bz^2$$

Solution:

$$Ax^2 - bx^2 + ay^2 - by^2 + az^2 - bz^2$$

Re – arranging the above we get,

$$Ax^2 + ay^2 + az^2 - bx^2 - by^2 - bz^2$$

Take out common in all terms,

$$A(x^2 + y^2 + z^2) - b(x^2 + y^2 + z^2)$$

$$(x^2 + y^2 + z^2)(a - b)$$

Question 13

$$X - 1 - 1 (x - 1)^2 + ax - a$$

Solution:

$$X - 1 - 1 (x - 1)^2 + ax - a$$

By expanding the above we get,

$$X - 1 - 1 (x^2 + 1 - 2x) + ax - a$$

$$X - 1 - 1 x^2 - 1 + 2x + ax - a$$

$$2x - x^2 + ax - 2 + x - a$$

Take out common in all terms,

$$X (2 - x + a) - 1 (2 - x + a)$$

$$(2 - x + a) (x - 1)$$

Exercise 4.3

Factorise the following (1 to 17) :

Question 1.

$$4x^2 - 25y^2$$

Solution :

We know that, $a^2 - b^2 = (a + b)(a - b)$

$$\text{So, } (2x)^2 - (5y)^2$$

$$\text{Then, } (2x + y)(2x - 5y)$$

$$(ii) 9x^2 - 1$$

Solution:

We know that, $a^2 - b^2 = (a + b)(a - b)$

$$\text{So, } (3x)^2 - 1^2$$

$$\text{Then, } (3x + 1)(3x - 1)$$

Question 2.

$$(i) 150 - 6a^2$$

Solution:

$$150 - 6a^2$$

Take out common in all terms,

$$6(25 - a^2)$$

$$6(5^2 - a^2)$$

We know that, $a^2 - b^2 = (a + b)(a - b)$

So, $6(5 + a)(5 - a)$

(ii) $32x^2 - 18y^2$

Solution:

$$32x^2 - 18y^2$$

Take out common in all terms,

$$2(16x^2 - 9y^2)$$

$$2((4x)^2 - (3y)^2)$$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$$2(4x + 3y)(4x - 3y)$$

Question 3.

(i) $(x - y)^2 - 9$

Solution:

$$(x - y)^2 - 9$$

$$(x - y)^2 - 3^2$$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$$(x - y + 3)(x - y - 3)$$

$$(ii) 9(x + y)^2 - x^2$$

Solution:

$$9[(x + y)^2 - x^2]$$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$$9[(x + y + x)(x + y - x)]$$

So, $9(2x + y)y$

$$9y(2x + y)$$

Question 4.

$$(i) 20x^2 - 45y^2$$

Solution;

$$20x^2 - 45y^2$$

Take out common in all terms,

$$5(4x^2 - 9y^2)$$

$$5((2x)^2 - (3y)^2)$$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$$5(2x + 3y)(2x - 3y)$$

$$(ii) 9x^2 - 4(y + 2x)^2$$

Solution:

$$9x^2 - 4(y + 2x)^2$$

Above question can be written as,

$$(3x)^2 - [2(y + 2x)]^2$$

$$(3x)^2 - (2y + 4x)^2$$

$$\text{We know that, } a^2 - b^2 = (a + b)(a - b)$$

$$(3x + 2y + 4x)(3x - 2y - 4x)$$

$$(7x + 2y)(-x - 2y)$$

Question 5.

$$(i) 2(x - 2y)^2 - 50y^2$$

Solution:

$$2(x - 2y)^2 - 50y^2$$

Take out common in all terms,

$$2[(x - 2y)^2 - 25y^2]$$

$$2[(x - 2y)^2 - (5y)^2]$$

$$\text{We know that, } a^2 - b^2 = (a + b)(a - b)$$

$$2[(x - 2y + 5y)(x - 2y - 5y)]$$

$$2[(x + 3y)(x - 7y)]$$

$$2(x + 3y)(x - 7y)$$

$$(ii) 32 - 2(x - 4)^2$$

Solution:

$$32 - 2(x - 4)^2$$

Take out common in all terms,

$$2[16 - (x - 4)^2]$$

$$2[42 - (x - 4)^2]$$

$$\text{We know that, } a^2 - b^2 = (a + b)(a - b)$$

$$2[(4 + x - 4)(4 - x + 4)]$$

$$2[(x)(8 - x)]$$

$$2x(8 - x)$$

Question 6.

$$(i) 108a^2 - 3(b - c)^2$$

Solution:

$$108a^2 - 3(b - c)^2$$

Take out common in all terms,

$$3[36a^2 - (b - c)^2]$$

$$3[(6a)^2 - (b - c)^2]$$

$$\text{We know that, } a^2 - b^2 = (a + b)(a - b)$$

$$3[(6a + b - c)(6a - b + c)]$$

$$(ii) \pi a^5 - \pi^3 ab^2$$

Solution:

$$\pi a^5 - \pi^3 ab^2$$

Take out common in all terms,

$$\pi a(a^4 - \pi^2 b^2)$$

$$\pi a((a^2)^2 - (\pi b)^2)$$

$$\text{We know that, } a^2 - b^2 = (a + b)(a - b)$$

$$\pi a(a^2 + \pi b)(a^2 - \pi b)$$

Question 7.

$$(i) 50x^2 - 2(x - 2)^2$$

Solution'

$$50x^2 - 2(x - 2)^2$$

Take out common in all terms,

$$2[25x^2 - (x - 2)^2]$$

$$2[(5x)^2 - (x - 2)^2]$$

$$\text{We know that, } a^2 - b^2 = (a + b)(a - b)$$

$$2[(5x + x - 2)(5x - x + 2)]$$

$$2[(6x - 2)(4x + 2)]$$

$$2(6x - 2)(4x + 2)$$

$$(ii) (x - 2)(x + 2) + 3$$

Solution:

$$\text{We know that, } a^2 - b^2 = (a + b)(a - b)$$

$$(x^2 - 2^2) + 3$$

$$X^2 - 4 + 3$$

$$X^2 - 1$$

Then,

$$(x + 1)(x - 1)$$

Question 8.

$$(i) x - 2y - x^2 + 4y^2$$

Solution:

$$X - 2y - x^2 + 4y^2$$

$$X - 2y - (x^2 + (2y)^2)$$

$$\text{We know that, } a^2 - b^2 = (a + b)(a - b)$$

$$X - 2y - [(x + 2y)(x - 2y)]$$

Take out common in all terms,

$$(x - 2y)(1 - (x + 2y))$$

$$(x - 2y)(1 - x - 2y)$$

$$(ii) 4a^2 - b^2 + 2a + b$$

Solution:

$$4a^2 - b^2 + 2a + b$$

$$(2a)^2 - b^2 + 2a + b$$

$$\text{We know that, } a^2 - b^2 = (a + b)(a - b)$$

$$((2a + b)(2a - b) + 1(2a + b))$$

Take out common in all terms,

$$(2a + b)(2a - b + 1)$$

Question 9.

$$(i) a(a - 2) - b(b - 2)$$

Solution:

$$a(a - 2) - b(b - 2)$$

above question can be written as,

$$a^2 - 2a - b^2 - 2b$$

re - arranging the above terms, we get,

$$a^2 - b^2 - 2a - 2b$$

we know that, $a^2 - b^2 = (a + b)(a - b)$

$$[(a + b)(a - b)] - 2(a - b)$$

Take out common in all terms,

$$(a - b)(a + b - 2)$$

$$(ii) a(a - 1) - b(b - 1)$$

Solution:

$$a(a - 1) - b(b - 1)$$

above question can be written as,

$$a^2 - a - b^2 + b$$

Re - arranging the above terms, we get,

$$A^2 - b^2 - a + b$$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$$[(a + b)(a - b)] - 1(a - b)$$

Take out common in all terms,

$$(a - b)(a + b - 1)$$

Question 10.

$$(i) 9 - x^2 + 2xy - y^2$$

Solution:

$$9 - x^2 + 2xy - y^2$$

Above terms can be written as,

$$9 - x^2 + xy + xy - y^2$$

Now,

$$9 - x^2 + xy + 3x - 3x + 3y - 3y + xy - y^2$$

Re – arranging the above terms, we get,

$$9 - 3x + 3y + 3x - x^2 + xy + xy - 3y - y^2$$

Take out common in all terms,

$$3(3 - x + y) + x(3 - x + y) + y(-3 - y + x)$$

$$3(3 - x + y) + x(3 - x + y) - y(3 - x + y)$$

$$(3 - x + y)(3 + x - y)$$

$$(ii) 9x^4 - (x^2 + 2x + 1)$$

Solution:

$$9x^4 - (x^2 + 2x + 1)$$

Above terms can be written as,

$$(3x^2)^2 - (x + 1)^2 \text{ [because } (a + b)^2 = a^2 + 2ab + b^2 \text{]}$$

$$\text{We know that, } a^2 - b^2 = (a + b)(a - b)$$

$$\text{So, } (3x^2 + x + 1)(3x^2 - x - 1)$$

Question 11.

$$(i) 9x^4 - x^2 - 12x - 36$$

Solution:

$$9x^4 - x^2 - 12x - 36$$

Above terms can be written as,

$$9x^4 - (x^2 + 12x + 36)$$

$$\text{We know that, } (a + b)^2 = a^2 + 2ab + b^2$$

$$(3x^2)^2 - (x^2 + (2 \times 6 \times x) + 6^2)$$

$$\text{So, } (3x^2)^2 - (x + 6)^2$$

$$\text{We know that, } a^2 - b^2 = (a + b)(a - b)$$

$$(3x^2 + x + 6)(3x^2 - x - 6)$$

$$(ii) x^3 - 5x^2 - x + 5$$

Solution :

$$x^3 - 5x^2 - x + 5$$

take out common in all terms,

$$x^2 (x - 5) - 1 (x - 5)$$

$$(x - 5) (x^2 - 1)$$

$$(x - 5) (x^2 - 1)$$

We know that, $a^2 - b^2 = (a + b) (a - b)$

$$(x - 5) (x + 1) (x - 1)$$

Question 12.

$$(i) a^4 - b^4 + 2b^2 - 1$$

Solution:

$$a^4 - b^4 + 2b^2 - 1$$

above terms can be written as,

$$a^4 - (b^4 - 2b^2 + 1)$$

we know that, $(a - b)^2 = a^2 - 2ab + b^2$

$$a^4 - ((b^2)^2) - (2 \times b^2 \times 1) + 1^2)$$

$$(a^2)^2 - (b^2 - 1)^2$$

We know that, $a^2 - b^2 = (a + b) (a - b)$

$$(a^2 + b^2 - 1) (a^2 - b^2 + 1)$$

(ii) $x^3 - 25x$

Solution;

$$X^3 - 25x$$

Take out common in all terms,

$$X (x^2 - 25)$$

Above terms can be written as,

$$X (x^2 - 5^2)$$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$$X (x + 5)(x - 5)$$

Question 13.

(i) $2x^4 - 32$

Solution:

$$2x^4 - 32$$

Take out common in all terms,

$$2(x^4 - 16)$$

Above terms can be written as,

$$2((x^2)^2 - 4^2)$$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$$2(x^2 + 4)(x^2 - 4)$$

$$2(x^2 + 4)(x^2 - 2^2)$$

$$2(x^2 + 4)(x + 2)(x - 2)$$

$$(ii) a^2 (b + c) - (b + c)^3$$

Solution:

$$a^2 (b + c) - (b + c)^3$$

take out common in all terms,

$$(b + c) (a^2 - (b + c)^3)$$

We know that , $a^2 - b^2 = (a + b) (a - b)$

$$(b + c) (a + b + c) (a - b - c)$$

Question 14.

$$(i) (a + b)^3 - a - b$$

Solution:

$$(a + b)^3 - a - b$$

Above terms can be written as,

$$(a + b)^3 - (a + b)$$

Take out common in all terms,

$$(a + b) [(a + b)^2 - 1]$$

$$(a + b) [(a + b)^2 - 1]$$

We know that, $a^2 - b^2 = (a + b) (a - b)$

$$(a + b) (a + b + 1) (a + b - 1)$$

$$(ii) X^2 - 2xy + y^2 - a^2 - 2ab - b^2$$

Solution:

$$X^2 - 2xy + y^2 - a^2 - 2ab - b^2$$

Above terms can be written as,

$$(x^2 - 2xy + y^2) - (a^2 + 2ab + b^2)$$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$

$$(x^2 - (2 \times x \times y) + y^2) - (a^2 + (2 \times a \times b) + b^2)$$

$$(x - y)^2 - (a + b)^2$$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$$[(x - y) + (a + b)][(x - y) - (a + b)]$$

$$(x - y + a + b)(x - y - a - b)$$

Question 15.

$$(i) (a^2 - b^2)(c^2 - d^2) - 4abcd$$

Solution:

$$(a^2 - b^2)(c^2 - d^2) - 4abcd$$

$$A^2(c^2 - d^2) - b^2(c^2 - d^2) - 4abcd$$

$$A^2c^2 - a^2d^2 - b^2c^2 + b^2d^2 - 4abcd$$

$$A^2c^2 + b^2d^2 - a^2d^2 - b^2c^2 - 2abcd - 2abcd$$

Re arranging the above terms, we get,

$$A^2c^2 + b^2d^2 - 2abcd - a^2d^2 - b^2c^2 - 2abcd$$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$

$$(ac - bd)^2 - (ad - bc)^2$$

$$(ac - bd + ad - bc)(ac - bd - ad + bc)$$

$$(ii) 4x^2 - y^2 - 3xy + 2x - 2y$$

Solution:

$$4x^2 - y^2 - 3xy + 2x - 2y$$

Above terms can be written as,

$$X^2 + 3x^2 - y^2 - 3xy + 2x - 2y$$

Rearranging the above terms, we get,

$$(x^2 - y^2) + (3x^2 - 3xy) + (2x - 2y)$$

We know that, $a^2 - b^2 = (a + b)(a - b)$ and take out common terms,

$$(x + y)(x - y) + 3x(x - y) + 2(x - y)$$

$$(x - y)[(x + y) + 3x + 2]$$

$$(x - y)(x + y + 3x + 2)$$

$$(x - y)(4x + y + 2)$$

Question 16.

$$(i) x^2 + \frac{1}{x^2} - 11$$

Solution:

$$x^2 + \frac{1}{x^2} - 11$$

above terms can be written as,

$$x^2 + \frac{1}{x^2} - 2 - 9$$

$$\text{then , } (x^2 + \frac{1}{x^2}) - 2 - 3^2$$

$$\text{we know that, } (a - b)^2 = a^2 - 2ab + b^2 ,$$

$$(x^2 - (2 \times x^2 \times \frac{1}{x^2}) + (\frac{1}{x})^2$$

$$(x - \frac{1}{x})^2 - 3^2$$

$$\text{We know that, } a^2 - b^2 = (a + b)(a - b)$$

$$(x - \frac{1}{x} + 3)(x - \frac{1}{x} - 3)$$

$$(ii) x^4 + 5x^2 + 9$$

Solution:

$$x^4 + 5x^2 + 9$$

$$x^4 + 6x^2 - x^2 + 9$$

$$(x^4 + 6x^2 + 9) - x^2$$

$$((x^2)^2 + (2 \times x^2 \times 3) + 3^2$$

$$\text{We know that, } (a + b)^2 = a^2 + 2ab + b^2,$$

$$((x^2)^2 + (2 \times x^2 \times 3) + 3^2)$$

$$\text{So, } (x^2 + 3)^2 - x^2$$

$$\text{We know that , } a^2 - b^2 = (a + b)(a - b)$$

$$(x^2 + 3 + x)(x^2 + 3 - x)$$

Question 17.

(i) $a^4 + b^4 - 7a^2b^2$

Solution:

$$A^4 + b^4 - 7a^2b^2$$

Above terms can be written as,

$$A^4 + b^4 + 2a^2b^2 - 9a^2b^2$$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$

$$[(a^2)^2 + (b^2)^2 + (2 \times a^2 \times b^2)] - (3ab)^2$$

$$(a^2 + b^2)^2 - (3ab)^2$$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$$(a^2 + b^2 + 3ab)(a^2 + b^2 - 3ab)$$

(ii) $x^4 - 14x^2 + 1$

Solution:

$$X^4 - 14x^2 + 1$$

Above terms can be written as,

$$X^4 + 2x^2 + 1 - 16x^2$$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$,

$$\text{So, } [(x^2)^2 + (2 \times x^2 \times 1) + 12] - 16x^2$$

$$(x^2 + 1)^2 - (4x)^2$$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$$(x^2 + 1 + 4x)(x^2 + 1 - 4x)$$

Question 18.

Express each of the following as the difference of two squares:

(i) $(x^2 - 5x + 7)(x^2 + 5x + 7)$

Solution:

$$(x^2 - 5x + 7)(x^2 + 5x + 7)$$

Rearranging the above terms, we get,

$$((x^2 + 7) - 5x)((x^2 + 7) + 5x)$$

As, we know that, $a^2 - b^2 = (a + b)(a - b)$

$$\text{So, } (x^2 + 7)^2 - (5x)^2$$

$$(x^2 + 7)^2 - 25x^2$$

(ii) $(x^2 - 5x + 7)(x^2 - 5x - 7)$

Solution :

$$(x^2 - 5x + 7)(x^2 - 5x - 7)$$

$$[(x^2 - 5x + 7)((x^2 - 5x) - 7)]$$

As, we know that, $a^2 - b^2 = (a + b)(a - b)$

$$(x^2 - 5x)^2 - 7^2$$

$$(x^2 - 5x)^2 - 49$$

(iii) $(x^2 + 5x - 7)(x^2 - 5x + 7)$

Solution:

$$(x^2 + 5x - 7)(x^2 - 5x + 7)$$

$$[x^2 + (5x - 7)(x^2 - (5x - 7))]$$

As, we know that, $a^2 - b^2 = (a + b)(a - b)$

$$X^2 - (5x - 7)^2$$

We know that, $(a - b)^2 = a^2 - 2ab + b^2$.

$$X^2 - [(5x)^2 - (2 \times 5x \times 7) + 7^2]$$

$$X^2 - (25x^2 - 70x + 49)$$

$$X^2 - 25x^2 + 70x - 49$$

$$-24x^2 + 70x - 49$$

Question 19.

Evaluate the following by using factors:

$$(i) (979)^2 - (21)^2$$

$$(ii) (99.9)^2 - (0.1)^2$$

Solution:

$$(i) (979)^2 - (21)^2$$

We know that

$$= (979 + 21)(979 - 21)$$

So we get

$$= 1000 \times 958$$

$$= 958000$$

$$(ii) (99.9)^2 - (0.1)^2$$

= we know that

$$= (99.9 + 0.1) (99.9 - 0.1)$$

So we get

$$= 100 \times 99.8$$

$$= 9980$$

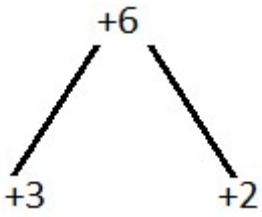
Exercise 4.4

Factorise the following (1 to 18) :

Question 1.

(i) $x^2 + 5x + 6$

Solution:


$$x^2 + 3x + 2x + 6$$

take out common in all terms we get,

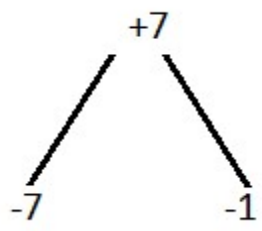
$$x(x + 3) + 2(x + 3)$$

$$(x + 3)(x + 2)$$

(ii) $x^2 - 8x + 7$

Solution :

$$x^2 - 8x + 7$$



$$x^2 - 7x - x + 7$$

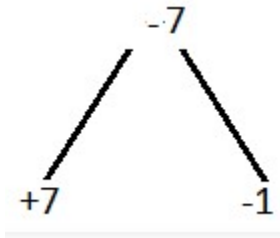
Take out common in all terms we get,

$$x(x - 7) - 1(x - 7)$$

$$(x - 7)(x - 1)$$

Question 2.

(i) $x^2 + 6x - 7$



$$x^2 + 6x - 7$$

$$x^2 + 7x - x - 7$$

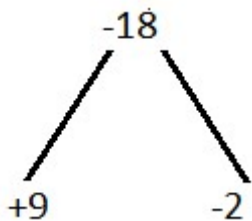
take out common in all terms we get,

$$x(x + 7) - 1(x + 7)$$

$$(x + 7)(x - 1)$$

(ii) $y^2 + 7y - 18$

Solution :



$$y^2 + 7y - 18$$

$$y^2 + 9y - 2y - 18$$

take out common in all terms we get,

$$y(y + 9) - 2(y + 9)$$

$$(y + 9)(y - 2)$$

Question 3.

(i) $y^2 - 7y - 18$

Solution:

$$Y^2 - 7y - 18$$

$$Y^2 + 2y - 9y - 18$$

Take out common in all terms we get,

$$Y(y + 2) - 9(y + 2)$$

$$(y + 2)(y - 9)$$

(ii) $a^2 - 3a - 54$

Solution:

$$a^2 - 3a - 54$$

$$a^2 + 6a - 9a - 54$$

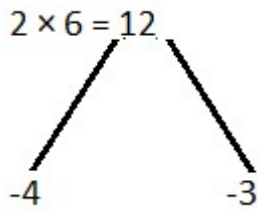
take out common in all terms we get,

$$a(a + 6) - 9(a + 6)$$

so, $(a + 6)(a - 9)$

Question 4.

(i) $2x^2 - 7x + 6$



$$2x^2 - 7x + 6$$

$$2x^2 - 4x - 3x + 6$$

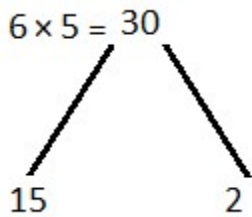
Take out common in all terms we get,

$$2x(x - 2) - 3(x - 2)$$

$$(x - 2)(2x - 3)$$

(ii) $6x^2 + 13x - 5$

Solution:



$$6x^2 + 15x - 2x - 5$$

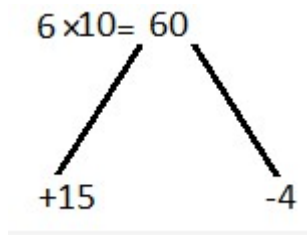
Take out common in all terms we get,

$$3x(2x + 5) - 1(2x + 5)$$

$$(2x + 5)(3x - 1)$$

Question 5.

$$6x^2 + 11x - 10$$



Solution:

$$6x^2 + 11x - 10$$

$$6x^2 + 15x - 4x - 10$$

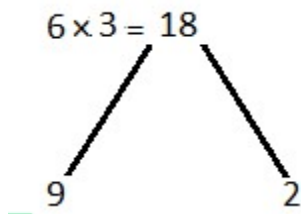
Take out common in all terms we get,

$$3x (2x + 5) -2 (2x + 5)$$

$$(2x + 5) (3x - 2)$$

(ii) $6x^2 - 7x - 3$

Solution:



$$6x^2 - 9x + 2x - 3$$

Take out common in all terms we get,

$$3x (2x - 3) +1(2x - 3)$$

$$(2x - 3) (3x + 1)$$

Question 6.

(i) $2x^2 - x - 6$

Solution: $2x^2 - x - 6$

$$2x^2 - 4x + 3x - 6$$

Take out common in all terms we get,

$$2x(x - 2) + 3(x - 2)$$

$$(x - 2)(2x + 3)$$

(ii) $1 - 18y - 63y^2$

Solution:

$$1 - 18y - 63y^2$$

$$1 - 21y + 3y - 63y^2$$

Take out common in all terms we get,

$$1(1 - 21y) + 3y(1 - 21y)$$

$$(1 - 21y)(1 + 3y)$$

Question 7

(i) $2y^2 + y - 45$

Solution:

$$2y^2 + y - 45$$

$$2y^2 + 10y - 9y - 45$$

Take out common in all terms we get,

$$2y (y + 5) - 9 (y + 5)$$

$$(y + 5) (2y - 9)$$

$$(ii) 5 - 4x - 12x^2$$

Solution :

$$5 - 4x - 12x^2$$

$$5 - 10x + 6x - 12x^2$$

Take out common in all terms we get,

$$5(1-2x) + 6x (1 - 2x)$$

$$(1 - 2x) (5 + 6x)$$

Question 8.

$$(i) x(12x + 7) - 10$$

Solution:

$$x(12x + 7) - 10$$

above terms can be written as,

$$12x^2 + 7x - 10$$

$$12x^2 + 15x - 8x - 10$$

Take out common in all terms we get,

$$3x (4x + 5) -2(4x + 5)$$

$$(4x + 5) (3x - 2)$$

$$(ii) (4 - x)^2 - 2x$$

Solution:

$$(4 - x)^2 - 2x$$

We know that, $(a - b)^2 = a^2 - 2ab + b^2$

$$\text{So, } (4^2 - (2 \times 4 \times x) + x^2) - 2x$$

$$16 - 8x + x^2 - 2x$$

$$x^2 - 10x + 16$$

$$x^2 - 8x - 2x + 16$$

Take out common in all terms we get,

$$x(x - 8) - 2(x - 8)$$

$$(x - 8)(x - 2)$$

Question 9.

$$(i) 60x^2 - 70x - 30$$

Solution:

$$60x^2 - 70x - 30$$

Take out common in all terms we get,

$$10(6x^2 - 7x - 3)$$

$$10(6x^2 - 9x + 2x - 3)$$

Again, take out common in all terms we get,

$$10(3x(2x - 3) + 1(2x - 3))$$

$$10(2x - 3)(3x + 1)$$

$$(ii) x^2 - 6xy - 7y^2$$

Solution:

$$X^2 - 6xy - 7y^2$$

$$X^2 - 7xy + xy - 7y^2$$

Take out common in all terms we get,

$$X(x - 7y) + y(x - 7y)$$

$$(x - 7y)(x + y)$$

Question 10.

$$(i) 2x^2 + 13xy - 24y^2$$

Solution:

$$2x^2 + 13xy - 24y^2$$

$$2x^2 + 16xy - 3xy - 24y^2$$

Take out common in all terms we get,

$$2x(x + 8y) - 3y(x + 8y)$$

$$(x + 8y)(2x - 3y)$$

$$(ii) 6x^2 - 5xy - 6y^2$$

Solution:

$$6x^2 - 5xy - 6y^2$$

$$6x^2 - 9xy + 4xy - 6y^2$$

Take out common in all terms we get,

$$3x (2x - 3y) + 2y(2x - 3y)$$

$$(2x - 3y) (3x + 2y)$$

Question 11.

$$(i) 5x^2 + 17xy - 12y^2$$

Solution:

$$5x^2 + 17xy - 12y^2$$

$$5x^2 + 20xy - 3xy - 12y^2$$

Take out common in all terms we get,

$$5x (x + 4y) - 3y(x + 4y)$$

$$(x + 4y) (5x - 3y)$$

$$(x + 4y) (5x - 3y)$$

$$(ii) x^2y^2 - 8xy - 48$$

Solution:

$$X^2y^2 - 8xy - 48$$

$$X^2y^2 - 12xy + 4xy - 48$$

Take out common in all terms we get,

$$Xy (xy - 12) + 4 (xy - 12)$$

$$(xy - 12) (xy + 4)$$

Question 12.

(i) $2a^2b^2 - 7ab - 30$

Solution:

$$2a^2b^2 - 7ab - 30$$

$$2a^2b^2 - 12ab + 5ab - 30$$

Take out common in all terms we get,

$$2ab (ab - 6) + 5 (ab - 6)$$

$$(ab - 6) (2ab + 5)$$

(ii) $a (2a - b) - b^2$

Solution:

$$(2a - b) - b^2$$

Above terms can be written as,

$$2a^2 - ab - b^2$$

$$2a^2 - 2ab + ab - b^2$$

Take out common in all terms we gets,

$$2a (a - b) + b (a - b)$$

$$(a - b) (2a + b)$$

Question 13.

$$(i) (x-y)^2 - 6(x-y) + 5$$

Solution:

$$(x-y)^2 - 6(x-y) + 5$$

Above terms can be written as,

$$(x-y)^2 - 5(x-y) - (x-y) + 5$$

$$(x-y)(x-y-5) - 1(x-y-5)$$

Then ,

$$(x-y-5)(x-y-1)$$

$$(ii) (2x-y)^2 - 11(2x-y) + 28$$

Solution:

$$(2x-y)^2 - 11(2x-y) + 28$$

Above terms can be written as,

$$(2x-y)^2 - 7(2x-y) - 4(2x-y) + 28$$

$$(2x-y)(2x-y-7) - 4(2x-y-7)$$

$$(2x-y-7)(2x-y-4)$$

Question 14.

$$(i) 4(a-1)^2 - 4(a-1) - 3$$

Solution:

$$4(a-1)^2 - 4(a-1) - 3$$

Above terms can be written as,

$$4(a-1)^2 - 6(a-1) + 2(a-1) - 3$$

Take out common in all terms we get,

$$2(a-1)[2(a-1)-3] + 1[2(a-1)-3]$$

$$(2(a-1)-3)(2(a-1)+1)$$

$$(2a-2-3)(2a-2+1)$$

$$(2a-5)(2a-1)$$

$$(ii) 1 - 2a - 2b - 3(a+b)^2$$

Solution:

$$1 - 2a - 2b - 3(a+b)^2$$

Above terms can be written as,

$$1 - 2(a+b) - 3(a+b)^2$$

$$1 - 3(a+b) + (a+b) - 3(a+b)^2$$

Take out common in all terms we get,

$$1(1-3(a+b)) + (a+b)(1-(a+b))$$

$$(1-3(a+b))(1+(a+b))$$

$$(1-3a+3b)(1+a+b)$$

Question 15.

(i) $3 - 5a - 12(a + b)^2$

Solution:

Above terms can be written as,

$$3 - 5(a + b) - 12(a + b)^2$$

$$3 - 9(a + b) + 4(a + b) - 12(a + b)^2$$

Take out common in all terms we get,

$$3(1 - 3(a + b)) + 4(a + b)(1 - 3(a + b))$$

$$(1 - 3(a + b))(3 + 4(a + b))$$

$$(1 - 3a - 3b)(3 + 4a + 4b)$$

(ii) $a^4 - 11a^2 + 10$

Solution:

$$A^4 - 10a^2 - a^2 + 10$$

Take out common in all terms we get,

$$A^2(a^2 - 10) - 1(a^2 - 10)$$

$$(a^2 - 10)(a^2 - 1)$$

Question 16.

(i) $(x + 4)^2 - 5xy - 20y - 6y^2$

Solution :

$$(x + 4)^2 - 5xy - 20y - 6y^2$$

Above terms can be written as,

$$(x + 4)^2 - 5y(x + 4) - 6y^2$$

$$(x + 4)^2 - 6y(x + 4) + y(x + 4) - 6y^2$$

Take out common in all terms we get ,

$$(x + 4)(x + 4 - 6y) + y(x + 4 - 6y)$$

$$(x - 6y + 4)(x + 4 + y)$$

$$(ii) (x^2 - 2x^2) - 23(x^2 - 2x) + 120$$

Solution:

$$(x^2 - 2x^2) - 23(x^2 - 2x) + 120$$

Above terms can be written as,

$$(x^2 - 2x)^2 - 15(x^2 - 2x) - 8(x^2 - 2x) + 120$$

Take out common in all terms we get,

$$(x^2 - 2x)(x^2 - 2x - 15) - 8(x^2 - 2x - 15)$$

$$(x^2 - 2x - 15)(x^2 - 2x - 8)$$

Question 17.

$$4(2a - 3)^2 - 3(2a - 3)(a - 1) - 7(a - 1)^2$$

Solution:

$$4(2a - 3)^2 - 3(2a - 3)(a - 1) - 7(a - 1)^2$$

Let us assume, $2a - 3 = p$ and $a - 1 = q$

$$\text{So } 4p^2 - 3pq - 7q^2$$

Then,

$$4p^2 - 7pq + 4pq - 7q^2$$

out common in all terms we get,

$$P(4p - 7q) + q(4p - 7q)$$

$$(4p - 7q)(p + q)$$

Now, substitute the value of p and q we get,

$$(4(2a - 3) - 7(a - 1))(2a - 3 + a - 1)$$

$$(8a - 12 - 7a + 7)(3a - 4)$$

$$(a - 5)(3a - 4)$$

Question 18.

$$(2x^2 + 5x)(2x^2 + 5x - 19) + 84$$

Solution:

$$(2x^2 + 5x)(2x^2 + 5x - 19) + 84$$

Let us assume, $2x^2 + 5x = p$

$$\text{So, } (P)(P - 19) + 84$$

$$P^2 - 19p + 84$$

$$P^2 - 12p - 7p + 84$$

$$P(p - 12) - 7(p - 12)$$

$$(p - 12)(p - 7)$$

Now, substitute the value of p we get,

$$(2x^2 + 5x - 12)(2x^2 + 5x - 7)$$

Exercise 4.5

Factorise the following (1 to 13) :

Question 1.

(i) $8x^3 + y^3$

Solution:

$$8x^3 + y^3$$

Above terms can be written as,

$$(2x)^3 + y^3$$

$$\text{We know that, } a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\text{Where, } a = 2x, b = y$$

$$\begin{aligned}\text{Then, } (2x)^3 + y^3 &= (2x + y)(2x)^2 - (2x \times y) + y^2 \\ &= (2x + y)(4x^2 - 2xy + y^2)\end{aligned}$$

(ii) $64x^3 - 125y^3$

Solution:

$$64x^3 - 125y^3$$

Above terms can be written as,

$$(4x)^3 - (5y)^3$$

$$\text{We know that, } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\text{Where, } a = 4x, b = 5y$$

$$\text{Then, } (4x)^3 - (5y)^3 = (4x - 5y)(4x)^2 + (4x \times 5y) + 5y^2$$

$$= (4x - 5y)(16x^2 + 20xy + 25y^2)$$

Question 2.

(i) $64x^3 + 1$

Solution:

$$64x^3 + 1$$

Above terms can be written as,

$$(4x)^3 + 1^3$$

$$\text{We know that, } a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\text{Where } a = 4x, b = 1$$

$$\text{Then, } (4x)^3 + 1^3 = (4x + 1)((4x)^2 - (4x \times 1) + 1)$$

$$= (4x + 1)(16x^2 - 4x + 1)$$

(ii) $7a^3 + 56b^3$

Solution:

$$7a^3 + 56b^3$$

Take out common in all terms we get,

$$7(a^3 + 8b^3)$$

Above terms can be written as,

$$7(a^3 + (2b)^3)$$

$$\text{We know that, } a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\text{Where, } a = a, b = 2b$$

$$\begin{aligned}\text{Then , } 7[(a)^3 + (2b)^3] &= 7[(a + 2b) (a)^2 - (a \times 2b) + (2b)^2] \\ &= 7(a + 2b) (a^2 - 2ab + 4b^2)\end{aligned}$$

Question 3.

$$(i) \left(\frac{x^6}{343}\right) + \left(\frac{343}{x^6}\right)$$

Solution:

$$\left(\frac{x^6}{343}\right) + \left(\frac{343}{x^6}\right)$$

Above terms can be written as,

$$\left(\frac{x^2}{7}\right)^3 + \left(\frac{7}{x^2}\right)^3$$

We know that, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$

Where , a =

$$a = \left(\frac{x^2}{7}\right) \quad b = \left(\frac{7}{x^2}\right)$$

Then

$$\begin{aligned}\left(\frac{x^2}{7}\right)^3 + \left(\frac{7}{x^2}\right)^3 &= \left[\left(\frac{x^2}{7}\right) + \left(\frac{7}{x^2}\right)\right] \left[\left(\frac{x^2}{7}\right)^2 - \left(\frac{x^2}{7}\right) \times \left(\frac{7}{x^2}\right) + \left(\frac{7}{x^2}\right)^2\right] \\ &= \left[\left(\frac{x^2}{7}\right) + \left(\frac{7}{x^2}\right)\right] \left[\left(\frac{x^4}{49}\right) - 1 + \left(\frac{49}{x^4}\right)\right]\end{aligned}$$

$$(ii) 8x^3 - \frac{1}{27}y^3$$

Solution:

$$8x^3 - \frac{1}{27}y^3$$

Above terms can be written as,

$$(2x)^3 - \left(\frac{1}{3y}\right)^3$$

We know that , $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Where, $a = 2x$, $b = \frac{1}{3y}$

$$\text{Then , } (2x)^3 - \left(\frac{1}{3y}\right)^3 = \left(2x - \frac{1}{3y}\right) \left((2x)^2 + \left(2x \times \frac{1}{3y}\right) + \left(\frac{1}{3y}\right)^2\right)$$

$$= \left(2x - \frac{1}{3y}\right) \left(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right)$$

Question 4.

(i) $x^2 + x^5$

Solution:

$$x^2 + x^5$$

Take out common in all terms we get,

$$x^2 (1 + x^3)$$

$$x^2 (1^3 + x^3)$$

We know that, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Where , $a = 1$, $b = x$

$$= x^2 [(1 + x)(1^2 - (1 \times x) + x^2)]$$

$$= x^2 (1 + x)(1 - x + x^2)$$

(ii) $32x^4 - 500$

Solution:

$$32x^4 - 500x$$

Take out common in all terms we get,

$$4x (8x^3 - 125)$$

Above terms can be written as,

$$4x (2x)^3 - 5^3$$

$$\text{We know that, } a^3 - b^3 = (a - b) (a^2 + ab + b^2)$$

$$\text{Where, } a = 2x , b = 5$$

$$= 4x (2x - 5) (2x)^2 + (2x \times 5) + 5^2$$

$$= 4x (2x - 5) (4x^2 + 10x + 25)$$

Question 5.

$$(i) 27x^3y^3 - 8$$

Solution:

$$27x^3y^3 - 8$$

Above terms can be written as,

$$(3xy)^3 - 2^3$$

$$\text{We know that, } a^3 - b^3 = (a - b) (a^2 + ab + b^2)$$

$$\text{Where , } a = 3xy , b = 2$$

$$(3xy - 2) (3xy)^2 + (3xy \times 2) + 2^2)$$

$$= (3xy - 2) (9x^2y^2 + 6xy + 4)$$

$$(ii) 27(x + y)^3 + 8(2x - y)^3$$

Solution:

$$27(x + y)^3 + 8(2x - y)^3$$

Above terms can be written as,

$$3^3(x + y)^3 + 2^3(2x - y)^3$$

$$(3(x + y))^3 + (2(2x - y))^3$$

We know that, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Where, $a = 3(x + y)$, $b = 2(2x - y)$

$$= [3(x + y) + 2(2x - y)][(3(x + y))^2 - (3(x + y) \times 2(2x - y)) + (2(2x - y))^2]$$

$$= [3x + 3y + 4x - 2y][9(x + y)^2 - 6(x + y)(2x - y) + 4(2x - y)^2]$$

$$= (7x - y)[9(x^2 + y^2 + 2xy) - 6(2x^2 - xy + 2xy - y^2) + 4(4x^2 + y^2 - 4xy)]$$

$$= (7x - y)[9x^2 + 9y^2 + 18xy - 12x^2 - 6xy - 6y^2 + 16x^2 + 4y^2 - 16xy]$$

$$= (7x - y)[13x^2 - 4xy + 19y^2]$$

Question 6.

$$(i) a^3 + b^3 + a + b$$

Solution :

$$A^3 + b^3 + a + b$$

$$(a^3 + b^3) + (a + b)$$

We know that, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$$[(a + b)(a^2 - ab + b^2)] + (a + b)$$

$$(a + b)(a^2 - ab + b^2 + 1)$$

(ii) $a^3 - b^3 - a + b$

Solution:

$$A^3 - b^3 - a + b$$

$$(a^3 - b^3) - (a - b)$$

We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$[(a - b)(a^2 + ab + b^2)] - (a - b)$$

$$(a - b)(a^2 + ab + b^2 - 1)$$

Question 7.

(i) $x^3 + x + 2$

Solution:

$$x^3 + x + 2$$

above terms can be written as,

$$x^3 + x + 1 + 1$$

rearranging the above terms, we get

$$(x^3 + 1)(x + 1)$$

$$(x^3 + 1^3)(x + 1)$$

We know that, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$$[(x+1)(x^2-x+1)]+(x+1)$$

$$(x+1)(x^2-x+1+1)$$

$$(x+1)(x^2-x+2)$$

$$(ii) a^3 - a - 120$$

Solution:

$$A^3 - a - 120$$

Above terms can be written as,

$$A^3 - a - 125 + 5$$

Rearranging the above terms, we get

$$A^3 - 125 - a + 5$$

$$(a^3 - 125) - (a - 5)$$

$$(a^3 - 5^3) - (a - 5)$$

We know that , $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$[(a - 5)(a^2 + 5a + 5^2)] - (a - 5)$$

$$(a - 5)(a^2 + 5a + 25) - (a - 5)$$

$$(a - 5)(a^2 + 5a + 25 - 1)$$

$$(a - 5)(a^2 + 5a + 24)$$

Question 8.

(i) $x^3 + 6x^2 + 12x + 16$

Solution:

$$x^3 + 6x^2 + 12x + 16$$

$$x^3 + 6x^2 + 12x + 8 + 8$$

above terms can be written as,

$$(x^3 + (3 \times 2 \times x^2) + (3 \times 2^2 \times x) + 2^3) + 8$$

We know that, $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

Now $a = x$ and $b = 2$

So, $(x + 2)^3 + 2^3$

We know that, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$$(x + 2 + 2)((x + 2)^2 - (2 \times (x + 2)) + 2^2)$$

$$(x + 4)(x^2 + 4 + 4x - 2x - 4 + 4)$$

$$(x + 4)(x^2 + 2x + 4)$$

(ii) $a^3 - 3a^2b + 3ab^2 - 2b^3$

Solution :

$$A^3 - 3a^2b + 3ab^2 - 2b^3$$

Above terms can be written as,

$$A^3 - 3a^2b + 3ab^2 - b^3 - b^3$$

We know that, $(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

So, $(a - b)^3 + b^3$

We also know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Where, $a = a - b$, $b = b$

$$(a - b - b)(a - b)^2 + (a - b)b + b^2$$

$$(a - 2b)(a^2 + b^2 - 2ab + ab - b^2 + b^2)$$

$$(a - 2b)(a^2 + b^2 - ab)$$

Question 9.

(i) $2a^3 + 16b^3 - 5a - 10b$

Solution:

$$2a^3 + 16b^3 - 5a - 10b$$

Above terms can be written as,

$$2(a^3 + 8b^3) - 5(a + 2b)$$

$$2(a^3 + (2b)^3) - 5(a + 2b)$$

We know that, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$$2[(a + 2b)(a^2 - 2ab + 4b^2)] - 5(a + 2b)$$

$$(a + 2b)(2a^2 - 4ab + 8b^2 - 5)$$

(ii) $a^3 - \frac{1}{a^3} - 2a + \frac{2}{a}$

Solution:

$$a^3 - \frac{1}{a^3} - 2a + \frac{2}{a}$$

$$= a^3 - \left(\frac{1}{a}\right)^3 - 2a + \frac{2}{a}$$

We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$\left[\left(a - \frac{1}{a}\right) - \left(a^2 + \left(a \times \frac{1}{a}\right) + \left(\frac{1}{a}\right)^2\right)\right] - 2\left(a - \frac{1}{a}\right)$$

$$\left(a - \frac{1}{a}\right)\left(a^2 + 1 + \frac{1}{a^2}\right) - 2\left(a - \frac{1}{a}\right)$$

$$\left(a - \frac{1}{a}\right)\left(a^2 + 1 + \frac{1}{a^2} - 2\right)$$

$$\left(a - \frac{1}{a}\right)\left(a^2 + \frac{1}{a^2} - 1\right)$$

Question 10.

(i) $a^6 - b^6$

Solution:

$$A^6 - b^6$$

Above terms can be written as,

$$(a^2)^3 - (b^2)^3$$

We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So, $a = a^2, b = b^2$

$$(a^2 - b^2)((a^2)^2 + a^2b^2 + (b^2)^2)$$

$$(a^2 - b^2)(a^4 + a^2b^2 + b^4)$$

$$(ii) x^6 - 1$$

Solution:

$$X^6 - 1$$

Above terms can be written as,

$$(x^2)^3 - 1^3$$

$$\text{We know that, } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\text{So, } a = x^2, b = 1$$

$$(x^2 - 1)((x^2)^2 + (x^2 \times 1) + 1^2)$$

$$(x^2 - 1)(x^4 + x^2 + 1)$$

Question 11.

$$(i) 64x^6 - 729y^6$$

Solution:

$$64x^6 - 729y^6$$

Above terms can be written as,

$$(2x)^6 - (3y)^6$$

$$[(2x)^2]^3 - [(3y)^2]^3$$

$$\text{We know that, } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\text{So, } a = (2x)^2, b = (3y)^2$$

$$[(2x)^2 - (3y)^2][(2x)^2 + ((2x)^2 \times (3y)^2 + ((3y)^2)^2)]$$

$$(4x^2 - 9y^2)[16x^4 + (4x^2 \times 9y^2) + (9y^2)^2]$$

$$(4x^2 - 9y^2)[16x^4 + 36x^2y^2 + 81y^4][(2x)^2 - (3y)^2][16x^4 + 36x^2y^2 + 81y^4]$$

$$(2x + 3y) (2x - 3y) (16x^4 + 36x^2y^2 + 81y^4)$$

$$(ii) x^3 - \frac{8}{x}$$

Solution:

$$x^3 - \frac{8}{x}$$

Above terms be written as,

$$\frac{1}{x} (x^3 - 8)$$

$$\frac{1}{x} [x^3 - 2^3]$$

We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So, $a = x$, $b = 2$

$$\frac{1}{x} (x - 2)(x^2 + 2x + 4)$$

Question 12.

$$(i) 250(a - b)^3 + 2$$

Solution:

$$250(a - b)^3 + 2$$

Take out common in all terms we get,

$$2(125(a - b)^3 + 1)$$

$$2[(5(a - b))^3 + 1^3]$$

$$\begin{aligned}
&\text{We know that, } a^3 + b^3 = (a + b)(a^2 - ab + b^2) \\
&= 2[(5a - 5b + 1)(5a - 5b)^2 - (5a - 5b)(1 + 12)] \\
&= 2(5a - 5b + 1)(25a^2 + 25b^2 - 50ab - 5a + 5b + 1)
\end{aligned}$$

$$(ii) 32a^2x^2 - 8b^2x^3 - 4a^2y^3 + b^2y^3$$

Solution:

$$32a^2x^2 - 8b^2x^3 - 4a^2y^3 + b^2y^3$$

Take out common in all terms we get,

$$\begin{aligned}
&8x^3(4a^2 - b^2) - y^3(4a^2 - b^2) \\
&(4a^2 - b^2)(8x^3 - y^3)
\end{aligned}$$

Above terms can be written as,

$$((2a)^2 - b^2)((2x)^3 - y^3)$$

We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ and $(a^2 - b^2) = (a + b)(a - b)$

$$\begin{aligned}
&(2a + b)(2a - b)[(2x - y)((2x)^2 + 2xy + y^2)] \\
&(2a + b)(2a - b)(2x - y)(4x^2 + 2xy + y^2)
\end{aligned}$$

Question 13.

$$(i) x^9 + y^9$$

Solution:

$$X^9 + y^9$$

Above terms can be written as,

$$(x^3)^3 + (y^3)^3$$

We know that, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Where, $a = x^3$, $b = y^3$

$$(x^3 + y^3)((x^3)^2 - x^3y^3 + (y^3)^2)$$

$$(x^3 + y^3)(x^6 - x^3y^3 + y^6)$$

Then, $(x^3 + y^3)$ in the form of $(a^3 + b^3)$

$$(x + y)(x^2 - xy + y^2)(x^6 - x^3y^3 + y^6)$$

$$(ii) x^6 - 7x^3 - 8$$

Solution:

$$X^6 - 7x^3 - 8$$

Above terms can be written as,

$$(x^2)^3 - 7x^3 - x^3 + x^3 - 8$$

$$(x^2)^3 - 8x^3 + x^3 - 2^3$$

$$(((x^2)^3 - (2x)^3) + (x^3 - 2^3))$$

We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$(x^2 - 2x)((x^2)^2 + (x^2 \times 2x) + (2x)^2) + (x - 2)(x^2 + 2x + 2^2)$$

$$X(x - 2)x^2(x^2 + 2x + 4) + (x - 2)(x^2 + 2x + 4)$$

Take out common in all terms we get,

$$(x - 2)(x^2 + 2x + 4)((x \times x^2) + 1)$$

$$(x - 2)(x^2 + 2x + 4)(x^3 + 1)$$

So, above terms are in the form of $a^3 + b^3$

$$\text{Therefore, } (x - 2)(x^2 + 2x + 4)(x + 1)(x^2 - x + 1)$$

Chapter test

Factorise the following (1 to 12) :

Question 1.

(i) $15 (2x - 3)^3 - 10 (2x - 3)$

Solution :

$$15 (2x - 3)^3 - 10 (2x - 3)$$

Take out common in both terms,

$$\text{Then, } 5(2x - 3) [3(3(2x - 3)^2 - 2)]$$

(ii) $a(a - c) (b + c) - d (c - d)$

Solution:

$$a (b - c) (b + c) - d (c - b)$$

above terms can be written as,

$$a(b - c) (b + c) + d (b - c)$$

take out common in both terms,

$$(b - c) [a (b + c) + d]$$

$$(b - c) (ab + ac + d)$$

Question 2.

(i) $2a^2x - bx + 2a^2 - b$

Solution:

$$2a^2x - bx + 2a^2 - b$$

Rearrange the above terms we get,

$$2a^2x + 2a - bx - b$$

Take out common in both terms,

$$2a^2 (x + 1) - b (x + 1)$$

$$(x + 1) (2a^2 - b)$$

$$(ii) p^2 - a (a + 2b) p + 2ab$$

Solution:

$$p^2 - a (a + 2b) p + 2ab$$

above terms can be written as,

$$p^2 - ap - 2bp + 2ab$$

take out common in both terms,

$$p(p - a) - 2b(p - a)$$

$$(p - a) (p - 2b)$$

Question 3.

$$(i) (x^2 - y^2) z + (y^2 - z^2) x$$

Solution:

$$(x^2 - y^2) z + (y^2 - z^2) x$$

Above terms can be written as,

$$Zx^2 - zy^2 + xy^2 - xz^2$$

Re arranging the above terms we get,

$$Zx^2 - xz^2 + xy^2 - zy^2$$

Take out common in both terms,

$$Zx (x - z) + y^2 (x - z)$$

$$(x - z) (zx + y^2)$$

$$(ii) 5a^4 - 5a^3 + 30a^2 - 30a$$

Solution:

$$5a^4 - 5a^3 + 30a^2 - 30a$$

Take out common in both terms,

$$5a (a^3 - a^2 + 6a - 6)$$

$$5a [a^2 (a - 1) + 6(a - 1)]$$

$$5a (a - 1) (a^2 + 6)$$

Question 4.

$$(i) b(c - d)^2 + a(d - c) + 3c - 3d$$

Solution:

$$b(c - d)^2 + a(d - c) + 3c - 3d$$

above terms can be written as,

$$b(c - d)^2 - a(c - d) + 3c - 3d$$

$$b(c - d)^2 - a(c - d) + 3(c - d)$$

take out common in both terms,

$$(c - d) [b(c - d) - a + 3]$$

$$(c - d) (bc - bd - a + 3)$$

$$(ii) x^3 - x^2 - xy + x + y - 1$$

Solution:

$$x^3 - x^2 - xy + x + y - 1$$

re arrange the above terms we get,

$$x^3 - x^2 - xy + y + x - 1$$

take out common in both terms,

$$x^2(x - 1) - y (x - 1) + 1(x - 1)$$

$$(x - 1) (x^2 - y + 1)$$

Question 5.

$$(i) x (x + z) - y(y + z)$$

Solution:

$$x (x + z) - y(y + z)$$

$$x^2 + xz - y^2 - yz$$

re arrange the above terms we get,

$$x^2 - y^2 + xz - yz$$

we know that, $(a^2 - b^2) = (a + b) (a - b)$

so , $(x + y) (x - y) + z(x - y)$

$$(x - y) (x + y + z)$$

$$(ii) a^{12}x^4 - a^4x^{12}$$

Solution:

$$a^{12}x^4 - a^4x^{12}$$

take out common in both terms,

$$a^4x^4 (a^8 - x^8)$$

$$a^4x^4((a^4)^2 - (x^4)^2)$$

we know that, $(a^2 - b^2) = (a + b)(a - b)$

$$a^4x^4(a^4 + x^4)(a^4 - x^4)$$

$$a^4x^4(a^4 + x^4)((a^2)^2 - (x^2)^2)$$

$$a^4x^4(a^4 + x^4)(a^2 + x^2)(a^2 - x^2)$$

$$a^4x^4(a^4 + x^4)(a^2 + x^2)(a + x)(a - x)$$

Question 6.

$$(i) 9x^2 + 12x + 4 - 16y^2$$

Solution:

$$9x^2 + 12x + 4 - 16y^2$$

Above terms can be written as,

$$(3x)^2 + (2 \times 3x \times 2) + 2^2 - 16y^2$$

$$\text{Then, } (3x + 2)^2 - (4y)^2$$

$$(3x + 2 + 4y)(3x + 2 - 4y)$$

$$(ii) x^4 + 3x^2 + 4$$

Solution:

$$X^4 + 3x^2 + 4$$

Above terms can be written as,

$$(x^2)^2 + 3(x^2) + 4$$

$$(x^2)^2 + (2)^2 + 4x^2 - x^2$$

$$(x^2 + 2)^2 - (x^2)$$

We know that,

$$(a^2 - b^2) = (a + b)(a - b)$$

$$(x^2 + 2 + x)(x^2 + 2 - x)$$

$$(x^2 + x + 2)(x^2 - x + 2)$$

Question 7.

$$(i) 21x^2 - 59xy + 40y^2$$

Solution:

$$21x^2 - 59xy + 40y^2$$

By multiplying first and last term we get, $12 \times 40 = 840$

$$\text{Then, } (-35) \times (-24) = 840$$

$$\text{So, } 21x^2 - 35xy - 24xy + 40y^2$$

$$7x(3x - 5y) - 8y(3x - 5y)$$

$$(3x - 5y)(7x - 8y)$$

(ii) $4x^3y - 44x^2y + 112xy$

solution:

take out common in all terms,

$$4xy(x^2 - 11x + 28)$$

$$\text{Then } 4xy(x^2 - 7x - 4x + 28)$$

$$4xy[x(x - 7) - 4(x + 7)]$$

$$4xy(x - 7)(x - 4)$$

Question 8.

(i) $x^2y^2 - xy - 72$

Solution:

$$X^2y^2 - xy - 72$$

$$X^2y^2 - 9xy + 8xy - 72$$

Take out common in all terms,

$$Xy(xy - 9) + 8(xy - 9)$$

$$(xy - 9)(xy + 8)$$

(ii) $9x^3y + 41x^2y^2 + 20xy^3$

Solution:

$$9x^3y + 41x^2y^2 + 20xy^3$$

Take out common in all terms,

$$Xy(9x^2 + 41xy + y^2)$$

Above terms can be written as,

$$Xy(9x^2 + 36xy + 5xy + 20y^2)$$

$$Xy [9x(x + 4y) + 5y(x + 4y)]$$

$$Xy (x + 4y) (9x + 5y)$$

Question 9.

$$(i) (3a - 2b)^2 + 3(3a - 2b) - 10$$

Solution:

$$(3a - 2b)^2 + 3(3a - 2b) - 10$$

Let us assume, $(3a - 2b) = p$

$$p^2 + 3p - 10$$

$$p^2 + 5p - 2p - 10$$

Take out common in all terms,

$$p(p + 5) - 2(p + 5)$$

$$(p + 5) (p - 2)$$

Now, substitute the value of p

$$(3a - 2b + 5) (3a - 2b - 2)$$

$$(ii) (x^2 - 3x) (x^2 - 3x + 7) + 10$$

Solution:

$$(x^2 - 3x) (x^2 - 3x + 7) + 10$$

Let us assume, $(x^2 - 3x) = q$

$$q (q + 7) + 10$$

$$q^2 + 7q + 10$$

$$q^2 + 5q + 2q + 10$$

$$q(q + 5) + 2(q + 5)$$

$$(q + 5) (q + 2)$$

Now, substitute the value of q

$$(x^2 - 3x + 5) (x^2 - 3x + 2)$$

Question 10.

$$(i) (x^2 - x) (4x^2 - 4x - 5) - 6$$

Solution:

$$(x^2 - x) (4x^2 - 4x - 5) - 6$$

$$(x^2 - x) [(4x^2 - 4x) - 5] - 6$$

$$(x^2 - x) [4(x^2 - x) - 5] - 6$$

Let us assume $x^2 - x = q$

$$\text{So, } q [4q - 5] - 6$$

$$4q^2 - 5q - 6$$

$$4q^2 - 5q - 6$$

$$4q^2 - 8q + 3q - 6$$

$$4q (q - 2) + 3 (q - 2)$$

$$(q - 2) (4q + 3)$$

Now, substitute the value of q

$$(x^2 - x - 2)(4(x^2 - x) + 3)$$

$$(x^2 - x - 2)(4x^2 - 4x + 3)$$

$$(x^2 - 2x + x - 2)(4x^2 - 4x + 3)$$

$$[x(x - 2) + 1(x - 2)](4x^2 - 4x + 3)$$

$$(x - 2)(x + 1)(4x^2 - 4x + 3)$$

$$(ii) x^4 + 9x^2y^2 + 81y^4$$

Solution :

$$x^4 + 9x^2y^2 + 81y^4$$

above terms can be written as,

$$((x^2)^2 + (2 \times x^2 \times 9y^2) + (9y^2)^2 - 9x^2y^2)$$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$

$$(x^2 + 9y^2)^2 - (3xy)^2$$

$$(x^2 + 9y^2 + 3xy)(x^2 + 9y^2 - 3xy)$$

Question 11.

$$(i) \left(\frac{8}{27}\right)x^3 - \left(\frac{8}{27}\right)y^3$$

Solution:

$$\left(\frac{8}{27}\right)x^3 - \left(\frac{8}{27}\right)y^3$$

Above terms can be written as,

$$\left(\frac{2}{3}x\right)^3 - \left(\frac{1}{3}y\right)^3$$

We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$\frac{2}{3}x - \frac{1}{2}y \left[\frac{2}{3}x + \frac{2}{3}x \times \frac{1}{2}y + \left(\frac{1}{2}y\right)^2 \right]$$

$$\frac{2}{3}x - \frac{1}{2}y \left[\frac{4}{9}x^2 + \frac{xy}{3} + \frac{y^2}{4} \right]$$

(ii) $x^6 + 63x^3 - 64$

Solution:

$$X^6 + 63x^3 - 64$$

Above terms can be written as,

$$X^6 + 64x^3 - 64$$

Take out common in all terms,

$$X^3 (x^3 + 64) - 1 (x^3 + 64)$$

$$(x^3 + 64)(x^3 - 1)$$

$$(x^3 + 43)(x^3 - 13)$$

We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ and $a^3 + b^3 = (a + b)$

$$(a^2 - ab + b^2)$$

$$\text{So, } (x + 4)[x^2 - 4x + 42](x - 1)[x^2 + x + 12]$$

$$(x + 4)(x^2 - 4x + 16)(x - 1)(x^2 + x + 1)$$

Question 12.

(i) $x^3 + x^2 - \frac{1}{x^2} + \frac{1}{x^3}$

Re arranging the above terms, we get

$$X^3 + \frac{1}{x^3} + x^2 - \frac{1}{x^2}$$

We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ and $(a^2 - b^2) = (a + b)(a - b)$

$$\left(x + \frac{1}{x}\right) \left(x^2 - 1 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right)$$

$$\left(x + \frac{1}{x}\right) \left[x^2 - 1 + \frac{1}{x^2} + x - \frac{1}{x}\right]$$

$$(ii) (x + 1)^6 - (x - 1)^6$$

Solution:

$$(x + 1)^6 - (x - 1)^6$$

Above terms can be written as,

$$((x + 1)^3)^2 - ((x - 1)^3)^2$$

We know that, $(a^2 - b^2) = (a + b)(a - b)$

$$[(x + 1)^3 + (x - 1)^3] [(x + 1)^3 - (x - 1)^3] [(x + 1) + (x - 1)] [(x + 1)^2 - (x - 1)(x + 1) + (x - 1)^2] [(x - 1)] [(x + 1)^2 + (x - 1)(x + 1) + (x - 1)^2]$$

$$(x + 1 + x - 1) [x^2 + 2x + 1 - x^2 + 1 + x^2 + 1 - 2x(x + 1) - x + 1] [x^2 + 2x + 1 + x^2 - 1 + x^2 - 2x + 1]$$

By simplifying we get,

$$2x(x^2 + 3)^2(3x^2 + 1)$$

$$4x(x^2 + 3)(3x^2 + 1)$$

Question 13.

Show that $(97)^3 + (14)^3$ is divisible by 111

Solution:

From the question,

$$(97)^3 + (14)^3$$

$$\text{We know that } a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

So

$$(97 + 14)[(97)^2 - (97 \times 14) + (14)^2]$$

$$111[(97)^2 - (97 \times 14) + (14)^2]$$

Therefore, it is clear that the given expression is divisible by 111

Question 14.

If $a + b = 8$ and $ab = 15$, find the value of $a^4 + a^2b^2 + b^4$.

Solution:

$$A^4 + a^2b^2 + b^4$$

Above terms can be written as,

$$A4 + 2a^2b^2 + b^4 - a^2b^2$$

$$(a^2)^2 + 2a^2b^2 + (b^2)^2 - (ab)^2$$

$$(a^2 + b^2)^2 - (ab)^2$$

$$(a^2 + b^2 + ab)(a^2 + b^2 - ab)$$

$$A + b = 8, ab = 15$$

$$\text{So, } (a + b)^2 = 8^2$$

$$A^2 + 2ab + b^2 = 64$$

$$A^2 + 2(15) + b^2 = 64$$

$$A^2 + b^2 + 30 = 64$$

By transposing ,

$$A^2 + b^2 = 64 - 30$$

$$A^2 + b^2 = 34$$

$$\text{Then , } a^4 + a^2b^2 + b^4$$

$$= (a^2 + b^2 + ab) (a^2 + b^2 - ab)$$

$$= (34 + 15) (34 - 15)$$

$$= 49 \times 19$$

$$= 931$$