

10 | CIRCLES

EXERCISE 10.1

Q.1. Fill in the blanks :

- (i) The centre of a circle lies in _____ of the circle. (exterior/interior)
- (ii) A point, whose distance from the centre of a circle is greater than its radius lies in _____ of the circle. (exterior/interior)
- (iii) The longest chord of a circle is a _____ of the circle.
- (iv) An arc is a _____ when its ends are the ends of a diameter.
- (v) Segment of a circle is the region between an arc and _____ of the circle.
- (vi) A circle divides the plane, on which it lies in _____ parts.

Sol. (i) interior (ii) exterior (iii) diameter (iv) semicircle (v) the chord (vi) three

Q.2. Write True or False: Give reasons for your answers.

- (i) Line segment joining the centre to any point on the circle is a radius of the circle.
- (ii) A circle has only finite number of equal chords.
- (iii) If a circle is divided into three equal arcs, each is a major arc.
- (iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.
- (v) Sector is the region between the chord and its corresponding arc.
- (vi) A circle is a plane figure.

Sol. (i) True (ii) False (iii) False (iv) True (v) False (vi) True

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EXERCISE 10.2

Q.1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Sol. Given : Two congruent circles with centres O and O'. AB and CD are equal chords of the circles with centres O and O' respectively.

To Prove : $\angle AOB = \angle COD$

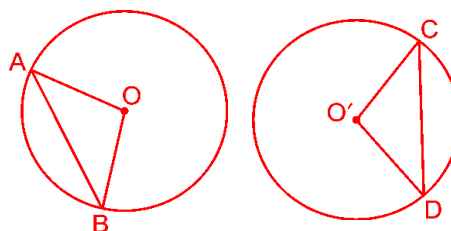
Proof : In triangles AOB and COD,

$$AB = CD \quad [\text{Given}]$$

$$\left. \begin{array}{l} AO = CO' \\ BO = DO' \end{array} \right\} \quad [\text{Radii of congruent circle}]$$

$$\Rightarrow \triangle AOB \cong \triangle CO'D \quad [\text{SSS axiom}]$$

$$\Rightarrow \angle AOB \cong \angle CO'D \quad \text{Proved.} \quad [\text{CPCT}]$$



Q.2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Ans. Given : Two congruent circles with centres O and O'. AB and CD are chords of circles with centre O and O' respectively such that $\angle AOB = \angle CO'D$

To Prove : $AB = CD$

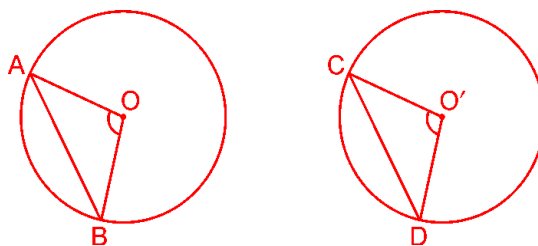
Proof : In triangles AOB and CO'D,

$$\left. \begin{array}{l} AO = CO' \\ BO = DO' \end{array} \right\} \quad [\text{Radii of congruent circle}]$$

$$\angle AOB = \angle CO'D \quad [\text{Given}]$$

$$\Rightarrow \triangle AOB \cong \triangle CO'D \quad [\text{SAS axiom}]$$

$$\Rightarrow AB = CD \quad \text{Proved.} \quad [\text{CPCT}]$$



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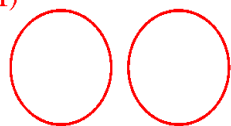
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EXERCISE 10.3

Q.1. Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

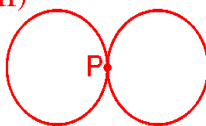
Ans.

(i)



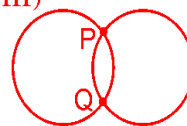
(i) 0 point

(ii)



(ii) 1 point

(iii)



(iii) 2 points

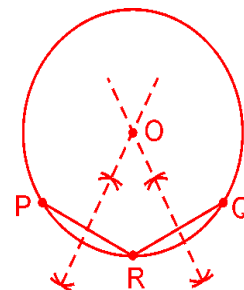
Maximum number of common points = 2 **Ans.**

Q.2. Suppose you are given a circle. Give a construction to find its centre.

Ans. Steps of Construction :

1. Take arc PQ of the given circle.
2. Take a point R on the arc PQ and draw chords PR and RQ.
3. Draw perpendicular bisectors of PR and RQ. These perpendicular bisectors intersect at point O.

Hence, point O is the centre of the given circle.



Q.3. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

Ans. Given : AB is the common chord of two intersecting circles (O, r) and (O', r') .

To Prove : Centres of both circles lie on the perpendicular bisector of chord AB, i.e., AB is bisected at right angle by OO' .

Construction : Join AO, BO, AO' and BO' .

Proof : In $\triangle AOO'$ and $\triangle BOO'$

$AO = BO$ (Radii of the circle (O, r))

$AO' = BO'$ (Radii of the circle (O', r'))

$OO' = OO'$ (Common)

$\therefore \triangle AOO' \cong \triangle BOO'$ (SSS congruency)

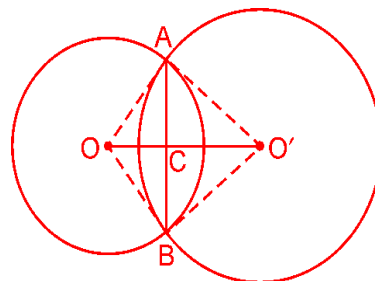
$\Rightarrow \angle AOO' = \angle BOO'$ (CPCT)

Now in $\triangle AOC$ and $\triangle BOC$

$\angle AOC = \angle BOC$ ($\angle AOO' = \angle BOO'$)

$AO = BO$ (Radii of the circle (O, r))

$OC = OC$ (Common)



$\therefore \triangle AOC \cong \triangle BOC$ (SAS congruency)

$\Rightarrow AC = BC$ and $\angle ACO = \angle BCO$... (i) (CPCT)

$\Rightarrow \angle ACO + \angle BCO = 180^\circ$.. (ii) (Linear pair)

$\Rightarrow \angle ACO = \angle BCO = 90^\circ$ (From (i) and (ii))

Hence, OO' lie on the perpendicular bisector of AB

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EXERCISE 10.4

Q.1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Sol. In $\triangle AOO'$,

$$AO^2 = 5^2 = 25$$

$$AO'^2 = 3^2 = 9$$

$$OO'^2 = 4^2 = 16$$

$$AO'^2 + OO'^2 = 9 + 16 = 25 = AO^2$$

$$\Rightarrow \angle AO'O = 90^\circ$$

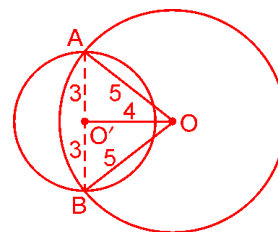
[By converse of pythagoras theorem]

Similarly, $\angle BO'O = 90^\circ$.

$$\Rightarrow \angle AO'B = 90^\circ + 90^\circ = 180^\circ$$

$\Rightarrow$ $AO'B$ is a straight line, whose mid-point is O .

$$\Rightarrow AB = (3 + 3) \text{ cm} = 6 \text{ cm} \text{ Ans.}$$



Q.2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Sol. Given : AB and CD are two equal chords of a circle which meet at E .

To prove : $AE = CE$ and $BE = DE$

Construction : Draw $OM \perp AB$ and $ON \perp CD$ and join OE . **Proof :**

In $\triangle OME$ and $\triangle ONE$

$OM = ON$ [Equal chords are equidistant]

$OE = OE$ [Common]

$\angle OME = \angle ONE$ [Each equal to 90°]

$\therefore \triangle OME \cong \triangle ONE$ [RHS axiom]

$\Rightarrow EM = EN$...(i) [CPCT]

Now $AB = CD$ [Given]

$$\Rightarrow \frac{1}{2} AB = \frac{1}{2} CD$$

$\Rightarrow AM = CN$..(ii) [Perpendicular from centre bisects the chord]

Adding (i) and (ii), we get

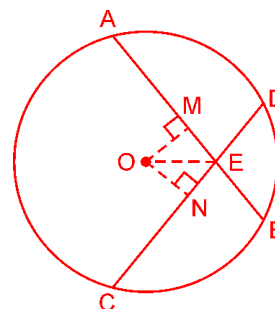
$$EM + AM = EN + CN$$

$$\Rightarrow AE = CE \text{ ..(iii)}$$

$$\text{Now, } AB = CD \text{ ..(iv)}$$

$$\Rightarrow AB - AE = CD - CE \text{ [From (iii)]}$$

$$\Rightarrow BE = CD - CE \text{ Proved.}$$



Q.3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Sol. Given : AB and CD are two equal chords of a circle which meet at E within the circle and a line PQ joining the point of intersection to the centre.

To Prove : $\angle AEQ = \angle DEQ$

Construction : Draw $OL \perp AB$ and $OM \perp CD$.

Proof : In $\triangle OLE$ and $\triangle OME$, we have

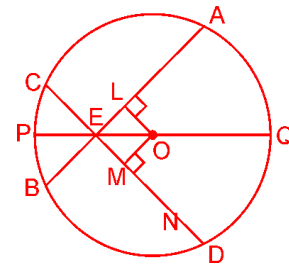
$$OL = OM \text{ [Equal chords are equidistant]}$$

$$OE = OE \quad \text{[Common]}$$

$$\angle OLE = \angle OME \quad \text{[Each} = 90^\circ \text{]}$$

$$\therefore \triangle OLE \cong \triangle OME \quad \text{[RHS congruence]}$$

$$\Rightarrow \angle LEO = \angle MEO \quad \text{[CPCT]}$$



Q.4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that $AB = CD$ (see Fig.)

Sol. Given : A line AD intersects two concentric circles at A, B, C and D, where O is the centre of these circles.

To prove : $AB = CD$

Construction : Draw $OM \perp AD$.

Proof : AD is the chord of larger circle.

$$\therefore AM = DM \quad \text{..(i) [OM bisects the chord]}$$

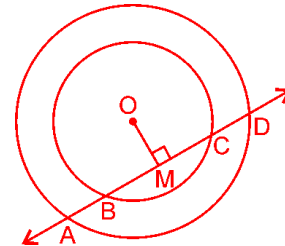
BC is the chord of smaller circle

$$\therefore BM = CM \quad \text{..(ii) [OM bisects the chord]}$$

Subtracting (ii) from (i), we get

$$AM - BM = DM - CM$$

$$\Rightarrow AB = CD \quad \text{Proved.}$$



Q.5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Sol. Let Reshma, Salma and Mandip be represented by R, S and M respectively.

Draw $OL \perp RS$,

$$OL^2 = OR^2 - RL^2$$

$$OL^2 = 5^2 - 3^2 \quad \text{[RL = 3 m, because } OL \perp RS \text{]}$$

$$= 25 - 9 = 16$$

$$OL = \sqrt{16} = 4$$

$$\text{Now, area of triangle ORS} = \frac{1}{2} \times KR \times OS$$

$$= \frac{1}{2} \times KR \times OS$$

$$\text{Also, area of } \triangle ORS = \frac{1}{2} \times RS \times OL = \frac{1}{2} \times 6 \times 4 = 12 \text{ m}^2$$

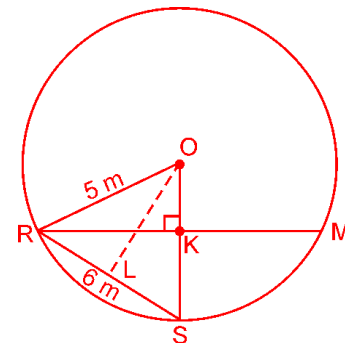
$$\Rightarrow \frac{1}{2} \times KR \times 5 = 12$$

$$\Rightarrow KR = \frac{12 \times 2}{5} = \frac{24}{5} = 4.8 \text{ m}$$

$$\Rightarrow RM = 2KR$$

$$\Rightarrow RM = 2 \times 4.8 = 9.6 \text{ m}$$

Hence, distance between Reshma and Mandip is 9.6 m **Ans.**



Q.6. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Sol. Let Ankur, Syed and David be represented by A, S and D respectively.

Let $PD = SP = SQ = QA = AR = RD = x$

In $\triangle OPD$,

$$OP^2 = 400 - x^2$$

$$\Rightarrow OP = \sqrt{400 - x^2}$$

$$\Rightarrow AP = 2\sqrt{400 - x^2} + \sqrt{400 - x^2}$$

[$\because$ centroid divides the median in the ratio 2 : 1]

$$= 3\sqrt{400 - x^2}$$

Now, in $\triangle APD$,

$$PD^2 = AD^2 - DP^2$$

$$\Rightarrow x^2 = (2x)^2 - (3\sqrt{400 - x^2})^2$$

$$\Rightarrow x^2 = 4x^2 - 9(400 - x^2)$$

$$\Rightarrow x^2 = 4x^2 - 3600 + 9x^2$$

$$\Rightarrow 12x^2 = 3600$$

$$\Rightarrow x^2 = \frac{3600}{12} = 300$$

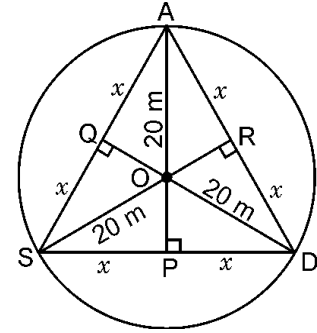
$$\Rightarrow x = 10\sqrt{3}$$

$$\text{Now, } SD = 2x = 2 \times 10\sqrt{3} = 20\sqrt{3}$$

$\therefore$ ASD is an equilateral triangle.

$$\Rightarrow SD = AS = AD = 20\sqrt{3}$$

Hence, length of the string of each phone is $20\sqrt{3}$ m **Ans.**



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EXERCISE 10.5

Q.1. In the figure, A , B and C are three points on a circle with centre O such that $\angle BOC = 30^\circ$ and $\angle AOB = 60^\circ$. If D is a point on the circle other than the arc ABC , find $\angle ADC$.

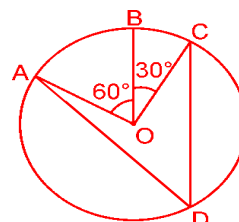
Sol. We have, $\angle BOC = 30^\circ$ and $\angle AOB = 60^\circ$

$$\angle AOC = \angle AOB + \angle BOC = 60^\circ + 30^\circ = 90^\circ$$

We know that angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc on the remaining part of the circle.

$$\therefore 2\angle ADC = \angle AOC$$

$$\Rightarrow \angle ADC = \frac{1}{2} \angle AOC = \frac{1}{2} \times 90^\circ \Rightarrow \angle ADC = 45^\circ \quad \text{Ans.}$$



Q.2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Sol. We have, $OA = OB = AB$

Therefore, $\triangle OAB$ is an equilateral triangle.

$$\Rightarrow \angle AOB = 60^\circ$$

We know that angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc on the remaining part of the circle.

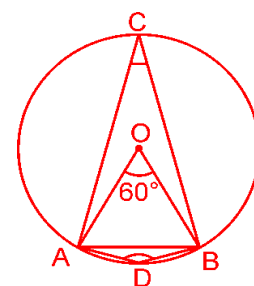
$$\therefore \angle AOB = 2\angle ACB$$

$$\Rightarrow \angle ACB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 60^\circ$$

$$\Rightarrow \angle ACB = 30^\circ$$

$$\begin{aligned} \text{Also, } \angle ADB &= \frac{1}{2} \text{ reflex } \angle AOB \\ &= \frac{1}{2} (360^\circ - 60^\circ) = \frac{1}{2} \times 300^\circ = 150^\circ \end{aligned}$$

Hence, angle subtended by the chord at a point on the minor arc is 150° and at a point on the major arc is 30° **Ans.**



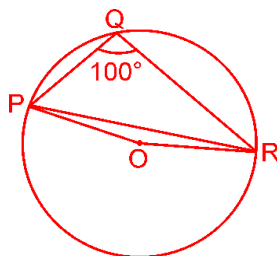
Q.3. In the figure, $\angle PQR = 100^\circ$, where P , Q and R are points on a circle with centre O . Find $\angle OPR$.

Sol. Reflex angle $POR = 2\angle PQR$

$$= 2 \times 100^\circ = 200^\circ$$

$$\text{Now, angle } POR = 360^\circ - 200^\circ = 160^\circ$$

Also,



$$PO = OR \quad [\text{Radii of a circle}]$$

$$\angle OPR = \angle ORP \quad [\text{Opposite angles of isosceles triangle}]$$

$$\text{In } \triangle OPR, \angle POR = 160^\circ$$

$$\therefore \angle OPR = \angle ORP = 10^\circ$$

[Angle sum property of a triangle]. **Ans.**

Q.4. In the figure, $\angle ABC = 69^\circ$, $\angle ACB = 31^\circ$, find $\angle BDC$.

Sol. In $\triangle ABC$, we have

$$\angle ABC + \angle ACB + \angle BAC = 180^\circ$$

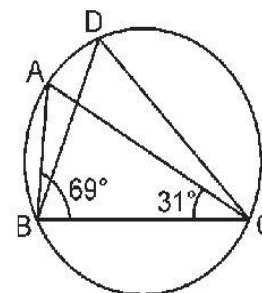
[Angle sum property of a triangle]

$$\Rightarrow 69^\circ + 31^\circ + \angle BAC = 180^\circ$$

$$\Rightarrow \angle BAC = 180^\circ - 100^\circ = 80^\circ$$

Also, $\angle BAC = \angle BDC$ [Angles in the same segment]

$$\Rightarrow \angle BDC = 80^\circ \quad \text{Ans.}$$



Q.5. In the figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130^\circ$ and $\angle ECD = 20^\circ$. Find $\angle BAC$.

Sol. $\angle BEC + \angle DEC = 180^\circ$ [Linear pair]

$$\Rightarrow 130^\circ + \angle DEC = 180^\circ$$

$$\Rightarrow \angle DEC = 180^\circ - 130^\circ = 50^\circ$$

Now, in $\triangle DEC$,

$$\Rightarrow \angle DEC + \angle DCE + \angle CDE = 180^\circ$$

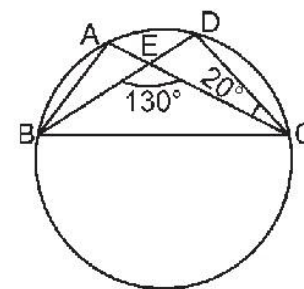
[Angle sum property of a triangle]

$$\Rightarrow 50^\circ + 20^\circ + \angle CDE = 180^\circ$$

$$\Rightarrow \angle CDE = 180^\circ - 70^\circ = 110^\circ$$

Also, $\angle CDE = \angle BAC$ [Angles in same segment]

$$\Rightarrow \angle BAC = 110^\circ \quad \text{Ans.}$$



Q.6. $ABCD$ is a cyclic quadrilateral whose diagonals intersect at a point E . If $\angle DBC = 70^\circ$, $\angle BAC = 30^\circ$, find $\angle BCD$. Further, if $AB = BC$, find $\angle ECD$.

Sol. $\angle CAD = \angle DBC = 70^\circ$ [Angles in the same segment]

$$\begin{aligned}\text{Therefore, } \angle DAB &= \angle CAD + \angle BAC \\ &= 70^\circ + 30^\circ = 100^\circ\end{aligned}$$

$$\text{But, } \angle DAB + \angle BCD = 180^\circ$$

[Opposite angles of a cyclic quadrilateral]

$$\text{So, } \angle BCD = 180^\circ - 100^\circ = 80^\circ$$

Now, we have $AB = BC$

Therefore, $\angle BCA = 30^\circ$ [Opposite angles of an isosceles triangle]

$$\text{Again, } \angle DAB + \angle BCD = 180^\circ$$

[Opposite angles of a cyclic quadrilateral]

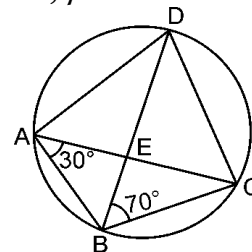
$$\Rightarrow 100^\circ + \angle BCA + \angle ECD = 180^\circ \quad [\because \angle BCD = \angle BCA + \angle ECD]$$

$$\Rightarrow 100^\circ + 30^\circ + \angle ECD = 180^\circ$$

$$\Rightarrow 130^\circ + \angle ECD = 180^\circ$$

$$\Rightarrow \angle ECD = 180^\circ - 130^\circ = 50^\circ$$

$$\text{Hence, } \angle BCD = 80^\circ \text{ and } \angle ECD = 50^\circ \quad \text{Ans.}$$



Q.7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Sol. Given : $ABCD$ is a cyclic quadrilateral, whose diagonals AC and BD are diameter of the circle passing through A, B, C and D .

To Prove : $ABCD$ is a rectangle.

Proof : In $\triangle AOD$ and $\triangle COB$

$$AO = CO \quad [\text{Radii of a circle}]$$

$$OD = OB \quad [\text{Radii of a circle}]$$

$$\angle AOD = \angle COB \quad [\text{Vertically opposite angles}]$$

$$\therefore \triangle AOD \cong \triangle COB \quad [\text{SAS axiom}]$$

$$\therefore \angle OAD = \angle OCB \quad [\text{CPCT}]$$

But these are alternate interior angles made by the transversal AC , intersecting AD and BC .

$$\therefore AD \parallel BC$$

Similarly, $AB \parallel CD$.

Hence, quadrilateral $ABCD$ is a parallelogram.

$$\text{Also, } \angle ABC = \angle ADC \quad \dots(i) \quad [\text{Opposite angles of a } \parallel\text{gm are equal}]$$

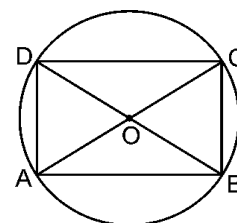
$$\text{And, } \angle ABC + \angle ADC = 180^\circ \dots(ii)$$

[Sum of opposite angles of a cyclic quadrilateral is 180°]

$$\Rightarrow \angle ABC = \angle ADC = 90^\circ \quad [\text{From (i) and (ii)}]$$

$$\therefore ABCD \text{ is a rectangle. [A } \parallel\text{gm one of whose angles is}$$

$$90^\circ \text{ is a rectangle]} \quad \text{Proved.}$$



Q.8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Sol. Given : A trapezium $ABCD$ in which $AB \parallel CD$ and $AD = BC$.

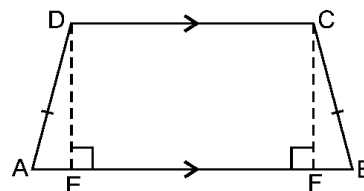
To Prove : $ABCD$ is a cyclic trapezium.

Construction : Draw $DE \perp AB$ and $CF \perp AB$.

Proof : In $\triangle DEA$ and $\triangle CFB$, we have

$$AD = BC \quad [\text{Given}]$$

$$\angle DEA = \angle CFB = 90^\circ \quad [DE \perp AB \text{ and } CF \perp AB]$$



$$DE = CF$$

[Distance between parallel lines remains constant]

$$\therefore \triangle DEA \cong \triangle CFB \quad [\text{RHS axiom}]$$

$$\Rightarrow \angle A = \angle B \quad \dots(i) \quad [\text{CPCT}]$$

$$\text{and, } \angle ADE = \angle BCF \quad \dots(ii) \quad [\text{CPCT}]$$

$$\text{Since, } \angle ADE = \angle BCF \quad [\text{From (ii)}]$$

$$\Rightarrow \angle ADE + 90^\circ = \angle BCF + 90^\circ$$

$$\Rightarrow \angle ADE + \angle CDE = \angle BCF + \angle DCF$$

$$\Rightarrow \angle D = \angle C \quad \dots(iii)$$

$$[\angle ADE + \angle CDE = \angle D, \angle BCF + \angle DCF = \angle C]$$

$$\therefore \angle A = \angle B \text{ and } \angle C = \angle D \quad [\text{From (i) and (iii)}] \quad (iv)$$

$$\angle A + \angle B + \angle C + \angle D = 360^\circ \quad [\text{Sum of the angles of a quadrilateral is } 360^\circ]$$

$$\Rightarrow 2(\angle B + \angle D) = 360^\circ \quad [\text{Using (iv)}]$$

$$\Rightarrow \angle B + \angle D = 180^\circ$$

$\Rightarrow$ Sum of a pair of opposite angles of quadrilateral ABCD is 180° .

$\Rightarrow$ ABCD is a cyclic trapezium **Proved.**

Q.9. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig.). Prove that $\angle ACP = \angle QCD$.

Sol. Given : Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively.

To Prove : $\angle ACP = \angle QCD$.

Proof : $\angle ACP = \angle ABP \quad \dots(i)$

[Angles in the same segment]

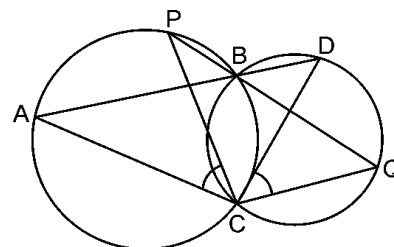
$$\angle QCD = \angle QBD \quad \dots(ii)$$

[Angles in the same segment]

$$\text{But, } \angle ABP = \angle QBD \quad \dots(iii) \quad [\text{Vertically opposite angles}]$$

By (i), (ii) and (iii) we get

$$\angle ACP = \angle QCD \quad \text{Proved.}$$



Q.10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Sol. Given : Sides AB and AC of a triangle ABC are diameters of two circles which intersect at D.

To Prove : D lies on BC.

Proof : Join AD

$$\angle ADB = 90^\circ \quad \dots(i) \quad [\text{Angle in a semicircle}]$$

$$\text{Also, } \angle ADC = 90^\circ \quad \dots(ii)$$

Adding (i) and (ii), we get

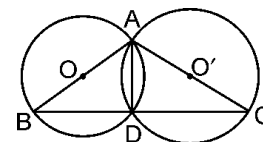
$$\angle ADB + \angle ADC = 90^\circ + 90^\circ$$

$$\Rightarrow \angle ADB + \angle ADC = 180^\circ$$

$\Rightarrow$ BDC is a straight line.

$\therefore$ D lies on BC

Hence, point of intersection of circles lie on the third side BC. **Proved.**



Q.11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

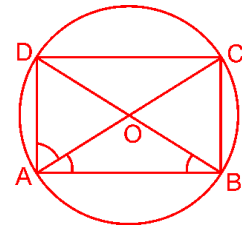
Sol. Given : ABC and ADC are two right triangles with common hypotenuse AC.

To Prove : $\angle CAD = \angle CBD$

Proof : Let O be the mid-point of AC.

Then $OA = OB = OC = OD$

Mid point of the hypotenuse of a right triangle is equidistant from its vertices with O as centre and radius equal to OA, draw a circle to pass through A, B, C and D.



We know that angles in the same segment of a circle are equal.

Since, $\angle CAD$ and $\angle CBD$ are angles of the same segment.

Therefore, $\angle CAD = \angle CBD$. **Proved.**

Q.12. Prove that a cyclic parallelogram is a rectangle.

Sol. Given : ABCD is a cyclic parallelogram.

To prove : ABCD is a rectangle.

Proof : $\angle ABC = \angle ADC$...(i)

[Opposite angles of a ||gm are equal]

But, $\angle ABC + \angle ADC = 180^\circ$... (ii)

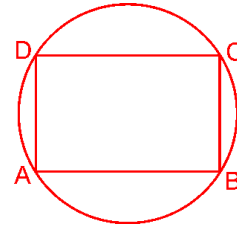
[Sum of opposite angles of a cyclic quadrilateral is 180°]

$\Rightarrow \angle ABC = \angle ADC = 90^\circ$ [From (i) and (ii)]

$\therefore$ ABCD is a rectangle

[A ||gm one of whose angles is 90° is a rectangle]

Hence, a cyclic parallelogram is a rectangle. **Proved.**

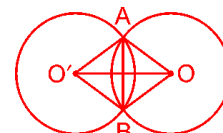


10 CIRCLES

EXERCISE 10.6 (Optional)

Q.1. Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Sol. Given : Two intersecting circles, in which OO' is the line of centres and A and B are two points of intersection.



To prove : $\angle OAO' = \angle OBO'$

Construction : Join AO , BO , AO' and BO' .

Proof : In $\triangle AOO'$ and $\triangle BOO'$, we have

$$AO = BO \quad [\text{Radii of the same circle}]$$

$$AO' = BO' \quad [\text{Radii of the same circle}]$$

$$OO' = OO' \quad [\text{Common}]$$

$$\therefore \triangle AOO' \cong \triangle BOO' \quad [\text{SSS axiom}]$$

$$\Rightarrow \angle OAO' = \angle OBO' \quad [\text{CPCT}]$$

Hence, the line of centres of two intersecting circles subtends equal angles at the two points of intersection. **Proved.**

Q.2. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

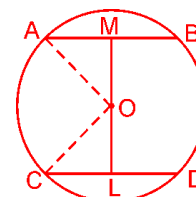
Sol. Let O be the centre of the circle and let its radius be r cm.

Draw $OM \perp AB$ and $OL \perp CD$.

$$\text{Then, } AM = \frac{1}{2} AB = \frac{5}{2} \text{ cm}$$

$$\text{and, } CL = \frac{1}{2} CD = \frac{11}{2} \text{ cm}$$

Since, $AB \parallel CD$, it follows that the points O , L , M are



collinear and therefore, $LM = 6$ cm.

Let $OL = x$ cm. Then $OM = (6 - x)$ cm

Join OA and OC . Then $OA = OC = r$ cm.

Now, from right-angled $\triangle OMA$ and $\triangle OLC$, we have

$$OA^2 = OM^2 + AM^2 \text{ and } OC^2 = OL^2 + CL^2 \quad [\text{By Pythagoras Theorem}]$$

$$\Rightarrow r^2 = (6 - x)^2 + \left(\frac{5}{2}\right)^2 \quad \dots (i) \text{ and } r^2 = x^2 + \left(\frac{11}{2}\right)^2 \quad \dots (ii)$$

$$\Rightarrow (6 - x)^2 + \left(\frac{5}{2}\right)^2 = x^2 + \left(\frac{11}{2}\right)^2 \quad [\text{From (i) and (ii)}]$$

$$\Rightarrow 36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$$

$$\Rightarrow -12x = \frac{121}{4} - \frac{25}{4} - 36$$

$$\Rightarrow -12x = \frac{96}{4} - 36$$

$$\Rightarrow -12x = 24 - 36$$

$$\Rightarrow -12x = -12$$

$$\Rightarrow x = 1$$

Substituting $x = 1$ in (i), we get

$$r^2 = (6 - x)^2 + \left(\frac{5}{2}\right)^2$$

$$\Rightarrow r^2 = (6 - 1)^2 + \left(\frac{5}{2}\right)^2$$

$$\Rightarrow r^2 = (5)^2 + \left(\frac{5}{2}\right)^2 = 25 + \frac{25}{4}$$

$$\Rightarrow r^2 = \frac{125}{4}$$

$$\Rightarrow r = \frac{5\sqrt{5}}{2}$$

Hence, radius $r = \frac{5\sqrt{5}}{2}$ cm. **Ans.**

Q.3. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?

Sol. Let PQ and RS be two parallel chords of a circle with centre O.

We have, PQ = 8 cm and RS = 6 cm.

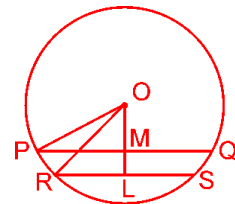
Draw perpendicular bisector OL of RS which meets PQ in M. Since, PQ || RS, therefore, OM is also perpendicular bisector of PQ.

Also, OL = 4 cm and $RL = \frac{1}{2} RS \Rightarrow RL = 3$ cm

and $PM = \frac{1}{2} PQ \Rightarrow PM = 4$ cm

In $\triangle ORL$, we have

$$OR^2 = RL^2 + OL^2 \quad [\text{Pythagoras theorem}]$$



$$\Rightarrow OR^2 = 3^2 + 4^2 = 9 + 16$$

$$\Rightarrow OR^2 = 25 \Rightarrow OR = \sqrt{25}$$

$$\Rightarrow OR = 5 \text{ cm}$$

$$\therefore OR = OP \quad [\text{Radii of the circle}]$$

$$\Rightarrow OP = 5 \text{ cm}$$

Now, in $\triangle OPM$

$$OM^2 = OP^2 - PM^2 \quad [\text{Pythagoras theorem}]$$

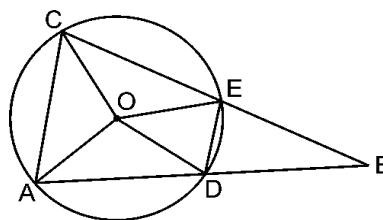
$$\Rightarrow OM^2 = 5^2 - 4^2 = 25 - 16 = 9$$

$$OM = \sqrt{9} = 3 \text{ cm}$$

Hence, the distance of the other chord from the centre is 3 cm. **Ans.**

Q.4. Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that $\angle ABC$ is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Sol. Given : Two equal chords AD and CE of a circle with centre O . When meet at B when produced.



$$\text{To Prove : } \angle ABC = \frac{1}{2} (\angle AOC - \angle DOE)$$

Proof : Let $\angle AOC = x$, $\angle DOE = y$, $\angle AOD = z$

$\angle EOC = z$ [Equal chords subtend equal angles at the centre]

$$\therefore x + y + 2z = 360^\circ \quad [\text{Angle at a point}] \quad \dots (i)$$

$$OA = OD \Rightarrow \angle OAD = \angle ODA$$

$\therefore$ In $\triangle OAD$, we have

$$\angle OAD + \angle ODA + z = 180^\circ$$

$$\Rightarrow 2\angle OAD = 180^\circ - z \quad [\because \angle OAD = \angle ODA]$$

$$\Rightarrow \angle OAD = 90^\circ - \frac{z}{2} \quad \dots (ii)$$

$$\text{Similarly } \angle OCE = 90^\circ - \frac{z}{2} \quad \dots (iii)$$

$$\Rightarrow \angle ODB = \angle OAD + \angle ODA \quad [\text{Exterior angle property}]$$

$$\Rightarrow \angle OEB = 90^\circ - \frac{z}{2} + z \quad [\text{From (ii)}]$$

$$\Rightarrow \angle ODB = 90^\circ + \frac{z}{2} \quad \dots (iv)$$

$$\text{Also, } \angle OEB = \angle OCE + \angle COE \quad [\text{Exterior angle property}]$$

$$\Rightarrow \angle OEB = 90^\circ - \frac{z}{2} + z \quad [\text{From (iii)}]$$

$$\Rightarrow \angle OEB = 90^\circ + \frac{z}{2} \quad \dots (v)$$

$$\text{Also, } \angle OED = \angle ODE = 90^\circ - \frac{y}{2} \quad \dots \text{ (vi)}$$

O from (iv), (v) and (vi), we have

$$\angle BDE = \angle BED = 90^\circ + \frac{z}{2} - \left(90^\circ - \frac{y}{2}\right)$$

$$\Rightarrow \angle BDE = \angle BED = \frac{y+z}{2}$$

$$\Rightarrow \angle BDE = \angle BED = y + z \quad \dots \text{ (vii)}$$

$$\therefore \angle BDE = 180^\circ - (y + z)$$

$$\Rightarrow \angle ABC = 180^\circ - (y + z) \quad \dots \text{ (viii)}$$

$$\text{Now, } \frac{y-z}{2} = \frac{360^\circ - y - 2z - y}{2} = 180^\circ - (y + z) \quad \dots \text{ (ix)}$$

From (viii) and (ix), we have

$$\angle ABC = \frac{x-y}{2} \quad \textbf{Proved.}$$

Q.5. Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.

Sol. Given : A rhombus ABCD whose diagonals intersect each other at O.

To prove : A circle with AB as diameter passes through O.

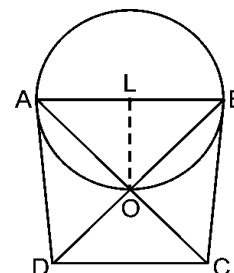
Proof : $\angle AOB = 90^\circ$

[Diagonals of a rhombus bisect each other at 90°]

$\Rightarrow \triangle AOB$ is a right triangle right angled at O.

$\Rightarrow AB$ is the hypotenuse of $\triangle AOB$.

$\Rightarrow$ If we draw a circle with AB as diameter, then it will pass through O. because angle in a semicircle is 90° and $\angle AOB = 90^\circ$ **Proved.**



Q.6. ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that $AE = AD$.

Sol. Given : ABCD is a parallelogram.

To Prove : $AE = AD$.

Construction : Draw a circle which passes through ABC and intersect CD (or CD produced) at E.

Proof : For fig (i)

$$\angle AED + \angle ABC = 180^\circ$$

[Linear pair] ... (ii)

$$\text{But } \angle ACD = \angle ADC = \angle ABC + \angle ADE$$

$$\Rightarrow \angle ABC + \angle ADE = 180^\circ \quad \text{[From (ii)]} \quad \dots \text{ (iii)}$$

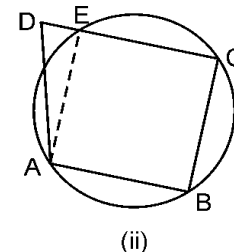
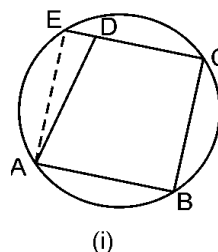
From (i) and (iii)

$$\angle AED + \angle ABC = \angle ABC + \angle ADE$$

$$\Rightarrow \angle AED = \angle ADE$$

$$\Rightarrow \angle AD = \angle AE \quad \text{[Sides opposite to equal angles are equal]}$$

Similarly we can prove for Fig (ii) **Proved.**



Q.7. *AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters, (ii) ABCD is rectangle.*

Sol. Given : A circle with chords AB and CD which bisect each other at O.

To Prove : (i) AC and BD are diameters
(ii) ABCD is a rectangle.

Proof : In $\triangle OAB$ and $\triangle OCD$, we have

$$OA = OC$$

[Given]

$$OB = OD$$

[Given]

$$\angle AOB = \angle COD$$

[Vertically opposite angles]

$$\Rightarrow \triangle AOB \cong \triangle COD$$

[SAS congruence]

$$\Rightarrow \angle ABO = \angle CDO \text{ and } \angle BAO = \angle BCO \quad [\text{CPCT}]$$

$$\Rightarrow AB \parallel DC \quad \dots (i)$$

$$\text{Similarly, we can prove } BC \parallel AD \quad \dots (ii)$$

Hence, ABCD is a parallelogram.

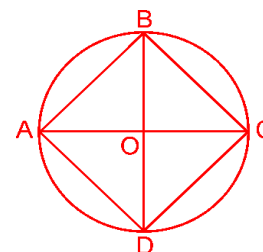
But ABCD is a cyclic parallelogram.

$\therefore$ ABCD is a rectangle. [Proved in Q. 12 of Ex. 10.5]

$$\Rightarrow \angle ABC = 90^\circ \text{ and } \angle BCD = 90^\circ$$

$$\Rightarrow AC \text{ is a diameter and } BD \text{ is a diameter}$$

[Angle in a semicircle is 90°] **Proved.**



Q.8. *Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are $90^\circ - \frac{1}{2}A$, $90^\circ - \frac{1}{2}B$ and $90^\circ - \frac{1}{2}C$.*

Sol. Given : $\triangle ABC$ and its circumcircle. AD, BE, CF are bisectors of $\angle A$, $\angle B$, $\angle C$ respectively.

Construction : Join DE, EF and FD.

Proof : We know that angles in the same segment are equal.

$$\therefore \angle 5 = \frac{\angle C}{2} \text{ and } \angle 6 = \frac{\angle B}{2} \quad \dots (i)$$

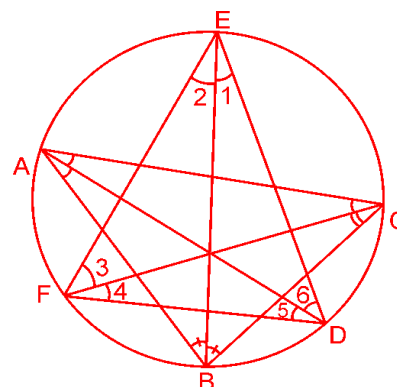
$$\angle 1 = \frac{\angle A}{2} \text{ and } \angle 2 = \frac{\angle C}{2} \quad \dots (ii)$$

$$\angle 4 = \frac{\angle A}{2} \text{ and } \angle 3 = \frac{\angle B}{2} \quad \dots (iii)$$

From (i), we have

$$\angle 5 + \angle 6 = \frac{\angle C}{2} + \frac{\angle B}{2}$$

$$\Rightarrow \angle D = \frac{\angle C}{2} + \frac{\angle B}{2} \quad \dots (iv)$$



$$[\because \angle 5 + \angle 6 = \angle D]$$

$$\text{But } \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle B + \angle C = 180^\circ - \angle A$$

$$\Rightarrow \frac{\angle B}{2} + \frac{\angle C}{2} = 90^\circ - \frac{\angle A}{2}$$

$\therefore$ (iv) becomes,

$$\angle D = 90^\circ - \frac{\angle A}{2}.$$

Similarly, from (ii) and (iii), we can prove that

$$\angle E = 90^\circ - \frac{\angle B}{2} \text{ and } \angle F = 90^\circ - \frac{\angle C}{2} \quad \textbf{Proved.}$$

Q.9. Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

Sol. Given : Two congruent circles which intersect at A and B. PAB is a line through A.

To Prove : BP = BQ.

Construction : Join AB.

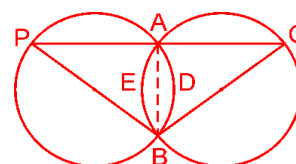
Proof : AB is a common chord of both the circles.

But the circles are congruent —

$$\Rightarrow \text{arc ADB} = \text{arc AEB}$$

$$\Rightarrow \angle APB = \angle AQB \quad \text{Angles subtended}$$

$$\Rightarrow BP = BQ \quad [\text{Sides opposite to equal angles are equal}] \quad \textbf{Proved.}$$



Q.10. In any triangle ABC, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.

Sol. Let angle bisector of $\angle A$ intersect circumcircle of $\triangle ABC$ at D.

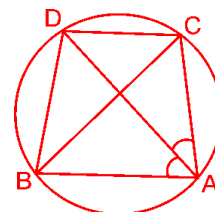
Join DC and DB.

$$\angle BCD = \angle BAD$$

[Angles in the same segment]

$$\Rightarrow \angle BCD = \angle BAD = \frac{1}{2} \angle A$$

[AD is bisector of $\angle A$] ... (i)



$$\text{Similarly } \angle DBC = \angle DAC = \frac{1}{2} \angle A \quad \dots \text{ (ii)}$$

From (i) and (ii) $\angle DBC = \angle BCD$

$$\Rightarrow BD = DC \quad [\text{sides opposite to equal angles are equal}]$$

$\Rightarrow$ D lies on the perpendicular bisector of BC.

Hence, angle bisector of $\angle A$ and perpendicular bisector of BC intersect on the circumcircle of $\triangle ABC$ **Proved.**