

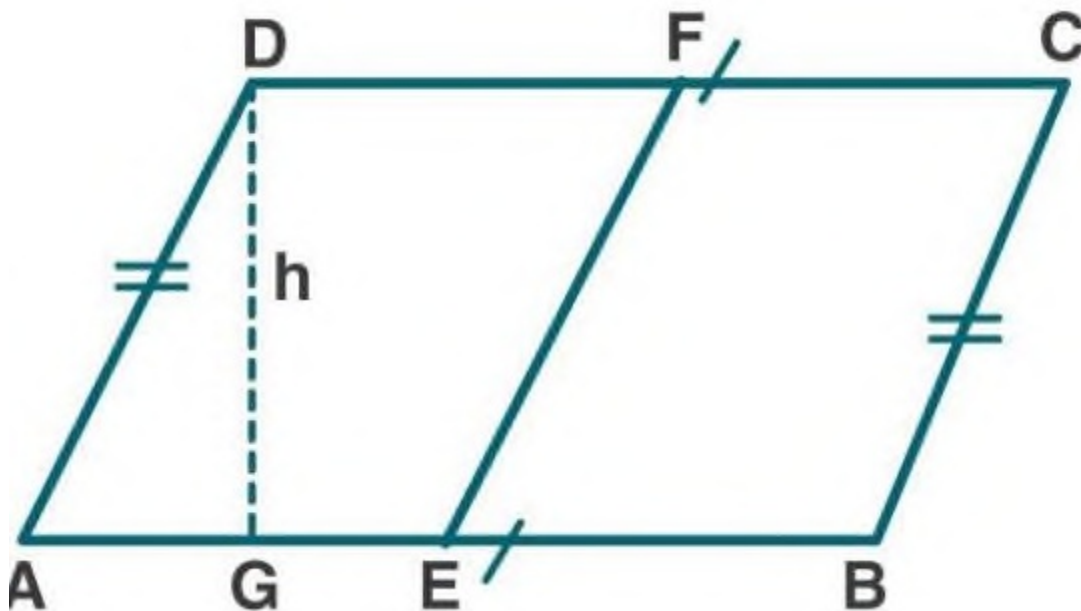
Chapter 14
Theorems on area
Exercise 14

1. prove that the line segments joining the mid – points of a pair of opposite sides of a parallelogram divides it into two equal parallelograms.

Solution

Let us consider ABCD be a parallelogram in which E and F are mid- points of AB and CD. join EF.

To prove : $\text{ar}(\parallel \text{AEFD}) = \text{ar}(\parallel \text{EBCF})$



Let us construct $DG \perp AG$ and let $DG = h$ where, h is the altitude on side AB.

Proof :

$$\text{ar}(\parallel ABCD) = AB \times h$$

$$\text{ar}(\parallel AEFD) = AE \times h$$

$$= \frac{1}{2} AB \times h \dots (1) \text{ [since, E is the mid- point of AB]}$$

$$\text{ar}(\parallel EBCF) = EF \times h$$

$$= \frac{1}{2} AB \times h \dots (2) \text{ [since , E is the mid point of AB]}$$

From(1) and (2)

$$\text{ar}(\parallel ABFD) = \text{ar}(\parallel EBCF)$$

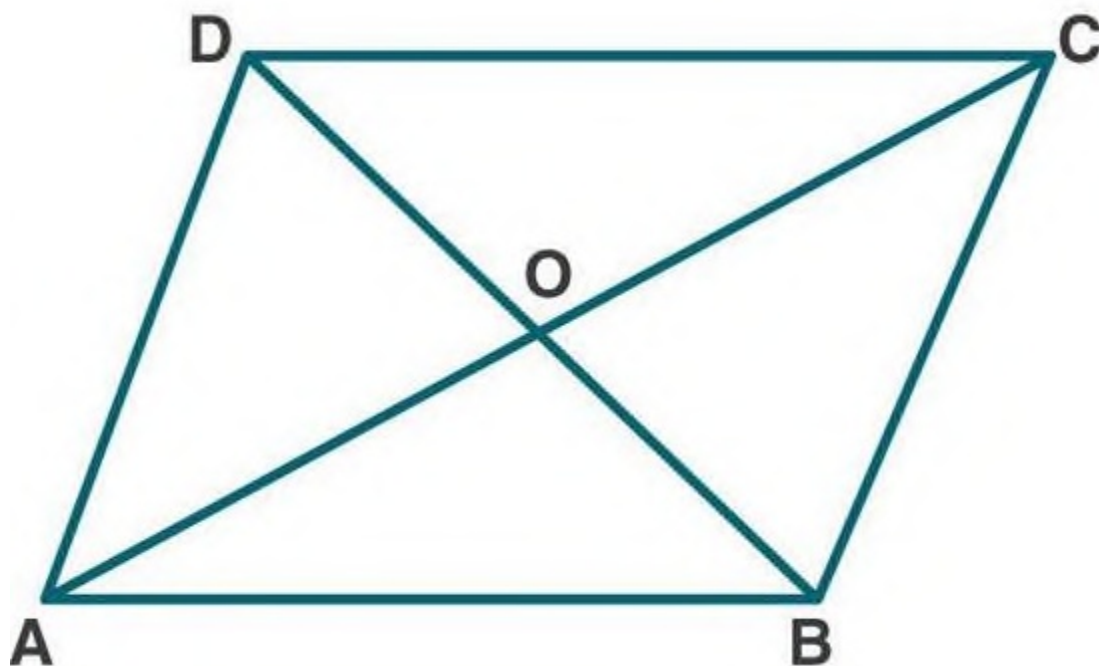
hence proved.

2. prove that the diagonals of a parallelogram divide it into four triangles of equal area.

Solution

Let us consider in a parallelogram ABCD the diagonals AC and BD are cut at point O.

To prove: $\text{ar}(\triangle AOB) = \text{ar}(\triangle BOC) = \text{ar}(\triangle COD) = \text{ar}(\triangle AOD)$



Proof:

In parallelogram ABCD the diagonals bisect each other

$$AO = OC$$

In $\triangle ACD$, O is the mid point of AC. DO is the median

$\text{ar}(\triangle AOD) = \text{ar}(\triangle COD) \dots\dots(1)$ [median of $\triangle$ divides it into two triangle of equal areas]

similarly , in $\triangle ABC$

$$\text{ar}(\triangle AOB) = \text{ar}(\triangle COB) \dots\dots(2)$$

in $\triangle ADB$

$$\text{ar}(\triangle AOD) = \text{ar}(\triangle AOB) \dots\dots(3)$$

in $\triangle CDB$

$$\text{ar}(\Delta COD) = \text{ar}(\Delta COB) \dots (4)$$

from (1),(2) ,(3) and (4)

$$\text{ar}(\Delta AOB) = \text{ar}(\Delta BOC) = \text{ar}(\Delta COD) = \text{ar}(\Delta AOD)$$

hence proved.

3. (a) in the figure (1) given below , AD is medium of ΔABC and P is any point on AD. Prove that

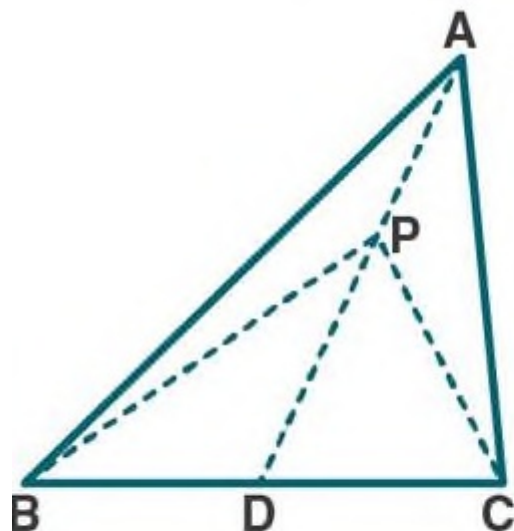
(i) area of ΔPBD = area of ΔPDC

(ii) area of ΔABP = area of ΔACP .

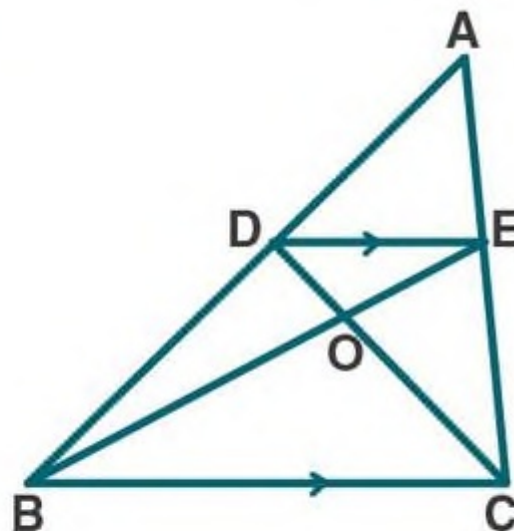
(b) in the figure(2) given below , $DE \parallel BC$. Proved that

(i) area of ΔACD = area of ΔABE

(ii) area of ΔOBD = area of ΔOCE .



(1)



(2)

Solution

(a) given :

ΔABC in which AD is the median . P is any point on AD . Join PB and PC .

To prove :

(i) area of ΔPBD = area of ΔPDC

(ii) area of ΔABP = area of ΔACP

Proof :

From fig(1)

AD is a median of ΔABC

So, $\text{ar}(\Delta ABD) = \text{ar}(\Delta ADC) \dots(1)$

Also, PD is the median of ΔBPD

Similarly , $\text{ar}(\Delta \text{PBD}) = \text{ar}(\Delta \text{PDC}) \dots(2)$

Now , let us subtract (2) from (1) we get

$$\text{ar}(\Delta \text{ABD}) - \text{ar}(\Delta \text{PBD}) = \text{ar}(\Delta \text{ADC}) - \text{ar}(\Delta \text{PDC})$$

$$\text{or } \text{ar}(\Delta \text{ABP}) = \text{ar}(\Delta \text{ACP})$$

hence proved.

(b) given:

ΔABC in which $\text{DE} \parallel \text{BC}$

To prove:

(i) area of ΔACD = area of ΔABE

(ii) area of ΔOBD = area of ΔOCE .

Proof:

From fig(2)

ΔDEC and ΔBDE are on the same base DE and between the same $\parallel$ line DE and BE .

$$\text{ar}(\Delta \text{DEC}) = \text{ar}(\Delta \text{BDE})$$

now add $\text{ar}(\Delta \text{ADE})$ on both sides , we get

$$\text{ar}(\Delta \text{DEC}) + \text{ar}(\Delta \text{ADE}) = \text{ar}(\Delta \text{BDE}) + \text{ar}(\Delta \text{ADE})$$

$$\text{ar}(\Delta \text{ACD}) = \text{ar}(\Delta \text{ABE})$$

hence proved.

Similarly ,ar (Δ DEC) = ar(ΔBDE)

Subtract ar(ΔDOE) from both sides, we get

$$\text{ar}(\Delta DEC) - \text{ar}(\Delta DOE) = \text{ar}(\Delta BDE) - \text{ar}(\Delta DOE)$$

$$\text{ar}(\Delta OBD) = \text{ar}(\Delta OCE)$$

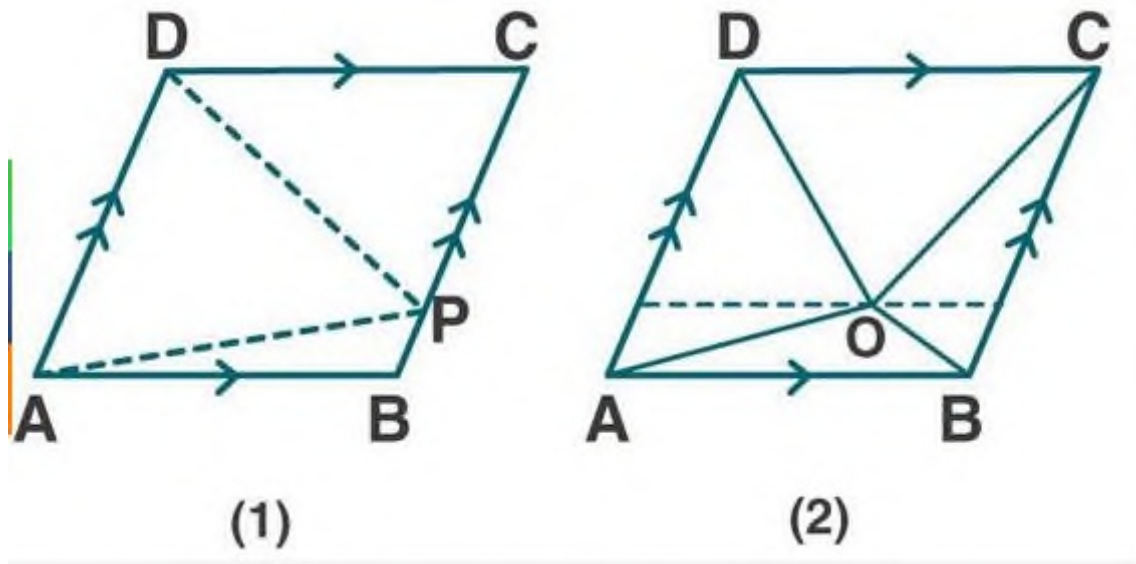
hence proved.

4. (a) in the figure(1) given below , ABCD is a parallelogram and P is any point in BC. Prove that : Area of ΔABP + area of ΔDPC = area of Δ APD.

(b) in the figure (2) given below, O is any point inside a parallelogram ABCD. Prove that

(i) area of ΔOAB + area of ΔOCD = $\frac{1}{2}$ area of || gm ABCD

(ii) area of Δ OBC + area of ΔOAD = $\frac{1}{2}$ area of || gm ABCD



Solution

(a) given :

From fig(1)

ABCD is a parallelogram and P is any point in BC

To prove :

Area of $\triangle ABP$ + area of $\triangle DPC$ = area of $\triangle APD$

Proof:

$\triangle APD$ and $\parallel$ gm ABCD are on the same base AD and between the same $\parallel$ lines AD and BC,

$$\text{ar}(\triangle APD) = \frac{1}{2} \text{ar}(\parallel \text{ gm ABCD}) \dots (1)$$

in parallelogram ABCD

$$\text{ar}(\parallel \text{ gm ABCD}) = \text{ar}(\triangle ABP) + \text{ar}(\triangle APD) + \text{ar}(\triangle DPC)$$

now , divide both sides by 2, we get

now , divide both sides by 2, we get

$$\frac{1}{2} \text{ar}(\parallel \text{gm ABCD}) = \frac{1}{2} \text{ar}(\Delta \text{ ABP}) + \frac{1}{2} \text{ar}(\Delta \text{APD}) + \frac{1}{2} \text{ar}(\Delta \text{DPC})$$

...(2)

From (1) and (2)

$$\text{Ar}(\Delta \text{ APD}) = \frac{1}{2} \text{ar}(\parallel \text{gm ABCD})$$

Substituting (2) in (1)

$$\text{ar}(\Delta \text{ APD}) = \frac{1}{2} \text{ar}(\Delta \text{ABP}) + \frac{1}{2} \text{ar}(\Delta \text{APD}) + \frac{1}{2} \text{ar}(\Delta \text{DPC})$$

$$\text{ar}(\Delta \text{ APD}) - \frac{1}{2} \text{ar}(\Delta \text{APD}) = \frac{1}{2} \text{ar}(\Delta \text{ ABP}) + \frac{1}{2} \text{ar}(\Delta \text{DPC})$$

$$\frac{1}{2} \text{ar}(\Delta \text{APD}) = \frac{1}{2} [\text{ar}(\Delta \text{ABP}) + \text{ar}(\Delta \text{ DPC})]$$

$$\text{Or } \text{ar}(\Delta \text{ABP}) + \text{ar}(\Delta \text{ DPC}) = \text{ar}(\Delta \text{ APD})$$

Hence proved.

(b) given:

From fig (2)

$\parallel$ gm ABCD in which O is any point inside it.

To prove:

$$(i) \text{ area of } \Delta \text{ OAB} + \text{area of } \Delta \text{OCD} = \frac{1}{2} \text{ area of } \parallel \text{ gm ABCD}$$

$$(ii) \text{ area of } \Delta \text{OBC} + \text{area of } \Delta \text{OAD} = \frac{1}{2} \text{ area of } \parallel \text{ gm ABCD}$$

Draw POQ $\parallel$ AB through o. It meets AD at P and BC at Q.

Proof :

(i) $AB \parallel PQ$ and $AP \parallel BQ$

ABQP is a $\parallel$ gm

Similarly , PQCD is a $\parallel$ gm

Now , ΔOAB and $\parallel$ gm ABQP are on same base AB and between same $\parallel$ lines AB and PQ

$$\text{ar}(\Delta OAB) = \frac{1}{2}\text{ar}(\parallel \text{ gm ABQP })....(1)$$

$$\text{similarly , ar} (\Delta OCD) = \frac{1}{2}\text{ar}(\parallel \text{ gm PQCD})...(2)$$

now adding (1) and (2)

$$\text{ar}(\Delta OAB) + \text{ar}(\Delta OCD) = \frac{1}{2}\text{ar}(\parallel \text{ gm ABQP}) + \frac{1}{2}\text{ar}(\parallel \text{ gm PQCD})$$

$$= \frac{1}{2} [\text{ar}(\parallel \text{ gm ABQP}) + \text{ar}(\parallel \text{ gm PQCD})]$$

$$= \frac{1}{2}\text{ar}(\parallel \text{ gm ABCD})$$

$$\text{ar}(\Delta OAB) + \text{ar}(\Delta OCD) = \frac{1}{2} \text{ar}(\parallel \text{ gm ABCD})$$

hence proved.

(ii) we know that,

$$\text{ar}(\Delta OAB) + \text{ar}(\Delta OBC) + \text{ar}(\Delta OCD) + \text{ar}(\Delta OAD) = \text{ar}(\parallel \text{ gm ABCD})$$

$$[\text{ar}(\Delta OAB) + \text{ar}(\Delta OCD)] + [\text{ar}(\Delta OBC) + \text{ar}(\Delta OAD)] = \text{ar}(\parallel \text{ gm ABCD})$$

$$\frac{1}{2} \text{ar}(\parallel \text{gm ABCD}) + \text{ar}(\Delta \text{ OBC}) + \text{ar}(\Delta \text{OAD}) = \text{ar}(\parallel \text{gm ABCD})$$

$$\text{ar}(\Delta \text{OBC}) + \text{ar}(\Delta \text{OAD}) = \text{ar}(\parallel \text{gm ABCD}) - \frac{1}{2} \text{ar}(\parallel \text{gm ABCD})$$

$$\text{ar}(\Delta \text{OBC}) + \text{ar}(\Delta \text{OAD}) = \frac{1}{2} \text{ar}(\parallel \text{gm ABCD})$$

hence proved.

5. if E,F,G and H are mid- points of the sides AB, BC,CD and DA respectively of a parallelogram ABCD, prove that area of quad. EFGH = $\frac{1}{2}$ area of $\parallel$ gm ABCD.

Solution

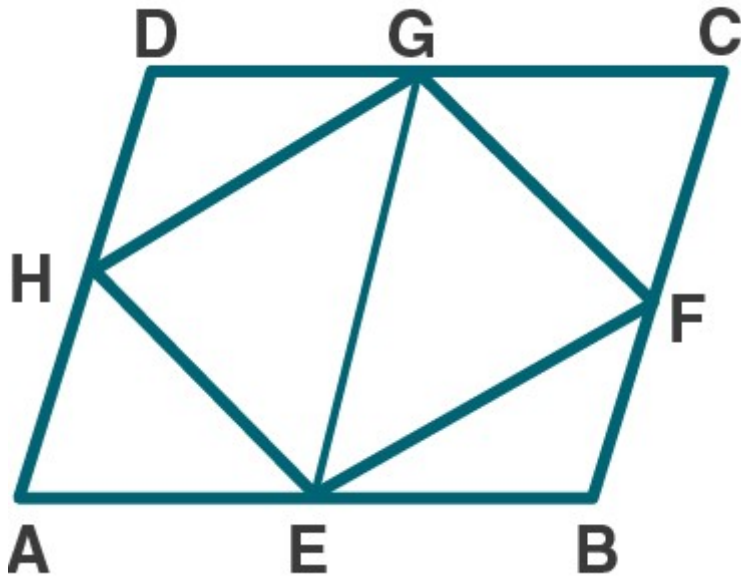
Given

In parallelogram ABCD, E,F,G,H are the mid – points of its sides AB,BC,CD and DA

Join EF,FG,GH and HE.

To prove :

$$\text{Area of quad. EFGH} = \frac{1}{2} \text{area of } \parallel \text{ gm ABCD}$$



Proof:

Let us join EG

We know that, E and G are mid- points of AB and CD

$EG \parallel AD \parallel BC$

AEGD and EBCG are parallelogram

Now, $\parallel$ gm AEGD and Δ EHG are on the same base and between the parallel lines.

$$\text{ar}\Delta\text{EHG} = \frac{1}{2} \text{ar } \parallel\text{gm AEGD} \dots (1)$$

similarly,

$$\text{ar}\Delta\text{EFG} = \frac{1}{2} \text{ar } \parallel\text{gm EBCG} \dots (2)$$

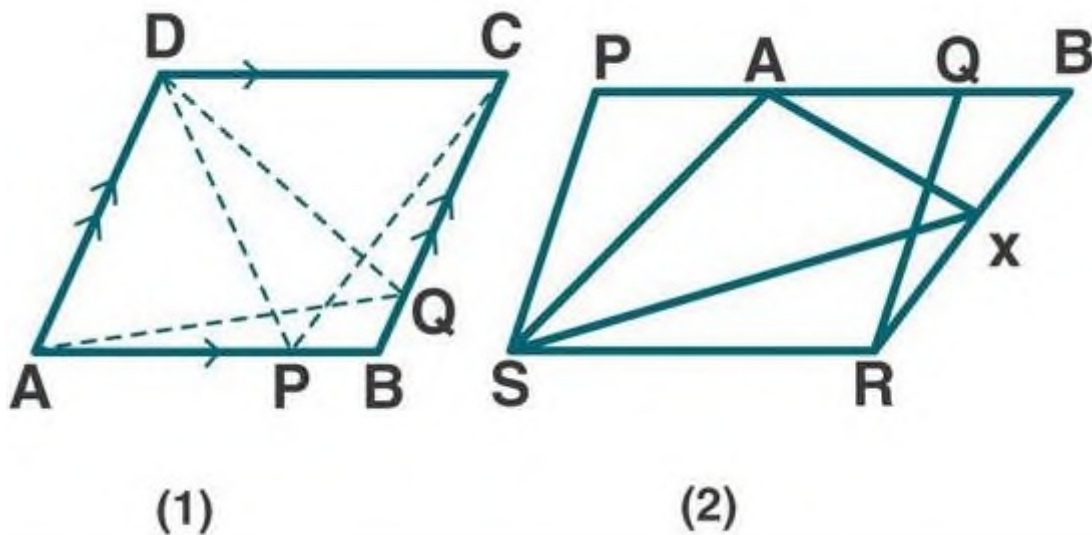
now by adding (1) and (2)

$$\text{ar}\Delta\text{EHG} + \text{ar}\Delta\text{EFG} = \frac{1}{2} \text{ar } \parallel\text{gm AEGD} + \frac{1}{2} \text{ar } \parallel\text{gm EBCG}$$

$$\text{area quad. EFGH} = \frac{1}{2} \text{ar } \parallel\text{gm ABCD}$$

Hence proved.

6(a) In the figure(1) given below ,ABCD is a parallelogram. P,Q are any two points on the sides AB and BC respectively. Prove that, area of ΔCPD = area of ΔAQD .



(b) in the figure(2) given below, PQRS and ABRS are parallelograms and x is any point on the side BR. Show that area of $\Delta AXS = \frac{1}{2}$ area of $\parallel\text{gm PQRS}$.

Solution

(a) given :

From fig (1)

|| gm ABCD in which P is a point on AB and Q is a point on BC.

To prove:

Area of ΔCPD = area of ΔAQD .

Proof:

ΔCPD and || gm ABCD are on the same base CD and between the same parallels AB and CD

$$\text{ar}(\Delta CPD) = \frac{1}{2} \text{ar}(\text{|| gm ABCD}) \dots (1)$$

ΔAQD and ||gm ABCD are on the same base AD and between the same parallels AD and BC.

$$\text{ar}(\Delta AQD) = \frac{1}{2} \text{ar}(\text{|| gm ABCD}) \dots (2)$$

from (1) and (2)

$$\text{ar}(\Delta CPD) = \text{ar}(\Delta AQD)$$

hence proved.

(b) from fig (2)

Given:

PQRS and ABRS are parallelograms on the same base SR. X is any point on the side BR.

Join AX and SX.

To prove:

$$\text{Area of } \Delta AXS = \frac{1}{2} \text{ area of ||gm PQRS}$$

We know that , $\parallel gm$ PQRS and ABRS are on the same SR and between the same parallels.

So , $ar \parallel gm PQRS = ar \parallel gm ABRS....(1)$

We know that, Δ AXS and $\parallel gm$ ABRS are on the same base AS and between the same parallels.

So, $ar \Delta AXS = \frac{1}{2} ar \parallel gm ABRS$

$= \frac{1}{2} ar \parallel gm PQRS$ [from (1)]

Hence proved.

7. D,E and F are mid – point of the sides BC, CA and AB respectively of a ΔABC . prove that

(i) FDCE is a parallelogram

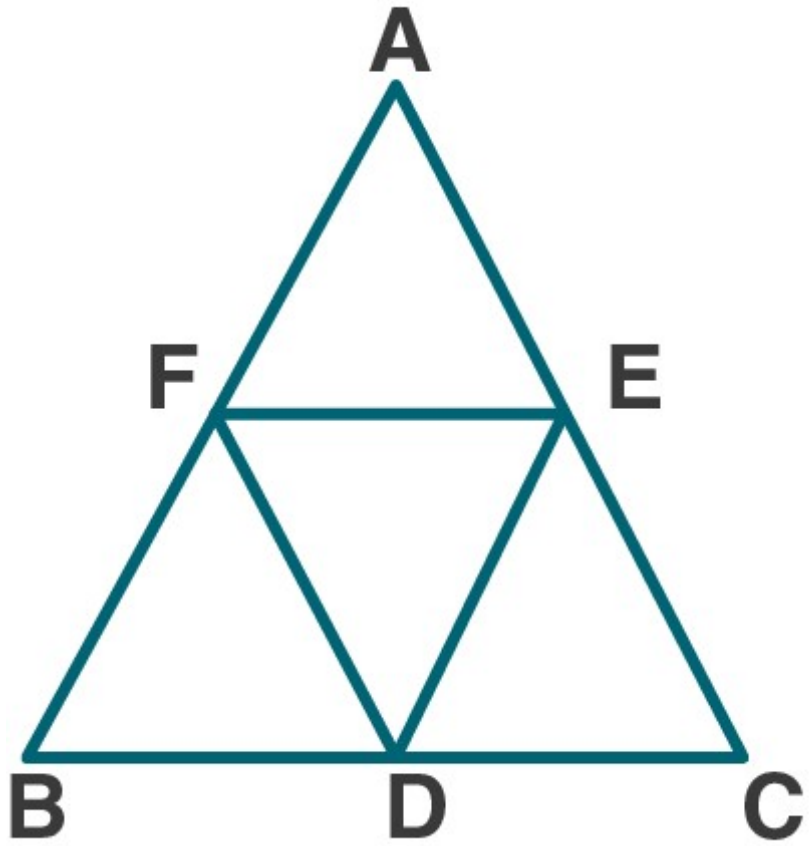
(ii) area of $\Delta DEF = \frac{1}{4}$ area of ΔABC

(iii) area of $\parallel gm FDCE = \frac{1}{2}$ area of ΔABC

Solution :

Given :

D, E and F are mid- point of the sides BC, CA and AB respectively of a ΔABC .



To prove:

(i) FDCE is a parallelogram

(ii) area of $\Delta DEF = \frac{1}{4}$ area of ΔABC

(iii) area of $\parallel$ gm FDCE $= \frac{1}{2}$ area of ΔABC

Proof :

(i) F and E are mid – points of AB and AC.

So, $FE \parallel BC$ and $FE = \frac{1}{2} BC \dots (1)$

Also, D is mid point of BC

$$CD = \frac{1}{2} BC \dots(2)$$

From (1) and (2)

FE $\parallel$ BC and FE = CD

FE $\parallel$ CD and FE = CD....(3)

Similarly,

D and F are mid- points of BC and AB.

So DF $\parallel$ EC is a parallelogram.

Hence proved.

(ii) we know that, FDCE is a parallelogram

And DE is a diagonal of $\parallel$ gm FDCE

So, $\text{ar}(\Delta DEF) = \text{ar}(\Delta DEC) \dots(4)$

Similarly, we know BDEF and DEAF are $\parallel$ gm

So, $\text{ar}(\Delta DEF) = \text{ar}(\Delta BDF) = \text{ar}(\Delta AFE) \dots(5)$

From(4) and (5)

$$\text{ar}(\Delta DEF) = \text{ar}(\Delta DEC) = \text{ar}(\Delta BDF) = \text{ar}(\Delta AFE)$$

$$\text{now, } \text{ar}(\Delta ABC) = \text{ar}(\Delta DEF) + \text{ar}(\Delta DEF) + \text{ar}(\Delta DEF) + \text{ar}(\Delta DEF)$$

$$= 4 \text{ar}(\Delta DEF)$$

$$\text{ar}(\Delta DEF) = \frac{1}{4} \text{ar}(\Delta ABC) \dots(6)$$

hence proved.

$$(iii) \text{ ar of } \parallel \text{ gm FDCE} = \text{ar}(\Delta DEF) + \text{ar}(\Delta DEC)$$

$$= \text{ar}(\Delta DEF) + \text{ar}(\Delta DEF)$$

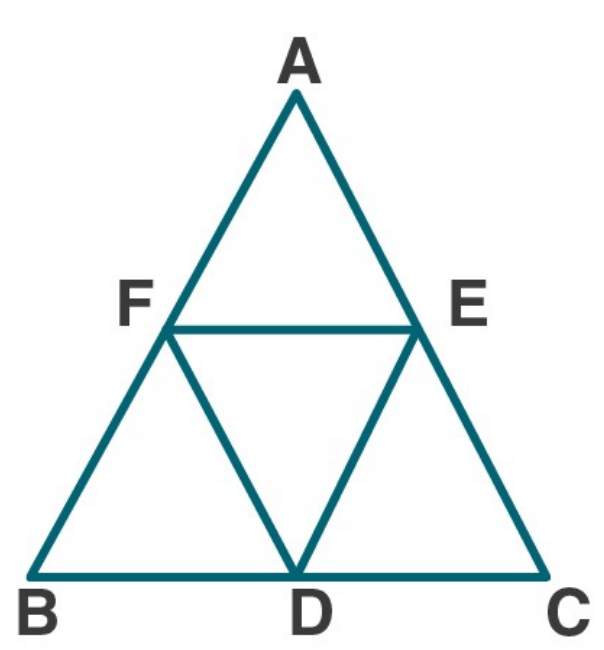
$$= 2 \text{ ar}(\Delta DEF) \text{ [from (4)]}$$

$$= 2 \left[\frac{1}{4} \text{ar}(\Delta ABC) \right] \text{ [from (6)]}$$

$$\text{ar of } \parallel \text{ gm FDCE} = \frac{1}{2} \text{ ar of } \Delta ABC$$

hence proved.

8. in the given figure, D,E and F are mid- points of the sides BC, CA and AB respectively of ΔABC . Prove that BCEF is a trapezium and area of trap. BCEF = $\frac{3}{4}$ area of ΔABC .



Solution

Given:

In ΔABC , D, E and F are mid – points of the sides BC, CA and AB.

To prove:

$$\text{Area of trap. BCEF} = \frac{3}{4} \text{area of } \Delta ABC$$

Proof:

We know that D and E are the mid – points of BC and CA.

So , DE $\parallel$ AB and $\frac{1}{2}$ AB

Similarly,

EF $\parallel$ BC and $\frac{1}{2}$ BC

And FD $\parallel$ AC and $\frac{1}{2}$ AC

$\therefore$ BDEF, CDFE , AFDE are parallelograms which are equal in area.

ED, DF , EF are diagonals of these $\parallel$ gm which divides the corresponding parallelogram into two triangle equal in area.

Hence , BCEF is a trapezium.

$$\text{Area of trap. BCEF} = \frac{3}{4} \text{area of } \Delta ABC$$

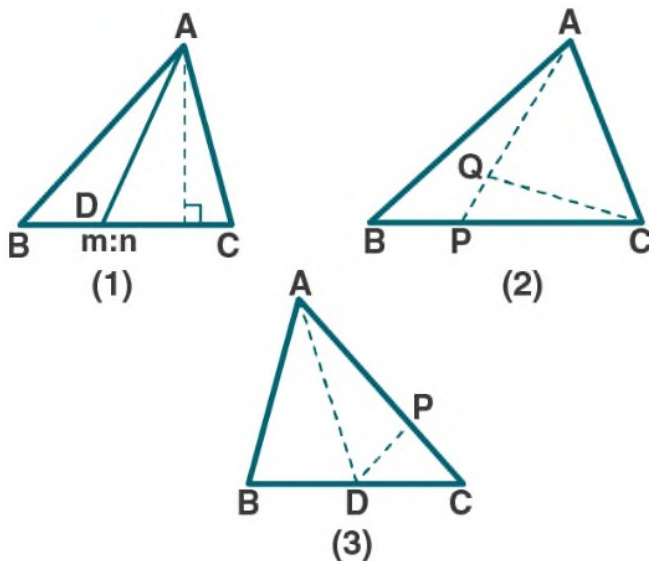
9. (a) in the figure (1) given below, the point D divides the side BC of $\triangle ABC$ in the ratio $m : n$. Prove that area of $\triangle ABD : \text{area of } \triangle ADC = m : n$.

(b) in the figure(2) given below, P is a point on the side BC of $\triangle ABC$ such that $PC = 2BP$, and Q is a point on AP such that $QA = 5PQ$, find area of $\triangle AQC : \text{area of } \triangle ABC$.

(C) In the figure(3) given below, AD is a median of $\triangle ABC$ and P is a point in AC such that area of $\triangle ADP : \text{area of } \triangle ABD = 2:3$ find

(i) AP: PC

(ii) area of $\triangle PDC : \text{area of } \triangle ABC$.



Solution

(a) given:

From fig(1)

In ΔABC , the point D divides the side BC in the ratio m:n.

$$BD : DC = m:n$$

To prove:

$$\text{Area of } \Delta ABD : \text{area of } \Delta ADC = m : n$$

Proof:

$$\text{Area of } \Delta ABD = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{ar}(\Delta ABD) = \frac{1}{2} \times BD \times AE \dots (1)$$

$$\text{ar}(\Delta ACD) = \frac{1}{2} \times DC \times AE \dots (2)$$

let us divide (1) by (2)

$$\frac{\left[\text{ar}(\Delta ABD) = \frac{1}{2} \times BD \times AE \right]}{\left[\text{ar}(\Delta ACD) = \frac{1}{2} \times DC \times AE \right]} = \frac{[\text{ar}(\Delta ABD)]}{[\text{ar}(\Delta ACD)]} = \frac{BD}{DC}$$
$$= \frac{m}{n} \text{ [it is given that, } BD : DC = m : n \text{]}$$

Hence proved.

(b) given:

From fig(2)

In ΔABC , P is a point on the side BC such that $PC = 2BP$, and Q is a point on AP such that $QA = 5PQ$.

To find:

Area of ΔAQC : area of ΔABC

Now,

It is given that: $PC = 2BP$

$$\frac{PC}{2} = BP$$

We know that , $BC = BP + PC$

Now substituting the values, we get

$$BC = BP + PC$$

$$= \frac{PC}{2} + PC$$

$$= \frac{PC + 2PC}{2}$$

$$= \frac{3PC}{2}$$

$$\frac{2BC}{3} = PC$$

$$\text{ar}(\Delta APC) = \frac{2}{3} \text{ar}(\Delta ABC) \dots (1)$$

it is given that $QA = 5PQ$

$$\frac{QA}{5} = PQ$$

We know that, $QA = QA + PQ$

$$\text{So, } QA = \frac{5}{6} AP$$

$$\begin{aligned} \text{ar}(\Delta AQC) &= \frac{5}{6} \text{ar}(\Delta APC) \\ &= \frac{5}{6} \left(\frac{2}{3} \text{ar}(\Delta ABC) \right) [\text{from (1)}] \end{aligned}$$

$$\text{ar}(\Delta AQC) = \frac{5}{9} \text{ar}(\Delta ABC)$$

$$\frac{\text{ar} \Delta AQC}{\text{ar}(\Delta AQC)} = \frac{5}{9}$$

Hence proved.

(c) Given:

From fig (3)

AD is a median of ΔABC and P is point in AC such that area of ΔADP : area of $\Delta ABD = 2 : 3$

To find:

(i) AP : PC

(ii) area of ΔPDC : area of ΔABC

Now,

(i) we know that AD is the median of ΔABC

$$\text{ar}(\Delta ABD) = \text{ar}(\Delta ADC) = \frac{1}{2} \text{ar}(\Delta ABC) \dots (1)$$

it is given that

$$\text{ar}(\Delta ADP) : \text{ar}(\Delta ABD) = 2 : 3$$

$$AP : AC = 2 : 3$$

$$\frac{AP}{AC} = \frac{2}{3}$$

$$AP = \frac{2}{3} AC$$

Now ,

$$PC = AC - AP$$

$$= AC - \frac{2}{3} AC$$

$$= \frac{3AC - 2AC}{3}$$

$$= \frac{AC}{3} \dots (2)$$

So,

$$\frac{AP}{PC} = \frac{\left(\frac{2}{3} AC\right)}{\frac{AC}{3}}$$

$$= \frac{2}{1}$$

$$AP : PC = 2 : 1$$

(ii) we know that from (2)

$$PC = \frac{AC}{3}$$

$$\frac{PC}{AC} = \frac{1}{3}$$

So,

$$\frac{ar(\Delta PDC)}{ar(\Delta ADC)} = \frac{PC}{AC} = \frac{1}{3}$$

$$\frac{\frac{ar(\Delta PDC)}{1}}{2ar(\Delta ABC)} = \frac{1}{3}$$

$$\frac{ar(\Delta PDC)}{ar(\Delta ABC)} = \frac{1}{3} \times \frac{1}{2}$$

$$= \frac{1}{6}$$

$$ar(\Delta PDC): ar(\Delta ABC) = 1:6$$

hence proved.

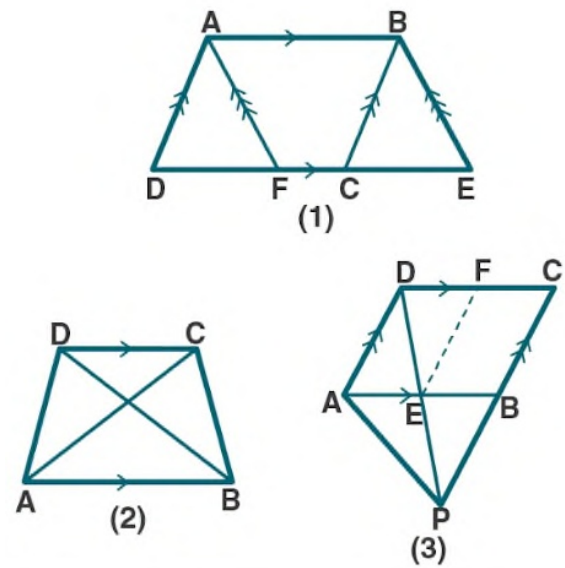
10.(a) in the figure (1) given below , area of parallelogram ABCD is 29 cm². Calculate the height of parallelogram ABEF if AB = 5.8 cm

(b) in the figure(2) given below , area of ΔABD is 24sq. units. If AB = 8 units, find the height of ABC.

(c) in the figure (3) given below, E and F are mid-points of sides AB and CD respectively of parallelogram ABCD. If the area of parallelogram ABC is 36cm².

(i) state the area of ΔAPD .

(ii) Name the parallelogram whose area is equal to the area of ΔAPD .



Solution

(a) given :

From fig(1)

$$\text{ar } \parallel \text{ gm } ABCD = 29 \text{ cm}^2$$

to find :

height of parallelogram ABEF if $AB = 5.8 \text{ cm}$

now let us find

we know that $\parallel \text{ gm } ABCD$ and $\parallel \text{ gm } ABEF$ with equal bases and between the same parallels so that their areas are same .

$$\text{ar}(\parallel \text{ gm } ABEF) = \text{ar}(\parallel \text{ gm } ABCD)$$

$$\text{ar}(\parallel \text{ gm } ABEF) = 29 \text{ cm}^2 \dots(1) \text{ [since , ar } \parallel \text{ gm } ABCD = 29 \text{ cm}^2 \text{]}$$

$$\text{also, ar}(\parallel \text{ gm } ABEF = \text{base} \times \text{height})$$

$$29 = AB \times \text{height [from (1)]}$$

$$29 = 5.8 \times \text{height}$$

$$\text{Height} = \frac{29}{5.8}$$

$$= 5$$

$\therefore$ height of parallelogram ABEF is 5 cm

(b) given:

From fig (2)

Area of ΔABD is 24 sq. units . $AB = 8$ units

To find :

Height of ABC

Now, let us find

We know that ar $\Delta ABD = 24$ sq. units(1)

So, ar $\Delta ABD = \Delta ABC$(2)

From (1) and (2)

ar $\Delta ABC = 24$ sq. units

$$\frac{1}{2} \times AB \times \text{height} = 24$$

$$\frac{1}{2} \times 8 \times \text{height} = 24$$

$$4 \times height = 24$$

$$height = \frac{24}{4}$$

$$= 6$$

$\therefore$ height of $\Delta ABC = 6$ sq. units

(c) given :

From fig (3)

In $\parallel$ gm ABCD, E and F are mid points of sides AB and CD respectively.

$$ar(\parallel gm ABCD) = 36cm^2$$

to find:

(i) state the area of ΔAPD .

(ii) Name the parallelogram whose area is equal to the area of ΔAPD .

Now , let us find

(i) we know that ΔAPD and $\parallel gm ABCD$ are on the same base AD and between the same parallel lines AD and BC.

$$ar(\Delta APD) = \frac{1}{2} ar(\parallel gm ABCD)....(1)$$

$$ar(\parallel gm ABCD) = 36cm^2.....(2)$$

from(1) and (2)

$$\begin{aligned} \text{ar}(\Delta \text{APD}) &= \frac{1}{2} \times 36 \\ &= 18 \text{ cm}^2 \end{aligned}$$

(ii) we know that E and F are mid – points of AB and CD

In ΔCPD , $\text{EF} \parallel \text{PC}$

Also, EF bisects the $\parallel \text{gm ABCD}$ in two equal parts.

So $\text{EF} \parallel \text{AD}$ and $\text{AE} \parallel \text{DF}$

AEFD is a parallelogram.

$$\text{ar}(\parallel \text{gm AEFD}) = \frac{1}{2} \text{ar}(\parallel \text{gm ABCD}) \dots (3)$$

from (1) and (3)

$$\text{ar}(\Delta \text{APD}) = \text{ar}(\parallel \text{gm AEFD})$$

$\therefore$ AEFD is the required parallelogram which is equal to area of ΔAPD .

11. (a) in the figure(1) given below, ABCD is a parallelogram.

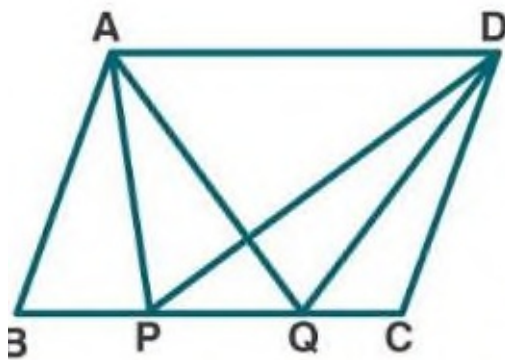
Points P and Q on BC trisect BC into three equal parts.

Prove that:

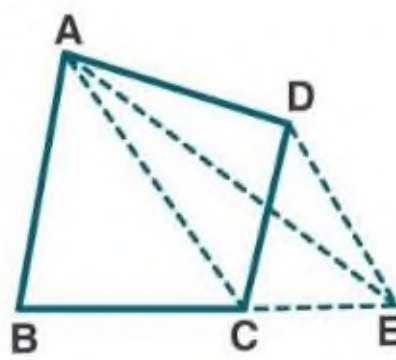
$$\text{Area of } \Delta \text{APQ} = \text{area of } \Delta \text{DPQ} = \frac{1}{6} (\text{area of } \parallel \text{gm ABCD})$$

(b) in the figure (2) given below, DE is drawn parallel to the diagonal AC of the quadrilateral ABCD to meet BC produced at the point E. Prove that area of quad. ABCD = area of ΔABE

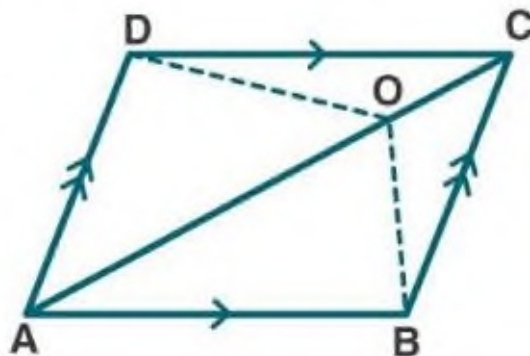
(c) in the figure(3) given below, ABCD is a parallelogram. O is any point on the diagonal AC of the parallelogram . show that the area of ΔAOB is equal to the area of ΔAOD .



(1)



(2)

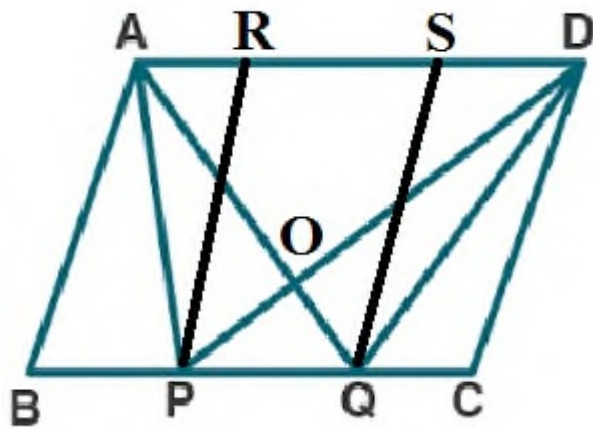


(3)

Solution

(a) Given:

From fig(1)



In $\parallel$ gm ABCD, points P and Q trisect BC into three equal parts.

To prove :

$$\text{Area of } \Delta APQ = \text{area of } \Delta DPQ = \frac{1}{6} (\text{area of } \parallel \text{ gm ABCD})$$

Firstly, let us construct: through p and Q, draw PR and QR parallel to AB and CD.

Proof:

$\text{ar}(\Delta APD) = \text{ar}(\Delta AQD)$ [since, ΔAPD and ΔAQD lie on the same base AD and between the same parallel lines AD and BC]

$\text{ar}(\Delta APD) - \text{ar}(\Delta AOD) = \text{ar}(\Delta AQD) - \text{ar}(\Delta AOD)$ [on subtracting $\text{ar } \Delta AOD$ on both sides]

$$\text{ar}(\Delta APO) = \text{ar}(\Delta OQD) \dots (1)$$

$\text{ar}(\Delta APO) + \text{ar}(\Delta OPQ) = \text{ar}(\Delta OQD) + \text{ar}(\Delta OPQ)$ [on adding ar ΔOPQ on both sides]

$$\text{ar}(\Delta APQ) = \text{ar}(\Delta DPQ) \dots (2)$$

we know that, ΔAPQ and $\parallel gm PQRS$ are on the same base PQ and between same parallel lines PQ and AD.

$$\text{ar}(\Delta APQ) = \frac{1}{2} \text{ar}(\parallel gm PQRS) \dots (3)$$

now,

$$\left[\frac{\text{ar}(\parallel gm ABCD)}{\text{ar}(\parallel gm PQRS)} \right] = \left[\frac{BC \times height}{PQ \times height} \right] = \left[\frac{3PQ \times height}{1PQ \times height} \right]$$

$$\text{ar}(\parallel gm PQRS) = \frac{1}{3} \text{ar}(\parallel gm ABCD) \dots (4)$$

by using (2),(3),(4) we get

$$\text{ar}(\Delta APQ) = \text{ar}(\Delta DPQ)$$

$$= \frac{1}{2} \text{ar}(\parallel gm PQRS)$$

$$= \frac{1}{2} \times \frac{1}{3} \text{ar}(\parallel gm ABCD)$$

$$= \frac{1}{6} \text{ar}(\parallel gm ABCD)$$

Hence proved.

(b) given:

In the figure (2) given below, $DE \parallel AC$ the diagonal of the quadrilateral ABCD to meet at point E on producing BC. Join AC, AE .

To prove:

Area of quad. ABCD = area of ΔABE

Proof:

We know that , ΔACE and ΔADE are on the same base AC and between the same parallelgram

$$\text{ar}(\Delta ACE) = \text{ar}(\Delta ADC)$$

now by adding $\text{ar}(\Delta ABC)$ on both sides, we get

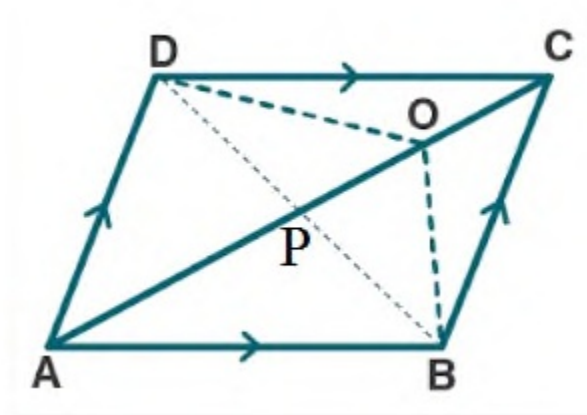
$$\text{ar}(\Delta ACE) + \text{ar}(\Delta ABC) = \text{ar}(\Delta ADC) + \text{ar}(\Delta ABC)$$

$$\text{ar}(\Delta ABE) = \text{ar quad. ABCD}$$

hence proved.

(c) Given

From fig (3)



In $\parallel$ gm ABCD, O is any point on diagonal AC.

To prove:

Area of ΔAOB is equal to the area of ΔAOD

Proof:

Let us join BD which meets AC at P.

In ΔABD , AP is the median .

$$\text{ar}(\Delta ABP) = \text{ar}(\Delta ADP) \dots\dots(1)$$

$$\text{similarly, ar}(\Delta PBO) = \text{ar}(\Delta PDO) \dots\dots(2)$$

now add (1) and (2) we get

$$\text{ar}(\Delta ABO) = \text{ar}(\Delta ADO) \dots\dots(3)$$

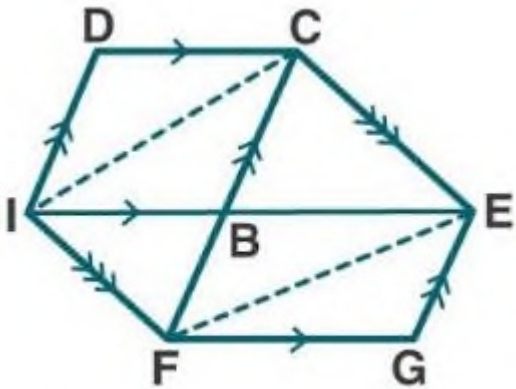
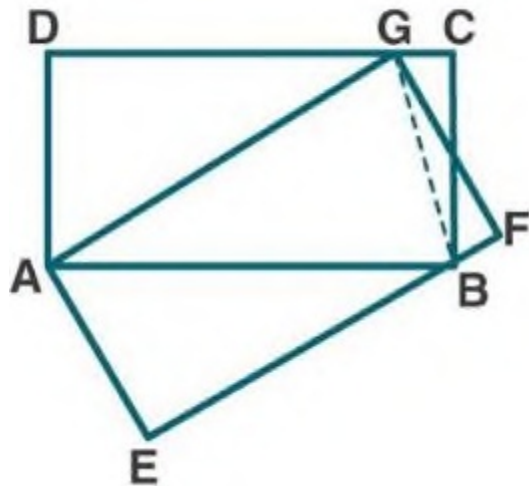
so,

$$\Delta AOB = \text{ar}\Delta AOD$$

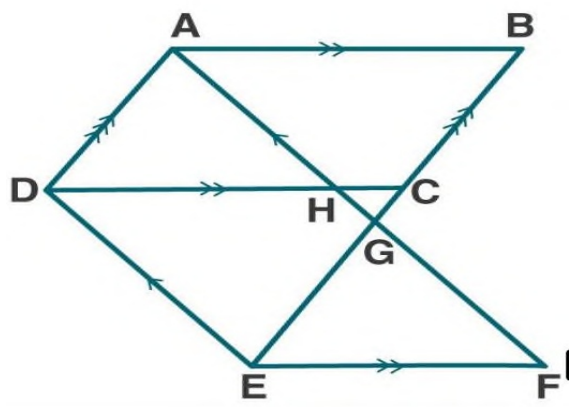
hence proved.

12. (a) In the figure given, ABCD and AEFG are two parallelograms. Prove that area of $\parallel$ gm ABCD = area of $\parallel$ gm AEFG.

(b) in the fig.(2) given below, the side AB of the parallelogram ABCD is produced to E. A straight line through A is drawn parallel to CE to meet CB produced at F and parallelogram BFGE is completed prove that area of $\parallel$ gm BFGE = area of $\parallel$ gm ABCD.



(c) in the figure (3) given below $AB \parallel DC \parallel EF$, $AD \parallel BE$ and $DE \parallel AF$. Prove the area of DEFH is equal to the area of ABCD.



Solution

(a) Given

From fig(1)

ABCD and AEFG are two parallelograms as shown in the figure.

To prove:

Area of $\parallel$ gm ABCD = area of $\parallel$ gm AEFG

Proof:

Let us join BG.

We know that,

$$\text{ar}(\Delta ABG) = \frac{1}{2} (\text{ar } \parallel \text{ gm } ABCD) \dots (1)$$

similarly,

$$\text{ar}(\Delta ABG) = \frac{1}{2} (\text{ar } \parallel \text{ gm } AEFG) \dots (2)$$

from (1) and (2)

$$\frac{1}{2} (\text{ar } \parallel \text{ gm } ABCD) = \frac{1}{2} (\text{ar } \parallel \text{ gm } AEFG)$$

so,

$$\text{ar}(\parallel \text{ gm ABCD} = \text{ar } \parallel \text{ gm AEFG})$$

hence proved.

(b) given:

From fig (2)

A parallelogram ABCD in which AB is produced to E. A straight line through A is drawn parallel to CE to meet CB produced at F and parallelogram BFGE is completed.

To prove:

$$\text{Area of } \parallel \text{ gm BFGE} = \text{area of } \parallel \text{ gm ABCD}$$

Proof:

Let us join AC and EF.

We know that,

$$\text{ar}(\triangle AFC) = \text{ar}(\triangle AFE) \dots (1)$$

now subtract $\text{ar}(\triangle ABF)$ on both sides, we get

$$\text{ar}(\triangle AFC) - \text{ar}(\triangle ABF) = \text{ar}(\triangle AFE) - \text{ar}(\triangle ABF)$$

$$\text{or } \text{ar}(\triangle ABC) = \text{ar}(\triangle BEF)$$

now multiply by 2 on both sides, we get

$$2. \text{ar}(\triangle ABC) = 2. \text{ar}(\triangle BEF)$$

$$\text{Or } \text{ar}(\parallel \text{ gm ABCD}) = \text{ar}(\parallel \text{ gm BFGE})$$

Hence proved

(c) Given:

From fig(3)

$AB \parallel DC \parallel EF, AD \parallel BE \text{ and } DE \parallel AF$

To prove:

Area of DEFH = area of ABCD

Proof:

We know that,

$DE \parallel AF \text{ and } AD \parallel BE$

It is given that ADEG is a parallelogram .

So,

$$\text{ar}(\parallel \text{ gm ABCD}) = \text{ar}(\parallel \text{ gm ADEG}) \dots (1)$$

again , DEFG is a parallelogram

$$\text{ar}(\parallel \text{ gm DEFH}) = \text{ar}(\parallel \text{ gm ADEG}) \dots (2)$$

From (1) and(2)

$$\text{ar}(\parallel \text{ gm ABCD}) = \text{ar}(\parallel \text{ gm DEFH})$$

$$\text{or ar ABCD} = \text{ar DEFH}$$

hence proved.

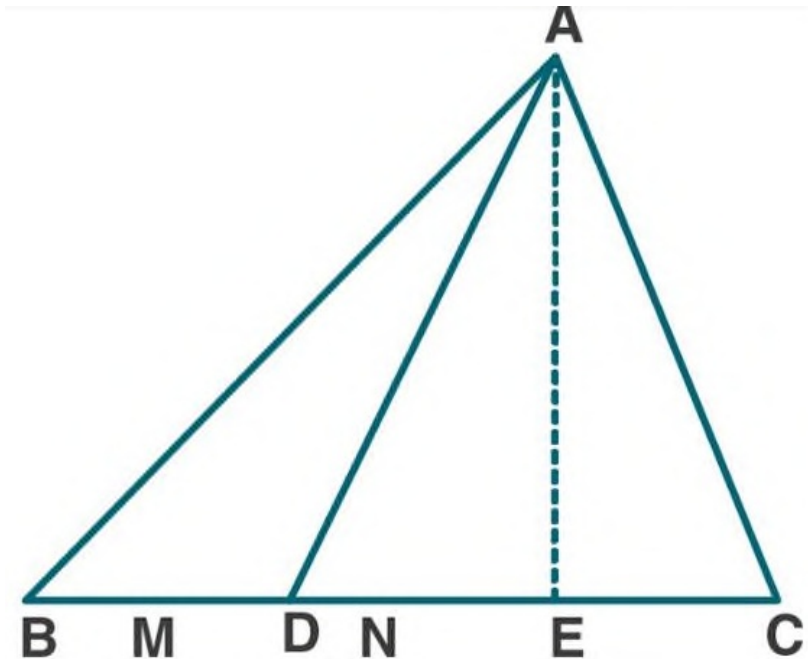
13. Any point D is taken on the side BC of , a ΔABC and AD is produced to E such $AD = DE$, prove that area of ΔBCE = area of ΔABC .

Solution

Given

In ΔABC , D is taken on the side BC.

AD produced to E such that $AD = DE$



To prove:

Area of ΔBCE = area of ΔABC

Proof:

In ΔABE , it is given that $AD = DE$

So, BD is the median of ΔABE

$\text{ar}(\Delta ABD) = \text{ar}(\Delta BED) \dots\dots(1)$

similarly,

in ΔACE , CD is the median of ΔACE

$$\text{ar}(\Delta ACD) = \text{ar}(\Delta CED) \dots\dots (2)$$

by adding (1) and (2) , we get

$$\text{ar}(\Delta ABD) + \text{ar}(\Delta ACD) = \text{ar}(\Delta BED) + \text{ar}(\Delta CED)$$

$$\text{or } \text{ar}(\Delta ABC) = \text{ar}(\Delta BCE)$$

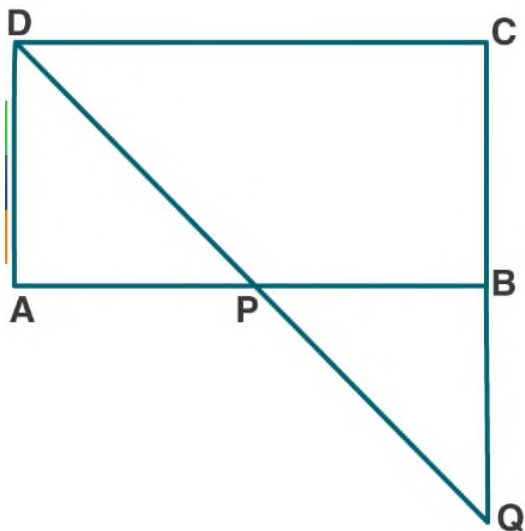
hence proved.

14. ABCD is a rectangle and P is mid- point of AB. DP is produced to meet CB at Q. Prove that area of rectangle $\Delta BCD = \text{area of } \Delta DQC$.

Solution

Given:

ABCD is a rectangle and P is mid-point of AB. DP is produced to meet CB at Q.



To prove :

Area of rectangle ΔBCD = area of ΔDQC

Proof:

In ΔAPD and ΔBQP

$AP = BP$ [since , D is the mid- point of AB]

$\angle DAP = \angle QBP$ [each angle is 90°]

$\angle APD = \angle BPQ$ [vertically opposite angles]

So, $\Delta APD \cong \Delta BQP$ [by using ASA postulate]

$\text{ar}(\Delta APD) = \text{ar}(\Delta BQP)$

now,

$\text{ar } ABCD = \text{ar}(\Delta APD) + \text{ar } PBCD$

$= \text{ar} (\Delta BQP) + \text{ar } PBCD$

$= \text{ar}(\Delta DQC)$

Hence proved.

15.(a) In the figure (1) given below, the perimeter of parallelogram is 42 cm. Calculate the lengths of the sides of the parallelogram.

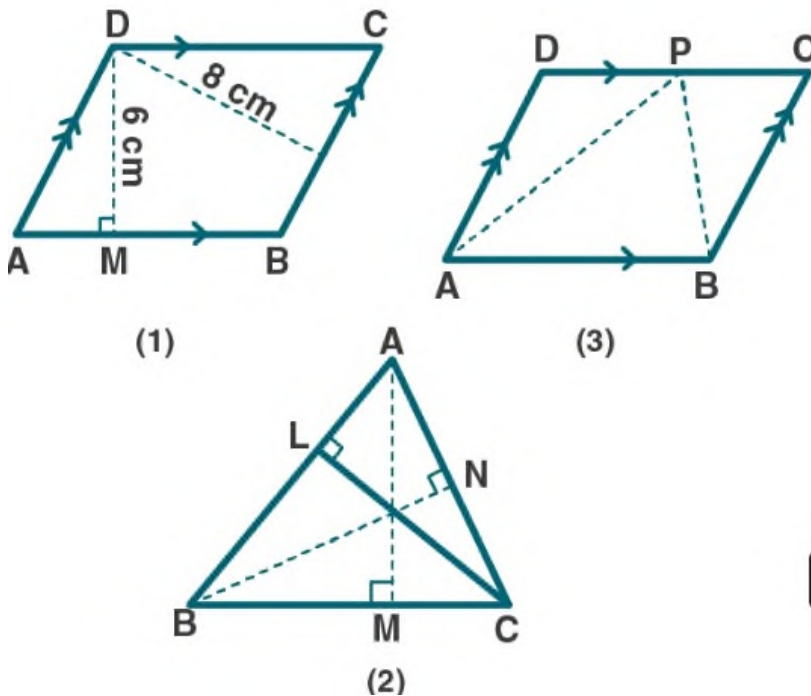
(b) In the figure(2) given below, the perimeter of ΔABC is 37 cm . if the length of the altitudes AM, BN and CL are $5x$, $6x$

and $4x$ respectively, calculate the lengths of the sides of $\triangle ABC$.

(c) In the fig.(3) Given below , ABCD is a parallelogram. P is a point on DC such that area of $\triangle DAP = 25\text{cm}^2$ and area of $\triangle BCP = 15\text{cm}^2$. Find

(i) area of $\parallel\text{gm ABCD}$

(ii) $DP : PC$



Solution

(a) given

The perimeter of parallelogram ABCD = 42cm

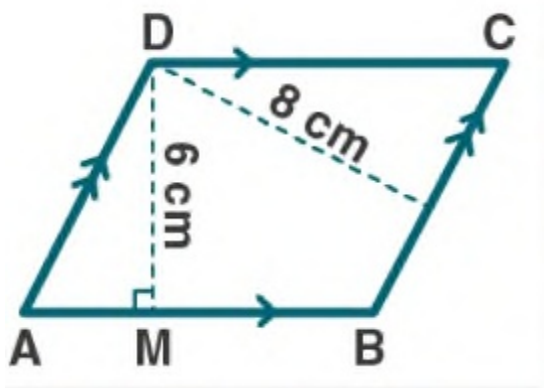
To find:

Lengths of the sides of the parallelogram ABCD.

From fig(1)

We know that,

$$AB = P$$



Then, perimeter of $\parallel$ gm ABCD $= 2(AB + BC)$

$$42 = 2(p + BC)$$

$$\frac{42}{2} = P + BC$$

$$21 = P + BC$$

$$BC = 21 - P$$

$$\text{So, ar}(\parallel \text{ gm ABCD}) = AB \times DM$$

$$= P \times 6$$

$$= 6P \dots\dots(1)$$

$$\text{Again, ar}(\parallel \text{ gm ABCD}) = BC \times DN$$

$$= (21 - P) \times 8$$

$$= 8(21 - P) \dots\dots(2)$$

From (1) and (2) we get

$$6P = 8(21 - P)$$

$$6P = 168 - 8P$$

$$6P + 8P = 168$$

$$14P = 168$$

$$P = \frac{168}{14}$$

$$= 12$$

Hence, sides of ΔABC are

$$AB = 12 \text{ cm and } BC = (21 - 12)\text{cm} = 9\text{cm}$$

(b) given :

The perimeter of ΔABC is 37 cm . the length of the altitudes

AM, BN and CL are $5x$, $6x$ and $4x$ respectively.

To find :

Length of the sides of ΔABC .i.e, BC, CA and AB

Let us consider $BC = P$ and $CA = Q$

From fig (2),

Then, perimeter of $\Delta ABC = AB + BC + CA$

$$= 37 = AB + P + Q$$

$$AB = 37 - P - Q$$

$$\text{Area } (\Delta ABC) = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times BC \times AM = \frac{1}{2} \times CA \times BN = \frac{1}{2} \times AB \times CL$$

$$= \frac{1}{2} \times P \times 5x = \frac{1}{2} \times Q \times 6x = \frac{1}{2} (37 - P - Q) \times 4x$$

$$= \frac{5p}{2} = 3Q = 2(37 - P - Q)$$

Let us consider first two parts:

$$\frac{5p}{2} = 3Q$$

$$5P = 6Q$$

$$5P - 6Q = 0 \dots\dots(1)$$

$$25P - 30Q \text{ (multiplying by 5) } \dots\dots(2)$$

Let us consider second and third parts:

$$3Q = 2(37 - P - Q)$$

$$3Q = 74 - 2P - 2Q$$

$$3Q + 2Q + 2P = 74$$

$$2P + 5Q = 74 \dots\dots(3)$$

$$12P + 30Q = 444 \text{ (multiplying by 6) } \dots\dots(4)$$

By adding (2) and (4), we get

$$37P = 444$$

$$P = \frac{444}{37}$$

$$= 12$$

Now, substituting the value of P in equation (1) , we get

$$5P - 6Q = 0$$

$$5(12) - 6Q = 0$$

$$60 = 6Q$$

$$Q = \frac{60}{6}$$

$$= 10$$

$$\text{Hence, } BC = P = 12\text{cm}$$

$$CA = Q = 10\text{cm}$$

$$\text{And } AB = 37 - P - Q = 37 - 12 - 10 = 15 \text{ cm}$$

(c) given :

ABCD is a parallelogram . P is a point on DC such that area of Δ

$$DAP = 25\text{cm}^2 \text{ and area of } \Delta BCP = 15\text{cm}^2$$

To find :

(i) area of $\parallel\text{gm ABCD}$

(ii) DP : PC

Now let us find ,

From fig (3)

(i) we know that ,

$$\text{ar}(\Delta APB) = \frac{1}{2} \text{ar}(\parallel \text{gm ABCD})$$

then,

$$\frac{1}{2} \text{ar}(\parallel \text{gm ABCD}) = \text{ar}(\Delta DAP) + \text{ar}(\Delta BCP)$$

$$= 25 + 15$$

$$= 40 \text{ cm}^2$$

$$\text{So, ar}(\parallel \text{ gm ABCD}) = 2 \times 40 = 80\text{cm}^2$$

(ii) we know that

ΔADP and ΔBCP are on the same base CD and between same parallel lines CD and AD.

$$\frac{\text{ar}(\Delta DAP)}{\text{ar}(\Delta BCP)} = \frac{DP}{PC}$$

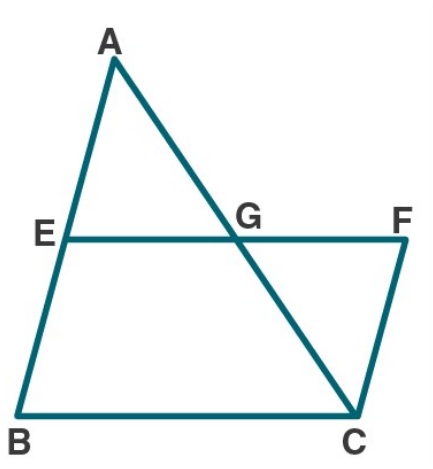
$$\frac{25}{15} = \frac{DP}{PC}$$

$$\frac{5}{3} = \frac{DP}{PC}$$

$$\text{So, DP : PC} = 5 : 3$$

16. In the adjoining figure, E is mid – point of the side AB of a triangle ABC and EBCF is a parallelogram. If the area of ΔABC is 25sq. units, find the area of $\parallel \text{ gm EBCF}$.

Solution :



Let us consider EF, side of $\parallel$ gm BCEF meets AC at G.

We know that, E is the mid- point and $EF \parallel BC$

G is the mid – point of AC.

So,

$$AG = GC$$

Now, in ΔAEG and ΔCFG ,

The alternate angles are : $\angle EAG, \angle GCF$

Vertically opposite angles are : $\angle EGA = \angle CGF$

So, $AG = GC$

Proved.

$$\therefore \Delta AEG \cong \Delta CFG$$

$$\text{ar}(\Delta AEG) = \text{ar}(\Delta CFG)$$

now,

$$\text{ar}(\parallel \text{gm EBCF}) = \text{ar BCGE} + \text{ar}(\Delta CFG)$$

$$= \text{ar BCGE} + \text{ar}(\Delta AEG)$$

$$= \text{ar}(\Delta ABC)$$

We know that, $\text{ar}(\Delta ABC) = 25 \text{ sq. units}$

Hence, $\text{ar}(\text{|| gm EBCF}) = 25 \text{sq. units}$

17. (a) In the figure (1) given below , $BC \parallel AE$ and $CD \parallel BE$.

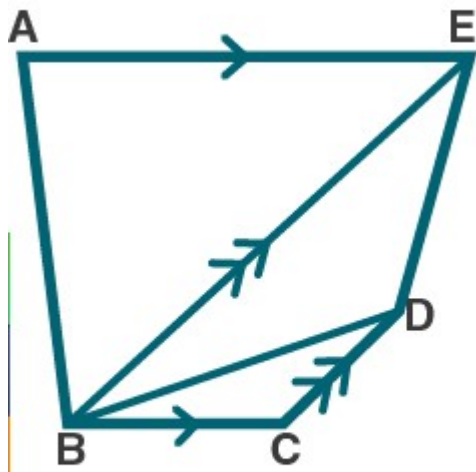
Prove that:

Area of ΔABC = area of ΔEBD .

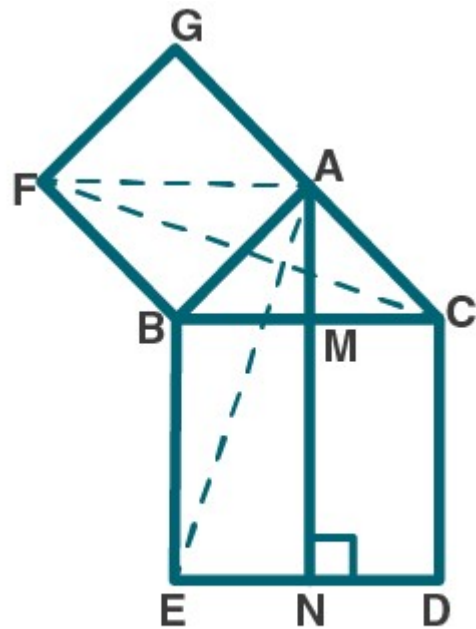
(b) In the figure(2) given below, ABC is right angled triangle at A . $AGFB$ is a square on the side AB and $BCDE$ is a square on the hypotenuse BC . If $AN \perp ED$, prove that :

(i) $\Delta BCF \cong \Delta ABE$.

(ii) area of square $ABFG$ = area of rectangle $BENM$.



(1)



(2)

Solution

(a) Given:

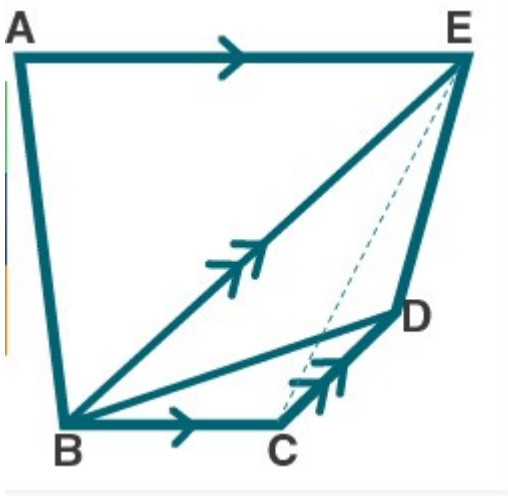
From fig (1)

$BC \parallel AE$ and $CD \parallel BE$

To prove:

Area of $\Delta ABC = \text{area of } \Delta EBD$

Proof:



By joining CE.

We know that, from ΔABC and ΔEBC

$$\text{ar}(\Delta ABC) = \text{ar}(\Delta EBC) \dots (1)$$

from EBC and ΔEBD

$$\text{ar}(\Delta EBC) = \text{ar}(\Delta EBD) \dots (2)$$

from (1) and (2), we get

$$\text{ar}(\Delta ABC) = \text{ar}(\Delta EBD)$$

hence proved.

(b) Given :

ΔABC is right angled triangle at A. Squares AGFB and BCDE are drawn on the side AB and hypotenuse BC of ΔABC . $AN \perp ED$ which meets BC at M.

To prove:

(i) $\Delta BCF \cong \Delta ABE$.

(ii) area of square ABFG = area of rectangle BENM

From the figure(2)

$$(i) \angle FBC = \angle FBA + \angle ABC$$

So,

$$\angle FBC = 90^\circ + \angle ABC \dots(1)$$

$$\angle ABE = \angle EAC + \angle ABC$$

So,

$$\angle ABE = 90^\circ + \angle ABC \dots\dots(2)$$

From (1) and (2) we get

$$\angle FBC = \angle ABE \dots\dots(3)$$

$$\text{So } BC = BE$$

Now in ΔBCF and ΔABE

$$BF = AB$$

By using SAS axiom rule of congruency.

$$\therefore \Delta BCF \cong \Delta ABE$$

Hence proved

(ii)we know that,

$$\Delta BCF \cong \Delta ABE$$

$$\text{So, } \text{ar}(\Delta BCF) = \text{ar}(\Delta ABE) \dots\dots(4)$$

$$\angle BAG + \angle BAC = 90^\circ + 90^\circ$$

$$= 180^\circ$$

So, GAC is a straight line.

Now from ΔBCF and square AGFB

$$\text{ar}(\Delta BCF) = \frac{1}{2} \text{ar}(\text{square AGFB}) \dots (5)$$

From ΔABE and rectangle BENM

$$\text{ar}(\Delta ABE) = \frac{1}{2} \text{ar}(\text{rectangle BENM}) \dots (6)$$

from (4), (5) and (6)

$$\frac{1}{2} \text{ar}(\text{square AGFB}) = \frac{1}{2} \text{ar}(\text{rectangle BENM})$$

$$\text{ar}(\text{square AGFB}) = \text{ar}(\text{rectangle BENM})$$

hence proved.