

Chapter 8
Indices
Exercise 8

1.

(i) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

Solution:

$$\begin{aligned}& \left(\frac{81}{16}\right)^{-\frac{3}{4}} \\&= \left[\left(\frac{3^4}{2^4}\right)\right]^{-\frac{3}{4}} \\&= \left[\left(\frac{3}{2}\right)^4\right]^{-\frac{3}{4}} \\&= \left(\frac{3}{2}\right)^{-\frac{3}{4} \times 4} \\&= \left(\frac{3}{2}\right)^{-3} \\&= \left(\frac{2}{3}\right)^3 \\&= \frac{2^3}{3^3} \\&= \frac{2 \times 2 \times 2}{3 \times 3 \times 3} \\&= \frac{8}{27}\end{aligned}$$

$$\text{(ii)} \left(1 \frac{61}{64}\right)^{-\frac{2}{3}}$$

Solution:

$$\begin{aligned} & \left(1 \frac{61}{64}\right)^{-\frac{2}{3}} \\ &= \left(\frac{125}{64}\right)^{-\frac{2}{3}} = \left(\frac{5^3}{4^3}\right)^{-\frac{2}{3}} = \left[\left(\frac{5}{4}\right)^3\right]^{-\frac{2}{3}} \\ &= \left(\frac{5}{4}\right)^{3 \times -\frac{2}{3}} \\ &= \left(\frac{5}{4}\right)^{-2} \\ &= \left(\frac{4}{5}\right)^2 \\ &= \frac{16}{25} \end{aligned}$$

$$\text{2. (i) } (2a^{-3}b^2)^3$$

Solution:

$$\begin{aligned} & (2a^{-3}b^2)^3 \\ &= 2^3 a^{-3 \times 3} b^{2 \times 3} \\ &= 8a^{-9}b^6 \end{aligned}$$

$$(ii) \frac{a^{-1} + b^{-1}}{ab^{-1}}$$

Solution:

$$= \frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{ab}} = \frac{\frac{b+a}{ab}}{\frac{1}{ab}} = \frac{b+a}{ab} \times \frac{ab}{1} = a + b$$

$$3. (i) \frac{x^{-1} y^{-1}}{x^{-1} + y^{-1}}$$

Solution:

$$= \frac{\frac{xy^{-1}}{1}}{x} + \frac{1}{y} = \frac{\frac{1}{xy}}{\frac{y+x}{xy}}$$

$$= \frac{1}{x+y}$$

$$(ii) \frac{(4 \times 10^7)(6 \times 10^{-5})}{8 \times 10^{10}}$$

Solution:

$$= \frac{4 \times 10^7 \times 6 \times 10^{-5}}{8 \times 10^{10}}$$

$$= \frac{3 \times 10^{7-5}}{10^{10}}$$

$$= \frac{3 \times 10^2}{10^{10}}$$

$$= \frac{3 \times 10^2}{10^{10}}$$

$$= \frac{3}{10^{10-2}}$$

$$= \frac{3}{10^8}$$

4. (i) $\frac{3a}{b^{-1}} + \frac{2b}{a^{-1}}$

Solution:

$$= \frac{3a}{b^{-1}} + \frac{2b}{a^{-1}}$$

$$= \frac{3a}{\frac{1}{b}} + \frac{2b}{\frac{1}{a}}$$

$$= \frac{3a \times b}{1} + \frac{2b \times a}{1}$$

$$= 3ab + 2ab = 5ab$$

(ii) $5^0 \times 4^{-1} + 8^{\frac{1}{3}}$

Solution:

$$5^0 \times 4^{-1} + 8^{\frac{1}{3}}$$

$$= 1 \times \left(\frac{1}{4}\right) + (2)^{3 \times \frac{1}{3}}$$

$$= \frac{1}{4} + 2$$

$$= \frac{1+8}{4}$$

$$= \frac{9}{4} = 2 \frac{1}{4}$$

5. (i) $\left(\frac{8}{125}\right)^{-\frac{1}{3}}$

Solution:

$$\begin{aligned} & \left(\frac{8}{125}\right)^{-\frac{1}{3}} \\ &= \left[\frac{2 \times 2 \times 2}{5 \times 5 \times 5}\right]^{-\frac{1}{3}} \\ &= \left(\frac{2^3}{5^3}\right)^{-\frac{1}{3}} \\ &= \left(\frac{2}{5}\right)^{3 \times -\frac{1}{3}} \\ &= \left(\frac{2}{5}\right)^{-1} \\ &= \frac{5}{2} = 2\frac{1}{2} \end{aligned}$$

(ii) $(0.027)^{-\frac{1}{3}}$

Solution:

$$\begin{aligned} & (0.027)^{-\frac{1}{3}} \\ &= \left(\frac{27}{1000}\right)^{-\frac{1}{3}} \\ &= \left[\frac{3 \times 3 \times 3}{10 \times 10 \times 10}\right]^{-\frac{1}{3}} \end{aligned}$$

$$= \left(\frac{3^3}{10^3} \right)^{-\frac{1}{3}}$$

$$= \left(\frac{3}{10} \right)^{3 \times -\frac{1}{3}}$$

$$= \left(\frac{3}{10} \right)^{-1}$$

$$= \frac{10}{3}$$

$$\mathbf{6. (i) \left(-\frac{1}{27} \right)^{-\frac{2}{3}}}$$

Solution:

$$\left(-\frac{1}{27} \right)^{-\frac{2}{3}}$$

$$= \left(-\frac{1}{3^3} \right)^{-\frac{2}{3}}$$

$$= \left(-\frac{1}{3} \right)^{3 \times -\frac{2}{3}}$$

$$= \left(-\frac{1}{3} \right)^{-2}$$

$$= (-3)^2$$

$$= 9$$

$$\mathbf{(ii) (64)^{-\frac{2}{3}} \div 9^{-\frac{3}{2}}}$$

Solution:

$$(4^3)^{-\frac{2}{3}} \div (9^2)^{-\frac{3}{2}}$$

$$= 4^{3 \times -\frac{2}{3}} \div 3^{7 \times -\frac{3}{2}}$$

$$= 4^{-2} \div 3^{-3}$$

$$= \frac{4^{-2}}{3^{-3}}$$

$$= \frac{\frac{1}{4^2}}{3^3}$$

$$= \frac{3^3}{4^2}$$

$$= \frac{27}{16}$$

7.

Solution

$$(i) \frac{(27)^{\frac{2n}{3}} \times 8^{-\frac{n}{6}}}{(18)^{-\frac{n}{2}}}$$

$$= \frac{(3^3)^{\frac{2n}{3}} \times (2^3)^{-\frac{n}{6}}}{2 \times 9^{-\frac{n}{2}}}$$

$$= \frac{3^{3 \times \frac{2n}{3}} \times 2^{3 \times -\frac{n}{6}}}{2^{-\frac{n}{2}} \times 3^{2 \times -\frac{n}{2}}}$$

$$= \frac{3^{2n} \times 2^{-\frac{n}{2}}}{2^{-\frac{n}{2}} \times 3^{-n}}$$

$$= \frac{3^{2n}}{2^{-\frac{n}{2}}} \times \frac{1}{2^{\frac{n}{2}}} \times 3^n$$

$$= \frac{3^{2n+n}}{2^{-\frac{n}{2}} \cdot 2^{\frac{n}{2}}}$$

$$= \frac{3^{3n}}{2^{-\frac{n}{2}} \cdot 2^{\frac{n}{2}}}$$

$$= \frac{3^{3n} \times 2^{\frac{n}{2}}}{2^{\frac{n}{2}}}$$

$$= 3^{3n}$$

$$(ii) \frac{5 \cdot (25)^{n+1} - 25 \cdot (5)^{2n}}{5 \cdot (5)^{2n+3} - (25)^{n+1}}$$

$$= \frac{5^1 \cdot (5^2)^{n+1} - 5^2 \cdot 5^{2n}}{5^1 \cdot 5^{2n+3} - (5^2)^{n+1}}$$

$$= \frac{5^{1+2n+2} - 5^{2+2n}}{5^{1+2n+3} - 5^{2+2n}}$$

$$= \frac{5^{2n+3} - 5^{2+2n}}{5^{2n+4} - 5^{2+2n}}$$

$$= \frac{5^{2n+3} - 5^{2+2n}}{5^{2n+4} - 5^{2+2n}}$$

$$= \frac{5^{2n} \cdot 5^3 - 5^2 \cdot 5^{2n}}{5^{2n} \cdot 5^4 - 5^2 \cdot 5^{2n}}$$

$$= \frac{5^{2n}[5^3 - 5^2]}{5^{2n}[5^4 - 5^2]}$$

$$= \frac{125 - 25}{625 - 25}$$

$$= \frac{100}{600}$$

$$= \frac{1}{6}$$

8.

Solution

$$(i) \quad (8^{-\frac{4}{3}} \div 2^{-2})^{\frac{1}{2}}$$

$$= ((2^3)^{-\frac{4}{3}} \div 2^{-2})^{\frac{1}{2}}$$

$$= \left(\frac{2^{-4}}{2^{-2}} \right)^{\frac{1}{2}}$$

$$= (2^{-4+2})^{\frac{1}{2}}$$

$$= (2^{-2})^{\frac{1}{2}}$$

$$= 2^{-1}$$

$$= \frac{1}{2}$$

$$\text{(ii)} \quad \left(\frac{27}{8}\right)^{\frac{2}{3}} - \left(\frac{1}{4}\right)^{-2} + 5^0$$

$$= \left(\frac{3^3}{2^3}\right)^{\frac{2}{3}} - \left(\frac{1}{2^2}\right)^{-2} + 1$$

$$= \left(\frac{3}{2}\right)^{3 \times \frac{2}{3}} - \frac{1}{2^{2 \times -2}} + 1$$

$$= \left(\frac{3}{2}\right)^2 - \frac{1}{2^{-4}} + 1$$

$$= \frac{9}{4} - 2^4 + 1$$

$$= \frac{9}{4} - 16 + 1$$

$$= \frac{9}{4} - 15$$

$$= \frac{9-60}{4}$$

$$= -\frac{51}{4}$$

9.

Solution

$$(i) (3x^2)^{-3} \times (x^9)^{\frac{2}{3}}$$

$$= \frac{1}{(3x^2)^3} \times x^{9 \times \frac{2}{3}}$$

$$= \frac{1}{3^3 \cdot x^{2 \times 3}} \times x^{3 \times 2}$$

$$= \frac{1}{27 \cdot x^6} \times x^6$$

$$= \frac{1}{27}$$

$$(ii) (8x^4)^{\frac{1}{3}} \div x^{\frac{1}{3}}$$

$$= (2^3 \cdot x^4)^{\frac{1}{3}} \div x^{\frac{1}{3}}$$

$$= \frac{2^{3 \cdot \frac{1}{3}} \cdot x^{4 \cdot \frac{1}{3}}}{x^{\frac{1}{3}}}$$

$$= \frac{2 \cdot x^{\frac{4}{3}}}{x^{\frac{1}{3}}}$$

$$= \frac{2 \cdot x^{\frac{4}{3}}}{x^{\frac{1}{3}}}$$

$$= 2 \cdot x^{4 \cdot \frac{1}{3} - \frac{1}{3}}$$

$$= x^{\frac{1}{3}} [2x^4 - 1]$$

10.

Solution

$$(i) (3^2)^0 + 3^{-4} \times 3^6 + \left(\frac{1}{3}\right)^{-2}$$

$$= 3^0 + 3^{-4+6} + \frac{1}{3^{-2}}$$

$$= 1 + 3^2 + 3^2$$

$$= 1 + 9 + 9$$

$$= 19$$

$$\begin{aligned}
\text{(ii)} \quad & 9^{\frac{5}{2}} - 3 \cdot (5)^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}} \\
&= 9^{\frac{5}{2}} - 3(1) - \left(\frac{1}{9^2}\right)^{-\frac{1}{2}} \\
&= 9^{\frac{5}{2}} - 3 - \frac{1}{9^{2 \times \frac{1}{2}}} \\
&= 3^{2 \times \frac{5}{2}} - 3 - \frac{1}{9^{-1}} \\
&= 3^5 - 3 - 9 \\
&= 3^5 - 3 - 3^2 \\
&= 3(3^4 - 1 - 3) \\
&= 3(81 - 1 - 3) \\
&= 3(77) \\
&= 231
\end{aligned}$$

11.

Solution

$$\begin{aligned}
\text{(i)} \quad & 16^{\frac{3}{4}} + 2 \left(\frac{1}{2}\right)^{-1} \cdot 3^0 \\
&= (2^4)^{\frac{3}{4}} + 2 \left(\frac{1}{2^{-1}}\right) \cdot 1 \\
&= 2^{4 \times \frac{3}{4}} + 2 \cdot 2 \\
&= 2^3 + 4
\end{aligned}$$

$$= 8 + 4$$

$$= 12$$

$$(ii) (81)^{\frac{3}{4}} - \left(\frac{1}{32}\right)^{-\frac{2}{5}} + (8)^{\frac{1}{3}} \left(\frac{1}{2}\right)^{-1} \cdot (2)^0$$

$$= (3^4)^{\frac{3}{4}} - \left(\frac{1}{2^5}\right)^{-\frac{2}{5}} + 2^{2 \times \frac{1}{3}} \left(\frac{1}{2^{-1}}\right)^1$$

$$= 3^3 - \frac{1}{2^{5 \times -\frac{2}{5}}} + 2 (2)^1$$

$$= 3^3 - \frac{1}{2^{-2}} + 2(2)$$

$$= 27 - 2^2 + 4$$

$$= 27 - 4 + 4$$

$$= 27$$

12

Solution

$$(i) \left(\frac{64}{125}\right)^{-\frac{2}{3}} \div \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^0$$

$$= \left(\frac{4^3}{5^3}\right)^{-\frac{2}{3}} \div \frac{1}{\left(\frac{4^4}{5^4}\right)^{\frac{1}{4}}} + 1$$

$$= \left(\frac{4}{5}\right)^{-2} \div \frac{1}{\left(\frac{4}{5}\right)} + 1$$

$$= \frac{\left(\frac{5}{4}\right)^2}{\left(\frac{4}{5}\right)} + 1$$

$$= \left(\frac{5}{4}\right)^2 \times \left(\frac{4}{5}\right) + 1$$

$$= \frac{5}{4} + 1$$

$$= \frac{9}{4}$$

$$\text{(ii)} \frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 2^2 \times 5^n}$$

$$= \frac{5^n \cdot 5^3 - 6 \times 5^n \cdot 5}{9 \times 5^n - 2^2 \times 5^n}$$

$$= \frac{5^n [5^3 - 6 \times 5]}{5^n [9 - 4]}$$

$$= \frac{125 - 30}{5}$$

$$= \frac{95}{5}$$

$$= 19$$

13.

Solution

$$(i) [(64)^{\frac{2}{3}} \cdot 2^{-2} \div 8^0]^{-\frac{1}{2}}$$

$$= \left((4^3)^{\frac{2}{3}} \cdot \frac{1}{2^2} \div 1 \right)^{-\frac{1}{2}}$$

$$= \left(\frac{4^2}{2^2} \right)^{-\frac{1}{2}}$$

$$= \left(\frac{4}{2} \right)^{2 \times -\frac{1}{2}}$$

$$= 2^{-1}$$

$$= \frac{1}{2}$$

$$(ii) 3^n \times 9^{n+1} \div 3^{n-1} \times 9^{n-1}$$

$$= 3^n \times 3^{2(n+1)} \div 3^{n-1} \times 3^{2(n-1)}$$

$$= 3^n \times 3^{2n+2} \div 3^{n-1} \times 3^{2n-2}$$

$$= \frac{3^{n+2n+2}}{3^{n-1+2n-2}}$$

$$= \frac{3^{3n+2}}{3^{3n-3}}$$

$$= \frac{3^{3n} \cdot 3^2}{3^{3n} \cdot 3^{-3}}$$

$$= 3^2 \times 3^3$$

$$= 3^{2+3}$$

$$= 3^5$$

$$= 243$$

14.

Solution

$$(i) \frac{\sqrt{2^2} \times \sqrt[4]{256}}{\sqrt[3]{64}} - \left(\frac{1}{2}\right)^{-2}$$

$$= \frac{(2^2)^{\frac{1}{2}} \times (4)^{4 \times \frac{1}{4}}}{4^{3 \times \frac{1}{3}}} - \frac{1}{2^{-2}}$$

$$= \frac{2 \times 4}{4} - 2^2$$

$$= 2 - 4$$

$$= -2$$

$$(ii) \frac{3^{-\frac{6}{7}} \times 4^{-\frac{3}{7}} \times 9^{\frac{3}{7}} \times 2^{\frac{6}{7}}}{2^2 + 2^0 + 2^{-2}}$$

$$= \frac{3^{-\frac{6}{7}} \times 3^{2 \times \frac{3}{7}} \times 2^{2 \times -\frac{3}{7}} \times 2^{\frac{6}{7}}}{4 + 1 + \frac{1}{2^2}}$$

$$= \frac{3^{-\frac{6}{7}} \times 3^{\frac{6}{7}} \times 2^{-\frac{6}{7}} \times 2^{\frac{6}{7}}}{4 + 1 + \frac{1}{4}}$$

$$= \frac{3^{-\frac{6}{7} + \frac{6}{7}} \times 2^{-\frac{6}{7} + \frac{6}{7}}}{\frac{16 + 4 + 1}{4}}$$

$$= \frac{3^0 \times 2^0}{\left(\frac{21}{4}\right)}$$

$$= \frac{1}{\frac{21}{4}}$$

$$= \frac{4}{21}$$

15.

Solution

$$(i) \frac{(32)^{\frac{2}{5}} \times (4)^{-\frac{2}{2}} \times (8)^{\frac{1}{3}}}{2^{-2} \div (64)^{-\frac{1}{3}}}$$

$$= \frac{(2^5)^{-\frac{2}{5}} \times (2^2)^{-\frac{1}{2}} \times (2^3)^{\frac{1}{3}}}{\frac{1}{2^2} \div 4^{3 \times -\frac{1}{3}}}$$

$$= \frac{2^{-2} \times 2^{-1} \times 2^1}{\frac{1}{2^2} \div 4^{-1}}$$

$$= \frac{2^{-1-2+1}}{\frac{\frac{1}{2^2}}{\frac{1}{2^2}}}$$

$$= 2^{-2}$$

$$= \frac{1}{2^2}$$

$$= \frac{1}{4}$$

$$(ii) \frac{5^{2(x+6)} \times 25^{-7+2x}}{125^{2x}}$$

$$= \frac{5^{2x+12} \times 5^{2(-7+2x)}}{(5^3)^{2x}}$$

$$= \frac{5^{2x+12-14+14x}}{5^{6x}}$$

$$= \frac{5^{6x-2}}{5^{6x}}$$

$$= 5^{-2}$$

$$= \frac{1}{5^2}$$

$$= \frac{1}{25}$$

16.

Solution

$$(i) \frac{7^{2n+3} - 49^{n+2}}{(343^{n+1})^{\frac{2}{3}}}$$

$$= \frac{7^{2n+3} - 7^{2(n+2)}}{(7^{3(n+1)})^{\frac{2}{3}}}$$

$$= \frac{7^{2n+3} - 7^{2n+4}}{7^{2(n+1)}}$$

$$= \frac{7^{2n+3} - 7^{2n+4}}{7^{2n} \cdot 7^2}$$

$$= \frac{7^{2n}[7^3-7^4]}{7^{2n}.7^2}$$

$$= \frac{343-2401}{49}$$

$$= -\frac{2058}{49}$$

$$= -42$$

$$(ii) (27)^{\frac{4}{3}} + (32)^{0.8} + (0.8)^{-1}$$

$$= 3^{3 \times \frac{4}{3}} + 2^{5 \times \frac{8}{10}} + \left(\frac{8}{10}\right)^{-1}$$

$$= 3^4 + 2^{5 \times \frac{4}{5}} + \left(\frac{4}{5}\right)^{-1}$$

$$= 3^4 + 2^4 + \frac{5}{4}$$

$$= 81 + 16 + \frac{5}{4}$$

$$= 97 + \frac{5}{4}$$

$$= \frac{388+5}{4}$$

$$= \frac{393}{4}$$

17.

Solution

$$(i) (\sqrt{32} - \sqrt{5})^{\frac{1}{3}} \cdot (\sqrt{32} + \sqrt{5})^{\frac{1}{3}}$$

$$= \left[(\sqrt{2}^5 - \sqrt{5}) \cdot (\sqrt{2}^5 + \sqrt{5}) \right]^{\frac{1}{3}}$$

$$= \left[(\sqrt{2}^5)^2 - (\sqrt{5})^2 \right]^{\frac{1}{3}}$$

$$= (2^5 - 5)^{\frac{1}{3}}$$

$$= (32 - 5)^{\frac{1}{3}}$$

$$= (27)^{\frac{1}{3}}$$

$$= (3^3)^{\frac{1}{3}}$$

$$= 3$$

$$(ii) \left(x^{\frac{1}{3}} - x^{-\frac{1}{3}} \right) \left(x^{\frac{2}{3}} + 1 + x^{-\frac{2}{3}} \right)$$

$$= \left(x^{\frac{1}{3}} - \frac{1}{x^{\frac{1}{3}}} \right) \left(x^{\frac{2}{3}} + 1 + x^{\frac{2}{3}} \right)$$

∴ it is in the form of

$$(a - b)(a^2 + ab + b^2) = a^3 - b^3$$

$$\text{Here } a = x^{\frac{1}{3}}; \quad b = \frac{1}{x^{\frac{1}{3}}}$$

$$\therefore \left(x^{\frac{1}{3}} \right)^3 - \left(\frac{1}{x^{\frac{1}{3}}} \right)^3$$

$$= x^{\frac{3}{3}} - \frac{1}{x^{\frac{3}{3}}}$$

$$= x - \frac{1}{x}$$

18.

Solution

$$(i) \left(\frac{x^m}{x^n} \right)^l \cdot \left(\frac{x^n}{x^l} \right)^m \cdot \left(\frac{x^l}{x^m} \right)^n$$

$$= (x^{m-n})^l \cdot (x^{n-l})^m \cdot (x^{l-m})^n$$

$$= x^{ml-nl} \cdot x^{mn-ml} \cdot x^{nl-nm}$$

$$= x^{ml-nl+mn-ml+nl-nm}$$

$$= x^0$$

$$= 1$$

$$(ii) \left(\frac{x^{a+b}}{x^c} \right)^{a-b} \cdot \left(\frac{x^{b+c}}{x^a} \right)^{b-c} \cdot \left(\frac{x^{c+a}}{x^b} \right)^{c-a}$$

$$= \frac{x^{(a+b)(a-b)}}{x^{c(a-b)}} \cdot \frac{x^{(b+c)(b-c)}}{x^{a(b-c)}} \cdot \frac{x^{(c+a)(c-a)}}{x^{b(c-a)}}$$

$$= \frac{x^{a^2-b^2}}{x^{ac-bc}} \cdot \frac{x^{b^2-c^2}}{x^{ab-ac}} \cdot \frac{x^{c^2-a^2}}{x^{bc-ab}}$$

$$= \frac{x^{a^2-b^2+b^2-c^2+c^2-a^2}}{x^{ac-bc+ab-ac+bc-ab}}$$

$$= \frac{x^0}{x^0}$$

$$= 1$$

Solution

$$\begin{aligned}
& \text{(i)} \quad \sqrt[lm]{\frac{x^l}{x^m}} \cdot \sqrt{mn}{\frac{x^m}{x^n}} \cdot \sqrt[nl]{\frac{x^n}{x^l}} \\
&= \sqrt[lm]{x^{l-m}} \cdot \sqrt{mn}{x^{m-n}} \cdot \sqrt[nl]{x^{n-l}} \\
&= (x^{l-m})^{\frac{1}{lm}} \cdot (x^{m-n})^{\frac{1}{mn}} \cdot (x^{n-l})^{\frac{1}{nl}} \\
&= x^{\frac{l-m}{lm}} \cdot x^{\frac{m-n}{nm}} \cdot x^{\frac{n-l}{nl}} \\
&= x^{\frac{n(l-m)+l(m-n)+m(n-l)}{lm.n}} \\
&= x^{\frac{nl-nm+lm-ln+mn-ln}{lmn}} \\
&= x^0 \\
&= 1
\end{aligned}$$

$$(ii) \left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \cdot \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \cdot \left(\frac{x^c}{x^a}\right)^{c^2+ac+a^2}$$

$$= x^{(a-b)(a^2+ab+b^2)} \cdot x^{(b-c)(b^2+bc+c^2)} \cdot x^{(c-a)(c^2+ac+a^2)}$$

$$= x^{a^3-b^3} \cdot x^{b^3-c^3} \cdot x^{c^3-a^3}$$

$$= x^{a^3-b^3+b^3-c^3+c^3-a^3}$$

$$= x^0$$

$$= 1.$$

$$(iii) \left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \cdot \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \cdot \left(\frac{x^c}{x^{-a}}\right)^{c^2-ac+a^2}$$

$$= x^{(a-c-b)(a^2-ab+b^2)} \cdot x^{(b-c-c)(b^2-bc+c^2)} \cdot x^{(c-c-a)(c^2-ac+a^2)}$$

$$= x^{a^3+b^3} \cdot x^{b^3+c^3} \cdot x^{c^3+a^3}$$

$$= x^{a^3+b^3+b^3+c^3+c^3+a^3}$$

$$= x^{2a^3+2b^3+2c^3}$$

$$= x^{2(a^3+b^3+c^3)}$$

20.

Solution

$$(i) (a^{-1} + b^{-1}) \div (a^{-2} - b^{-2})$$

$$= \left(\frac{1}{a} + \frac{1}{b} \right) \div \left(\frac{1}{a^2} - \frac{1}{b^2} \right)$$

$$= \left(\frac{b+a}{ab} \right) \div \left(\frac{b^2-a^2}{a^2b^2} \right)$$

$$= \frac{\frac{b+a}{ab}}{\frac{b^2-a^2}{a^2b^2}}$$

$$= \frac{b+a}{ab} \times \frac{(ab)^2}{(b^2-a^2)}$$

$$= \frac{b+a}{ab} \cdot \frac{ab^2}{(b+a)(b-a)}$$

$$= \frac{ab}{b-a}$$

$$(ii) \frac{1}{1+a^{m-n}} + \frac{1}{a^{n-m}+1}$$

$$= \frac{1}{1+a^{m-n}} + \frac{1}{1+a^{-(m-n)}}$$

$$= \frac{1}{1+a^{m-n}} + \frac{1}{1+\frac{1}{a^{(m-n)}}}$$

$$= \frac{1}{1+a^{m-n}} + \frac{1}{\frac{a^{m-n}+1}{a^{m-n}}}$$

$$= \frac{1}{1+a^{m-n}} + \frac{a^{m-n}}{a^{m-n}+1}$$

$$= \frac{1+a^{m-n}}{1+a^{m-n}}$$

$$=1$$

21.

Solution

$$(i) (a+b)^{-1} (a^{-1} + b^{-1}) = \frac{1}{ab}$$

$$\text{LHS} = (a+b)^{-1} (a^{-1} + b^{-1})$$

$$= \frac{1}{a+b} \left(\frac{1}{a} + \frac{1}{b} \right)$$

$$= \frac{1}{a+b} \left(\frac{b+a}{ab} \right)$$

$$= \frac{1}{a+b} \left(\frac{a+b}{ab} \right)$$

$$= \frac{1}{ab}$$

$$= \text{RHS}$$

$$(ii) \frac{x+y+z}{x^{-1}y^{-1}+y^{-1}z^{-1}+z^{-1}x^{-1}} = xyz$$

$$\text{LHS} = \frac{x+y+z}{x^{-1}y^{-1}+y^{-1}z^{-1}+z^{-1}x^{-1}}$$

$$= \frac{x+y+z}{\frac{1}{xy} + \frac{1}{yz} + \frac{1}{xz}}$$

$$= \frac{x+y+z}{\frac{z+xy}{xyz}}$$

$$= \frac{x+y+z}{\frac{x+y+z}{xyz}}$$

$$= xyz$$

$$= \text{RHS}$$

22

Solution

Given $a = c^z$; $b = a^x$; $c = b^y$

$$= a = c^z$$

$$a = (b^y)^z \quad (\because c = b^y)$$

$$a = b^{yz}$$

$$a = (a^x)^{yz} \quad (\because b = a^x)$$

$$a^1 = a^{xyz}$$

$\therefore$ bases are equal so exponents are also equal

$$\therefore xyz = 1$$

Hence proved

23.

Solution

Given $a = xy^{p-1}$; $b = xy^{q-1}$; $c = xy^{r-1}$

$$\text{LHS} = a^{q \cdot r} \cdot b^{r \cdot p} \cdot c^{p \cdot q}$$

$$= (xy^{p-1})^{q \cdot r} \cdot (xy)^{q \cdot 1(r-p)} \cdot (xy^{r-1})^{(p-q)}$$

$$= xy^{(p-1)(q-r)} \cdot xy^{(q-1)(r-p)} \cdot xy^{(r-1)(p-q)}$$

$$= xy^{pq-pr-q+r} \cdot xy^{qr-qp-r+p} \cdot xy^{rp-rq-p+q}$$

$$= xy^{pq-pr-q+r+qr-qp-r+p+rp-rq-p+q}$$

$$= xy^0$$

$$= 1$$

$$= \text{RHS}$$

Hence proved

24.

Solution

$$\text{Given } 2^x = 3^y = 6^{-z}$$

$$\text{Let } 2^x = 3^y = 6^{-z} = k$$

$$2 = k^{\frac{1}{x}}$$

$$3 = k^{\frac{1}{y}}$$

$$6 = k^{-\frac{1}{z}}$$

$$\frac{1}{6} = k^{\frac{1}{z}}$$

$$= \frac{1}{2 \times 3} = k^{\frac{1}{z}}$$

$$= \frac{1}{k^{\frac{1}{x}} \cdot k^{\frac{1}{y}}} = k^{\frac{1}{z}}$$

$$1 = k^{\frac{1}{z}} \cdot k^{\frac{1}{x}} \cdot k^{\frac{1}{y}}$$

$$k = k^{\frac{1}{x} + \frac{1}{y} + \frac{1}{z}}$$

$$\therefore \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

25.

Solution

$$\text{Given } 2^x = 3^y = 12^z$$

$$\text{Let } 2^x = 3^y = 12^z = k$$

$$= 2 = k^{\frac{1}{x}}$$

$$= 3 = k^{\frac{1}{y}}$$

$$= 12 = k^{\frac{1}{z}}$$

$$= 2^2 \cdot 3 = k^{\frac{1}{z}}$$

$$= \left(k^{\frac{1}{x}}\right)^2 \cdot k^{\frac{1}{y}} = k^{\frac{1}{z}}$$

$$= k^{\frac{2}{x}} \cdot k^{\frac{1}{y}} = k^{\frac{1}{z}}$$

$$= \frac{2}{x} + \frac{1}{y} = \frac{1}{z}$$

$$= \frac{2y+x}{xy} = \frac{1}{z}$$

$$\text{Or } \frac{2}{x} = \frac{1}{z} - \frac{1}{y}$$

$$\frac{2}{x} = \frac{y-z}{yz}$$

$$x = \frac{2yz}{y-z}$$

Hence proved

26.

Solution

$$(i) (3x^2)^0$$

$$= 1$$

$$(ii) (xy)^{-2}$$

$$= \frac{1}{(xy)^2}$$

$$= \frac{1}{x^2y^2}$$

$$\begin{aligned}
& \text{(iii) } (-27a^a)^{\frac{2}{3}} \\
&= (3^3 a^a)^{\frac{2}{3}} \\
&= (3a^3)^{3 \times \frac{2}{3}} \\
&= (3a^3)^2 \\
&= 3^2 a^{3 \times 2} \\
&= 9a^6
\end{aligned}$$

27

Solution

Given $a=3$; $b = -2$

$$\begin{aligned}
& \text{(i) } a^a + b^b \\
&= 3^3 + (-2)^{-2} \\
&= 3^3 + \frac{1}{(-2)^2} \\
&= 27 + \frac{1}{4}
\end{aligned}$$

$$= \frac{108+1}{4}$$

$$= \frac{109}{4}$$

$$(ii) a^b + b^a$$

$$= 3^{-2} + (-2)^3$$

$$= \frac{1}{3^2} - 8$$

$$= \frac{1}{9} - 8$$

$$= \frac{1-72}{9}$$

$$= -\frac{71}{9}$$

28.

Solution

$$\text{Given } x = 10^3 \times 0.99 \quad ; \quad y = 10^{-2} \times 110$$

$$= \sqrt{\frac{x}{y}} = \sqrt{10^3 \times \frac{0.0099}{10^{-2} \times 110}}$$

$$= \sqrt{10^{3+2} \times \frac{0.0099}{110}}$$

$$= \sqrt{10^5 \times \frac{0.0099}{110}}$$

$$= \sqrt{\frac{990}{110}}$$

$$= \sqrt{9}$$

$$= 3$$

29.

Solution

Given $x = 9$, $y = 2$, $z = 8$

$$x^{\frac{1}{2}} \cdot y^{-1} \cdot z^{\frac{2}{3}}$$

$$= 9^{\frac{1}{2}} \cdot 2^{-1} \cdot 8^{\frac{2}{3}}$$

$$= (3^2)^{\frac{1}{2}} \cdot \left(\frac{1}{2}\right) (2^3)^{\frac{2}{3}}$$

$$= 3^{\frac{1}{2}} \cdot 2^2$$

$$= 3 \cdot \frac{1}{2} \cdot 4^2$$

$$= 6$$

30

Solution

Given $x^4y^2z^3 = 49392$

2	49392
2	24696
2	12348
2	6174
2	3087
3	1029
7	343
7	49
	7

$$x^4y^2z^3 = 2^4 \cdot 3^2 \cdot 7^4$$

$\therefore x, y, z$ are different primes

$$\therefore x = 2 ; y = 3 ; z = 7$$

31.

Solution

$$\text{Given } \sqrt[3]{a^6b^{-4}} = a^x \cdot b^2y$$

$$(a^6b^{-4})^{\frac{1}{3}} = a^x b^{2y}$$

$$a^{\frac{6}{3}} \cdot b^{-\frac{4}{3}} = a^x b^{2y}$$

$$\therefore x = \frac{6}{3} \qquad 2y = -\frac{4}{3}$$

$$x = 2 \qquad y = -\frac{4}{3 \times 2}$$

$$y = -\frac{2}{3}$$

32.

Solution

$$\text{Given } (p+q)^{-1} (p^{-1}+q^{-1}) = p^a \cdot q^b$$

$$\frac{1}{p+q} \left(\frac{1}{p} + \frac{1}{q} \right) = p^a \cdot q^b$$

$$\frac{1}{p+q} \left(\frac{q+p}{qp} \right) = p^a \cdot q^b$$

$$\frac{1}{qp} = p^a q^b$$

$$(qp)^{-1} = p^a \cdot q^b$$

$$p^{-1} \cdot q^{-1} = p^a \cdot q^b$$

$$a = -1$$

$$b = -1$$

$$\text{LHS} = a + b + 2$$

$$= -1 + 2 - 1$$

$$= 0$$

$$= \text{RHS}$$

33.

Solution

$$\text{Given } \left(\frac{p^{-1}q^2}{p^2q^{-4}} \right)^7 \div \left(\frac{p^3q^{-5}}{p^{-2}q^3} \right)^{-5} = p^x q^y$$

$$= \left(\frac{p^{-7} \cdot q^{2 \times 7}}{p^{2 \times 7} q^{-4 \times 7}} \right) \div \frac{p^{3 \times 5} \cdot q^{-5 \times -5}}{p^{-2 \times -5} q^{3 \times -5}} = p^x q^y$$

$$= \left(\frac{p^{-7} q^{14}}{p^{14} \cdot q^{-28}} \right) \div \left(\frac{p^{15} \cdot q^{25}}{p^{10} q^{-15}} \right) = p^x \cdot q^y$$

$$= (p^{-7-14} \cdot q^{14+28}) \div (p^{15-10} \cdot q^{25+15}) = p^x \cdot q^y$$

$$= (p^{-21} \cdot q^{42}) \div (p^5 \cdot q^{40}) = p^x \cdot q^y$$

$$= \frac{p^{-21} \cdot q^{42}}{p^5 \cdot q^{40}} = p^x \cdot q^y$$

$$= (p^{-21-5} \cdot q^{42-20}) = p^x q^y$$

$$= (p^{-26} \cdot q^2) = p^x \cdot q^y$$

$$= \therefore x = -26 \quad ; \quad y = 2$$

$$\therefore x + y = -26 + 2$$

$$= -24$$

34.

Solution

$$(i) \quad 5^{2x+3} = 1$$

$$5^{2x+3} = 5^0 \quad (\because 5^0 = 10)$$

$$\therefore 2x + 3 = 0$$

$$2x = -3$$

$$x = -\frac{3}{2}$$

$$(ii) \quad (13)^{\sqrt{x}} = 4^4 - 3^4 - 6$$

$$(13)^{\sqrt{x}} = 256 - 81 - 6$$

$$(13)^{\sqrt{x}} = 169$$

$$(13)^{\sqrt{x}} = (13)^2$$

$$\sqrt{x} = 2$$

$$x = 2^{\frac{1}{2}}$$

$$\text{(iii)} \left(\sqrt{\frac{3}{5}} \right)^{x+1} = \frac{125}{27}$$

$$\left(\frac{3}{5} \right)^{\frac{x+1}{2}} = \frac{5^3}{3^3}$$

$$\left(\frac{3}{5} \right)^{\frac{x+1}{2}} = \left(\frac{3}{5} \right)^{-3}$$

$$\frac{x+1}{2} = -3$$

$$x + 1 = -6$$

$$x = -6 - 1$$

$$x = -7$$

$$\text{(iv)} \left(\sqrt[3]{4} \right)^{2x+\frac{1}{2}} = \frac{1}{32}$$

$$\left[(2^2)^{\frac{1}{3}} \right]^{4x+\frac{1}{2}} = \frac{1}{32}$$

$$\left(2^{\frac{2}{3}} \right)^{4x+\frac{1}{2}} = \frac{1}{2^5}$$

$$2^{\frac{4x+1}{3}} = 2^{-5}$$

$$= \frac{4x+1}{3} = -5$$

$$4x + 1 = -15$$

$$4x = -15 - 1$$

$$4x = -16$$

$$x = -\frac{16}{4}$$

$$x = -4$$

35.

Solution

$$(i) \sqrt{\frac{p}{q}} = \left(\frac{q}{p}\right)^{1-2x}$$

$$\left(\frac{p}{q}\right)^{\frac{1}{2}} = \frac{p^{-(1-2x)}}{q}$$

$$\left(\frac{p}{q}\right)^{\frac{1}{2}} = \left(\frac{p}{q}\right)^{-1+2x}$$

$$\frac{1}{2} = -1 + 2x$$

$$-1 + 2x = \frac{1}{2}$$

$$2x = \frac{1}{2} + 1$$

$$2x = \frac{1+2}{2}$$

$$2x = \frac{3}{2}$$

$$x = \frac{3}{2 \times 2}$$

$$x = \frac{3}{4}$$

$$(ii) \quad 4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^x$$

$$2^{2(x-1)} \times \left(\frac{1}{2}\right)^{3-2x} = \left(\frac{1}{2^3}\right)^x$$

$$2^{2x-2} \times \frac{1}{2^{2x-3}} = \frac{1}{2^{3x}}$$

$$2^{2x-2} \times 2^{2x-3} = 2^{-3x}$$

$$2x-2 + 2x-3 = -3x$$

$$4x - 5 = -3x$$

$$4x + 3x = 5$$

$$7x = 5$$

$$x = \frac{5}{7}$$

36.

Solution

$$\text{Given } 5^{3x} = 125$$

$$10^y = 0.001$$

$$= 5^{3x} = 125$$

$$5^{3x} = 5^3$$

$$3x = 3$$

$$X = \frac{3}{3}$$

$$X = 1$$

$$10^y = 0.001$$

$$10^y = \left(\frac{10}{1000}\right)$$

$$10^y = \left(\frac{1}{10^3}\right)$$

$$10^y = 10^{-3}$$

$$Y = -3$$

$$\therefore x = 1 ; y = -3$$

37.

Solution

Given

$$\frac{9^n 3^2 \cdot 3^n - 27^n}{3^{3m} \cdot 2^3} = \frac{1}{27}$$

$$\frac{3^{2n} 3^2 \cdot 3^n - 3^{3n}}{3^{3m} \cdot 2^3} = \frac{1}{3^3}$$

$$\frac{3^{2n+2+n} - 3^{3n}}{3^{3m} \cdot 8} = \frac{1}{3^3}$$

$$\frac{3^{3n+2} - 3^{3n}}{3^{3m} \cdot 8} = \frac{1}{3^3}$$

$$\frac{3^{3n} \cdot (3^2 - 1)}{3^{3m} \cdot 8} = \frac{1}{3^3}$$

$$\frac{3^{3n} (9 - 1)}{3^{3m} \cdot 8} = \frac{1}{3^3}$$

$$\frac{3^{3n} \cdot 8}{3^{3m} \cdot 8} = \frac{1}{3^3}$$

$$3^{3n} \cdot 3^3 = 3^{3m}$$

$$3^{3(n+1)} = 3^{3m}$$

$$\therefore m = n + 1$$

38.

Solution

Given

$$3^{4x} = (81)^{-1}$$

$$3^{4x} = (3^4)^{-1}$$

$$4x = -4$$

$$x = -\frac{4}{4}$$

$$x = -1$$

and

$$10^{\frac{1}{y}} = 0.0001$$

$$10^{\frac{1}{y}} = \frac{1}{10000}$$

$$10^{\frac{1}{y}} = \frac{1}{10^4}$$

$$10^{\frac{1}{y}} = 10^{-4}$$

$$\frac{1}{y} = -4$$

$$y = -\frac{1}{4}$$

$$2^{-x} \cdot (16)^y$$

$$2^{-(-1)} (16)^{-\frac{1}{4}}$$

$$2^1 \cdot (2^4)^{-\frac{1}{4}}$$

$$2^1 \cdot 2^{-1}$$

$$2 \cdot \frac{1}{2}$$

$$1$$

39

Solution

$$\text{Given } 3^{x+1} = 9^{x-2}$$

$$3^{x+1} = 3^{2(x-2)}$$

$$3^{x+1} = 3^{2x-4}$$

$$x + 1 = 2x - 4$$

$$2x - x = 1 + 4$$

$$x = 5$$

$$2^{1+x} = 2^{1+5}$$

$$= 2^6$$

$$= 64$$

40.

Solution

$$(i) 3(2^x + 1) - 2^{x+2} + 5 = 0$$

$$3 \cdot 2^x + 3 - 2^x \cdot 2^2 + 5 = 0$$

$$3 \cdot 2^x + 3 - 2^x \cdot 4 + 5 = 0$$

$$3 \cdot 2^x - 4 \cdot 2^x + 8 = 0$$

$$-2^x + 8 = 0$$

$$2^x = 8$$

$$2^x = 2^3$$

$$x = 3$$

$$(ii) 3^x = 9 \cdot 3^y$$

$$3^x = 3^2 \cdot 3^y$$

$$3^x = 3^{2+y}$$

$$x = 2 + y$$

$$\text{and } 8 \cdot 2^y = 4^x$$

$$2^3 \cdot 2^y = 2^{2x}$$

$$3 + y = 2x$$

$$X = \frac{3+y}{2}$$

Chapter test

1. if $2^x \cdot 3^y \cdot 5^z = 2160$ find the value of x, y and z hence compute the value of $3^x \cdot 2^{-y} \cdot 5^{-z}$

Solution

$$2^x \cdot 3^y \cdot 5^z = 2160$$

$$2^x \cdot 3^y \cdot 5^z = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$$

$$2^x \cdot 3^y \cdot 5^z = 2^4 \cdot 3^3 \cdot 5^1$$

Comparing powers of 2, 3 and 5 on both sides of above equation we get $x = 4$, $y = 3$, $z = 1$

$$\text{Also } 2^x \cdot 3^{-y} \cdot 5^{-z} = (3)^4 \times (2)^{-3} \times (5)^{-1}$$

$$= 3 \times 3 \times 3 \times 3 \times \frac{1}{2 \times 2 \times 2} \times \frac{1}{5} = \frac{81}{40} = 2 \frac{1}{40}$$

2	2160
2	1080
2	540
2	270
3	130
3	45
3	15
5	5

2.

If $x = 2$ and $y = -3$ find the values of

(i) $x^x + y^y$

(ii) $x^y + y^x$

Solution

(i) $x^x + y^y$ given that $x = 2$ and $y = 3$

$$\text{Then } x^x + y^y = (2)^2 + (-3)^{-3} = 2 \times 2 + \frac{1}{(-3)^3}$$

$$= 4 + \frac{1}{-3 \times (-3) \times (-3)} = 4 + \frac{1}{-27}$$

$$= 4 - \frac{1}{27} = \frac{108-1}{27} = \frac{107}{27} = 3\frac{26}{27}$$

(ii) $x^y + y^x = (2)^{-3} + (-3)^2 = \frac{1}{(2)^3} + (-3) \times (-3)$

$$= \frac{1}{2 \times 2 \times 2} + 9 = \frac{1}{8} + 9 = \frac{1+9 \times 8}{8} = \frac{1+72}{8}$$

$$= \frac{73}{8} = 9\frac{1}{8}$$

3. if $p = x^{m+n} \cdot y^t$, $q = x^{n+1} \cdot y^m$ and $r = x^{l+m} \cdot y^n$. prove that $p^{m-n} \cdot q^{n-1} \cdot r^{l-n} = 1$

solution

given that $p = x^{m+n} \cdot y^t$ (1)

$q = x^{n+1} \cdot y^m$ (2)

$r = x^{l+m} \cdot y^n$ (3)

L.H.S = $p^{m-n} \cdot q^{n-1} \cdot r^{l-n}$ (4)

Putting the value of a, b, c from (1) (2) (3) respectively in (4)
we get

$$\begin{aligned}
 \text{L.H.S} &= (x^{m+n}y^t)^{m-n}, (x^{n+1}, y^m)^{n-1} \cdot (x^{l+m} y^n)^{l-n} \\
 &= (x^{m+n})^{m-n} \cdot y^{l(m-n)} \cdot (x)^{(n+1)(n-1)} \cdot y^{m(n-1)} \cdot (x)^{(l+m)(l-n)} \cdot y^{m(l-n)} \\
 &= (x^{(m+n)(m-n)} \cdot y^{lm+ln} (x)^{(n+1)(n-1)} \cdot y^{mn-1} \cdot (x)^{(l+m)(l-n)} \cdot y^{ln+nm} \\
 &= (x)^{m^2-n^2} \cdot y^{lm-ln} \cdot (x)^{n^2-l^2} \cdot y^{mn-ml} \cdot (x)^{l^2-m^2} \cdot y^{nl-nm} \\
 &= (x)^{m^2-n^2+n^2-l^2+l^2=m^2} (y)^{lm-ln+mn-ml+nl-nm} \\
 &= (x)^0 (y)^0 = 1 \times 1 = 1
 \end{aligned}$$

Hence proved L.H.S = R.H.S

4. if $x = a^{m+n}$, $y = a^{n+l}$ and $z = a^{l+m}$, prove that $x^m \cdot y^n z^l = x^n y^l z^m$

Answer

$$x = a^{m+n} , y = a^{n+l} , z = a^{l+m}$$

$$\text{L.H.S} = x^m y^n z^l$$

$$= a^{m[m+n]} \cdot y^{n[n+l]} \cdot z^{l[l+m]}$$

$$= a^{m^2+mn} \cdot y^{n^2+nl} \cdot z^{l^2+lm}$$

$$= a^{m^2+mn+n^2+nl+l^2+lm} = a^{l^2+m^2+n^2+lm+mn+nl}$$

$$\text{R.H.S} = x^n \cdot y^l \cdot z^m$$

$$= a^{n(m+n)} \cdot a^{l(n+l)} \cdot a^{m(l+m)}$$

$$= a^{mn+n^2} \cdot a^{ln+l^2} \cdot a^{ml+m^2}$$

$$= a^{mn+n^2+ln+l^2+lm+m^2}$$

$$= a^{l^2+m^2+n^2+lm+mn+nl}$$

$$\therefore \text{L.H.S} = \text{R.H.S}$$

5.Show that

$$\frac{\left[p+\frac{1}{q}\right]^m \times \left[p-\frac{1}{q}\right]^n}{\left[q+\frac{1}{p}\right]^m \times \left[q-\frac{1}{p}\right]^n} = \left(\frac{p}{q}\right)^{m+n}$$

Solution

$$\begin{aligned}\text{L.H.S} &= \frac{\left[p+\frac{1}{q}\right]^m \times \left[p-\frac{1}{q}\right]^n}{\left[q+\frac{1}{p}\right]^m \times \left[q-\frac{1}{p}\right]^n} \\&= \frac{\left(\frac{pq+1}{q}\right)^m \times \left(\frac{pq-1}{q}\right)^n}{\left(\frac{pq+1}{p}\right)^m \times \left(\frac{pq-1}{p}\right)^n} \\&= \frac{\frac{(pq+1)^m}{(q)^m} \times \frac{(pq-1)^n}{(q)^n}}{\frac{(pq+1)^m}{(p)^m} \times \frac{(pq-1)^n}{(p)^n}} \\&= \frac{(pq+1)^m}{(q)^m} \times \frac{(p)^m}{(pq+1)^m} \times \frac{(pq-1)^n}{(q)^n} \times \frac{(p)^n}{(pq-1)^n} \\&= \frac{1}{(q)^m} \times \frac{(p)^m}{1} \times \frac{1}{q^n} \times \frac{p^n}{1} = \frac{p^m \times p^n}{q^m \times q^n} = \frac{(p)^{m+n}}{(q)^{m+n}} \\&= \left(\frac{p}{q}\right)^{m+n} = R.H.S\end{aligned}$$

Hence , L.H.S = R.H.S proved the result

6.

If x is a positive real number and exponents are rational numbers then simplify the following

$$(i) \frac{(x^{(a+b)})^2 (x^{(b+c)})^2 (x^{(c+a)})^2}{(x^a x^b x^c)^4}$$

$$(ii) \left[\frac{x^{a^2}}{x^{b^2}} \right]^{\frac{1}{a+b}} \left[\frac{x^{b^2}}{x^{c^2}} \right]^{\frac{1}{b+c}} \left[\frac{x^{c^2}}{x^{a^2}} \right]^{\frac{1}{c+a}}$$

$$(iii) \left(\frac{x^b}{x^c} \right)^{b+c-a} \left(\frac{x^c}{x^a} \right)^{c+a-b} \left(\frac{x^a}{x^b} \right)^{a+b-c}$$

Solution

$$(i) \frac{(x^{(a+b)})^2 (x^{(b+c)})^2 (x^{(c+a)})^2}{(x^a x^b x^c)^4}$$

$$= \frac{x^{2a+2b} \cdot x^{2b+2c} \cdot x^{2c+2a}}{x^{4a} \cdot x^{4b} \cdot x^{4c}} \left\{ \begin{array}{l} \because (a^n)^m = a^{mn} \\ x^m \cdot x^n = x^{m+n} \end{array} \right\}$$

$$= \frac{x^{2a+2b+2b+2c+2c+2a}}{x^{4a+4b+4c}}$$

$$= \frac{x^{4a+4b+4c}}{x^{4a+4b+4c}} = x^{4a+4b+4c-4a-4b-4c}$$

$$= x^0 = 1$$

$$(ii) \left[\frac{x^{a^2}}{x^{b^2}} \right]^{\frac{1}{a+b}} \left[\frac{x^{b^2}}{x^{c^2}} \right]^{\frac{1}{b+c}} \left[\frac{x^{c^2}}{x^{a^2}} \right]^{\frac{1}{c+a}}$$

$$= (x^{a^2-b^2})^{\frac{1}{a+b}} \times (x^{b^2-c^2})^{\frac{1}{b+c}} \times (x^{c^2-a^2})^{\frac{1}{c+a}}$$

$$\{\because (x^m)^n = x^{mn}\}$$

$$= x^{\frac{a^2-b^2}{a+b}} \times x^{\frac{b^2-c^2}{b+c}} \times x^{\frac{c^2-a^2}{c+a}}$$

$$= x^{\frac{(a+b)(a-b)}{a+b}} \times x^{\frac{(b+c)(b-c)}{b+c}} \times x^{\frac{(c+a)(c-a)}{c+a}}$$

$$\{\because a^2 - b^2 = (a+b)(a-b)\}$$

$$= x^{a-b} \cdot x^{b-c} \cdot x^{c-a}$$

$$= x^{a-b+b-c+c-a} = x^0 = 1 \quad \{\because x^0 = 1\}$$

$$(iii) \left(\frac{x^b}{x^c} \right)^{b+c-a} \left(\frac{x^c}{x^a} \right)^{c+a-b} \left(\frac{x^a}{x^b} \right)^{a+b-c}$$

$$= (x^{b-c})^{b+c-a} \cdot (x^{c-a})^{c+a-b} \cdot (x^{a-b})^{a+b-c}$$

$$= (x^{b-c})^{(b+c-a)} \cdot (x^{c-a})^{(c+a-b)} \cdot (x^{a-b})^{(a+b-c)}$$

$$\left\{ \begin{array}{l} \because x^m \div x^n = x^{m-n} \\ (x^m)^n = x^{mn} \end{array} \right\}$$

$$= x^{b^2-c^2-ab+ca} \cdot x^{c^2-a^2-bc+ab} \cdot x^{a^2-b^2-ca+bc}$$

$$= x^{b^2-c^2-ab+ca+c^2-a^2-bc+ab+a^2-b^2-ca+bc}$$

$$= x^0 = 1 \quad \left\{ \begin{array}{l} \because (x^m)^n = x^{mn} \\ x^0 = 1 \end{array} \right\}$$