

Chapter 15

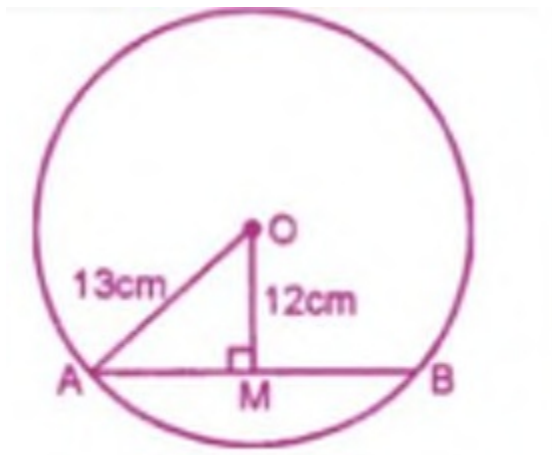
Circle

Exercise 15.1

1 . calculate the length of a chord which is at a distance of 12cm from the centre of circle of radius 13 cm.

Solution

AB is chord of a circle with centre O and OA is its radius $OM \perp AB$



Therefore $OA = 13\text{cm}$, $OM = 12\text{ cm}$

Now from right angled triangle OAM,

$OA^2 = OM^2 + AM^2$ by using Pythagoras theorem,

$$13^2 = 12^2 + AM^2$$

$$AM^2 = 13^2 - 12^2$$

$$AM^2 = 25$$

$$AM = 5$$

We know that OM perpendicular to AB

Therefore, M is the mid point of AB

$$AB = 2AM$$

$$AB = 2(5)$$

$$AB = 10\text{cm}$$

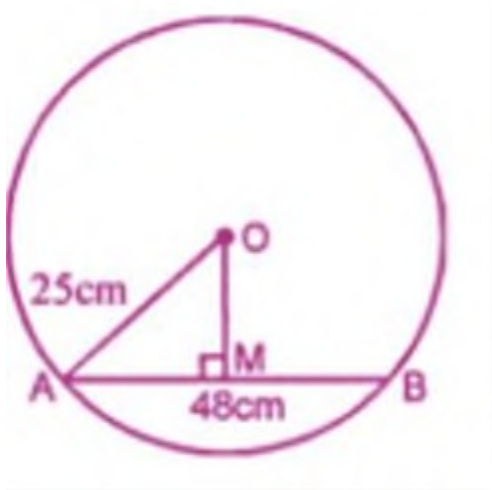
2.A chord of length 48cm is drawn in a circle of radius 25cm.

Calculate its distance from the centre of the circle

Solution

AB is the chord of the circle with centre O and radius OA

OM is perpendicular to AB



Therefore, $AB = 48\text{ cm}$

$$OA = 25\text{cm}$$

$$OM \perp AB$$

M is the mid – point of AB

$$AM = \frac{1}{2} AB = \frac{1}{2} \times 48 = 24 \text{ cm}$$

Now right Δ OAM,

$$OA^2 = OM^2 + AM^2$$

(by Pythagoras Axiom)

$$(25)^2 = OM^2 + (24)^2$$

$$OM^2 = (25)^2 - (24)^2 = 625 - 576$$

$$= 49 = (7)^2$$

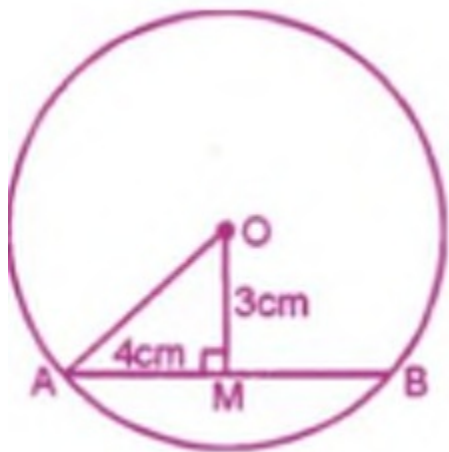
$$OM = 7 \text{ cm}$$

3. A chord of length 8 cm is at a distance of 3 cm from the centre of circle. Calculate the radius of the circle.

Solution

AB is the chord of a circle with centre O

And radius OA and $OM \perp AB$



$$AB = 8\text{cm}$$

$$OM = 3\text{cm}$$

$$OM \perp AB$$

M is the mid point of AB

$$AM = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4\text{cm}$$

Now in right ΔOAM

$$OA^2 = OM^2 + AM^2$$

(by Pythagoras axiom)

$$=(3)^2 + (4)^2 = 9 + 16 = 25$$

$$=(5)^2$$

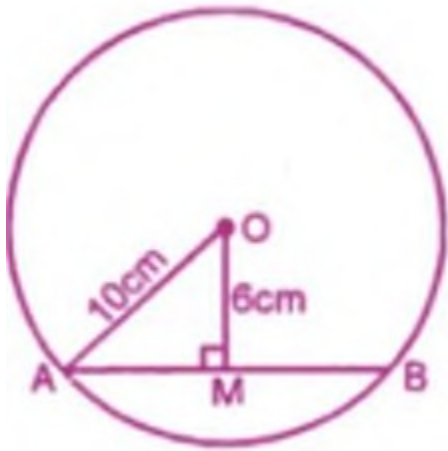
$$OA = 5\text{cm}$$

4. Calculate the length of the chord which is at a distance of 6cm from the centre of a circle of diameter 20 cm.

Solution

AB is the chord of the circle with centre O

And radius OA and $OM \perp AB$



Diameter of the circle = 20 cm

$$\text{Radius} = \frac{20}{2} = 10 \text{ cm}$$

$$OA = 10 \text{ cm}, OM = 6 \text{ cm}$$

Now in right ΔOAM ,

$$OA^2 = AM^2 + OM^2$$

(by Pythagoras axiom)

$$(10)^2 = AM^2 + (6)^2$$

$$AM^2 = 10^2 - 6^2$$

$$AM^2 = 100 - 36 = 64 = (8)^2$$

$$AM = 8 \text{ cm}$$

$$OM \perp AB$$

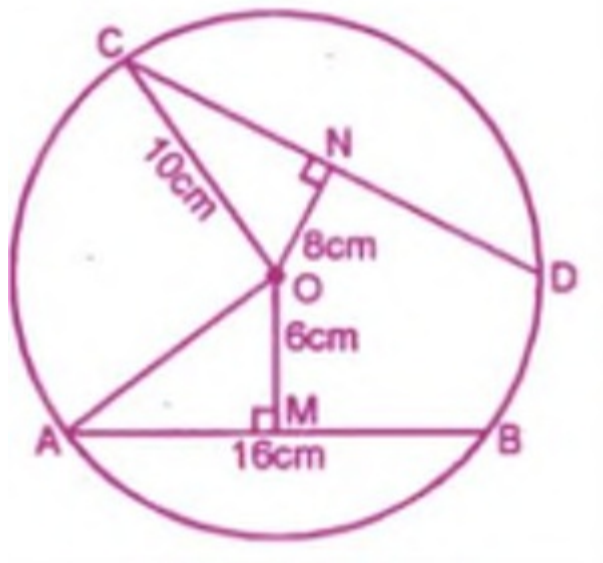
M is the mid point of AB

$$AB = 2AM = 2 \times 8 = 16 \text{ cm}$$

5. A chord of length 16 cm is at a distance of 6cm from the centre of the circle. Find the length of the chord of the same circle which is at a distance of 8 cm from the centre.

Solution

AB is a chord a circle with centre O and OA is the radius of the circle and $OM \perp AB$



$$AB = 16\text{cm} , OM = 6\text{ cm}$$

$$OM \perp AB$$

$$AM = \frac{1}{2} AB = \frac{1}{2} \times 16 = 8\text{ cm}$$

Now in right ΔOAM

$$OA^2 = AM^2 + OM^2$$

(by Pythagoras axiom)

$$= (8)^2 + (6)^2$$

$$64 + 36 = 100 = (10)^2$$

Now CD is another chord of the same circle

ON $\perp$ CD and OC is the radius

In right $\triangle ONC$

$$OC^2 = ON^2 + NC^2$$

(by Pythagoras axioms)

$$(10)^2 = (8)^2 + (NC)^2$$

$$100 = 64 + NC^2$$

$$NC^2 = 100 - 64 = 36 = (6)^2$$

$$NC = 6$$

But ON $\perp$ AB

N is the mid – point of CD

$$CD = 2NC = 2 \times 6 = 12 \text{ cm}$$

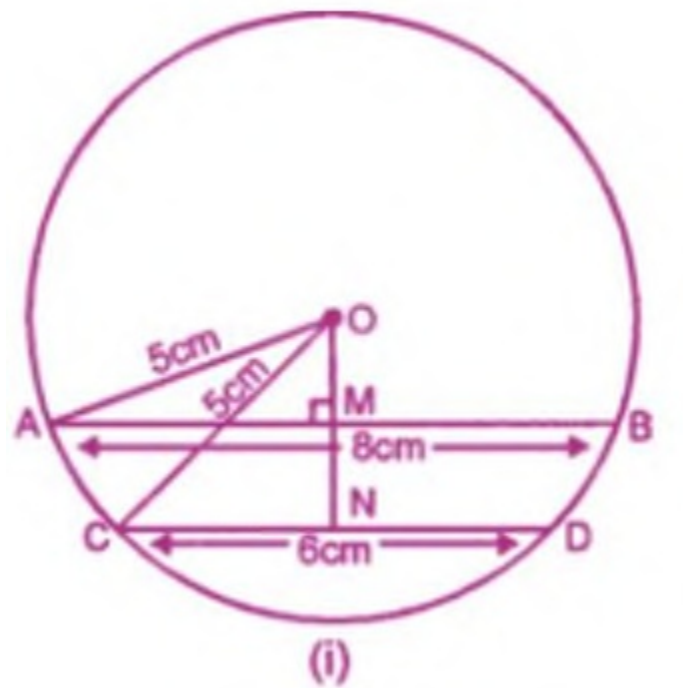
6. in a circle of radius 5 cm , AB and CD are two parallel chords of length 8 cm and 6 cm respectively. Calculate the distance between the chords if they are on :

(i) the same side of the centre.

(ii) the opposite sides of the centre

Solution

Two chords AB and CD of a circle with centre O and radius OA or OC



$$OA = OC = 5\text{cm}$$

$$AB = 8\text{cm}$$

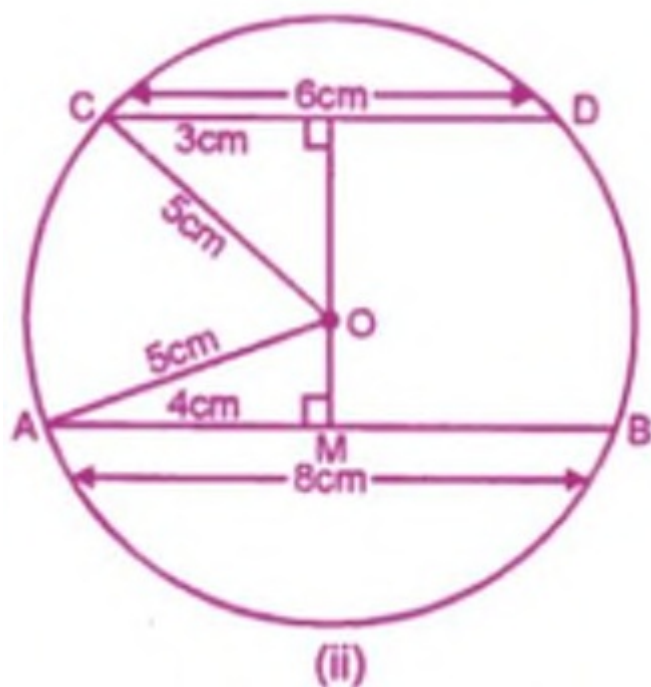
$$CD = 6\text{cm}$$

OM and ON are perpendicular from O to AB and CD respectively

M and N are the mid points of AB and CD respectively

In figure (i) chord are on the same side

And in figure (ii) chord are on the opposite sides of the centre



In right ΔOAM

$$OA^2 = AM^2 + OM^2$$

(by Pythagoras axiom)

$$(5)^2 = (4)^2 + OM^2$$

$$AM = \frac{1}{2}AB$$

$$25 = 16 + OM^2$$

$$OM^2 = 25 - 16 = 9 = (3)^2$$

$$OM = 3\text{cm}$$

Again in right ΔOCN ,

$$OC^2 = CN^2 + ON^2$$

$$(5)^2 = (3)^2 + ON^2$$

$$(CN = \frac{1}{2} CD)$$

$$25 = 9 + ON^2$$

$$ON^2 = 25 - 9 = 16 = (4)^2$$

$$ON = 4$$

In figure(i) distance $MN = ON - OM$

$$= 4 - 3 = 1 \text{ cm}$$

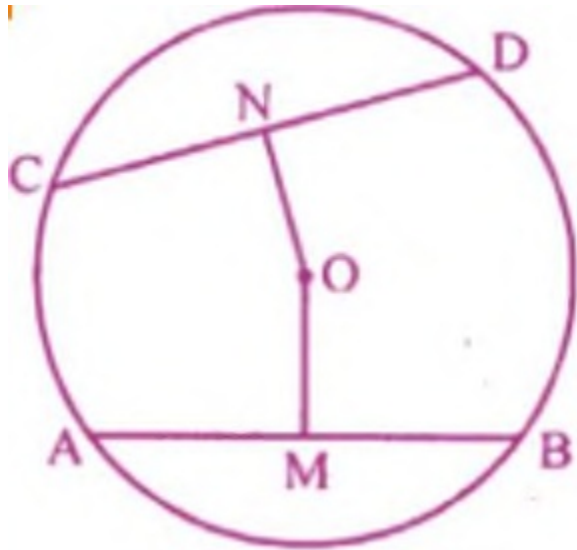
In fig(ii)

$$MN = OM + ON = 3 + 4 = 7 \text{ cm}$$

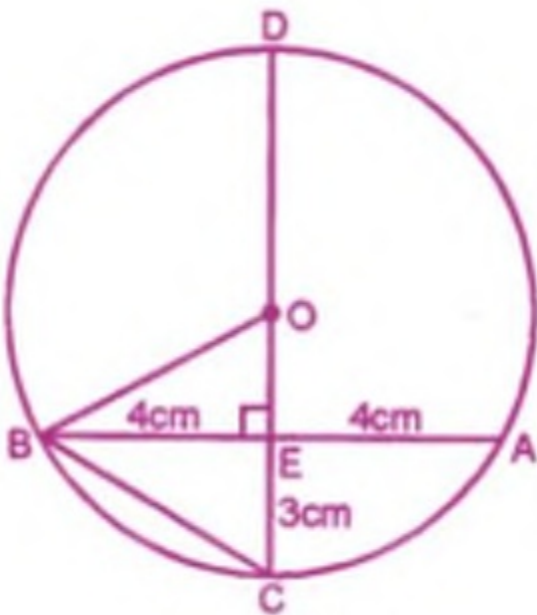
7.(a) in the figure given below , o is the centre of the circle. AB and CD are two chords of the circle, OM is perpendicular to AB and ON is perpendicular to CD . AB =24 cm, OM = 5 cm , ON = 12cm . find the :

(i) radius of the circle

(ii) length of chord CD



(b) In the figure (ii) given below , CD is the diameter which meets the chord AB in E such that $AE = BE = 4\text{cm}$. if $CE = 3\text{cm}$, find the radius of circle.



Solution

(a) given : $AB = 24 \text{ cm}$, $OM = 5 \text{ cm}$, $ON = 12 \text{ cm}$

$OM \perp AB$

M is midpoint of AB

$AM = 12 \text{ cm}$

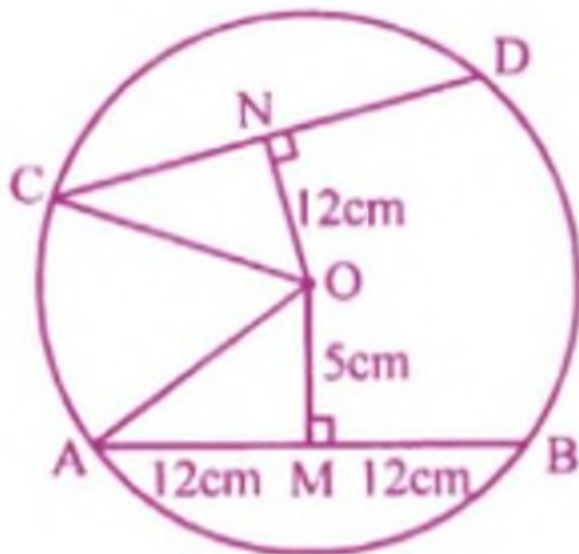
(i) Radius of circle $OA = \sqrt{OM^2 + AM^2}$

(ii) again $OC^2 = ON^2 + CN^2$

$$13^2 = 12^2 + CN^2$$

$$CN = \sqrt{13^2 - 12^2} = \sqrt{169 - 144} = \sqrt{25}$$

$$CN = 5 \text{ cm}$$



As $ON \perp CD$, N is midpoint of CD

$$CD = 2CN = 2 \times 5 = 10\text{cm}$$

(b) $AB = 8\text{cm}$, $EC = 3\text{cm}$

Let radius $OB = OC = r$

$$OE = (r - 3) \text{ cm}$$

Now in right $\triangle OBE$

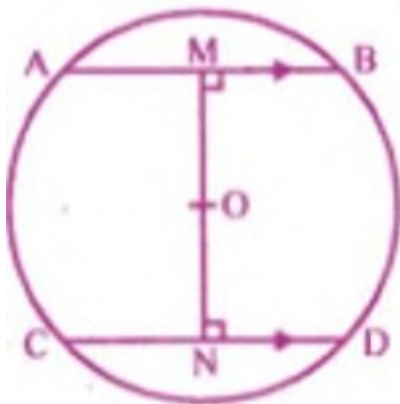
$$OB^2 = BE^2 + OE^2$$

$$r^2 = (4)^2 + (r - 3)^2$$

$$r^2 = 16 + r^2 - 6r + 9$$

$$6r = 25 \left(r = \frac{25}{6} = 4\frac{1}{6} \right) \text{cm}$$

8. In the adjoining figure, AB and CD are two parallel chords and O is the centre. If the radius of the circle is 15 cm, find the distance MN between the two chords of length 24 cm and 18cm respectively.



Solution

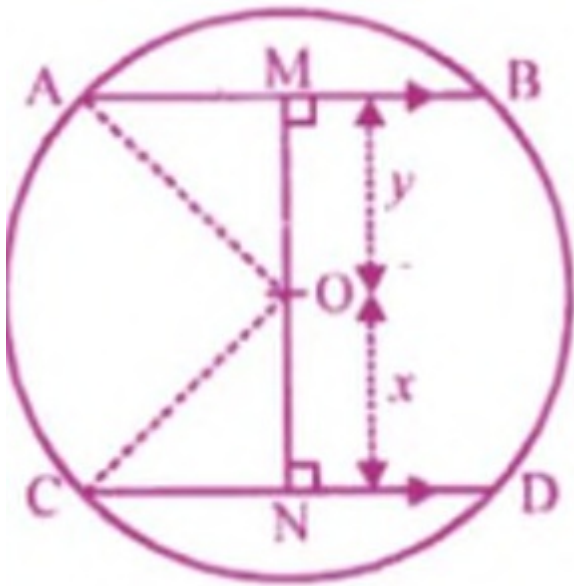
In the figure chord $AB \parallel CD$

O is the centre of the circle

Radius of the circle = 15cm

Length of $AB = 24\text{cm}$ and $CD = 18\text{cm}$

Join OA and OC



$AB = 24\text{cm}$ and $OM \perp AB$

$$AM = MB = \frac{24}{2} = 12\text{cm}$$

Similarly $ON \perp CD$

$$CN = ND = \frac{18}{2} = 9\text{cm}$$

Similarly in right ΔCNO

$$OC^2 = CN^2 + ON^2 \quad (15)^2 = (9)^2 + ON^2$$

$$225 = 81 + ON^2$$

$$ON^2 = 225 - 81 = 144 = (12)^2$$

$$ON = 12\text{cm}$$

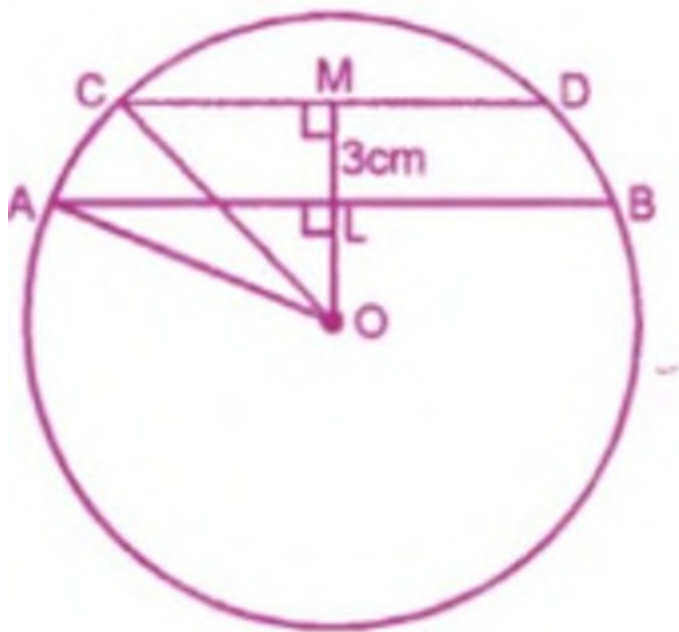
$$\text{Now } MN = OM + ON = 9 + 12 = 21 \text{ cm}$$

9. AB and CD are two parallel chords of a circle of lengths 10 cm and 4 cm respectively. If the chords lie on the same side of the centre and the distance between them is 3cm , find the diameter of the circle.

Solution

AB and CD are two parallel chords and $AB = 10 \text{ cm}$, $CD = 4\text{cm}$
and distance between

AB and CD = 3cm



Let radius of circle $OA = OC = r$

$OM \perp CD$ which intersects AB in L .

Let $OL = x$, then $OM = x + 3$

Now right $\triangle OLA$

$$OA^2 = AL^2 + OL^2$$

$$r^2 = (5)^2 + x^2 = 25 + x^2$$

(L is mid point of AB)

Again in right $\triangle OCM$

$$OC^2 = CM^2 + OM^2$$

$$r^2 = (2)^2 + (x + 3)^2$$

(M is mid – point of CD)

$$r^2 = 4 + (x + 3)^2$$

(M is mid-point of CD)

From (i) and (ii)

$$25 + x^2 = 4 + (x + 3)^2$$

$$25 + x^2 = 4 + x^2 + 9 + 6x$$

$$6x = 25 - 13 = 12$$

$$x = \frac{12}{6} = 2\text{cm}$$

Substituting the value of x in (i)

$$r^2 = 25 + x^2 = 25 + (2)^2 = 25 + 4$$

$$r^2 = 29$$

$$r = \sqrt{29} \text{ cm}$$

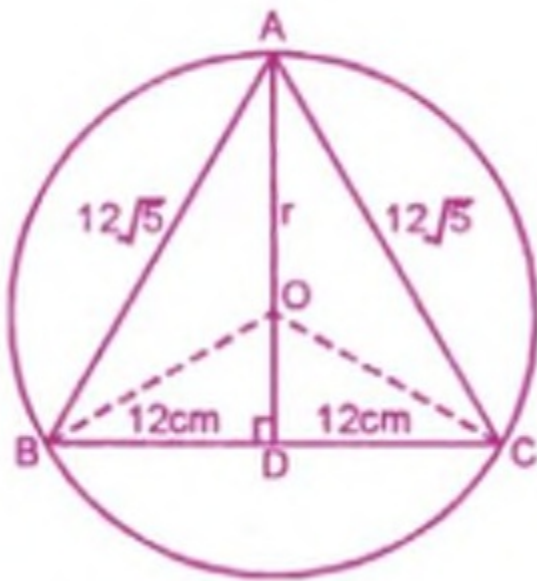
diameter of the circle = $2r$

$$= 2 \times \sqrt{29} \text{ cm} = 2\sqrt{29} \text{ cm}$$

10. ABC is an isosceles triangle inscribed in a circle. If $AB = AC = 12\sqrt{5}$ cm and $BC = 24$ cm, find the radius of the circle

Solution

$AB = AC = 12\sqrt{5}$ and $BC = 24$ cm



Join OB and OC and OA

Draw $AD \perp BC$ which will pass through

Centre O

OD bisect BC in D

$$BD = DC = 12 \text{ cm}$$

In right $\triangle ABD$

$$AB^2 = AD^2 + BD^2$$

$$(12\sqrt{5})^2 = AD^2 + BD^2$$

$$(12\sqrt{5})^2 = AD^2 + (12)^2$$

$$144 \times 5 = AD^2 + 144$$

$$720 - 144 = AD^2$$

$$AD^2 = 576 \quad (AD = \sqrt{576} = 24)$$

Let radius of the circle = $OA = OB = OC = r$

$$OD = AD - AO = 24 - r$$

Now in right ΔOBD

$$OB^2 = BD^2 + OD^2$$

$$r^2 = 144 + 576 + r^2 - 48r$$

$$48r = 720$$

$$r = \frac{720}{48}$$

$$r = 15\text{cm}$$

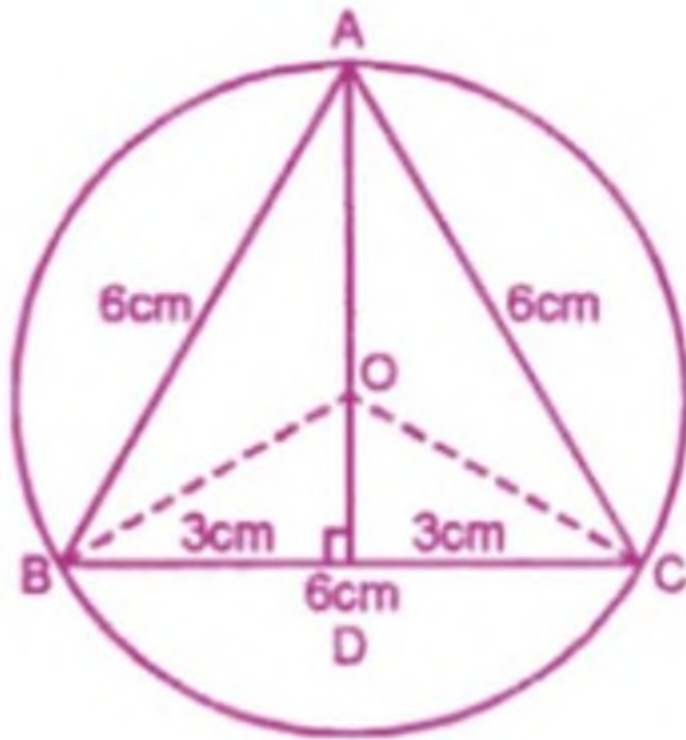
$$\text{radius} = 15\text{cm}.$$

**11. An equilateral triangle of side 6 cm is inscribed in a circle.
Find the radius of the circle.**

Solution

ABC is an equilateral triangle inscribed in a

Circle with centre O. Join OB and OC,
 From A, draw $AD \perp BC$ which will pass
 Through the centre O of the circle .



Each side of $\Delta ABC = 6\text{cm}$

$$AD = \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{2} \times 6 = 3\sqrt{3}\text{cm}$$

$$OD = AD - AO = 3\sqrt{3} - r$$

Now in right ΔOBD

$$OB^2 = BD^2 + OD^2$$

$$r^2 = (3)^2 + (3\sqrt{3} - r)^2$$

$$r^2 = 9 + 27 + r^2 - 6\sqrt{3}r$$

(D is mid – point of BC)

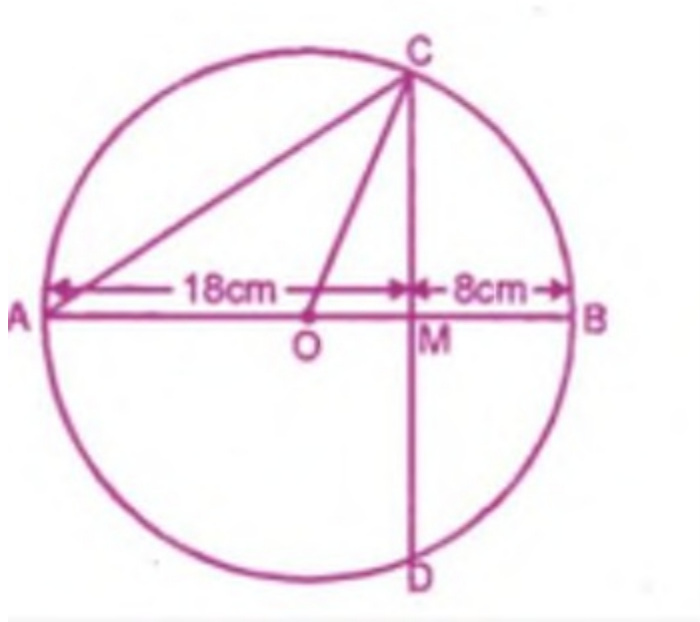
$$6\sqrt{3}r = 36$$

$$R = \frac{36}{6\sqrt{3}} = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3} \text{ cm}$$

$$\text{Radius} = 2\sqrt{3} \text{ cm}$$

12. AB is a diameter of a circle. M is a point in AB such that AM = 18 cm and MB = 8 cm. Find the length of the shortest chord through M.

Solution



$$AM = 18 \text{ cm and } MB = 8 \text{ cm}$$

$$AB = AM + MB = 18 + 8 = 26 \text{ cm}$$

$$\text{Radius of the circle} = \frac{26}{2} = 13 \text{ cm}$$

Let CD is the shortest chord drawn through M.

$$CD \perp AB$$

Join OC

$$OM = AM - AO = 18 - 13 = 5\text{cm}$$

$$OC = OA = 13\text{ cm}$$

Now in right ΔOMC

$$OC^2 = OM^2 + MC^2$$

$$(13)^2 = (5)^2 + MC^2 \quad (MC^2 = 13^2 - 5^2)$$

$$MC^2 = 169 - 25 = 144 = (12)^2$$

$$MC = 12$$

M is mid – point of CD

$$CD = 2 \times MC = 2 \times 12 = 24\text{ cm}$$

Exercise 15.2

1. if arcs APB and CQD of a circle are congruent, then find the ratio of AB : CD

Solution

arc APB = arc CQD(given)

AB = CD

(if two arcs are congruent, then their corresponding chords are equal)

$$\text{Ratio of AB and CD} = \frac{AB}{CD} = \frac{AB}{AB} = \frac{1}{1}$$

$$AB : CD = 1:1$$

2. A and B are points on a circle with centre o. C is a point on the circle such that OC bisects $\angle AOB$, prove that OC bisects the arc AB.

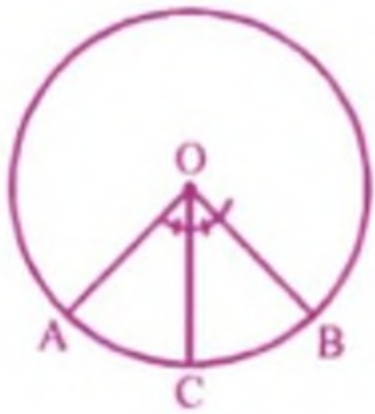
Solution

Given : in a given circle with centre O,A

A and B are two points on the circle . C

Another point on the circle such that

$$\angle AOC = \angle BOC$$



To prove : arc AC = arc BC

Proof : OC is the bisector of $\angle AOB$

Or $\angle AOC = \angle BOC$

But these are the angle subtended by the
arc AC and BC

arc AC = arc BC.

3. prove that the angle subtended at the centre of a circle is bisected by the radius passing through the mid- point of the arc.

Solution :

Given : AB is the arc of the circle with

Centre o and C is the mid – point od arc AB

To prove : OC bisects the $\angle AOB$

i.e, $\angle AOC = \angle BOC$

proof : C is the mid – point of arc AB

arc AC = arc BC



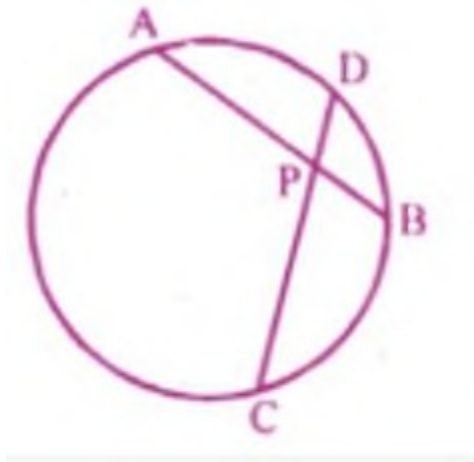
But arc AC and arc BC subtend $\angle AOC$ and $\angle BOC$ at the centre

$\angle AOC = \angle BOC$

Hence OC bisects the $\angle AOB$.

4. in the given figure, two chords AB and CD of a circle intersect at P. If $AB = CD$, prove that arc AD = arc CB.

Solution



Given : two chord AB and CD of a circle

Intersect at P and $AB = CD$

To prove : $\text{arc AD} = \text{arc CB}$

Proof : $AB = CD$ (given)

Minor arc $AB = \text{minor arc } CD$

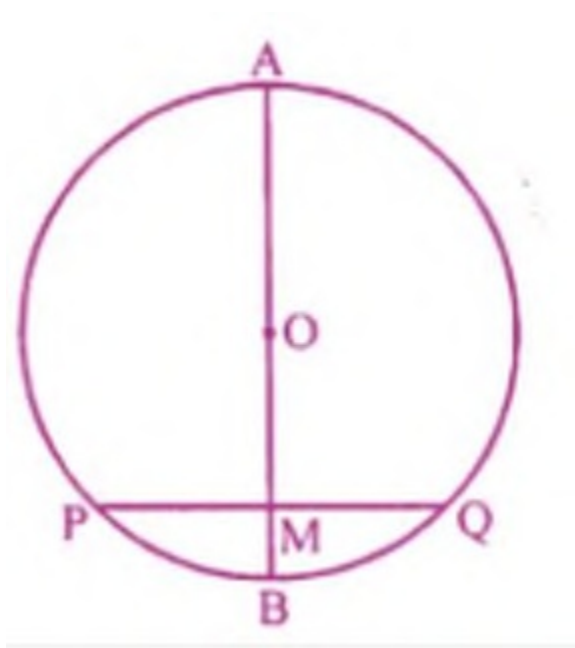
Subtracting arc BD from both sides

$\text{arc AB} - \text{arc BD} = \text{arc CD} - \text{arc BD}$

$\text{arc AD} = \text{arc CB}$

Chapter test

1. in the given figure , a chord PQ of a circle with centre O and radius 15 cm is bisected at M by a diameter AB. If $OM = 9\text{cm}$, find the length of : (i) PQ (ii) AP (iii) BP



Solution

Given , radius = 15 cm

$OA = OB = OP = OQ = 15\text{cm}$

Also, $OM = 9\text{cm}$



$$MB = OB - OM = 15 - 9 = 6\text{cm}$$

$$AM = OA + OM = 15 + 9 \text{ cm} = 24 \text{ cm}$$

In ΔOMP , by using Pythagoras theorem,

$$OP^2 = OM^2 + PM^2$$

$$15^2 + 9^2 + PM^2$$

$$PM^2 = 255 - 81$$

$$PM = \sqrt{144} = 12\text{cm}$$

Also , in ΔOMQ

By using Pythagoras theorem

$$OQ^2 = OM^2 + QM^2$$

$$15^2 = OM^2 + QM^2$$

$$15^2 = 9^2 + QM^2 \text{ (QM}^2 = 225 - 81\text{)}$$

$$QM = \sqrt{144} = 12\text{cm}$$

$$PQ = PM + QM$$

(as radius is bisected at M)

$$PQ = 12 + 12 \text{ cm} = 24 \text{ cm}$$

(ii) now in ΔAPM

$$AP^2 = AM^2 + OM^2$$

$$AP^2 = 24^2 + 12^2$$

$$AP^2 = 567 + 144$$

$$AP = \sqrt{720} = 12\sqrt{5}\text{cm}$$

(iii) now in ΔBMP

$$BP^2 = BM^2 + PM^2$$

$$BP^2 = 6^2 + 12^2$$

$$BP^2 = 36 + 144$$

$$BP = \sqrt{180} = 6\sqrt{5}\text{cm}$$

2. the radii of two concentric circles are 17 cm and 10 cm; a line PQRS cuts the larger circle at P and S and the smaller circle at Q and R. If QR = 12 cm , calculate PQ

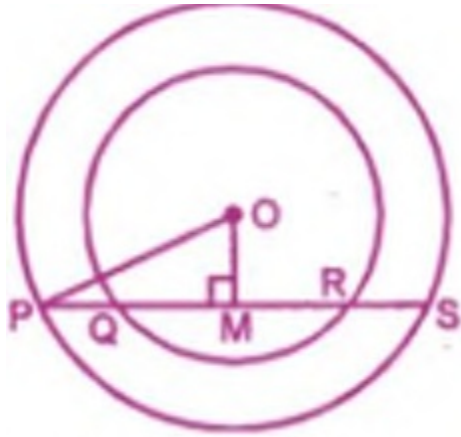
Solution

A line PQRS intersects the outer circle at P

And S and inner circle at Q and R radius of

Outer circle OP = 17 cm and radius of inner

Circle OQ = 10cm



QR = 12cm

From O, draw $OM \perp PS$

$$QM = \frac{1}{2} QR = \frac{1}{2} \times 12 = 6\text{cm}$$

In right ΔOQM

$$OQ^2 = OM^2 + QM^2$$

$$(10)^2 = OM^2 + (6)^2$$

$$OM^2 = 10^2 - 6^2$$

$$= 100 - 36 = 64 = (8)^2$$

$$OM = 8 \text{ cm}$$

Now in right ΔOPM

$$OP^2 = OM^2 + PM^2$$

$$(17)^2 = OM^2 + PM^2$$

$$PM^2 = (17)^2 - (8)^2$$

$$= 289 - 64 = 225 = (15)^2$$

$$PM = 15\text{cm}$$

$$PQ = PM - QM = 15 - 6 = 9 \text{ cm}$$