

Chapter 9
logarithms
Exercise 9.1

1. convert the following to logarithmic form :

(i) $5^2 = 25$

(ii) $a^5 = 64$

(iii) $7^x = 100$

(iv) $9^0 = 1$

(v) $6^1 = 6$

(vi) $3^{-2} = \frac{1}{9}$

(vii) $10^{-2} = 0.01$

(viii) $(81)^{\frac{3}{4}} = 27$

Solution

(i) $5^2 = 25 = \log_5 25 = 2$

(ii) $a^5 = 64 = \log_a 64 = 5$

(iii) $7^x = 100 = \log_7 100 = x$

(iv) $9^0 = 1 = \log_9 1 = 0$

(v) $6^1 = 6 = \log_6 6 = 1$

(vi) $3^{-2} = \frac{1}{9} = \log_3 \frac{1}{9} = -2$

(vii) $10^{-2} = 0.01 = \log_{10} 0.01 = -2$

$$\text{(viii)} \quad (81)^{\frac{3}{4}} = 27 = \log_{81} 27 = \frac{3}{4}$$

2. convert the following into exponential form :

$$\text{(i)} \quad \log_2 32 = 5$$

$$\text{(ii)} \quad \log_3 81 = 4$$

$$\text{(iii)} \quad \log_3 \frac{1}{3} = -1$$

$$\text{(iv)} \quad \log_3 4 = \frac{2}{3}$$

$$\text{(v)} \quad \log_8 32 = \frac{5}{3}$$

$$\text{(vi)} \quad \log_{10}(0.001) = -3$$

$$\text{(vii)} \quad \log_2 0.25 = -2$$

$$\text{(viii)} \quad \log_a \left(\frac{1}{a} \right) = -1$$

Solution

$$\text{(i)} \quad \log_2 32 = 5 \quad = 2^5 = 32$$

$$\text{(ii)} \quad \log_3 81 = 4 \quad = 3^4 = 81$$

$$\text{(iii)} \quad \log_3 \frac{1}{3} = -1 \quad = 3^{-1} = \frac{1}{3}$$

$$\text{(iv)} \quad \log_3 4 = \frac{2}{3} \quad = (8)^{\frac{2}{3}} 4$$

$$\text{(v)} \quad \log_8 32 = \frac{5}{3} \quad = (8)^{\frac{5}{3}} = 32$$

$$\text{(vi)} \quad \log_{10}(0.001) = -3 \quad = 10^{-3} = 0.001$$

$$\text{(vii) } \log_2 0.25 = -2 = 2^{-2} = 0.25$$

$$\text{(viii) } \log_a \left(\frac{1}{a}\right) = -1 = a^{-1} = \frac{1}{a}$$

3. by converting to exponential form , find the values of :

$$\text{(i) } \log_2 16$$

$$\text{(ii) } \log_5 125$$

$$\text{(iii) } \log_4 8$$

$$\text{(iv) } \log_9 27$$

$$\text{(v) } \log_{10} (.01)$$

$$\text{(vi) } \log_7 \frac{1}{7}$$

$$\text{(vii) } \log_5 256$$

$$\text{(viii) } \log_2 0.25$$

Solution

$$\text{(i) let , } \log_2 16 = x$$

$$= 2^x = 16$$

$$= 2^x = 2 \times 2 \times 2 \times 2$$

$$= 2^x = 2^4$$

$$\therefore x = 4$$

$$\begin{aligned}
 \text{(ii) let } \log_5 125 &= x = 5^x = 125 \\
 &= 5^x = 5 \times 5 \times 5 = 5^x = 5^3 \\
 x &= 3
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii) let } \log_4 8 &= x = 4^x = 8 \\
 &= (2 \times 2)^x = 2 \times 2 \times 2 = (2)^{2x} = (2)^3 \\
 &= 2x = 3 \\
 x &= \frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv) } \log_9 27 &= x \\
 &= 9^x = 27 \\
 &= (3 \times 3)^x = 3 \times 3 \times 3 = (3)^{2x} = (3)^3 \\
 &= 2x = 3 \\
 \therefore x &= \frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(v) } \log_{10} (.01) &= x = (10)^x = .01 \\
 &= (10)^x = \frac{1}{100} = (10)^x = \frac{1}{10} \times \frac{1}{10} \\
 &= (10)^x = \frac{1}{(10)^2} = (10)^x = (10)^{-2} \\
 \therefore x &= -2
 \end{aligned}$$

$$\text{(vi) } \log_7 \frac{1}{7} = x = (7)^x = \frac{1}{7}$$

$$= (7)^x = (7)^{-1}$$

$$x = -1$$

$$\text{(vii) let } \log_5 256 = x$$

$$= (.5)^x = 256 = \left(\frac{5}{10}\right)^x = 256$$

$$= \left(\frac{1}{2}\right)^x = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= (2)^x = 2^8$$

$$= -x = 8$$

$$x = -8$$

$$\text{(viii) let , } \log_3 0.25 = x$$

$$= (2)^x = 0.25$$

$$= (2)^x = \frac{25}{100} = (2)^x = \frac{1}{4}$$

$$= (2)^x = (2)^{-2}$$

$$X = -2$$

4. solve the following equations for x.

(i) $\log_3 x = 2$

(ii) $\log_x 25 = 2$

(iii) $\log_{10} x = -2$

(iv) $\log_4 x = \frac{1}{2}$

(v) $\log_x 11 = 2.5$

(vi) $\log_x \frac{1}{4} = -1$

(vii) $\log_{81} x = \frac{3}{2}$

(viii) $\log_9 x = 2.5$

(ix) $\log_4 x = -1.5$

Solution

(i) $\log_3 x = 2$

Let us simplify the expression

$$(3)^2 = x$$

$$X = 9$$

(ii) $\log_x 25 = 2$

Let us simplify the expression

$$(x)^2 = 25$$

$$= 5 \times 5$$

$$= x^2 = 5^2$$

Since the powers are same

So,

$$X = 5$$

$$(iii) \log_{10} x = -2$$

Let us simplify the expression ,

$$(10)^{-2} = x$$

$$X = \frac{1}{10^2}$$

$$= \frac{1}{100}$$

$$= 0.01$$

$$(iv) \log_4 x = \frac{1}{2}$$

Let us simplify the expression,

$$4^{\frac{1}{2}} = x$$

$$x = (2 \times 2)^{\frac{1}{2}}$$

$$= 2^{2 \times \frac{1}{2}}$$

$$= 2$$

$$(v) \log_x 11 = 2.5$$

Let us simplify the expression,

$$(x)^1 = 11$$

$$X = 11$$

$$(vi) \log_x \frac{1}{4} = -1$$

Let us simplify the expression

$$(x)^{-1} = \frac{1}{4}$$

$$x^{-1} = 4^{-1}$$

since the powers are same,

so,

$$x = 4$$

$$(vii) \log_{81} x = \frac{3}{2}$$

Let us simplify the expression

$$81^{\frac{3}{2}} = x$$

$$x = 81^{\frac{3}{2}}$$

$$= (3^4)^{\frac{3}{2}}$$

$$= 3^{4 \times \frac{3}{2}}$$

$$= 3^6$$

$$= 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

$$= 729$$

$$X = 729$$

$$\text{(viii)} \log_9 x = 2.5$$

$$\log_9 x = \frac{5}{2}$$

Let us simplify the expression ,

$$9^{\frac{5}{2}} = x$$

$$x = (3^2)^{\frac{5}{2}}$$

$$= 3^{2 \times \frac{5}{2}}$$

$$= 3^5$$

$$= 3 \times 3 \times 3 \times 3 \times 3$$

$$= 234$$

$$X = 234$$

$$\text{(ix)} \log_4 x = -1.5$$

$$\log_4 x = -\frac{3}{2}$$

Let us simplify the expression

$$(4)^{-\frac{3}{2}} = x$$

$$x = (2^2)^{-\frac{3}{2}}$$

$$= 2^{2 \times -\frac{3}{2}}$$

$$= 2^{-3}$$

$$= \frac{1}{2^3}$$

$$= \frac{1}{2 \times 2 \times 2}$$

$$= \frac{1}{8}$$

$$= \frac{1}{8}$$

5. given $\log_{10} a = b$, express 10^{2b-3} in terms of a .

Solution

Given :

$$\log_{10} a = b$$

$$(10)^b = a$$

Now ,

$$10^{2b-3} = \frac{(10)^{2b}}{(10)^3}$$

$$\frac{(10^b)^2}{10 \times 10 \times 10}$$

Substitute the value of $(10)^b = a$, we get

$$= \frac{a^2}{1000}$$

6. given $\log_{10} x = a$, $\log_{10} y = b$ and $\log_{10} z = c$,

(i) write down 10^{2a-3} in terms of x.

(ii) write down 10^{3b-1} in terms of y.

(iii) if $\log_{10} p = 2a + \frac{b}{2} - 3c$ express p in terms of x, y and z

Solution

Given :

$$\log_{10} x = a$$

- $(10)^a = x$

$$\log_{10} y = b$$

- $(10)^b = y$

$$\log_{10} z = c$$

- $(10)^c = z$

(i) write down 10^{2a-3} in terms of x.

$$10^{2a-3} = \frac{(10)^{2a}}{(10)^3}$$

$$= \frac{(10)^{2a}}{(10 \times 10 \times 10)}$$

Substitute the value of $(10)^a = x$, we get

$$= \frac{x^2}{1000}$$

(ii) write down 10^{3b-1} in terms of y.

$$10^{3b-1} = \frac{(10)^{3b}}{(10)^1}$$

$$= \frac{(10)^{3b}}{10}$$

Substitute the value of $(10)^b = y$ we get

$$\frac{y^3}{10}$$

(iii) if $\log_{10} p = 2a + \frac{b}{2} - 3c$, express p in term of x, y and z

We know that ,

$$(10)^a = x$$

$$(10)^b = y$$

$$(10)^c = z$$

By substituting the values

$$\log_{10} p = 2a + \frac{b}{2} - 3c$$

$$= 2 \log_{10} x + \frac{1}{2} \log_{10} y - 3 \log_{10} z$$

$$= \log_{10} x^2 + \log_{10} y^{\frac{1}{2}} - \log_{10} z^3$$

$$= \log_{10} \left(x^2 + y^{\frac{1}{2}} \right) - \log_{10} z^3$$

$$= \frac{\log_{10} x^2 \sqrt{y}}{z^3}$$

$$P = \frac{(x^2 \sqrt{y})}{z^3}$$

7. if $\log_{10} x = a$ and $\log_{10} y = b$, find the value of xy .

solution

given

$$\log_{10} x = a$$

$$(10)^a = x$$

$$\log_{10} y = b$$

$$(10)^b = y$$

Then ,

$$\begin{aligned} xy &= (10)^a \times (10)^b \\ &= (10)^{a+b} \end{aligned}$$

8. given $\log_{10} a = m$ and $\log_{10} b = n$, express $\frac{a^3}{b^2}$

In terms of m and n

Solution

Given:

$$\log_{10} a = m$$

$$(10)^m = a$$

$$\log_{10} b = n$$

$$(10)^n = b$$

So,

$$\frac{a^3}{b^2} = \frac{(10^m)^3}{(10^n)^2}$$

$$= \frac{10^{3m}}{10^{2n}}$$

$$= 10^{3m-2n}$$

9. given $\log_{10} a = 2a$ and $\log_{10} y = -\frac{b}{2}$

(i) write 10^a in terms of x.

(ii) write 10^{2b+1} in terms of y.

(iii) if $\log_{10} p = 3a - 2b$ express p in terms of x and y

solution

given :

$$\log_{10} a = 2a$$

$$(10)^{2a} = a$$

$$\log_{10} y = -\frac{b}{2}$$

$$(10)^{-\frac{b}{2}} = y$$

(i) write 10^a in terms of x.

$$\begin{aligned} 10^a &= (10^{2a})^{\frac{1}{2}} \\ &= x^{\frac{1}{2}} \\ &= \sqrt{x} \end{aligned}$$

(ii) write 10^{2b+1} in terms of y .

$$\begin{aligned} 10^{2b+1} &= 10^{2b} \times 10^1 \\ &= 10^{4\left(\frac{b}{2}\right)} \times 10^1 \\ &= y^4 \times 10^1 \\ &= 10y^4 \end{aligned}$$

(iii) if $\log_{10} p = 3a - 2b$, express p in terms of x and y.

$$\log_{10} p = 3a - 2b$$

Substitute the values ,

$$\begin{aligned} \log_{10} p &= \frac{3}{2} (2a) - 4 \left(\frac{b}{2}\right) \\ &= \frac{3}{2} \log_{10} x - 4 \log_{10} y \end{aligned}$$

$$= \log_{10} \frac{x^{\frac{3}{2}}}{y^4}$$

$$P = \frac{x^{\frac{3}{2}}}{y^4}$$

10 . if $\log_2 y = x$ and $\log_3 z = x$, find 72^x in terms of y and z

solution

Given

$$\log_2 y = x$$

$$2^x = y$$

$$\log_3 z = x$$

$$3^x = z$$

So,

$$72^x = (2 \times 2 \times 2 \times 3 \times 3)^x$$

$$= (2^3 \times 3^2)^x$$

$$= 2^{3x} \times 3^{2x}$$

$$= (2^x)^3 \times (3^x)^2$$

$$= y^3 \times z^2$$

$$= y^3 z^2$$

11. if $\log_2 x = a$ and $\log_5 y = a$, write 100^{2a-1} in terms of x and y .

solution

given

$$\log_2 x = a$$

$$2^a = x$$

$$\log_5 y = a$$

$$5^a = y$$

So,

$$100^{2a-1} = (2 \times 2 \times 5 \times 5)^{2a-1}$$

$$= (2^2 \times 5^2)^{2a-1}$$

$$= 2^{4a-2} \times 5^{4a-2}$$

$$= \frac{(2^{4a})}{2^2} \times \frac{(5^{4a})}{5^2}$$

$$= \frac{(2^a)^4 \times (5^a)^4}{4 \times 25}$$

$$= \frac{x^4 y^4}{100}$$

Exercise 9.2

1. simplify the following

(i) $\log a^3 - \log a^2$

(ii) $\log a^3 \div \log a^2$

(iii) $\frac{\log 4}{\log 2}$

(iv) $\frac{\log 8 \log 9}{\log 27}$

(v) $\frac{\log 27}{\log \sqrt{3}}$

(vi) $\frac{\log 9 - \log 3}{\log 27}$

Solution

(i) $\log a^3 - \log a^2$

By using quotient law,

$$\begin{aligned}\log a^3 - \log a^2 &= \log \frac{a^3}{a^2} \\ &= \log a\end{aligned}$$

(ii) $\log a^3 \div \log a^2$

By using power law,

$$\begin{aligned}\log a^3 \div \log a^2 &= 3\log a \div 2\log a \\ &= \frac{3\log a}{2\log a} \\ &= \frac{3}{2}\end{aligned}$$

$$(iii) \frac{\log 4}{\log 2}$$

Let us simplify the expression ,

$$\frac{\log 4}{\log 2} = \frac{\log(2 \times 2)}{\log 2}$$

By using power law,

$$= \frac{2\log 2}{\log 2}$$

$$= 2$$

$$(iv) \frac{\log 8 \log 9}{\log 27}$$

Let us simplify the expression

$$\frac{\log 8 \log 9}{\log 27} = \frac{\log 2^3 \cdot \log 3^2}{\log 3^3}$$

By using power law,

$$= \frac{(3\log 2) \cdot (2\log 3)}{3\log 3}$$

$$= \frac{\log 2 \cdot 2}{1}$$

$$= 2 \log 2$$

$$= \log 2^2$$

$$= \log 4$$

$$(v) \frac{\log 27}{\log \sqrt{3}}$$

Let us simplify the expression,

$$\frac{\log 27}{\log \sqrt{3}} = \frac{\log(3 \times 3 \times 3)}{\log 3^{\frac{1}{2}}}$$

$$= \frac{\log 3^3}{\log 3^{\frac{1}{2}}}$$

= by using power law

$$= \frac{3 \log 3}{\frac{1}{2} \log 3}$$

$$= \frac{3 \times 2}{1 \left(\frac{\log 3}{\log 3} \right)}$$

$$= (6)(1)$$

$$= 6$$

$$\text{(vi)} \quad \frac{\log 9 - \log 3}{\log 27}$$

Let us simplify the expression

$$\frac{\log 9 - \log 3}{\log 27} = \frac{\log(3 \times 3) - \log 3}{\log(3 \times 3 \times 3)}$$

$$= \frac{\log 3^2 - \log 3}{\log 3^3}$$

$$= \frac{\log 3}{3 \log 3}$$

$$= \frac{1}{3}$$

2. evaluate the following

$$\text{(i)} \quad \log(10 \div \sqrt[3]{10})$$

$$\text{(ii)} \quad 2 + \frac{1}{2} \log(10^{-3})$$

$$\text{(iii)} \quad 2 \log 5 + \log 8 - \frac{1}{2} \log 4$$

$$(iv) 2 \log 10^3 + 3 \log 10^{-2} - \frac{1}{3} \log 5^{-3} + \frac{1}{2} \log 4$$

$$(v) 2 \log 2 + \log 5 - \frac{1}{2} \log 36 - \log \frac{1}{30}$$

$$(vi) 2 \log 5 + \log 3 + 3 \log 2 - \frac{1}{2} \log 36 - 2 \log 10$$

$$(vii) \log 2 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 7 \log \frac{81}{80}$$

$$(viii) 2 \log 10^5 + \log 10^8 - \frac{1}{2} \log 10^4$$

Solution

$$(i) \log(10 \div \sqrt[3]{10})$$

Let us simplify the expression,

$$\log(10 \div \sqrt[3]{10}) = \log(10 \div 10^{\frac{1}{3}})$$

$$= \log(10^{1-\frac{1}{3}})$$

$$= \log(10^{\frac{2}{3}})$$

$$= \frac{2}{3} \log 10$$

$$= \frac{2}{3} (1)$$

$$= \frac{2}{3}$$

(ii) $2 + \frac{1}{2} \log(10^{-3})$

Let us simplify the expression ,

$$2 + \frac{1}{2} \log(10^{-3}) = 2 + \frac{1}{2} \times (-3) \log 10$$

$$= 2 - \frac{3}{2} \log 10$$

$$= 2 - \frac{3}{2} (1)$$

$$= 2 - \frac{3}{2}$$

$$= \frac{4-3}{2}$$

$$= \frac{1}{2}$$

(iii) $2 \log 5 + \log 8 - \frac{1}{2} \log 4$

Let us simplify the expression

$$2 \log 5 + \log 8 - \frac{1}{2} \log 4 = \log 5^2 + \log 8 - \frac{1}{2} \log 2^2$$

$$= \log 25 + \log 8 - \frac{1}{2} 2 \log 2$$

$$= \log 25 + \log 8 - \log 2$$

$$= \log \frac{(25 \times 8)}{2}$$

$$= \log (25 \times 4)$$

$$\begin{aligned}
&= \log 100 \\
&= \log 10^2 \\
&= 2 \log 10 \\
&= 2(1) \\
&= 2
\end{aligned}$$

$$\text{(iv) } 2 \log 10^3 + 3 \log 10^{-2} - \frac{1}{3} \log 5^{-3} + \frac{1}{2} \log 4$$

Let us simplify the expression,

$$\begin{aligned}
&2 \log 10^3 + 3 \log 10^{-2} - \frac{1}{3} \log 5^{-3} + \frac{1}{2} \log 4 = 2 \times 3 \log 10 + 3 \\
&(-2) \log 10 - \frac{1}{3}(-3) \log 5 + \frac{1}{2} \log 2^2
\end{aligned}$$

$$= 6 \log 10 - 6 \log 10 + \log 5 + \frac{1}{2} 2 \log 2$$

$$= 6 \log 10 - 6 \log 10 + \log 5 + \log 2$$

$$= 0 + \log 5 + \log 2$$

$$= \log(5 \times 2)$$

$$= \log 10$$

$$= 1$$

$$(v) 2 \log 2 + \log 5 - \frac{1}{2} \log 36 - \log \frac{1}{30}$$

let us simplify the expression ,

$$2 \log 2 + \log 5 - \frac{1}{2} \log 36 - \log \frac{1}{30} = \log 2^2 + \log 5 - \frac{1}{2} \log 6^2 - \log \frac{1}{30}$$

$$= \log 4 + \log 5 - \log 6 - \log \frac{1}{30}$$

$$= \log 4 + \log 5 - \log 6 - (\log 1 - \log 30)$$

$$= \log 4 + \log 5 - \log 6 - \log 1 + \log 30$$

$$= \log 4 + \log 5 + \log 30 - (\log 6 + \log 1)$$

$$= \log(4 \times 5 \times 30) - \log(6 \times 1)$$

$$= \log \frac{4 \times 5 \times 30}{6 \times 1}$$

$$= \log(4 \times 5 \times 5)$$

$$= \log 100$$

$$= \log 10^2$$

$$= 2 \log 10$$

$$= 2 \log 10$$

$$= 2(1)$$

$$= 2$$

(vi) $2 \log 5 + \log 3 + 3 \log 2 - \frac{1}{2} \log 36 - 2 \log 10$

Let us simplify the expression ,

$$2 \log 5 + \log 3 + 3 \log 2 - \frac{1}{2} \log 36 - 2 \log 10 = \log 5^2 + \log 3 + \log 2^3 - \frac{1}{2} \log 6^2 - \log 10^2$$

$$= \log 25 + \log 3 + \log 8 - \log 6 - \log 100$$

$$= \log(25 \times 3 \times 8) - \log(6 \times 100)$$

$$= \log \frac{25 \times 3 \times 8}{6 \times 100}$$

$$= \log \frac{1 \times 3 \times 8}{6 \times 100}$$

$$= \log \frac{1 \times 3 \times 8}{6 \times 4}$$

$$= \log \frac{24}{24}$$

$$= \log 1$$

$$= 0$$

(vii) $\log 2 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 7 \log \frac{81}{80}$

Let us simplify the expression,

$$\log 2 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 7 \log \frac{81}{80} = \log 2 + 16(\log 16 - \log 15) + 12(\log 25 - \log 24) + 7(\log 81 - \log 80)$$

$$= \log 2 + 16(\log 2^4 - \log(3 \times 5)) + 12(\log 5^2 - \log(3 \times 2 \times 2 \times 2)) + 7(\log(3 \times 3 \times 3 \times 3) - \log(2^4 \times 5))$$

$$= \log 2 + 16(4 \log 2 - (\log 3 + \log 5)) + 12(2 \log 5 - \log(3 \times 2^3)) + 7(\log 3^4 - (\log 2^4 + \log 5))$$

$$= \log 2 + 16(4 \log 2 - \log 3 - \log 5) + 12(2 \log 5 - (\log 3 + 3 \log 2)) + 7(4 \log 3 - 4 \log 2 - \log 5)$$

$$= \log 2 + 64 \log 2 - 16 \log 3 - 16 \log 5 + 24 \log 5 - 12 \log 3 - 36 \log 2 + 28 \log 3 - 28 \log 2 - 7 \log 5$$

$$= (\log 2 + 64 \log 2 - 36 \log 2 - 28 \log 2) + (-16 \log 3 - 12 \log 3 + 28 \log 3) + (-16 \log 5 + 24 \log 5 - 7 \log 5)$$

$$= (65 \log 2 - 64 \log 2) + (-28 \log 3 + 28 \log 3) + (-23 \log 5 + 24 \log 5)$$

$$= \log 2 + 0 + \log 5$$

$$= \log(2 \times 5)$$

$$= \log 10$$

$$= 1$$

(viii) $2 \log 10^5 + \log 10^8 - \frac{1}{2} \log 10^4$

Let us simplify the expression

$$2 \log 10^5 + \log 10^8 - \frac{1}{2} \log 10^4 = \log_{10} 5^2 + \log 10^8 - \log 10^{4 \times \frac{1}{2}}$$

$$= \log 10^{25} + \log 10^8 - \log 10^{2 \times \frac{1}{2}}$$

$$= \log 10^{25} + \log 10^8 - \log 10^2$$

$$= \log 10^{\frac{25 \times 8}{2}}$$

$$= \log 10^{25 \times 4}$$

$$= \log 10^{100}$$

$$= \log 10^{10^2}$$

$$= 2 \log 10^{10}$$

$$= 2(1)$$

$$= 2$$

3. express each of the following as a single logarithm :

(i) $2 \log 3 - \frac{1}{2} \log 16 + \log 12$

(ii) $2 \log 10^5 - \log 10^2 + 3 \log 10^{4+1}$

(iii) $\frac{1}{2} \log 36 + 2 \log 8 - \log 1.5$

$$(iv) \frac{1}{2} \log 25 - 2 \log 3 + 1$$

$$(v) \frac{1}{2} \log 9 + 2 \log 3 - \log 6 + \log 2 - 2$$

Solution

$$(i) 2 \log 3 - \frac{1}{2} \log 16 + \log 12$$

Let us simplify the expression into single logarithm ,

$$2 \log 3 - \frac{1}{2} \log 16 + \log 12 = 2 \log 3 - \frac{1}{2} \log 4^2 + \log 12$$

$$= 2 \log 3 - \log 4 + \log 12$$

$$= \log 3^2 - \log 4 + \log 12$$

$$= \log 9 - \log 4 + \log 12$$

$$= \log \frac{9 \times 12}{4}$$

$$= \log (9 \times 3)$$

$$= \log 27$$

$$(ii) 2 \log 10^5 - \log 10^2 + 3 \log 10^{4+1}$$

Let us simplify the expression into single logarithm ,

$$2 \log 10^5 - \log 10^2 + 3 \log 10^{4+1} = \log 10^{5^2} - \log 10^2 + \log 10^{4^3} + \log 10^{10}$$

$$= \log_{10} 25 - \log_{10} 2 + \log_{10} 64 + \log_{10} 10$$

$$= \log_{10}(25 \times 64 \times 10) - \log_{10} 2$$

$$= \log_{10} (16000) - \log_{10} 2$$

$$= \log_{10} \left(\frac{16000}{2} \right)$$

$$= \log_{10} 8000$$

$$(iii) \frac{1}{2} \log 36 + 2 \log 8 - \log 1.5$$

Let us simplify the expression into single logarithm,

$$\frac{1}{2} \log 36 + 2 \log 8 - \log 1.5 = \log 36^{\frac{1}{2}} + \log 8^2 - \log 1.5$$

$$= \log 6^{2 \times \frac{1}{2}} + \log 64 - \log 1.5$$

$$= \log 6 + \log 64 - \log \left(\frac{15}{10} \right)$$

$$= \log 6 + \log 64 - (\log 15 - \log 10)$$

$$= \log (6 \times 64) - \log 15 + \log 10$$

$$= \log(6 \times 64 \times 10) - \log 15$$

$$= \log \frac{6 \times 64 \times 10}{15}$$

$$= \log (4 \times 64)$$

$$= \log 256$$

$$(iv) \frac{1}{2} \log 25 - 2 \log 3 + 1$$

Let us simplify the expression into single logarithm,

$$\frac{1}{2} \log 25 - 2 \log 3 + 1 = \log 25^{\frac{1}{2}} - \log 3^2 + \log 10$$

$$= \log 5^{2 \times \frac{1}{2}} - \log 9 + \log 10$$

$$= \log (5 \times 10) - \log 9$$

$$= \log \left(\frac{5 \times 10}{9} \right)$$

$$= \log \frac{50}{9}$$

$$(v) \frac{1}{2} \log 9 + 2 \log 3 - \log 6 + \log 2 - 2$$

Let us simplify the expression into single logarithm,

$$\frac{1}{2} \log 9 + 2 \log 3 - \log 6 + \log 2 - 2 = \log 9^{\frac{1}{2}} + \log 3^2 - \log 6 + \log 2 - \log 100$$

$$= \log 3^{2 \times \frac{1}{2}} + \log 9 - \log 6 + \log 2 - \log 100$$

$$= \log 3 + \log 9 - \log 6 + \log 2 - \log 100$$

$$= \log \frac{3 \times 9 \times 2}{6 \times 100}$$

$$= \log \frac{9}{100}$$

4. prove the following :

(i) $\log_{10} 4 \div \log_{10} 2 = \log_3 9$

(ii) $\log_{10} 25 + \log_{10} 4 = \log_5 25$

Solution

(i) $\log_{10} 4 \div \log_{10} 2 = \log_3 9$

$$\begin{aligned}\text{Let us consider LHS, } & \log_{10} 4 \div \log_{10} 2 \\ \log_{10} 4 \div \log_{10} 2 &= \log_{10} 2^2 \div \log_{10} 2 \\ &= 2 \log_{10} 2 \div \log_{10} 2 \\ &= 2 \frac{\log_{10} 2}{\log_{10} 2}\end{aligned}$$

$$= 2(1)$$

$$= 2$$

Now let us consider RHS,

$$\log_3 9 = \log_3 3^2$$

$$= 2 \log_3 3$$

$$= 2(1)$$

$$= 2$$

$\therefore \text{LHS} = \text{RHS}$

Hence proved.

(ii) $\log_{10} 25 + \log_{10} 4 = \log_5 25$

Let us consider LHS, $\log_{10} 25 + \log_{10} 4$

$$\log_{10} 25 + \log_{10} 4 = \log_{10} (25 \times 4)$$

$$= \log_{10} 100$$

$$= \log_{10} 10^2$$

$$= 2 \log_{10} 10$$

$$=2(1)$$

$$= 2$$

Now let us consider RHS,

$$\log_5 25 = \log_5 5^2$$

$$= 2\log_5 5$$

$$= 2(1)$$

$$= 2$$

$$\text{LHS} = \text{RHS}$$

Hence proved .

5. if $x = (100)^a$, $y = (10000)^b$ and $z = (10)^c$, express $\log \left[\frac{10\sqrt{y}}{x^2 z^3} \right]$ in terms of a,b, c.

Solution

Given :

$$x = (100)^a = (10^2)^a = 10^{2a}$$

$$y = (10000)^b = (10^4)^b = 10^{4b}$$

$$z = (10)^c$$

it is given that , $\log \left[\frac{10\sqrt{y}}{x^2 z^3} \right]$

$$\log \left[\frac{10\sqrt{y}}{x^2 z^3} \right] = (\log 10 + \log \sqrt{y}) - (\log x^2 + \log z^3)$$

$$= (1 + \log y^{\frac{1}{2}}) - (\log x^2 + \log z^3) \quad [\text{ we know that , } \log 10 = 1]$$

$$= (1 + \frac{1}{2} \log y) - (2 \log x + 3 \log z)$$

Now substitute the values of x,y, z we get

$$= \left(1 + \frac{1}{2} \log 10^{4b} \right) - (2 \log 10^{2a} + 3 \log 10^c)$$

$$\begin{aligned}
&= (1 + \frac{1}{2} 4b \log 10) - (2 \times 2a \log 10 + 3 \times c \log 10) \\
&= \left(1 + \frac{1}{2} 4b\right) - (2 \times 2a + 3c) [\text{since } \log 10 = 1] \\
&= (1 + 2b) - (4a + 3c) \\
&= 1 + 2b - 4a - 3c
\end{aligned}$$

6. if $a = \log_{10} x$, find the following in terms of a :

(i) x

(ii) $\log_{10} \sqrt[5]{x^2}$

(iii) $\log_{10} 5x$

Solution

Given :

$$a = \log_{10} x$$

(i) x

$$10^a = x$$

$$x = 10^a$$

(ii) $\log_{10} \sqrt[5]{x^2}$

$$\log_{10} \sqrt[5]{x^2} = \log_{10} (x^2)^{\frac{1}{5}}$$

$$= \log_{10} x^{\frac{2}{5}}$$

$$= \frac{2}{5} \log_{10} x$$

$$= \frac{2}{5} (a)$$

$$= \frac{2a}{5}$$

$$(iii) \log_{10} 5x$$

$$x = (10)^a$$

$$= \log_{10} 5x$$

$$= \log_{10} 5(10)^a$$

$$= \log_{10} 5 + \log_{10} 10$$

$$= \log_{10} 5 + a(1)$$

$$= a + \log_{10} 5$$

7. if $a = \log \frac{2}{3}$, $b = \log \frac{3}{5}$ and $c = 2 \log \sqrt{\frac{5}{2}}$. find the values of

(i) $a + b + c$

(ii) 5^{a+b+c}

Solution

Given:

$$a = \log \frac{2}{3}$$

$$b = \log \frac{3}{5}$$

$$c = 2 \log \sqrt{\frac{5}{2}}$$

(i) $a + b + c$

Let us substitute the given values we get

$$a + b + c = \log \frac{2}{3} + \log \frac{3}{5} + 2 \log \sqrt{\frac{5}{2}}$$

$$= (\log 2 - \log 3) + (\log 3 - \log 5) + 2 \log \frac{5^{\frac{1}{2}}}{2}$$

$$= \log 2 - \log 3 + \log 3 - \log 5 + 2 \times \frac{1}{2} (\log 5 - \log 2)$$

$$= \log 2 - \log 3 + \log 3 - \log 5 + \log 5 - \log 2 \\ = 0$$

$$(ii) 5^{a+b+c}$$

$$5^{a+b+c} = 5^0$$

$$= 1$$

8. if $x = \log \frac{3}{5}$, $y = \log \frac{5}{4}$ and $z = 2 \log \frac{\sqrt{3}}{2}$, find the value of

$$(i) x + y - z$$

$$(ii) 3^{x+y-z}$$

Solution

Given

$$X = \log \frac{3}{5} = \log 3 - \log 5$$

$$Y = \log \frac{5}{4} = \log 5 - \log 4$$

$$Z = 2 \log \frac{\sqrt{3}}{2} = \log \left(\frac{\sqrt{3}}{2} \right)^2 = \log \frac{3}{4} = \log 3 - \log 4$$

$$(i) x + y - z$$

Let us substitute the given values we get

$$\begin{aligned} x + y - z &= \log 3 - \log 5 + \log 5 - \log 4 - (\log 3 - \log 4) \\ &= \log 3 - \log 5 + \log 5 - \log 4 - \log 3 + \log 4 \\ &= 0 \end{aligned}$$

$$\begin{aligned} (ii) 3^{x+y-z} \\ 3^{x+y-z} &= 3^0 \\ &= 1 \end{aligned}$$

9. if $x = \log_{10} 12$, $y = \log_4 2 \times \log_{10} 9$ and $z = \log_{10} 0.4$, find the values of

$$(i) x - y - z$$

$$(ii) 7^{x-y-z}$$

Solution

Given

$$X = \log_{10} 12$$

$$Y = \log_4 2 \times \log_{10} 9$$

$$Z = \log_{10} 0.4$$

$$(i) x - y - z$$

Let us substitute the given values we get

$$\begin{aligned} x - y - z &= \log_{10} 12 - \log_4 2 \times \log_{10} 9 - \log_{10} 0.4 \\ &= \log_{10} (3 \times 4) - \log_4 4^{\frac{1}{2}} \times \log_{10} 3^2 - \log_{10} \frac{4}{10} \end{aligned}$$

$$\begin{aligned} &= \log_{10} 3 + \log_{10} 4 - \frac{1}{2} \log_4 4 \times 2 \log_{10} 3 - (\log_{10} 4 - \log_{10} 10) \\ &= \log_{10} 3 + \log_{10} 4 - \frac{1}{2} \times 1 \times 2 \log_{10} 3 - (\log_{10} 4 - \log_{10} 10) \end{aligned}$$

$$= \log_{10} 3 + \log_{10} 4 - \frac{1}{2} \times 1 \times 2 \log_{10} 3 - \log_{10} 4 + 1$$

$$= \log_{10} 3 + \log_{10} 4 - \log_{10} 3 - \log_{10} 4 + 1$$

$$= 1$$

$$(ii) 7^{x-y-z}$$

$$7^{x-y-z} = 7^1$$

$$= 7$$

10. if $\log V + \log 3 = \log \pi + \log 4 + 3 \log r$, find v in terms of other quantities .

Solution

Given

$$\log v + \log 3 = \log \pi + \log 4 + 3 \log r$$

Let us simplify the given expression to find V,

$$\log(v \times 3) = \log(\pi \times 4 \times r^3)$$

$$\log 3v = \log 4\pi r^3$$

$$3V = 4\pi r^3$$

$$V = \frac{4\pi r^3}{3}$$

11. given $3(\log 5 - \log 3) - (\log 5 - 2 \log 6) = 2 - \log n$, find n.

Solution

Given :

$$3(\log 5 - \log 3) - (\log 5 - 2 \log 6) = 2 - \log n$$

Let us simplify the given expression to find n,

$$3 \log 5 - 3 \log 3 - \log 5 + 2 \log 6 = 2 - \log n$$

$$2 \log 5 - 3 \log 3 + 2 \log 6 = 2(1) - \log n$$

$$\log 5^2 - \log 3^3 + \log 6^2 = 2 \log 10 - \log n \text{ [since , } 1 = \log 10 \text{]}$$

$$\log 25 - \log 27 + \log 36 - \log 10^2 = - \log n$$

$$\log n = - \log 25 + \log 27 - \log 36 + \log 100$$

$$= (\log 100 + \log 27) - (\log 25 + \log 36)$$

$$= \log (100 \times 27) - \log (25 \times 36)$$

$$= \log \frac{(100 \times 27)}{25 \times 36}$$

$$= \log n = \log 3$$

$$n = 3$$

12. given that $\log_{10} y + 2 \log_{10} x = 2$, express y in terms of x .

Solution

Given :

$$\log_{10} y + 2 \log_{10} x = 2$$

Let us simplify the given expression,

$$\log_{10} y + \log_{10} x^2 = 2(1)$$

$$\log_{10} y + \log_{10} x^2 = 2 \log_{10} 10$$

$$\log_{10}(y \times x^2) = \log_{10} 10^2$$

$$yx^2 = 100$$

$$y = \frac{100}{x^2}$$

13. express $\log_{10} 2 + 1$ in the form $\log_{10} x$.

Solution

Given

$$\log_{10} 2 + 1$$

Let us simplify the given expression

$$\log_{10} 2 + 1 = \log_{10} 2 + \log_{10} 10 \text{ [since } 1 = \log_{10} 10 \text{]}$$

$$= \log_{10}(2 \times 10)$$

$$= \log_{10} 20$$

**14. if $a^2 = \log_{10} x$, $b^2 = \log_{10} y$ and $\frac{a^2}{2} - \frac{b^2}{3} = \log_{10} z$.
express z in terms of x and y .**

Solution

Given

$$a^2 = \log_{10} x$$

$$b^2 = \log_{10} y$$

$$\frac{a^2}{2} - \frac{b^2}{3} = \log_{10} z$$

Let us substitute the given values in the expression we get

$$\log_{10} \frac{x}{2} - \log_{10} \frac{y}{3} = \log_{10} z$$

$$\log_{10} x^{\frac{1}{2}} - \log_{10} y^{\frac{1}{3}} = \log_{10} z$$

$$\log_{10} \sqrt{x} - \log_{10} \sqrt[3]{y} = \log_{10} z$$

$$\log_{10} \frac{\sqrt{x}}{\sqrt[3]{y}} = \log_{10} z$$

$$\frac{\sqrt{x}}{\sqrt[3]{y}} = z$$

$$Z = \frac{\sqrt{x}}{\sqrt[3]{y}}$$

15. given that $\log m = x + y$ and $\log n = x - y$, express the value of $\log m^2n$ in terms of x and y .

Solution

Given :

$$\log m = x + y$$

$$\log n = x - y$$

$$\log m^2n$$

Let us simplify the given expression,

$$\log m^2n = \log m^2 + \log n$$

$$= 2 \log m + \log n$$

By substituting the given values, we get

$$= 2(x + y) + (x - y)$$

$$= 2x + 2y + x - y$$

$$= 3x + y$$

16. given that $\log x = m + n$ and $\log y = m - n$, express the value of $\log\left(\frac{10x}{y^2}\right)$ in terms of m and n .

Solution

Given

$$\log x = m + n$$

$$\log y = m - n$$

$$\log\left(\frac{10x}{y^2}\right)$$

let us simplify the given expression,

$$\log\left(\frac{10x}{y^2}\right) = \log 10x - \log y^2$$

$$= \log 10 + \log x - 2 \log y$$

$$= 1 + \log x - 2 \log y$$

$$= 1 + (m + n) - 2(m - n)$$

$$= 1 + m + n - 2m + 2n$$

$$= 1 - m + 3n$$

17. if $\log \frac{x}{2} = \log \frac{y}{3}$ find the value of $\frac{y^4}{x^6}$

Solution

Given :

$$\text{Log } \frac{x}{2} = \log \frac{y}{3}$$

Let us simplify the given expression ,

By cross multiplying , we get

$$3 \log x = 2 \log y$$

$$\text{Log } x^3 = \log y^2$$

$$\text{So , } x^3 = y^2$$

Now square on both sides we get

$$(x^3)^2 = (y^2)^2$$

$$x^6 = y^4$$

$$\frac{y^4}{x^6} = 1$$

18. solve for x :

(i) $\log x + \log 5 = 2 \log 3$

(ii) $\log_3 x - \log_3 2 = 1$

(iii) $x = \frac{\log 125}{\log 25}$

(iv) $\left(\frac{\log 8}{\log 2}\right) \times \left(\frac{\log 3}{\log \sqrt{3}}\right) = 2 \log x$

Solution

$$(i) \log x + \log 5 = 2 \log 3$$

Let us solve for x,

$$\log x = 2 \log 3 - \log 5$$

$$= \log 3^2 - \log 5$$

$$= \log 9 - \log 5$$

$$= \log \left(\frac{9}{5} \right)$$

$$x = \frac{9}{5}$$

$$(ii) \log_3 x - \log_3 2 = 1$$

Let us solve for x ,

$$\log_3 x = 1 + \log_3 2$$

$$= \log_3 3 + \log_3 2 \text{ [since , 1 can be written as } \log_3 3 = 1 \text{]}$$

$$= \log_3 (3 \times 2)$$

$$= \log_3 6$$

$$x = 6$$

$$(iii) x = \frac{\log 125}{\log 25}$$

$$x = \frac{\log 5^3}{\log 5^2}$$

$$= \frac{3 \log 5}{2 \log 5}$$

$$= \frac{3}{2} \text{ [since , } \frac{\log 5}{\log 5} = 1 \text{]}$$

$$\therefore x = \frac{3}{2}$$

$$(iv) \left(\frac{\log 8}{\log 2} \right) \times \left(\frac{\log 3}{\log \sqrt{3}} \right) = 2 \log x$$

$$\left(\frac{\log 2^3}{\log 2} \right) \times \left(\frac{\log 3}{\log 3^{\frac{1}{2}}} \right) = 2 \log x$$

$$\left(\frac{3 \log 2}{\log 2}\right) \times \left(\frac{\log 3}{\frac{1}{2} \log 3}\right) = 2 \log x$$

$$\frac{3 \times 1}{\frac{1}{2}} = 2 \log x$$

$$3 \times 2 = 2 \log x$$

$$6 = 2 \log x$$

$$\log x = \frac{6}{2}$$

$$\log x = 3$$

$$x = (10)^3$$

$$= 1000$$

$$x = 1000$$

19. given $2 \log_{10} x + 1 = \log_{10} 250$, find

(i) x

(ii) $\log_{10} 2x$

Solution

Given :

$$2 \log_{10} x + 1 = \log_{10} 250$$

(i) let us simplify the above expression ,

$$\log_{10} x^2 + \log_{10} 10 = \log_{10} 250 \quad [\text{since } 1 \text{ can be written as } \log_{10} 10]$$

$$\log_{10} (x^2 \times 10) = \log_{10} 250$$

$$(x^2 \times 10) = 250$$

$$x^2 = \frac{250}{10}$$

$$x^2 = 25$$

$$x = \sqrt{25}$$

$$= 5$$

$$\therefore x = 5$$

$$(ii) \log_{10} 2x$$

We know that , $x = 5$

$$\begin{aligned} \text{So } \log_{10} 2x &= \log_{10} 2 \times 5 \\ &= \log_{10} 10 \\ &= 1 \end{aligned}$$

$$20 . \text{ if } \frac{\log x}{\log 5} = \frac{\log y^2}{\log 2} = \frac{\log 9}{\log\left(\frac{1}{3}\right)} \text{ find } x \text{ and } y$$

Solution

Given :

$$\frac{\log x}{\log 5} = \frac{\log y^2}{\log 2} = \frac{\log 9}{\log\left(\frac{1}{3}\right)}$$

Let us consider,

$$\frac{\log x}{\log 5} = \frac{\log 9}{\log\left(\frac{1}{3}\right)}$$

$$\log x = \frac{(\log 9 \times \log 5)}{\log\left(\frac{1}{3}\right)}$$

$$= \frac{\log 3^2 \times \log 5}{\log 1 - \log 3}$$

$$= \frac{(2 \log 3 \times \log 5)}{-\log 3} [\log 1 = 0]$$

$$= -2 \times \log 5$$

$$= \log 5^{-2}$$

$$x = 5^{-2}$$

$$= \frac{1}{5^2}$$

$$= \frac{1}{25}$$

Now,

$$\frac{\log y^2}{\log 2} = \frac{\log 9}{\log\left(\frac{1}{3}\right)}$$

$$\text{Log } y^2 = \frac{\log 9 \times \log 2}{\log\left(\frac{1}{3}\right)}$$

$$= \frac{\log 3^2 \times \log 2}{\log 1 - \log 3}$$

$$= \frac{2\log 3 \times \log 2}{-\log 3} [\log 1 = 0]$$

$$= -2 \times \log 2$$

$$= \log 2^{-2}$$

$$y^2 = 2^{-2}$$

$$= \frac{1}{2^2}$$

$$= \frac{1}{4}$$

$$= \sqrt{\frac{1}{4}}$$

$$= \frac{1}{2}$$

21. prove the following

(i) $3^{\log 4} = 4^{\log 3}$

(ii) $27^{\log 2} = 8^{\log 3}$

Solution

(i) $3^{\log 4} = 4^{\log 3}$

Let us take log on both sides,

If $\log 3^{\log 4} = \log 4^{\log 3}$

$\text{Log } 4 \cdot \log 3 = \log 3 \cdot \text{Log } 4$

$\text{Log } 2^2 \cdot \log 3 = \log 3 \cdot \text{Log } 2^2$

$2\log 2 \cdot \log 3 = \log 3 \cdot 2\log 2$

Which is true

Hence proved.

(ii) $27^{\log 2} = 8^{\log 3}$

Let us take log on both sides,

$$\text{If } \log 27^{\log 2} = \log 8^{\log 3}$$

$$\log 2 \cdot \log 27 = \log 3 \cdot \log 8$$

$$\log 2 \cdot \log 3^3 = \log 3 \cdot \log 2^3$$

$$\log 2 \cdot 3 \log 3 = \log 3 \cdot 3 \log 2$$

$$3 \log 2 \cdot \log 3 = 3 \log 2 \cdot \log 3$$

Which is true.

Hence proved.

22. solve the following equations:

(i) $\log(2x+3) = \log 7$

(ii) $\log(x+1) + \log(x-1) = \log 24$

(iii) $\log(10x+5) - \log(x-4) = 2$

(iv) $\log_{10} 5 + \log_{10}(5x+1) = \log_{10}(x+5) + 1$

(v) $\log(4y-3) = \log(2y+1) - \log 3$

(vi) $\log_{10}(x+2) + \log_{10}(x-2) = \log_{10} 3 + 3 \log_{10} 4$

(vii) $\log(3x+2) + \log(3x-2) = 5 \log 2$

Solution

(i) $\log(2x+3) = \log 7$

Let us simplify the expression ,

$$2x + 3 = 7$$

$$2x = 7 - 3$$

$$2x = 4$$

$$x = \frac{4}{2}$$

$$= 2$$

(ii) $\log(x+1) + \log(x-1) = \log 24$

Let us simplify the expression,

$$\log[(x+1)(x-1)] = \log 24$$

$$\log(x^2 - 1) = \log 24$$

$$(x^2 - 1) = 24$$

$$x^2 = 24 + 1$$

$$= 25$$

$$X = \sqrt{25}$$

$$= 5$$

$$(iii) \log(10x+5) - \log(x-4) = 2$$

Let us simplify the expression ,

$$\text{Log} \frac{10x+5}{x-4} = 2 \log 10$$

$$\log \frac{10x+5}{x-4} = \log 10^2$$

$$\frac{10x+5}{x-4} = 100$$

$$10x + 5 = 100(x-4)$$

$$10x + 5 = 100x - 400$$

$$5 + 400 = 100x - 10x$$

$$90x = 405$$

$$X = \frac{405}{90}$$

$$= \frac{81}{18}$$

$$= \frac{9}{2}$$

$$= 4.5$$

$$(iv) \log_{10} 5 + \log_{10}(5x+1) = \log_{10}(x+5) + 1$$

Let us simplify the expression

$$\text{Log}_{10}[5 \times (5x+1)] = \log_{10}(x+5) + \log_{10} 10$$

$$\text{Log}_{10}[5 \times (5x+1)] = \log_{10}[(x+5) \times 10] + [5 \times (5x+1)] = [(x+5) \times 10]$$

$$25x + 5 = 10x + 50$$

$$25x - 10x = 50 - 5$$

$$15x = 45$$

$$X = \frac{45}{15}$$

$$= 3$$

$$(v) \log(4y - 3) = \log(2y + 1) - \log 3$$

Let us simplify the expression ,

$$\text{Log}(4y - 3) = \log \frac{2y + 1}{3}$$

$$(4y - 3) = \frac{2y + 1}{3}$$

By cross multiplying we get

$$3(4y - 3) = 2y + 1$$

$$12y - 9 = 2y + 1$$

$$12y - 2y = 9 + 1$$

$$10y = 10$$

$$Y = \frac{10}{10}$$

$$= 1$$

$$(vi) \log_{10} (x+2) + \log_{10} (x - 2) = \log_{10} 3 + 3 \log_{10} 4$$

Let us simplify the expression,

$$\text{Log}_{10} [(x + 2) \times (x - 2)] = \log_{10} 3 + \log 10^{4^3}$$

$$\text{Log}_{10} [(x + 2) \times (x - 2)] = \log_{10} (3 \times 4^3)$$

$$[(x + 2) \times (x - 2)] = (3 \times 4^3)$$

$$(x^2 - 4) = (3 \times 4 \times 4 \times 4)$$

$$(x^2 - 4) = 192$$

$$x^2 = 192 + 4$$

$$= 196$$

$$x = \sqrt{196}$$

$$= 14$$

$$(vii) \log (3x+2) + \log(3x - 2) = 5 \log 2$$

Let us simplify the expression

$$\text{Log}(3x + 2) + \log(3x - 2) = \log 2^5$$

$$\text{Log}[(3x + 2) \times (3x - 2)] = \log 32$$

$$\text{Log}(9x^2 - 4) = \log 32$$

$$(9x^2 - 4) = 32$$

$$9x^2 = 32 + 4$$

$$9x^2 = 36$$

$$x^2 = \frac{36}{9}$$

$$x^2 = 4$$

$$x = \sqrt{4}$$

$$= 2$$

23. solve for x :

$$\text{Log}_3(x+1) - 1 = 3 + \log_3(x-1)$$

Solution :

Given :

$$\text{Log}_3(x+1) - 1 = 3 + \log_3(x-1)$$

Let us simplify the expression,

$$\text{Log}_3(x+1) - \log_3(x-1) = 3 + 1$$

$$\text{Log}_3 \frac{x+1}{x-1} = 4 \log_3 3 \text{ [since } \log_3 3 = 1 \text{]}$$

$$\text{Log}_3 \frac{x+1}{x-1} = \log_3 3^4$$

$$\frac{x+1}{x-1} = 3^4$$

By cross multiplying , we get

$$(x+1) = 81(x-1)$$

$$X+1 = 81x - 81$$

$$81x - x = 1 + 81$$

$$80x = 82$$

$$X = \frac{82}{80}$$

$$= \frac{41}{40}$$

$$= 1 \frac{1}{40}$$

24. solve for x :

$$5^{\log x} + 3^{\log x} = 3^{\log x + 1} - 5^{\log x - 1}$$

Solution

Given

$$5^{\log x} + 3^{\log x} = 3^{\log x + 1} - 5^{\log x - 1}$$

Let us simplify the expression,

$$5^{\log x} + 3^{\log x} = 3^{\log x} \cdot 3^1 - 5^{\log x} \cdot 5^{-1}$$

$$5^{\log x} + 3^{\log x} = 3 \cdot 3^{\log x} - \frac{1}{5} \cdot 5^{\log x}$$

$$5^{\log x} + \frac{1}{5} \cdot 5^{\log x} = 3 \cdot 3^{\log x} - 3^{\log x}$$

$$\left(1 + \frac{1}{5}\right) 5^{\log x} = (3 - 1) 3^{\log x}$$

$$\left(\frac{6}{5}\right) 5^{\log x} = 2(3^{\log x})$$

$$\frac{5^{\log x}}{3^{\log x}} = \frac{2 \times 5}{6}$$

$$\left(\frac{5}{3}\right)^{\log x} = \frac{10}{6}$$

$$\left(\frac{5}{3}\right)^{\log x} = \frac{5}{3}$$

$$\left(\frac{5}{3}\right)^{\log x} = \left(\frac{5}{3}\right)^1$$

So , by comparing the powers

$$\log x = 1$$

$$\log x = \log 10$$

$$x = 10$$

25. if $\log \frac{x-y}{2} = \frac{1}{2} (\log x + \log y)$, prove that $x^2 + y^2 = 6xy$

Solution

Given

$$\log \frac{x-y}{2} = \frac{1}{2} (\log x + \log y)$$

Let us simplify

$$\log \frac{x-y}{2} = \frac{1}{2} (\log x + \log y)$$

$$\log \frac{x-y}{2} = \frac{1}{2} \log xy$$

$$\log \frac{x-y}{2} = \log (xy)^{\frac{1}{2}}$$

$$\frac{x-y}{2} = xy^{\frac{1}{2}}$$

By squaring on both sides we get

$$\left[\frac{x-y}{2} \right]^2 = \left[xy^{\frac{1}{2}} \right]^2$$

$$\frac{(x-y)^2}{4} = xy$$

By cross multiplying , we get

$$(x - y)^2 = 4xy$$

$$x^2 + y^2 - 2xy = 4xy$$

$$x^2 + y^2 = 4xy + 2xy$$

$$x^2 + y^2 = 6xy$$

hence proved

26. if $x^2 + y^2 = 23xy$, prove that $\log \frac{x+y}{5} = \frac{1}{2} (\log x + \log y)$

Solution

Given

$$x^2 + y^2 = 23xy$$

so the above equation can be written as

$$x^2 + y^2 = 25xy - 2xy$$

$$x^2 + y^2 + 2xy = 25xy$$

$$(x+y)^2 = 25xy$$

$$\frac{(x+y)^2}{25} = xy$$

Now by taking log on both sides we get

$$\log \frac{(x+y)^2}{25} = \log xy$$

$$\log \left[\frac{x+y}{5} \right]^2 = \log xy$$

$$2 \log \frac{x+y}{5} = \log x + \log y$$

$$\text{Log} \frac{x+y}{5} = \frac{1}{2} \log x + \log y$$

Hence proved .

27. if $p = \log_{10} 20$ and $q = \log_{10} 25$, find the value of x if $2 \log_{10}(x+1) = 2p-q$

Solution

Given

$$P = \log_{10} 20$$

$$Q = \log_{10} 25$$

Then ,

$$2 \log_{10} (x + 1) = 2p - q$$

Now substitute the values of p and q we get

$$2 \log_{10} (x + 1) = 2 \log_{10} 20 - \log_{10} 25$$

$$= 2 \log_{10} 20 - \log_{10} 5^2$$

$$= 2 \log_{10} 20 - 2 \log_{10} 5$$

$$2 \text{Log}_{10} (x + 1) = 2(\log_{10} 20 - \log_{10} 5)$$

$$\text{Log}_{10} (x + 1) = (\log_{10} 20 - \log_{10} 5)$$

$$= \log_{10} \left(\frac{20}{5} \right)$$

$$= \log_{10} (x + 1) = \log_{10} 4$$

$$(x + 1) = 4$$

$$x = 4 - 1$$

$$= 3$$

28. show that :

$$(i) \frac{1}{\log_2 42} + \frac{1}{\log_3 42} + \frac{1}{\log_7 42} = 1$$

$$(ii) \frac{1}{\log_8 36} + \frac{1}{\log_9 36} + \frac{1}{\log_{18} 36} = 2$$

Solution

$$(i) \frac{1}{\log_2 42} + \frac{1}{\log_3 42} + \frac{1}{\log_7 42} = 1$$

Let us consider LHS:

$$\frac{1}{\log_2 42} + \frac{1}{\log_3 42} + \frac{1}{\log_7 42}$$

By using the formula , $\log_n m = \frac{\log m}{\log n}$

$$\frac{1}{\log_2 42} + \frac{1}{\log_3 42} + \frac{1}{\log_7 42} = \frac{1}{\left(\frac{\log 42}{\log 2}\right)} + \frac{1}{\left(\frac{\log 42}{\log 3}\right)} + \frac{1}{\left(\frac{\log 42}{\log 7}\right)}$$

$$= \frac{\log_2}{\log 42} + \frac{\log_3}{\log 42} + \frac{\log_7}{\log 42}$$

$$= \frac{\log_2 + \log_3 + \log_7}{\log 42}$$

$$= \frac{\log 2 \times 3 \times 7}{\log 42}$$

$$= \frac{\log 42}{\log 42}$$

$$= 1$$

$$= \text{RHS}$$

$$(ii) \frac{1}{\log_8 36} + \frac{1}{\log_9 36} + \frac{1}{\log_{18} 36} = 2$$

Let us consider LHS:

$$\frac{1}{\log_8 36} + \frac{1}{\log_9 36} + \frac{1}{\log_{18} 36}$$

By using the formula , $\log_n m = \frac{\log m}{\log n}$

$$\frac{1}{\log_8 36} + \frac{1}{\log_9 36} + \frac{1}{\log_{18} 36} = \frac{1}{\frac{\log 36}{\log 8}} + \frac{1}{\frac{\log 36}{\log 9}} + \frac{1}{\frac{\log 36}{\log 18}}$$

$$= \frac{\log_8}{\log 36} + \frac{\log_9}{\log 36} + \frac{\log_{18}}{\log 36}$$

$$= \frac{\log_8 + \log_9 + \log_{18}}{\log 36}$$

$$= \frac{\log 8 \times 9 \times 18}{\log 36}$$

$$= \frac{\log 36^2}{\log 36}$$

$$= \frac{2 \log 36}{\log 36}$$

$$= 2$$

$$= \text{RHS}$$

29. prove the following identities :

(i) $\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc} = 1$

(ii) $\log_b a \cdot \log_c b \cdot \log_d c = \log_d a$

Solution

(i) $\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc} = 1$

Let us consider LHS:

$$\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$$

By using the formula, $\log_n m = \frac{\log m}{\log n}$

$$\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc} = \frac{\frac{1}{\log abc}}{\log a} + \frac{\frac{1}{\log abc}}{\log b} + \frac{\frac{1}{\log abc}}{\log c}$$

$$= \frac{\log a}{\log abc} + \frac{\log b}{\log abc} + \frac{\log c}{\log abc}$$

$$= \frac{\log a + \log b + \log c}{\log abc}$$

$$\begin{aligned}
&= \frac{(\log a \times b \times c)}{\log abc} \\
&= \frac{\log abc}{\log abc} \\
&= 1 \\
&\text{RHS}
\end{aligned}$$

$$(ii) \log_b a \cdot \log_c b \cdot \log_d c = \log_d a$$

Let us consider LHS

$$\begin{aligned}
\log_b a \cdot \log_c b \cdot \log_d c &= \frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log d} \\
&= \frac{\log a}{\log d} \\
&= \log_d a \\
&= \text{RHS}
\end{aligned}$$

30. given that $\log_a x = \frac{1}{\alpha}$, $\log_b x = \frac{1}{\beta}$, $\log_c x = \frac{1}{\gamma}$, find $\log_{abc} x$.

Solution

It is given that :

$$\log_a x = \frac{1}{\alpha}, \log_b x = \frac{1}{\beta}, \log_c x = \frac{1}{\gamma}$$

so,

$$\log_a x = \frac{1}{\alpha} = \frac{\log x}{\log a} = \frac{1}{\alpha} \Rightarrow \log a = \alpha \log x$$

$$\log_b x = \frac{1}{\beta} = \frac{\log x}{\log b} = \frac{1}{\beta} \Rightarrow \log b = \beta \log x$$

$$\log_c x = \frac{1}{\gamma} = \frac{\log x}{\log c} = \frac{1}{\gamma} \Rightarrow \log c = \gamma \log x$$

now ,

$$\begin{aligned}
\log_{abc} x &= \frac{\log x}{\log abc} \\
&= \frac{\log x}{\log a + \log b + \log c}
\end{aligned}$$

$$= \frac{\log x}{a \log x + b \log x + c \log x}$$

$$= \frac{\log x}{\log_x(\alpha + \beta + \gamma)}$$

$$= \frac{1}{(\alpha + \beta + \gamma)}$$

31. solve for x :

(i) $\log_3 x + \log_9 x + \log_{81} x = \frac{7}{4}$

(ii) $\log_2 x + \log_8 x + \log_{32} x = \frac{23}{15}$

Solution

(i) $\log_3 x + \log_9 x + \log_{81} x = \frac{7}{4}$

Let us simplify the expression ,

$$\frac{1}{\log_x 3} + \frac{1}{\log_x 9} + \frac{1}{\log_x 81} = \frac{7}{4}$$

$$\frac{1}{\log_x 3^1} + \frac{1}{\log_x 3^2} + \frac{1}{\log_x 3^4} = \frac{7}{4}$$

$$\frac{1}{\log_x 3} + \frac{1}{2} \log_x 3 + \frac{1}{4} \log_x 3 = \frac{7}{4}$$

$$\frac{1}{\log_x 3 \left[1 + \frac{1}{2} + \frac{1}{4} \right]} = \frac{7}{4}$$

$$\frac{1}{\log_x 3 \left[\frac{4+2+1}{4} \right]} = \frac{7}{4}$$

$$\log_3 x \frac{7}{4} = \frac{7}{4}$$

$$\log_3 x = 1$$

$$\log_3 x = \log_3 3 \text{ [since } 1 = \log_a a \text{]}$$

On comparing we get

$$x = 3$$

$$(ii) \log_2 x + \log_8 x + \log_{32} x = \frac{23}{15}$$

Let us simplify the expression ,

$$\frac{1}{\log_x 2} + \frac{1}{\log_x 8} + \frac{1}{\log_x 32} = \frac{23}{15}$$

$$\frac{1}{\log_x 2^1} + \frac{1}{\log_x 2^3} + \frac{1}{\log_x 2^5} = \frac{23}{15}$$

$$\frac{1}{\log_x 2} + \frac{1}{3} \log_x 2 + \frac{1}{5} \log_x 2 = \frac{23}{15}$$

$$\frac{1}{\log_x 2} \left[1 + \frac{1}{3} + \frac{1}{5} \right] = \frac{23}{15}$$

$$\text{Log}_2 x \frac{15+5+3}{15} = \frac{23}{15}$$

$$\text{Log}_2 x \frac{23}{15} = \frac{23}{15}$$

$$\text{Log}_2 x = \frac{23}{15} \times \frac{15}{23}$$

$$\text{Log}_2 x = 1$$

$$\text{Log}_2 x = \text{Log}_2 2 \text{ [since , } 1 = \log_a a \text{]}$$

On comparing , we get

$$X = 2$$