

Chapter 12

Pythagoras Theorems

Exercise 12.1

1. Lengths of sides of triangles are given below. Determine which of them are right triangles. In case of a right triangles, write the length of its hypotenuse:

(i) 3 cm, 8cm, 6cm

(ii) 13cm, 12cm, 5cm

(iii) 1.4cm, 4.8cm, 5cm

Solution:

We use the Pythagoras theorem to check whether the triangles are right triangles.

We have $h^2 = b^2 + a^2$ [Pythagoras theorem]

Where h is the hypotenuse, b is the base and a is the altitude.

(i) Given sides are 3cm, 8cm and 6cm

$$b^2 + a^2 = 3^2 + 6^2 = 9 + 36 = 45$$

$$h^2 = 8^2 = 64$$

here $45 \neq 64$

Hence the given triangle is not a right triangle.

(ii) Given sides are 13cm, 12cm and 5cm

$$b^2 + a^2 = 12^2 + 5^2 = 144 + 25 = 169$$

$$h^2 = 13^2 = 169$$

$$\text{here } b^2 + a^2 = h^2$$

Hence the given triangle is a right triangle.

Length of the hypotenuse is 13 cm.

(ii) Given sides are 1.4cm, 4.8cm and 5cm

$$b^2 + a^2 = 1.4^2 + 4.8^2 = 1.96 + 23.04 = 25$$

$$h^2 = 5^2 = 25$$

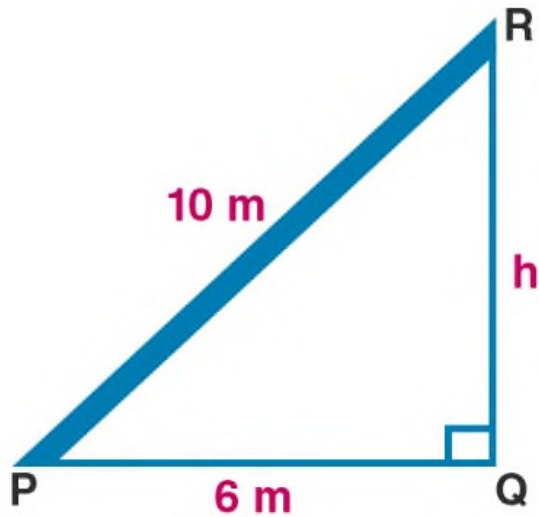
$$\text{here } b^2 + a^2 = h^2$$

Hence the given triangle is a right triangle.

Length of the hypotenuse is 5cm.

2. Foot of a 10m long ladder leaning against a vertical wall is 6m away from the base of the wall. Find the height of the point on the wall where the top of the ladder reaches.

Solution:



Let PR be the ladder and QR be the vertical wall.

Length of the ladder $PR = 10\text{m}$

$PQ = 6\text{m}$

Let height of the wall, $QR = h$

According to Pythagoras theorem,

$$PR^2 = PQ^2 + QR^2$$

$$10^2 = 6^2 + QR^2$$

$$100 = 36 + QR^2$$

$$QR^2 = 100 - 36$$

$$QR^2 = 64$$

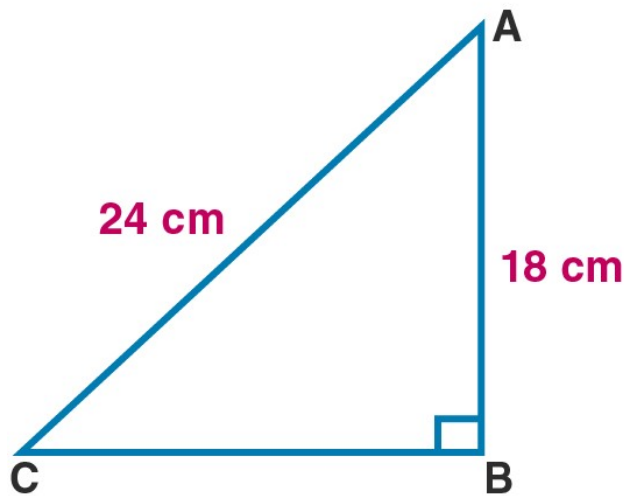
Taking square root on both sides,

$$QR = 8$$

Hence the height of the wall where the top of the ladder reaches is 8m.

3. A guy attached a wire 24m long to a vertical pole of height 18m and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be tight ?

Solution:



Let AC be the wire and AB be the height of the pole.

$$AC = 24\text{cm}$$

$$AB = 18\text{cm}$$

According to Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$24^2 = 18^2 + BC^2$$

$$576 = 324 + BC^2$$

$$BC^2 = 576 - 324$$

$$BC^2 = 252$$

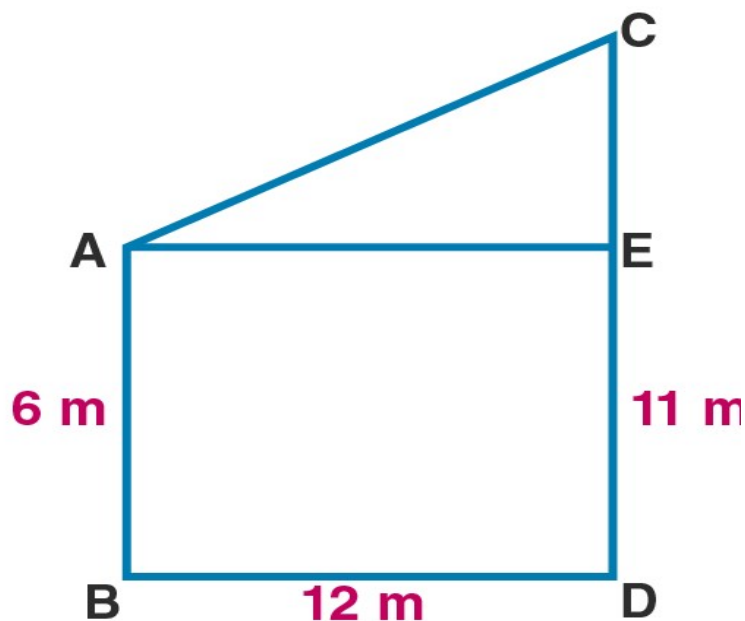
Taking square root on both sides,

$$\begin{aligned}
 BC &= \sqrt{252} \\
 &= \sqrt{4 \times 9 \times 7} \\
 &= 2 \times \sqrt{7} \\
 &= 6\sqrt{7} \text{ cm}
 \end{aligned}$$

Hence the distance is $6\sqrt{7}$ cm.

4. Two poles of heights 6m and 11m stand on a plane ground. If the distance between their feet is 12m, find the distance between their tops.

Solution:



Let AB and CD be the poles which are 12m apart.

$$AB = 6\text{m}$$

$$CD = 11\text{m}$$

$$BD = 12\text{m}$$

Draw AE BD

$$CE = 11 - 6 = 5\text{m}$$

$$AE = 12\text{m}$$

According to Pythagoras theorem,

$$AC^2 = AE^2 + CE^2$$

$$AC^2 = 12^2 + 5^2$$

$$AC^2 = 144 + 25$$

$$AC^2 = 169$$

Taking square root on both sides

$$AC = 13$$

Hence the distance between their tops is 13m.

5. In a right-angles triangle, if hypotenuse is 20 cm and the ratio of the other two sides is 4:3, find the sides.

Solution:

Given hypotenuse, $h = 20\text{cm}$

Ratio of other two sides, $a:b = 4 : 3$

Let altitude of the triangle be $4x$ and base be $3x$.

According to Pythagoras theorem,

$$h^2 = b^2 + a^2$$

$$20^2 = (3x)^2 + (4x)^2$$

$$400 = 9x^2 + 16x^2$$

$$25x^2 = 400$$

$$x^2 = \frac{400}{25}$$

$$x^2 = 16$$

Taking square root on both sides

$$x = 4$$

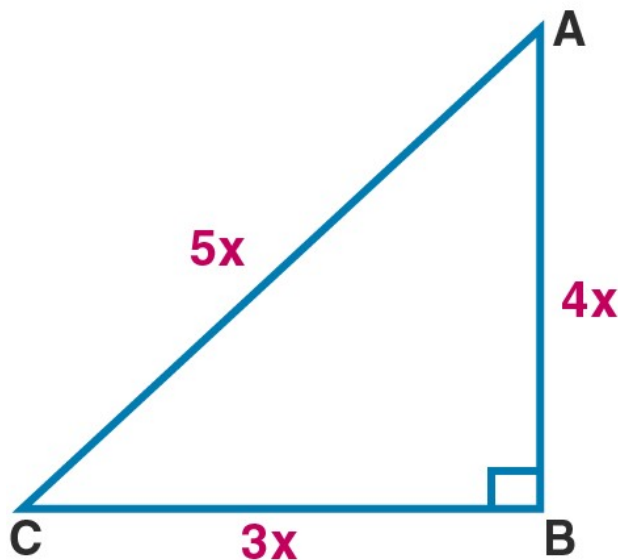
$$\text{so base, } b = 3x = 3 \times 4 = 12$$

$$\text{altitude, } a = 4x = 4 \times 4 = 16$$

Hence the other sides are 12cm and 16cm

6. If the sides of a triangle are in the ratio 3:4:5, prove that it is right-angles triangle.

Solution:



Given the sides are in the ratio 3:4:5.

Let ABC be the given triangle.

Let the sides be $3x$, $4x$ and hypotenuse be $5x$.

According to Pythagoras theorem,

$$AC^2 = BC^2 + AB^2$$

$$BC^2 + AB^2 = (3x)^2 + (4x)^2$$

$$= 9x^2 + 16x^2$$

$$= 25x^2$$

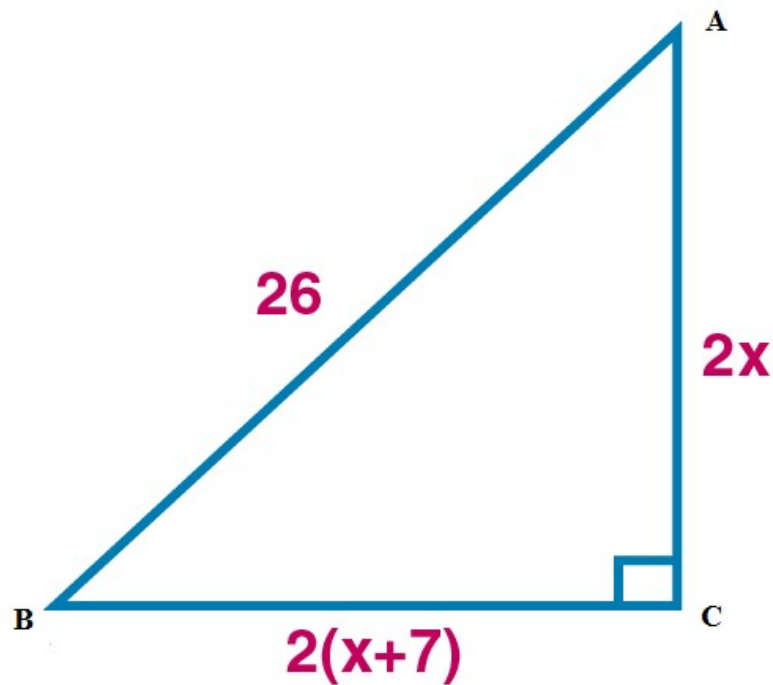
$$AC^2 = (5x)^2 = 25x^2$$

$$AC^2 = BC^2 + AB^2$$

Hence ABC is a right angles triangle.

7. For going to a city B from city A, there is route via city C such that $AC \perp CB$, $AC = 2x$ km and $CB = 2(x + 7)$ km. It is proposed to construct a 26 km highway which directly connects the two cities A and B. Find how much distance will be saved in reaching city B from city A after the construction of highway.

Solution:



Given $AC = 2x \text{ km}$

$CB = 2(x + 7) \text{ km}$

$AB = 26$

Given AC CB.

According to Pythagoras theorem,

$$AB^2 = CB^2 + AC^2$$

$$26^2 = (2(x + 7))^2 + (2x)^2$$

$$676 = 4(x^2 + 14x + 49) + 4x^2$$

$$4x^2 + 56x + 196 + 4x^2 = 676$$

$$8x^2 + 56x + 196 = 676$$

$$8x^2 + 56x + 196 - 676 = 0$$

$$8x^2 + 56x - 480 = 0$$

$$x^2 + 7x - 60 = 0$$

$$(x - 5)(x + 12) = 0$$

$$(x - 5) = 0 \quad \text{or} \quad (x + 12) = 0$$

$$x = 5 \quad \text{or} \quad x = -12$$

Length cannot be negative. So $x = 5$

$$BC = 2(x + 7) = 2(5 + 7) = 2 \times 12 = 24\text{km}$$

$$AC = 2x = 2 \times 5 = 10\text{km}$$

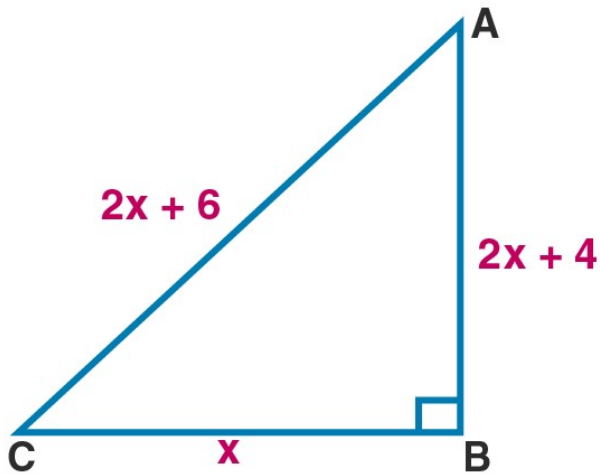
$$\text{Total distance} = AC + BC = 10 + 24 = 34 \text{ km}$$

$$\text{Distance saved} = 34 - 26 = 8 \text{ km}$$

Hence the distance saved is 8km.

8. The hypotenuse of right triangle is 6m more than twice the shortest side. If the third side is 2m less than the hypotenuse, find the sides of the triangle.

Solution:



Let the shortest side be x .

Then hypotenuse = $2x + 6$

Third side = $2x + 6 - 2 = 2x + 4$

According to Pythagoras theorem,

$$AB^2 = CB^2 + AC^2$$

$$(2x + 6)^2 = x^2 + (2x + 4)^2$$

$$4x^2 + 24x + 36 = x^2 + 4x^2 + 16x + 16$$

$$x^2 - 8x - 20 = 0$$

$$(x - 10)(x + 2) = 0$$

$$x - 10 = 0 \quad \text{or} \quad x + 2 = 0$$

$$x = 10 \quad \text{or} \quad x = -2$$

x cannot be negative.

So, shortest side is 10m.

$$\text{Hypotenuse} = 2x + 6$$

$$= 2 \times 10 + 6$$

$$= 20 + 6$$

$$= 26 \text{ m}$$

$$\text{Third side} = 2x + 4$$

$$= 2 \times 10 + 4$$

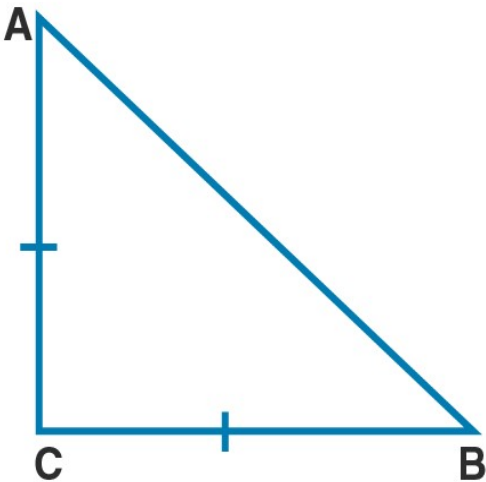
$$= 20 + 4$$

$$= 24 \text{ m}$$

Hence the shortest side, hypotenuse and third side of the triangle are 10m, 26m, and 24m respectively.

9. ABC is an isosceles triangle right angles at C. Prove that $AB^2 = 2AC^2$.

Solution:



Let ABC be the isosceles right angled triangle.

$$\angle C = 90^\circ$$

$$AC = BC \text{ [isosceles triangle]}$$

According to Pythagoras theorem,

$$AB^2 = BC^2 + AC^2$$

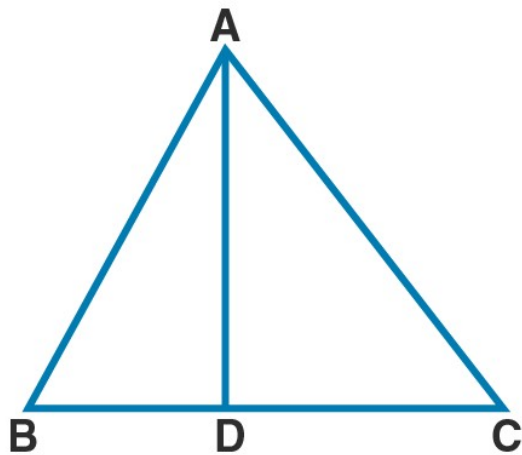
$$AB^2 = AC^2 + AC^2 \quad [\because AC = BC]$$

$$AB^2 = 2AC^2$$

Hence proved.

10. In a triangle ABC, AD is perpendicular to BC. Prove that $AB^2 + CD^2 = AC^2 + BD^2$.

Solution:



Given AD $\perp$ BC.

So ADB and ADC are right triangles.

In ADB,

$$AB^2 = AD^2 + BD^2 \quad [\text{Pythagoras theorem}]$$

$$AD^2 = AB^2 - BD^2 \quad \dots(i)$$

In ADC,

$$AC^2 = AD^2 + CD^2 \quad [\text{Pythagoras theorem}]$$

$$AD^2 = AC^2 - CD^2 \quad \dots(ii)$$

Comparing (i) and (ii)

$$AB^2 - BD^2 = AC^2 - CD^2$$

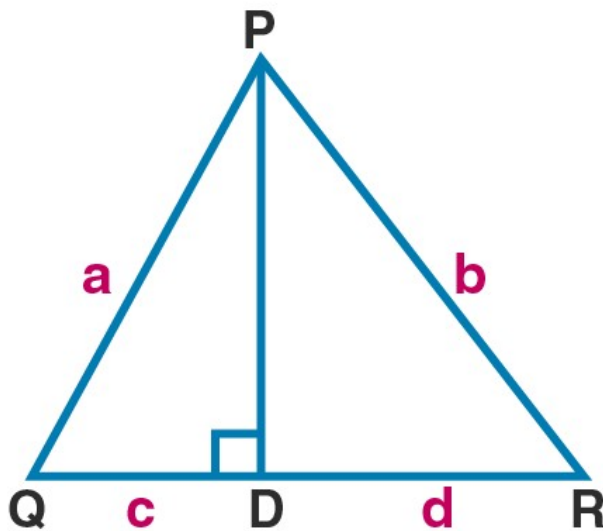
$$AB^2 + CD^2 = AC^2 + BD^2$$

Hence proved.

11. In $\triangle PQR$, $PD \perp QR$, such that D lies on QR. If $PQ = a$, $PR = b$, $QD = c$ and $DR = d$,

prove that $(a+b)(a-b) = (c+d)(c-d)$.

Solution:



Given $PQ = a$, $PR = b$, $QD = c$ and $DR = d$.

$PD \perp QR$.

So $\triangle PDQ$ and $\triangle PDR$ are right triangles.

In $\triangle PDQ$,

$$PQ^2 = PD^2 + QD^2 \text{ [Pythagoras theorem]}$$

$$PD^2 = PQ^2 - QD^2$$

$$PD^2 = a^2 - c^2 \dots(i) \quad [\because PQ = a \text{ and } QD = c]$$

In PDR,

$$PR^2 = PD^2 + DR^2 \quad [\text{Pythagoras theorem}]$$

$$PD^2 = PR^2 - DR^2$$

$$PD^2 = b^2 - d^2 \dots(ii) \quad [\because PR = b \text{ and } DR = d]$$

Comparing (i) and (ii)

$$a^2 - c^2 = b^2 - d^2$$

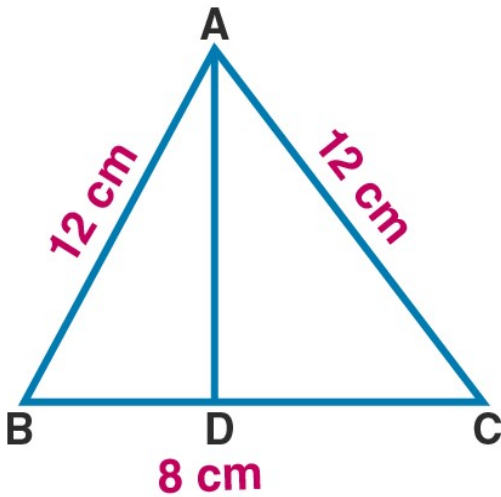
$$a^2 - b^2 = c^2 - d^2$$

$$(a + b)(a - b) = (c + d)(c - d)$$

Hence proved.

12. ABC is an isosceles triangle with AB = AC = 12cm and BC = 8cm. Find the altitude on BC and Hence, calculate its area.

Solution:



Let AD be the altitude of ABC.

Given $AB = AC = 12\text{cm}$

$BC = 8\text{cm}$

The altitude to the base of an isosceles triangle bisects the base.

So $BD = DC$

$$BD = \frac{8}{2} = 4\text{cm}$$

$$DC = 4\text{cm}$$

ADC is a right triangle.

$$AB^2 = BD^2 + AD^2 \text{ [Pythagoras theorem]}$$

$$AD^2 = 12^2 - 4^2$$

$$AD^2 = 144 - 16$$

$$AD^2 = 128$$

Taking square root on both sides,

$$AD = \sqrt{128} = \sqrt{2 \times 64} = 8\sqrt{2} \text{ cm}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 8 \times 8\sqrt{2}$$

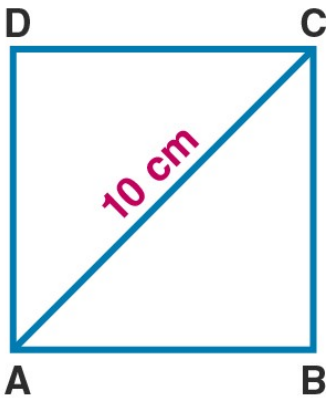
$$= 4 \times 8\sqrt{2}$$

$$= 32\sqrt{2} \text{ cm}^2$$

Hence the area of triangle is $32\sqrt{2} \text{ cm}^2$.

13. Find the area and the perimeter of a square whose diagonal is 10cm long.

Solution:



Given length of the diagonal of the square is 10cm.

$$AC = 10$$

Let $AB = BC = x$ [sides of square are equal in measure]

$\angle B = 90^\circ$ [all angles of a square are 90°]

ABC is a right triangle.

$$AC^2 = AB^2 + BC^2$$

$$10^2 = x^2 + x^2$$

$$100 = 2x^2$$

$$x^2 = \frac{100}{2}$$

$$x^2 = 50$$

$$x = \sqrt{50}$$

$$x = \sqrt{(25 \times 2)}$$

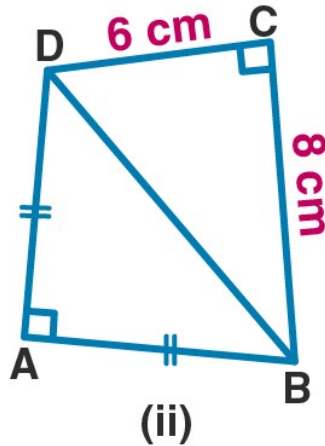
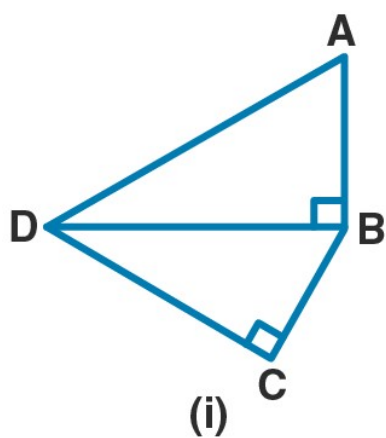
$$x = 4 \times 5\sqrt{2}$$

$$= 20\sqrt{2} \text{ cm}$$

Hence area and perimeter of the square are 50cm^2 and $20\sqrt{2}$ cm.

14. (a) In fig. (i) given below, ABCD is a quadrilateral in which AD = 13cm, DC = 12cm, BC = 3cm, $\angle ABD = \angle BCD = 90^\circ$. Calculate the length of AB.

(b) In fig. (ii) given below, ABCD is a quadrilateral in which AB = AD, $\angle A = 90^\circ = \angle C$, BC = 8cm and CD = 6cm. Find AB and calculate the area of $\triangle ABD$.



Solution:

(i) Given $AD = 13\text{cm}$, $DC = 12\text{m}$

$BC = 3\text{m}$

$\angle ABD = \angle BCD = 90^\circ$

BCD is a right triangle.

$BD^2 = BC^2 + DC^2$ [Pythagoras theorem]

$BD^2 = 3^2 + 12^2$

$BD^2 = 9 + 144$

$BD^2 = 153$

ABD is a right triangle.

$AD^2 = AB^2 + BD^2$ [Pythagoras theorem]

$13^2 = AB^2 + 153$

$169 = AB^2 + 153$

$AB^2 = 169 - 153$

$AB^2 = 16$

Taking square root on both sides,

$$AB = 4\text{cm}$$

Hence the length of AB is 4cm.

(ii) Given $AB = AD$, $\angle A = 90^\circ = \angle C$, $BC = 8\text{ cm}$ and $DC = 6\text{cm}$

BCD is a right triangle.

$$BD^2 = BC^2 + DC^2 \quad [\text{Pythagoras theorem}]$$

$$BD^2 = 8^2 + 6^2$$

$$BD^2 = 64 + 36$$

$$BD^2 = 100$$

Taking square root on both sides,

$$BD = 10\text{ cm}$$

ABD is a right triangle.

$$BD^2 = AB^2 + AD^2 \quad [\text{Pythagoras theorem}]$$

$$10^2 = 2AB^2 \quad [\because AB = AD]$$

$$100 = 2AB^2$$

$$AB^2 = \frac{100}{2}$$

$$AB^2 = 50$$

Taking square root on both sides,

$$AB = \sqrt{50}$$

$$AB = \sqrt{(2 \times 25)}$$

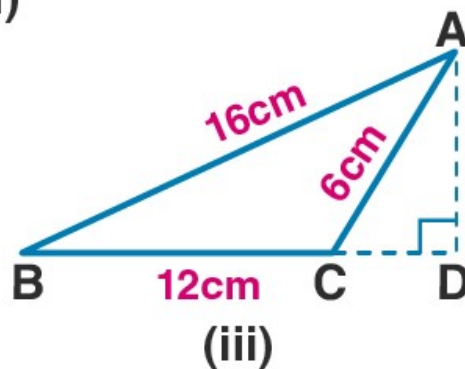
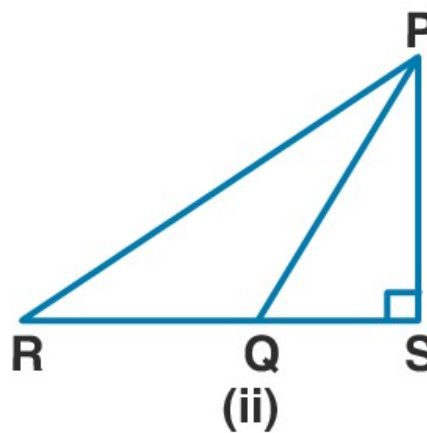
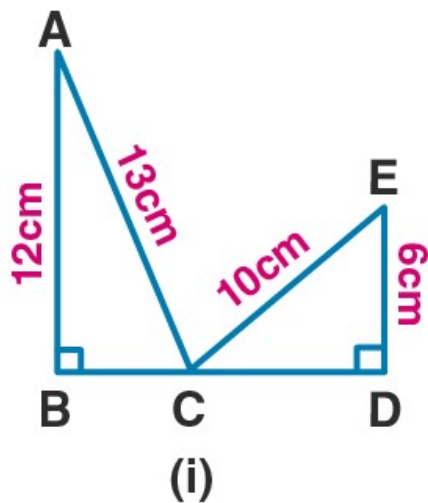
$$AB = 5\sqrt{2} \text{ cm}$$

Hence the length of AB is $5\sqrt{2}$ cm.

15. (a) In figure (i) given below, $AB = 12\text{cm}$, $AC = 13\text{cm}$, $CD = 10\text{cm}$ and $DE = 6\text{cm}$. Calculate the length of BD .

(b) In figure (ii) given below, $\angle PSR = 90^\circ$, $PQ = 10\text{ cm}$,

(c) In figure (iii) given below, $\angle D = 90^\circ$, $AB = 16\text{cm}$, $BC = 12\text{cm}$ and $CA = 6\text{cm}$, Find CD .



Solution :

(a) Given $AB = 12\text{cm}$, $AC = 13\text{cm}$, $CE = 10\text{ cm}$ and $DE = 6\text{cm}$

ABC is a right triangle.

$$AC^2 = AB^2 + BC^2 \text{ [Pythagoras theorem]}$$

$$13^2 = 12^2 + BC^2$$

$$BC^2 = 13^2 - 12^2$$

$$BC^2 = 169 - 144$$

$$BC^2 = 25$$

Taking square root on both sides,

$$BC = 5\text{ cm}$$

CDE is a right triangle.

$$CE^2 = CD^2 + DE^2 \text{ [Pythagoras theorem]}$$

$$10^2 = CD^2 + 6^2$$

$$100 = CD^2 + 36$$

$$CD^2 = 100 - 36$$

$$CD^2 = 64$$

Taking square root on both sides,

$$CD = 8\text{cm}$$

$$BD = BC + CD$$

$$BD = 5 + 8$$

$$BD = 13\text{ cm}$$

Hence the length of BD is 13 cm .

(b) Given $\angle PSR = 90^\circ$, $PQ = 10\text{cm}$, $QS = 6\text{cm}$ and $RQ = 9\text{cm}$

$\triangle PSQ$ is a right triangle.

$$PQ^2 = PS^2 + QS^2 \quad [\text{Pythagoras theorem}]$$

$$10^2 = PS^2 + 6^2$$

$$100 = PS^2 + 36$$

$$PS^2 = 100 - 36$$

$$PS^2 = 64$$

Taking square root on both sides,

$$PS = 8\text{cm}$$

$\triangle PSR$ is a right triangle.

$$RS = RQ + QS$$

$$RS = 9 + 6$$

$$RS = 15\text{ cm}$$

$$PR^2 = PS^2 + RS^2 \quad [\text{Pythagoras theorem}]$$

$$PR^2 = 8^2 + 15^2$$

$$PR^2 = 64 + 225$$

$$PR^2 = 289$$

Taking square root on both sides,

$$PR = 17\text{ cm}$$

Hence the length of PR is 17 cm .

(c) $\angle D = 90^\circ$, $AB = 16\text{cm}$, $BC = 12\text{cm}$ and $CA = 6\text{cm}$

$\triangle ADC$ is a right triangle.

$$AC^2 = AD^2 + CD^2 \quad [\text{Pythagoras theorem}]$$

$$6^2 = AD^2 + CD^2 \dots\dots (i)$$

$\triangle ABD$ is a right triangle.

$$AB^2 = AD^2 + BD^2 \quad [\text{Pythagoras theorem}]$$

$$16^2 = AD^2 + (BC + CD)^2$$

$$16^2 = AD^2 + (12 + CD)^2$$

$$256 = AD^2 + 144 + 24CD + CD^2$$

$$256 - 144 = AD^2 + CD^2 + 24CD$$

$$AD^2 + CD^2 = 112 - 24CD$$

$$6^2 = 112 - 24CD \quad [\text{from (i)}]$$

$$36 = 112 - 24CD$$

$$24CD = 112 - 36$$

$$24CD = 76$$

$$CD = \frac{76}{24} = \frac{19}{6}$$

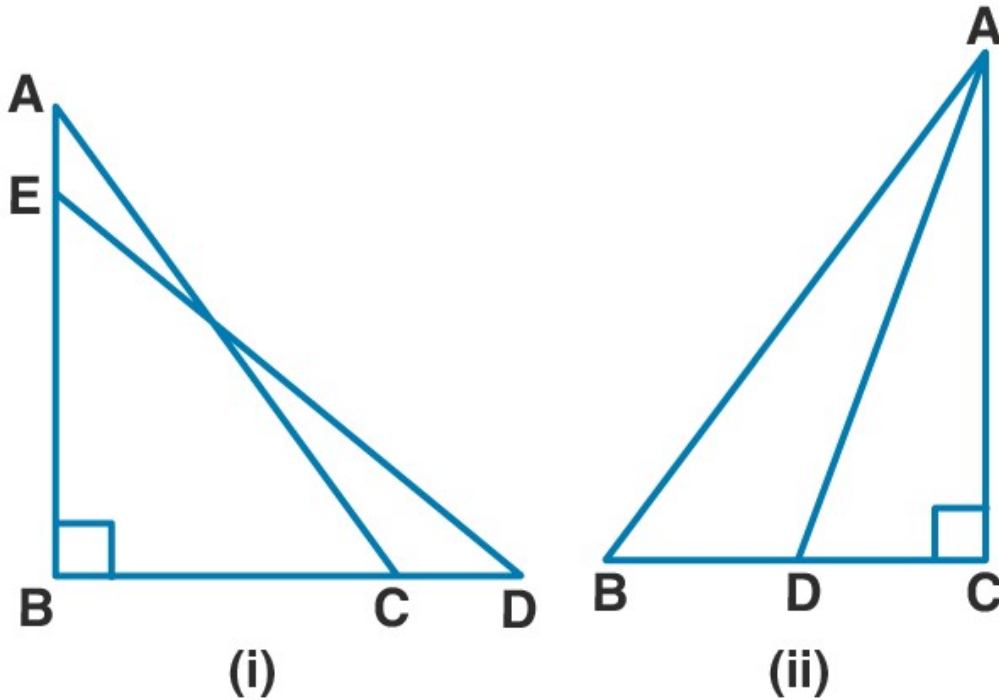
$$\therefore CD = 3\frac{1}{6}$$

Hence the length of CD is $3\frac{1}{6}\text{ cm}$

16. (a) In figure (i) given below, $BC = 5\text{cm}$,

$\angle B = 90^\circ$, $AB = 5AE$, $CD = 2AE$ and $AC = ED$. Calculate the lengths of EA , CD , AB and AC .

(b) In the figure (ii) given below, ABC is a right triangle right angles at C . If D is mid-point of BC , prove that $AB^2 = 4AD^2 - 3AC^2$.



Solution:

(a) Given $BC = 5\text{cm}$,

$\angle B = 90^\circ$, $AB = 5AE$,

$CD = 2AE$ and $AC = ED$

ABC is a right triangle.

$AC^2 = AB^2 + BC^2 \dots(i)$ [Pythagoras theorem]

BED is a right triangle.

$$ED^2 = BE^2 + BD^2 \quad [\text{Pythagoras theorem}]$$

$$AC^2 = BE^2 + BD^2 \dots(\text{ii}) \quad [\because AC = ED]$$

Comparing (i) and (ii)

$$AB^2 + BC^2 = BE^2 + BD^2$$

$$(5AE)^2 + 5^2 = (4AE)^2 + (BC + CD)^2 \quad [\because BE = AB - AE = 5AE - AE = 4AE]$$

$$(5AE)^2 + 25 = (4AE)^2 + (5 + 2AE)^2 \dots(\text{iii})$$

$$[\because BC = 5, CD = 2AE]$$

Let $AE = x$. So (iii) becomes,

$$(5x)^2 + 25 = (4x)^2 + (5 + 2x)^2$$

$$25x^2 + 25 = 16x^2 + 25 + 20x + 4x^2$$

$$25x^2 = 20x + 20x$$

$$5x^2 = 20x$$

$$x = \frac{20}{5} = 4$$

$$AE = 4\text{cm}$$

$$CD = 2AE = 2 \times 4 = 8\text{cm}$$

$$AB = 5AE$$

$$AB = 5 \times 4 = 20\text{cm}$$

ABC is a right triangle.

$$AC^2 = AB^2 + BC^2 \quad [\text{Pythagoras theorem}]$$

$$AC^2 = 20^2 + 5^2$$

$$AC^2 = 400 + 25$$

$$AC^2 = 425$$

Taking square root on both sides,

$$AC = \sqrt{425} = \sqrt{(25 \times 17)}$$

$$AC = 5\sqrt{17} \text{ cm}$$

Hence EA = 4cm, CD = 8cm, AB = 20cm and $AC = 5\sqrt{17} \text{ cm}$.

(b) Given D is the midpoint of BC.

$$DC = \frac{1}{2}BC$$

ABC is a right triangle.

$$AB^2 = AC^2 + BC^2 \dots(i) \text{ [Pythagoras theorem]}$$

ADC is a right triangle.

$$AD^2 = AC^2 + DC^2 \dots(ii) \text{ [Pythagoras theorem]}$$

$$AC^2 = AD^2 - DC^2$$

$$AC^2 = AD^2 - \left(\frac{1}{2} BC\right)^2 \quad [\because DC = \frac{1}{2} BC]$$

$$AC^2 = AD^2 - \frac{1}{4}BC^2$$

$$4AC^2 = 4AD^2 - BC^2$$

$$AC^2 + 3AC^2 = 4AD^2 - BC^2$$

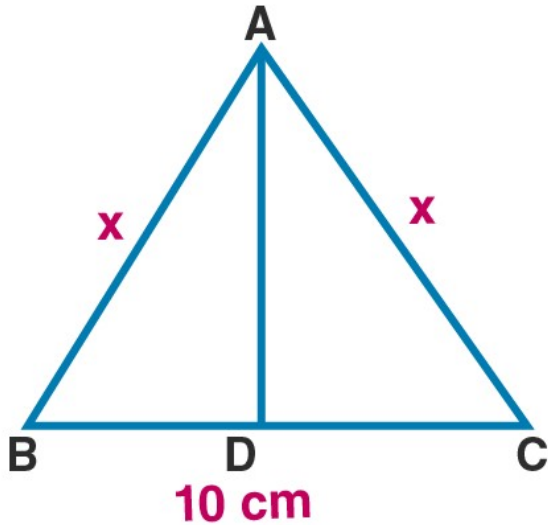
$$AC^2 + BC^2 = 4AD^2 - 3AC^2$$

$$AB^2 = 4AD^2 - 3AC^2 \text{ [from(i)]}$$

Hence proved.

17. In $\triangle ABC$, $AB = AC = x$, $BC = 10\text{cm}$ and the area of $\triangle ABC$ is 60cm^2 . Find x .

Solution:



Given $AB = AC = x$

So ABC is an isosceles triangle.

$AD \perp BC$

The altitude to the base of an isosceles triangle bisects the base.

$$BD = DC = \frac{10}{2} = 5\text{ cm}$$

Given area = 60 cm^2

$$\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 10 \times AD = 60$$

$$AD = 60 \times \frac{2}{10}$$

$$AD = \frac{60}{5}$$

$$AD = 12\text{cm}$$

ADC is a right triangle.

$$AC^2 = AD^2 + DC^2$$

$$x^2 = 12^2 + 5^2$$

$$x^2 = 144 + 25$$

$$x^2 = 169$$

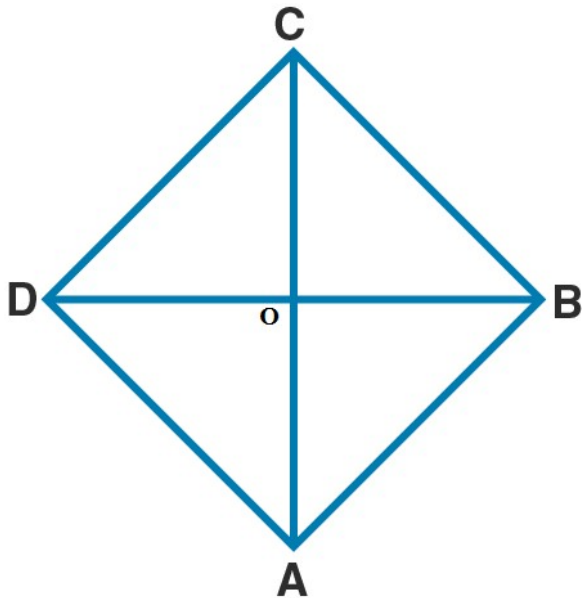
Taking square root on both sides

$$x = 13\text{cm}$$

Hence the value of x is 13 cm.

18. In a rhombus, if diagonals are 30cm and 40cm, find its perimeter.

Solution:



Let ABCD be the rhombus.

Given $AC = 30\text{cm}$

$BD = 40\text{ cm}$

Diagonals of a rhombus are perpendicular bisectors of each other.

$$OB = \frac{1}{2}BD = \frac{1}{2} \times 40 = 20\text{cm}$$

$$OC = \frac{1}{2}AC = \frac{1}{2} \times 30 = 15\text{cm}$$

OCB is a right triangle.

$$BC^2 = OC^2 + OB^2 \text{ [Pythagoras theorem]}$$

$$BC^2 = 15^2 + 20^2$$

$$BC^2 = 225 + 400$$

$$BC^2 = 625$$

Taking square root on both sides

$$BC = 25\text{cm}$$

So side of a rhombus, $a = 25\text{cm}$

$$\text{Perimeter} = 4a = 4 \times 25 = 100\text{ cm}$$

Hence the perimeter of the rhombus is 100 cm.

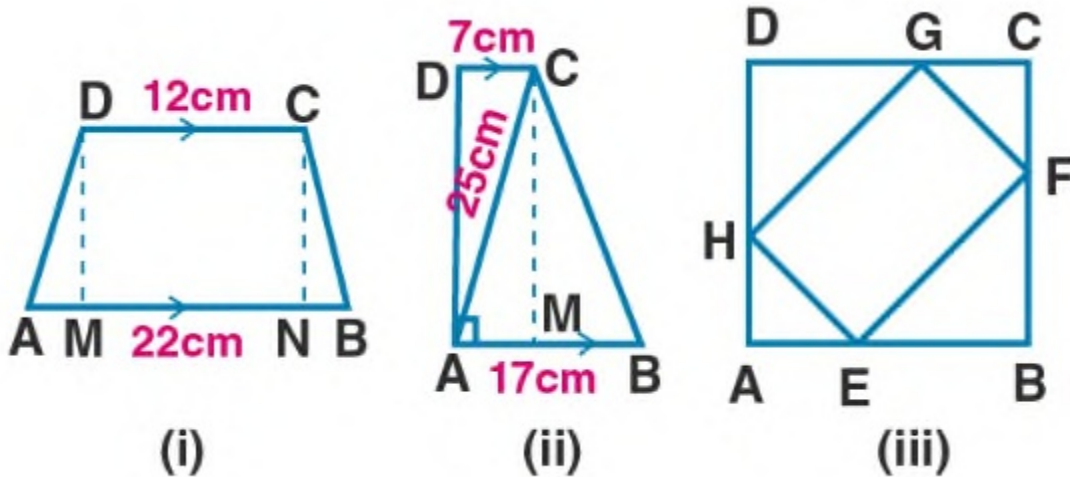
19. (a) In figure (i) given below, $AB \parallel DC$, $BC = AD = 13$ cm. $AB = 22$ cm and $DC = 12$ cm. Calculate the height of the trapezium ABCD.

(b) In figure (ii) given below, $AB \parallel DC$, $\angle A = 90^\circ$, $DC = 7$ cm, $AB = 17$ cm and $AC = 25$ cm. Calculate BC.

(c) In figure (iii) given below, ABCD is a square of side 7 cm. If $AE = FC = CG = HA = 3$ cm,

(i) Prove that EFGH is a rectangle.

(ii) find the area and perimeter of EFGH.



Solution:

(i) Given $AB \parallel DC$, $BC = AD = 13$ cm.

$AB = 22$ cm and $DC = 12$ cm

Here $DC = 12$

$MN = 12$ cm

$AM = BN$

$AB = AM + MN + BN$

$22 = AM + 12 + AM$ [$\because AM = BN$]

$$2AM = 22 - 12 = 10$$

$$AM = \frac{10}{2}$$

$$AM = 5\text{cm}$$

AMD is a right triangle.

$$AD^2 = AM^2 + DM^2 \text{ [Pythagoras theorem]}$$

$$13^2 = 5^2 + DM^2$$

$$DM^2 = 13^2 - 5^2$$

$$DM^2 = 169 - 25$$

$$DM^2 = 144$$

Taking square root on both sides,

$$DM = 12\text{cm}$$

Hence the height of the trapezium is 12 cm.

(b) Given $AB \parallel DC$, $\angle A = 90^\circ$, $DC = 7\text{cm}$, $AB = 17\text{cm}$ and $AC = 25\text{cm}$.

ADC is a right triangle.

$$AC^2 = AD^2 + DC^2 \text{ [Pythagoras theorem]}$$

$$25^2 = AD^2 + 7^2$$

$$AD^2 = 25^2 - 7^2$$

$$AD^2 = 625 - 49$$

$$AD^2 = 576$$

Taking square root on both sides

$$AD = 24\text{cm}$$

$$CM = 24 \text{ cm } [\because ABCD]$$

$$DC = 7\text{cm}$$

$$AM = 7 \text{ cm}$$

$$BM = AB - AM$$

$$BM = 17 - 7 = 10 \text{ cm}$$

BMC is a right triangle.

$$BC^2 = BM^2 + CM^2$$

$$BC^2 = 10^2 + 24^2$$

$$BC^2 = 100 + 576$$

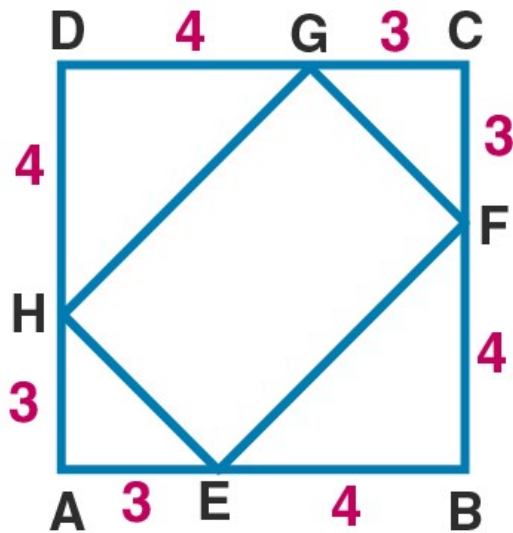
$$BC^2 = 676$$

Taking square root on both sides

$$BC = 26 \text{ cm}$$

Hence length of BC is 26 cm.

(c) (i) Proof:



Given ABCD is a square of side 7cm.

So $AB = BC = CD = AD = 7\text{cm}$

Also given $AE = FC = CG = HA = 3\text{cm}$

$BE = AB - AE = 7 - 3 = 4\text{cm}$

$BF = BC - FC = 7 - 3 = 4\text{ cm}$

$GD = CD - CG = 7 - 3 = 4\text{ cm}$

$DH - AD - HA = 7 - 3 = 4\text{ cm}$

$A = 90^\circ$ [Each angle of a square equals 90°]

AHE is a right triangle.

$HE^2 = AE^2 + AH^2$ [Pythagoras theorem]

$$HE^2 = 3^2 + 3^2$$

$$HE^2 = 9 + 9 = 18$$

$$HE = \sqrt{9 \times 2} = 3\sqrt{2}\text{ cm}$$

Similarly $GF = 3\sqrt{2}\text{ cm}$

EBF is a right triangle.

$$EF^2 = BE^2 + BF^2 \text{ [Pythagoras theorem]}$$

$$EF^2 = 4^2 + 4^2$$

$$EF^2 = 16 + 16 = 32$$

Taking square root on both sides

$$EF = \sqrt{16 \times 2} = 4\sqrt{2} \text{ cm}$$

$$\text{Similarly } HG = 4\sqrt{2} \text{ cm}$$

Now join EG

In EFG

$$EG^2 = EF^2 + GF^2$$

$$EG^2 = (4\sqrt{2})^2 + (3\sqrt{2})^2$$

$$EG^2 = 32 + 18 = 50$$

$$EG = \sqrt{50} = 5\sqrt{2} \text{ cm....(i)}$$

Join HF.

$$\text{Also } HF^2 = EH^2 + HG^2$$

$$= (3\sqrt{2})^2 + (4\sqrt{2})^2$$

$$= 18 + 32$$

$$= 50$$

$$HF = \sqrt{50}$$

$$= 5\sqrt{2} \text{ cm.....(ii)}$$

From (i) and (ii)

$$EG = HF$$

Diagonals of the quadrilateral are congruent.. So EFGH is a rectangle.

Hence proved.

(ii) Area of rectangle EFGH = length $\times$ *breadth*

$$= HE \times EF$$

$$= 3\sqrt{2} \times 4\sqrt{2}$$

$$= 24 \text{ cm}^2$$

Perimeter of rectangle EFGH = 2(length + breadth)

$$= 2 \times (4\sqrt{2} + 3\sqrt{2})$$

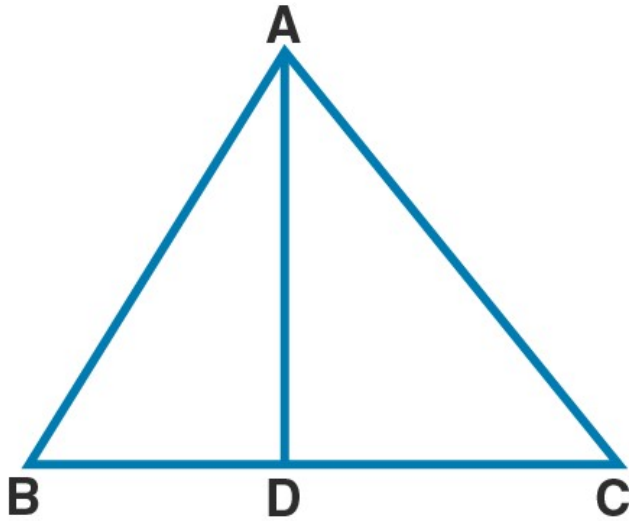
$$= 2 \times 7\sqrt{2}$$

$$= 14\sqrt{2} \text{ cm}$$

Hence area of the rectangle is 24cm^2 and perimeter is $14\sqrt{2}$ cm.

**20. AD is perpendicular to the side BC of an equilateral $\triangle ABC$.
Prove that $4AD^2 = 3AB^2$.**

Solution:



Given $AD \perp BC$

$\angle D = 90^\circ$

Proof :

Since $\triangle ABC$ is an equilateral triangle,

$$AB = AC = BC$$

$\triangle ABD$ is a right triangle.

According to Pythagoras theorem,

$$AB^2 = AD^2 + BD^2$$

$$BD = \frac{1}{2} BC$$

$$AB^2 = AD^2 + \left(\frac{1}{2} BC\right)^2$$

$$AB^2 = AD^2 + \left(\frac{1}{2}AB\right)^2 [\because BC = AB]$$

$$AB^2 = AD^2 + \frac{1}{4}AB^2$$

$$AB^2 = \frac{(4AD^2 + AB^2)}{4}$$

$$4AB^2 = 4AD^2 + AB^2$$

$$4AD^2 = 4AB^2 - AB^2$$

$$4AD^2 = 3AB^2$$

Hence proved.

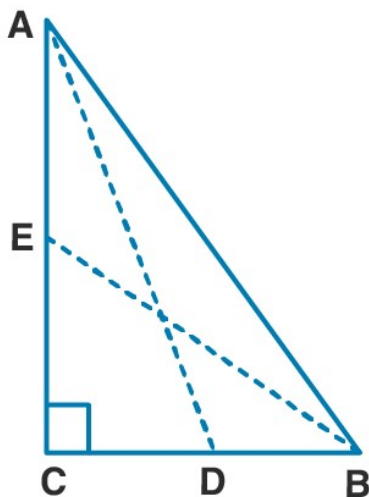
21. In figure (i) given below, D and E are mid-points of the sides BC and CA respectively of a $\triangle ABC$, right angles at C.

Prove that :

(i) $4AD^2 = 4AC^2 + BC^2$

(ii) $4BE^2 = 4BC^2 + AC^2$

(iii) $4(AD^2 + BE^2) = 5AB^2$



Solution:

Proof :

(i) $C = 90^\circ$

So ACD is a right triangle.

$$AD^2 = AC^2 + CD^2 \quad [\text{Pythagoras theorem}]$$

Multiply both sides by 4, we get

$$4AD^2 = 4AC^2 + 4CD^2$$

$$4AD^2 = 4AC^2 + 4BD^2$$

$$[\because D \text{ is the midpoint of } BC, CD = BD = \frac{1}{2} BC]$$

$$4AD^2 = 4AC^2 + (2BD)^2$$

$$4AD^2 = 4AC^2 + BC^2 \dots\dots(i) \quad [\because BC = 2BD]$$

Hence proved.

(ii) BCE is right triangle.

$$BE^2 = BC^2 + CE^2 \quad [\text{Pythagoras theorem}]$$

Multiply both sides by 4, we get

$$4BE^2 = 4BC^2 + 4CE^2$$

$$4BE^2 = 4BC^2 + (2CE)^2$$

$$4BE^2 = 4BC^2 + AC^2 \dots\dots(ii)$$

$$[\because E \text{ is the midpoint of } AC, AE = CE = \frac{1}{2} AC]$$

Hence proved.

(iii) Adding (i) and (ii)

$$4AD^2 + 4BE^2 = 4AC^2 + BC^2 + 4BC^2 + AC^2$$

$$4AD^2 + 4BE^2 = 5AC^2 + 5BC^2$$

$$4(AD^2 + BE^2) = 5(AC^2 + BC^2)$$

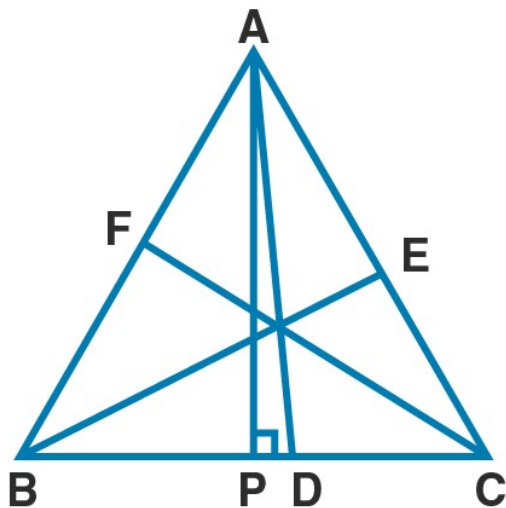
$$4(AD^2 + BE^2) = 5(AB^2)$$

$$[\because ABC \text{ is a right triangle, } AB^2 = AC^2 + BC^2]$$

Hence proved.

22. If AD, BE and CF are medians of ABC, prove that $3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$.

Solution:



Construction :

Draw APBC

Proof :

APB is a right triangle.

$$AB^2 = AP^2 + BP^2 \text{ [Pythagoras theorem]}$$

$$AB^2 = AP^2 + (BD - PD)^2$$

$$AB^2 = AP^2 + BD^2 + PD^2 - 2BD \times PD$$

$$AB^2 = (AP^2 + PD^2) + BD^2 - 2BD \times PD$$

$$AB^2 = AD^2 + \left(\frac{1}{2} BC\right)^2 - 2 \times \left(\frac{1}{2} BC\right) \times PD$$

$$\left[\because AP^2 + PD^2 = AD^2 \text{ and } BD = \frac{1}{2} BC \right]$$

$$AB^2 = AD^2 + \frac{1}{4} BC^2 - BC \times PD \dots\dots (i)$$

APC is a right triangle.

$$AC^2 = AP^2 + PC^2 \text{ [Pythagoras theorem]}$$

$$AC^2 = AP^2 + (PD^2 + DC^2)$$

$$AC^2 = AP^2 + PD^2 + DC^2 + 2 \times PD \times DC$$

$$AC^2 = (AP^2 + PD^2) + \left(\frac{1}{2} BC\right)^2 + 2 \times PD \times \left(\frac{1}{2} BC\right)$$

$$\left[DC = \frac{1}{2} BC \right]$$

$$AC^2 = AD^2 + \frac{1}{4} BC^2 + PD \times DC \dots(ii)$$

$$[\text{In APD, } AP^2 + PD^2 = AD^2]$$

Adding (i) and (ii), we get

$$AB^2 + AC^2 = 2AD^2 + \frac{1}{2} BC^2 \dots(iii)$$

Draw perpendicular from B and C to AC and AB respectively.

Similarly we get,

$$BC^2 + CA^2 = 2CF^2 + \frac{1}{2} AB^2 \dots\dots\dots(\text{iv})$$

$$AB^2 + BC^2 = 2BE^2 + \frac{1}{2} AC^2 \dots\dots\dots(\text{v})$$

Adding (iii), (iv) and (v) we get

$$2(AB^2 + BC^2 + CA^2) = 2(AD^2 + BE^2 + CF^2) + \frac{1}{2}(BC^2 + AB^2 + AC^2)$$

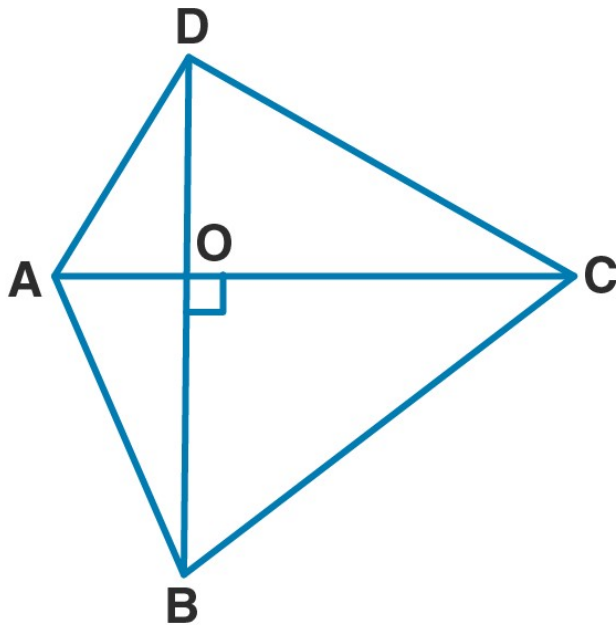
$$2(AB^2 + BC^2 + CA^2) = 2(AB^2 + BC^2 + CA^2) - \frac{1}{2}(AB^2 + BC^2 + CA^2)$$

$$2(AD^2 + BE^2 + CF^2) = \left(\frac{3}{2}\right) \times (AB^2 + BC^2 + CA^2)$$

$$4(AD^2 + BE^2 + CF^2) = 3(AB^2 + BC^2 + CA^2)$$

Hence proved.

23. (a) In fig. (i) given below, the diagonals AC and BD of a quadrilateral ABCD intersect at O, at right angles. Prove that $AB^2 + CD^2 = AD^2 + BC^2$.



Solution:

Given diagonals of quadrilateral ABCD, AC and BD intersect at O at right angles.

Proof :

AOB is a right triangle.

$$AB^2 = OB^2 + OA^2 \dots(i) \text{ [Pythagoras theorem]}$$

COD is a right triangle.

$$CD^2 = OC^2 + OD^2 \dots(ii) \quad \text{[Pythagoras theorem]}$$

Adding (i) and (ii), we get

$$AB^2 + CD^2 = OB^2 + OA^2 + OC^2 + OD^2$$

$$AB^2 + CD^2 = (OA^2 + OD^2) + (OC^2 + OB^2) \dots(iii)$$

AOD is a right triangle.

$$AD^2 = OA^2 + OD^2 \text{ ..(iv) [Pythagoras theorem]}$$

BOC is a right triangle.

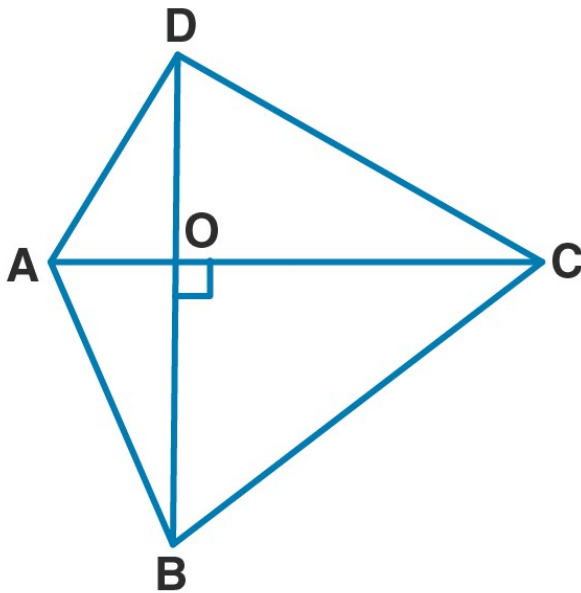
$$BC^2 = OC^2 + OB^2 \text{ ..(v) [Pythagoras theorem]}$$

Substitute (iv) and (v) in (iii), we get

$$AB^2 + CD^2 = AD^2 + BC^2$$

Hence proved.

23. (a) In fig. (i) given below, the diagonals AC and BD of a quadrilateral ABCD intersect at O, at right angles. Prove that $AB^2 + CD^2 = AD^2 + BC^2$.



Solution:

Given diagonals of quadrilateral ABCD, AC and BD intersect O at right angles.

Proof :

Given diagonals of quadrilateral ABCD, AC and BD intersect at O at right angles.

Proof :

AOB is a right triangle.

$$AB^2 = OB^2 + OA^2 \dots (i) \text{ [Pythagoras theorem]}$$

COD is a right triangle.

$$CD^2 = OC^2 + OD^2 \dots (ii) \text{ [Pythagoras theorem]}$$

Adding (i) and (ii), we get

$$AB^2 + CD^2 = OB^2 + OA^2 + OC^2 + OD^2$$

$$AB^2 + CD^2 = (OA^2 + OD^2) + (OC^2 + OB^2) \dots (iii)$$

AOD is a right triangle.

$$AD^2 = OA^2 + OD^2 \dots (iv) \text{ [Pythagoras theorem]}$$

BOC is a right triangle.

$$BC^2 = OC^2 + OB^2 \dots (v) \text{ [Pythagoras theorem]}$$

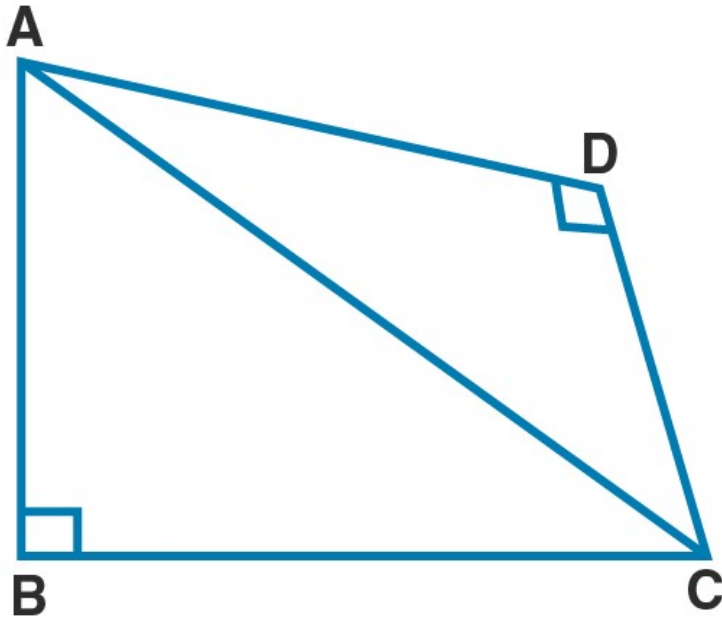
Substitute (iv) and (v) in (iii), we get

$$AB^2 + CD^2 = AD^2 + BC^2$$

Hence proved.

24. In a quadrilateral ABCD, $B = 90^\circ = D$. Prove that $2AC^2 - BC^2 = AB^2 + AD^2 + DC^2$.

Solution:



Given $B = D = 90^\circ$

So, ABC and ADC are right triangles.

In ABC,

$$AC^2 = AB^2 + BC^2 \dots(i) \text{ [Pythagoras theorem]}$$

In ADC,

$$AC^2 = AD^2 + DC^2 \dots(ii) \text{ [Pythagoras theorem]}$$

Adding (i) and (ii)

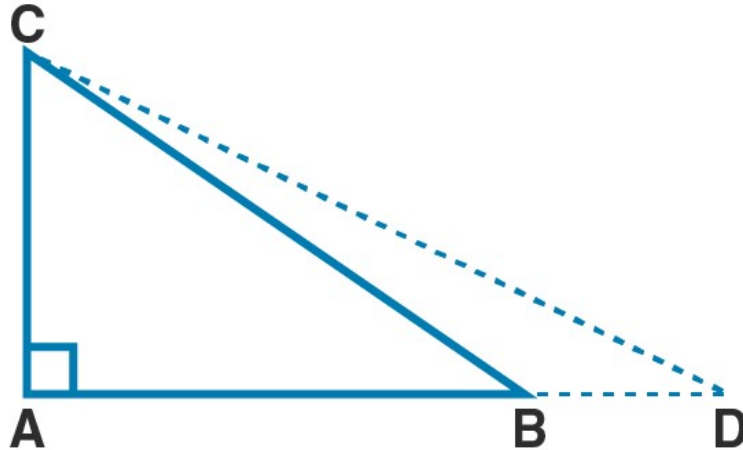
$$2AC^2 = AB^2 + BC^2 + AD^2 + DC^2$$

$$2AC^2 - BC^2 = AB^2 + AD^2 + DC^2$$

Hence proved.

25. In a $\triangle ABC$, $A = 90^\circ$, $CA = AB$ and D is a point on AB produced. Prove that : $DC^2 - BD^2 = 2AB \times AD$.

Solution:



Given $A = 90^\circ$

$CA = AB$

Proof :

in $\triangle ACD$,

$$DC^2 = CA^2 + AD^2 \text{ [Pythagoras theorem]}$$

$$DC^2 = CA^2 + (AB + BD)^2$$

$$DC^2 = CA^2 + AB^2 + BD^2 + 2AB \times BD$$

$$DC^2 - BD^2 = CA^2 + AB^2 + 2AB \times BD$$

$$DC^2 - BD^2 = AB^2 + AB^2 + 2AB \times BD \quad [\because CA = AB]$$

$$DC^2 - BD^2 = 2AB^2 + 2AB \times BD$$

$$DC^2 - BD^2 = 2AB (AB + BD)$$

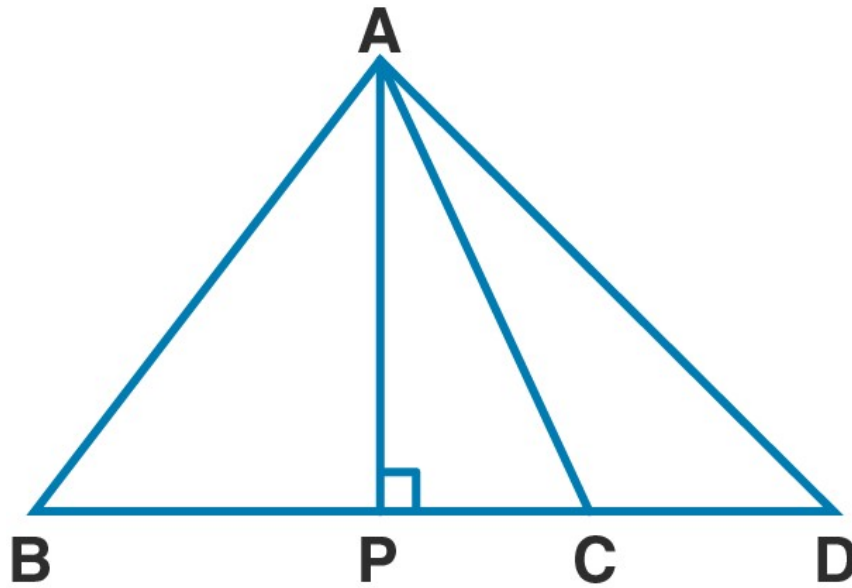
$$DC^2 - BD^2 = 2AB \times AD [A - B - D]$$

Hence proved.

26. In an isosceles triangle ABC, $AB = AC$ and D is a point on BC produced.

Prove that $AD^2 = AC^2 + BD \cdot CD$.

Solution:



Given ABC is an isosceles triangle.

$AB = AC$

Construction : Draw AP $\perp$ BC

Proof :

APD is a right triangle.

$$AD^2 = AP^2 + PD^2 \quad [\text{Pythagoras theorem}]$$

$$AD^2 = AP^2 + (PC + CD)^2 \quad [PD = PC + CD]$$

$$AD^2 = AP^2 + PC^2 + CD^2 + 2PC \times CD \quad \dots(i)$$

APC is a right triangle.

$$AC^2 = AP^2 + PC^2 \dots(ii) \text{ [Pythagoras theorem]}$$

Substitute (ii) in (i)

$$AD^2 = AC^2 + CD^2 + 2PC \times CD \dots (iii)$$

Since ABC is an isosceles triangle,

$$PC = \frac{1}{2} BC$$

[The altitude to the base of an isosceles triangle bisects the base]

$$AD^2 = AC^2 + CD^2 + 2 \times \frac{1}{2} BC \times CD$$

$$AD^2 = AC^2 + CD^2 + BC \times CD$$

$$AD^2 = AC^2 + CD(CD + BC)$$

$$AD^2 = AC^2 + CD \times BD \text{ [} CD + BC = BD \text{]}$$

$$AD^2 = AC^2 + BD \times CD$$

Hence proved.

Chapter Test

1. a) In fig. (i) given below, $AD \perp BC$, $AB = 25\text{ cm}$, $AC = 17\text{ cm}$ and $AD = 15\text{ cm}$. Find the length of BC .

b) In figure (ii) given below, $\angle BAC = 90^\circ$, $\angle ADC = 90^\circ$, $AD = 6\text{ cm}$, $CD = 8\text{ cm}$ and $BC = 26\text{ cm}$.

Find : (i) AC

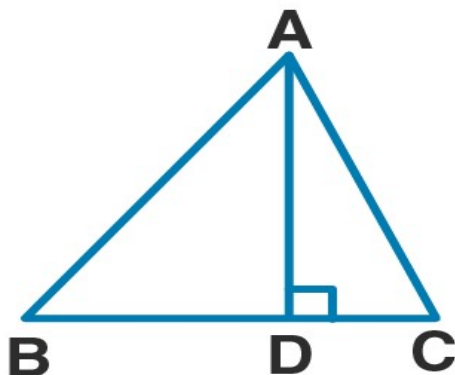
(ii) AB

(iii) area of the shaded region.

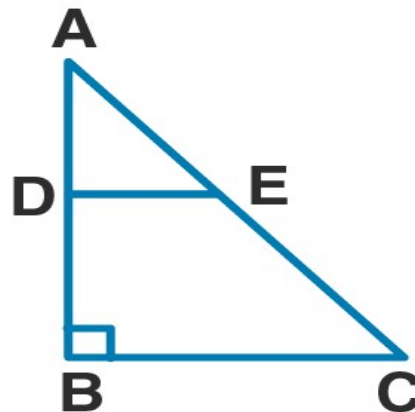
(c) In figure (iii) given below, triangle ABC is right angles at B . Given that $AB = 9\text{ cm}$, $AC = 15\text{ cm}$ and D, E are mid-points of the sides AB and AC respectively, calculate

(i) the length of BC

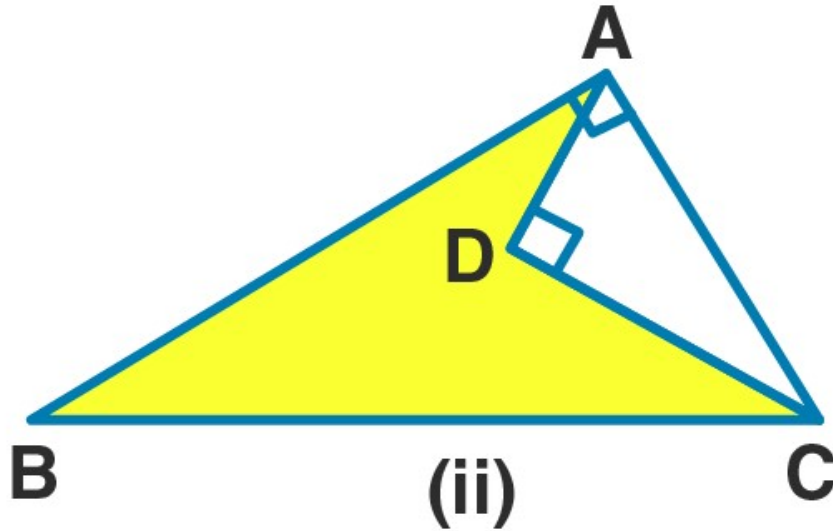
(ii) the area of $\triangle ADE$.



(i)



(iii)



Solution:

(a) Given $AD \perp BC$, $AB = 25\text{cm}$, $AC = 17\text{ cm}$ and $AD = 15\text{ cm}$

ADC is a right triangle.

$$AC^2 = AD^2 + DC^2 \text{ [Pythagoras theorem]}$$

$$17^2 = 15^2 + DC^2$$

$$289 = 225 + DC^2$$

$$DC^2 = 289 - 225$$

$$DC^2 = 64$$

Taking square root on both sides,

$$DC = 8\text{cm}$$

ADB is a right triangle.

$$AB^2 = AD^2 + BD^2 \text{ [Pythagoras theorem]}$$

$$25^2 = 15^2 + BD^2$$

$$625 = 225 + BD^2$$

$$BD^2 = 625 - 225 = 400$$

Taking square root on both sides,

$$BD = 20 \text{ cm}$$

$$BC = BD + DC$$

$$= 20 + 8$$

$$= 28 \text{ cm}$$

Hence the length of BC is 28 cm.

(b) Given $\angle BAC = 90^\circ$, $\angle ADC = 90^\circ$, $AD = 6\text{cm}$, $CD = 8\text{cm}$ and $BC = 26 \text{ cm}$.

(i) ADC is a right triangle.

$$AC^2 = AD^2 + DC^2 \text{ [Pythagoras theorem]}$$

$$AC^2 = 6^2 + 8^2$$

$$AC^2 = 36 + 64$$

$$AC^2 = 100$$

Taking square root on both sides,

$$AC = 10 \text{ cm}$$

Hence length of AC is 10 cm.

(ii) ABC is a right triangle.

$$BC^2 = AC^2 + AB^2 \text{ [Pythagoras theorem]}$$

$$26^2 = 10^2 + AB^2$$

$$AB^2 = 26^2 - 10^2$$

$$AB^2 = 676 - 100$$

$$AB^2 = 576$$

Taking square root on both sides,

$$AB = 24 \text{ cm}$$

Hence length of AB is 24 cm.

$$\text{(iii) Area of ABC} = \frac{1}{2} \times AB \times AC$$

$$= \frac{1}{2} \times 24 \times 10$$

$$= 120 \text{ cm}^2$$

$$\text{Area of ADC} = \frac{1}{2} \times AD \times DC$$

$$= \frac{1}{2} \times 6 \times 8$$

$$= 24 \text{ cm}^2$$

$$\begin{aligned}\text{Area of shaded region} &= \text{area of ABC} - \text{area of ADC} \\ &= 120 - 24 \\ &= 96 \text{ cm}^2\end{aligned}$$

Hence the area of shaded region is 96 cm^2 .

(c) Given $B = 90^\circ$

$AB = 9 \text{ cm}$, $AC = 15 \text{ cm}$.

D, E are mid- points of the sides AB and AC respectively.

(i) ABC is a right triangle.

$$AC^2 = AB^2 + BC^2 \text{ [Pythagoras theorem]}$$

$$15^2 = 9^2 + BC^2$$

$$225 = 81 + BC^2$$

$$BC^2 = 225 - 81$$

$$BC^2 = 144$$

Taking square root on both sides,

$$BC = 12 \text{ cm}$$

Hence the length of BC is 12 cm.

(ii) $AD = \frac{1}{2} AB$ [D is the midpoint of AB]

$$AD = \frac{1}{2} \times 9 = \frac{9}{2}$$

$AE = \frac{1}{2} AC$ [E is the midpoint of AC]

$$AE = \frac{1}{2} \times 15 = \frac{15}{2}$$

ADE is a right triangle.

$$AE^2 = AD^2 + DE^2 \text{ [Pythagoras theorem]}$$

$$\left(\frac{15}{2}\right)^2 = \left(\frac{9}{2}\right)^2 + DE^2$$

$$DE^2 = \left(\frac{15}{2}\right)^2 - \left(\frac{9}{2}\right)^2$$

$$DE^2 = \frac{225}{4} - \frac{81}{4}$$

$$DE^2 = \frac{144}{4}$$

Taking square root on both sides,

$$DE = \frac{12}{2} = 6 \text{ cm.}$$

$$\text{Area of ADE} = \frac{1}{2} \times DE \times AD$$

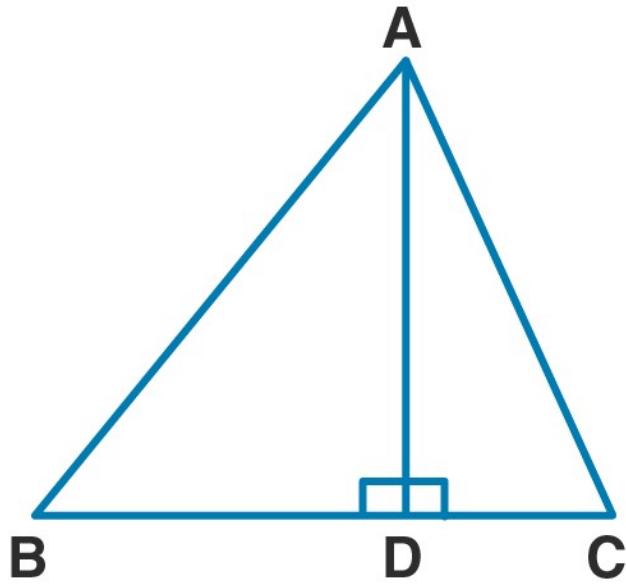
$$= \frac{1}{2} \times 6 \times \frac{9}{2}$$

$$= 13.5 \text{ cm}^2$$

Hence the area of the ADE is 13.5 cm^2 .

2. If in $\triangle ABC$, $AB > AC$ and $AD \perp BC$, prove that $AB^2 - AC^2 = BD^2 - CD^2$

Solution:



Given $AD \perp BC$, $AB > AC$

So $\triangle ADB$ and $\triangle ADC$ are right triangles.

Proof :

In $\triangle ADB$,

$$AB^2 = AD^2 + BD^2 \quad [\text{Pythagoras theorem}]$$

$$AD^2 = AB^2 - BD^2 \quad \text{..(i)}$$

In ADC,

$$AC^2 = AD^2 + CD^2 \text{ [Pythagoras theorem]}$$

$$AD^2 = AC^2 - CD^2 \quad \text{..(ii)}$$

Equation (i) and (ii)

$$AB^2 - BD^2 = AC^2 - CD^2$$

$$AB^2 - AC^2 = BD^2 - CD^2$$

Hence proved.

3. In a right angles triangle ABC, right angles at C, P and Q are the points on the sides CA and CB respectively which divide these sides in the ratio 2:1.

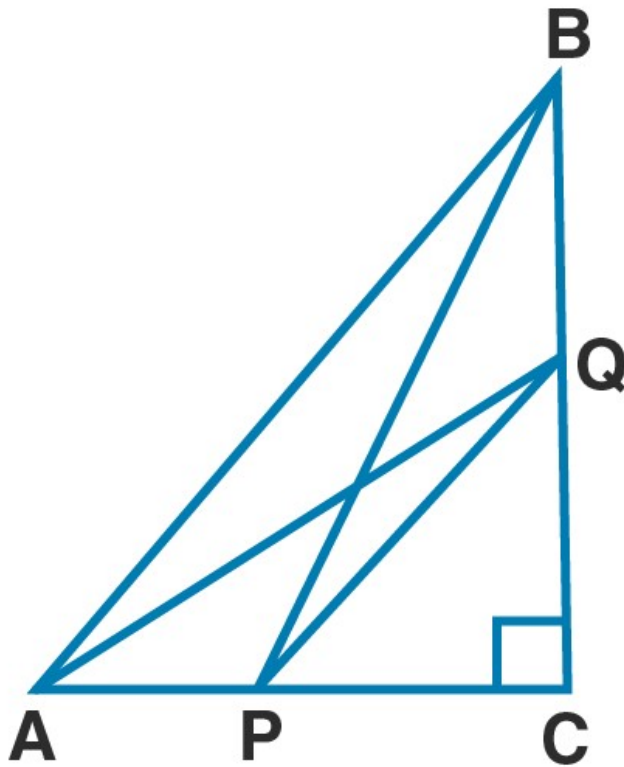
Prove that

$$\text{(i) } 9AQ^2 = 9AC^2 + 4BC^2$$

$$\text{(ii) } 9BP^2 = 9BC^2 + 4AC^2$$

$$\text{(iii) } 9(AQ^2 + BP^2) = 13AB^2$$

Solution:



Construction :

Join AQ and BP.

Given $C = 90^\circ$

Proof :

(i) In ACQ,

$$AQ^2 = AC^2 + CQ^2 \quad [\text{Pythagoras theorem}]$$

Multiplying both sides by 9, we get

$$9AQ^2 = 9AC^2 + 9CQ^2 \dots (i)$$

Given $BQ : CQ = 1 : 2$

$$\frac{CQ}{BC} = \frac{CQ}{(BQ+CQ)}$$

$$\frac{CQ}{BC} = \frac{2}{3}$$

$$3CQ = 2BC \dots(ii)$$

Substitute (ii) in (i)

$$9AQ^2 = 9AC^2 + (2BC)^2$$

$$9AQ^2 = 9AC^2 + 4BC^2 \dots(iii)$$

Hence proved.

(ii) In $\triangle BPC$,

$$BP^2 = BC^2 + CP^2 \quad [\text{Pythagoras theorem}]$$

Multiplying both sides by 9, we get

$$9BP^2 = 9BC^2 + 9CP^2$$

$$9BP^2 = 9BC^2 + (3CP)^2 \dots(iv)$$

Given $AP : PC = 1 : 2$

$$\frac{CP}{AC} = \frac{CP}{AP} + PC$$

$$\frac{CP}{AC} = \frac{2}{3}$$

$$3CP = 2AC \dots(v)$$

Substitute (v) in (iv)

$$9BP^2 = 9BC^2 + (2AC)^2$$

$$9BP^2 = 9BC^2 + 4AC^2 \dots(\text{vi})$$

Hence proved.

(iii) Adding (iii) and (vi), we get

$$9AQ^2 = 9BP^2 + 9AC^2 + 4BC^2 + 9BC^2 + 4AC^2$$

$$9(AQ^2 + BP^2) = 13(AC^2 + BC^2) \dots(\text{vii})$$

In ABC,

$$AB^2 = AC^2 + BC^2 \dots(\text{viii})$$

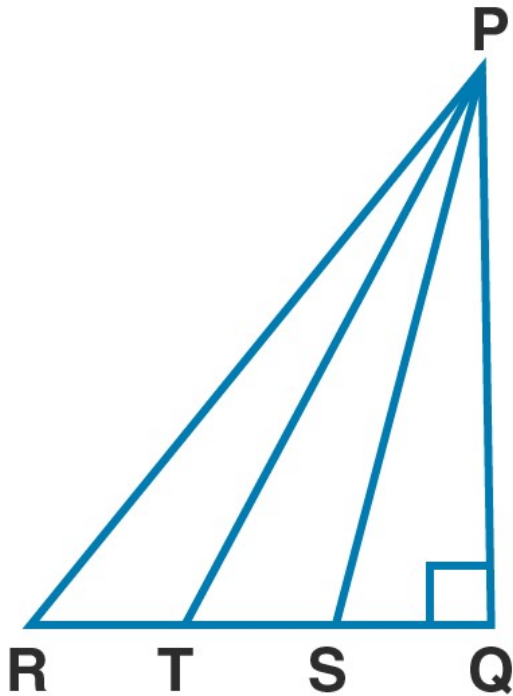
Substitute (viii) in (vii), we get

$$9(AQ^2 + BP^2) = 13AB^2$$

Hence proved.

4. In the given figure, ΔPQR is right angles at Q and points S and T trisect side QR. Prove that $8PT^2 = 3PR^2 + 5PS^2$.

Solution:



Given $\angle Q = 90^\circ$

S and T are points on RQ such that these points trisect it.

So $RT = TS = SQ$

To prove : $8PT^2 = 3PR^2 + 5PS^2$

Proof :

Let $RT = TS = SQ = x$

In $\triangle PQR$,

$$PR^2 = RQ^2 + PQ^2 \text{ [Pythagoras theorem]}$$

$$PR^2 = (3x)^2 + PQ^2$$

$$PR^2 = 9x^2 + PQ^2$$

Multiply above equation by 3

$$3PR^2 = 27x^2 + 3PQ^2 \dots\dots(i)$$

Similarly in PTs,

$$PT^2 = TQ^2 + PQ^2 \quad [\text{Pythagoras theorem}]$$

$$PT^2 = (2x)^2 + PQ^2$$

$$PT^2 = 4x^2 + PQ^2$$

Multiply above equation by 8

$$8PT^2 = 32x^2 + 8PQ^2 \dots\dots(ii)$$

Similarly in PSQ,

$$PS^2 = SQ^2 + PQ^2 \quad [\text{Pythagoras theorem}]$$

$$PS^2 = x^2 + PQ^2$$

Multiply above equation by 5

$$5PS^2 = 5x^2 + 5PQ^2 \dots\dots(iii)$$

Add (i) and (iii), we get

$$3PR^2 + 5PS^2 = 27x^2 + 3PQ^2 + 5x^2 + 5PQ^2$$

$$3PR^2 + 5PS^2 = 32x^2 + 8PQ^2$$

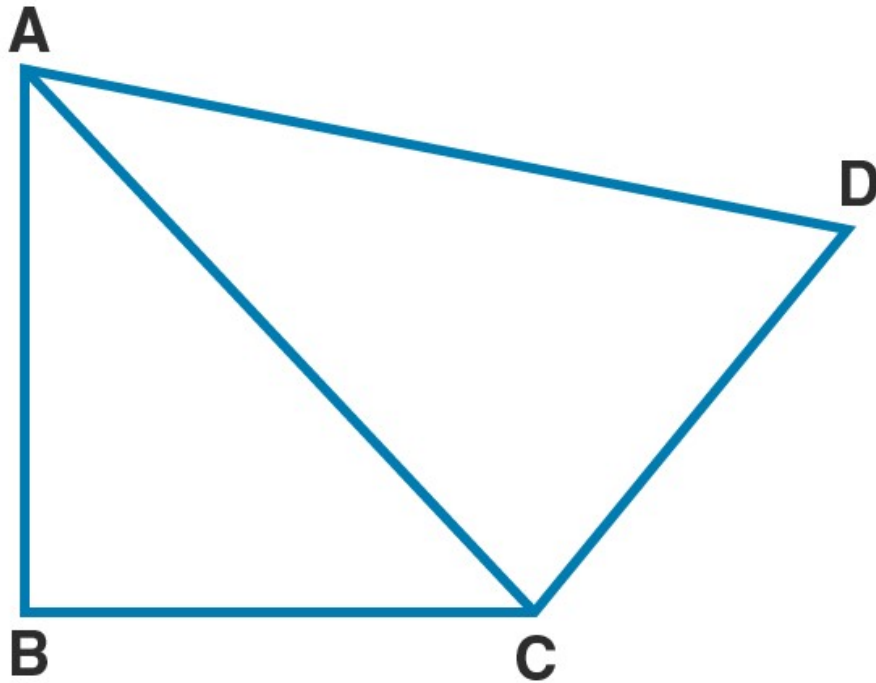
$$3PR^2 + 5PS^2 = 8PT^2 \quad [\text{from (ii)}]$$

$$8PT^2 = 3PR^2 + 5PS^2$$

Hence proved.

**5. In a quadrilateral ABCD, $B = 90^\circ$, if $AD^2 = AB^2 + BC^2 + CD^2$
To prove : $\angle ACD = 90^\circ$**

Solution:



Given : $B = 90^\circ$ in quadrilateral ABCD

$$AD^2 = AB^2 + BC^2 + CD^2$$

To prove : $\angle ACD = 90^\circ$

Proof :

In ABC,

$$AC^2 = AB^2 + BC^2 \text{ ..(i) [Pythagoras theorem]}$$

Given

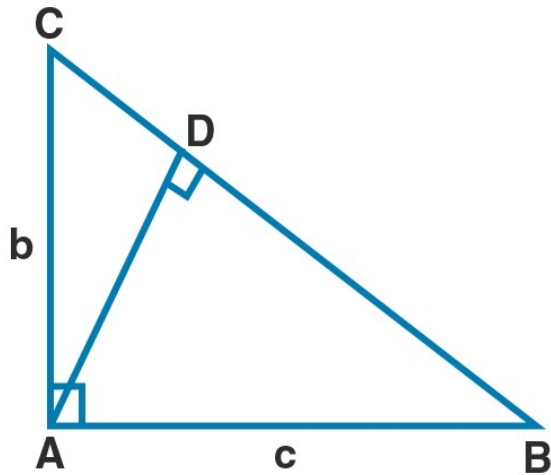
$$AD^2 = AB^2 + BC^2 + CD^2$$

$$AD^2 = AC^2 + CD^2 \text{ [from (i)]}$$

In $\triangle ACD$, $\angle ACD = 90^\circ$ [Converse of Pythagoras theorem]

Hence proved.

6. In the given figure, find the length of AD in terms of b and c.



Solution:

Given : $\angle A = 90^\circ$

$$AB = c$$

$$AC = b$$

$$\angle ADB = 90^\circ$$

In $\triangle ABC$,

$$BC^2 = AC^2 + AB^2 \quad [\text{Pythagoras theorem}]$$

$$BC^2 = b^2 + c^2$$

$$BC = \sqrt{b^2 + c^2} \quad \dots(i)$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \times AB \times AC$$

$$= \frac{1}{2} \times BC \dots (iii)$$

$$\text{Also, Area of } \triangle ABC = \frac{1}{2} \times BC \times AD$$

$$= \frac{1}{2} \times \sqrt{b^2 + c^2} \times AD \dots (iii)$$

Equating (ii) and (iii)

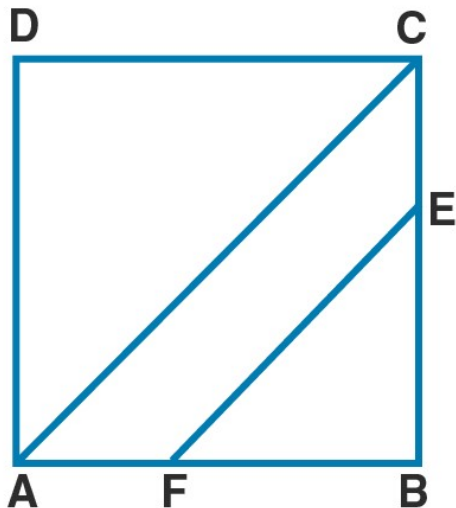
$$\frac{1}{2} \times bc = \frac{1}{2} \times (b^2 + c^2) \times AD$$

$$AD = \frac{bc}{(\sqrt{b^2 + c^2})}$$

$$\text{Hence AD is } \frac{bc}{(\sqrt{b^2 + c^2})}.$$

7. ABCD is a square, F is mid-point of AB and BE is one-third of BC. If area of $\triangle FBE$ is 108 cm^2 , find the length of AC.

Solution:



Let x be each side of the square ABCD.

$$FB = \frac{1}{2}AB \quad [\because F \text{ is the midpoint of } AB]$$

$$FB = \frac{1}{2}x \quad \dots(i)$$

$$BE = \left(\frac{1}{3}\right)BC$$

$$BE = \left(\frac{1}{3}\right)x \quad \dots(ii)$$

$$AC = \sqrt{2} \times \text{side} \quad [\text{Diagonal of a square}]$$

$$AC = \sqrt{2}x$$

$$\text{Area of FBE} = \frac{1}{2} FB \times BE$$

$$108 = \frac{1}{2} \times \frac{1}{2}x \times \left(\frac{1}{3}\right)x \quad [\text{given area of FBE} = 108\text{cm}^2]$$

$$108 = \left(\frac{1}{12}\right)x^2$$

$$x^2 = 108 \times 12$$

$$x^2 = 1296$$

Taking squares root on both sides.

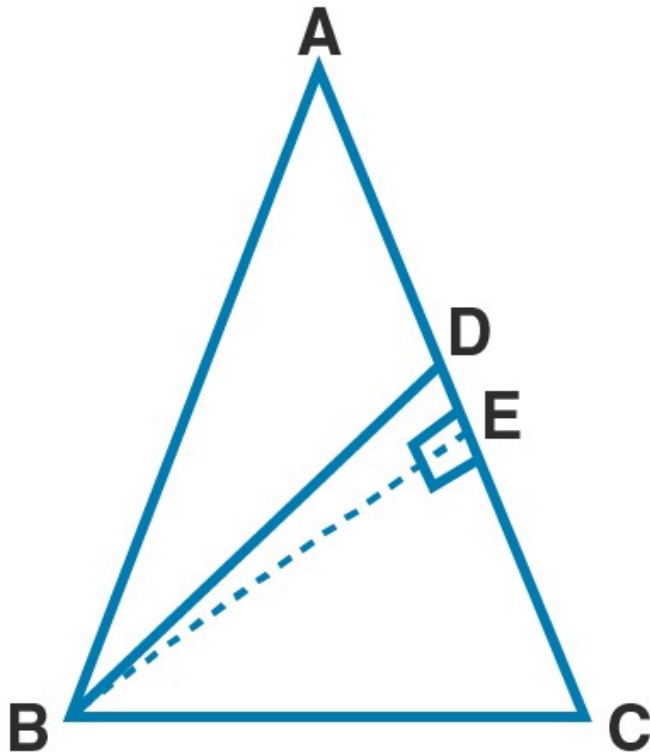
$$x = 36$$

$$AC = \sqrt{2} \times 36 = 36\sqrt{2}$$

Hence length of AC is $36\sqrt{2}$ cm.

8. In a triangles ABC, $AB = AC$ and D is a point on side AC such that $BC^2 = AC \times CD$, prove that $BD = BC$.

Solution:



Given : In ABC, $AB = AC$

D is a point on sides AC such that $BC^2 = AC \times CD$

To prove : $BD = BC$

Construction : Draw BEAC

Proof :

In BCE,

$$BC^2 = BE^2 + EC^2 \text{ [Pythagoras theorem]}$$

$$BC^2 = BE^2 + (AC - AE)^2$$

$$BC^2 = BE^2 + AC^2 + AE^2 - 2AC \times AE$$

$$BC^2 = BE^2 + AE^2 + AC^2 - 2AC \times AE \dots(i)$$

In ABC,

$$AB^2 = BE^2 + AE^2 \dots(ii)$$

Substitute (ii) in (i)

$$BC^2 = AB^2 + AC^2 - 2AC \times AE$$

$$BC^2 = AC^2 + AC^2 - 2AC \times AE \quad [\because AB = AC]$$

$$BC^2 = 2AC^2 - 2AC \times AE$$

$$BC^2 = 2AC (AC - AE)$$

$$BC^2 = 2AC \times EC$$

$$\text{Given } BC^2 = AC \times CD$$

$$2AC \times EC = AC \times CD$$

$$2EC = CD \dots(ii)$$

E is the midpoint of CD.

$$EC = DE \dots(iii)$$

In BED and BEC,

$$EC = DE \quad [\text{from (iii)}]$$

$$BE = BE \quad [\text{common side}]$$

$$\angle BED = \angle BEC$$

BED BEC [BY SAS congruency rule]

$$BD = BD \quad [\text{c.p.c.t.}]$$

Hence proved.