

CHAPTER 13
RECTILINEAR FIGURE

Exercise 13.1

- 1. If two angles of a quadrilateral are 40° and 110° and the other two are in the ratio 3: 4, find these angles.**

Solution:

We know that,

Sum of all four angles of a quadrilateral = 360°

Sum of two given angles = $40^\circ + 110^\circ = 150^\circ$

So, the sum of remaining two angles = $360^\circ - 150^\circ = 210^\circ$

Also given,

Ratio in these angles = 3: 4

Hence,

$$\text{Third angle} = \frac{(210 \times 3)}{(3+4)}$$

$$= \frac{(210 \times 3)}{7}$$

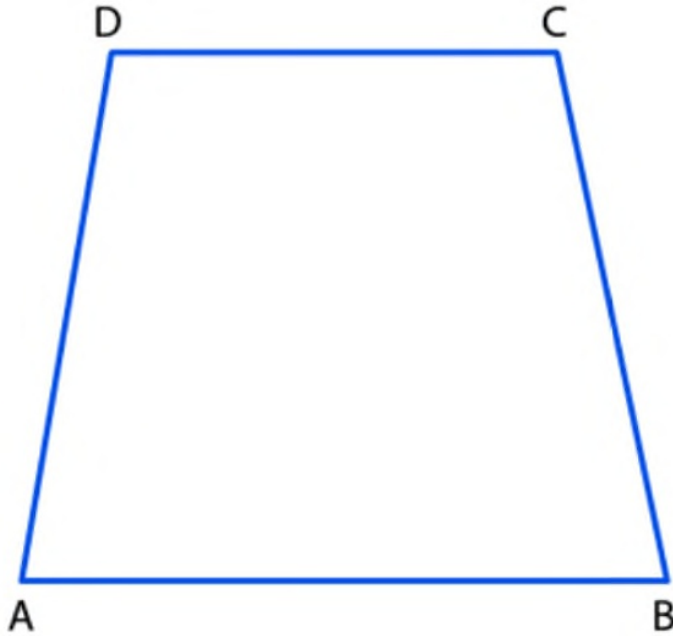
$$= 90^\circ$$

And,

$$\text{Fourth angle} = \frac{(210 \times 4)}{(3+4)}$$

$$= \frac{(210 \times 4)}{7}$$

$$= 120^\circ$$



2. If the angles of a quadrilateral, taken in order, are in the ratio 1: 2: 3: 4, Prove that it is a trapezium.

Solution:

Given,

In trapezium ABCD in which

$$\angle A : \angle B : \angle C : \angle D = 1 : 2 : 3 : 4$$

We know,

The sum of angles of the quad. ABCD = 360°

$$\angle A = \frac{(360^\circ \times 1)}{10} = 36^\circ$$

$$\angle B = \frac{(360^\circ \times 2)}{10} = 72^\circ$$

$$\angle C = \frac{(360^\circ \times 3)}{10} = 108^\circ$$

$$\angle D = \frac{(360^\circ \times 4)}{10} = 144^\circ$$

Now,

$$\angle A + \angle D = 36^\circ + 114^\circ = 180^\circ$$

Since the sum of angles $\angle A$ and $\angle D$ is 180° and these are co-interior angles

Thus, $AB \parallel DC$

Therefore, ABCD is a trapezium.

3. If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.

Solution:

Here ABCD is a parallelogram

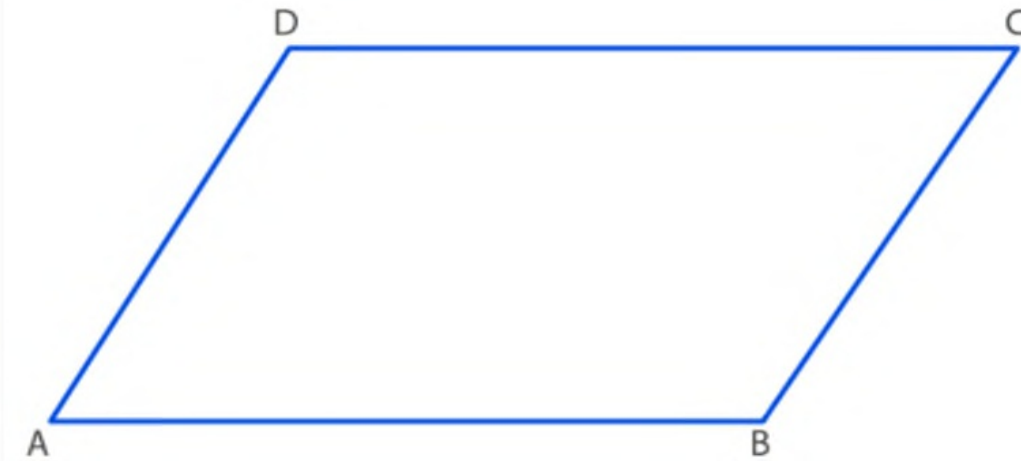
Let $\angle A = x^\circ$

Then, $\angle A = \left(\frac{2x}{3}\right)^\circ$ (Given condition)

So,

$$\angle A + \angle B = 180^\circ$$

(As the sum of adjacent angle in a parallelogram is 180°)



$$x^\circ + \frac{2}{3}x^\circ = 180^\circ$$

$$\Rightarrow \frac{3x + 2x}{3} = 180$$

$$\Rightarrow \frac{5x}{3} = 180$$

$$\Rightarrow 5x = 180 \times 3$$

$$\Rightarrow x = \frac{180 \times 3}{5}$$

$$\Rightarrow x = 36 \times 3$$

$$\Rightarrow x = 108$$

Hence, $\angle A = 108^\circ$

$$\angle B = \frac{2}{3} \times 108^\circ = 2 \times 36^\circ = 72^\circ$$

$\angle B = \angle D = 72^\circ$ (Opposite angle in a parallelogram is same)

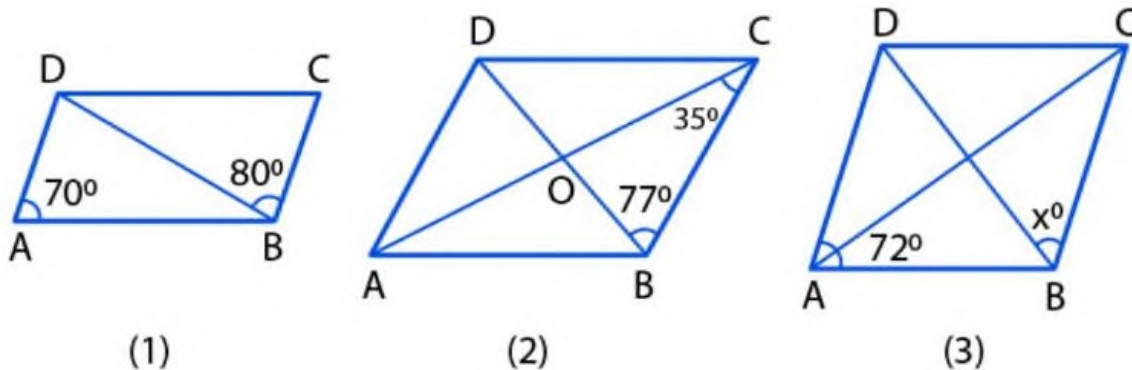
Also,

$\angle A = \angle C = 108^\circ$ (Opposite angle in a parallelogram is same)

Therefore, angles of parallelogram are 108° , 72° , 108° and 72° .

4.

- (a) In figure (1) given below, ABCD is a parallelogram in which $\angle DAB = 72^\circ$, $\angle DBC = 80^\circ$. Calculate angles CDB and ADB.
- (b) In figure (2) given below, ABCD is a parallelogram. Find the angles of the $\triangle AOD$.
- (c) In figure (3) given below, ABCD is a rhombus. Find the value of x .



Solution:

(a) Since, ABCD is a || gm

We have, $AB \parallel CD$

$$\angle ADB = \angle DBC \quad (\text{Alternate angles})$$

$$\angle ADB = 80^\circ \quad (\text{Given, } \angle DBC = 80^\circ)$$

Now,

In $\triangle ADB$, we have

$$\angle A + \angle ADB + \angle ABD = 180^\circ \quad (\text{Angle sum property of a triangle})$$

$$70^\circ + 80^\circ + \angle ABD = 180^\circ$$

$$150^\circ + \angle ABD = 180^\circ$$

$$\angle ABD = 180^\circ - 150^\circ = 30^\circ$$

Now, $\angle CDB = \angle ABD$ (Since, $AB \parallel CD$ and alternate angles)

So,

$$\angle CDB = 30^\circ$$

Hence, $\angle ADB = 80^\circ$ and $\angle CDB = 30^\circ$

(b) Given, $\angle BOC = 35^\circ$ and $\angle CBO = 77^\circ$

In $\triangle BOC$, we have

$$\angle BOC + \angle BCO + \angle CBO = 180^\circ \quad (\text{Angle sum property of a triangle})$$

$$\angle BOC = 180^\circ - 112^\circ = 68^\circ$$

Now, in $\parallel$ gm ABCD

We have,

$$\angle AOD = \angle BOC \text{ (Vertically opposite angles)}$$

$$\text{Hence, } \angle AOD = 68^\circ$$

(c) ABCD is a rhombus

So, $\angle A + \angle B = 180^\circ$ (Sum of adjacent angles of a rhombus is 180°)

$$72^\circ + \angle B = 180^\circ \quad (\text{Given, } \angle A = 72^\circ)$$

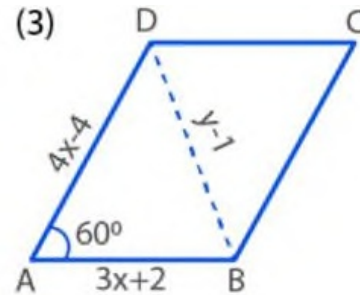
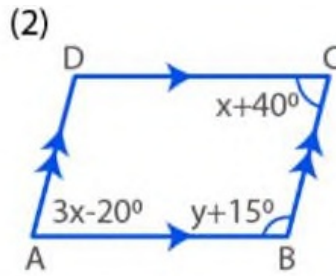
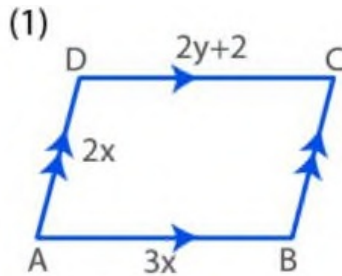
$$\angle B = 180^\circ - 72^\circ = 108^\circ$$

Hence,

$$x = \frac{1}{2} B = \frac{1}{2} \times 108^\circ = 54^\circ$$

5.

- (a) In figure (1) given below, ABCD is a parallelogram with perimeter 40. Find the values of x and y.
- (b) In figure (2) given below, ABCD is a parallelogram. Find the values of x and y.
- (c) In figure (3) given below. ABCD is a rhombus. Find x and y.



Solution:

(a) Since, ABCD is a parallelogram

So, $AB = CD$ and $BC = AD$

$$\Rightarrow 3x = 2y + 2$$

$$3x - 2y = 2 \quad \dots (i)$$

Also, $AB + BC + CD + DA = 40$

$$\Rightarrow 3x + 2x + 2y + 2 + 2x = 40$$

$$7x + 2y = 40 - 2$$

$$7x + 2y = 38 \quad \dots (ii)$$

Now, adding (i) and (ii) we get

$$3x - 2y = 2$$

$$7x + 2y = 38$$

$$10x = 40$$

$$\Rightarrow x = \frac{40}{10} = 4$$

On substituting the value of x in (i), we get

$$3(4) - 2y = 2$$

$$12 - 2y = 2$$

$$2y = 12 - 2$$

$$\Rightarrow y = \frac{10}{2} = 5$$

Hence, $x = 4$ and $y = 5$

(b) In parallelogram ABCD, we have

$\angle A = \angle C$ (Opposite angles are same in || gm)

$$3x - 20^\circ = x + 40^\circ$$

$$3x - x = 40^\circ + 20^\circ$$

$$2x = 60^\circ$$

$$x = \frac{60^\circ}{2} = 30^\circ \quad \dots (i)$$

Also, $\angle A + \angle B = 180^\circ$ (Sum of adjacent angles in || gm is equal to 180°)

$$3x - 20^\circ + y - 15^\circ = 180^\circ$$

$$3x + y = 180^\circ + 20^\circ - 15^\circ$$

$$3x + y = 185^\circ$$

$$3(30^\circ) + y = 185^\circ \quad [\text{Using (i)}]$$

$$90^\circ + y = 185^\circ$$

$$y = 185^\circ - 90^\circ = 95^\circ$$

Hence,

$$x = 30^\circ \text{ and } 95^\circ$$

(c) ABCD is a rhombus

So, $AB = CD$

$$3x + 2 = 4x - 4$$

$$3x - 4x = -4 - 2$$

$$-x = -6$$

$$x = 6$$

Now, in $\triangle ABD$ we have

$$\angle BAD = 60 \text{ and } AB = AD$$

$$\angle ADB = \angle ABD$$

So,

$$\angle ADB = \frac{(180^\circ - \angle BAD)}{2}$$

$$= \frac{(180^\circ - 60^\circ)}{2}$$

$$= \frac{120}{2} = 60^\circ$$

As $\triangle ABD$ is an equilateral triangle, all the angles of the triangle are 60°

Hence, $AB = BD$

$$3x + 2 = y - 1$$

$$3(6) + 2 = y - 1 \quad (\text{Substituting the value of } x)$$

$$18 + 2 = y - 1$$

$$20 = y - 1$$

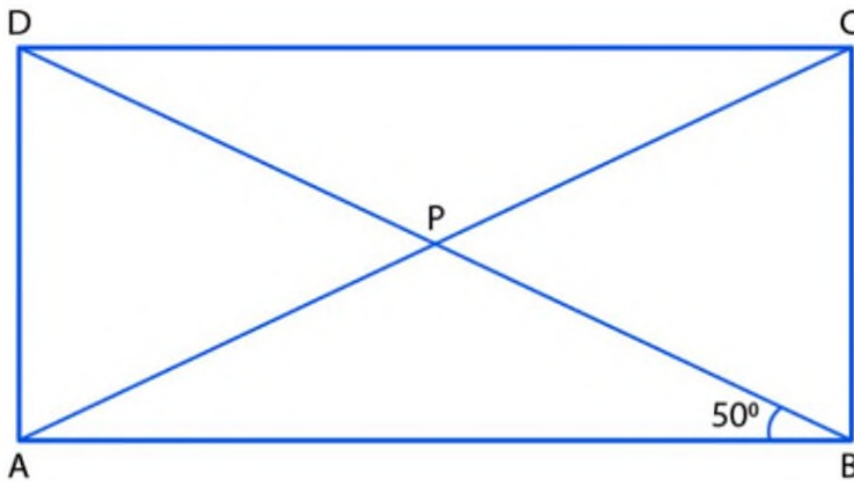
$$y = 20 + 1$$

$$y = 21$$

Thus, $x = 6$ and $y = 21$.

6. The diagonals AC and BD of a rectangle ABCD intersect each other at P. If $\angle ABD = 50^\circ$, find $\angle DPC$.

Solution:



Given, ABCD is a rectangle

We know that the diagonals of rectangle are same and bisect each other

So, we have

$$AP = BP$$

$$\angle PAB = \angle PBA \text{ (Equal sides have equal opposite angles)}$$

$$\angle PAB = 50^\circ \quad (\text{Since, given } \angle PBA = 50^\circ)$$

Now, in $\triangle APB$,

$$\angle APB + \angle ABP + \angle BAP = 180^\circ$$

$$\angle APB + 50^\circ + 50^\circ = 180^\circ$$

$$\angle APB = 180^\circ - 100^\circ$$

$$\angle APB = 80^\circ$$

Then,

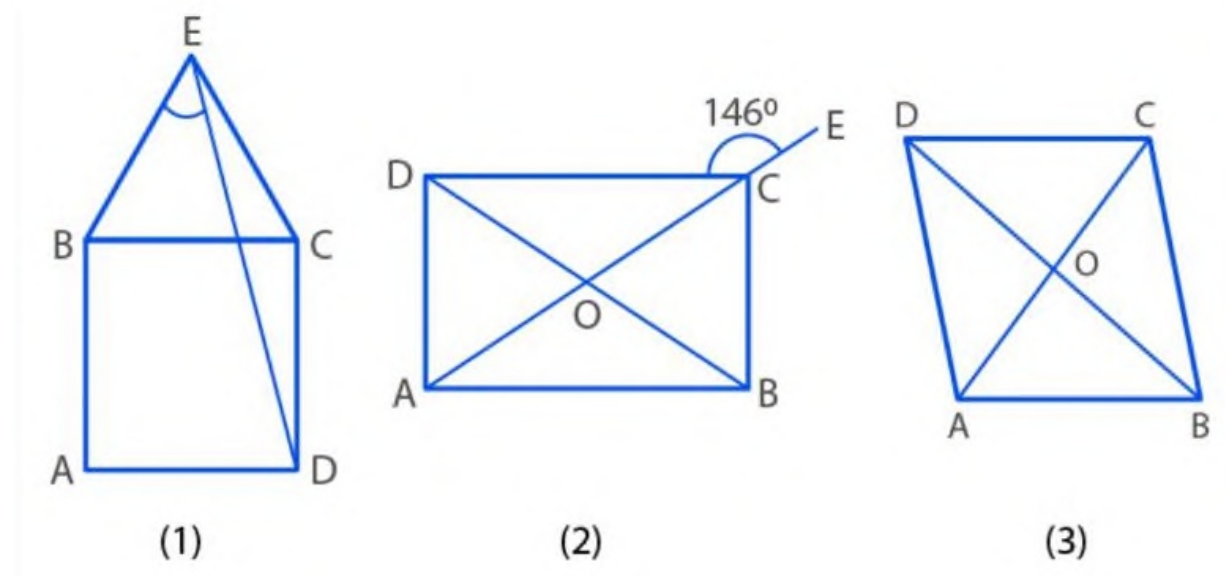
$$\angle DPB = \angle APB \quad (\text{Vertically opposite angles})$$

Hence,

$$\angle DPB = 80^\circ$$

7.

- (a) In figure (1) given below, equilateral triangle EBC surmounts square ABCD. Find angle Bed represented by x.
- (b) In figure, (2) given below, ABCD is a rectangle and diagonals intersect at O. AC is produced to E. If $\angle ECD = 146^\circ$. Find the angles of the $\triangle AOB$.
- (c) If figure (3) given below, ABCD is rhombus and diagonals intersect at O. If $\angle OAB : \angle OBA = 3 : 2$, find the angles of the $\angle AOD$.



Solution:

(a) Since, EBC is an equilateral triangle, we have

$$EB = BC = EC \quad \dots (i)$$

Also, ABCD is a square

$$\text{So, } AB = BC = CD = AD \quad \dots (ii)$$

From (i) and (ii), we get

$$EB = EC = AB = BC = CD = AD \quad \dots (iii)$$

Now, in $\triangle ECD$

$$\angle ECD = \angle BCD + \angle ECB$$

$$= 90^\circ + 60^\circ$$

$$= 150^\circ \quad \dots (iv)$$

Also, $EC = CD$ [From (iii)]

$$\text{So, } \angle DEC = \angle CDE \quad \dots (v)$$

$$\angle ECD + \angle DEC + \angle CDE = 180^\circ \quad [\text{Angles sum property of a triangle}]$$

$$150^\circ + \angle DEC + \angle DEC = 180^\circ \quad [\text{Using (iv) and (v)}]$$

$$2\angle DEC = 180^\circ - 150^\circ = 30^\circ$$

$$\angle DEC = \frac{30^\circ}{2}$$

$$\angle DEC = 15^\circ \quad \dots (vi)$$

Now, $\angle BEC = 60^\circ$ [BEC is an equilateral triangle]

$$\angle BEC + \angle DEC = 60^\circ$$

$$x^\circ + 15^\circ = 60^\circ \quad [\text{From (vi)}]$$

$$x = 60^\circ - 15^\circ$$

$$x = 45^\circ$$

Hence, the value of x is 45° .

(b) Given, ABCD is a rectangle

$$\angle ECD = 146^\circ$$

As ACE is a straight line, we have

$$146^\circ + \angle ACD = 180^\circ \quad [\text{Linear pair}]$$

$$\angle ACD = 180^\circ - 146^\circ = 34^\circ \quad \dots (i)$$

$$\text{And, } \angle CAB = \angle ACD \quad [\text{Alternate angles}] \quad \dots (ii)$$

From (i) and (ii), we have

$$\angle CAB = 34^\circ \Rightarrow \angle OAB = 34^\circ \quad \dots (iii)$$

In $\triangle AOB$

$AO = OB$ [Diagonals of a rectangle are equal and bisect each other]

$$\angle OAB = \angle OBA \quad \dots (iv)$$

[Equal sides have equal angles opposite to them]

From (iii) and (iv),

$$\angle OAB = 34^\circ \quad \dots (v)$$

Now,

$$\angle AOB + \angle OBA + \angle OAB = 180^\circ$$

$$\angle AOB + 34^\circ + 34^\circ = 180^\circ \quad [\text{Using (3) and (5)}]$$

$$\angle AOB + 68^\circ = 180^\circ$$

$$\angle AOB = 180^\circ - 68^\circ = 112^\circ$$

Hence, $\angle AOB = 112^\circ$, $\angle OAB = 34^\circ$ and $\angle OBA = 34^\circ$

- (c) Here, ABCD is a rhombus and diagonals intersect at O and $\angle OAB$:
 $\angle OBA = 3:2$.

Let $\angle OAB = 2x^\circ$

Then,

$$\angle OBA = 3x^\circ$$

We know that diagonals of rhombus intersect at right angle.

$$\text{So, } \angle AOB = 90^\circ$$

Now, in $\triangle AOB$

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$90^\circ + 3x^\circ + 2x^\circ = 180^\circ$$

$$90^\circ + 5x^\circ = 180^\circ$$

$$5x^\circ = 180^\circ - 90^\circ = 90^\circ$$

$$x^\circ = \frac{90^\circ}{5} = 18^\circ$$

Hence,

$$\angle OAB = 2x^\circ = 2 \times 18^\circ = 36^\circ$$

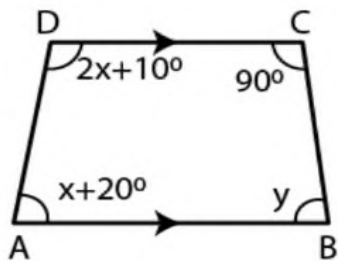
$$\angle OBA = 3x^\circ = 3 \times 18^\circ = 54^\circ \text{ and}$$

$$\angle AOB = 90^\circ$$

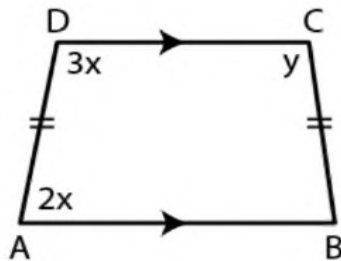
8.

- (a) In figure (1) given below, ABCD is a trapezium. Find the values of x and y.
- (b) In figure (2) given below, ABCD is an isosceles trapezium. Find the values of x and y.

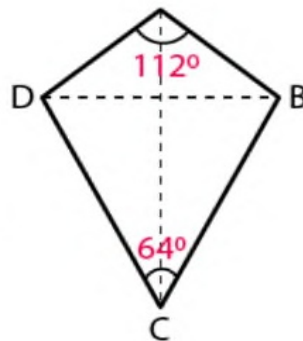
- (c) In figure (3) given below, ABCD is a kite and diagonals intersect at O. If $\angle DAB = 112^\circ$ and $\angle DCB = 64^\circ$, find $\angle ODC$ and $\angle OBA$.



(1)



(2)



(3)

Solution:

- (a) Given: ABCD is a trapezium

$$\angle A = x + 20^\circ, \angle B = y, \angle C = 92^\circ, \angle D = 2x + 10^\circ$$

We have,

$$\angle B + \angle C = 180^\circ \quad [\text{Since } AB \parallel DC]$$

$$y + 92^\circ = 180^\circ$$

$$y = 180^\circ - 92^\circ = 88^\circ$$

$$\text{Also, } \angle A + \angle D = 180^\circ$$

$$x + 20^\circ + 2x + 10^\circ = 180^\circ$$

$$3x + 30^\circ = 180^\circ$$

$$3x = 180^\circ - 30^\circ = 150^\circ$$

$$x = \frac{150^\circ}{3} = 50^\circ$$

Hence, the value of $x = 50^\circ$ and $y = 88^\circ$.

- (b) Given: ABCD is a isosceles trapezium $BC = AD$

$$\angle A = 2x, \angle C = y, \angle D = 3x$$

Since, ABCD is a trapezium and $AB \parallel DC$

$$\Rightarrow \angle A + \angle D = 180^\circ$$

$$2x + 3x = 180^\circ$$

$$5x = 180^\circ$$

$$x = \frac{180^\circ}{5} = 36^\circ \quad \dots (i)$$

Also, $AB = BC$ and $AB \parallel DC$

So, $\angle A + \angle C = 180^\circ$

$$2x + y = 180^\circ$$

$$2 \times 36^\circ + y = 180^\circ$$

$$72^\circ + y = 180^\circ$$

$$y = 180^\circ - 72^\circ = 108^\circ$$

Hence, value of $x = 36^\circ$ and $y = 108^\circ$.

(c) Given: ABCD is a kite and diagonal intersect at O.

$$\angle DAB = 112^\circ \text{ and } \angle DCB = 64^\circ$$

As AC is the diagonal of kite ABCD, we have

$$\angle DCO = \frac{64}{2} = 32^\circ$$

And, $\angle DOC = 90^\circ$ [Diagonal of kites bisect at right angles]

In $\triangle OCD$, we have

$$\angle ODC = 180^\circ - (\angle DCO + \angle DOC)$$

$$= 180^\circ - (32^\circ + 90^\circ)$$

$$= 180^\circ - 122^\circ$$

$$= 58^\circ$$

In $\triangle DAB$, we have

$$\angle OAB = \frac{112}{2} = 56^\circ$$

$$\angle AOB = 90^\circ \quad [\text{Diagonal of kites bisect at right angles}]$$

In $\triangle OAB$, we have

$$\angle OBA = 180^\circ - (\angle OAB + \angle AOB)$$

$$= 180^\circ - (56^\circ + 90^\circ)$$

$$= 180^\circ - 146^\circ$$

$$= 34^\circ$$

Hence, $\angle ODC = 58^\circ$ and $\angle OBA = 34^\circ$.

9.

- (i) Prove that each angle of a rectangle is 90° .**
- (ii) If the angle of a quadrilateral are equal, prove that it is a rectangle.**
- (iii) If the diagonals of a rhombus are equal, prove that it is a rectangle.**
- (iv) If the diagonals of a rhombus are equal, prove that it is a square.**
- (v) Prove that every diagonal of a rhombus bisects the angles at the vertices.**

Solution:

- (i) Given: ABCD is a rectangle**



To prove: Each angles of rectangle = 90°

Proof:

In a rectangle opposite angles of a rectangle are equal

So, $\angle A = \angle C$ and $\angle B = \angle C$

But, $\angle A + \angle B + \angle C + \angle D = 360^\circ$ [Sum of angles of a quadrilateral]

$$\angle A + \angle B + \angle A + \angle B = 360^\circ$$

$$2 (\angle A + \angle B) = 360^\circ$$

$$(\angle A + \angle B) = 360^\circ$$

$$\angle A + \angle B = \frac{360}{2}$$

$$\angle A + \angle B = 180^\circ$$

But, $\angle A = \angle B$ [Angles of a rectangle]

So, $\angle A = \angle B = 90^\circ$

Thus,

$$\angle A = \angle B = \angle C = \angle D = 90^\circ$$

Hence, each angle of a rectangle is 90° .



(ii) Given: In quadrilateral ABCD,

We have

$$\angle A = \angle B = \angle C = \angle D$$

To prove: ABCD is a rectangle

Proof:

$$\angle A = \angle B = \angle C = \angle D$$

$$\Rightarrow \angle A = \angle C \text{ and } \angle B = \angle D$$

But these are opposite angles of the quadrilateral

So, ABCD is a parallelogram

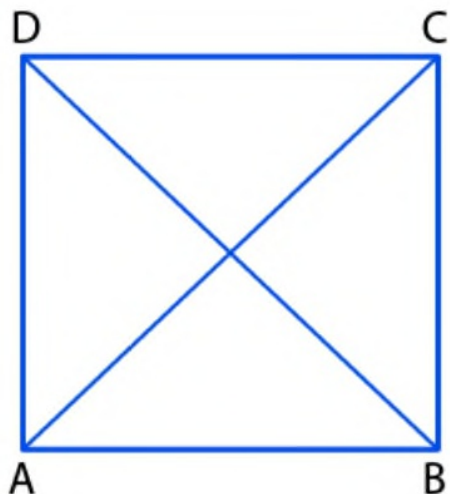
And, as $\angle A = \angle B = \angle C = \angle D$

Therefore, ABCD is a rectangle.

(iii) Given: ABCD is a rhombus in which $AC = BD$

To prove: ABCD is a square

Proof:



Join AC and BD.

Now, in $\triangle ABC$ and $\triangle DCB$ we have

$$\angle AB = \angle DC \quad [\text{Sides of a rhombus}]$$

$$\angle BC = \angle BC \quad [\text{Common}]$$

$$\angle AC = \angle BD \quad [\text{Given}]$$

So, $\triangle ABC \cong \triangle DCB$ by S.S.S. axiom of congruency

Thus,

$$\angle ABC = \angle DBC \quad [\text{By C.P.C.T}]$$

But these are made by transversal BC on the same side of parallel line AB and CD.

$$\text{So, } \angle ABC = \angle DBC = 180^\circ$$

$$\angle ABC = 90^\circ$$

Hence, ABCD is square.

(iv) Given: ABCD is rhombus.

To Prove: Diagonals AC and BD bisect $\angle A$, $\angle C$, $\angle B$ and $\angle D$ respectively

Proof:

In $\triangle AOD$ and $\triangle COD$, we have

$$AD = CD \quad [\text{Sides of a rhombus are all equal}]$$

$$OD = OD \quad [\text{Common}]$$

$$AO = OC \quad [\text{Diagonal of rhombus bisect each other}]$$

So, $\triangle AOD \cong \triangle COD$ by S.S.S. axiom of congruency

Thus,

$$\angle AOD = \angle COD \quad [\text{By C.P.C.T}]$$

$$\text{So, } \angle AOD = \angle COD = 180^\circ \quad [\text{Linear pair}]$$

$$\angle AOD = 180^\circ$$

$$\angle AOD = 90^\circ$$

$$\text{And, } \angle COD = 90^\circ$$

Thus,

$$OD \perp AC = BD \perp AC$$

$$\text{Also, } \angle ADO = \angle CDO \quad [\text{By C.P.C.T}]$$

So,

OD bisect $\angle D$

BD bisect $\angle D$

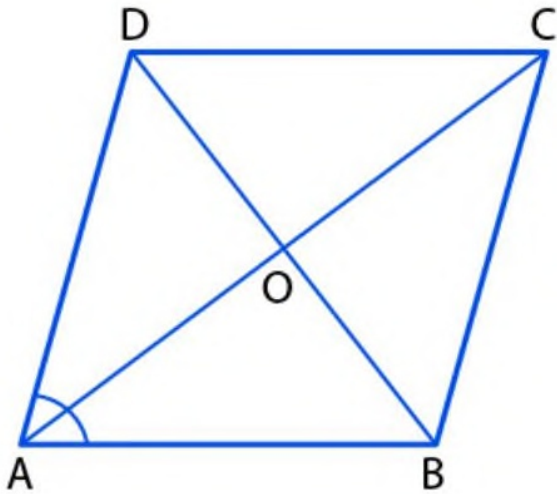
Similarly, we can prove that BD bisect $\angle B$ and AC bisect the $\angle A$ and $\angle C$.

10. ABCD is parallelogram. If the diagonal AC bisects $\angle A$, then prove that:

- (i) AC bisect $\angle C$**
- (ii) ABCD is a rhombus**
- (iii) $AC \perp BD$.**

Solution:

Given: In parallelogram ABCD in which diagonal AC bisect $\angle A$.



To prove:

- (i) AC bisects $\angle C$**
- (ii) ABCD is a rhombus**
- (iii) $AC \perp BD$.**

Proof:

- (i) As $AB \parallel CD$, we have [Opposite sides of a || gm]**

$$\angle DCA = \angle CAB$$

$$\text{Similarly, } \angle DAC = \angle DCB$$

$$\text{But, } \angle CAB = \angle DCA \quad [\text{Since, AC bisects } \angle A]$$

Hence,

$\angle DCA = \angle ACB$ and AC bisects $\angle C$.

(ii) As AC bisect $\angle A$ and $\angle C$

And, $\angle A = \angle C$

Hence, ABCD is a rhombus.

(iii) Since, AC and BD are the diagonals of a rhombus and AC and BD bisect each other at right angles

Hence, $AC \perp BD$.

11.

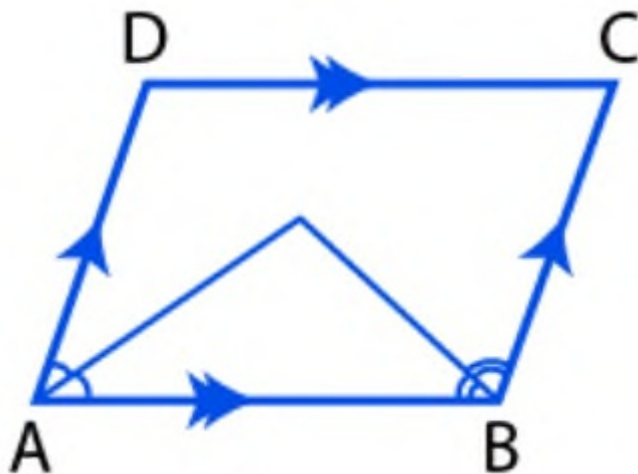
(i) **Prove that bisectors of any two adjacent angles of a parallelogram are at right angles.**

(ii) **Prove the bisector of any two opposite angles of a parallelogram are parallel.**

(iii) **If the diagonals of a quadrilateral are equal and bisect each other at right angles, then prove that it is a square.**

Solution:

(i) Given AM bisect angle A and BM bisects angle of || gm ABCD.



To prove: $\angle AMB = 90^\circ$

Proof:

We have,

$$\angle A + \angle B = 180^\circ \quad [\text{AD} \parallel \text{BC and AB is the transversal}]$$

$$\Rightarrow \frac{1}{2}(\angle A + \angle B) = \frac{180}{2}$$

$$\frac{1}{2}\angle A + \frac{1}{2}\angle B = 90^\circ$$

$$\angle MAB + \angle MBA = 90^\circ \quad [\text{Since, AM bisects } \angle A \text{ and BM bisects } \angle B]$$

Now, in $\triangle AMB$

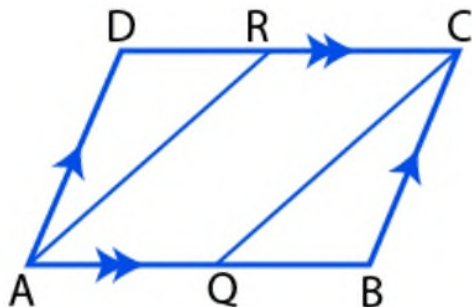
$$\angle AMB + \angle MAB + \angle MBA = 180^\circ \quad [\text{Angle sum property of a triangle}]$$

$$\angle AMB + 90^\circ = 180^\circ$$

$$\angle AMB = 180^\circ - 90^\circ = 90^\circ$$

Hence, bisectors of any two adjacent angles of a parallelogram are at right angles.

- (ii) Given: A $\parallel$ gm ABCD in which bisector AR of $\angle A$ meet DC in R and bisector CQ of $\angle C$ meets AB in Q



To prove: $AR \parallel CQ$

Proof:

In $\parallel$ gm ABCD, we have

$$\angle A = \angle C \quad [\text{Opposite angles of } \parallel \text{ gm are equal}]$$

$$\frac{1}{2}\angle A = \frac{1}{2}\angle C$$

$$\angle DAR = \angle BCQ$$

[Since, AR is bisector of $\frac{1}{2}\angle A$ and CQ is the bisector of $\frac{1}{2}\angle C$]

Now, in $\triangle ADR$ and $\triangle CBQ$

$$\angle DAR = \angle BCQ \quad [\text{Proved above}]$$

$$AD = BC \quad [\text{Opposite sides of } \parallel \text{ gm ABCD are equal}]$$

So, $\triangle ADR \cong \triangle CBQ$, by A.S.A axiom of congruency

Then by C.P.C.T, we have

$$\angle DAR = \angle BCQ$$

And,

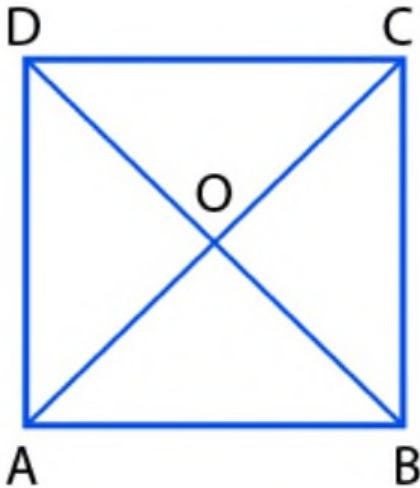
$$\angle DAR = \angle RAQ \quad [\text{Alternate angles since, } DC \parallel AB]$$

$$\text{Thus, } \angle RAQ = \angle BCQ$$

But these are corresponding angles,

Hence, $AR \parallel CQ$.

- (iii) Given: In quadrilateral ABCD, diagonals AC and BD are equal and bisect each other at right angles



To prove: ABCD is a square

Proof:

In $\triangle AOB$ and $\triangle COD$, we have

$$AO = OC \quad [\text{Given}]$$

$$BO = OD \quad [\text{Given}]$$

$$\angle AOB = \angle COD \quad [\text{Vertically opposite angles}]$$

So, $\triangle AOB \cong \triangle COD$ by S.A.S. axiom of congruency

By C.P.C.T, we have

$$AB = CD \text{ and } \angle OAB = \angle OCD$$

But these are alternate angles

$$AB \parallel CD$$

Thus, ABCD is a parallelogram

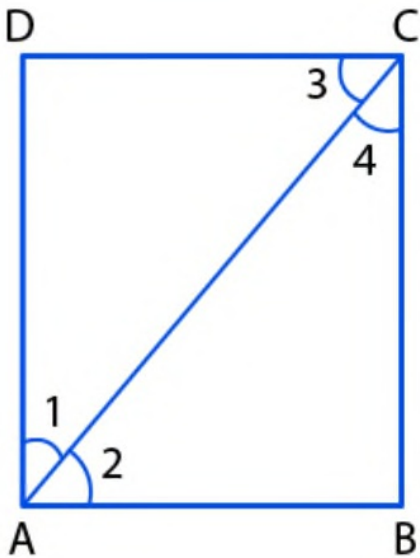
In a parallelogram, the diagonal bisect each other and are equal

Hence, ABCD is a square.

12.

- (i) If ABCD is a rectangle in which the diagonal BD bisect $\angle B$, then show that ABCD is a square.
- (ii) Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Solution:



- (i) ABCD is a rectangle and its diagonals AC bisects $\angle A$ and $\angle C$.

To prove: ABCD is a square

Proof:

We know that the opposite sides of a rectangle are equal and each angle is 90° .

As AC bisects $\angle A$ and $\angle C$

So, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

But, $\angle A = \angle C = 90^\circ$

$\angle 2 = 45^\circ$ and $\angle 4 = 45^\circ$

And, $AB = BC$ [Opposite sides of equal angles]

But, $AB = CD$ and $BC = AD$

So, $AB = BC = CD = DA$

Therefore, ABCD is a square.

(ii) In quadrilateral ABCD diagonals AC and BD are equal and bisect each other at right angles

To Prove: ABCD is a square

Proof:

In $\triangle AOB$ and $\triangle BOC$, we have

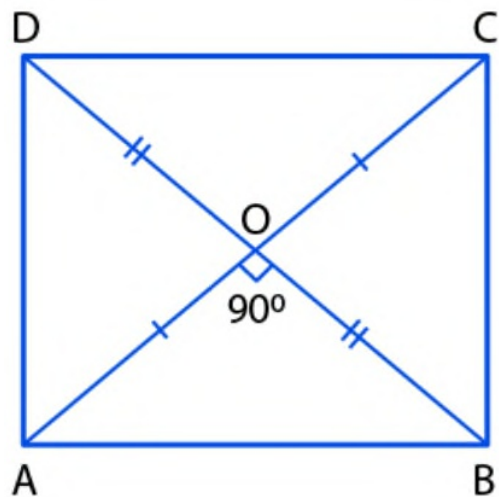
$AO = CO$ [Diagonals bisect each other at right angles]

$OB = OB$ [Common]

$\angle AOB = \angle COB$ [Each 90°]

So, $\triangle AOB \cong \triangle BOC$ by S.A.S. axiom of congruency

By C.P.C.T, we have



$AB = BC$ (i)

Similarly, in $\triangle BOC$ and $\triangle COD$

$OB = OD$ [Diagonals bisect each other at right angles]

$$OC = OC \quad [\text{Common}]$$

$$\angle BOC = \angle COD \quad [\text{Each } 90^\circ]$$

So, $\triangle BOC \cong \triangle COD$ by S.A.S. axiom of congruency

By C.P.C.T, we have

$$BC = CD \quad \dots (ii)$$

From (i) and (ii), we have

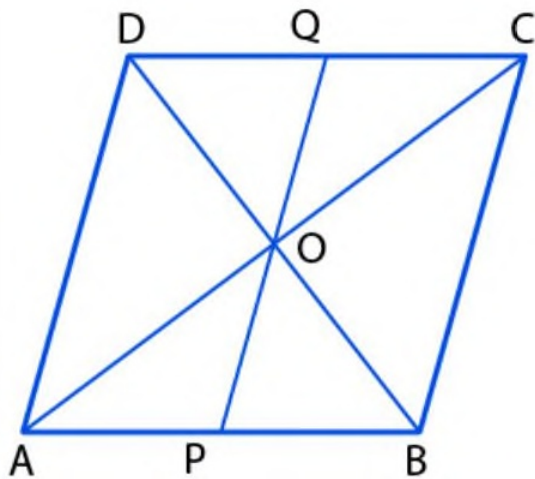
$$AB = BC = CD = DA$$

Hence, ABCD is a square.

13. P and Q are points on opposite sides AD and BC of a parallelogram ABCD such that PQ passes through the point of intersection O of its diagonal AC and BD. Show that PQ is bisected at O.

Solution:

Given: ABCD is a parallelogram, P and Q are the points on AB and DC. Diagonals AC and BD intersect each other at O.



To prove:

Diagonals of || gm ABCD bisect each other at O

So, $AO = OC$ and $BO = OD$

Now, in $\triangle AOP$ and $\triangle COQ$ we have

$AO = OC$ [Proved]

$\angle OAP = \angle OCQ$ [Alternate angles]

$\angle AOP = \angle COQ$ [Vertically opposite angles]

So, $\triangle AOP \cong \triangle COQ$ by S.A.S. axiom

Thus, by C.P.C.T, we have

$OP = OQ$

Hence, O bisects PQ.

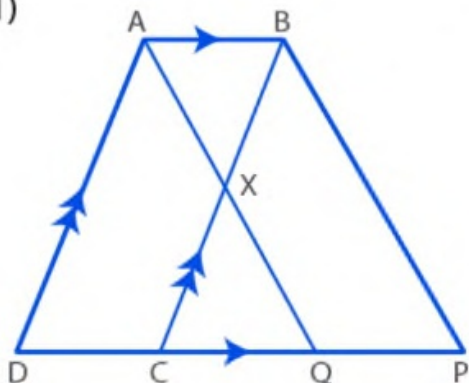
14.

- (a) In figure (1) given below, ABCD is a parallelogram and X is mid-point of BC. The line AX produced meets DC produced at Q. The parallelogram ABPQ is completed.**

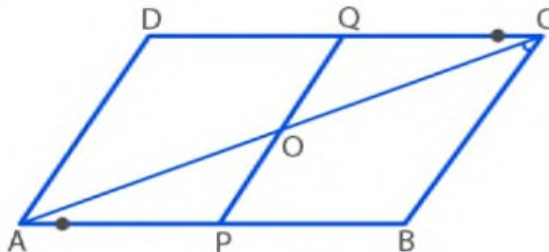
Prove that:

- (i) The triangle ABX and QCX are congruent:**
(ii) $DC = CQ = QP$
(b) In figure (2) given below, points P and Q have been taken on opposite sides AB and CD respectively of a parallelogram ABCD such that $AP = CQ$. Show that AC and PQ bisect each other.

(1)



(2)



Solution:

- (a) Given: ABCD is parallelogram and X is midpoint of BC. The line AX produced meets DC produced at Q and ABPQ is a $\parallel$ gm.

To Prove:

- (i) $\triangle ABX \cong \triangle QCX$
(ii) $DC = CQ = QP$

Proof:

In $\triangle ABX$ and $\triangle QCX$, we have

$$BX = XC \quad [X \text{ is the midpoint of } BC]$$

$$\angle AXB = \angle CXQ \quad [\text{Vertically opposite angles}]$$

$$\angle XCQ = \angle XBA \quad [\text{Alternate angle, since } AB \parallel CQ]$$

So, $\triangle ABX \cong \triangle QCX$ by A.S.A. axiom of congruence

Now, by C.P.C.T

$$CQ = AB$$

But,

$$AB = DC \text{ and } AB = QP \quad [\text{As } ABCD \text{ and } ABPQ \text{ are } \parallel \text{ gm}]$$

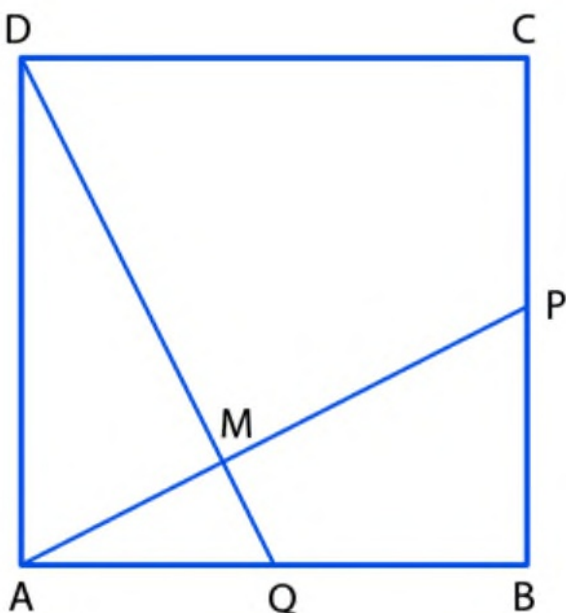
Hence,

$$DC = CQ = QP$$

15. ABCD is a square. A is joined to a point P on BC and D is joined to a point Q on AB. If $AP = DQ$. Prove that AP and DQ are perpendicular to each other.

Solution:

Given: ABCD is a square. P is any point on BC and Q is any point on AB and these point are taken such that $AP = DQ$.



To prove: $AP \perp DQ$

Proof:

In $\triangle ABP$ and $\triangle ADQ$, we have

$$AP = DQ \quad [\text{Given}]$$

$$AD = AB \quad [\text{Sides of square ABCD}]$$

$$\angle DAQ = \angle ABP \quad [\text{Each } 90^\circ]$$

So, $\triangle ABP \cong \triangle ADQ$ by R.H.S. axiom of congruence

Now, by C.P.C.T

$$\angle BAP = \angle ADQ$$

But, $\angle BAD = 90^\circ$

$$\angle BAD = \angle BAP + \angle PAD \quad \dots (i)$$

$$90^\circ = \angle BAP + \angle PAD$$

$$\angle BAP + \angle PAD = 90^\circ$$

$$\angle BAP + \angle ADQ = 90^\circ$$

Now, in $\triangle ADB$ we have

$$(\angle MAD + \angle ADM) + \angle AMD = 180^\circ \quad [\text{Angles sum property of a triangle}]$$

$$90^\circ + \angle AMD = 180^\circ \quad [\text{From (i)}]$$

$$\angle AMD = 180^\circ - 90^\circ = 90^\circ$$

So, $DM \perp AP$

$$\Rightarrow DQ \perp AP$$

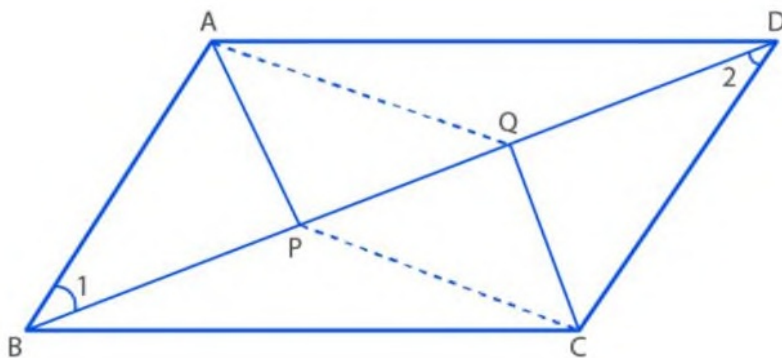
Hence, $AP \perp DQ$

16. If P and Q are points of trisection of the diagonal BD of a parallelogram ABCD, prove that $CQ \parallel AP$.

Solution:

Given : ABCD is a ||gm in which $BP = PQ = QD$

To prove : $CQ \parallel AP$



Proof :

In $\parallel gm$ ABCD, we have

$AB = CD$ [Opposite sides of a $\parallel gm$ are equal]

And BD is the transversal

So, $\angle 1 = \angle 2$ [Alternate interior angles](i)

Now, in $\triangle ABP$ and $\triangle DCQ$

$AB = CD$ [Opposite sides of a $\parallel gm$ are equal]

$\angle 1 = \angle 2$ [From (i)]

$BP = QD$ [Given]

So, $\triangle ABP \cong \triangle DCQ$ by S.A.S. axiom of congruency

Then by C.P.C.T, we have

$AP = QC$

Also, $\angle APB = \angle DQC$ [By C.P.C.T]

$-\angle APB = -\angle DQC$ [Multiplying both sides by -1]

$180^\circ - \angle APB = 180^\circ - \angle DQC$ [Adding 180° both sides]

$$\angle APQ = \angle CQP$$

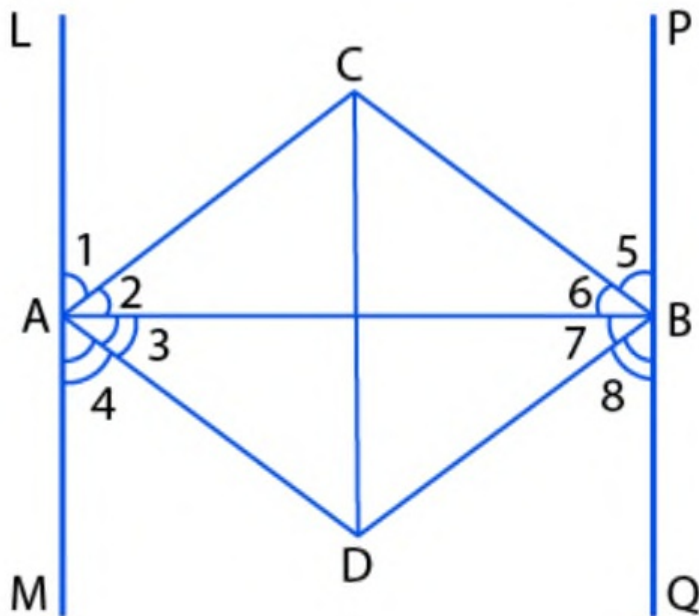
But, these are alternate angles

Hence, $AP \parallel QC \Rightarrow CQ \parallel AP$.

17. A transversal cuts two parallel lines at A and B. The two interior angles at A are bisected and so are the two interior angles at B; the four bisectors form a quadrilateral ABCD. Prove that

(i) ABCD is a rectangle.

(ii) CD is parallel to the original parallel lines.



Solution:

Given : $LM \parallel PQ$ and AB is the transversal line cutting $\angle M$ at A and PQ at B

AC , AD , BC and BD is the bisector of $\angle LAB$, $\angle BAM$, $\angle PAB$ and $\angle ABQ$ respectively.

AC and BC intersect at C and AD and BD intersect at D .

A quadrilateral $ABCD$ is formed.

To prove : (i) $ABCD$ is a rectangle

(ii) $CD \parallel LM$ and PQ

Proof:

(i) $\angle LAB + \angle BAM = 180^\circ$ [LAM is a straight line]

$$\frac{1}{2}(\angle LAB + \angle BAM) = 90^\circ$$

$$\frac{1}{2}\angle LAB + \frac{1}{2}\angle BAM = 90^\circ$$

$\angle 2 + \angle 3 = 90^\circ$ [Since, AC and AD is bisector of $\angle LAB$ & $\angle BAM$ respectively]

$$\angle CAD = 90^\circ$$

$$\angle A = 90^\circ$$

(2) Similarly, $\angle PBA + \angle QBA = 180^\circ$ [PBQ is a straight line]

$$\frac{1}{2}(\angle PBA + \angle QBA) = 90^\circ$$

$$\frac{1}{2}\angle PBA + \frac{1}{2}\angle QBA = 90^\circ$$

$\angle 6 + \angle 7 = 90^\circ$ [Since, BC and BD is bisector of $\angle PAB$ & $\angle QBA$ respectively.]

$$\angle CBD = 90^\circ$$

$$\angle B = 90^\circ$$

(3) $\angle LAB + \angle ABP = 180^\circ$ [Sum of co-interior angles is 180° and given $LM \parallel PQ$]

$$\frac{1}{2}\angle LAB + \frac{1}{2}\angle ABP = 90^\circ$$

$$\angle 2 + \angle 6 = 90^\circ$$

[Since, AC and BC is bisector of $\angle LAB$ & $\angle PBA$ respectively]

(4) In $\triangle ACB$,

$$\angle 2 + \angle 6 + \angle C = 180^\circ \text{ [Angles sum property of a triangle]}$$

$$(\angle 2 + \angle 6) + \angle C = 180^\circ$$

$$90^\circ + \angle C = 180^\circ \text{ [Using (3)]}$$

$$\angle C = 180^\circ - 90^\circ$$

$$\angle C = 90^\circ$$

(5) $\angle MAB + \angle ABQ = 180^\circ$ [Sum of co-interior angles is 180° and given $LM \parallel PQ$]

$$\frac{1}{2}\angle MAB + \frac{1}{2}\angle ABQ = 90^\circ$$

$\angle 3 + \angle 7 = 90^\circ$ [Since, AD and BD is bisector of $\angle MAB$ & $\angle ABQ$ respectively]

(6) In $\triangle ADB$,

$$\angle 3 + \angle 7 + \angle D = 180^\circ \text{ [Angles sum property of a triangle]}$$

$$(\angle 3 + \angle 7) + \angle C = 180^\circ$$

$$90^\circ + \angle D = 180^\circ \text{ [Using (5)]}$$

$$\angle D = 180^\circ - 90^\circ$$

$$\angle D = 90^\circ$$

$$(7) \angle LAB + \angle BAM = 180^\circ$$

$$\angle BAM = \angle ABP \text{ [From (1) and (2)]}$$

$$\frac{1}{2} \angle BAM = \frac{1}{2} \angle ABP$$

$\angle 3 = \angle 6$ [Since, AD and BC is bisector of $\angle BAM$ & $\angle ABP$ respectively]

Similarly, $\angle 2 = \angle 7$

(8) in $\triangle ABC$ and $\triangle ABD$,

$$\angle 2 = \angle 7 \text{ [From (7)]}$$

$$AB = AB \text{ [common]}$$

$$\angle 6 = \angle 3 \text{ [from (7)]}$$

So, $\triangle ABC \cong \triangle ABD$ by A.S.A. axiom of congruency

Then, by C.P.C.T. we have

$$AC = DB$$

$$\text{Also, } CB = AD$$

$$(9) \angle A = \angle B = \angle C = \angle D = 90^\circ \text{ [From (1), (2), (3) and (4)]}$$

$$AC = DB \text{ [Proved in (8)]}$$

$$CB = AD \text{ [Proved in (8)]}$$

Hence, ABCD is a rectangle.

(10) Since, ABCD is a rectangle [From (9)]

OA = OD [Diagonals of rectangle bisect each other]

(11) In $\triangle AOD$, we have

OA = OD [From (10)]

$\angle 9 = \angle 3$ [Angles opposite to equal sides are equal]

(12) $\angle 3 = \angle 4$ [AD bisects $\angle MAB$]

(13) $\angle 9 = \angle 4$ [From (11) and (12)]

But these are alternate angles.

$OD \parallel LM \Rightarrow CD \parallel LM$

Similarly, we can prove that

$$\angle 10 = \angle 8$$

But these are alternate angles,

So, $OD = PQ \Rightarrow CD \parallel PQ$

(14) $CD \parallel LM$ [Proved in (13)]

$CD \parallel PQ$ [Proved in (13)]

18. In a parallelogram ABCD, the bisector of $\angle A$ meets DC in E and $AB = 2 AD$. Prove that :

(i) BE bisects $\angle B$

(ii) $\angle AEB$ is a right angle

Solution:

Given: ABCD is a ||gm in which bisectors of angles A and B meet in E and $AB = 2AD$

To prove : (i) BE bisects $\angle B$

(ii) $\angle AEB = 90^\circ$

Proof:

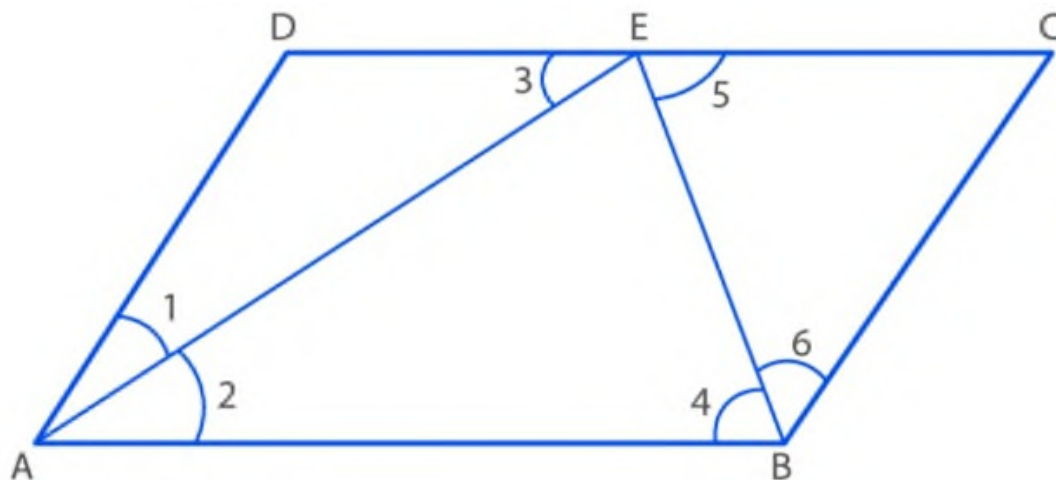
(1) In || gm ABCD

$\angle 1 = \angle 2$ [AD bisects angles $\angle A$]

(2) $AB \parallel DC$ and AE is the transversal

$\angle 2 = \angle 3$ [Alternate angles]

(3) $\angle 1 = \angle 2$ [From (1) and (2)]



(4) in $\triangle ADE$, we have

$\angle 1 = \angle 3$ [Proved in (3)]

$DE = AD$ [Sides opposite to equal angles are equal]

$\Rightarrow AD = DE$

$$(5) AB = 2AD \text{ [Given]}$$

$$\frac{AB}{2} = AD$$

$$\frac{AB}{2} = DE \text{ [Using (4)]}$$

$$\frac{DC}{2} = DE \text{ [AB = DC, Opposite sides of a || gm are equal]}$$

So, E is the mid-point of D.

$$\Rightarrow DE = EC$$

$$(6) AD = BC \text{ [Opposite sides of a || gm are equal]}$$

$$(7) DE = BC \text{ [From (4) and (6)]}$$

$$(8) EC = BC \text{ [From (5) and (7)]}$$

(9) In $\triangle BCE$, we have

$$EC = BC \text{ [Proved in (8)]}$$

$$\angle 6 = \angle 5 \text{ [Angles opposite to equal sides are equal]}$$

(10) $AB \parallel DC$ and BE is the transversal

$$\angle 4 = \angle 5 \text{ [Alternate angles]}$$

(11) $\angle 4 = \angle 6$ [From (9) and (10)]

So, BE is bisector of $\angle B$

(12) $\angle A + \angle B = 180^\circ$ [Sum of co-interior angles is equal to 180° , $AD \parallel BC$]

$$\frac{1}{2}\angle A + \frac{1}{2}\angle B = \frac{180^\circ}{2}$$

$\angle 2 + \angle 4 = 90^\circ$ [AE is bisector of $\angle A$ and BE is bisector of $\angle B$]

(13) In $\triangle AEB$,

$$\angle AEB + \angle 2 + \angle 4 = 180^\circ$$

$$\angle AEB + 90^\circ = 180^\circ$$

Hence, $\angle AEB = 90^\circ$

19. ABCD is a parallelogram, bisectors of angles A and B meet at E which lie on DC. Prove that AB = 2AD.

Solution:

Given : ABCD is a parallelogram in which bisector of $\angle A$ and $\angle B$ meets DC in E

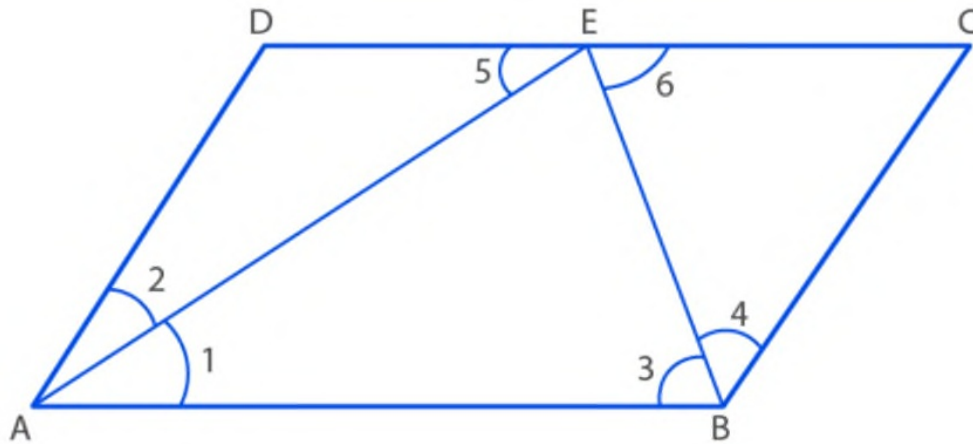
To prove : $AB = 2AD$

Proof :

In Parallelogram ABCD, we have

$AB \parallel DC$

$\angle 1 = \angle 5$ [Alternate angles, AE is transversal]



$\angle 1 = \angle 2$ [AE is bisector of $\angle A$, given]

Thus,

$$\angle 2 = \angle 5 \dots(i)$$

Now, in $\triangle AED$

$DE = AD$ [Sides opposite to equal angles are equal]

$\angle 3 = \angle 6$ [Alternate angles]

$\angle 3 = \angle 4$ [Since, BE is bisector of $\angle B$ (given)]

Thus, $\angle 4 = \angle 6 \dots(ii)$

In $\triangle BCE$, we have

$BC = EC$ [Sides opposite to equal angles are equal]

$AD = BC$ [Opposite sides of $\parallel$ gm are equal]

$AD = DE = EC$ [From (i) and (ii)]

$AB = DC$ [Opposite sides of a $\parallel$ gm are equal]

$$AB = DE + EC$$

$$= AD + AD$$

Hence,

$$AB = 2AD$$

20. ABCD is a square and the diagonals intersect at O. If P is a point on AB such that $AO = AP$, prove that $3\angle POB = \angle AOP$.

Solution:

Given : ABCD is a square and the diagonals intersect at O. P is the point on AB such that $AO = AP$

To prove : $3\angle POB = \angle AOP$

Proof :

(1) In square, ABCD, AC is a diagonal

So, $\angle CAB = 45^\circ$

$$\angle OAP = 45^\circ$$

(2) In $\triangle AOP$,

$$\angle OAP = 45^\circ \text{ [From(1)]}$$

$AO = AP$ [Sides oppsite to equal angles are equal]

Now,

$$\angle AOP + \angle APO + \angle OAP = 180^\circ \text{ [Angles sum property of a triangle]}$$

$$\angle AOP + \angle AOP + 45^\circ = 180^\circ$$

$$2\angle AOP = 180^\circ - 45^\circ$$

$$\angle AOP = \frac{135^\circ}{2}$$

(3) $\angle AOB = 90^\circ$ [Diagonals of a square bisect at right angles]

$$\text{So, } \angle AOP + \angle POB = 90^\circ$$

$$\frac{135^\circ}{2} + \angle POB = 90^\circ \text{ [From (2)]}$$

$$\angle POB = 90^\circ - \frac{135^\circ}{2}$$

$$= \frac{(180^\circ - 135^\circ)}{2}$$

$$= \frac{45^\circ}{2}$$

$$3\angle POB = \frac{135^\circ}{2} \text{ [Multiplying both sides by 3]}$$

Hence,

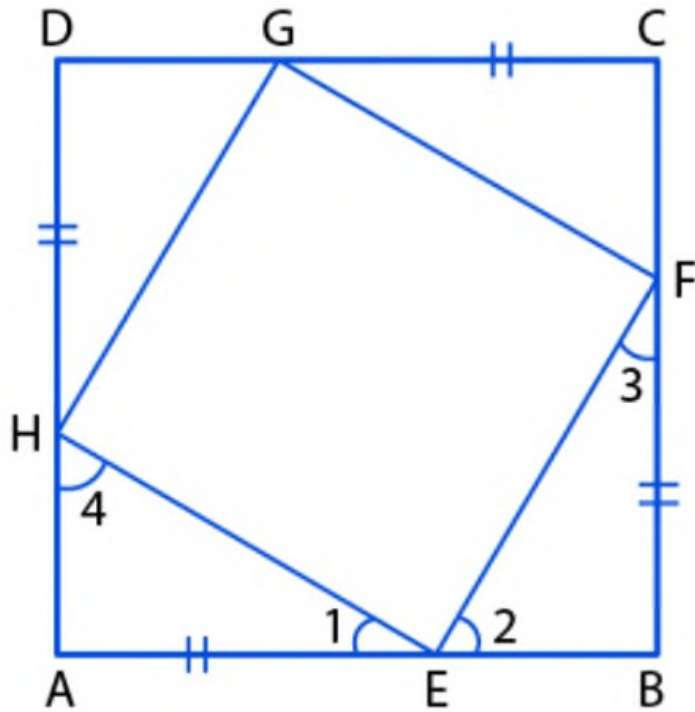
$$\angle AOP = 3\angle POB \text{ [From (2) and (3)]}$$

21. ABCD is a square. E, F, G and H are points on the sides AB, BC, CD and DA respectively such that AE = BF = CG = DH. Prove that EFGH is a square.

Solution:

Given : ABCD is a square in which E, F, G and H are points on AB, BC, CD and DA

such that AE = BF = CG = DH



EF, FG, GH and HE are joined

To Prove : EFGH is a square

Proof :

Since, $AE = BF = CG = DH$

So, $EB = FC = GD = HA$

Now, in $\triangle AEH$ and $\triangle BFE$

$AE = BF$ [Given]

$AH = EB$ [Proved]

$\angle A = \angle B$ [Each 90°]

So, $\triangle AEH \cong \triangle BFE$ by S.A.S. axiom of congruency

Then, by C.P.C.T. we have

$EH = EF$

And $\angle 4 = \angle 2$

But $\angle 1 + \angle 4 = 90^\circ$

$$\angle 1 + \angle 2 = 90^\circ$$

Thus, $\angle HEF = 90^\circ$

Hence, EFGH is a square.

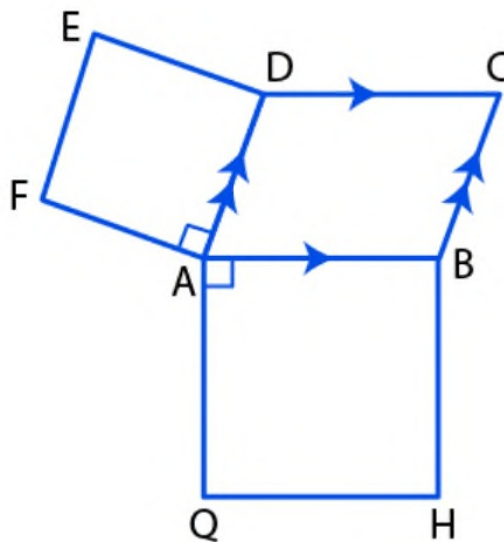
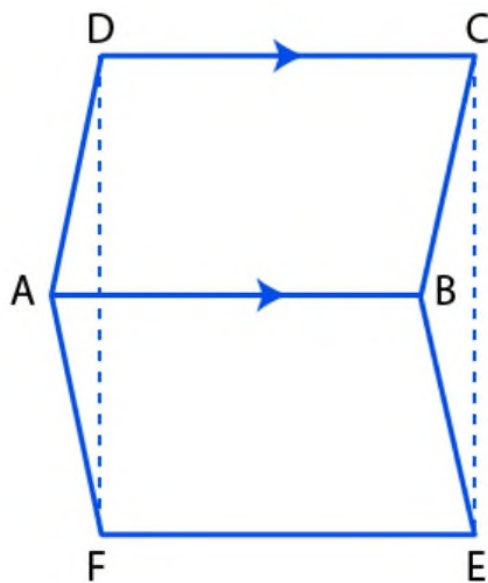
22. (a) In the figure (1) given below, ABCD and ABEF are parallelograms, Prove that

(i) CDEF is a parallelogram

(ii) $FD = EC$

(iii) $\triangle AFD = \triangle BEC$

(b) In the figure (2) given below, ABCD is a parallelogram, ADEF and AGHB are two squares. Prove that $FG = AC$.



Solution:

Given : ABCD and ABEF are $\parallel$ gms

To prove : (i) CDEF is a parallelogram

(ii) $FD = EC$

(iii) $\triangle AFD \cong \triangle BEC$

Proof :

(1) $DC \parallel AB$ and $DC = AB$ [ABCD is a $\parallel$ gm]

(2) $FE \parallel AB$ and $FE = AB$ [ABEF is a $\parallel$ gm]

(3) $DC \parallel FE$ and $DC = FE$ [From (1) and (2)]

Thus, CDEF is a $\parallel$ gm

(4) CDEF is a $\parallel$ gm

So, $FD = EC$

(5) In $\triangle AFD$ and $\triangle BEC$, we have

$AD = BC$ [Opposite sides of $\parallel$ gm ABCD are equal]

$AF = BE$ [Opposite sides of $\parallel$ gm ABEF are equal]

$FD = EC$ [From (4)]

Hence, $\triangle AFD \cong \triangle BEC$ by S.S.S, axiom of congruency

(b) Given : ABCD is a || gm, ADEF and AGHB are two squares

To Prove : $FG = AC$

Proof :

(1) $\angle FAG + 90^\circ + 90^\circ + \angle BAD = 360^\circ$ [At a point total angle is 360°]

$$\angle FAG = 360^\circ - 90^\circ - 90^\circ - \angle BAD$$

$$\angle FAG = 180^\circ - \angle BAD$$

(2) $\angle B + \angle BAD = 180^\circ$ [Adjacent angle in || gm is equal to 180°]

$$\angle B = 180^\circ - \angle BAD$$

(3) $\angle FAG = \angle B$ [From (1) and (2)]

(4) In $\triangle AFG$ and $\triangle ABC$, we have

$AF = BC$ [FADE and ABCD both are squares on the same base]

Similarly, $AG = AB$

$$\angle FAG = \angle B \text{ [From (3)]}$$

So, $\triangle AFG \cong \triangle ABC$ by S.A.S. axiom of congruency

Hence, by C.P.C.T

$$FG = AC$$

23. ABCD is a rhombus in which $\angle A = 60^\circ$. Find the ratio AC : BD.

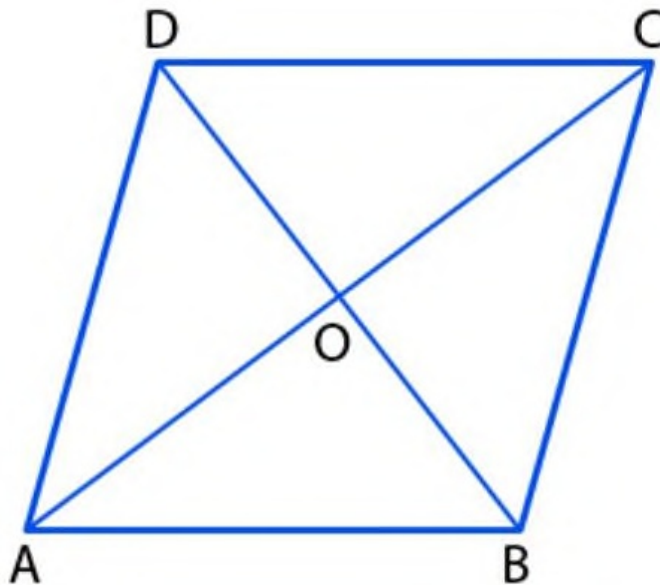
Solution:

Let each side of the rhombus ABCD be a

$$\angle A = 60^\circ$$

So, ABD is an equilateral triangle

$$\Rightarrow BD = AB = a$$



We know that, the diagonals of a rhombus bisect each other at right angles

So, in right triangle AOB, we have

$$AO^2 + OB^2 = AB^2 \text{ [By Pythagoras Theorem]}$$

$$AO^2 = AB^2 - OB^2$$

$$= a^2 - \left(\frac{1}{2}a\right)^2$$

$$= a^2 - \frac{a^2}{4}$$

$$= \frac{3a^2}{4}$$

$$AO = \sqrt{\frac{3a^2}{4}} = \frac{\sqrt{3}a}{2}$$

$$\text{But, } AC = 2 AO = 2 \times \frac{3a}{2} = 3a$$

Hence,

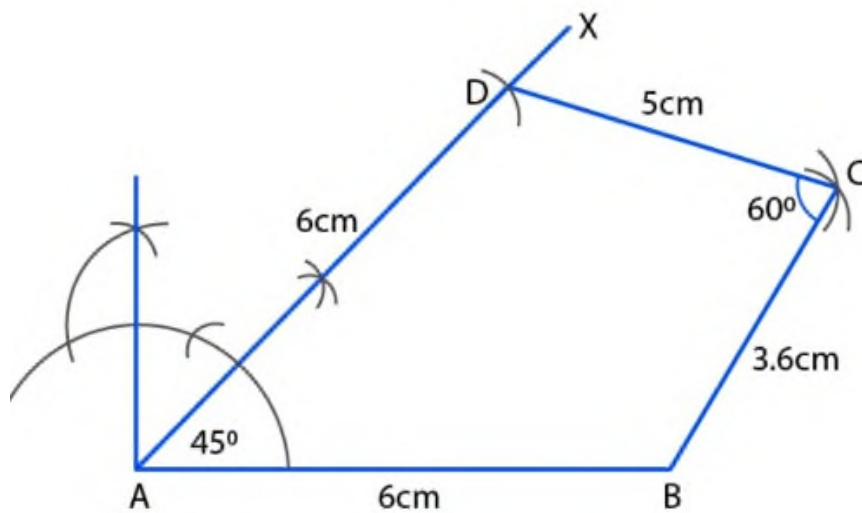
$$AC : BD = \sqrt{3}a : a = \sqrt{3} : 1$$

Exercise 13.2

Question 1

Using ruler and compasses only , construct the quadrilateral ABCD in which $\angle BAD = 45^\circ$, $AD = AB = 6\text{cm}$, $BC = 3.6\text{cm}$, $CD = 5\text{cm}$. measure $\angle BCD$.

Solution:



Steps of construction:

- (i) Draw a line segment $AB = 6\text{cm}$
- (ii) At A, draw a ray AX making an angle of 45° and cut of $AD = 6\text{cm}$
- (iii) with centre B and radius. 3.6 and with centre D and radius 5cm, draw two arcs intersecting each other at C.
- (iv) join BC and DC.

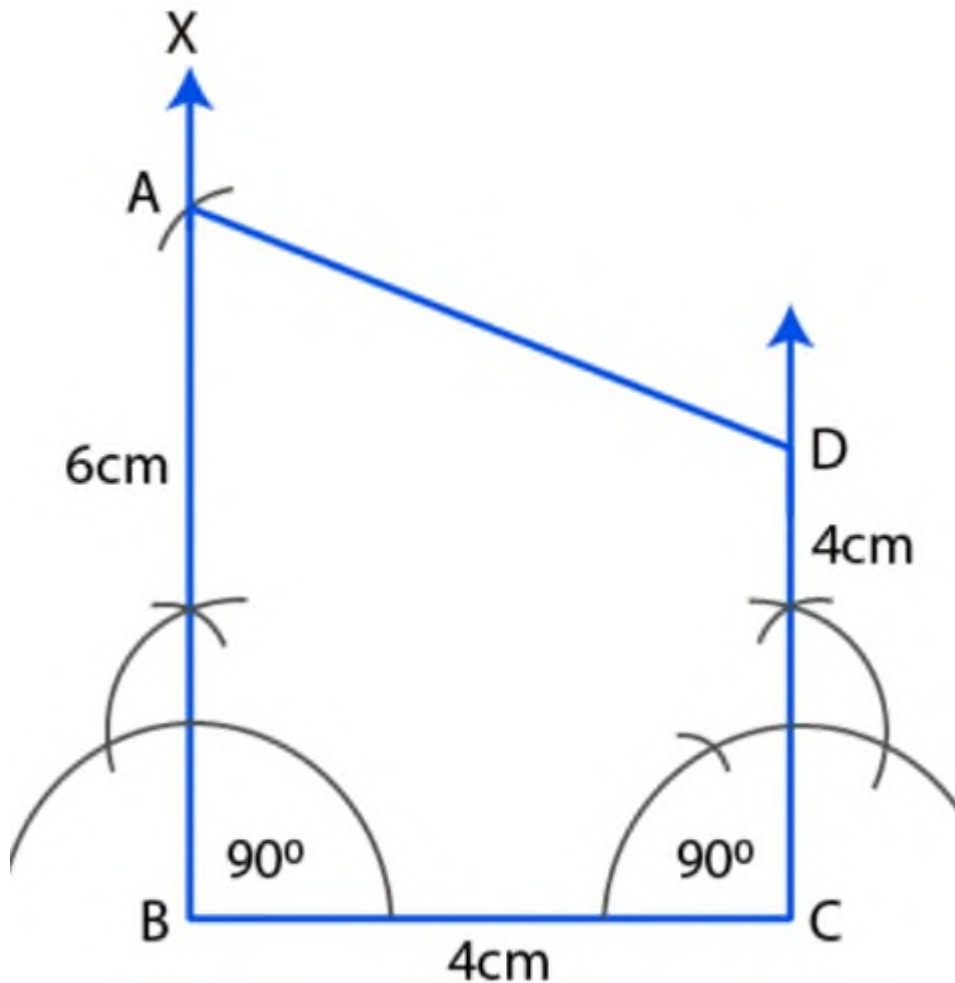
Thus, ABCD is the required quadrilateral.

On measuring $\angle BCD$, it is 60°

Question 2.

Draw a quadrilateral ABCD with $AB = 6\text{cm}$, $BC = 4\text{cm}$, $CD = 4\text{cm}$ and $\angle B = \angle C = 90^\circ$

Solution :



Steps of construction:

- (i) Draw a line segment $BC = 4\text{cm}$
- (ii) At B and C draw rays BX and CY making an angle of 90° each

(iii) From BX, cut off $BA = 6\text{cm}$ and from CY, cut off $CD = 4\text{cm}$

(iv) join AD.

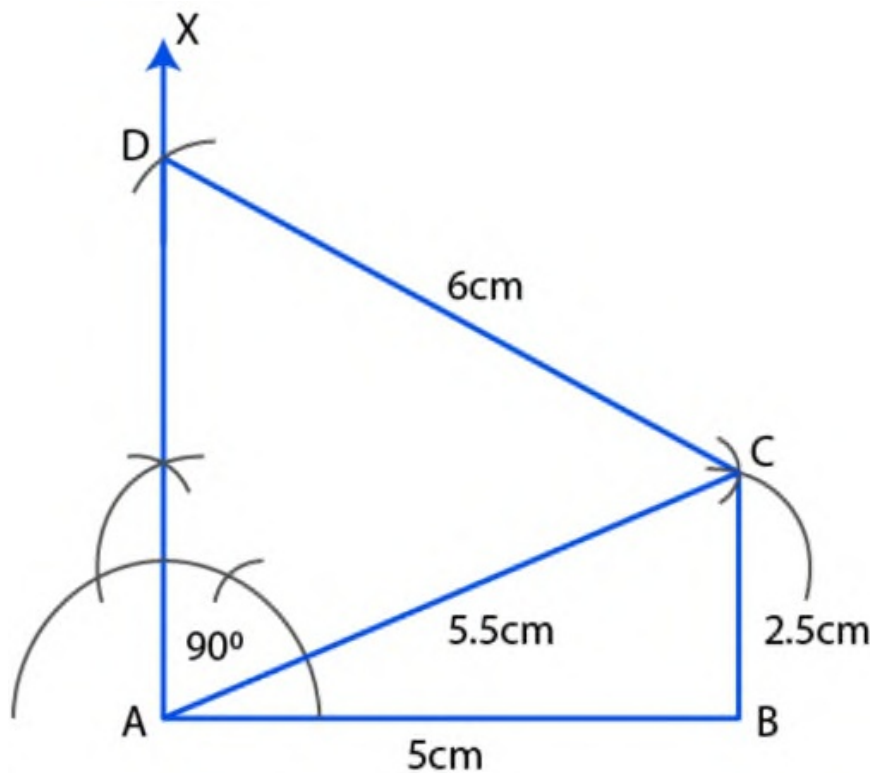
Thus, ABCD is the required quadrilateral.

Question 3.

Using ruler and compasses only, construct the quadrilateral ABCD given that $AB = 5\text{cm}$, $BC = 2.5\text{cm}$, $CD = 6\text{cm}$, $\angle BAD = 90^\circ$

And the diagonal $AC = 5.5\text{cm}$.

Solution:



Steps of construction:

(i) Draw a line segment $AB = 5\text{cm}$

(ii) with centre A and radius 5.5cm and with centre B and radius 2.5 cm draw arcs which intersect each other at C.

(iii) join AC and BC.

(iv) At A draw a ray AX making an angle of 90° .

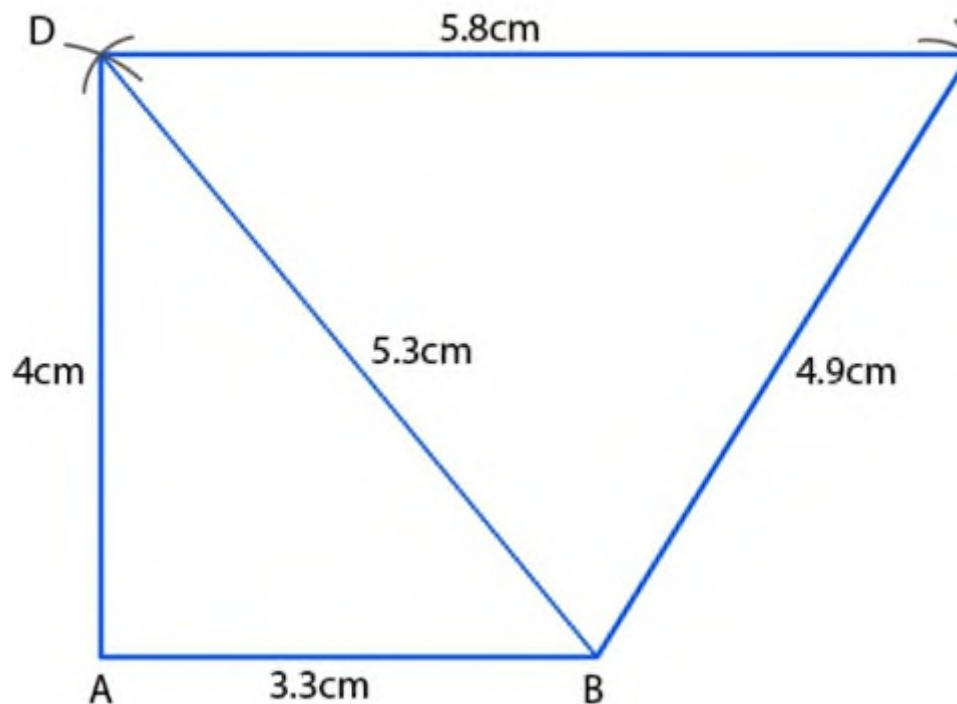
(v) with centre C and radius 6cm, draw an arc intersecting AX at D

(vi) join CD.

Thus, ABCD is the required quadrilateral .

Question 4.

Construct a quadrilateral ABCD in which $AB = 3.3$ cm, $BC = 4.9$ cm , $CD = 5.8$ cm, $DA = 4$ and $BD = 5.3$ cm.



Solution:

steps of construction:

(i) Draw a line segment $AB = 3.3$ cm

(ii) with centre A and radius 4cm, and with centre B and radius 5.3cm, draw arcs intersecting each other at

D.

(iii) join AD and BD.

(iv) with centre B and radius 4.9 cm and with centre D and radius 5.8cm, draw arcs intersecting each other at C.

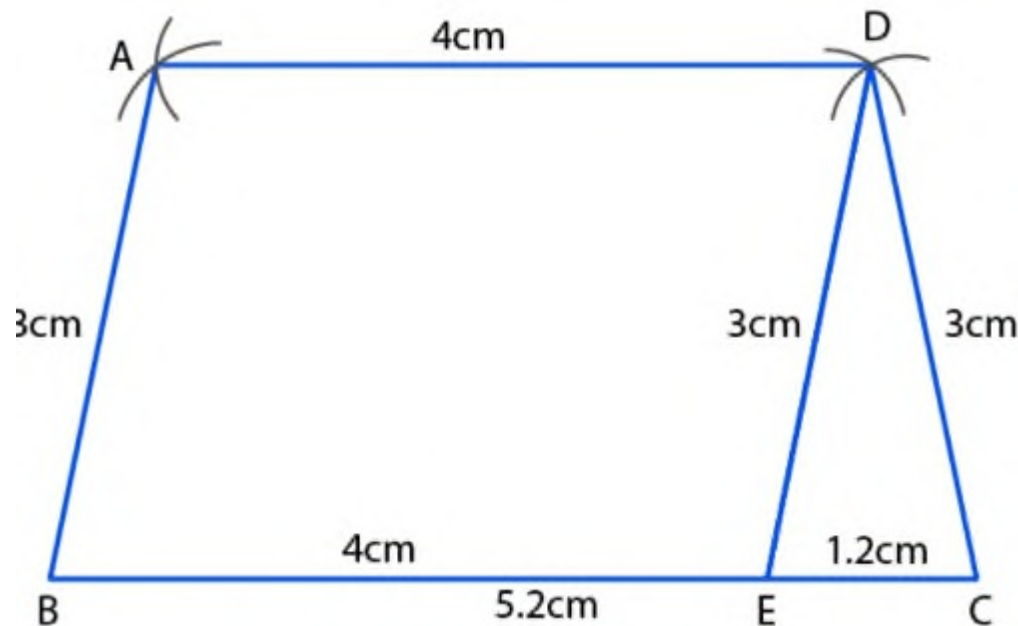
(v) join BC and DC.

Thus, ABCD is the required quadrilateral.

Question 5.

Construct a trapezium ABCD in which $AD \parallel BC$, $AB = CD = 3$ Cm , $BC = 5.2$ cm and $AD = 4$ cm.

Solution:



Steps of construction:

(i) Draw a line segment $BC = 5.2$ cm

(ii) From BC, cut off $BE = AD = 4$ cm

(iii) with centre E and C , and radius 3cm, draw arcs intersecting each other at D.

(iv) Join ED and CD.

(v) With centre D and radius 4cm and with centre B and radius 3cm, draw arcs intersecting each other at A.

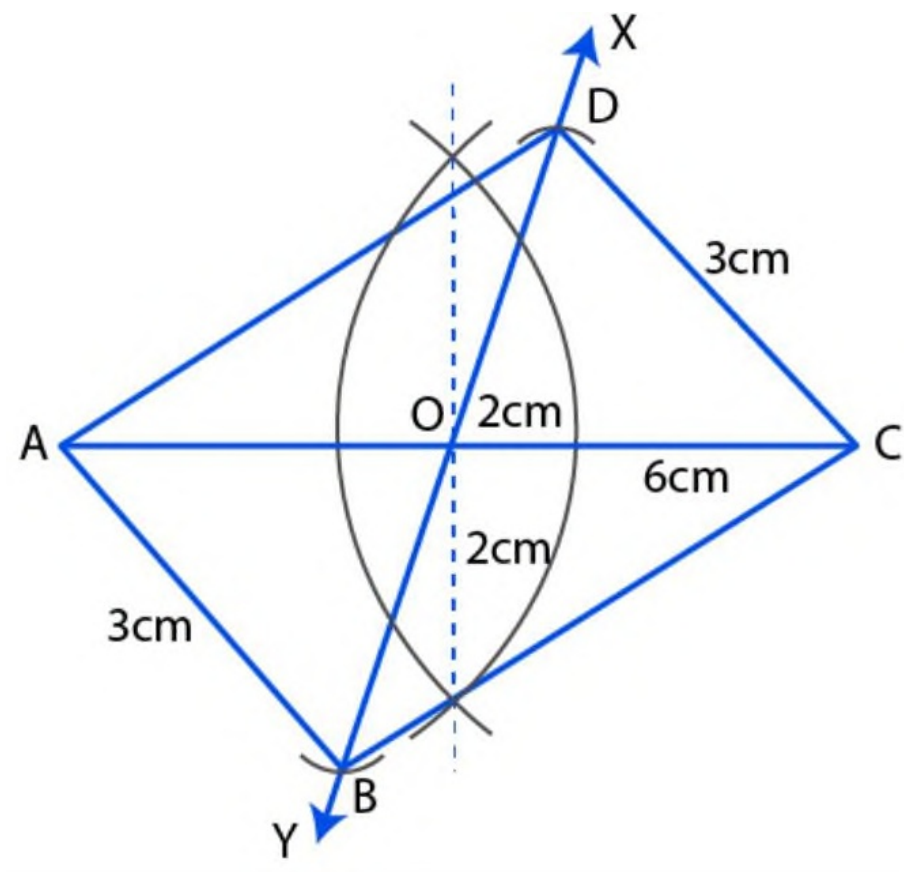
(vi) Join BA and DA

Thus, ABCD is the required trapezium.

Question 6.

Construct a trapezium ABCD in which $AD \parallel BC$, $\angle B = 60^\circ$, $AB = 5\text{cm}$.
 $BC = 6.2\text{cm}$ and $CD = 4.8\text{cm}$.

Solution:



Steps of construction:

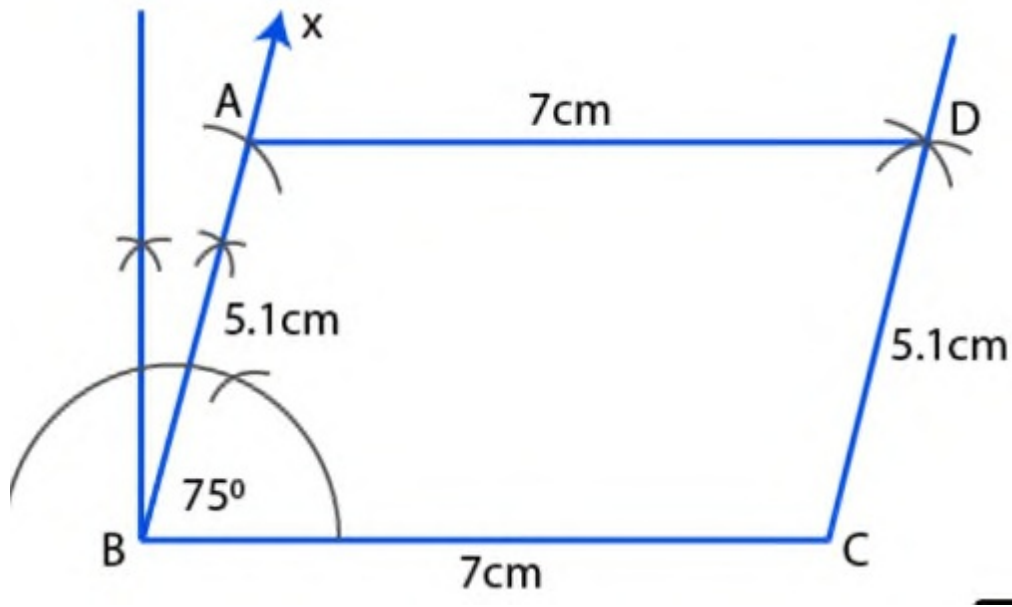
- (i) Draw a line segment $BC = 6.2\text{cm}$
- (ii) At B, draw a ray BX making an angle of 60° and cut off $AB = 5\text{cm}$
- (iii) from A, draw a line AY parallel to BC.
- (iv) with centre C and radius 4.8cm , draw an arc which intersects AY at D and D'.
- (v) join CD and CD'

Thus, ABCD and ABCD' are the required two trapeziums.

Question 7.

Using ruler and compasses only, construct a parallelogram ABCD with $AB = 5.1\text{ cm}$, $BC = 7\text{cm}$ and $\angle ABC = 75^\circ$.

Solution:



Steps of construction:

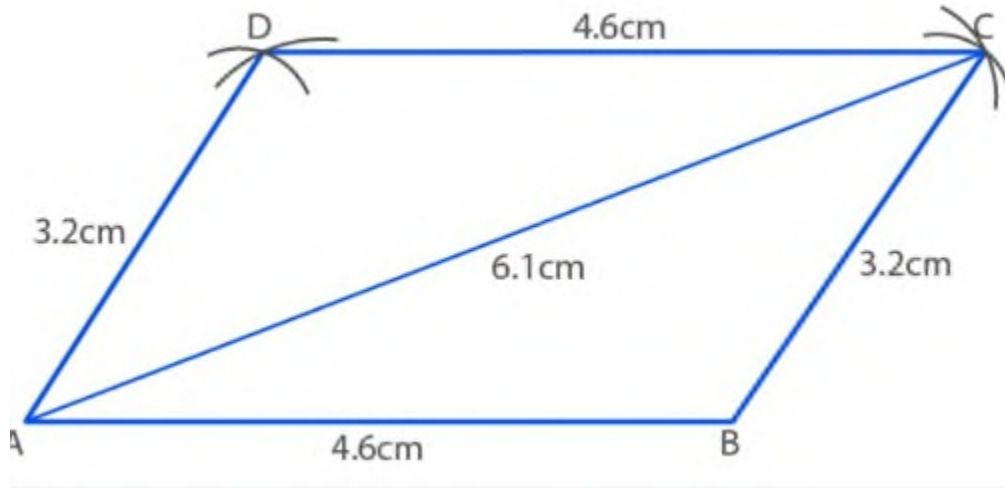
- (i) Draw a line segment $BC = 7\text{cm}$
- (ii) A to B, draw a ray Bx making an angle of 75° and cut off $AB = 5.1\text{cm}$
- (iii) with centre A and radius 7cm with centre c and radius 5.1cm ,
Draw arcs intersecting each other at D.
- (iv) join AD and CD.

Thus, ABCD is the required parallelogram.

Question 8.

Using ruler and compasses only, construct a parallelogram ABCD in which $AB = 4.6\text{cm}$ $BC = 3.2$ and $AC = 6.1\text{cm}$.

Solution:



Steps of construction:

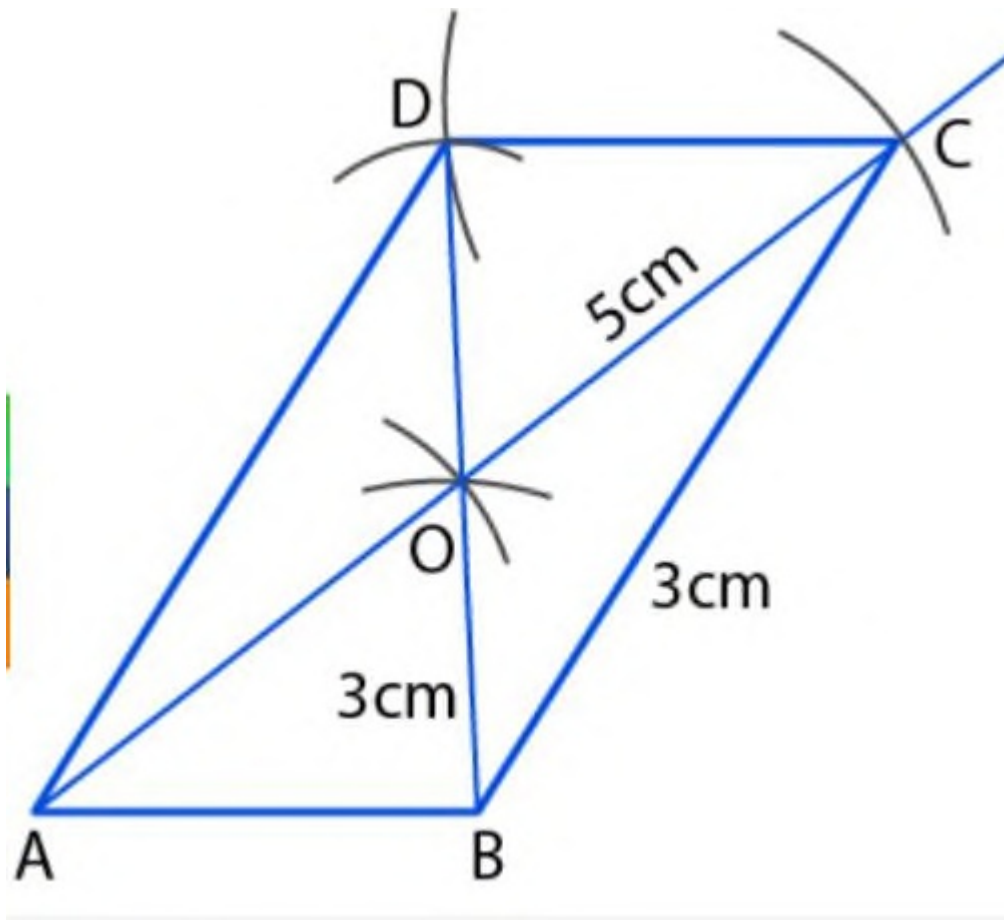
- (i) Draw a line segment $AB = 4.6\text{cm}$
- (ii) with centre A and radius 6.1cm and with centre B and radius 3.2cm , draw arcs intersecting each other at C.
- (iii) Join AC and BC.
- (iv) Again , with centre A and radius 3.2cm and with centre C and radius 4.6cm , draw arcs intersecting each other at D.
- (v) Join AD and CD.

Thus, ABCD is the required parallelogram.

Question 9.

Using ruler and compasses, construct a parallelogram ABCD give that $AB = 4\text{cm}$, $AC = 10\text{cm}$, $BD = 6\text{cm}$. Measure BC.

Solution:



Steps of construction:

(i) construct triangle OAB such that

$$OA = \frac{1}{2} \times AC = \frac{1}{2} \times 10cm = 5cm$$

$$OB = \frac{1}{2} \times BD = \frac{1}{2} \times 6cm = 3cm$$

As, diagonals of $\parallel$ gm bisect each other and $AB = 4cm$

(ii) Produce AO to C such that $OA = OC = 5cm$

(iii) Produce BO to D such that $OB = OD = 3cm$

(iv) join AD, BC and CD

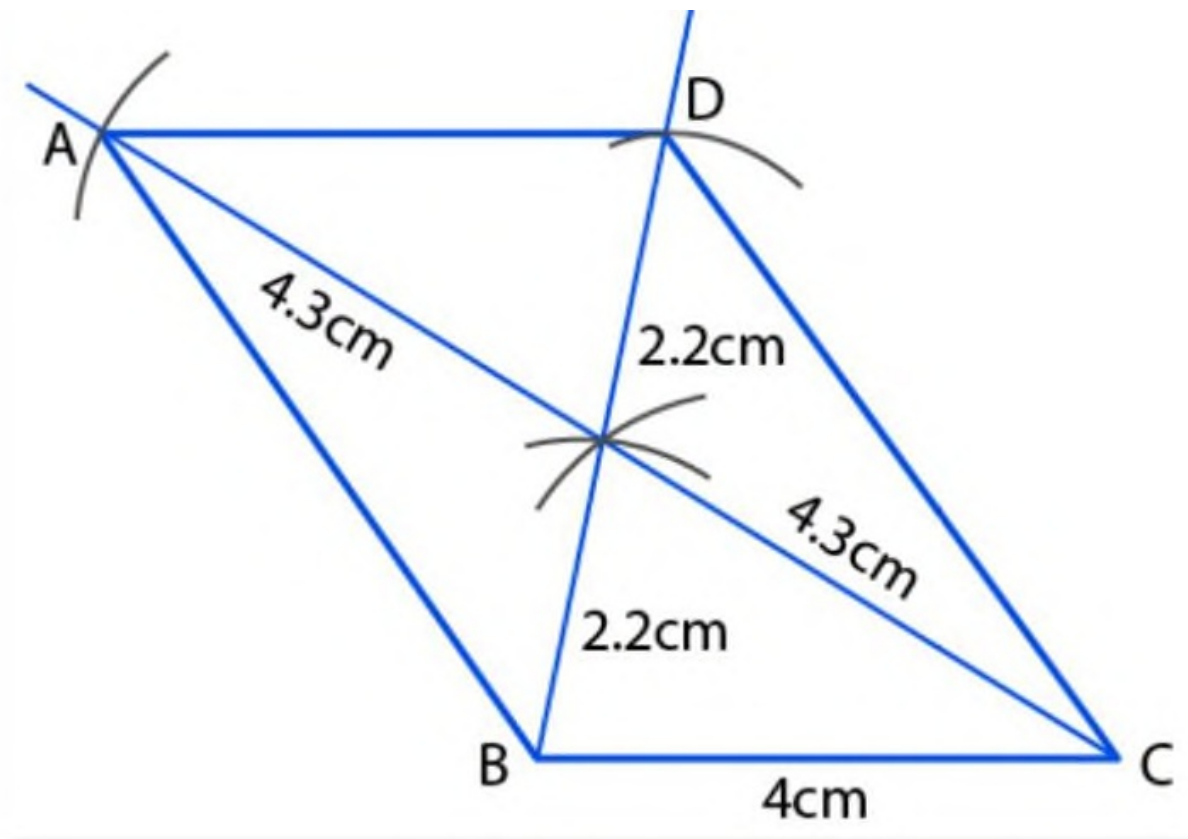
Thus , ABCD is the required parallelogram

(v) measure BC which is equal to 7.2cm

Question 10.

Using ruler and compasses only, construct a parallelogram ABCD such that $BC = 4\text{cm}$, diagonal $AC = 8.6\text{ cm}$ and diagonal $BD = 4.4\text{cm}$. measure the side AB.

Solution:



Steps of construction;

(i) Construct triangle OBC such that

$$OB = \frac{1}{2} \times BD = \frac{1}{2} \times 4.4\text{cm} = 2.2\text{cm}$$

$$OC = \frac{1}{2} \times AC = \frac{1}{2} \times 8.6\text{cm} = 4.3$$

Since, diagonals of || gm bisect each other and $BC = 4\text{cm}$

- (ii) produce BO to D such that $BO = OD = 2.2\text{cm}$
- (iii) Produce CO to A such that $CO = OA = 4.3\text{cm}$
- (iv) Join AB, AD and CD

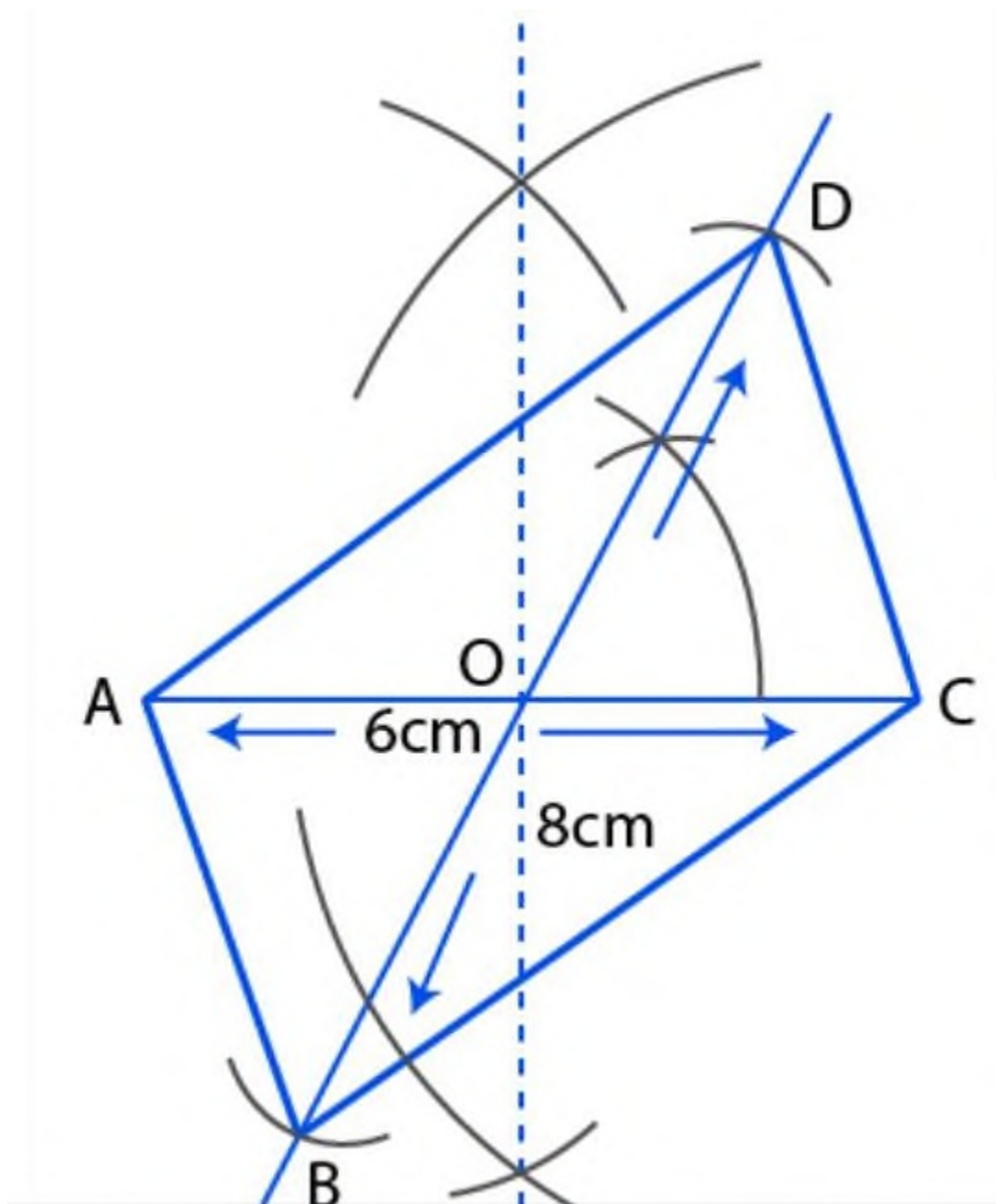
Thus, ABCD is the required parallelogram.

- (v) Measure the side AB, $AB = 5.6\text{cm}$

Question 11.

Use ruler and compasses to construct a parallelogram with diagonals 6cm and 8 cm in length having given the acute angle between them is 60° . Measure one of the longer sides.

Solution:



Steps of construction:

(i) Draw $AC = 6\text{cm}$

(ii) Find the mid – point O of AC . [since , diagonals of $\parallel$ gm bisect each other]

(iii) Draw line POQ such that $\angle POC = 60^\circ$ and $OB = OD = \frac{1}{2} BD = \frac{1}{2} \times 8\text{cm} = 4\text{cm}$

8cm = 4cm

So, from OP cut OD = 4cm and from OQ cut OB = 4cm

(iv) Join AB, BC, CD and DA.

Thus, ABCD is the required parallelogram.

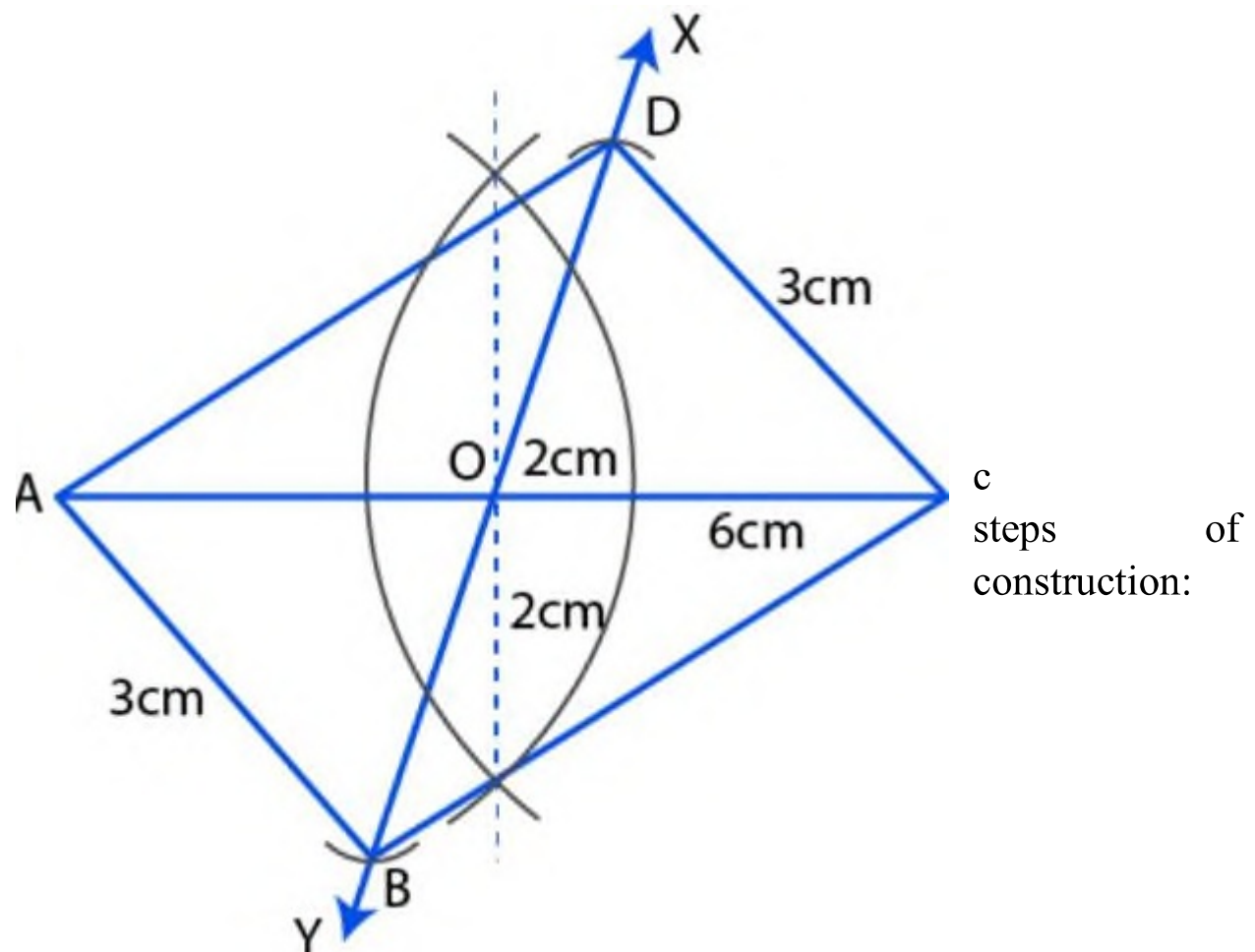
(v) Measure the length of side AD = 6.1cm

Question 12.

Using ruler and compasses only, draw a parallelogram whose diagonals are 4cm and 6cm long and contain an angle of 75°

Measure and write down the length of one of the shorter sides of the parallelogram.

Solution:



- (i) Draw a line segment $AC = 6\text{cm}$
- (ii) Bisect AC at O .
- (iii) At O , Draw a ray XY making an angle of 75° at O .
- (iv) From OX and OY , cut off $OD = OB = \frac{4}{2} = 2\text{cm}$
- (v) Join AB , BC , CD and DA

Thus, $ABCD$ is the required parallelogram

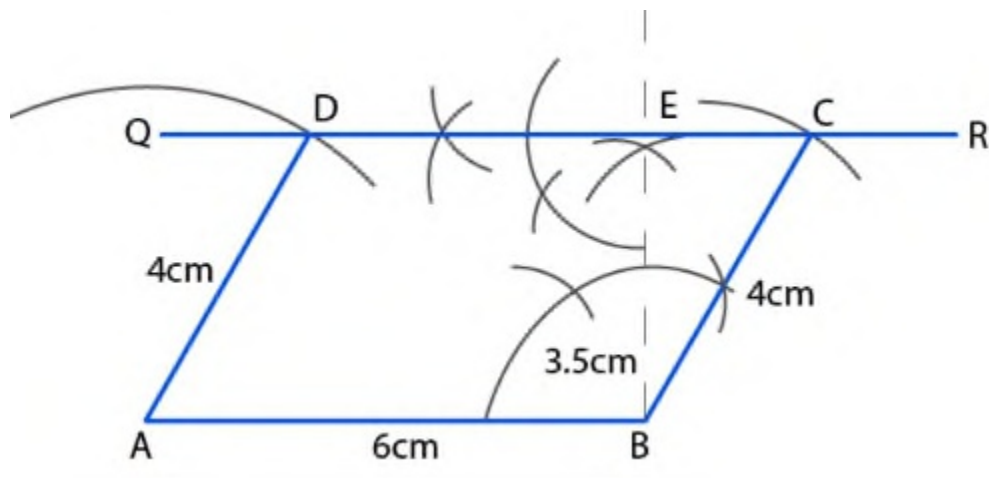
On measuring one of the shorter sides, we get

$$AB = CD = 3\text{cm}$$

Question 13.

Using ruler and compasses only, construct a parallelogram $ABCD$ with $AB = 6\text{cm}$, altitude $= 3.5\text{cm}$ and side $BC = 4\text{cm}$. Measure the acute angles of the parallelogram.

Solution:



Steps of construction:

- (i) Draw $AB = 6\text{cm}$
- (ii) At B , Draw $BP \perp AB$

(iii) From BP, cut $BE = 3.5 \text{ cm} = \text{height of } \parallel \text{ gm}$

(iv) through E draw QR parallel to AB

(v) With B as centre and radius $BC = 4\text{cm}$ draw an arc which cuts QR at C.

(vi) Since, opposite sides of $\parallel \text{ gm}$ are equal

So, $AD = BC = 4\text{cm}$

(vii) with A as centre and radius $= 4\text{cm}$ draw an arc which cuts QR at D.

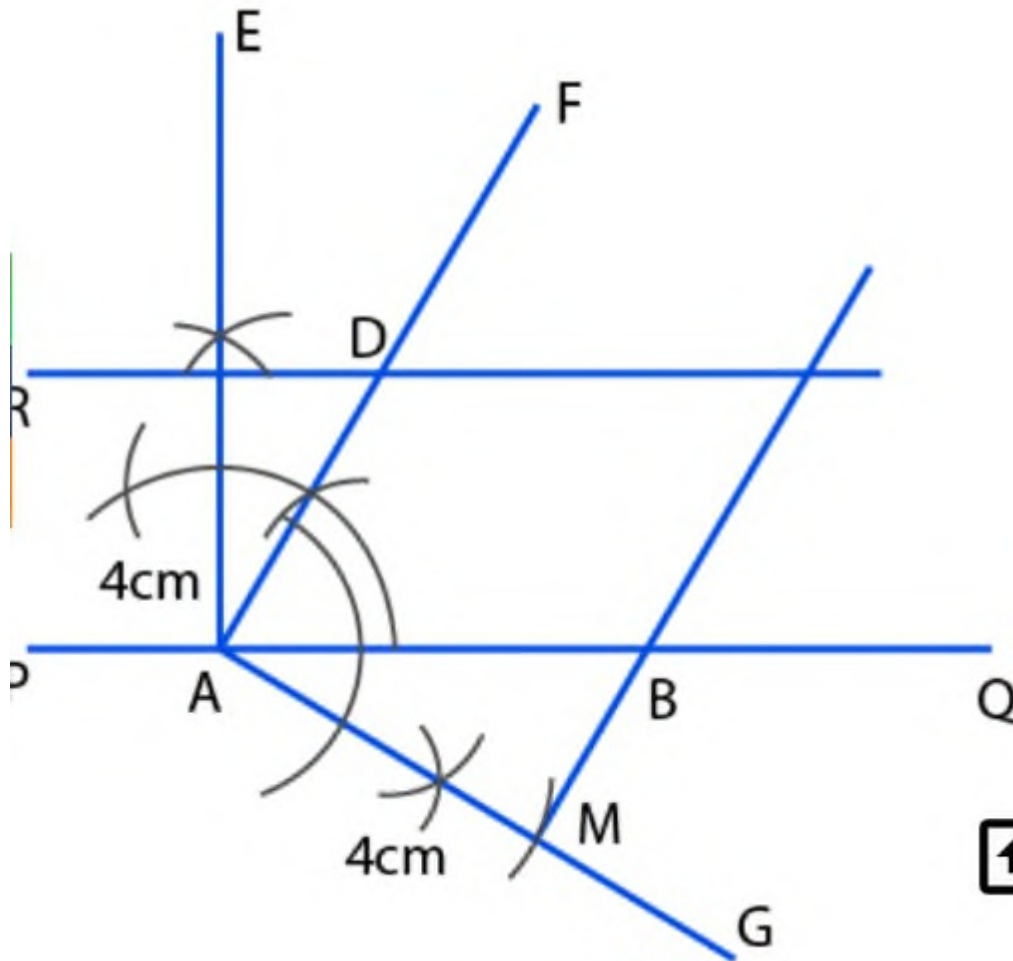
Thus, ABCD is the required parallelogram.

(viii) to measure the acute angle of parallelogram which is equal to 61°

Question 14.

The perpendicular distances between the pairs of opposite sides of a parallelogram ABCD are 3cm and 4cm and one of its angles measures 60° . Using ruler and compasses only, construct ABCD.

Solution:



Steps of construction:

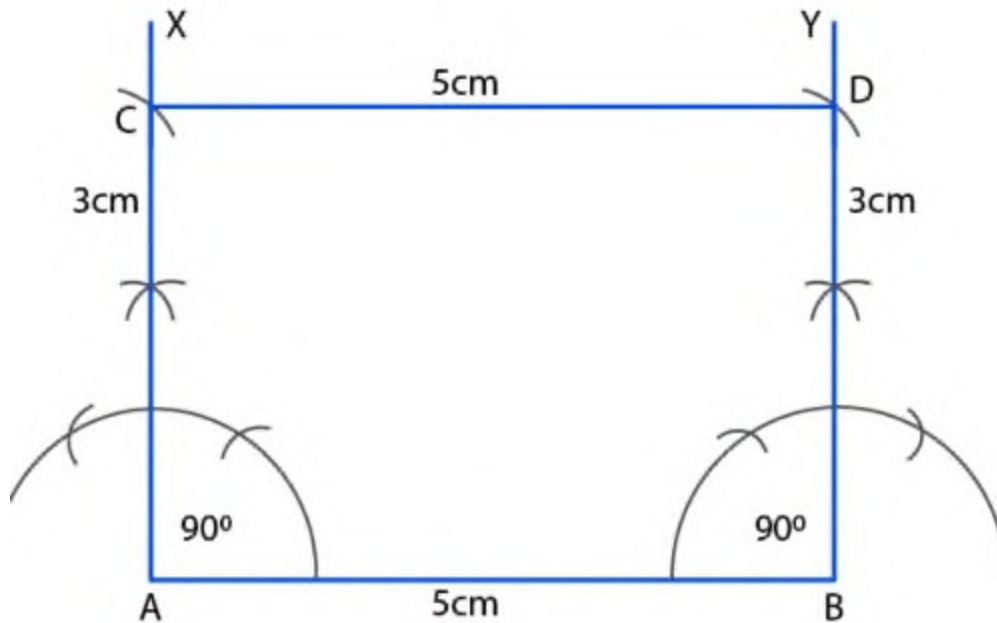
- (i) Draw a straight – line PQ, take a point A on it.
- (ii) At A, construct $\angle QAF = 60^\circ$
- (iii) At A draw $AE \perp PQ$ from AE cut $AN = 3\text{cm}$
- (iv) Through N draw a straight line to PQ to meet AF at D.
- (v) At D, Draw $AG \perp AD$, from AG cut of $AM = 4\text{cm}$
- (vi) Through M, Draw a straight line parallel to meet AQ in B and ND in C.

Then , ABCD is the required parallelogram

Question 15

Using ruler and compasses, construct a rectangle ABCD with $AB = 5\text{cm}$ and $AD = 3\text{cm}$.

Solution:



Steps of construction:

- (i) Draw a straight – line $AB = 5\text{cm}$
- (ii) At A and B construct $\angle XAB$ and $\angle YBA = 90^\circ$
- (iii) From A and B cut off AC and BD = 3cm each
- (iv) join CD

Thus, ABCD is the required rectangle.

Question 16.

Using ruler and compasses only, construct a rectangle each of whose diagonals measures 6cm and the diagonals intersect at an angle of 45° .

Solution:

Steps of construction:

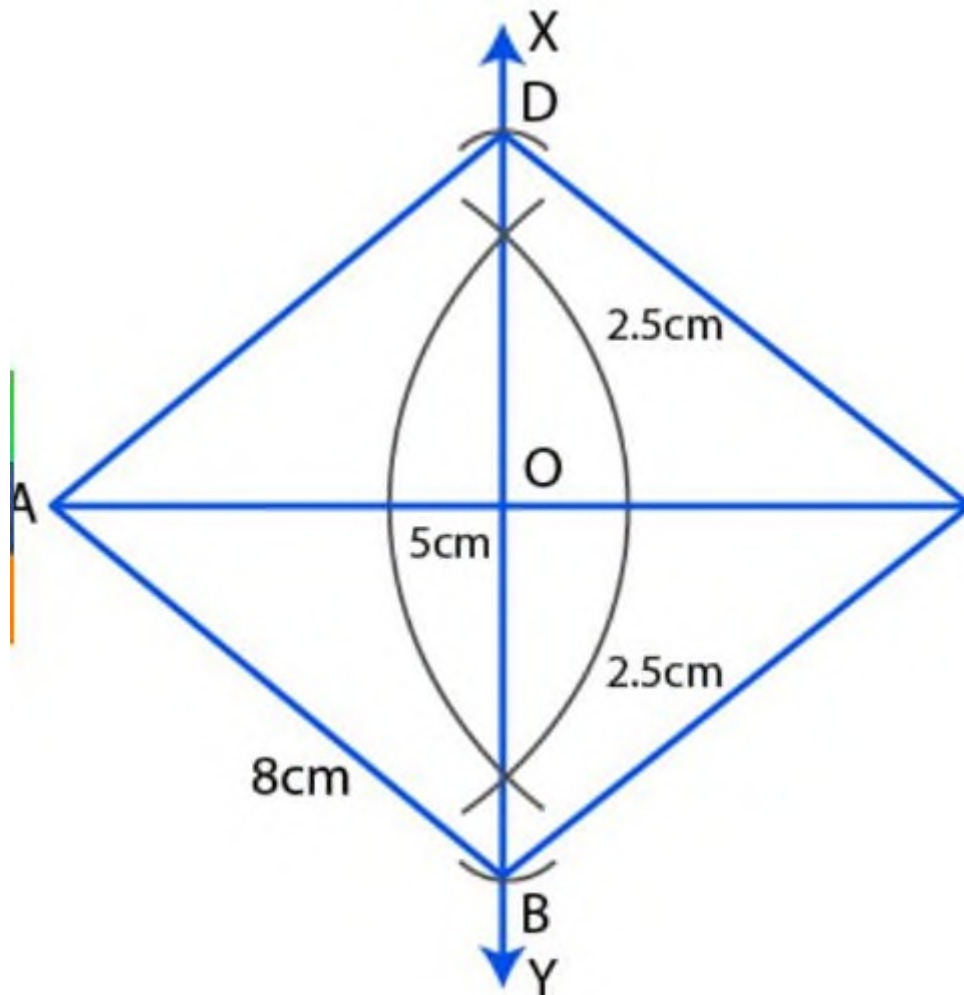
- (i) Draw a line segment $AC = 6\text{cm}$
- (ii) Bisect AC at O .
- (iii) At O draw a ray XY making an angle of 45° at O .
- (iv) From xy , cut off $OB = OD = \frac{6}{2} = 3\text{cm}$ each
- (v) join AB, BC, CD and DA .

Thus, $ABCD$ is the required rectangle.

Question 17.

Using ruler and compasses only, construct a square having a diagonal of length 5cm. Measure its sides correct to the nearest millimetre.

Solution:



Steps of construction:

- (i) Draw a line segment $AC = 5\text{cm}$

(ii) Draw its perpendicular bisector XY bisecting it at O

(iii) From XY, cut off

$$OB = OD = \frac{5}{2} = 2.5\text{cm}$$

(iv) join AB, BC, CD and DA.

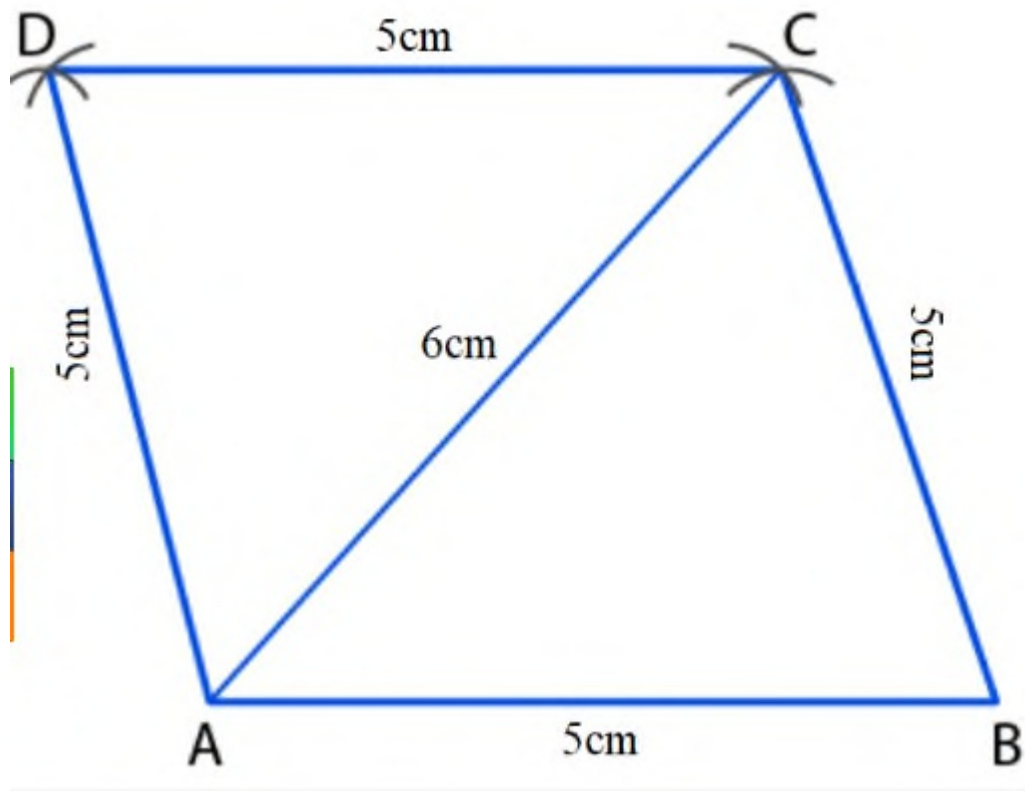
Thus, ABCD is the required square

On measuring its sides, each = 3.6cm (approximately)

Question 18.

Using ruler and compasses only construct A rhombus ABCD given that AB 5cm, AC = 6cm measure $\angle BAD$.

Solution:



Steps of construction:

- (i) Draw a line segment $AB = 5\text{cm}$
- (ii) with centre A and radius 6cm, with centre B and radius 5cm draw arcs intersecting each other at C.
- (iii) Join AC and BC
- (iv) with centre A and C and radius 5cm, draw arc intersecting each other 5cm, draw arcs intersecting each other at D
- (v) Join AD and CD.

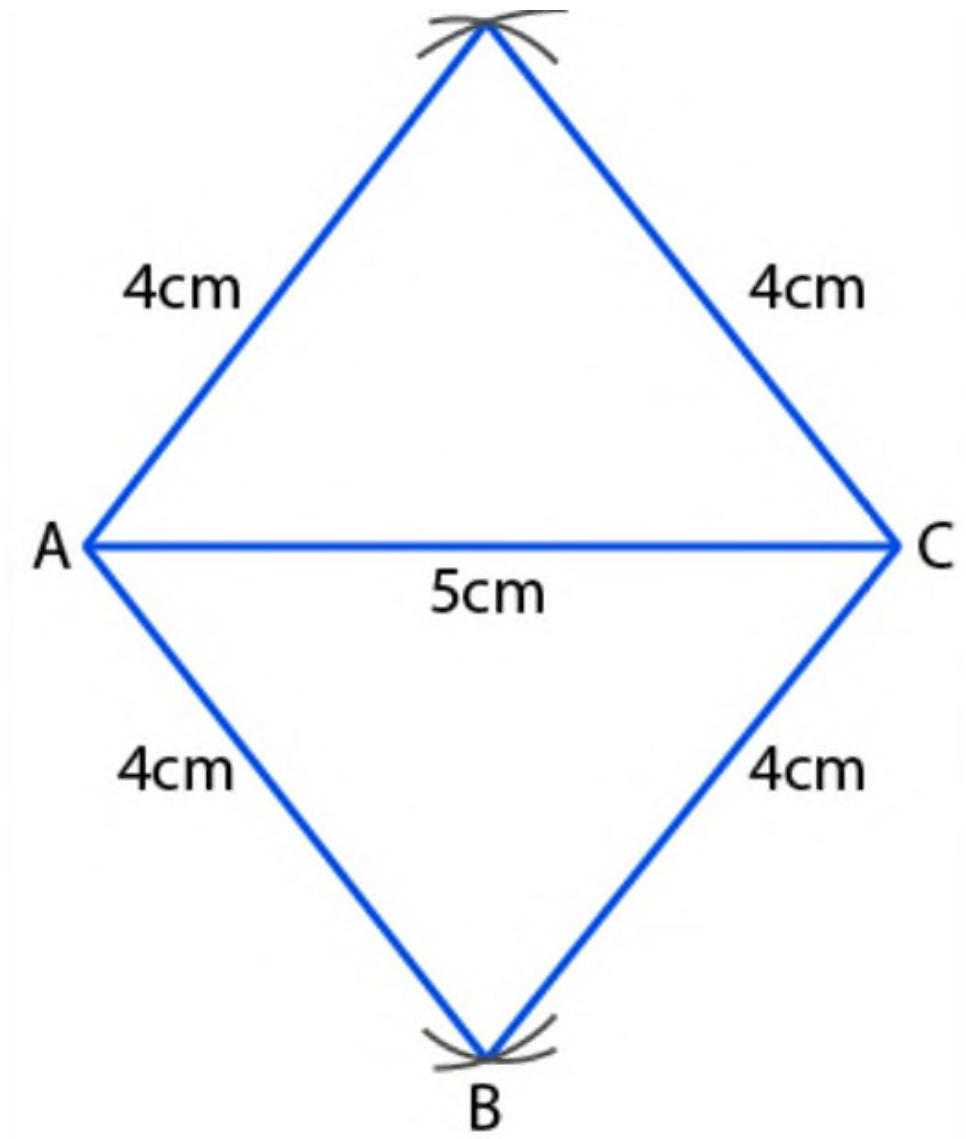
Thus, ABCD is a rhombus

On measuring , $\angle BAD = 106^\circ$.

Question 19.

Using ruler and , construct rhombus ABCD with sides of length 4cm and diagonal AC of length 5cm . measure $\angle ABC$.

Solution: D



Steps of construction:

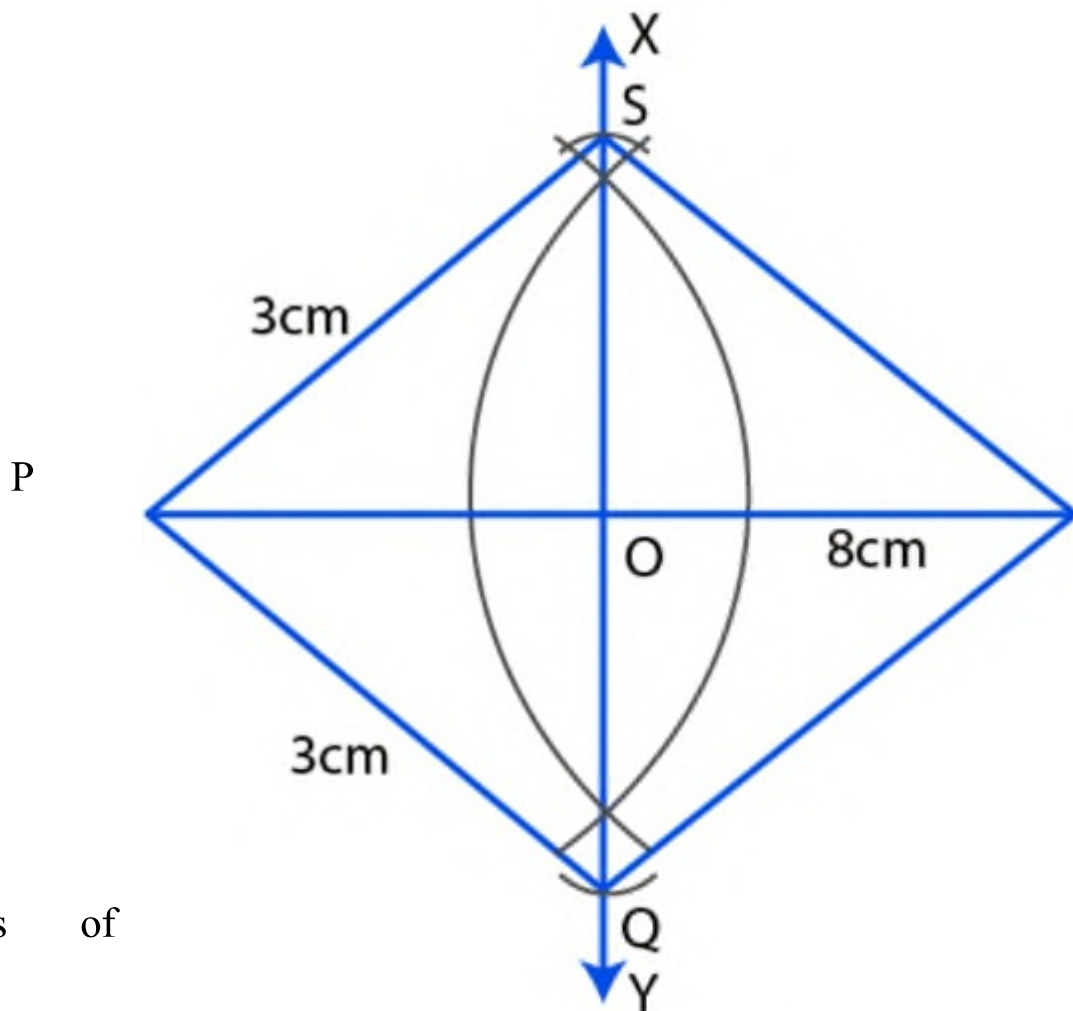
- (i) Draw a line segment $AC = 5\text{cm}$
- (ii) with centre A and c and radius 4cm , draw arcs intersecting each other above and below AC at D and B.
- (iii) join AB, BC , CD and DA.

Thus, ABCD is the required rhombus.

Question 20.

Construct a rhombus PQRS whose diagonals PR and QS are 8 cm and 6 cm respectively

Solution:



Steps of

construction:

(i) Draw a line segment $PR = 8\text{ cm}$

(ii) Draw its perpendicular bisector XY intersecting it at O.

(iii) From XY, cut off $OQ = OS = \frac{6}{2} = 3\text{cm each}$

(iv) join PQ, PR, RS and SP

Thus, PQRS and SP

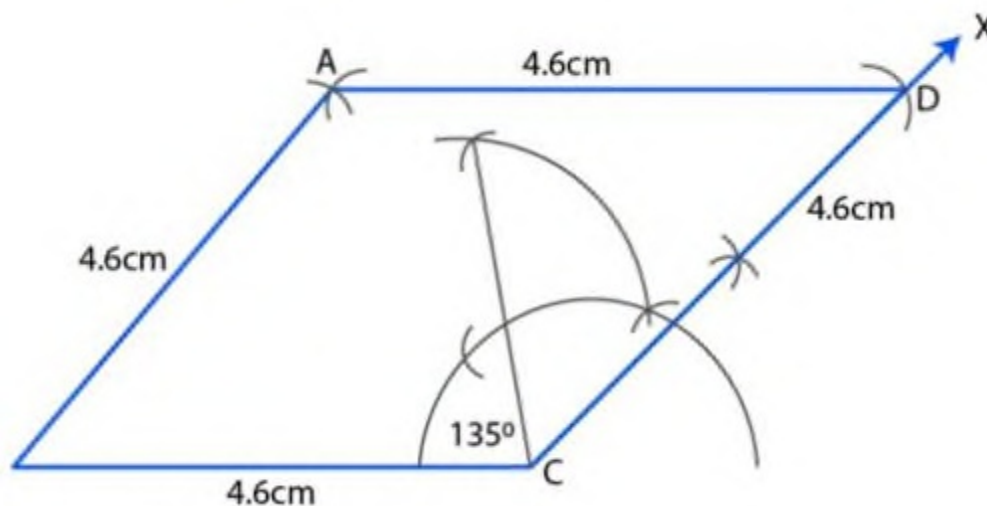
Thus, PQRS is the required rhombus.

Question 21.

Construct a rhombus ABCD of side 4.6 cm and $\angle BCD = 135^\circ$.

By using ruler and compasses only.

Solution:



Steps of construction:

(i) Draw a line segment $BC = 4.6\text{cm}$

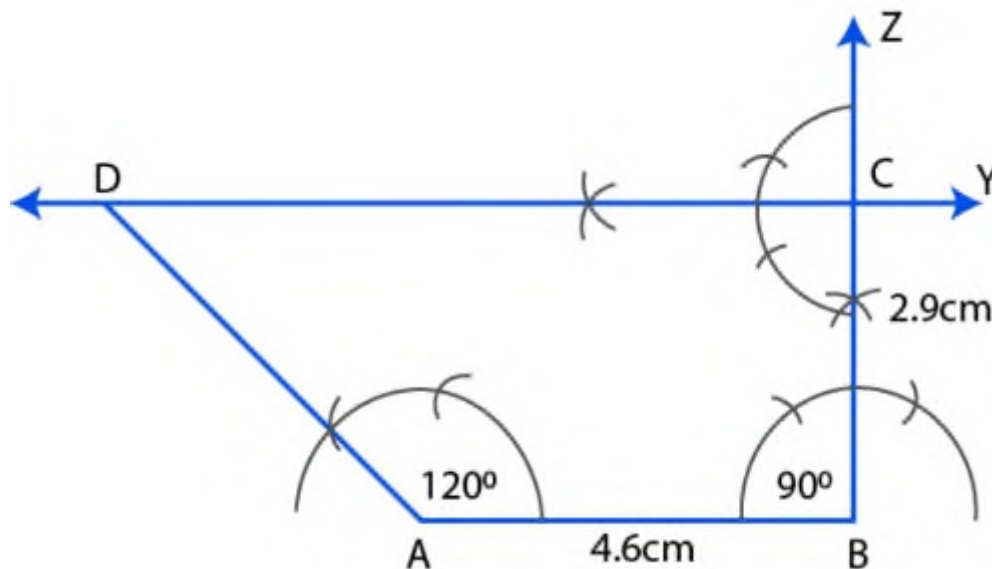
(ii) At C, Draw a ray CX making an angle of 135° and cut off $CD = 4.6\text{cm}$

(iii) with centres B and D, and radius 4.6cm draw arcs intersecting each other at A.

Thus, ABCD is the required rhombus.

Question 22.

Construct a trapezium in which $AB \parallel CD$, $AB = CD$, $AB = 4.6\text{cm}$, $\angle ABC = 90^\circ$ $\angle DAB = 120^\circ$ and the distance between parallel sides is 2.9 cm.



Steps of construction:

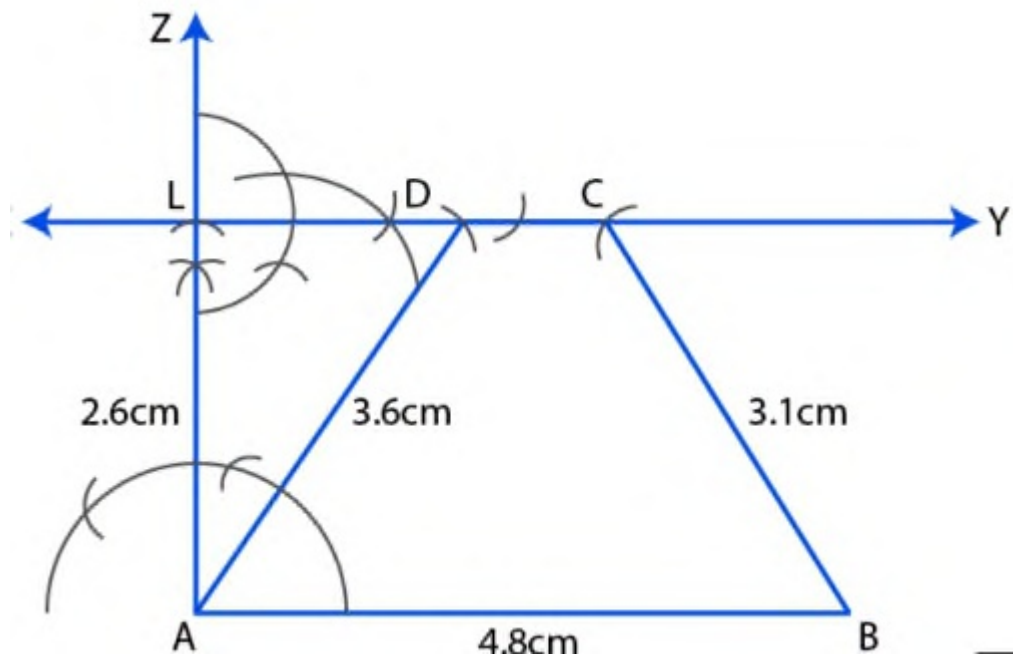
- (i) Draw a line segment $AB = 4.6\text{cm}$
- (ii) At B, Draw a ray BZ making an angle of 90° and cut off $BC = 2.9\text{cm}$ (distance between AB and CD)
- (iii) At C, Draw a parallel line XY to AB.
- (iv) At A, Draw a ray making an angle of 120° meeting XY at D.

Thus. ABCD is the required trapezium.

Question 23.

Construct a trapezium ABCD when one of parallel sides $AB = 4.8$ cm, height = 2.6cm, $BC = 3.1$ cm and $AD = 3.6$ cm

Solution:



Step construction:

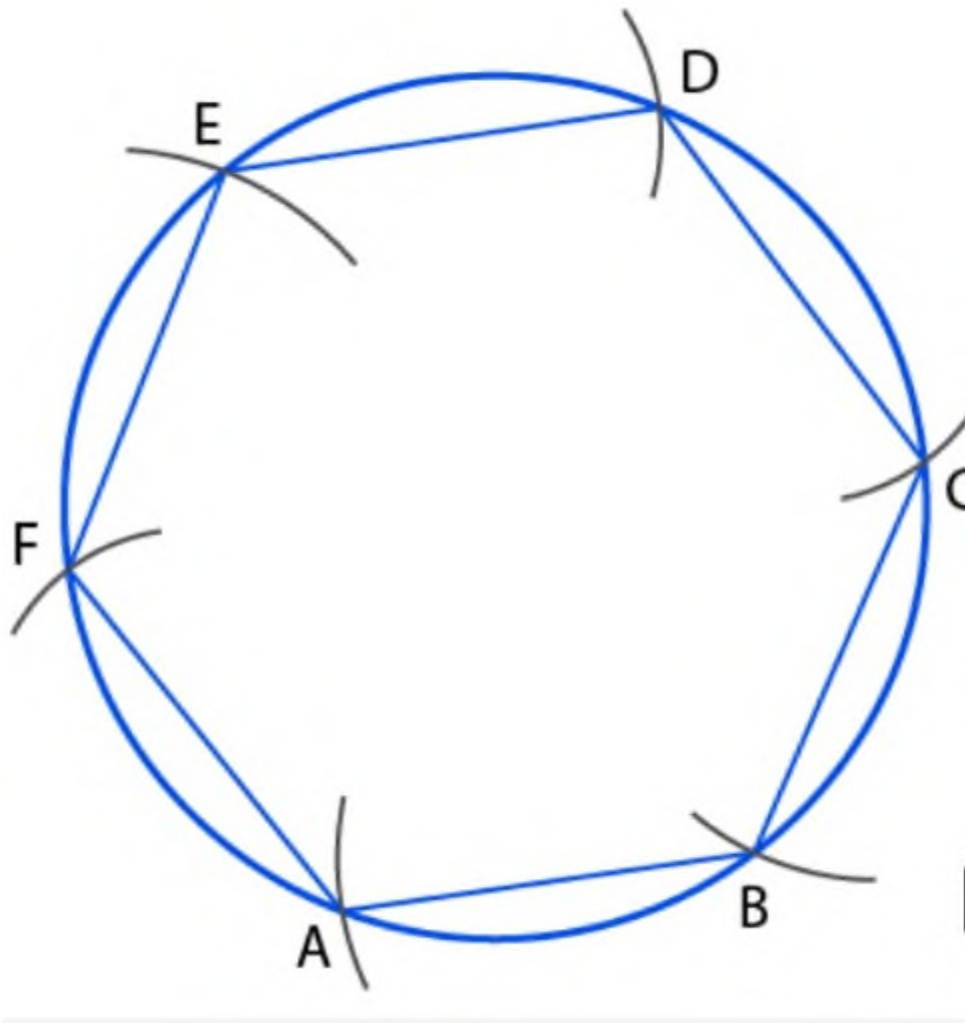
- (i) Draw a line segment $AB = 4.8$ cm
- (ii) At A, draw a ray AZ making an angle of 90° cut off $AL = 2.6$ cm
- (iii) At L, draw a line XY parallel to AB .
- (iv) with centre A and radius 3.6cm and with centre B and radius 3.1cm, draw arcs intersecting XY at D and C respectively.
- (v) join AD and BC

Thus, ABCD is the required trapezium.

Question 24.

Construct a regular hexagon of side 2.5cm.

Solution:



Steps of construction:

- (i) With O as centre and radius = 2.5cm, draw a circle
- (ii) take any point A on the circumference of circle.
- (iii) with A as centre and radius = 2.5cm, draw an arc which cuts the circumference of circle at B.

(iv) with B as centre and radius = 2.5cm, draw an arc which cuts the circumference of circle at C.

(v) with C as centre and radius = 2.5cm, draw an arc which cuts the circumference of circle at D.

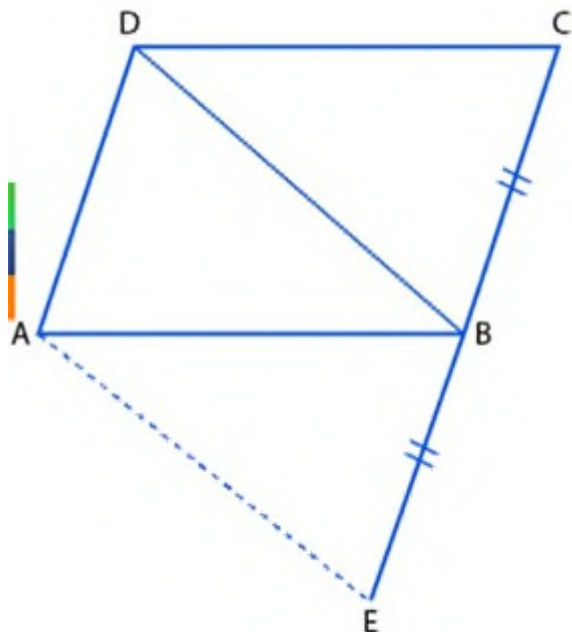
(vi) with D as centre and radius = 2.5cm, draw an arc which cuts the circumference of circle at E.

(vii) with E as centre and radius = 2.5cm. draw an arc which cuts the circumference of circle at F.

(viii) join AB, BC, CD, DE, EF and FA.

Chapter test

1. in the given figure, ABCD is a parallelogram . CB is produced to E such that $BE = BC$. Prove that AEBD is a parallelogram.



Solution

Given ABCD is a $\parallel$ gm in which CB is produced to E such that $BE = BC$
BD and AE are joined

To prove : AEBD is a parallelogram

Proof:

In ΔAEB and ΔBDC

$EB = BC$ [given]

$\angle ABE = \angle DCB$ [corresponding angles]

$AB = DC$ [opposite sides of $\parallel$ gm]

Thus, $\triangle AEB \cong \triangle BDC$ by S.A.S axiom

So, by C.P.C.T

But , $AD = CB = BE$ [given]

As the opposite sides are equal and $\angle AEB = \angle DBC$

But these are corresponding angles

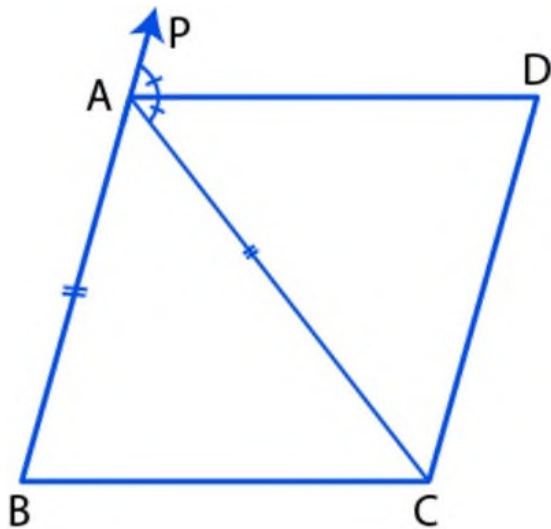
Hence, $AEBD$ is a parallelogram .

2. in the given figure , ABC is an isosceles triangle in which $AB = AC$. AD bisects exterior angle PAC and $CD \parallel BA$. show that (i) $\angle DAC = \angle BCA$ (ii) $ABCD$ is parallelogram.

Solution

Given: In isosceles triangle ABC , $AB = AC$. AD is the bisector of ext. $\angle PAC$ and $CD \parallel BA$

To prove: (i) $\angle DAC = \angle BCA$



(ii) ABCD is a || gm

Proof:

In ΔABC

$AB = AC$ [given]

$\angle C = \angle B$ [angle opposite to equal sides]

Since , ext. $\angle PAC = \angle B + \angle C$

$= \angle C + \angle C$

$= 2\angle C$

$= 2\angle BCA$

So, $\angle DAC = 2 \angle BCA$

$\angle DAC = \angle BCA$

But these are alternate angles

Thus, $AD \parallel BC$

But, $AB \parallel AC$

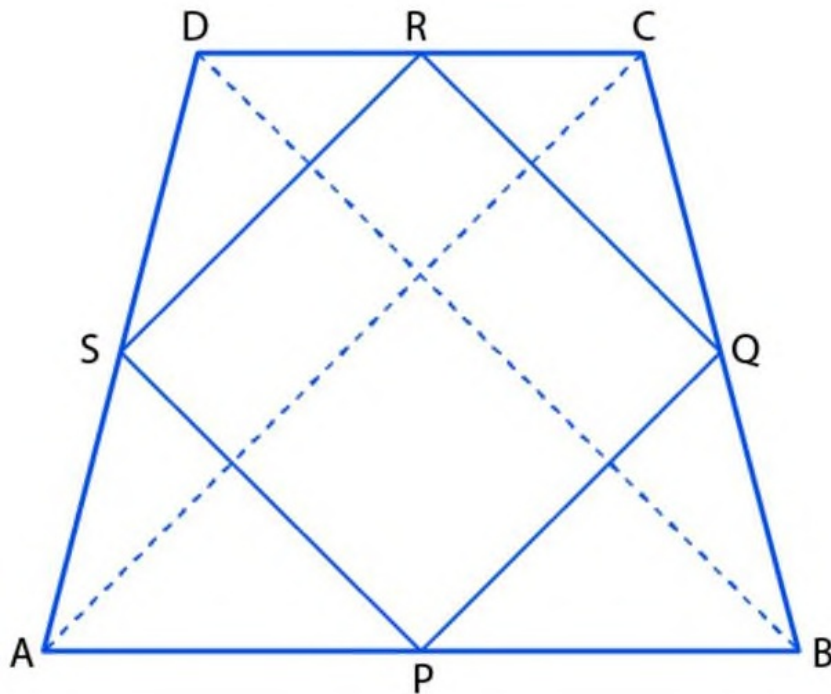
Hence ABCD is a || gm .

3. prove that the quadrilateral obtained by joining the mid-points of an isosceles trapezium is a rhombus.

Solution

Given : ABCD is an isosceles trapezium in which $AB \parallel DC$ and $AD = BC$

P, Q, R and S are the mid – points of the sides AB, BC, CD and DA respectively PQ, QR, RS and SP are joined.



To prove : PQRS is a rhombus

Construction : join AC and BD

Proof :

Since, ABCD is an isosceles trapezium

Its diagonals are equal

$$AC = BD$$

Now, in ΔABC

P and Q are the mid- points of AB and BC

$$\text{So, } PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \dots\dots(i)$$

Similarly, in ΔADC

S and R mid – point of CD and AD

So, $SR \parallel AC$ and $SR = \frac{1}{2} AC \dots (ii)$

From (i) and (ii) we have

$PQ \parallel SR$ and $PQ = SR$

Thus, PQRS is a parallelogram.

Now, in ΔAPS and ΔBPQ

$AP = BP$ [p is the mid – point]

$AS = BQ$ [half of equal sides]

$\angle A = \angle B$ [as ABCD is an isosceles triangle]

So, $\Delta APS \cong \Delta BPQ$ by SAS axiom of congruency

Thus by C.P.C.T we have

$PS = PQ$

But there are the adjacent sides of a parallelogram

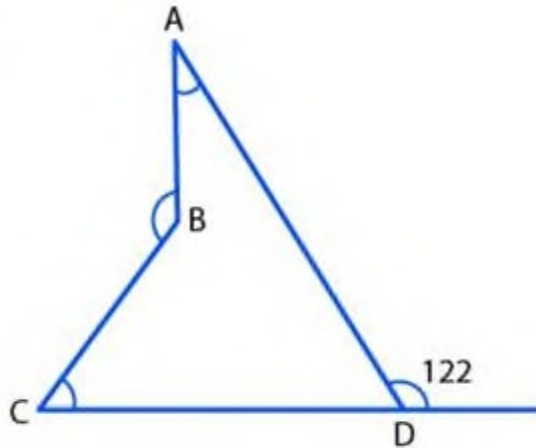
So, sides of PQRS are equal

Hence, PQRS is a rhombus

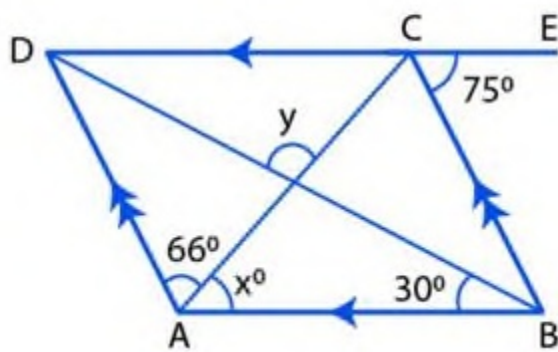
Hence proved

4. Find the size of each lettered angle in the following figures.

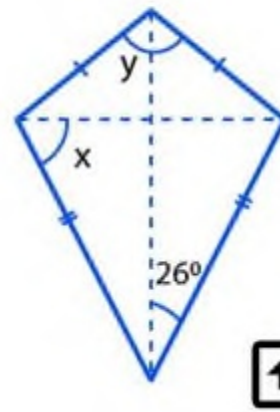
(i)



(ii)



(iii)



Solution

(i) as CDE is a straight line

$$\angle ADE + \angle ADC = 180^\circ$$

$$122^\circ + \angle ADC = 180^\circ$$

$$\angle ADC = 180^\circ - 122^\circ = 58^\circ \dots (i)$$

$$\angle ABC = 360^\circ - 140^\circ = 220^\circ \dots (ii)$$

[at any point the angle is 360°]

Now, in quadrilateral ABCD we have

$$\angle ADC + \angle BCD + \angle BDA + \angle ABC = 360^\circ$$

$$58^\circ + 53^\circ + x + 220^\circ = 360^\circ \text{ [using (i) and (ii)]}$$

$$331^\circ + x = 360^\circ$$

$$x = 360^\circ - 331^\circ$$

$$x = 29^\circ$$

(ii) As $DE \parallel AB$ [given]

$$\angle ECB = \angle CBA \text{ [alternate angles]}$$

$$75^\circ = \angle CBA$$

$$= \angle CBA = 75^\circ$$

Since , $AD \parallel BC$ we have

$$(x + 66^\circ) + 75^\circ = 180^\circ$$

$$x + 141^\circ = 180^\circ$$

$$x = 180^\circ - 141^\circ$$

$$x = 39^\circ \dots (i)$$

Now in ΔAMB

$$x + 30^\circ + \angle AMB = 180^\circ \text{ [Angles sum property of a triangle]}$$

$$39^\circ + 30^\circ + \angle AMB = 180^\circ \text{ [from (i)]}$$

$$69^\circ + \angle AMB = 180^\circ$$

$$\angle AMB = 180^\circ - 69^\circ = 111^\circ \dots (ii)$$

Since, $\angle AMB = y$ [vertically opposite angles]

$$= y = 111^\circ$$

Hence $x = 39^\circ$ and $y = 111^\circ$

(iii) In $\triangle ABD$

$$AB = AD \text{ [given]}$$

$$\angle ABD = \angle ADB \text{ [angles opposite to equal sides are equal]}$$

$$\angle ABD = 42^\circ \text{ [since, given } \angle ADB = 42^\circ \text{]}$$

And ,

$$\angle ABD + \angle ADB + \angle BAD = 180^\circ \text{ [angles sum property of a triangle]}$$

$$42^\circ + 42^\circ + y = 180^\circ$$

$$84^\circ + y = 180^\circ$$

$$Y = 180^\circ - 84^\circ$$

$$Y = 96^\circ$$

$$\angle BCD = 2 \times 26^\circ = 52^\circ$$

In $\triangle BCD$,

$$\text{As } BC = CD \text{ [given]}$$

$$\angle CBD = \angle CDB = X \text{ [angle opposite to equal sides are equal]}$$

$$\angle CBD + \angle CDB + \angle BCD = 180^\circ$$

$$X + x + 52^\circ = 180^\circ$$

$$2x + 52^\circ = 180^\circ$$

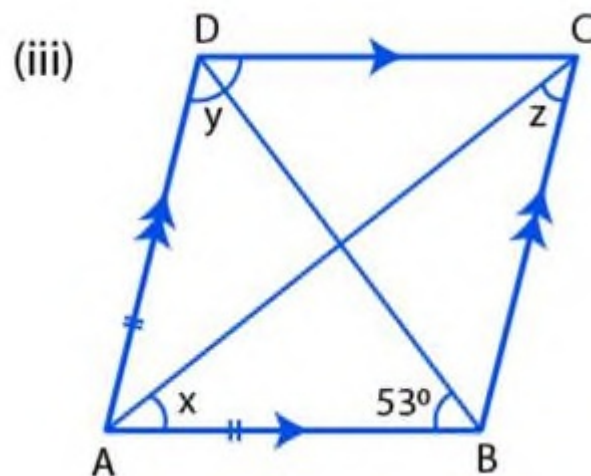
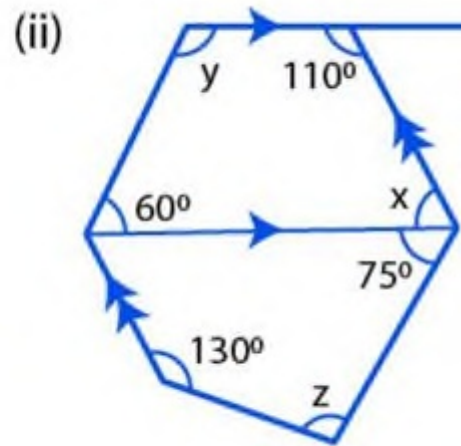
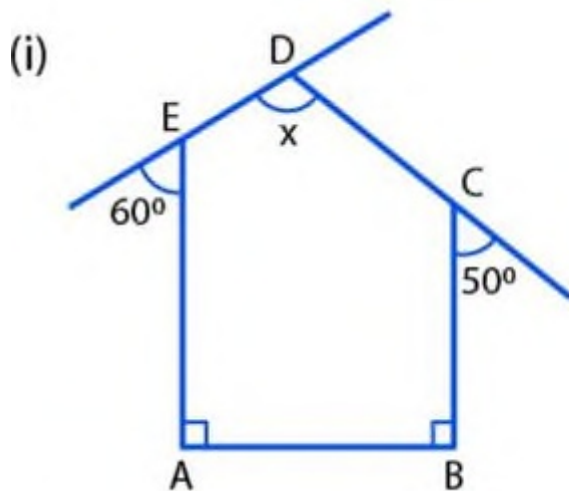
$$2x = 180^\circ - 52^\circ$$

$$X = \frac{120^\circ}{2}$$

$$X = 64^\circ$$

Hence , $x = 64^\circ$ and $y = 90^\circ$

5. find the size of each lettered angle in the following figures:



Solution

(i) here , $AB \parallel CD$ and $BC \parallel AD$

So, ABCD is a $\parallel$ gm

$$Y = 2 \times \angle ABD$$

$$Y = 2 \times 53^\circ = 106^\circ \dots (1)$$

Also , $y + \angle DAB = 180^\circ$

$$\angle DAB = 180^\circ - 106^\circ$$

$$= 74^\circ$$

Thus , $x = \frac{1}{2} \angle DAB$ [as AC bisects $\angle DAB$]

$$X = \frac{1}{2} \times 74^\circ = 37^\circ$$

And $\angle DAB = x = 37^\circ \dots (ii)$

Also , $\angle DAB = z \dots (iii)$ [Alternative angles]

From (ii) and (iii)

$$Z = 37^\circ$$

Hence $x = 37^\circ$, $y = 106^\circ$ and $z = 37^\circ$

(ii) As ED is a straight line, we have

$$60^\circ + \angle AED = 180^\circ \text{ [linear pair]}$$

$$\angle AED = 180^\circ - 60^\circ$$

$$\angle AED = 120^\circ \dots (i)$$

Also , as CD is a straight line

$$50^\circ + \angle BCD = 180^\circ \text{ [linear pair]}$$

$$\angle BCD = 180^\circ - 50^\circ$$

$$\angle BCD = 130^\circ \dots (ii)$$

In pentagon ABCDE, we have

$$\angle A + \angle B + \angle AED + \angle BCD + \angle x = 540^\circ \text{ [sum of interior angles in pentagon is } 540^\circ \text{]}$$

$$90^{\circ} + 90^{\circ} + 120^{\circ} + 130^{\circ} + x = 540^{\circ}$$

$$430^{\circ} + x = 540^{\circ}$$

$$x = 540^{\circ} - 430^{\circ}$$

$$x = 110^{\circ}$$

Hence , value of $x = 110^{\circ}$

(iii) in given figure, $AD \parallel BC$ [given]

$$60^{\circ} + y = 180^{\circ} \text{ and } x + 110^{\circ} = 180^{\circ}$$

$$y = 180^{\circ} - 60^{\circ} \text{ and } x = 180^{\circ} - 110^{\circ}$$

$$y = 120^{\circ} \text{ and } x = 70^{\circ}$$

Since , $CD \parallel AF$ [given]

$$\angle FAD + 75^{\circ} + z + 130^{\circ} = 360^{\circ}$$

$$70^{\circ} + 75^{\circ} + z + 130^{\circ} = 360^{\circ}$$

$$275^{\circ} + z = 360^{\circ}$$

$$z = 360^{\circ} - 275^{\circ} = 85^{\circ}$$

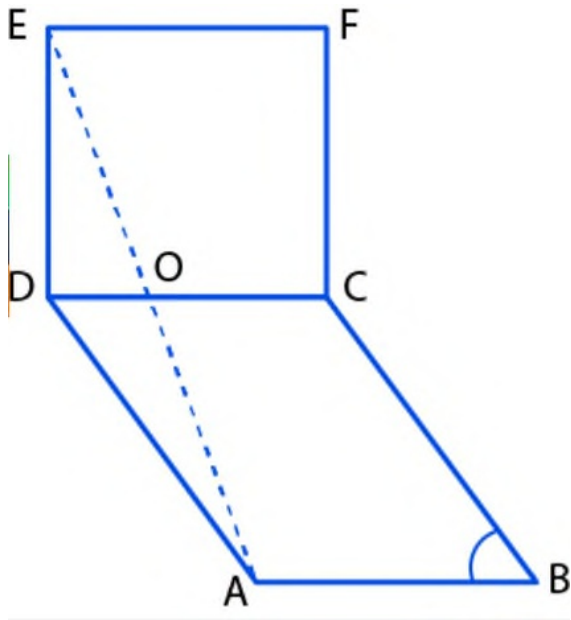
Hence ,

$$x = 70^{\circ} , y = 120^{\circ} \text{ and } z = 85^{\circ}$$

6. in the adjoining figure, ABCD is a rhombus and DCFE is a square. If $\angle ABC = 56^{\circ}$, find (i) $\angle DAG$ (ii) $\angle FEG$ (iii) $\angle GAC$ (iv) $\angle AGC$.

Solution

Here ABCD and DCEF is a rhombus and square respectively .



So $AB = BC = DC = AD \dots (i)$

Also , $DC = EF = FC = EF \dots (ii)$

From (i) and (ii) , we have

$AB = BC = DC = AD = EF = FC = EF \dots (iii)$

$\angle ABC = 56^\circ$ [Given]

$\angle ADC = 56^\circ$ [opposite angle in rhombus are equal]

So, $\angle EDA = \angle EDC + \angle ADC = 90^\circ + 56^\circ = 146^\circ$

In $\triangle ADE$,

$DE = AD$ [from (iii)]

$\angle DEA = \angle DAE$ [equal sides have equal opposite angles]

$$\angle DEA = \angle DAG = \frac{180^\circ - \angle EDA}{2}$$

$$= \frac{180^\circ - 146^\circ}{2}$$

$$= \frac{34^\circ}{2} = 17^\circ$$

$$\angle DAG = 17^\circ$$

Also

$$\angle DEG = 17^\circ$$

$$\angle FEB = \angle E - \angle DEG$$

$$= 90^\circ - 17^\circ$$

$$= 73^\circ$$

In rhombus ABCD

$$\angle DAB = 180^\circ - 56^\circ = 124^\circ$$

$$\angle DAC = \frac{124^\circ}{2} \text{ [since , AC diagonals bisect the } \angle A \text{]}$$

$$\angle DAC = 62^\circ$$

$$\angle GAC = \angle DAC - \angle DAG$$

$$= 62^\circ - 17^\circ$$

$$= 45^\circ$$

In ΔEDG ,

$$\angle D + \angle DEG + \angle DGE = 180^\circ \text{ [angles sum property of a triangle]}$$

$$90^\circ + 17^\circ + \angle DGE = 180^\circ$$

$$\angle DGE = 180^\circ - 107^\circ = 73^\circ \text{(iv)}$$

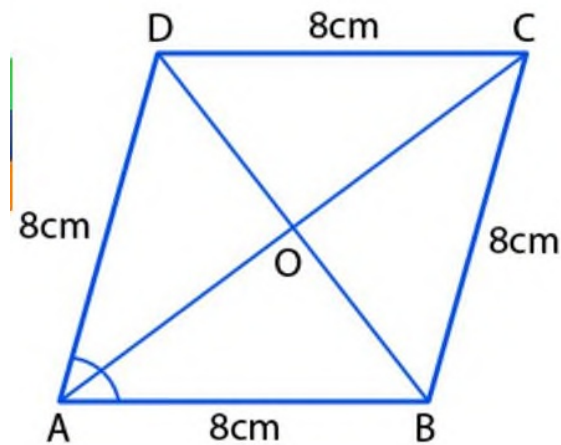
$$\text{Thus, } \angle AGC = \angle DGE \text{(v) [vertically opposite angles]}$$

Hence from (iv) and (v) we have

$$\angle AGC = 73^\circ$$

7. if one angle of a rhombus is 60° and the length of a side is 8 cm, find the lengths of its diagonals.

Solution



Each side of rhombus ABCD is 8 cm

So, $AB = BC = CD = DA = 8 \text{ cm}$

Let $\angle A = 60^\circ$

So, $\triangle ABD$ is an equilateral triangle

Then ,

$AB = BD = AD = 8 \text{ cm}$

As we know the diagonals of a rhombus bisect each other at right angle

$AO = OC$, $BO = OD = 4 \text{ cm}$ and $\angle AOB = 90^\circ$

Now, in right $\triangle AOB$

By Pythagoras theorem

$$AB^2 = AO^2 + OB^2$$

$$8^2 = AO^2 + 4^2$$

$$64 = AO^2 + 16$$

$$AO^2 = 64 - 16 = 48$$

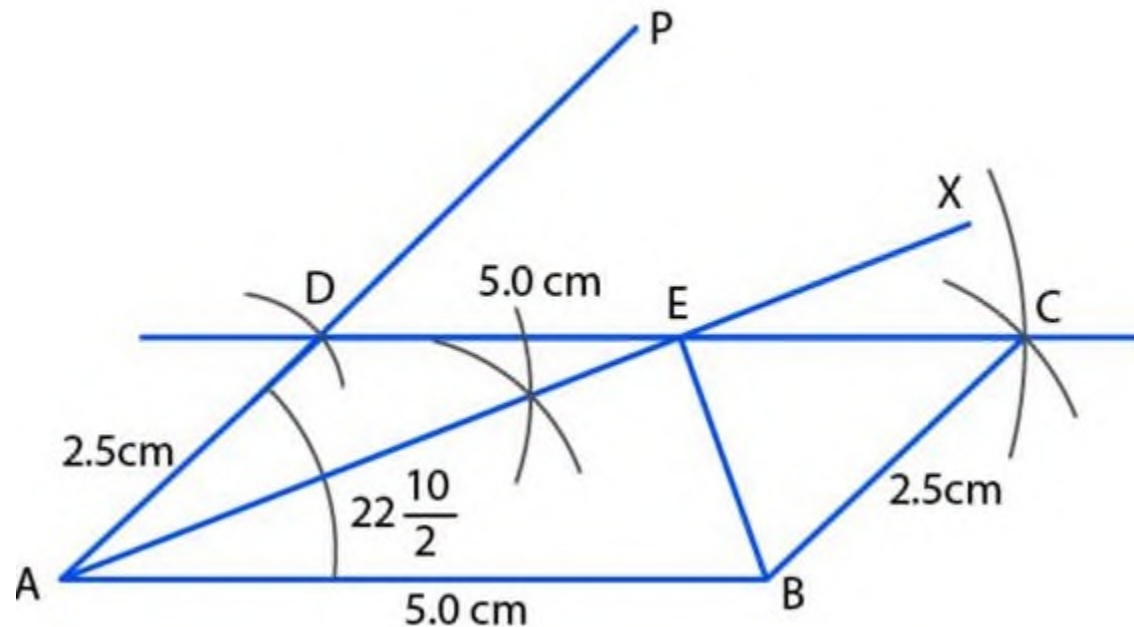
$$AO = \sqrt{48} = 3\sqrt{3}\text{CM}$$

$$\text{But , } AC = 2AO$$

$$\text{Hence , } AC = 2 \times 3\sqrt{3} = 6\sqrt{3} \text{ cm .}$$

8. using ruler and compasses only , construct a parallelogram ABCD with AB = 5 cm , AD = 2.5 cm and $\angle BAD = 45^\circ$. If the bisector of $\angle BAD$ meets DC at E, prove that $\angle AEB$ is a right angle.

Solution



Steps of construction :

(i) Draw AB = 5.0 cm

(ii) draw $\angle BAP = 45^\circ$ on side AB

- (iii) take A as centre and radius 2.5 cm cut the line AP at D
- (iv) take D as a centre and radius 5.0 cm draw an arc
- (v) take B as a centre and radius equal to 2.5 cm cut the arc of step (iv) at c .
- (vi) join BC and CD
- (vii) ABCD is the required parallelogram
- (viii) draw the bisector of $\angle BAD$, which cuts the DC at E
- (ix) join EB
- (x) measure $\angle AEB$ which is equal to 90° .