CHAPTER – 10

TRIANGLES

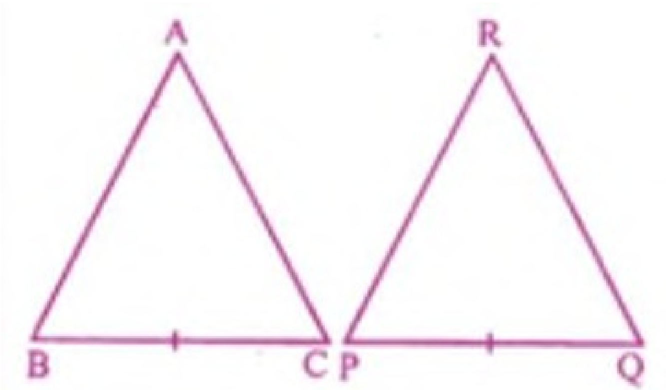
Exercise 10.1

1. It is given that $\Delta ABC \cong \Delta RPQ$. is it true to say that BC = QR? Why?

Solution:

Given $\Delta ABC \cong \Delta RPQ$

Therefore, their corresponding sides and angles are equal.



Therefore BC = PQ

Hence it is not true to say that BC = QR.

2. "If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent." Is the statement true? Why?

Solution:

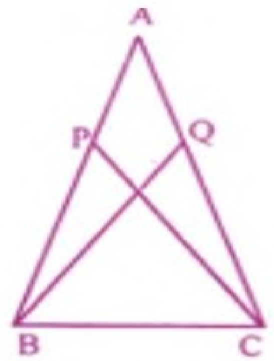
No, it is not true statement as the angles should be included angle of there two given sides.

3. In the given figure, AB = AC and AP = AQ. Prove that

(i) $\Delta APC \cong \Delta AQB$.

(ii) $CP = BQ$

(iii) $\angle APC = \angle AQB$.



Solution:

(i) In ΔAPC and ΔAQB

AB = AC and AP = AQ [Given]

From the given figure, $\angle A = \angle A$ [Common in both the triangles]

Therefore, using SAS axiom we have $\Delta APC \cong \Delta AQB$.

(ii) In ΔAPC and ΔAQB

AB = AC and AP = AQ [Given]

From the given figure, $\angle A = \angle A$ [Common in both the triangles]

By using corresponding parts of congruent triangle concept we have

BQ = CP

(iii) In ΔAPC and ΔAQB

AB = AC and AP = AQ [Given]

From the given figure, $\angle A = \angle A$ [Common in both the triangles]

By using corresponding parts of congruent triangle concept we have

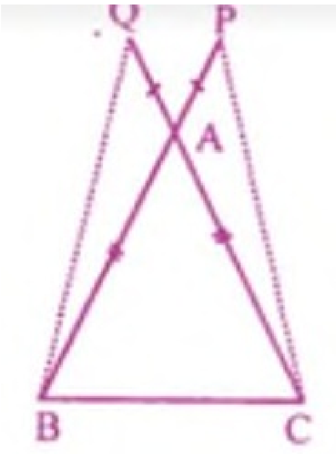
$$\angle APC = \angle AQB.$$

4. In the given figure, AB = AC, P and Q are points on BA and CA respectively such that AP = AQ. Prove that:

(i) $\Delta APC \cong \Delta AQB$.

(ii) $CP = BQ$

(iii) $\angle APC = \angle ABQ$.



Solution:

(i) In the given figure AB = AC

P and Q are point on BA and CA produced respectively such that AP = AQ.

Now we have to prove $\Delta APC \cong \Delta AQB$.

By using corresponding parts of congruent triangle concept we have

CP = BQ

$$\angle APC = \angle ABQ.$$

(ii) CP = BQ.

(iii) $\angle APC = \angle ABQ$.

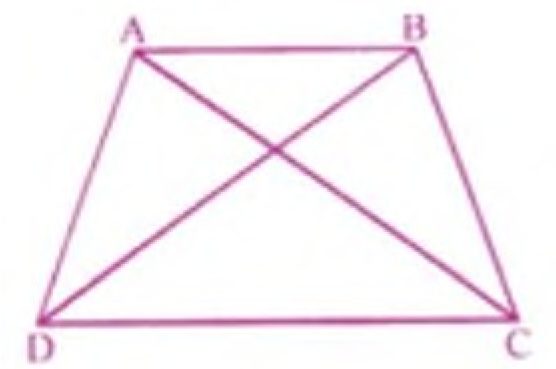
AC = AB (Given)

AP = AQ (Given)

$\angle PAC = \angle QAB$. (Vertically opposite angle)

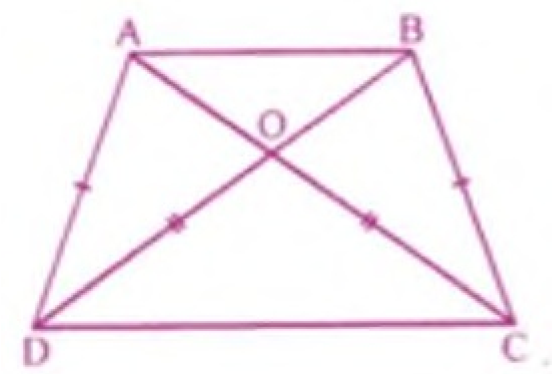
5. In the given figure, AD = BC and BD = AC. Prove that:

$$\angle ADB = \angle BCA \text{ and } \angle DBA = \angle CBA.$$



Solution:

Given: In the figure, AD = BC, BD = AC.



To prove:

(i) $\angle ADB = \angle BCA$

(ii) $\angle DBA = \angle CBA$.

Proof: In ΔADB and ΔACB

AB = AB (common)

AD = BC (given)

DB = AC (given)

$\Delta ADB \cong \Delta ACD$. (SSS axiom)

$$\angle ADB = \angle BCA$$

$$\angle DBA = \angle CBA$$

6. In the given figure, ABCD is a quadrilateral in which AD = BC and $\angle DBA = \angle CBA$. Prove that

(i) $\Delta \mathbf{ABD} \cong \Delta \mathbf{BAC}$.

(ii) $\mathbf{BD} = \mathbf{AC}$

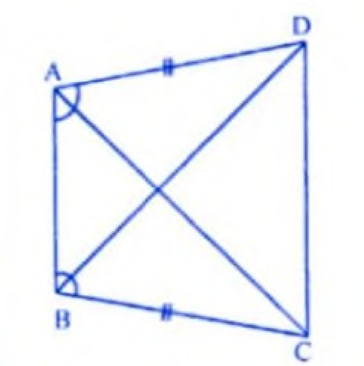
(iii) $\angle \mathbf{ABC} = \angle \mathbf{BAC}$.

Solution:

Given: In the figure ABCD is a quadrilateral

In which AD = BC

$$\angle DBA = \angle CBA.$$



To prove:

(i) $\Delta ABD \cong \Delta BAC$.

(ii) $\angle ABD = \angle BAC$.

Proof: In ΔABD and ΔABC

AB = AB (common)

$\angle DAB = \angle CBA$(Given)

AD = BC (given)

(i) $\Delta ABD \cong \Delta ABC$. (SAS axiom)

(ii) $BD = AC$

(iii) $\angle ABD = \angle BAC$

7. In the given figure, AB = DC and AB || DC. Prove that AD = BC.

Solution:

Given: In the given figure,

AB = DC, AB || DC

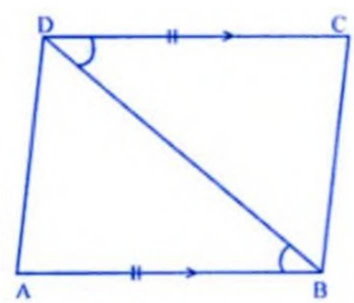
To prove: AD = BC

Proof: AB || DC

$\angle ABD = \angle CDB$ (Alternate angles)

In ΔABD and ΔCDB

AB = DC



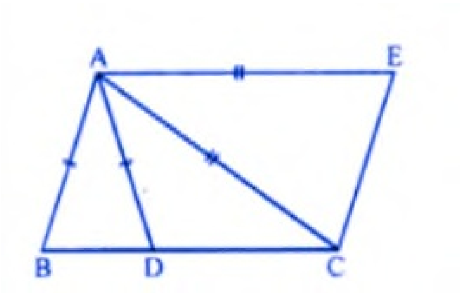
$\angle ABD = \angle CDB$ (Alternate angles)

BD = BD (common)

$\Delta ABD \cong \Delta CDB$ (SAS axiom)

AD = BC

8. In the given figure, AC = AE, AB = AD and ∠BAD = ∠CAE. Such that BC = DE.

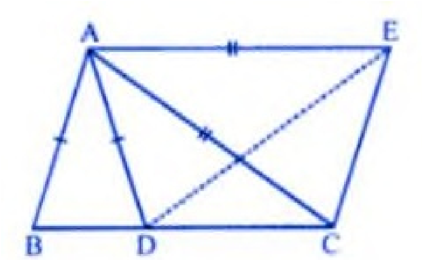


Solution:

Given: In the figure, AC = AE, AB = AD

$$\angle BAD = \angle CAE.$$

To prove: BC = DE



Proof: In ΔABC and ΔADE

AB = AD (given)

AC = AE (given)

$$\angle BAD + \angle DAC + \angle CAE$$

$$\angle BAD = \angle DAC$$

$\Delta ABC = \Delta ADE$ (SAS axiom)

BC = DE

9. In the adjoining figure, AB = CD, CE = BF and ∠ACE = ∠DBF. Prove that

(i) ΔACE ≅ ΔDBF.

(ii) AE = DF

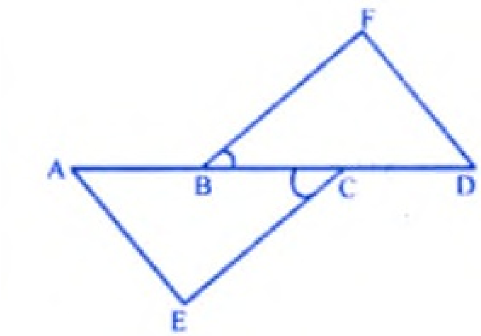
Solution:

Given: In the given figure

AB = CD

CE = BF

$$\angle ACE = \angle DBF$$



To prove: (i) ΔACE ≅ ΔDBF.

(i) ΔACE ≅ ΔDBF (SAS axiom)

AE = DE

(ii) AE = DF

Proof: AB = CD

Adding BC to both sides

AB + BC = BC + CD

AC = BD

Now in ΔACE and ΔDBF

AC = BD (Proved)

CE = BF (Given)

∠ACE = ∠DBF (SAS axiom)

Exercise – 10.2

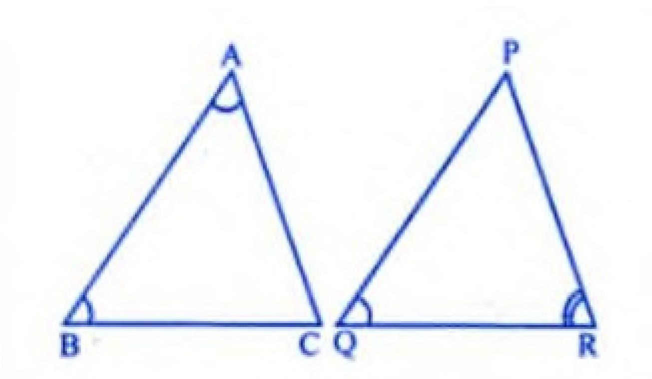
1. In triangles ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$. which side of APQR should be equal to side AB of AABC so that the two triangle are congruent? Give reason for your answer.

Solution:

In triangle ABC and triangle PQR

$$\angle A = \angle Q$$

$$\angle B = \angle R$$



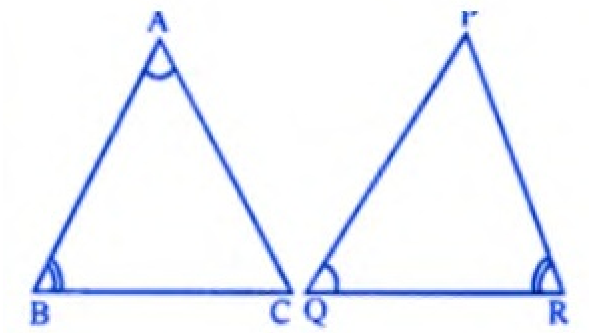
AB = QP

Because triangles are congruent of their corresponding two angles and included sides are equal.

2. In triangle ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$. which side of APQR should be equal to side BC of AABC so that the two triangles are congruent? Give reason for your answer.

Solution:

In triangle ABC and triangle PQR



$$\angle A = \angle Q$$

$$\angle B = \angle R$$

Their included sides AB and QR will be equal for their congruency.

Therefore, BC = PR by corresponding parts of congruent triangles.

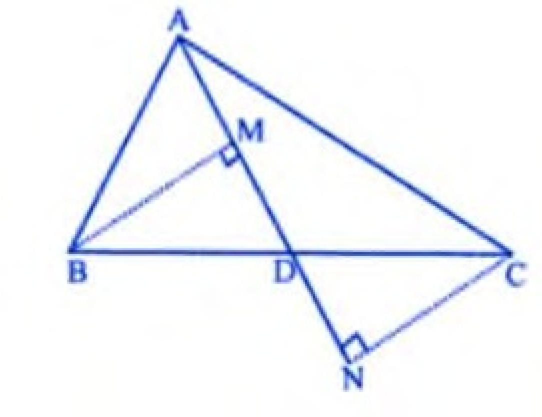
3. "If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangle must be congruent". Is the statement true? Why?

Solution:

The given statement can be true only if the corresponding (included) sides are equal otherwise not.

4. In the given figure, AD is median of ΔABC, BM and CN are perpendiculars drawn from B and C respectively on AD and AD produced. Prove that BM = CN.

Solution:



Given in ΔABC, AD is median BM and CN are perpendicular to AD from B and C respectively.

To prove:

$BM = CN$

Proof: In ΔBMD and ΔCND

$BD = CD$ (because AD is median)

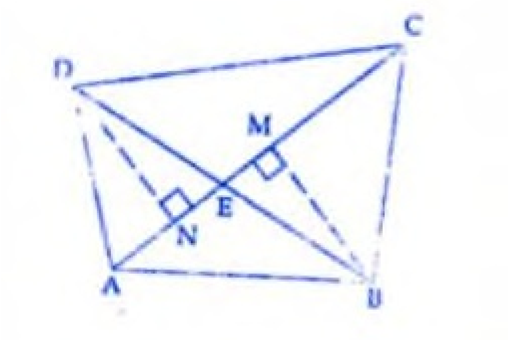
$$\angle M = \angle N$$

$\angle BDM = \angle CDN$ (Vertically opposite angles)

$\Delta BMD \cong \Delta CND$ (AAS axiom)

Therefore, $BM = CN$.

5. In the given figure, BM and DN are perpendiculars to the line segment AC. If BM = DN, prove that AC bisects BD.



Solution:

Given in figure BM and DN are perpendicular to AC

BM = DN

To prove:

AC bisects BD that is BE = ED

Construction:

Join BD which intersects AC at E

Proof:

In ΔBEM and ΔDEN

BM = DN

$\angle M = \angle N$ (Given)

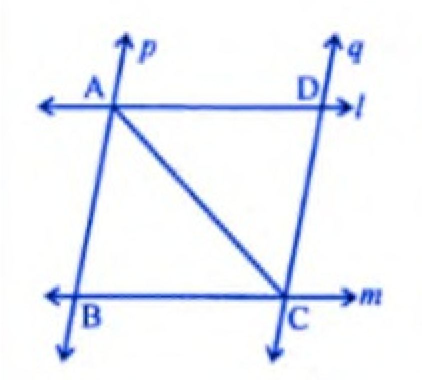
$\angle DEN = \angle BEM$ (Vertically opposite angles)

$\Delta BEM \cong \Delta DEN$

BE = ED

Which implies AC bisects BD

6. In the given figure, l and m are two parallel lines intersected by another pair of parallel lines p and q. Show that $\Delta ABC \cong \Delta CDA$.



Solution:

In the given figure, two lines l and m are parallel to each other and lines p and q are also a pair of parallel lines intersecting each other at A, B, C and D. AC is joined.

To prove:

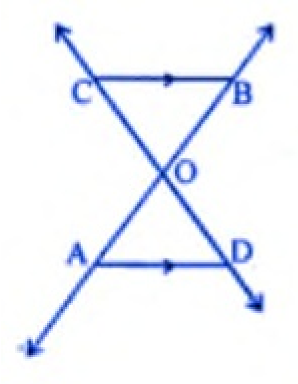
$\Delta ABC \cong \Delta CDA$

Proof:

In ΔABC and ΔCDA

$AC = AC$	(common)
$\angle ACB = \angle CAD$	(Alternate angles)
$\angle BAC = \angle ACD$	(Alternate angles)
$\Delta ABC \cong \Delta DCA$	(ASA axiom)

7. In the given figure, two lines AB and CD intersect each other at the point O such that BC || DA and BC = DA. Show that O is the mid-point of both the line segments AB and CD.



Solution:

In the given figure, lines AB and CD intersect each other at O such that BC || AD and BC = DA

To prove:

O is the midpoint of AB and CD

Proof:

ConsiderΔAOD and ΔBOC

AD = BC	(given)
$\angle OAD = \angle OBC$	(Alternate angles)
$\angle ODA = \angle OCB$	(Alternate angles)
$\Delta AOD \cong \Delta BOC$	(SAS axiom)

Therefore, OA = OB and OD = OC

Therefore, O is the midpoint of AB and CD.

Exercise – 10.3

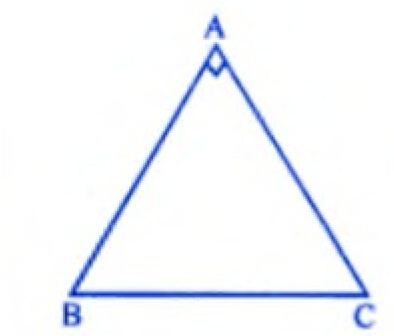
1. ABC is a right angled triangle in which ∠A = 90° and AB = AC. Find ∠B and ∠C.

Solution:

In right angled triangle ABC, ∠A = 90°

$$\angle B + \angle C = 180^\circ - \angle A$$

= 180° – 90° = 90°



Because AB = AC

$\angle C = \angle B$ (Angles opposite to equal sides)

$$\angle B + \angle B = 90^\circ (2\angle B = 90^\circ)$$

$$\angle B = \frac{90}{2^\circ} = 45^\circ$$

$$\angle B = \angle C = 45^\circ$$

$$\angle B = \angle C = 45^\circ$$

2. Show that the angles of an equilateral triangle are 60° each.

Solution:

ΔABC is an equilateral triangle

AB = BC = CA

$\angle A = \angle B = \angle C$ (Opposite to equal sides)

But $\angle A + \angle B + \angle C = 180°$ (Sum of angles of a triangle)

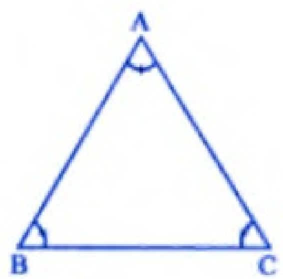
$$3\angle A = 180° \left(\angle A = \frac{180°}{3} = 60°\right)$$

$$\angle A = \angle B = \angle C = 60°$$

3. Show that every equiangular triangle is equilateral.

Solution:

ΔABC is an equiangular



$$\angle A = \angle B = \angle C$$

In ΔABC

$$\angle B = \angle C$$

AC = AB (Sides opposite to equal angles)

Similarly, $\angle C = \angle A$

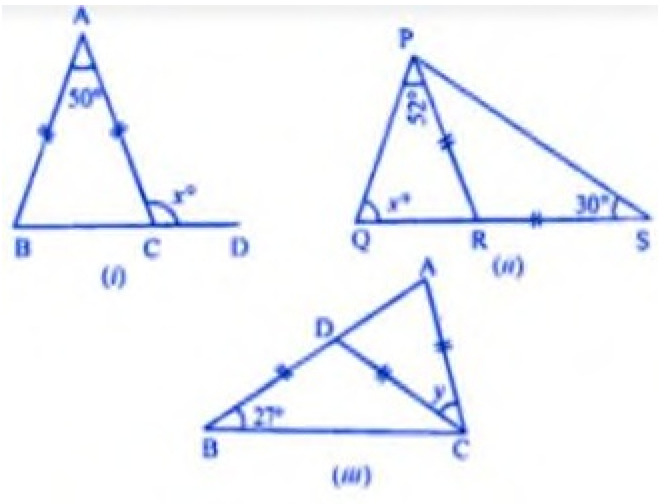
BC = AB

From (i) and (ii)

AB = BC = AC

ΔABC is an equilateral triangle.

4. In the following diagram, find the value of x:



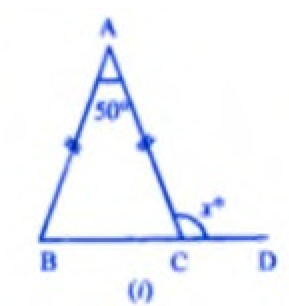
Solution:

(i) In following diagram given that AB = AC

That is $\angle B = \angle ACB$ (angles opposite to equal sides in a triangle are equal)

Now, $\angle A + \angle B + \angle ACB = 180°$

(Sum of all angles in a triangle is 180°).



$$50 + \angle B + \angle B = 180°$$

$$(\angle A = 50°(\text{given})\angle B = \angle ACB)$$

$50° + 2\angle B = 180° \quad (2\angle B = 180° - 50°)$

$2\angle B = 130°$ $\left(\angle B = \frac{130}{2} = 65°\right)$

$$\angle ACB = 65°$$

Also $\angle ACB + x° = 180°$ (Linear pair)

$65° + x° = 180°$ $(x° = 180° - 65°)$

$$x° = 115°$$

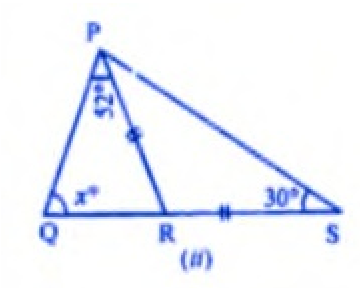
Hence, value of $x = 115$

(ii) In ΔPRS,

Given that PR = RS

$$\angle PSR = \angle RPS$$

(Angles opposite in a triangle, equal sides are equal)



$30° = \angle RPS$ $(\angle PSR = 30°)$ … (1)

$$\angle QPS = \angle QPR + \angle RPS$$

$$\angle QPS = 52° + 30°$$

(Given, $\angle QPR = 52°$ and from (i),$\angle RPS = 30°$)

$$\angle QPS = 82°$$

Now, in ΔPQS

$$\angle QPS = \angle QSP + \angle PQS = 180°$$

(Sum of all angles in a triangles is 180°)

= $82° + 30° + x° = 180°$

From (2) $\angle QPS = 82°$ and $\angle QSP = 30°$ (given)

$112° + x° = 180°$ $(x° = 180° - 112°)$

Hence, value of $x = 68$

(iii) In the following figure, Given

That, BD = CD = AC and $\angle DBC = 27°$

Now in ΔBCD

BD = CD (Given)

$\angle DBC = \angle BCD$ …. (1)

(In a triangle sides opposite equal angles are equal)

Also, $\angle DBC = 27°$ (Given) …. (2)

From (1) and (2) we get

$$\angle BCD = 27°$$

Now, ext. $\angle CDA = \angle DBC + \angle BCD$

(Exterior angles is equal to sum of two interior opposite angles)

Ext. $\angle CDA = 27° + 27°$ (From (2) and (3))

$\angle CDA = 54°$ (From (4)) …. (5)

Also, in ΔACD

$$\angle CDA + \angle CDA + \angle ACD = 180°$$

(Sum of all angles in a triangle is 180°)

$$54° + 54° + Y = 180°$$

$$108° + Y = 180° \ (Y = 180° - 108°)$$

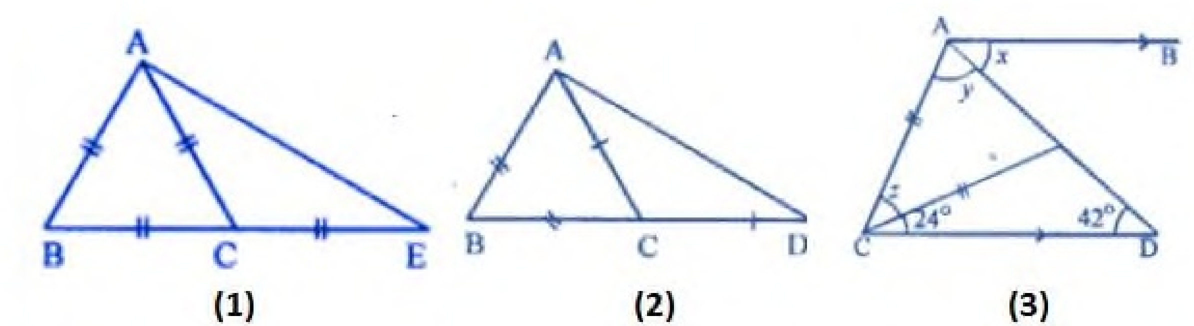
$$Y = 72°$$

5.

(a) In the figure (1) given below, ABC is an equilateral triangle. Base BC is produced to E, such that BC' = CE. Calculate ∠ACE and ∠AEC.

(b) In the figure (2) given below, prove that ∠BAD : ∠ADB = 3 : 1.

(c) In the figure (2) given below, AB || CD. Find the values of x, y and ∠.

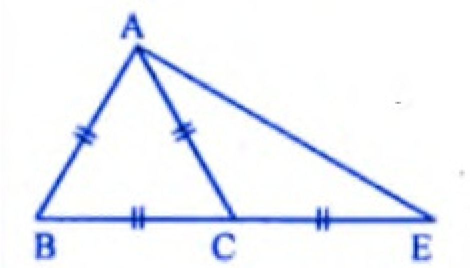


Solution:

(a) In following figure

Given: ABC is an equilateral triangle BC = CE

To find: ∠ACE and ∠AEC



(1)

As given that ABC is an equilateral triangle,

That is $\angle BAC = \angle B = \angle ACB = 60°$ …. (1)

(Each angle of an equilateral triangle is 60°)

Now, $\angle ACE = \angle BAC + CB$

(Exterior angle is equal to sum of two interior opposite angles)

$$(\angle ACE = 60° + 60°)$$

$$\angle ACE = 120°$$

Then, in ΔACE

Given, AC = CE … [because AC = BC = CE]

$\angle CAE = \angle AEC$ ….. (2)

We know that, in a triangle equal sides have equal angle opposite to them.

So, $\angle CAE + \angle AEC + 120° = 80°$

$\angle AEC + \angle AEC + 120° = 180°$ … [By equation (2) we get]

$$2\angle AEC = 180° - 120°$$

$$2\angle AEC = 60°$$

$$\angle AEC = \frac{60°}{2}$$

$$\angle AEC = 30°$$

Therefore, $\angle AEC = 120°$ and $\angle AEC = 30°$.

(b) In given figure,

Given: ΔABD, AC meets BD in C. AB = BC, AC = CD.

We have to prove that, $\angle BAD : \angle ADB = 3 : 1$.

Then, Consider ΔABC,

AB = BC … [Given]

Therefore, $\angle ACE = \angle BAC$ … (1)

(In a triangle, equal angle opposite to them)

In ΔACD,

AC = CD … [Given]

Therefore, $\angle ADC = \angle CAD$

(In a triangle, equal sides have equal angles opposite to them)

$\angle CAD = \angle ADC$ … (2)

From, adding (1) and (2), we get

$$\angle BAC + \angle CAD = \angle ACB + \angle ADC$$

$\angle BAD = \angle ACB + \angle ADC$ … (3)

Now, inΔACD,

Exterior $\angle ACB = \angle CAD + \angle ADC$… (4)

(In a triangle, exterior angle is equal to sum of two interior opposite angles)

Therefore, $\angle ACB = \angle ADC + \angle ADC$ [From (2) and (4)]

$\angle ACB = 2\angle ADC$ … (5)

Now, $\angle BAD = 2\angle ADC + \angle ADC$ [From (3) and (4)]

$$\angle BAD = 3\angle ADC = \left(\frac{\angle BAD}{\angle ADC}\right) = \frac{3}{1}$$

$$\angle BAD : \angle ADC = 3 : 1$$

(c) In given figure,

Given, AB parallel to CD, $\angle ECD = 24°, \angle CDE = 42°$

We have to find the value of x, y and z.

Consider, ΔCDE

Exterior, $\angle CEA = 24° + 42°$

(In a triangle exterior angle is equal to sum of two interior opposite angle)

$\angle CEA = 66°$ .... (1)

Then, in ΔACE,

AC = CE … [Given]

Therefore, $\angle CAE = \angle CEA$

(In a triangle equal side have equal angles opposite to them)

By equation (1),

$Y = 66°$ .... (2)

Also, $y + x + \angle CAE = 180°$

We know that, sum of all angles in a triangle is 180°.

$$66° + z + 66° = 180°$$

$$z + 132° = 180°$$

$$z = 180° - 132°$$

$z = 48°$ …. (3)

Then it is given that, AB is parallel to CD.

$\angle x = \angle ADC$ [Alternate angles]

$x = 42°$ …. (4)

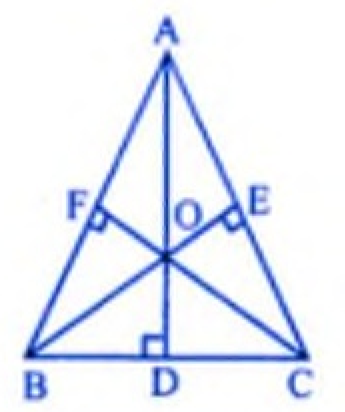
Therefore, from (2), (3) and (4) equation given $x = 42°, y = 66°$ and $z = 48°$.

6. In the given figure, AD, BE and CF are altitudes of ΔABC. If AD = BE = CF, prove that ABC is an equilateral triangle.

Solution:

Given: in the figure given,

AD, BE and CF are altitudes of ΔABC and AD = BE = CF



To prove: ΔABC is an equilateral triangle

Proof: In the right ΔBEC and ΔBFC

Hypotenuse BC = BC (Common)

Side BE = CF (Given)

$\Delta BEC \cong \Delta BFC$ (RHS axiom)

$$\angle C = \angle B$$

AB = AC (Sides opposite to equal angles)

Similarly we can prove that $\Delta CFA \cong \Delta ADC$

$$\angle A = \angle B$$

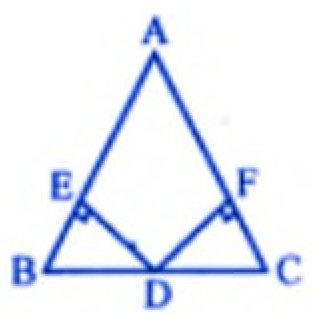
AB = BC

From (i) and (ii)

AB = BC = AC

ΔABC is an equilateral triangle.

7. In the given figure, D is mid-point of BC, DE and DF are perpendiculars to AB and AC respectively such that DE = DF. Prove that ABC is an isosceles triangle.



Solution:

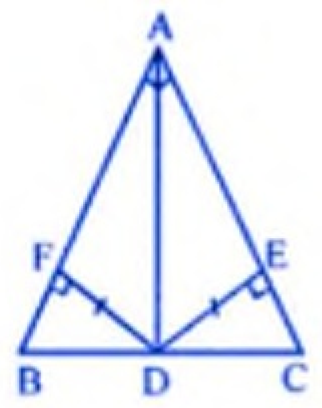
In triangle ABC

D is the midpoint of BC

DE perpendicular to AB

And DF perpendicular to AC

DE = DE



To prove:

Triangle ABC is an isosceles triangle

Proof:

In the right angled triangle BED and CDF

Hypotenuse BD = DC (because D is a midpoint)

Side DF = DE (Given)

$\Delta BED \cong \Delta CDA$ (RHS axiom)

$$\angle C = \angle B$$

AB = AC (Sides opposite to equal angles)

ΔABC is an isosceles triangle.

Exercise – 10.4

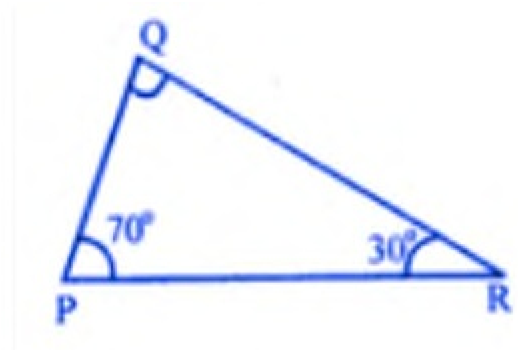
1. In ΔPQR, $\angle P = 70°$ and $\angle R = 30°$. Which side of this triangle is longest? Give reason for your answer.

Solution:

In ΔPQR, $\angle P = 70°$ and $\angle R = 30°$.

But $\angle P + \angle Q + \angle R = 180°$

$$100° + \angle Q = 180°$$



$$\angle Q = 180° - 100° = 80°$$

$\angle Q = 80°$ The greatest angle

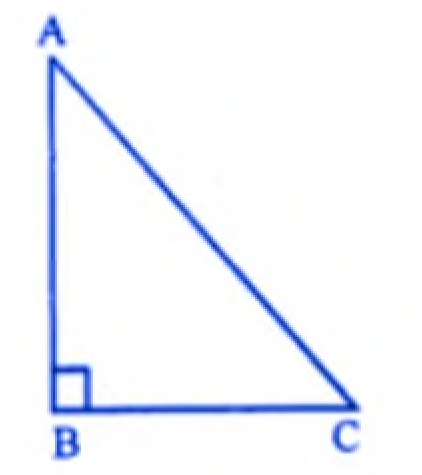
Its opposite side PR is the longest side

(Side opposite to greatest angle is longest)

2. Show that in a right angled triangle, the hypotenuse is the longest side.

Solution:

Given: in right angled ΔABC, $\angle B = 90°$



To prove: AC is the longest side

Proof: in ΔABC

$\angle B = 90°$

$\angle A$ and $\angle C$ are acute angles

That is less than 90°

$\angle B$ is the greatest angle

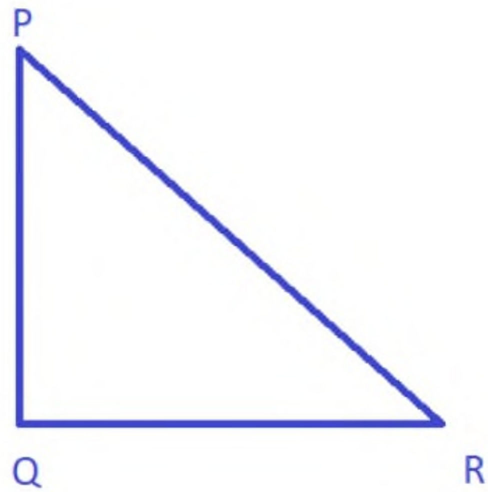
Or $\angle B > \angle C$ and $\angle B > \angle A$

$AC > AB$ and $AC > BC$

Hence AC is the longest side

3. PQR is a right angle triangle at Q and PQ: QR = 3: 2. Which is the least angle.

Solution:



Here, PQR is a right angle triangle at Q. Also given that

PQ: QR = 3: 2

Let PQ = $3x$, then, QR = $2x$

It is clear that QR is the least side,

Then, we know that the least angle has least side

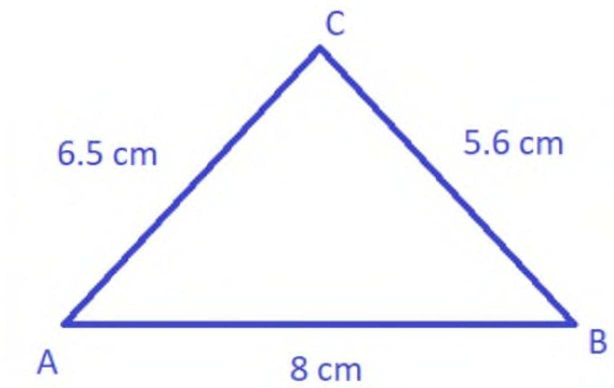
Opposite to it.

Hence ∠P is the least angle

4. In ΔABC, AB = 8 cm, BC = 5.6 cm and CA = 6.5 cm. Which is

(i) The greatest angle?

(ii) The smallest angle?



Solution:

Given that AB = 8 cm, BC = 5.6 cm, CA = 6.5 cm.

Here AB is the greatest side

Then ∠C is the least angles

The greatest side has greatest angle opposite to it

Also, BC is the least side

Then, ∠A is the least angle

(The least side has least opposite to it.)

CHAPTER TEST

1. In triangle ABC and DEF, $\angle A = \angle D, \angle B = \angle E$ and AB = EF. Will the two triangles be congruent? Give reasons for your answer.

Solution:

In ΔABC and ΔDEF

$$\angle A = \angle D$$

$$\angle B = \angle E$$

AB = EF

In ΔABC, two angles and included side is

Given but in ΔDEF, corresponding angles are equal but side is not included of their angle.

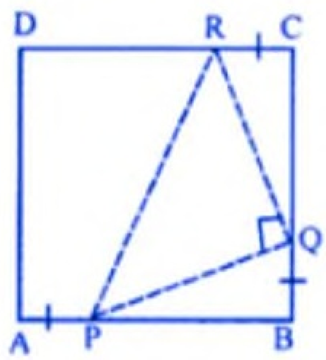
Triangle cannot be congruent.

2. In the adjoining figure, ABCD is a square, P, Q and *R*are points on the sides AB, BC and CD respectively such that AP = BQ = CR and ∠PQR = 90°. Prove that

(a) ΔPBQ ≅ ΔQCR

(b) PQ = QR

(c) ∠PRQ = 45°



Solution:

Given: In the given figure, ABCD is a square

P, Q and R are the points on the sides AB, BC and CD respectively such that

AP = BQ = CR, $\angle PQR = 90°$

To prove:

(i) $\Delta PBQ \cong \Delta QCR$

(ii) PQ = QR

(iii) $\angle PRQ = 45°$

Proof: AB = BC = CD (Sides of square)

And AP = BQ = CR (Given)

Subtracting, we get

AB – AP = BC – BQ = CD – CR

(PB = QC = RD)

Now in ΔPBQ and ΔQCR

PB = QC (Proved)

BQ = CR (Given)

$\angle B = \angle C$ (Each 90°)

$\Delta PBQ \cong \Delta QCR$

PQ = QR

But $\angle PQR = 90°$

$$\angle RPQ = \angle PQR$$

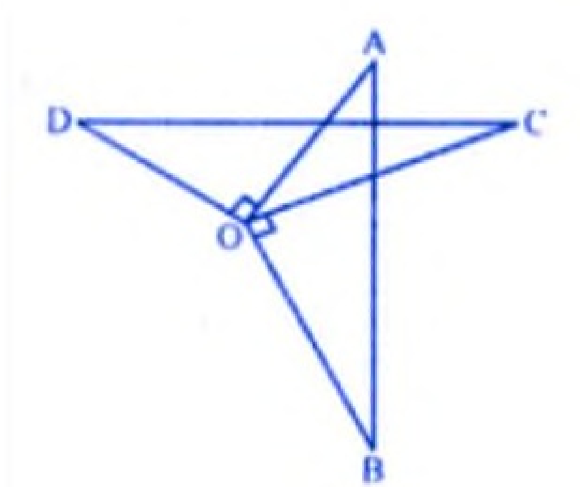
(Angles opposite to equal angles)

But $\angle RPQ = \angle PQR = 90°$

$$\angle RPQ = \angle PQR = 90°$$

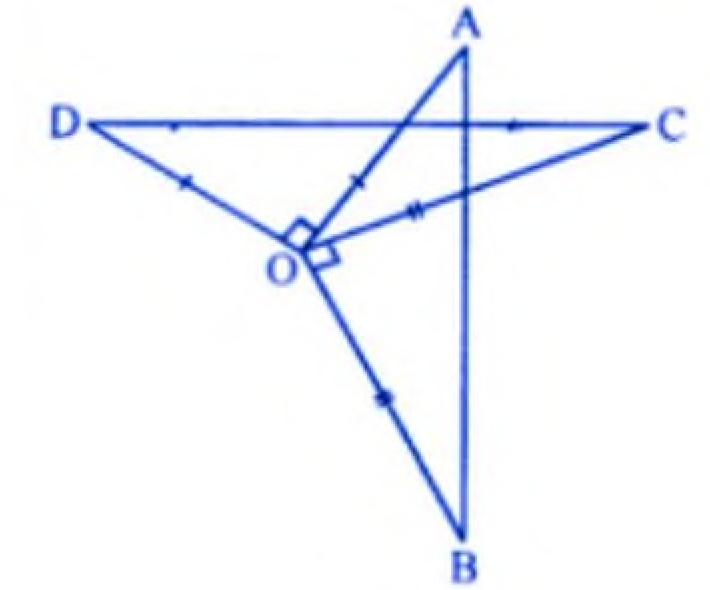
$$\angle RPQ = \angle PQR = \frac{90°}{2} = 45°$$

3. In the given figure, $\mathbf{OA \perp OD, OC \times OB, OD = OA}$ and $\boldsymbol{OB = OC}$. Prove that AB = CD



Solution:

Given: In the figure, $OA \perp OD, OC \times OB$.



To prove: AB = CD

Proof: $\angle AOD = \angle COB$ (Each 90°)

Adding $\angle AOC$

$$\angle AOD + \angle AOC = \angle AOC + \angle COB$$

$$\angle COD = \angle AOB$$

Now, in ΔAOB and ΔDOC

$OA = OD$ (Given)

$OB = OC$ (Given)

$\angle AOB = \angle COD$ (Proved)

$\Delta AOB \cong \Delta DOC$

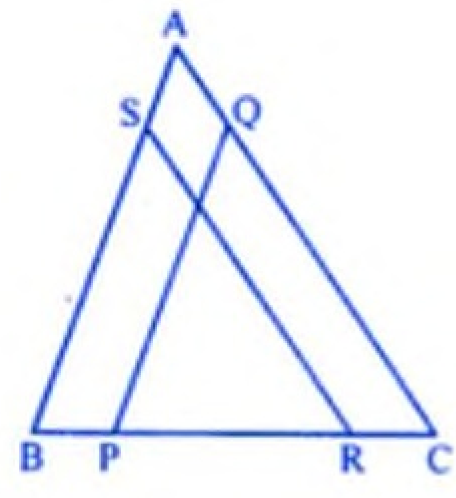
AB = CD

4. In the given figure, PQ || BA and RS || CA. If BP = RC, Prove that:

(i) ΔBSR≅ ΔPQC

(ii) BS = PQ

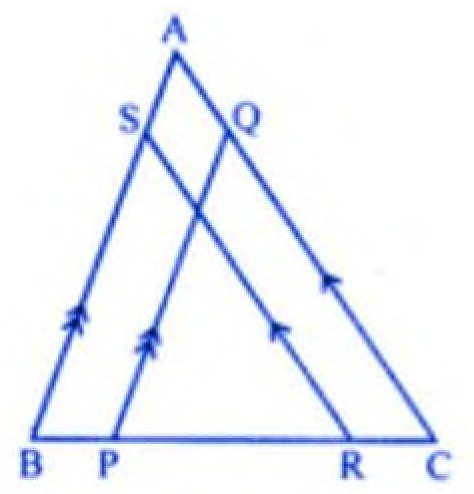
(iii) RS = CQ.



Solution:

PQ || BA, RS || CA

BP = RC



To prove:

(i) ΔBSR≅ ΔPQC

(ii) RS = CQ

Proof: BP = RS

BC – RC = BC – BP

BR = PC

Now, in ΔBSR and ΔPQC

$\angle B = \angle P$ (Corresponding angles)

$\angle R = \angle C$ (Corresponding angles)

BR = PC (Proved)

$\Delta BSR \cong \Delta PQC$

BS = PQ

RS = CQ.