

Chapter 17
Trigonometric Ratio

Exercise 17

1. (a) from the figure (1) given below , find the values of:

(i) $\sin \theta$

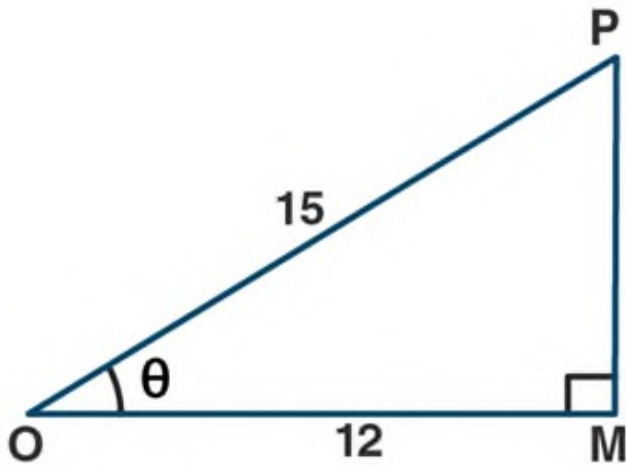
(ii) $\cos \theta$

(iii) $\tan \theta$

(iv) $\cot \theta$

(v) $\sec \theta$

(vi) $\operatorname{cosec} \theta$



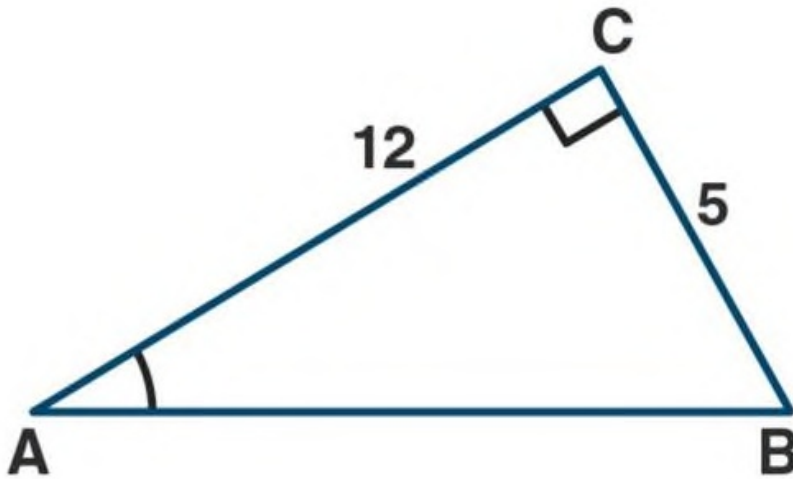
(b) from the figure (2) given below , find the values of :

(i) $\sin A$

(ii) $\cos A$

(iii) $\sin^2 A + \cos^2 A$

(iv) $\sec^2 A - \tan^2 A$.



Solution

(a) from right angled triangle OMP ,

By Pythagoras theorem , we get

$$OP^2 = OM^2 + MP^2$$

$$MP^2 = OP^2 - OM^2$$

$$MP^2 = (15)^2 - (12)^2$$

$$MP^2 = 225 - 144$$

$$MP^2 = 81$$

$$MP^2 = 92$$

$$MP = 9$$

$$(i) \sin \theta = \frac{MP}{OP}$$

$$= \frac{9}{15}$$

$$= \frac{3}{5}$$

$$(ii) \cos \theta = \frac{OM}{OP}$$

$$= \frac{12}{15}$$

$$= \frac{4}{5}$$

$$(iii) \tan \theta = \frac{MP}{OP}$$

$$= \frac{9}{12}$$

$$= \frac{3}{4}$$

$$(iv) \cot \theta = \frac{OM}{MP}$$

$$= \frac{12}{9}$$

$$= \frac{4}{3}$$

$$(v) \sec \theta = \frac{op}{om}$$

$$= \frac{15}{12}$$

$$= \frac{5}{4}$$

$$(vi) \operatorname{cosec} \theta = \frac{OP}{MP}$$

$$= \frac{15}{9}$$

$$= \frac{5}{3}$$

(B) from right angled triangle ABC,

By Pythagoras theorem , we get

$$AB^2 = AC^2 + BC^2$$

$$AB^2 = (12)^2 + (5)^2$$

$$AB^2 = 144 + 25$$

$$AB^2 = 169$$

$$AB^2 = 13^2$$

$$AB = 13$$

$$(i) \sin A = \frac{BC}{AB}$$

$$= \frac{5}{13}$$

$$(ii) \cos A = \frac{AC}{AB}$$

$$= \frac{12}{13}$$

$$(iii) \sin^2 A + \cos^2 A = \left(\frac{BC}{AB}\right)^2 + \left(\frac{AC}{AB}\right)^2$$

$$= \left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2$$

$$= \left(\frac{25}{169}\right) + \left(\frac{144}{169}\right)$$

$$= \frac{25+144}{169}$$

$$= \frac{169}{169}$$

$$= 1$$

$$\sin^2 A + \cos^2 A = 1$$

$$(iv) \sec^2 A - \tan^2 A = \left(\frac{AB}{AC}\right)^2 - \left(\frac{BC}{AC}\right)^2$$

$$= \left(\frac{13}{12}\right)^2 - \left(\frac{5}{12}\right)^2$$

$$= \frac{169}{144} - \frac{25}{144}$$

$$= \frac{144}{144}$$

$$= 1$$

$$\sec^2 A - \tan^2 A = 1$$

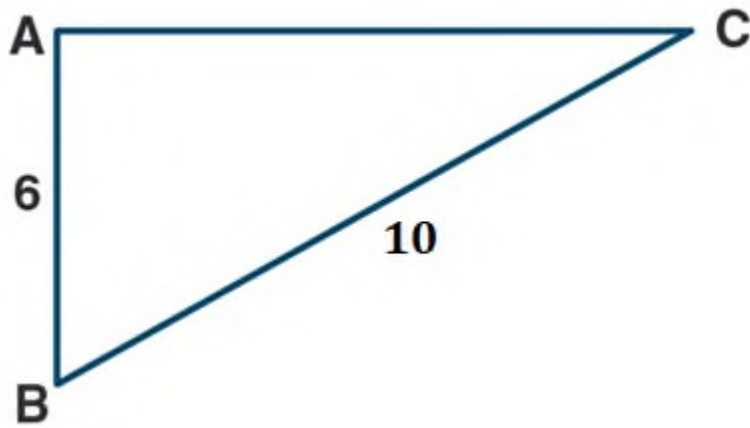
2. (a) from the figure (1) given below , find the values of :

(i) $\sin B$

(ii) $\cos c$

(iii) $\sin B + \sin C$

(iv) $\sin B \cos C + \sin C \cos B$.



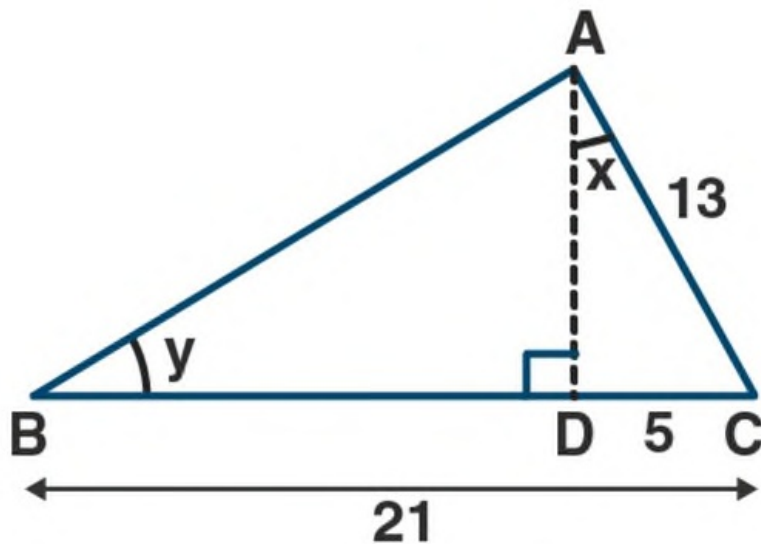
(b) from the figure (2) given below , find the values of :

(i) $\tan x$

(ii) $\cos y$

(iii) $\operatorname{cosec}^2 y - \cot^2 y$

(iv) $\frac{5}{\sin x} + \frac{3}{\sin y} - 3 \cot y$



Solution

From right angled triangle ABC ,

By Pythagoras theorem , we get

$$BC^2 = AC^2 + AB^2$$

$$AC^2 = BC^2 - AB^2$$

$$AC^2 = 10^2 - 6^2$$

$$AC^2 = 100 - 36$$

$$AC^2 = 64$$

$$AC^2 = 8^2$$

$$AC = 8$$

$$(i) \sin B = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$= \frac{AC}{BC}$$

$$= \frac{8}{10}$$

$$= \frac{4}{5}$$

$$(ii) \cos c = \frac{\text{base}}{\text{hypotenuse}}$$

$$= \frac{AC}{BC}$$

$$= \frac{8}{10}$$

$$= \frac{4}{5}$$

$$(iii) \sin B = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$= \frac{AC}{BC}$$

$$= \frac{8}{10}$$

$$= \frac{4}{5}$$

$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$= \frac{AB}{BC}$$

$$= \frac{6}{10}$$

$$= \frac{3}{5}$$

Now ,

$$\sin B + \sin C = \frac{4}{5} + \frac{3}{5}$$

$$= 4 + \frac{3}{5}$$

$$= \frac{7}{5}$$

$$(iv) \sin B = \frac{4}{5}$$

$$\cos C = \frac{4}{5}$$

$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$= \frac{AB}{BC}$$

$$= \frac{6}{10}$$

$$= \frac{3}{5}$$

$$\cos B = \frac{\text{base}}{\text{hypotenuse}}$$

$$= \frac{AB}{BC}$$

$$= \frac{6}{10}$$

$$= \frac{3}{5}$$

$$\sin B \cos C + \sin C \cos B$$

$$= \frac{4}{5} \times \frac{4}{5} \times \frac{3}{5} \times \frac{3}{5}$$

$$= \frac{26}{25} \times \frac{9}{25}$$

$$= \frac{16+9}{25}$$

$$= \frac{25}{25}$$

$$= 1$$

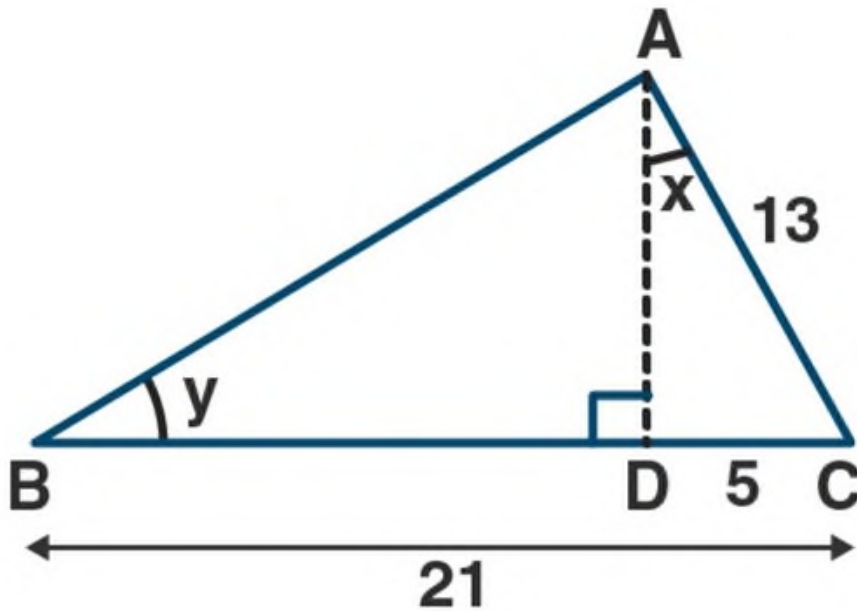
From figure

$$AC = 13, CD = 5, BC = 21$$

$$BD = BC - CD$$

$$= 21 - 5$$

$$= 16$$



From right angle $\triangle ACD$,

By Pythagoras theorem we get

$$AC^2 = AD^2 + CD^2$$

$$AD^2 = AC^2 - CD^2$$

$$AD^2 = (13)^2 - (5)^2$$

$$AD^2 = 169 - 25$$

$$AD^2 = 144$$

$$AD^2 = 12^2$$

$$AD = 12$$

From right angled $\triangle ABD$,

By Pythagoras angled $\triangle ABD$

By Pythagoras theorem we get

$$AB^2 = AD^2 + BD^2$$

$$AB^2 = 400$$

$$AB^2 = (20)^2$$

$$AB = 20$$

$$(i) \tan x = \frac{\text{perpendicular}}{\text{base}} \text{ (in right angled } \Delta ACD \text{)}$$

$$= \frac{CD}{AD}$$

$$= \frac{5}{12}$$

$$(ii) \cos y = \frac{\text{base}}{\text{hypotenuse}} \text{ (in right angled } \Delta ABD \text{)}$$

$$= \frac{BD}{AB}$$

$$= \frac{20}{12} - \frac{5}{3}$$

$$\cot y = \frac{\text{base}}{\text{perpendicular}} \text{ (in right angled } \Delta ABD \text{)}$$

$$= \frac{BD}{AB}$$

$$= \frac{16}{20} = \frac{4}{5}$$

$$(iii) \cos y = \frac{\text{hypotenuse}}{\text{perpendicular}} \text{ (in right angled } \Delta ABD \text{)}$$

$$\frac{BD}{AB}$$

$$= \frac{20}{12}$$

$$= \frac{5}{3}$$

$$\cot y = \frac{\text{base}}{\text{perpendicular}} \text{ (in right angled } \Delta ABD \text{)}$$

$$\frac{AB}{AD}$$

$$= \frac{16}{12}$$

$$= \frac{4}{3}$$

$$\operatorname{Cosec}^2 y - \cot^2 y = \left(\frac{5}{3}\right)^2 - \left(\frac{4}{3}\right)^2$$

$$= \left(\frac{25}{9}\right) - \left(\frac{16}{9}\right)$$

$$= \frac{25-16}{9}$$

$$= \frac{9}{9}$$

$$= 1$$

$$\text{Hence , } \operatorname{cosec}^2 y - \cot^2 y = 1$$

$$\text{(iv) } \sin x = \frac{\text{perpendicular}}{\text{hypotenuse}} \text{ (in right angled } \Delta ACD \text{)}$$

$$= \frac{AD}{AB}$$

$$= \frac{12}{20}$$

$$= \frac{3}{5}$$

$$\cot y = \frac{\text{base}}{\text{perpendicular}} \text{ (in right angled } \Delta \text{ ABD)}$$

$$= \frac{BD}{AD}$$

$$= \frac{16}{12}$$

$$= \frac{4}{3}$$

$$\left(\frac{5}{\sin x} \right) + \left(\frac{3}{\sin y} \right) - 3 \cot y$$

$$= \frac{5}{\frac{5}{13}} + \frac{3}{\frac{3}{5}} - 3 \times \frac{4}{3}$$

$$= 5 \times \frac{13}{5} + 3 \times \frac{5}{3} - 3 \times \frac{4}{3}$$

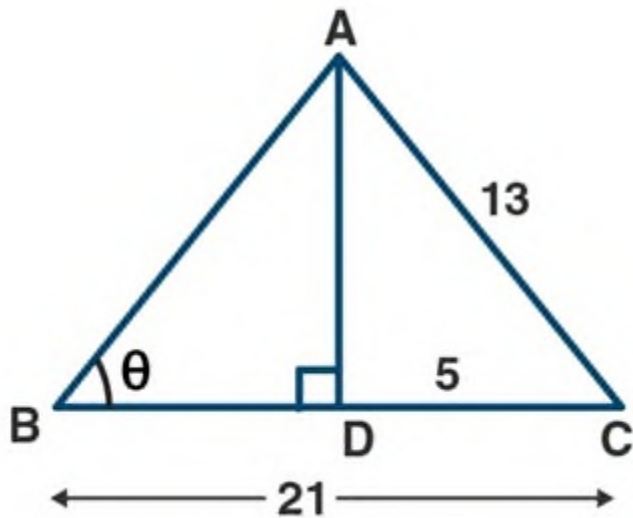
$$= 1 \times \frac{13}{1} + 1 \times \frac{5}{1} - 1 \times \frac{4}{1}$$

$$= 13 + 5 - 4 = 18 - 4$$

$$= 14$$

$$\text{Hence } \frac{5}{\sin x} + \frac{3}{\sin y} - 3 \cot y = 14$$

3. (a) from the figure (1) given below , find the value of $\sec\theta$



(b) from the figure (2) given below, find the values of :

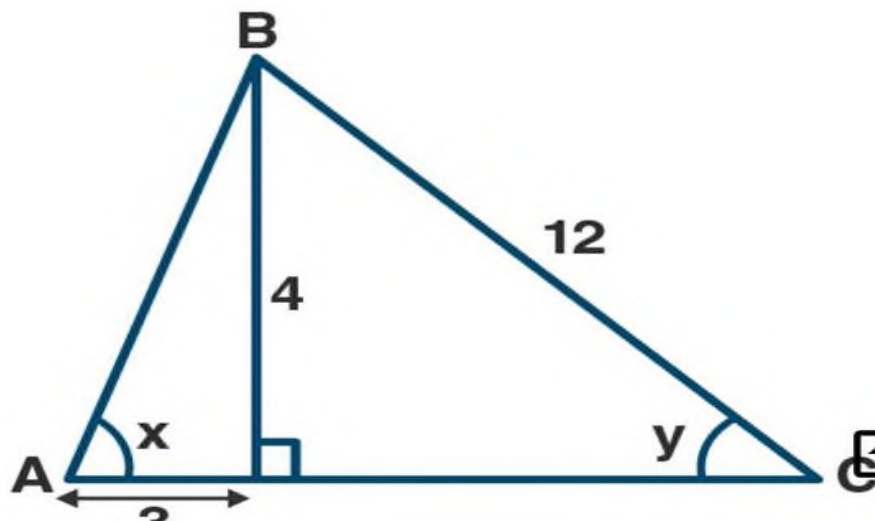
(i) $\sin x$

(ii) $\cot x$

(iii) $\cot^2 x - \operatorname{cosec}^2 x$

(iv) $\sec y$

(v) $\tan^2 y - \frac{1}{\cos^2 y}$



Solution

(a) from the figure , $\sec \theta = \frac{AB}{BD}$

But in ΔADC , $\angle D = 90^\circ$

$$AC^2 = AD^2 + DC^2 \text{ (Pythagoras theorem)}$$

$$(13)^2 = AD^2 + 25$$

$$AD^2 = 169 - 25$$

$$= 144$$

$$= (12)^2$$

$$AD = 12$$

(in right ΔABD)

$$AB^2 = AD^2 + BD^2$$

$$= (12)^2 + (16)^2$$

$$= 144 + 256$$

$$= 400$$

$$= (20)^2$$

$$AB = 20$$

Now , $\sec \theta = \frac{AB}{BD}$

$$= \frac{20}{16}$$

$$= \frac{5}{4}$$

(b) let given ΔABC

$$BD = 3, AC = 12, AD = 4$$

In right angled ΔABD

By Pythagoras theorem

$$AB^2 = AD^2 + BD^2$$

$$AB^2 = (4)^2 + (3)^2$$

$$AB^2 = 16 + 9$$

$$AB^2 = 25$$

$$AB = 5$$

In right angled triangle ACD

By Pythagoras theorem ,

$$AC^2 = AD^2 + CD^2$$

$$CD^2 = AC^2 - AD^2$$

$$CD^2 = 12^2 - 4^2$$

$$CD^2 = 128$$

$$CD = \sqrt{128}$$

$$CD = \sqrt{64} \times 2 CD$$

$$= 8\sqrt{2}$$

$$(i) \sin x = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$= \frac{AD}{AB}$$

$$= \frac{4}{5}$$

$$(ii) \cot x = \frac{\text{base}}{\text{perpendicular}}$$

$$= \frac{BD}{AD}$$

$$= \frac{3}{4}$$

$$(iii) \cot x = \frac{\text{base}}{\text{perpendicular}}$$

$$= \frac{BD}{AD}$$

$$= \frac{3}{4}$$

$$(iv) \operatorname{cosec} x = \frac{\text{hypotenuse}}{\text{perpendicular}}$$

$$\frac{AB}{BD}$$

$$= \frac{5}{4}$$

$$\cot^2 x - \operatorname{cosec}^2 x$$

$$= \left(\frac{3}{4}\right)^2 - \left(\frac{5}{4}\right)^2$$

$$= \frac{9}{16} - \frac{25}{16}$$

$$= \frac{9-25}{16}$$

$$= -\frac{16}{16}$$

$$= -1$$

$$\text{Perpendicular} = \frac{\text{hypotenuse}}{\text{base}} \text{ (in right angled } \Delta \text{ ACD)}$$

$$= \frac{AD}{CD}$$

$$= \frac{12}{8\sqrt{2}}$$

$$= \frac{3}{2\sqrt{2}}$$

$$\text{Cot } y = \frac{\text{base}}{\text{hypotenuse}}$$

$$= \frac{AD}{CD}$$

$$= \frac{4}{8} \sqrt{2}$$

$$= \frac{1}{2} \sqrt{2}$$

$$\text{Cot } y = \frac{\text{base}}{\text{hypotenuse}} \text{ (in right angled } \Delta \text{ ACD)}$$

$$= \frac{CD}{AC}$$

$$= \frac{8\sqrt{2}}{12}$$

$$= \frac{\sqrt{2}}{3}$$

$$\text{Now } \tan^2 y = \frac{1}{\cos^2 y}$$

$$= \left(\frac{1}{2\sqrt{2}} \right)^2 - \left(\frac{1}{\frac{2\sqrt{2}}{3}} \right)^2$$

$$= \frac{1}{4} \times -\frac{1}{4} \times 2$$

$$= \frac{1}{8} - \frac{9}{8}$$

$$= \frac{1-9}{8}$$

$$= -\frac{8}{8}$$

$$= -1$$

$$\tan^2 y - \frac{1}{\cos^2 y} = -1$$

4. (a) from the figure (1) given below , find the value of :

(i) $2 \sin y - \cos y$

(ii) $2 \sin x - \cos x$

(iii) $1 - \sin x + \cos y$

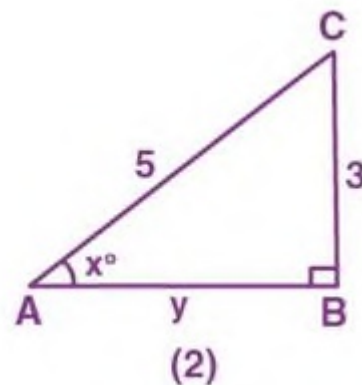
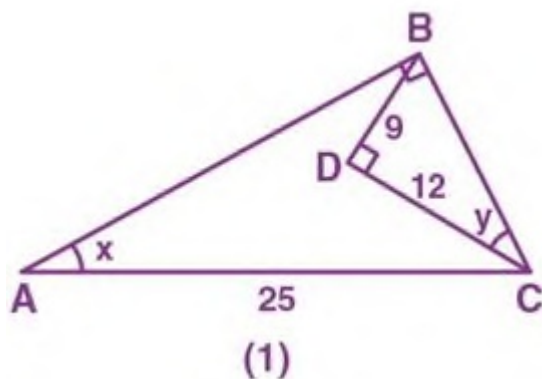
(iv) $2 \cos x - 3 \sin y + 4 \tan x$

(b) in figure (2) given below , ΔABC is right angled at B. If

$AB = Y$ units , $BC = 3$ units and $CA = 5$ units , find

(i) $\sin x^\circ$

(ii) y .



Solution :

(a) in a right angled Δ BCD ,

Using Pythagoras theorem

$$BC^2 = BD^2 + CD^2$$

Substituting the values

$$BC^2 = 9^2 + 12^2$$

By further calculation

$$BC^2 = 81 + 144 = 225$$

$$BC^2 = 15^2$$

$$BC = 15$$

In a right angled Δ ABC ,

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

We can write it as

$$AB^2 = 25^2 - 15^2$$

By further calculation

$$AB^2 = 625 - 225 = 400$$

So we get

$$AB^2 = 20^2$$

$$AB = 20$$

(i) we know that

In right angled ΔBCD

$$\sin y = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\sin y = \frac{BD}{BC}$$

Substituting the values

$$\sin y = \frac{9}{15} = \frac{3}{5}$$

In right angled ΔBCD

$$\cos y = \frac{\text{base}}{\text{hypotenuse}}$$

$$\cos y = \frac{CD}{BC}$$

Substituting the values

$$\cos y = \frac{12}{15} = \frac{4}{5}$$

Here

$$2 \sin y - \cos y = 2 \times \frac{3}{5} - \frac{4}{5}$$

We can write it as

$$= \frac{6}{5} - \frac{4}{5}$$

$$= \frac{2}{5}$$

$$\text{Therefore, } 2 \sin y - \cos y = \frac{2}{5}$$

(ii) in right angled ΔABC

$$\sin x = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\sin x = \frac{BC}{AC}$$

Substituting the values

$$\sin x = \frac{15}{25} = \frac{3}{5}$$

In right angled ΔABC

$$\cos x = \frac{\text{base}}{\text{hypotenuse}}$$

$$\cos x = \frac{AB}{AC}$$

Substituting the values

$$\cos x = \frac{20}{25} = \frac{4}{5}$$

Here

$$2 \sin x - \cos x = 2 \times \frac{3}{5} - \frac{4}{5}$$

We can write it as

$$= \frac{6}{5} - \frac{4}{5}$$

$$= \frac{2}{5}$$

$$\text{Therefore, } 2 \sin x - \cos x = \frac{2}{5}$$

(iii) in right angled ΔABC

$$\sin x = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\sin x = \frac{BC}{AC}$$

Substituting the values

$$\sin x = \frac{12}{25} = \frac{3}{5}$$

In right angled ΔBCD

$$\cos y = \frac{\text{base}}{\text{hypotenuse}}$$

$$\cos y = \frac{CD}{BC}$$

Substituting the values

$$\cos y = \frac{12}{15} = \frac{4}{5}$$

Here

$$1 - \sin x + \cos y = 1 - \frac{3}{5} + \frac{4}{5}$$

By further calculation

$$= \frac{5-3+4}{5}$$

So we get

$$= \frac{9-3}{5}$$

$$= \frac{6}{5}$$

$$\text{Therefore, } 1 - \sin x + \cos y = \frac{6}{5}$$

(iv) in right angled Δ BCD

$$\cos x = \frac{\text{base}}{\text{hypotenuse}}$$

$$\cos x = \frac{AB}{AC}$$

Substituting the values

$$\cos x = \frac{20}{25} = \frac{4}{5}$$

In right angled Δ BCD

$$\sin y = \frac{\text{perpendicular}}{\text{base}}$$

$$\sin y = \frac{BD}{BC}$$

Substituting the values

$$\sin y = \frac{9}{15} = \frac{3}{5}$$

In right angled Δ ABC

$$\tan x = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan x = \frac{BC}{AB}$$

substituting the values

$$\tan x = \frac{15}{20} = \frac{3}{4}$$

here

$$2 \cos x - 3 \sin y + 4 \tan x = 2 \times \frac{4}{5} - 3 \times \frac{3}{5} + 4 \times \frac{3}{4}$$

By further calculation

$$= \frac{8}{5} - \frac{9}{5} + \frac{3}{1}$$

Taking LCM

$$= \frac{8-9+15}{5}$$

$$= \frac{14}{5}$$

(b) it is given that

AB = y units , BC = 3units , CA = 5 units

(i) in right angled Δ ABC

$$\sin x = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\sin x = \frac{BC}{AC}$$

Substituting the values

$$\sin x = \frac{3}{5}$$

(ii) in right angled Δ ABC

Using Pythagoras theorem

$$AC^2 = BC^2 + AB^2$$

We can write it as

$$AB^2 = AC^2 - BC^2$$

Substituting the values

$$AB^2 = 5^2 - 3^2$$

By further calculation

$$AB^2 = 25 - 9 = 16$$

So we get

$$AB^2 = 4^2$$

$$AB = 4$$

$$y = 4 \text{ units}$$

therefore, $y = 4$ units

5. In a right angled triangle , it is given that angle A is an acute angle and that

$\tan A = \frac{5}{12}$. find the values of :

(i) $\cos A$

(ii) $\operatorname{cosec} A - \cot A$.

Solution

Here ABC is right angled triangle

$\angle A$ is an acute angle and $\angle C = 90^\circ$

$$\tan A = \frac{5}{12}$$

$$\frac{BC}{AC} = \frac{5}{12}$$

Let $BC = 5x$ and $AC = 12x$

From right angled $\triangle ABC$

By Pythagoras theorem , we get

$$AB^2 = (5x)^2 + (12x)^2$$

$$AB^2 = 25x^2 + 144x^2$$

$$AB^2 = 169x^2$$

$$(i) \cos A = \frac{\text{base}}{\text{hypotenuse}}$$

$$= \frac{AC}{AB}$$

$$= \frac{12x}{13x}$$

$$= \frac{12}{13}$$

$$(i) \operatorname{cosec} A = \frac{\text{hypotenuse}}{\text{perpendicular}}$$

$$= \frac{AC}{BC}$$

$$= \frac{13x}{5x}$$

$$= \frac{13}{5}$$

$$\operatorname{Cosec} A - \cot A = \frac{13}{5} - \frac{12}{5}$$

$$= \frac{13-12}{5}$$

$$= \frac{1}{5}$$

6. (a) in ΔABC , $\angle A = 90^\circ$. if $AB = 7\text{cm}$ and $BC - AC = 1\text{ cm}$, find :

(i) $\sin C$

(ii) $\tan B$

(b) in ΔPQR $\angle Q = 90^\circ$. if $PQ = 40\text{ cm}$ and $PR + QR = 50\text{ cm}$, find:

(i) $\sin P$

(ii) $\cos P$

(iii) $\tan R$.

Solution :

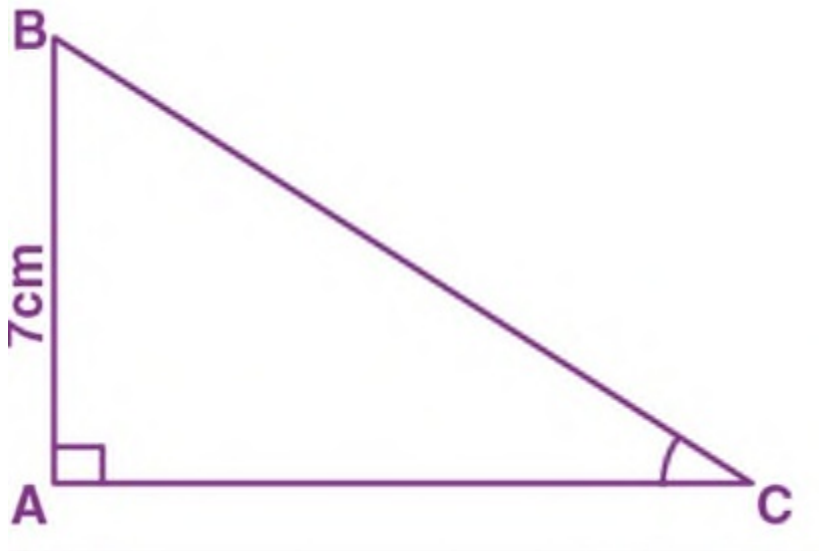
(a) in right ΔABC

$$\angle A = 90^\circ$$

$$AB = 7\text{ cm}$$

$$BC - AC = 1\text{cm}$$

$$BC = 1 + AC$$



We know that

$$BC^2 = AB^2 + AC^2$$

Substituting the value of BC

$$(1 + AC)^2 = AB^2 + AC^2$$

$$1 + AC^2 + 2AC = 7^2 + AC^2$$

By further calculation

$$1 + AC^2 + 2AC = 49 + AC^2$$

$$2AC = 49 - 1 - 48$$

So we get

$$AC = \frac{48}{2} = 24\text{cm}$$

Here

$$BC = 1 + AC$$

Substituting the value

$$BC = 1 + 24 = 25 \text{ cm}$$

$$(i) \sin C = \frac{AB}{BC} = \frac{7}{25}$$

$$(ii) \tan B = \frac{AC}{AB} = \frac{24}{7}$$

(b) in right ΔPQR

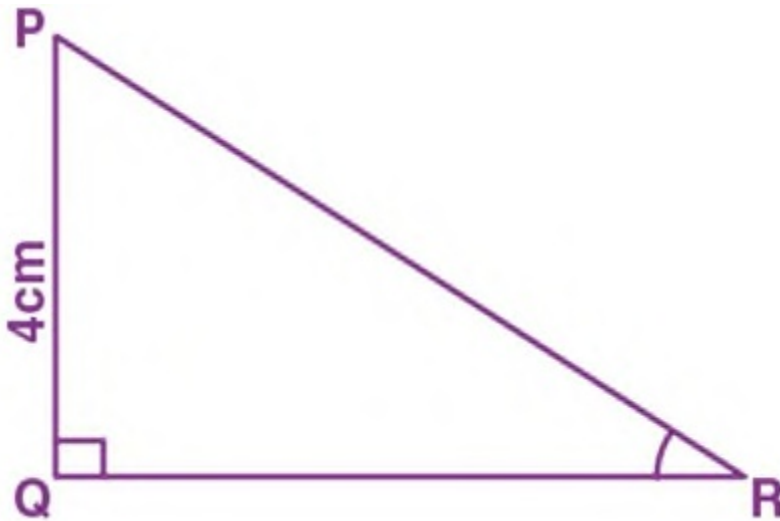
$$\angle Q = 90^\circ$$

$$PQ = 40 \text{ cm}$$

$$PQ + QR = 50 \text{ cm}$$

We can write it as

$$PQ = 50 - QR$$



Using Pythagoras theorem

$$PR^2 = PQ^2 + QR^2$$

$$(50 - QR)^2 = (40)^2 + QR^2$$

By Further calculation

$$2500 + QR^2 - 100QR = 1600 + QR^2$$

So we get

$$2500 - 1600 = 100QR$$

$$100QR = 900$$

By division

$$QR = \frac{900}{100} = 9$$

We get

$$PR = 50 - 9 = 41$$

$$(i) \sin P = \frac{QR}{PR} = \frac{9}{41}$$

$$(ii) \cos P = \frac{PQ}{PR} = \frac{40}{41}$$

$$(iii) \tan R = \frac{PQ}{QR} = \frac{40}{9}$$

7. in triangle ABC , AB = 15cm , AC = 15 cm and BC = 18cm . find

(i) cos ∠ABC

(ii) sin ∠ ACB

Solution

Here ABC is a triangle in which

AB = 15cm , AC = 15 cm and BC = 18cm

Draw AD perpendicular to BC, D is mid -point of BC.

Then BD – DC = 9cm

In right angled triangle ABD

By Pythagoras theorem we get

$$AB^2 = AD^2 + BD^2$$

$$AD^2 = AB^2 - BD^2$$

$$AD^2 = (15)^2 - (9)^2$$

$$AD^2 = 225 - 81$$

$$AD^2 = 144$$

$$AD = 12 \text{ cm}$$

$$(i) \cos \angle ABC = \frac{\text{base}}{\text{hypotenuse}}$$

(in right angled ΔABD , $\angle ABC = \angle ABD$)

$$= \frac{BD}{AD}$$

$$= \frac{9}{15}$$

$$= \frac{3}{5}$$

(ii) $\sin \angle ACB = \sin \angle ACD$

$$= \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$= \frac{ad}{ac}$$

$$= \frac{12}{15}$$

$$= \frac{4}{5}$$

8. (a) in the figure (1) given below , ΔABC is isosceles with $AB = AC = 5\text{cm}$ and $BC = 6\text{cm}$. find

(i) $\sin C$

(ii) $\tan B$

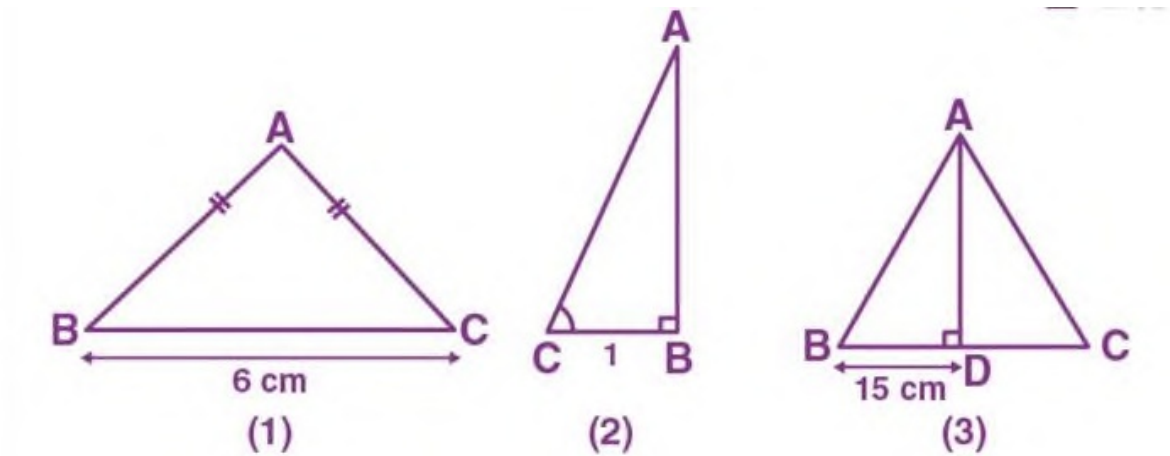
(iii) $\tan C - \cot B$.

(b) in the figure (2) given below , ΔABC is right angled at B. Given that $\angle ACB = \theta$, side $AB = 2$ units and side $BC = 1$ unit , find the value of $\sin^2 \theta + \tan^2 \theta$

(c) in the figure (3) given below AD is perpendicular to BC , $BD = 15 \text{ cm}$, $\sin B = \frac{4}{5}$ and $\tan C = 1$

(i) calculate the lengths of AD , AB , DC and AC

(ii) show that $\tan^2 B - \frac{1}{\cos^2 B} = -1$



Solution

(a) it is given that

ΔABC is isosceles with $AB = AC = 5\text{cm}$ and $BC = 6\text{cm}$

Construct AD perpendicular to BC

D is the mid point of BC

So $BD = CD$

Here

$$BD = CD = \frac{6}{2} = 3\text{ cm}$$

In right angled ΔABD

Using Pythagoras theorem

$$AB^2 = AD^2 + BD^2$$

We can write it as

$$AD^2 = AB^2 - BD^2$$

Substituting the values

$$AD^2 = 5^2 - 3^2$$

By further calculation

$$AD^2 = 25 - 9 = 16$$

So we get

$$AD^2 = 4^2$$

$$AD = 4 \text{ cm}$$

(i) in right angled ΔACD

$$\sin c = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\sin c = \frac{AD}{AC} = \frac{4}{5}$$

(ii) in right angled ΔABD

$$\tan B = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan B = \frac{AD}{BD} = \frac{4}{3}$$

(iii) in right angled ΔACD

$$\tan C = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan c = \frac{AD}{CD} = \frac{4}{3}$$

in right angled ΔABD

$$\cot B = \frac{\text{base}}{\text{perpendicular}}$$

$$\cot B = \frac{BD}{AD} = \frac{3}{4}$$

here

$$\tan C - \cot B = \frac{4}{3} - \frac{3}{4}$$

taking LCM

$$\tan C - \cot B = \frac{16-9}{12} = \frac{7}{12}$$

(b) it is given that

ΔABC is right angled at B

$AB = 2$ units and $BC = 1$ unit

In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$AC^2 = 2^2 + 1^2$$

$$AC^2 = 4 + 1 = 5$$

So we get

$$AC^2 = 5$$

$$AC = \sqrt{5} \text{ units}$$

In right angled ΔABC

$$\sin \theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\sin \theta = \frac{AB}{AC} = \frac{2}{\sqrt{5}}$$

In right angled ΔABC

$$\tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan \theta = \frac{AB}{BC} = \frac{2}{1}$$

we know that

$$\sin^2 \theta + \tan^2 \theta = \left(\frac{2}{\sqrt{5}}\right)^2 + \left(\frac{2}{1}\right)^2$$

by further calculation

$$= \frac{4}{5} + \frac{4}{1}$$

Taking LCM

$$= \frac{4+20}{5}$$

$$= \frac{24}{5}$$

$$= 4 \frac{4}{5}$$

(c) (i) in ΔABC

AD is perpendicular to BC

$$BD = 15\text{cm}$$

$$\sin B = \frac{4}{5}$$

$$\tan c = 1$$

in ΔABD

$$\sin B = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\sin B = \frac{AD}{AB} = \frac{4}{5}$$

consider $AD = 4x$ and $AB = 5x$

using Pythagoras theorem

in right angled ΔABD

$$AB^2 = AD^2 + BD^2$$

We can write it as

$$BD^2 = AB^2 - AD^2$$

Substituting the values

$$(15)^2 = (5x)^2 - (4x)^2$$

$$225 = 25x^2 - 16x^2$$

By further calculation

$$225 = 9x^2$$

$$x^2 = \frac{225}{9} = 25$$

So we get

$$x = \sqrt{25} = 5$$

Here

$$AD = 4 \times 5 = 20$$

$$AB = 5 \times 5 = 25$$

In right angled ΔACD

$$\tan C = \frac{\text{perpendicular}}{\text{base}}$$

so we get

$$\tan C = \frac{AD}{AC} = \frac{1}{1}$$

consider $AD = x$ then $CD = x$

in right angled ΔADC

using Pythagoras theorem

$$AC^2 = AD^2 + CD^2$$

Substituting the values

$$AC^2 = x^2 + x^2 \dots\dots(1)$$

So the equation becomes

$$AC^2 = 20^2 + 20^2$$

$$AC^2 = 400 + 400 = 800$$

So we get

$$AC = \sqrt{800} = 20\sqrt{2}$$

Length of $AD = 20$ cm

Length of $AB = 25$ cm

Length of $DC = 20$ cm

Length of $AC = 20\sqrt{2}$ cm

(ii) in right angled ΔABD

$$\tan B = \frac{\text{perpendicular}}{\text{base}}$$

so we get

$$\tan B = \frac{AD}{BD}$$

substituting the values

$$\tan B = \frac{20}{15} = \frac{4}{3}$$

in right angled ΔABD

$$\cos B = \frac{\text{base}}{\text{hypotenuse}}$$

so we get

$$\cos B = \frac{BD}{AB}$$

substituting the values

$$\cos B = \frac{15}{25} = \frac{3}{5}$$

here

$$\text{LHS} = \tan^2 B - \frac{1}{\cos^2 B}$$

Substituting the values

$$= \left(\frac{4}{3}\right)^2 - \frac{1}{\left(\frac{3}{5}\right)^2}$$

By further calculation

$$= \frac{4^2}{3^2} - \frac{5^2}{3^2}$$

$$= \frac{16}{9} - \frac{25}{9}$$

So we get

$$= \frac{16-25}{9}$$

$$= -\frac{9}{9}$$

$$= -1$$

$$= \text{RHS}$$

Hence , proved

9. if $\sin \theta = \frac{3}{5}$ and θ is acute angle , find

(i) $\cos \theta$

(ii) $\tan \theta$

Solution

Let ΔABC be a right angled at B

Let $\angle ACB = \theta$

Given that , $\sin \theta = \frac{3}{5}$

$$\frac{AB}{AC} = \frac{3}{5}$$

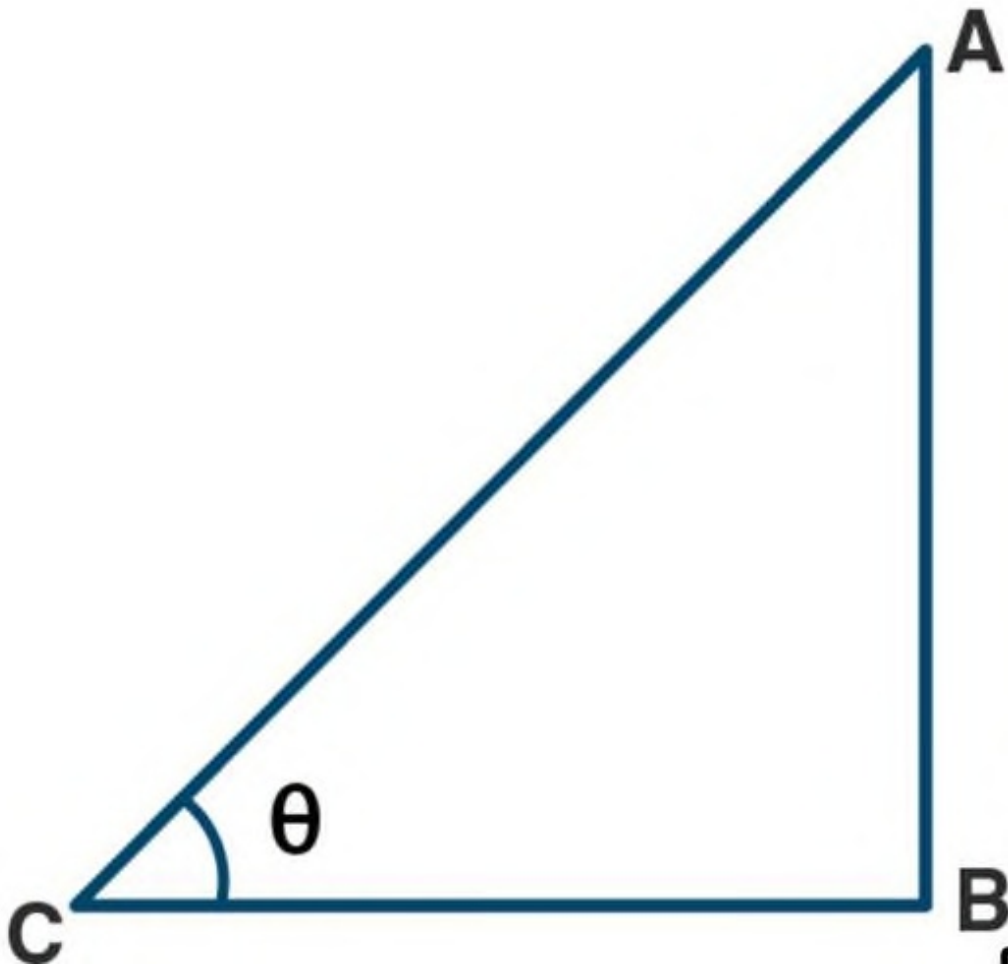
Let $AB = 3x$

Then $AC = 5x$

In right angled ΔABC ,

By Pythagoras theorem,

We get



$$(5x)^2 = (3x)^2 + BC^2$$

$$BC^2 = (5x)^2 - (3x)^2$$

$$BC^2 = (2x)^2$$

$$BC = 4x$$

$$(i) \cos \theta = \frac{\text{base}}{\text{hypotenuse}}$$

$$= \frac{BC}{AC}$$

$$= \frac{4x}{5x}$$

$$= \frac{4}{5}$$

$$(ii) \tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

$$= \frac{AB}{BC}$$

$$= \frac{3x}{4x}$$

$$= \frac{3}{4}$$

10. given that $\tan \theta = \frac{5}{12}$ and θ is an acute angle, find $\sin \theta$ and $\cos \theta$

Solution

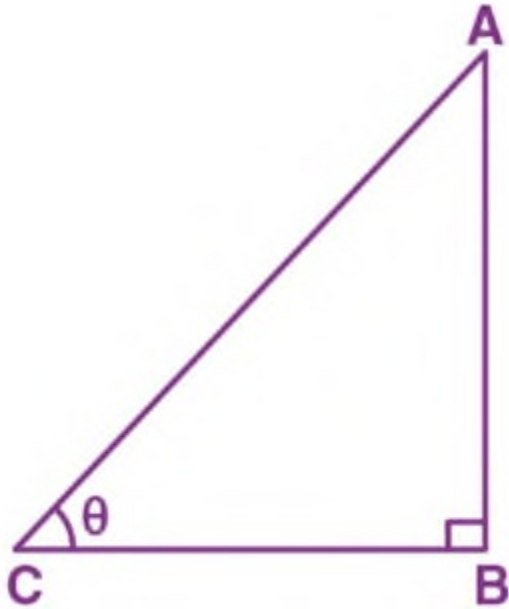
Consider ΔABC be right angled at B and $\angle ACB = \theta$

It is given that

$$\tan \theta = \frac{5}{12}$$

$$\frac{AB}{BC} = \frac{5}{12}$$

Consider $AB = 5x$ and $BC = 12x$



In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$AC^2 = 25x^2 + 144x^2 = 169x^2$$

So we get

$$AC^2 = (13x)^2$$

$$AC = 13x$$

In right angled ΔABC

$$\sin \theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

So we get

$$\sin \theta = \frac{AB}{AC} = \frac{5x}{13x} = \frac{5}{13}$$

In right angled ΔABC

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}}$$

So we get

$$\cos \theta = \frac{BC}{AC}$$

Substituting the values

$$\cos \theta = \frac{12x}{13x} = \frac{12}{13}$$

11. if $\sin \theta = \frac{6}{10}$, find the value of $\cos \theta + \tan \theta$

Solution

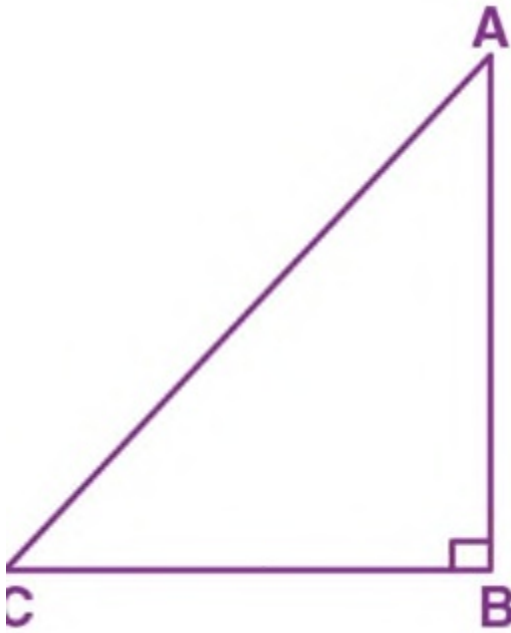
Consider ΔABC be right angled at B and $\angle ACB = \theta$

It is given that

$$\sin \theta = \frac{AB}{AC}$$

$$\sin \theta = \frac{6}{10}$$

Take $AB = 6x$ then $AC = 10x$



In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$(10x)^2 = (6x)^2 + BC^2$$

By further calculation

$$BC^2 = 100x^2 - 36x^2 = 64x^2$$

So we get

$$BC^2 = (8x)^2$$

$$BC = 8x$$

In right angled ΔABC

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{BC}{AC}$$

Substituting the values

$$\cos \theta = \frac{8x}{10x} = \frac{4}{5}$$

In right angled ΔABC

$$\tan \theta = \frac{AB}{BC}$$

substituting the values

$$\tan \theta = \frac{6x}{8x} = \frac{3}{4}$$

here

$$\cos \theta + \tan \theta = \frac{4}{5} + \frac{3}{4}$$

taking LCM

$$= 16 + \frac{15}{20}$$

$$= \frac{31}{20}$$

$$= 1 \frac{11}{20}$$

12. if $\tan \theta = \frac{4}{3}$, find the value of $\sin \theta + \cos \theta$ (both $\sin \theta$ and $\cos \theta$ are positive)

Solution

Let ΔABC be a right angled

$$\angle ACB = \theta$$

$$\text{Given that , } \tan \theta = \frac{4}{3}$$

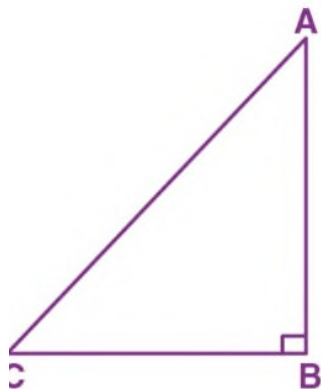
$$\left(\frac{AB}{BC} = \frac{4}{3} \right)$$

$$\text{Given that , } \tan \theta = \frac{4}{3}$$

$$\left(\frac{AB}{BC} = \frac{4}{3} \right)$$

$$\text{Let } AB = 4x$$

$$\text{Then } BC = 3x$$



In right angled ΔABC

By Pythagoras theorem , we get

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = (4x)^2 + (3x)^2$$

$$AC^2 = 16x + 9x$$

$$AC^2 = 25x$$

$$AC^2 = 5x^2$$

$$AC = 5x$$

$$\sin \theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$= \frac{AB}{AC}$$

$$= \frac{4x}{5x}$$

$$= \frac{4}{5}$$

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}}$$

$$= \frac{BC}{AC}$$

$$= \frac{3x}{5x}$$

$$= \frac{3}{5}$$

$$\sin \theta + \cos \theta$$

$$= \frac{4}{5} + \frac{3}{5}$$

$$= \frac{4+3}{5}$$

$$= \frac{7}{5}$$

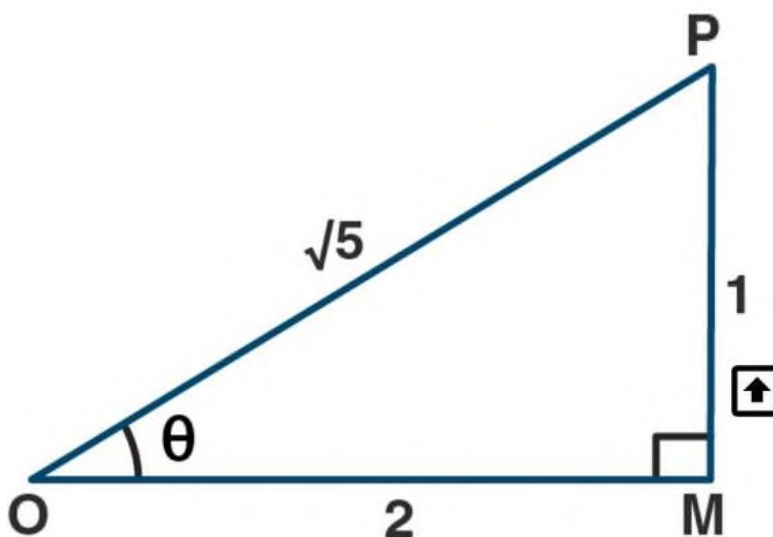
$$\text{Hence, } \sin \theta + \cos \theta = \frac{7}{5} = 1\frac{2}{5}$$

13. if $\operatorname{cosec} \theta = \sqrt{5}$ and θ is less than 90° , find the value of $\cot \theta - \cos \theta$.

Solution

$$\text{Given } \operatorname{cosec} \theta = \frac{\sqrt{5}}{1} = \frac{OP}{PM}$$

$$\text{Op} = \sqrt{5} \text{ and } PM = 1$$



Now $OP^2 = OM^2 + PM^2$ using Pythagoras theorem

$$\sqrt{5}^2 = OM^2 + 1^2$$

$$5 = OM^2 + 1^2$$

$$OM^2 = 5 - 1$$

$$OM^2 = 4$$

$$OM = 2$$

$$\text{Now } \cot \theta = \frac{OM}{PM}$$

$$= \frac{2}{1}$$

$$= 2$$

$$\cos \theta = \frac{OM}{OP}$$

$$= \frac{2}{\sqrt{5}}$$

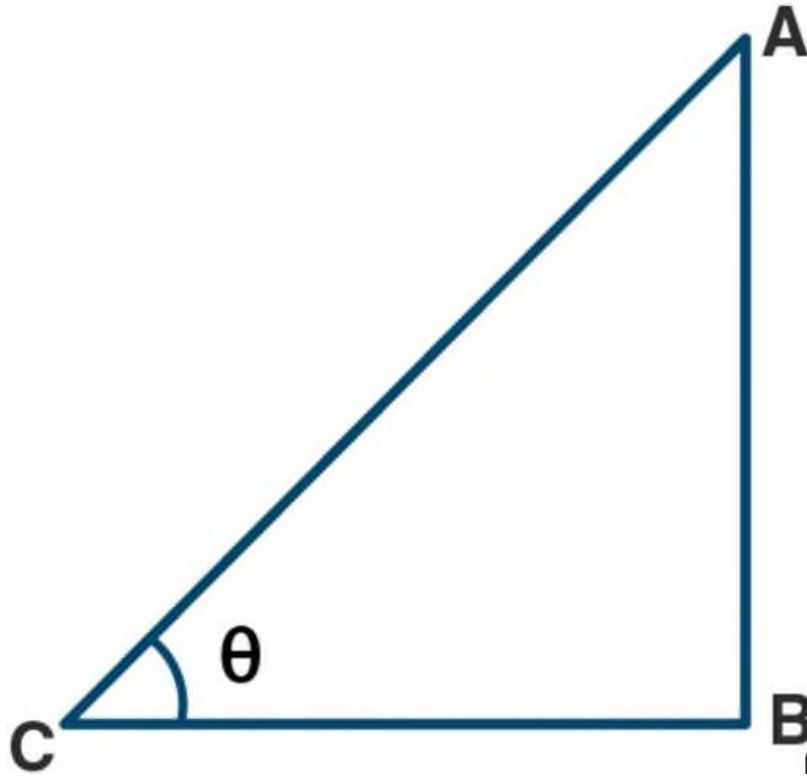
$$\text{Now } \cot \theta - \cos \theta = 2 - \left(\frac{2}{\sqrt{5}}\right)$$

$$= 2\frac{\sqrt{5}-1}{\sqrt{5}}$$

14. given $\sin \theta = \frac{p}{q}$, find $\cos \theta + \sin \theta$ in terms of p and q

Solution

Given that $\sin \theta = \frac{p}{q}$



Which implies ,

$$\frac{AB}{AC} = \frac{p}{q}$$

Let $AB = px$

And then $AC = qx$

In right angled triangle ABC

By Pythagoras theorem ,

We get

$$AC^2 = AB^2 + BC^2$$

$$BC^2 = AC^2 - AB^2$$

$$BC^2 = q^2x^2 - p^2x^2$$

$$BC^2 = (q^2 - p^2)x^2$$

$$BC = \sqrt{(q^2 - p^2)}x$$

In right angled triangle ABC,

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}}$$

$$= \frac{BC}{AC}$$

$$= \frac{\sqrt{(q^2 - p^2)}x}{qx}$$

$$= \frac{\sqrt{(q^2 - p^2)}}{q}$$

Now ,

$$\sin \theta + \cos \theta = \frac{p}{q} + \frac{\sqrt{q^2 - p^2}}{q}$$

$$= p + \frac{\sqrt{q^2 - p^2}}{q}$$

15. if θ is an acute angle and $\tan \theta = \frac{8}{15}$, find the value of $\sec \theta + \operatorname{cosec} \theta$.

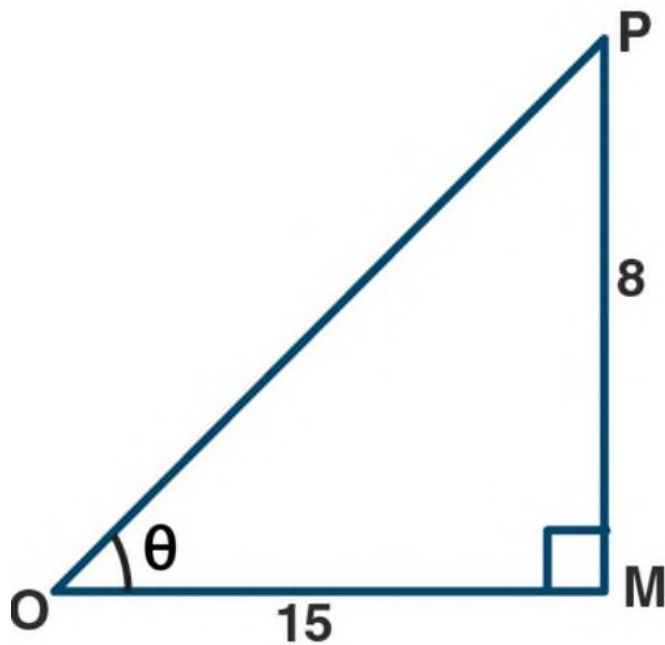
Solution

Given $\tan \theta = \frac{8}{15}$

θ is an acute angle

In the figure triangle OMP is a right angled triangle,

$\angle M = 90^\circ$ and $\angle Q = \theta$



$$\tan \theta = \frac{PM}{OM} = \frac{8}{15}$$

therefore, $PM = 8$, $OM = 15$

but $OP^2 = OM^2 + PM^2$ using Pythagoras theorem

$$= 15^2 + 8^2$$

$$= 225 + 64$$

$$= 289$$

$$= 17^2$$

Therefore $OP = 17$

$$\sec \theta = \frac{OP}{OM}$$

$$= \frac{17}{15}$$

Now ,

$$\sec \theta + \operatorname{cosec} \theta = \left(\frac{17}{15}\right) + \left(\frac{17}{8}\right)$$

$$= \frac{136+255}{120}$$

$$= \frac{391}{120}$$

$$3 \frac{31}{120}$$

16. given A is an acute angle and $13 \sin A = 5$, evaluate :

$$\frac{(5 \sin A - 2 \cos A)}{\tan A}$$

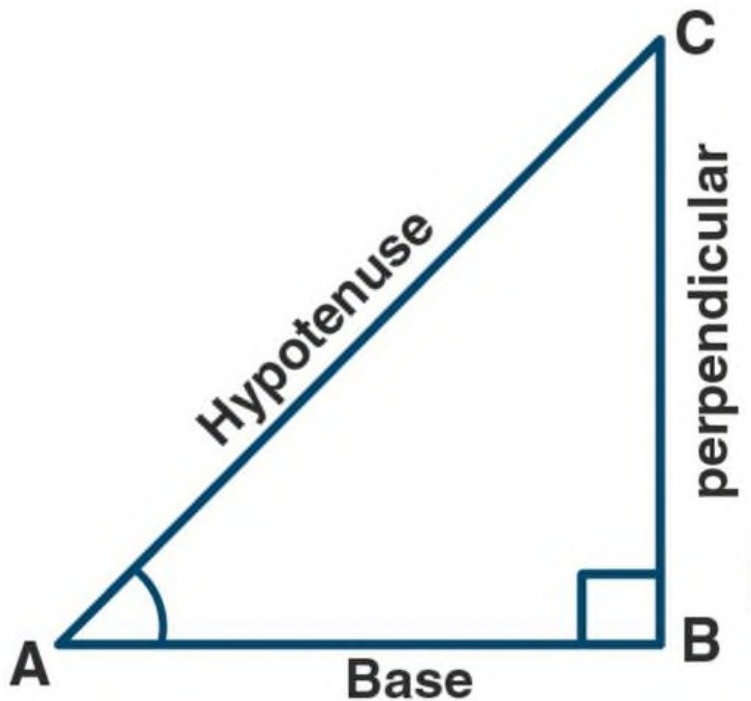
Solution

Let triangle ABC be a right angled triangle at B and A is an acute angle

Given that $13 \sin A = 5$

$$\sin A = \frac{5}{13}$$

$$\frac{AB}{AC} = \frac{5}{13}$$



Let $AB = 5x$

$AC = 13x$

In right angle triangle ABC

Using Pythagoras theorem ,

We get

$$AC^2 = AB^2 + BC^2$$

$$BC^2 = AC^2 - AB^2$$

$$BC^2 = (13x)^2 - (5x)^2$$

$$BC^2 = 169x^2 - 25x^2$$

$$BC^2 = 144x^2$$

$$BC = 12x$$

$$\sin A = \frac{5}{13}$$

$$\cos A = \frac{\text{base}}{\text{hypotenuse}}$$

$$= \frac{BC}{AC}$$

$$= \frac{12X}{13X}$$

$$= \frac{12}{13}$$

$$\tan A = \frac{\text{perpendicular}}{\text{base}}$$

$$\frac{AB}{BC}$$

$$\frac{5x}{12x}$$

$$\frac{5}{12}$$

Now ,

$$\frac{5 \sin A - 2 \cos A}{\tan A} = \frac{\left[(5) \left(\frac{5}{13} \right) - (2) \left(\frac{12}{13} \right) \right]}{\frac{5}{12}}$$

$$= \frac{\frac{1}{13}}{\frac{5}{12}}$$

$$= \frac{12}{65}$$

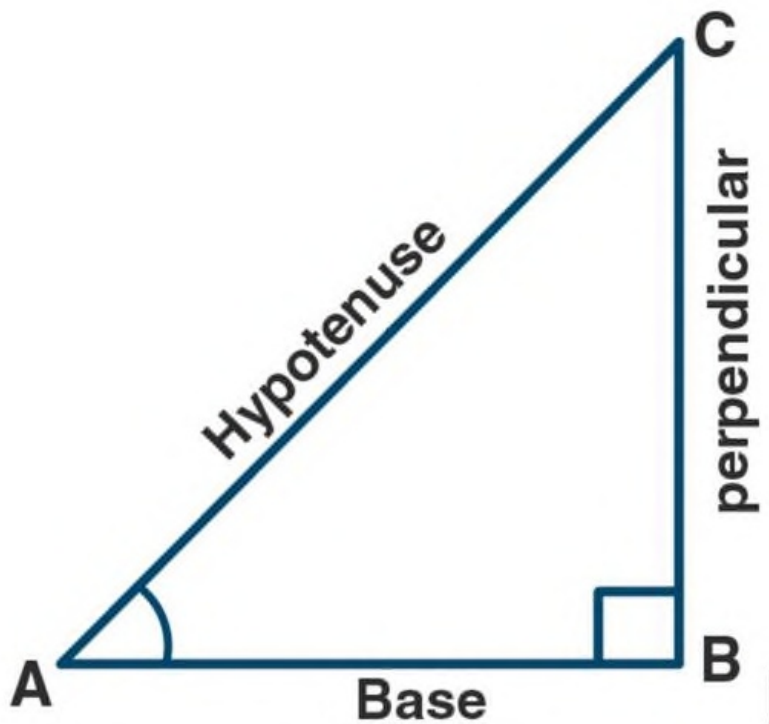
$$\text{Hence } \frac{5 \sin A - 2 \cos A}{\tan A} = \frac{12}{65}$$

17. given A is an acute angle and $\operatorname{cosec} A = \sqrt{2}$, find the value of $\frac{2 \sin^2 A + 3 \cot^2 A}{\tan^2 A - \cos^2 A}$

Solution

Let triangle ABC be a right angled at B and A is a acute angle .

Given that $\operatorname{cosec} A = \sqrt{2}$



Which implies ,

$$\frac{AC}{BC} = \frac{\sqrt{2}}{1}$$

Let $AC = \sqrt{2}x$

Then $BC = x$

In right angled triangle ABC

By using Pythagoras theorem

We get

$$AC^2 = AB^2 + BC^2$$

$$(\sqrt{2}x)^2 = AB^2 + x^2$$

$$AB^2 = 2x^2 - x^2$$

$$AB = x$$

$$\sin A = \frac{\text{Perpendicular}}{\text{hypotenuse}}$$

$$= \frac{BC}{AC}$$

$$= \frac{1}{\sqrt{2}}$$

$$\cot A = \frac{\text{Base}}{\text{perpendicular}}$$

$$= \frac{x}{x}$$

$$= 1$$

$$\tan A = \frac{\text{Perpendicular}}{\text{base}}$$

$$= \frac{BC}{AB}$$

$$= \frac{x}{x}$$

$$= 1$$

$$\cos A = \frac{\text{base}}{\text{hypotenuse}}$$

$$= \frac{AB}{AC}$$

$$= \frac{x}{\sqrt{2}x}$$

$$= \frac{1}{\sqrt{2}}$$

Substituting these values we get

$$\frac{2 \sin^2 A + 3 \cot^2 A}{\tan^2 A - \cos^2 A} = 8$$

18. the diagonals AC and BD of a rhombus ABCD meet at O.

If AC = 8 cm and BD = 6 cm , find $\sin \angle OCD$.

Solution

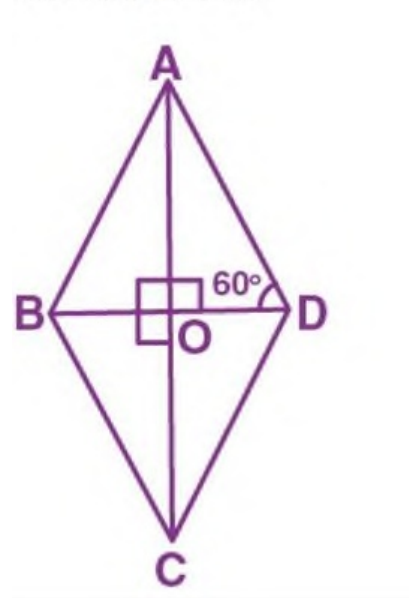
It is given that

Diagonals AC and BD of rhombus ABCD meet at O

AC = 8 cm and BD = 6 cm

O is the mid - point of AC

O is the mid point of BD



We know that

$$AO = OC = \frac{AC}{2} = \frac{8}{2} = 4 \text{ cm}$$

O is the mid point of BD

$$BO = OD = \frac{BD}{2} = \frac{6}{2} = 3 \text{ cm}$$

In right angled ΔCOD

$$CD^2 = OC^2 + OD^2$$

Substituting the values

$$CD^2 = 4^2 + 3^2$$

So we get

$$CD^2 = 16 + 9 = 25$$

$$CD^2 = 5^2$$

$$CD = 5 \text{ cm}$$

In right angled ΔCOD

$$\sin \angle OCD = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

So we get

$$\sin \angle OCD = \frac{OD}{CD} = \frac{3}{5}$$

19. if $\tan \theta = \frac{5}{12}$, find the value of $\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$

Solution

Consider ΔABC be right angled at B and $\angle ACB = \theta$

It is given that

$$\tan \theta = \frac{AB}{BC} = \frac{5}{12}$$

Take $AB = 5x$ then $BC = 12x$

In right angled ΔABC ,

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$AC^2 = (5x)^2 + (12x)^2$$

By further calculation

$$AC^2 = 25x^2 + 144x^2 = 169x^2$$

So we get

$$AC^2 = (13x)^2$$

$$AC = 13x$$

In right angled ΔABC

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{BC}{AC}$$

Substituting the values

$$\cos \theta = \frac{12x}{13x} = \frac{12}{13}$$

In right angled ΔABC

$$\sin \theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\sin \theta = \frac{AB}{AC}$$

Substituting the values

$$\sin \theta = \frac{5x}{13x} = \frac{5}{13}$$

Here

$$\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} = \frac{\left[\frac{12}{13} + \frac{5}{13}\right]}{\left[\frac{12}{13} - \frac{5}{13}\right]}$$

Taking LCM

$$= \frac{\left[\frac{12+5}{13}\right]}{\left[\frac{12-5}{13}\right]}$$

So we get

$$= \frac{\frac{17}{13}}{\frac{7}{13}}$$

$$= \frac{17}{13} \times \frac{13}{7}$$

$$= \frac{17}{7}$$

Therefore, $\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} = \frac{17}{7} = 2\frac{3}{7}$

20. given $5 \cos A - 12 \sin A = 0$, find the value of $\frac{\sin A + \cos A}{2 \cos A - \sin A}$

Solution

It is given that

$$5 \cos A - 12 \sin A = 0$$

We can write it as

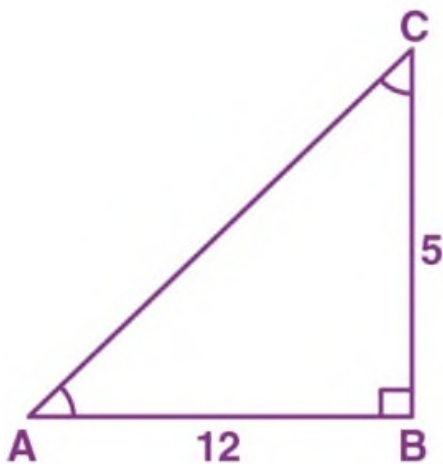
$$5 \cos A = 12 \sin A$$

So we get

$$\frac{\sin A}{\cos A} = \frac{5}{12}$$

We know that $\frac{\sin A}{\cos A} = \tan A$

$$\tan A = \frac{5}{12}$$



Consider ΔABC right angled at B and $\angle A$ is acute angle

Here

$$\tan A = \frac{BC}{AB} = \frac{5}{12}$$

take $BC = 5x$ then $AB = 12x$

in right angled ΔABC

using Pythagoras theorem

$$AC^2 = BC^2 + AB^2$$

Substituting the values

$$AC^2 = (5x)^2 + (12x)^2$$

$$AC^2 = 25x^2 + 144x^2 = 169x^2$$

So we get

$$AC^2 = (13x)^2$$

$$AC = 13x$$

In right angled ΔABC

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

So we get

$$\sin A = \frac{BC}{AC} = \frac{5x}{13x} = \frac{5}{13}$$

In right angled ΔABC

$$\cos A = \frac{\text{base}}{\text{hypotenuse}}$$

So we get

$$\cos A = \frac{AB}{AC} = \frac{12x}{13x} = \frac{12}{13}$$

Here

$$\frac{\sin A + \cos A}{2 \cos A - \sin A} = \frac{\frac{5}{13} + \frac{12}{13}}{2 \times \frac{12}{13} - \frac{5}{13}}$$

By further calculation

$$= \frac{\left[\frac{5+12}{13} \right]}{\left[\frac{24-5}{13} \right]}$$

So we get

$$= \frac{\left[\frac{5+12}{13} \right]}{\left[\frac{24-5}{13} \right]}$$

$$= \frac{\frac{17}{13}}{\frac{19}{13}}$$

$$= \frac{17}{13} \times \frac{13}{19}$$

$$= \frac{17}{19}$$

Therefore, $\frac{\sin A + \cos A}{2 \cos A - \sin A} = \frac{17}{19}$

21. if $\tan \theta = \frac{p}{q}$ find the value of $\frac{p \sin \theta - q \cos \theta}{p \sin \theta + q \cos \theta}$

Solution

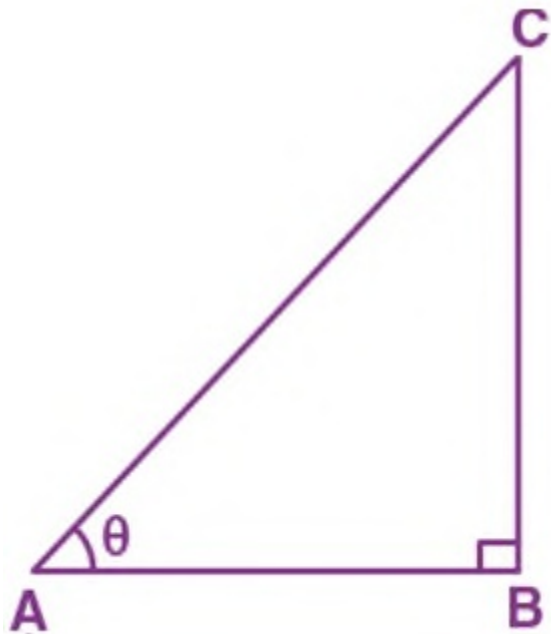
It is given that

$$\tan \theta = \frac{p}{q}$$

consider ΔABC be right angled at B and $\angle BCA = \theta$

$$\tan \theta = \frac{BC}{AB} = \frac{p}{q}$$

BC = px then AB = qx



In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = BC^2 + AB^2$$

Substituting the values

$$AC^2 = (px)^2 + (qx)^2$$

$$AC^2 = p^2x^2 + q^2x^2$$

$$AC^2 = x^2 (p^2 + q^2)$$

So we get

$$AC = \sqrt{x^2}(p^2 + q^2)$$

$$AC = x(\sqrt{p^2 + q^2})$$

In right angled ΔABC

$$\sin \theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\sin \theta = \frac{px}{x}(\sqrt{p^2 + q^2})$$

So we get

$$\sin \theta = \frac{p}{\sqrt{p^2 + q^2}}$$

In right angled ΔABC

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{AB}{AC}$$

Substituting the values

$$\cos \theta = \frac{qx}{x(\sqrt{p^2 + q^2})}$$

So we get

$$\cos \theta = \frac{q}{\sqrt{p^2 + q^2}}$$

Here

$$\frac{p \sin \theta - q \cos \theta}{p \sin \theta + q \cos \theta} = \frac{p \left(\frac{p}{\sqrt{p^2 + q^2}} \right) - q \left(\frac{q}{\sqrt{p^2 + q^2}} \right)}{p \left(\frac{p}{\sqrt{p^2 + q^2}} \right) + q \left(\frac{q}{\sqrt{p^2 + q^2}} \right)}$$

By further calculation

$$\begin{aligned} &= \frac{\frac{p^2}{\sqrt{p^2 + q^2}} - \frac{q^2}{\sqrt{p^2 + q^2}}}{\frac{p^2}{\sqrt{p^2 + q^2}} + \frac{q^2}{\sqrt{p^2 + q^2}}} \\ &= \frac{p^2 - q^2}{\sqrt{p^2 + q^2}} \end{aligned}$$

$$= \frac{p^2 + q^2}{\sqrt{p^2 + q^2}}$$

So we get

$$= \frac{p^2 - q^2}{\sqrt{p^2 + q^2}} \times \frac{\sqrt{p^2 + q^2}}{p^2 + q^2}$$

$$= \frac{p^2 - q^2}{p^2 + q^2}$$

$$\text{Therefore, } \frac{p \sin \theta - q \cos \theta}{p \sin \theta + q \cos \theta} = \frac{p^2 - q^2}{p^2 + q^2}$$

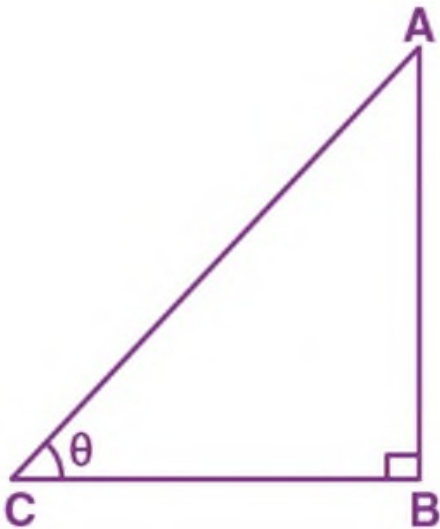
22. if $3 \cot \theta = 4$, find the value of $\frac{5 \sin \theta - 3 \cos \theta}{5 \sin \theta + 3 \cos \theta}$

Solution

It is given that

$$3 \cot \theta = 4$$

$$\cot \theta = \frac{4}{3}$$



Consider ΔABC be right angled at B and $\angle ACB = \theta$

$$\cot \theta = \frac{BC}{AB} = \frac{4}{3}$$

Take $BC = 4x$ then $AB = 3x$

In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$AC^2 = (3x)^2 + (4x)^2$$

$$AC^2 = 9x^2 + 16x^2 = 25x^2$$

So we get

$$AC^2 = (5x)^2$$

$$AC = 5x$$

In right angled ΔABC

$$\sin \theta = \frac{AB}{AC}$$

Substituting the values

$$\sin \theta = \frac{3x}{5x} = \frac{3}{5}$$

In right angled ΔABC

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{BC}{AC}$$

Substituting the values

$$\cos \theta = \frac{4x}{5x} = \frac{4}{5}$$

$$\frac{5 \sin \theta - 3 \cos \theta}{5 \sin \theta + 3 \cos \theta} = \frac{5 \times \frac{3}{5} - 3 \times \frac{4}{5}}{5 \times \frac{3}{5} + 3 \times \frac{4}{5}}$$

By further calculation

$$= \frac{\frac{15}{5} - \frac{12}{5}}{\frac{15}{5} + \frac{12}{5}}$$

$$= \frac{15-12}{\frac{15+12}{5}}$$

So we get

$$= \frac{\frac{3}{5}}{5}$$

$$= \frac{3}{5} \times \frac{5}{27}$$

$$= \frac{3}{27}$$

$$= \frac{1}{9}$$

Therefore, $\frac{5 \sin \theta - 3 \cos \theta}{5 \sin \theta + 3 \cos \theta} = \frac{1}{9}$

23. (i) if $5 \cos \theta - 12 \sin \theta = 0$, find the value of $\frac{\sin \theta + \cos \theta}{2 \cos \theta - \sin \theta}$

(ii) if $\operatorname{cosec} \theta = \frac{13}{12}$, find the value of $\frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta}$

Solution

(i) it is given that

$$5 \cos \theta - 12 \sin \theta = 0$$

We can write it as

$$5 \cos \theta = 12 \sin \theta$$

$$\frac{\sin \theta}{\cos \theta} = \frac{5}{12}$$

$$\tan \theta = \frac{5}{12}$$

dividing both numerator and denominator by $\cos \theta$

$$\frac{\sin \theta + \cos \theta}{2 \cos \theta - \sin \theta} = \frac{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\cos \theta}}{\frac{2 \cos \theta}{\cos \theta} - \frac{\sin \theta}{\cos \theta}}$$

$$= \frac{\tan \theta + 1}{2 - \tan \theta}$$

Substituting the values

$$= \frac{\frac{5}{12} + 1}{2 - \frac{5}{12}}$$

Taking LCM

$$= \frac{\frac{5+12}{12}}{24 - \frac{5}{12}}$$
$$= \frac{\frac{17}{12}}{\frac{19}{12}}$$

So we get

$$= \frac{17}{12} \times \frac{12}{19}$$

$$= \frac{17}{19}$$

(ii) it is given that

$$\operatorname{Cosec} \theta = \frac{13}{12}$$

$$\text{We know that } \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\frac{1}{\sin \theta} = \frac{13}{12}$$

$$\sin \theta = \frac{12}{13}$$

$$\text{Here } \cos^2 \theta = 1 - \sin^2 \theta$$

Substituting the values

$$= 1 - \left(\frac{12}{13}\right)^2$$

By further calculation

$$= 1 - \frac{144}{169}$$

Taking LCM

$$= \frac{169-144}{169}$$

$$= \frac{25}{169}$$

So we get

$$= \left(\frac{5}{13}\right)^2$$

$$\cos \theta = \frac{5}{13}$$

Here

$$\frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta} = \frac{2\left(\frac{12}{13}\right) - 3\left(\frac{5}{13}\right)}{4\left(\frac{12}{13}\right) - 9\left(\frac{5}{13}\right)}$$

By further calculation

$$\frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}}$$

So we get

$$= \frac{\frac{24-15}{13}}{48 - \frac{45}{13}}$$

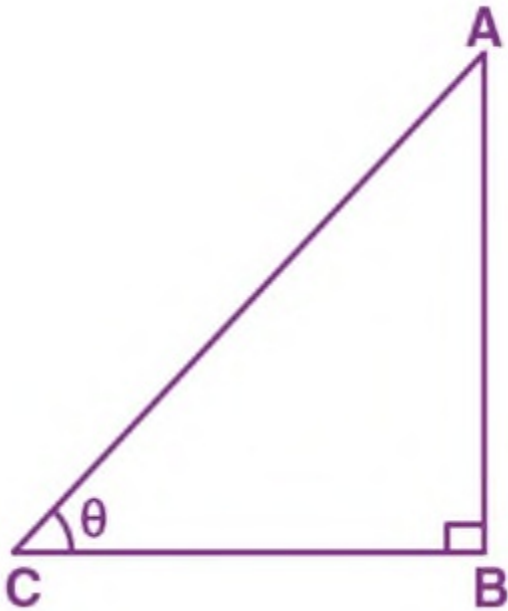
$$= \frac{\frac{9}{13}}{13}$$

$$= \frac{9}{13} \times \frac{13}{3}$$

$$= 3$$

24. if $5 \sin \theta = 3$, find the value of $\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta}$

Solution



Consider ΔABC be right angled at B and $\angle ACB = \theta$

It is given that

$$5 \sin \theta = 3$$

$$\sin \theta = \frac{AB}{AC} = \frac{3}{5}$$

Take $AB = 3x$ then $AC = 5x$

In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$BC^2 = AC^2 - AB^2$$

Substituting the values

$$BC^2 = (5x)^2 - (3x)^2$$

So we get

$$BC^2 = 25x^2 - 9x^2 = 16x^2$$

$$BC^2 = (4x)^2$$

$$BC = 4x$$

In right angled ΔABC

$$\tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan \theta = \frac{AB}{BC} = \frac{3x}{4x} = \frac{3}{4}$$

$$\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta} = \frac{\frac{5}{4} - \frac{3}{4}}{\frac{5}{4} + \frac{3}{4}}$$

By further calculation

$$= \frac{\frac{5-3}{4}}{5+\frac{3}{4}}$$

So we get

$$= \frac{\frac{2}{4}}{4}$$

$$= \frac{2}{4} \times \frac{4}{8}$$

$$= \frac{2}{8}$$

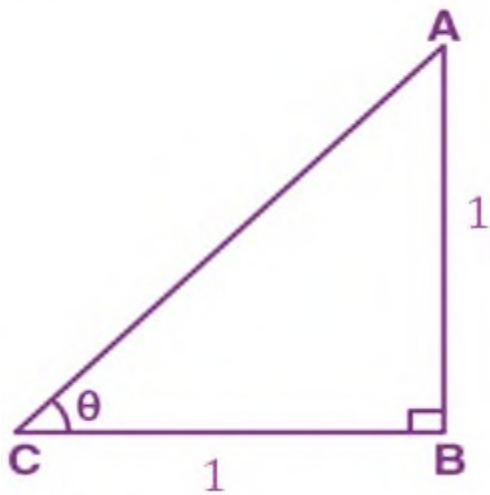
$$= \frac{1}{4}$$

$$\text{Therefore, } \frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta} = \frac{1}{4}$$

25. if θ is an acute angle and $\sin \theta = \cos \theta$ find the value of $2 \tan^2 \theta + \sin^2 \theta - 1$

Solution

Diagram.



Consider ΔABC be right angled at B and $\angle ACB = \theta$

It is given that

$$\sin \theta = \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = 1$$

$$\tan \theta = \frac{AB}{BC} = 1$$

take $AB = x$ then $BC = x$

in right angled ΔABC

using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = x^2 + x^2 = 2x^2$$

So we get

$$AC = \sqrt{2x^2}$$

$$AC = \sqrt{2x}$$

In right angled ΔABC

$$\sin \theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\sin \theta = \frac{AB}{AC} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$$

Here

$$2 \tan^2 \theta + \sin^2 \theta - 1 = 2 \times (1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 1$$

By further calculation

$$= 2 \times 1 + \frac{1}{2} - 1$$

$$= 2 + \frac{1}{2} - 1$$

$$= 1 + \frac{1}{2}$$

Taking LCM

$$= \frac{2+1}{2}$$

$$= \frac{3}{2}$$

$$\text{Therefore } 2 \tan^2 \theta + \sin^2 \theta - 1 = \frac{3}{2}$$

26. prove the following

(i) $\cos \theta \tan \theta = \sin \theta$

(ii) $\sin \theta \cot \theta = \cos \theta$

(iii) $\frac{\sin^2 \theta}{\cos \theta} + \cos \theta = \frac{1}{\cos \theta}$

Solution

(i) $\cos \theta \tan \theta = \sin \theta$

$$\text{LHS} = \cos \theta \tan \theta$$

$$\text{We know that } \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$= \cos \theta \left(\frac{\sin \theta}{\cos \theta} \right)$$

So we get

$$= 1 \times \sin \frac{\theta}{1}$$

$$= \sin \theta$$

$$= \text{RHS}$$

Therefore , LHS = RHS

$$(ii) \sin \theta \cot \theta = \cos \theta$$

$$\text{LHS} = \sin \theta \cot \theta$$

$$\text{We know that } \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$= \sin \theta \left(\frac{\cos \theta}{\sin \theta} \right)$$

$$= 1 \times \frac{\cos \theta}{1}$$

$$= \cos \theta$$

$$= \text{RHS}$$

Therefore , LHS = RHS

$$(iii) \frac{\sin^2 \theta}{\cos \theta} + \cos \theta = \frac{1}{\cos \theta}$$

$$\text{LHS} = \frac{\sin^2 \theta}{\cos \theta} + \frac{\cos \theta}{1}$$

Taking LCM

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta}$$

$$\text{We know that } \sin^2 \theta + \cos^2 \theta = 1$$

$$= \frac{1}{\cos \theta}$$

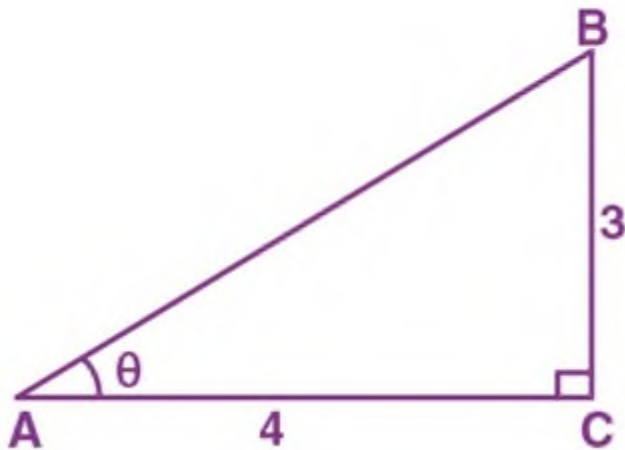
Therefore , LHS = RHS

27 . if in ΔABC , $\angle C = 90^\circ$ and $\tan A = \frac{3}{4}$ prove that $\sin A \cos B + \cos A \sin B = 1$

Solution

It is given that

$$\tan A = \frac{BC}{AC} = \frac{3}{4}$$



Using Pythagoras theorem

$$AB^2 = AC^2 + BC^2$$

Substituting the values

$$= 4^2 + 3^2$$

$$= 16 + 9$$

$$= 25$$

$$= 5^2$$

So we get $AB = 5$

Here

$$\sin A = \frac{BC}{AC} = \frac{3}{5}$$

$$\cos A = \frac{AC}{AB} = \frac{4}{5}$$

$$\cos B = \frac{BC}{AB} = \frac{3}{5}$$

$$\sin B = \frac{AC}{AB} = \frac{4}{5}$$

$$\text{LHS} = \sin A \cos B + \cos A \sin B$$

Substituting the values

$$= \frac{3}{5} \times \frac{3}{5} + \frac{4}{5} \times \frac{4}{5}$$

By further calculation

$$= \frac{9}{25} + \frac{16}{25}$$

$$= \frac{9+16}{25}$$

$$= \frac{25}{25}$$

$$= 1$$

$$= \text{RHS}$$

Therefore , $\text{LHS} = \text{RHS}$

28 . (a) in figure (1) given below , ΔABC is right angled at B and ΔBRS is right angled at R. If $AB = 18 \text{ cm}$, $BC = 7.5 \text{ cm}$, $RS = 5 \text{ cm}$, $\angle BSR = x^\circ$ and $\angle SAB = y^\circ$, then find

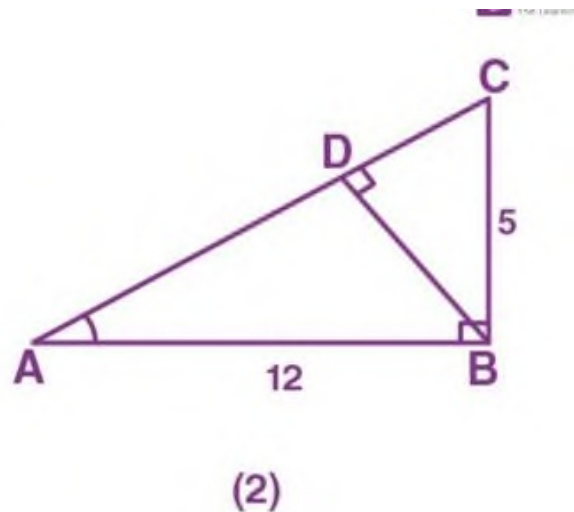
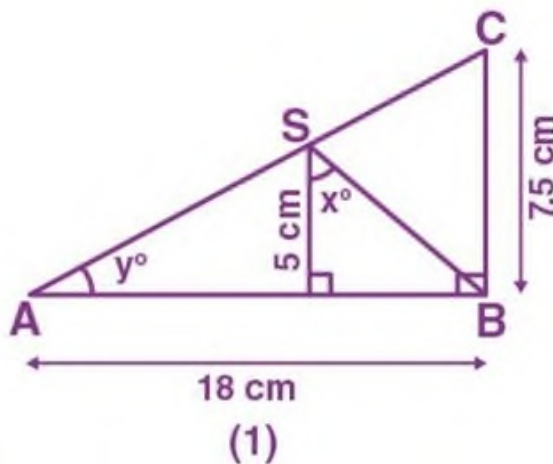
(i) $\tan x^\circ$

(ii) $\sin y^\circ$

(b) in the figure (2) given below ΔABC is right angled at B and BD is perpendicular to AC . find

(i) $\cos \angle CBD$

(ii) $\cot \angle ABD$



Solution

(a) ΔABC is right angled at B , ΔBSC is right angled at S and ΔBRS is right angled at R.

It is given that

$AB = 18 \text{ cm}$, $BC = 7.5 \text{ cm}$, $RS = 5 \text{ cm}$, $\angle BSR = x^\circ$ and $\angle SAB = y^\circ$

By geometry ΔARS and ΔABC are similar

$$\frac{AR}{AB} = \frac{RS}{BC}$$

Substituting the values

$$\frac{AR}{18} = \frac{5}{7.5}$$

By further calculation

$$AR = \frac{5 \times 18}{7.5} = \frac{1 \times 18}{1.5}$$

Multiply both numerator and denominator by 10

$$AR = \frac{18 \times 10}{15}$$

$$AR = \frac{10 \times 6}{5}$$

$$AR = \frac{2 \times 6}{1} = 12$$

So we get

$$RB = AB - AR$$

$$RB = 18 - 12 = 6$$

In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$AC^2 = 18^2 + 7.5^2$$

By further calculation

$$AC^2 = 324 + 56.25 = 380.25$$

$$AC = \sqrt{380.25} = 19.5 \text{ cm}$$

(i) in right angled Δ BSR

$$\tan x^\circ = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan x^\circ = \frac{RB}{RS} = \frac{6}{5}$$

(ii) in right angled Δ ASR

$$\sin y^\circ = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

Using Pythagoras theorem

$$AS^2 = 12^2 + 5^2$$

By further calculation

$$AS^2 = 144 + 25 = 169$$

$$AS = \sqrt{169} = 13 \text{ cm}$$

So we get

$$\sin y^\circ = \frac{RS}{AS} = \frac{5}{13}$$

(b) we know that

Δ ABC is right angled at B and BD is perpendicular to AC

In right angled Δ ABC

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$AC^2 = 12^2 + 5^2$$

By further calculation

$$AC^2 = 144 + 25 = 169$$

So we get

$$AC^2 = (13)^2$$

$$AC = 13$$

By geometry $\angle CBD = \angle A$ and $\angle ABD = \angle C$

$$(i) \cos \angle CBD = \cos \angle A = \frac{\text{base}}{\text{hypotenuse}}$$

In right angled ΔABC

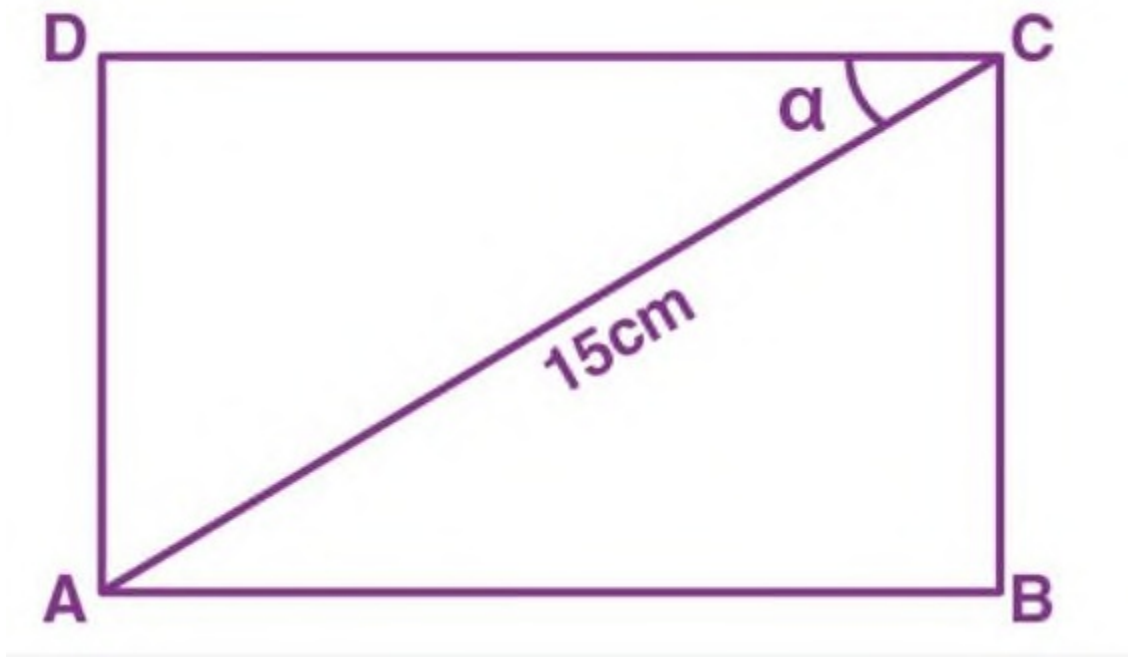
$$\cos \angle CBD = \cos \angle A = \frac{AB}{AC} = \frac{12}{13}$$

$$(ii) \cos \angle ABD = \cos \angle C = \frac{\text{base}}{\text{perpendicular}}$$

In right angled ΔABC

$$\cos \angle ABD = \cos \angle C = \frac{BC}{AB} = \frac{5}{12}$$

29. in the adjoining figure , ABCD is a rectangle . its diagonal $AC = 15$ cm and $\angle ACD = \alpha$. If $\cot \alpha = \frac{3}{2}$, find the perimeter and the area of the rectangle .



Solution

In right ΔADC

$$\cot \alpha = \frac{CD}{AD} = \frac{3}{2}$$

Take $CD = 3x$ then $AD = 2x$

Using Pythagoras theorem

$$AC^2 = CD^2 + AD^2$$

Substituting the values

$$(15)^2 = (3x)^2 + (2x)^2$$

By further calculation

$$13x^2 = 225$$

$$x^2 = \frac{225}{13}$$

So we get

$$x = \frac{\sqrt{225}}{13} = \frac{15}{\sqrt{13}}$$

$$\text{Length of rectangle (l)} = 3x = \frac{(3 \times 15)}{\sqrt{13}} = \frac{45}{\sqrt{13}} \text{ cm}$$

$$\text{Breath of rectangle (b)} = 2x = \frac{2 \times 15}{\sqrt{13}} = \frac{30}{\sqrt{13}} \text{ cm}$$

$$(i) \text{ perimeter of rectangle} = 2(l+b)$$

Substituting the values of l and b

$$= 2 \left(\frac{45}{\sqrt{13}} + \frac{30}{\sqrt{13}} \right)$$

So we get

$$= 2 \times \frac{75}{\sqrt{13}}$$

$$= \frac{150}{\sqrt{13}} \text{ cm}$$

(ii) area of rectangle = $l \times b$

Substituting the values of l and b

$$= \frac{45}{\sqrt{13}} \times \frac{30}{\sqrt{13}}$$

So we get

$$= \frac{1350}{13}$$

$$= 103 \frac{11}{13} \text{cm}^2$$

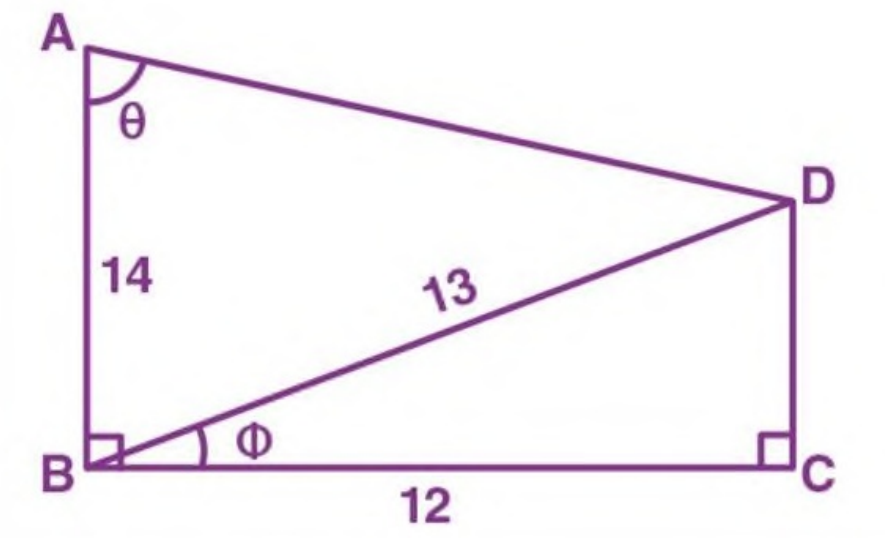
30 . using the measurements given in the figure alongside,

(a) find the values of :

(i) $\sin \theta$

(ii) $\tan \theta$

(b) write an expression for AD in terms of θ .



Solution

From the figure

$$BC = 12, BD = 13$$

In right angled ΔBCD

Using Pythagoras theorem

$$BD^2 = BC^2 + CD^2$$

It can be written as

$$CD^2 = BD^2 - BC^2$$

Substituting the values

$$CD^2 = BD^2 - BC^2$$

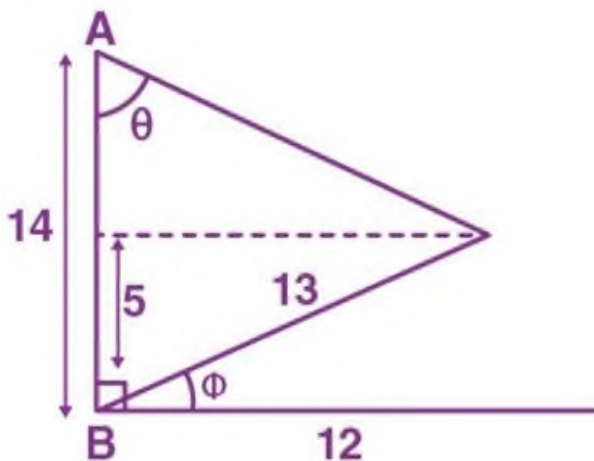
Substituting the values

$$CD^2 = (13)^2 - (12)^2$$

$$CD^2 = 169 - 144 = 25$$

So we get

$$CD = \sqrt{25} = 5$$



Construct BE perpendicular to AB

$$CD = BE = 5 \text{ and } EA = AE = 14 - 5 = 9$$

$$(a) (i) \sin \phi = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

In right angled ΔBCD

$$\sin \phi = \frac{CD}{BD} = \frac{5}{13}$$

$$(ii) \tan \theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

In right angled ΔAED

$$\tan \theta = \frac{ED}{AE} = \frac{BC}{AE} = \frac{12}{9} = \frac{4}{3} \text{ (since } ED = BC \text{)}$$

(b) in right angled ΔAED

$$\sin \theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{base}}{\text{perpendicular}}$$

We can write it as

$$\sin \theta = \frac{ED}{AD} \text{ or } \cos \theta = \frac{AE}{AD}$$

$$AD = \frac{ED}{\sin \theta} \text{ or } AD = \frac{AE}{\cos \theta}$$

Substituting the values

$$AD = \frac{12}{\sin \theta} \text{ or } AD = \frac{9}{\cos \theta}$$

$$\text{Therefore, } AD = \frac{12}{\sin \theta} \text{ or } AD = \frac{9}{\cos \theta}$$

31. prove the following

(i) $(\sin A + \cos A)^2 + (\sin A - \cos A)^2 = 2$

(ii) $\cot^2 A - \frac{1}{\sin^2 A + 1} = 0$

(iii) $\frac{1}{1 + \tan^2 A} + \frac{1}{1 + \cot^2 A} = 1$

Solution

(i) $(\sin A + \cos A)^2 + (\sin A - \cos A)^2 = 2$

LHS = $(\sin A + \cos A)^2 + (\sin A - \cos A)^2$

Using the formula

$(a+b)^2 = a^2 + b^2 + 2ab$ and $(a-b)^2 = a^2 + b^2 - 2ab$

$= [(\sin A)^2 + (\cos A)^2 + 2 \sin A \cos A] + [(\sin A)^2 + (\cos A)^2 - 2 \sin A \cos A]$

By further calculation

$= \sin^2 A + \cos^2 A + 2 \sin A \cos A + \sin^2 A + \cos^2 A - 2 \sin A \cos A$

$= \sin^2 A + \cos^2 A + \sin^2 A + \cos^2 A$

$= 2 \sin^2 A + 2 \cos^2 A$

We know that $\sin^2 A + \cos^2 A = 1$

$= 2(\sin^2 A + \cos^2 A)$

$= 2(1)$

$= 2$

$= \text{RHS}$

Therefore, LHS=RHS

$$(ii) \cot^2 A - \frac{1}{\sin^2 A} + 1 = 0$$

$$\text{LHS} = \cot^2 A - \frac{1}{\sin^2 A} + 1$$

We know that

$$\frac{1}{\sin A} = \operatorname{cosec} A$$

$$= \cot^2 A - \operatorname{cosec}^2 A + 1$$

$$= (1 + \cot^2 A) - \operatorname{cosec}^2 A$$

$$\text{We know that } 1 + \cot^2 A = \operatorname{cosec}^2 A$$

$$= \operatorname{cosec}^2 A - \operatorname{cosec}^2 A$$

$$= 0$$

$$= \text{RHS}$$

Therefore, LHS = RHS

$$(iii) \frac{1}{1+\tan^2 A} + \frac{1}{1+\cot^2 A} = 1$$

$$\text{LHS} = \frac{1}{1+\tan^2 A} + \frac{1}{1+\cot^2 A}$$

We know that

$$\sec^2 A - \tan^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{Cosec}^2 A - \cot^2 A = 1$$

$$\operatorname{Cosec}^2 A = 1 + \cot^2 A$$

So we get

$$= \frac{1}{\sec^2 A} + \frac{1}{\operatorname{cosec}^2 A}$$

$$\text{Here } \frac{1}{\sec A} = \cos A \text{ and } \frac{1}{\operatorname{cosec} A} = \sin A$$

$$= \cos^2 A + \sin^2 A$$

$$= 1$$

$$= \text{RHS}$$

Therefore, LHS = RHS

32. simplify

$$\sqrt{\frac{1 - \sin^2 \theta}{1 - \cos^2 \theta}}$$

Solution :

$$\sqrt{\frac{1 - \sin^2 \theta}{1 - \cos^2 \theta}} = \sqrt{\frac{\sin^2 \theta + \cos^2 \theta - \sin^2 \theta}{\sin^2 \theta + \cos^2 \theta - \cos^2 \theta}}$$

We know that $1 = \sin^2 \theta + \cos^2 \theta$

$$= \frac{\sqrt{\cos^2 \theta}}{\sin^2 \theta}$$

$$\frac{\cos \theta}{\sin \theta}$$

$$\text{Here } \frac{\cos \theta}{\sin \theta} = \cot \theta$$

$$= \cot \theta$$

Therefore,

$$\sqrt{\frac{1-\sin^2 \theta}{1-\cos^2 \theta}} = \cot \theta$$

33. if $\sin \theta + \operatorname{cosec} \theta = 2$, find the value of $\sin^2 \theta + \operatorname{cosec}^2 \theta$.

Solution

It is given that

$$\sin \theta + \operatorname{cosec} \theta = 2$$

$$\sin \theta + \frac{1}{\sin \theta} = 2$$

By further calculation

$$\sin^2 \theta + 1 = 2 \sin \theta$$

$$\sin^2 \theta - 2 \sin \theta + 1 = 0$$

So we get

$$(\sin \theta - 1)^2 = 0$$

$$\sin \theta - 1 = 0$$

$$\sin \theta = 1$$

Here

$$\sin^2 \theta + \operatorname{cosec}^2 \theta = \sin^2 \theta + \frac{1}{\sin^2 \theta}$$

Substituting the values

$$= 1^2 + \frac{1}{1^2}$$

$$= 1 + \frac{1}{1}$$

$$= 1 + 1$$

$$= 2$$

34. if $x = a \cos \theta + b \sin \theta$ and $y = a \sin \theta - b \cos \theta$, prove that $x^2 + y^2 = a^2 + b^2$

Solution

It is given that

$$X = a \cos \theta + b \sin \theta \dots(1)$$

$$Y = a \sin \theta - b \cos \theta \dots(2)$$

By squaring and adding both the equations

$$X^2 + y^2 = (a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2$$

Using the formula

$$(a+b)^2 = a^2 + b^2 + 2ab \text{ and } (a-b)^2 = a^2 + b^2 - 2ab$$

$$= [(a \cos \theta)^2 + (b \sin \theta)^2 + 2(a \cos \theta)(b \sin \theta)] + [(a \sin \theta)^2 + (b \cos \theta)^2 - 2(a \sin \theta)(b \cos \theta)]$$

By further calculation

$$= a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2 ab \sin \theta \cos \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta$$

$$= a^2 \cos^2 \theta + b^2 \sin^2 \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta$$

So we get

$$= a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta)$$

$$\text{Here } \sin^2 \theta + \cos^2 \theta = 1$$

$$a^2 (1) + b^2 (1)$$

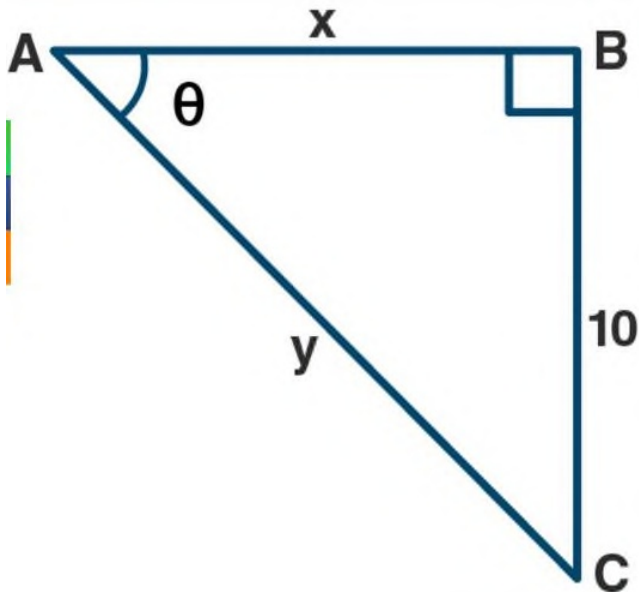
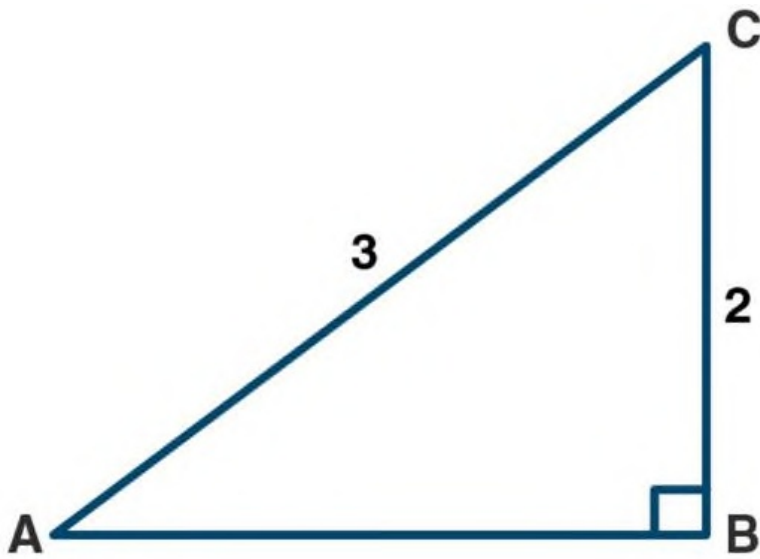
$$= a^2 + b^2$$

$$\text{Therefore, } x^2 + y^2 = a^2 + b^2$$

Chapter test

1. (a) from the figure (i) given below , calculation all the six t – ratios for both acute

(b) from the figure (ii) given below , find the values of x and y in terms of t – ratios



Solution

(a) from right angled triangle ABC ,

By Pythagoras theorem , we get

$$AC^2 = AB^2 + BC^2$$

$$AB^2 = AC^2 - BC^2$$

$$AB^2 = (3)^2 - (2)^2$$

$$AB^2 = 9 - 4$$

$$AB^2 = 5$$

$$AB = \sqrt{5}$$

$$(i) \sin A = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$= \frac{BC}{AC}$$

$$= \frac{2}{3}$$

$$(ii) \cos A = \frac{\text{base}}{\text{hypotenuse}}$$

$$= \frac{AB}{AC}$$

$$= \frac{\sqrt{5}}{3}$$

$$(iii) \tan A = \frac{\textit{perpendicular}}{\textit{base}}$$

$$= \frac{BC}{AB}$$

$$= \frac{2}{\sqrt{5}}$$

$$(iv) \sec A = \frac{\textit{hypotenuse}}{\textit{base}}$$

$$= \frac{AC}{AB}$$

$$= \frac{3}{\sqrt{5}}$$

$$(v) \sec A = \frac{\textit{hypotenuse}}{\textit{base}}$$

$$= \frac{AB}{BC}$$

$$= \frac{\sqrt{5}}{2}$$

$$(v) \sec A = \frac{\textit{hypotenuse}}{\textit{base}}$$

$$= \frac{AC}{AB}$$

$$= \frac{3}{\sqrt{5}}$$

$$\begin{aligned}
 \text{(vi) cosec } A &= \frac{\text{hypotenuse}}{\text{perpendicular}} \\
 &= \frac{AC}{BC} \\
 &= \frac{3}{2}
 \end{aligned}$$

(b) from right angled triangle ABC

$$\angle BAC = \theta$$

Then we know that,

$$\begin{aligned}
 \text{Cot } \theta &= \frac{\text{base}}{\text{perpendicular}} \\
 &= \frac{AB}{BC} \\
 &= \frac{x}{10}
 \end{aligned}$$

$$x = 10 \cot \theta$$

$$\begin{aligned}
 \text{also, cosec } \theta &= \frac{\text{hypotenuse}}{\text{perpendicular}} \\
 &= \frac{AC}{BC} \\
 &= \frac{y}{10}
 \end{aligned}$$

$$Y = 10 \operatorname{cosec} \theta$$

Therefore, $x = 10 \cot \theta$ and $y = 10 \operatorname{cosec} \theta$.

2. (a) from the figure (1) given below , find the value of :

(i) $\sin \angle ABC$

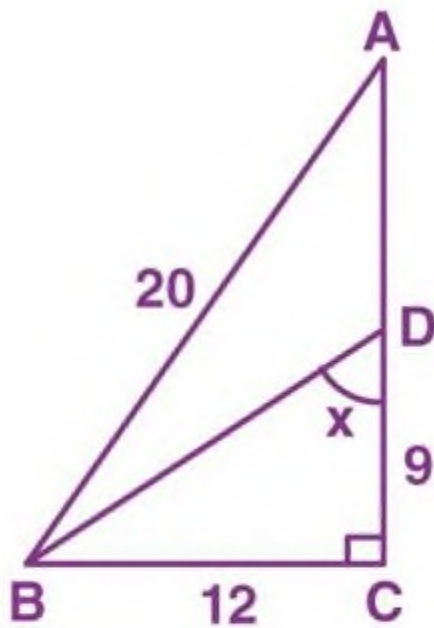
(ii) $\tan x - \cos x + 3 \sin x$.

(b) from the figure (2) given below , find the values of :

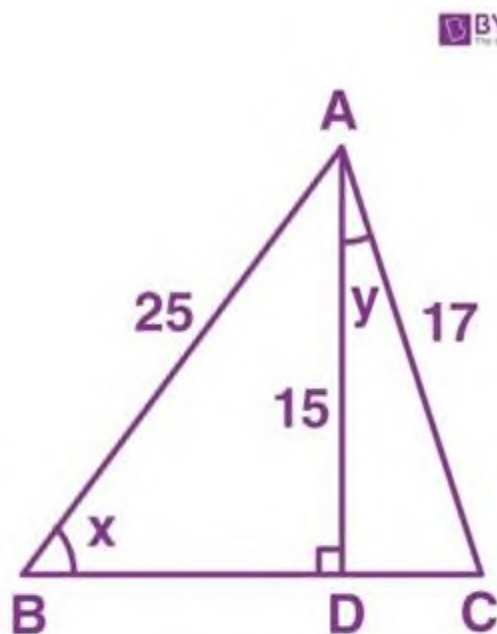
(i) $5 \sin x$

(ii) $7 \tan x$

(iii) $5 \cos x - 17 \sin y - \tan x$



(i)



(ii)

Solution

(a) from the figure

$$BC = 12, CD = 9 \text{ and } AC = 20$$

In right angled ΔABC ,

Using Pythagoras theorem

$$AB^2 = AC^2 + BC^2$$

It can be written as

$$AC^2 = AB^2 - BC^2$$

Substituting the values

$$AC^2 = (20)^2 - (12)^2$$

By further calculation

$$AC^2 = 400 - 144 = 256$$

So we get

$$AC^2 = (16)^2$$

$$AC = 16$$

In right angled ΔBCD

Using Pythagoras theorem

$$BD^2 = BC^2 + CD^2$$

Substituting the values

$$BD^2 = 12^2 + 9^2$$

By further calculation

$$BD^2 = 144 + 81 = 225$$

So we get

$$BD^2 = (15)^2$$

$$BD = 15$$

(i) in right angled ΔBCD

$$\sin \angle ABC = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

so we get

$$\sin \angle ABC = \frac{AC}{AB} = \frac{16}{20} = \frac{4}{5}$$

(ii) in right angled ΔBCD

$$\tan x = \frac{\text{perpendicular}}{\text{base}}$$

So we get

$$\tan x = \frac{BC}{CD} = \frac{12}{9} = \frac{4}{3}$$

In right angled ΔBCD

$$\cos x = \frac{\text{base}}{\text{hypotenuse}}$$

So we get

$$\cos x = \frac{CD}{BD} = \frac{9}{15} = \frac{3}{5}$$

In right angled ΔBCD

$$\sin x = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

So we get

$$\sin x = \frac{BC}{BD} = \frac{12}{15} = \frac{4}{5}$$

$$\tan x - \cos x + 3 \sin x = \frac{4}{3} - \frac{3}{5} + 3 \times \frac{4}{5}$$

By further calculation

$$= \frac{4}{3} - \frac{3}{5} + \frac{12}{5}$$

Taking LCM

$$= \frac{4 \times 5 - 3 \times 3 + 12 \times 3}{15}$$

So we get

$$= \frac{20 - 9 + 36}{15}$$

$$= \frac{56 - 9}{15}$$

$$= \frac{27}{15}$$

$$= 3 \frac{2}{15}$$

$$\text{Therefore, } \tan x - \cos x + 3 \sin x = 3 \frac{2}{15}$$

(b) in the figure

$$AC = 17, AB = 25, AD = 15$$

In right angled ΔACD

Using Pythagoras theorem

$$AC^2 = AD^2 + CD^2$$

Substituting the values

$$(17)^2 = (15)^2 + (CD)^2$$

By further calculation

$$CD^2 = (17)^2 - (15)^2$$

$$CD^2 = 289 - 225 = 64$$

So we get

$$CD^2 = 8^2$$

$$CD = 8$$

In right angled ΔABD

Using Pythagoras theorem

$$AB^2 = AD^2 + BD^2$$

Substituting the values

$$(25)^2 = (15)^2 + BD^2$$

By further calculation

$$BD^2 = 625 - 225 = 400$$

So we get

$$BD^2 = (20)^2$$

$$BD = 20$$

(i) in right angled ΔABD

$$5 \sin x = 5 \left(\frac{\text{perpendicular}}{\text{hypotenuse}} \right)$$

So we get

$$= 5 \left(\frac{AD}{AB} \right)$$

$$= 5 \times \frac{15}{25}$$

$$= \frac{15}{5}$$

$$= 3$$

(ii) in right angled ΔABD

$$7 \tan x = 7 \left(\frac{\text{perpendicular}}{\text{base}} \right)$$

So we get

$$= 7 \left(\frac{AD}{AB} \right)$$

$$= 7 \times \frac{15}{20}$$

$$= 7 \times \frac{3}{4}$$

$$= \frac{21}{4}$$

$$= 5\frac{1}{4}$$

(iii) in right angled ΔABD

$$\cos x = \frac{\text{base}}{\text{hypotenuse}}$$

So we get

$$\cos x = \frac{BD}{AB} = \frac{20}{25} = \frac{4}{5}$$

In right angled ΔACD

$$\sin y = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

So we get

$$\sin y = \frac{CD}{AC} = \frac{8}{17}$$

In right angled ΔABD

$$\tan x = \frac{\text{perpendicular}}{\text{base}}$$

So we get

$$\tan x = \frac{AD}{BD} = \frac{15}{20} = \frac{3}{4}$$

It can be written as

$$= \frac{4}{1} - \frac{8}{1} - \frac{3}{4}$$

Taking LCM

$$= \frac{16 - 32 - 3}{4}$$

$$= \frac{16 - 35}{4}$$

So we get

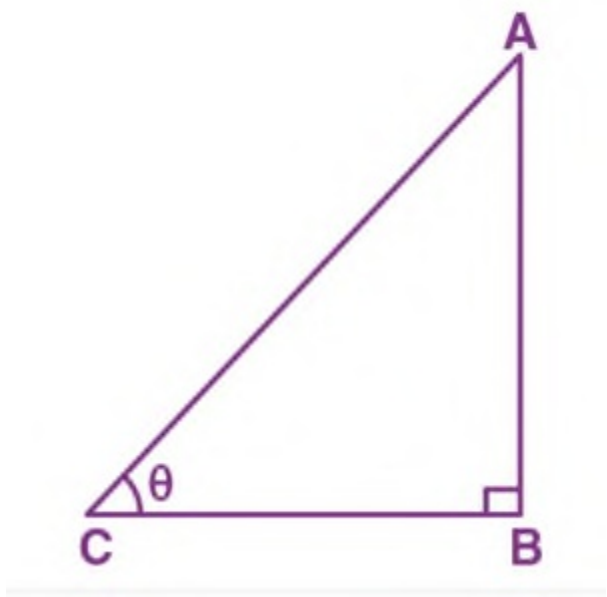
$$= -\frac{19}{4}$$

$$= -4\frac{3}{4}$$

Therefore $5 \cos x - 17 \sin y - \tan x = -4\frac{3}{4}$

3. if $q \cos \theta = p$, find $\tan \theta - \cot \theta$ in terms of p and q .

Solution



Consider ABC as a triangle right angled at B and $\angle ACB = \theta$

It is given that

$$q \cos \theta = p$$

$$\cos \theta = \frac{BC}{AC} = \frac{p}{q},$$

Take $BC = px$ then $AC = qx$

In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

It can be written as

$$AB^2 = AC^2 - BC^2$$

Substituting the values

$$AB^2 = (qx)^2 - (px)^2$$

$$AB^2 = q^2x^2 - p^2x^2$$

Taking out the common terms

$$AB^2 = (q^2 - p^2)x^2$$

So we get

$$AB = \sqrt{(q^2 - p^2)} x^2$$

$$AB = \sqrt{(q^2 - p^2)} x$$

In right angled ΔABC

$$\tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

so we get

$$\tan \theta = \frac{AB}{BC} = \frac{[\sqrt{(q^2 - p^2)}x]}{px}$$

$$\tan \theta = \frac{[\sqrt{(q^2 - p^2)}]}{p}$$

in right angled ΔABC

$$\cot \theta = \frac{\text{base}}{\text{perpendicular}}$$

so we get

$$\cot \theta = \frac{BC}{AB} = \frac{px}{\left[\sqrt{(q^2-p^2)x}\right]}$$

$$\cot \left[\sqrt{(q^2-p^2)x}\right] = \frac{p}{\left[\sqrt{(q^2-p^2)}\right]}$$

$$\tan \theta - \cot \theta = \frac{\left[\sqrt{(q^2-p^2)}\right]}{p} - \frac{p}{\left[\sqrt{(q^2-p^2)}\right]}$$

taking LCM

$$= \frac{\sqrt{(q^2-p^2)}(q^2-p^2) - p \times p}{p\sqrt{(q^2-p^2)}}$$

So we get

$$= \frac{q^2 - p^2 - p^2}{p\sqrt{(q^2-p^2)}}$$

$$= \text{therefore, } \tan \theta - \cot \theta = \frac{q^2 - 2p^2}{p\sqrt{q^2-p^2}}$$

4. given $4 \sin \theta = 3 \cos$, find the values of :

(i) $\sin \theta$

(ii) $\cos \theta$

(iii) $\cot^2 \theta - \operatorname{cosec}^2 \theta$

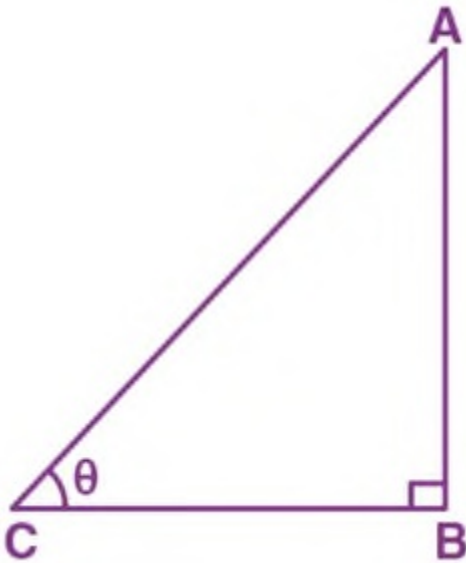
Solution

It is given that

$$4 \sin \theta = 3 \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = \frac{3}{4}$$

$$\tan \theta = \frac{3}{4}$$



Consider ΔABC right angled at B and $\angle ACB = \theta$

$$\tan \theta = \frac{\text{perpendicular}}{\text{base}},$$

substituting the values

$$\frac{3}{4} = \frac{AB}{BC}$$

$$\frac{AB}{BC} = \frac{3}{4}$$

Take $AB = 3x$ then $BC = 4x$

In right angled ΔABC

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$AC^2 = (3x)^2 + (4x)^2$$

By further calculation

$$AC^2 = 9x^2 + 16x^2 = 25x^2$$

So we get

$$AC^2 = (5x)^2$$

$$AC = 5x$$

(i) in right angled ΔABC

$$\sin \theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

so we get

$$\sin \theta = \frac{AB}{AC} = \frac{3x}{5x} = \frac{3}{5}$$

(ii) in right angled ΔABC

$$\cos \theta = \frac{\text{base}}{\text{Hypotenuse}}$$

So we get

$$\cos \theta = \frac{BC}{AC} = \frac{4x}{5x} = \frac{4}{5}$$

(iii) in right angled ΔABC

$$\cot \theta = \frac{\text{base}}{\text{perpendicular}}$$

So we get

$$\cot \theta = \frac{BC}{AB} = \frac{4x}{3x} = \frac{4}{3}$$

In right angled ΔABC

$$\operatorname{Cosec} \theta = \frac{\text{Hypotneuse}}{\text{perpendicular}}$$

So we get

$$\operatorname{Cosec} \theta = \frac{AC}{AB} = \frac{5x}{3x} = \frac{5}{3}$$

Here

$$\cot^2 \theta - \operatorname{cosec}^2 \theta = \left(\frac{4}{3}\right)^2 - \left(\frac{5}{3}\right)^2$$

By further calculation

$$= \frac{16}{9} - \frac{25}{9}$$

$$= \frac{16-25}{9}$$

$$= -\frac{9}{9}$$

$$= -1$$

Therefore, $\cot^2 \theta - \operatorname{cosec}^2 \theta = -1$

5. if $2 \cos \theta = \sqrt{3}$, prove that $3 \sin \theta - 4 \sin^3 \theta = 1$

Solution

It is given that

$$2 \cos \theta = \sqrt{3}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

We know that

$$\sin^2 \theta = 1 - \cos^2 \theta$$

Substituting the values

$$= 1 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 1 - \frac{3}{4}$$

$$= \frac{1}{4}$$

$$\sin \theta = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Consider

$$\text{LHS} = 3 \sin \theta - 4 \sin^3 \theta$$

It can be written as

$$= \sin \theta (3 - 4 \sin^2 \theta)$$

Substituting the values

$$= \frac{1}{2} \left(3 - 4 \times \frac{1}{4} \right)$$

$$= \frac{1}{2} (3 - 1)$$

$$= \frac{1}{2} \times 1$$

$$= 1$$

$$= \text{RHS}$$

Therefore, proved

6. if $\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta} = \frac{1}{4}$, find $\sin \theta$

Solution

We know that

$$\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta} = \frac{\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta}}$$

Taking LCM

$$= \frac{\frac{1 - \sin \theta}{\cos \theta}}{\frac{1 + \sin \theta}{\cos \theta}}$$

So we get

$$= \frac{1 - \sin \theta}{\cos \theta} \times \frac{\cos \theta}{1 + \sin \theta}$$

$$= \frac{1 - \sin \theta}{1 + \sin \theta}$$

Here

$$\frac{1 - \sin \theta}{1 + \sin \theta} = \frac{1}{4}$$

By cross multiplication

$$4 - 4 \sin \theta = 1 + \sin \theta$$

We get

$$4 - 1 = \sin \theta + 4 \sin \theta$$

$$3 = 5 \sin \theta$$

$$\sin \theta = \frac{3}{5}$$

7. if $\sin \theta + \operatorname{cosec} \theta = 3\frac{1}{3}$, find the value of $\sin^2 \theta + \operatorname{cosec}^2 \theta$

Solution

It is given that

$$\sin \theta + \operatorname{cosec} \theta = 3\frac{1}{3} = \frac{10}{3}$$

By squaring on both sides

$$(\sin \theta + \operatorname{cosec} \theta)^2 = \left(\frac{10}{3}\right)^2$$

Expanding using formula $(a+b)^2 = a^2 + b^2 + 2ab$

$$\sin^2 \theta + \operatorname{cosec}^2 \theta + 2 \sin \theta \operatorname{cosec} \theta = \frac{100}{9}$$

$$\text{We know that } \sin \theta = \frac{1}{\operatorname{cosec} \theta}$$

$$\sin^2 \theta + \operatorname{cosec}^2 \theta + 2 \sin \theta \times \frac{1}{\sin \theta} = \frac{100}{9}$$

By further calculation

$$\sin^2 \theta + \operatorname{cosec}^2 \theta + 2 = \frac{100}{9}$$

$$\sin^2 \theta + \operatorname{cosec}^2 \theta = \frac{100}{9} - 2$$

Taking LCM

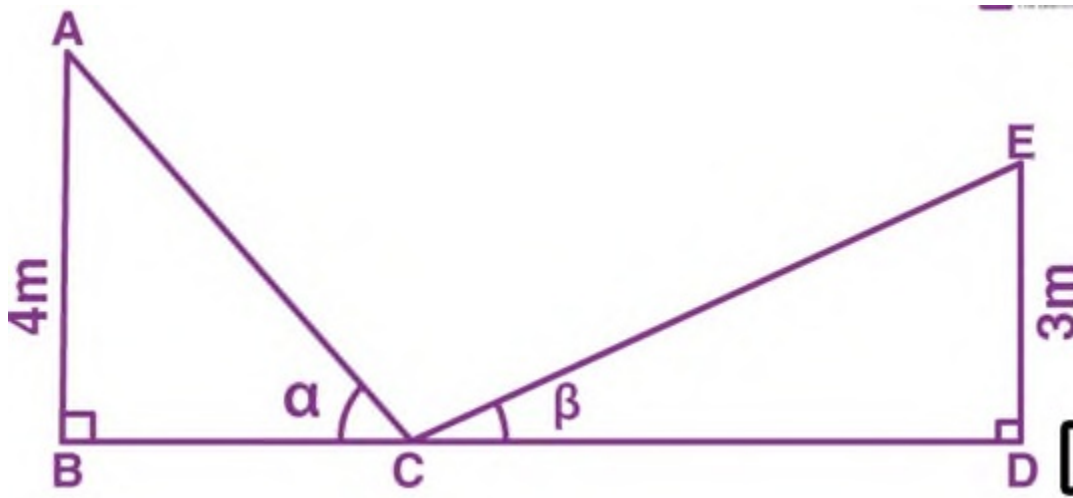
$$\sin^2 \theta + \operatorname{cosec}^2 \theta = \frac{100-18}{9} = \frac{82}{9}$$

So we get

$$\sin^2 \theta + \operatorname{cosec}^2 \theta = 9\frac{1}{9}$$

8. in the adjoining figure , $AB = 4\text{m}$ and $ED = 3\text{m}$.

If $\sin \alpha = \frac{3}{5}$ and $\cos \beta = \frac{12}{13}$, find the length of BD .



Solution

It is given that

$$\sin \alpha = \frac{AB}{AC} = \frac{3}{5}$$

$$AB = 3 \text{ and } AC = 5$$

Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

Substituting the values

$$5^2 = 3^2 + BC^2$$

By further calculation

$$25 = 9 + BC^2$$

$$BC^2 = 25 - 9 = 16$$

So we get

$$BC^2 = 4^2$$

$$BC = 4$$

We know that

$$\tan \alpha = \frac{AB}{BC} = \frac{4}{5}$$

$$\cos \beta = \frac{CD}{CE} = \frac{12}{13}$$

$$CD = 12 \text{ and } CE = 13$$

Using Pythagoras theorem

$$CE^2 = CD^2 + ED^2$$

Substituting the values

$$13^2 = 12^2 + ED^2$$

By further calculation

$$ED^2 = 13^2 - 12^2$$

$$ED^2 = 169 - 144 = 25$$

So we get

$$ED^2 = (5)^2$$

$$ED = 5$$

$$\tan \beta = \frac{ED}{CD} = \frac{5}{12}$$

from the figure

$$\tan \alpha = \frac{AB}{BC} = \frac{4}{BC}$$

so we get

$$\frac{3}{4} = \frac{4}{BC}$$

$$BC = \frac{(4 \times 4)}{3} = \frac{16}{3} \text{ m}$$

$$\tan \beta = \frac{ED}{CD} = \frac{3}{CD}$$

$$\frac{5}{12} = \frac{3}{CD}$$

So we get

$$CD = \frac{12 \times 3}{5} = \frac{36}{5} \text{ m}$$

Here

$$BD = BC + CD$$

Substituting the values

$$= \frac{16}{3} + \frac{36}{5}$$

Taking LCM

$$= \frac{80+108}{15}$$

$$= \frac{188}{15} \text{ m}$$

$$= 12 \frac{8}{15} \text{ m}$$