

Chapter 18

trigonometric Ratios and standard Angles

Exercise 18.1

1. find the values of

(i) $7 \sin 30^\circ \cos 60^\circ$

(ii) $3 \sin^2 45^\circ + 2 \cos^2 60^\circ$

(iii) $\cos^2 45^\circ + \sin^2 60^\circ + \sin^2 30^\circ$

(iv) $\cos 90^\circ + \cos^2 45^\circ \sin 30^\circ \tan 45^\circ$

Solution

(i) $7 \sin 30^\circ \cos 60^\circ$

Substituting the values

$$= 7 \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{7 \times 1 \times 1}{2 \times 2}$$

$$= \frac{7}{4}$$

(ii) $3 \sin^2 45^\circ + 2 \cos^2 60^\circ$

Substituting the values

$$= 3 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 2 \times \left(\frac{1}{2}\right)^2$$

By further calculation

$$= 3 \times \frac{1}{2} + 2 \times \frac{1}{4}$$

$$= \frac{3}{2} + \frac{1}{2}$$

So we get

$$= \frac{3+1}{2}$$

$$= \frac{4}{2}$$

$$= 2$$

$$(iii) \cos^2 45^\circ + \sin^2 60^\circ + \sin^2 30^\circ$$

Substituting the values

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

By further calculation

$$= \frac{1}{2} + \frac{3}{4} + \frac{1}{4}$$

Taking LCM

$$\frac{2+3+1}{4}$$

$$\frac{6}{4}$$

$$\frac{3}{2}$$

$$(iv) \cos 90^\circ + \cos^2 45^\circ \sin 30^\circ \tan 45^\circ$$

Substituting the values

$$0 + \left(\frac{1}{\sqrt{2}}\right)^2 \times \frac{1}{2} \times 1$$

By further calculation

$$\frac{1}{2} \times \frac{1}{2} \times 1$$

$$\frac{1}{4}$$

2. find the values of

(i) $\frac{\sin^2 45^\circ + \cos^2 45^\circ}{\tan^2 60^\circ}$

(ii) $\frac{\sin 30^\circ - \sin 90^\circ + 2\cos 0^\circ}{\tan 30^\circ \times \tan 60^\circ}$

(iii) $\frac{4}{3} \tan^2 30^\circ + \sin^2 60^\circ - 3\cos^2 60^\circ + \frac{3}{4} \tan^2 60^\circ - 2\tan^2 45^\circ$

Solution

(i) $\frac{\sin^2 45^\circ + \cos^2 45^\circ}{\tan^2 60^\circ}$

Substituting the values

$$= \frac{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}{\sqrt{3}^2}$$

By further calculation

$$= \frac{\frac{1}{2} + \frac{1}{2}}{3}$$

$$= \frac{1}{3}$$

$$(ii) \frac{\sin 30^\circ - \sin 90^\circ + 2 \cos 0^\circ}{\tan 30^\circ \times \tan 60^\circ}$$

Substituting the values

$$= \frac{\frac{1}{2} - 1 + 2 \times 1}{\frac{1}{\sqrt{3}} \times \sqrt{3}}$$

$$= \frac{\frac{1}{2} - 1 + 2}{1}$$

So we get

$$= \frac{1}{2} - 1 + 2$$

$$= \frac{1}{2} + 1$$

$$= \frac{1+2}{2}$$

$$= \frac{3}{2}$$

$$(iii) \frac{4}{3} \tan^2 30^\circ + \sin^2 60^\circ - 3 \cos^2 60^\circ + \frac{3}{4} \tan^2 60^\circ - 2 \tan^2 45^\circ$$

Substituting the values

$$= \frac{4}{3} \left(\frac{1}{\sqrt{3}} \right)^2 + \left(\frac{\sqrt{3}}{2} \right)^2 - 3 \left(\frac{1}{2} \right)^2 + \frac{3}{4} \times (\sqrt{3})^2 - 2 \times 1^2$$

By further calculation

$$= \frac{4}{3} \times \frac{1}{3} + \frac{3}{4} - 3 \times \frac{1}{4} + \frac{3}{4} \times 3 - 2 \times 1$$

$$= \frac{4}{9} + \frac{3}{4} - \frac{3}{4} + \frac{9}{4} - 2$$

So we get

$$= \frac{4}{9} + \frac{9}{4} - 2$$

Taking LCM

$$= \frac{16 + 81 - 72}{36}$$

$$= \frac{97 - 72}{36}$$

$$= \frac{25}{36}$$

3. find the values of

(i) $\frac{\sin 60^\circ}{\cos^2 45^\circ} - 3 \tan 30^\circ + 5 \cos 90^\circ$

(ii) $2\sqrt{2} \cos 45^\circ \cos 60^\circ + 2\sqrt{3} \sin 30^\circ \tan 60^\circ - \cos 0^\circ$

(iii) $\frac{4}{5} \tan^2 60^\circ - \frac{2}{\sin^2 30^\circ} - \frac{3}{4} \tan^2 30^\circ$

Solution

$$(i) \frac{\sin 60^\circ}{\cos^2 45^\circ} - 3 \tan 30^\circ + 5 \cos 90^\circ$$

Substituting the values

$$\begin{aligned} &= \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)^2} - 3 \times \frac{1}{\sqrt{3}} + 5 \times 0 \\ &= \frac{\left(\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}} - \sqrt{3} - 0 \end{aligned}$$

So we get

$$\begin{aligned} &\frac{\sqrt{3}}{2} \times \frac{2}{1} - \sqrt{3} \\ &= \sqrt{3} - \sqrt{3} \\ &= 0 \end{aligned}$$

$$(ii) 2\sqrt{2} \cos 45^\circ \cos 60^\circ + 2\sqrt{3} \sin 30^\circ \tan 60^\circ - \cos 0^\circ$$

Substituting the values

$$= 2\sqrt{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{2} + 2\sqrt{3} \times \frac{1}{2} \times \sqrt{3} - 1$$

By further calculation

$$\begin{aligned} &= 2 \times \frac{1}{1} \times \frac{1}{2} + 2 \times 3 \times \frac{1}{2} - 1 \\ &= 1 + 3 - 1 \\ &= 3 \end{aligned}$$

$$(iii) \frac{4}{5} \tan^2 60^\circ - \frac{2}{\sin^2 30^\circ} - \frac{3}{4} \tan^2 30^\circ$$

Substituting the values

$$= \frac{4}{5} \times \sqrt{3}^2 - \frac{2}{\left(\frac{1}{2}\right)^2} - \frac{3}{4} \left(\frac{1}{\sqrt{3}}\right)^2$$

By further calculation

$$= \frac{4}{5} \times 3 - \frac{2}{\frac{1}{4}} - \frac{3}{4} \times \frac{1}{3}$$

So we get

$$= \frac{12}{5} - \frac{2 \times 4}{1} - \frac{1}{4}$$

$$= \frac{12}{5} - \frac{8}{1} - \frac{1}{4}$$

Taking LCM

$$= \frac{48 - 160 - 5}{20}$$

$$= \frac{43 - 160}{20}$$

$$= -\frac{117}{20}$$

$$= -5\frac{17}{20}$$

4. prove that

(i) $\cos^2 30^\circ + \sin 30^\circ + \tan^2 45^\circ = 2\frac{1}{4}$

(ii) $4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) = 2$

(iii) $\cos 60^\circ = \cos^2 30^\circ - \sin^2 30^\circ$

Solution

(i) $\cos^2 30^\circ + \sin 30^\circ + \tan^2 45^\circ = 2\frac{1}{4}$

Consider

$$\text{LHS} = \cos^2 30^\circ + \sin 30^\circ + \tan^2 45^\circ$$

Substituting the values

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2} + 1^2$$

By further calculation

$$= \frac{3}{4} + \frac{1}{2} + 1$$

Taking LCM

$$= \frac{3+2+4}{4}$$

$$= \frac{9}{4}$$

$$= 2\frac{1}{4}$$

$$= \text{RHS}$$

Therefore LHS = RHS

$$(ii) 4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) = 2$$

Consider

$$\text{LHS} = 4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ)$$

Substituting the values

$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - 1^2\right]$$

It can be written as

$$= 4\left[\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right] - 3\left[\frac{1}{2} - 1\right]$$

By further calculation

$$= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left(-\frac{1}{2}\right)$$

$$= 4\left[\frac{1+1}{16}\right] + \frac{3}{2}$$

So we get

$$= \frac{4 \times 2}{16} + \frac{3}{2}$$

$$= \frac{8}{16} + \frac{3}{2}$$

$$= \frac{1}{2} + \frac{3}{2}$$

$$= \frac{1+3}{2}$$

$$= \frac{4}{2}$$

$$= 2$$

$$= \text{RHS}$$

Therefore LHS=RHS

$$(iii) \cos 60^\circ = \cos^2 30^\circ - \sin^2 30^\circ$$

Consider

$$\text{LHS} = \cos 60^\circ = \frac{1}{2}$$

$$\text{RHS} = \cos^2 30^\circ - \sin^2 30^\circ$$

Substituting the values

$$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

By further calculation

$$= \frac{3}{4} - \frac{1}{4}$$

$$= \frac{3-1}{4}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

$$= \text{RHS}$$

Therefore , LHS = RHS

5. (i) if $x = 30^\circ$, verify that $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

(ii) if $x = 15^\circ$, verify that $4 \sin 2x \cos 4x \sin 6x = 1$

Solution

(i) it is given that

$$X = 30^\circ$$

Consider LHS = $\tan 2x$

Substituting the value of x

$$= \tan 60^\circ$$

$$= \sqrt{3}$$

$$\text{RHS} = \frac{2\tan x}{1-\tan^2 x}$$

Substituting the value of x

$$= \frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$$

By further calculation

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2}$$

So we get

$$= \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{3 - \frac{1}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{2}$$

$$= \frac{3}{\sqrt{3}}$$

$$= \sqrt{3}$$

Therefore , LHS = RHS

(ii) if $x = 15^\circ$, verify that $4 \sin 2x \cos 4x \sin 6x = 1$

It is given that

$$x = 15^\circ$$

$$2x = 15 \times 2 = 30^\circ$$

$$4x = 15 \times 4 = 60^\circ$$

$$6x = 15 \times 6 = 90^\circ$$

Here

$$\text{LHS} = 4 \sin 2x \cos 4x \sin 6x$$

It can be written as

$$= 4 \sin 30^\circ \cos 60^\circ \sin 90^\circ$$

So we get

$$= 4 \times \frac{1}{2} \times \frac{1}{2} \times 1$$

$$= 1$$

$$= \text{RHS}$$

Therefore , LHS = RHS

6. find the values of

$$(i) \sqrt{\frac{1 - \cos^2 30^\circ}{1 - \sin^2 30^\circ}}$$

$$(ii) \frac{\sin 45^\circ \cos 45^\circ \cos 60^\circ}{\sin 60^\circ \cos 30^\circ \tan 45^\circ}$$

Solution

$$(i) \sqrt{\frac{1 - \cos^2 30^\circ}{1 - \sin^2 30^\circ}}$$

Substituting the values

$$= \sqrt{\frac{1 - \left(\frac{\sqrt{3}}{2}\right)^2}{1 - \left(\frac{1}{2}\right)^2}}$$

By further calculation

$$= \sqrt{\frac{1 - \frac{3}{4}}{1 - \frac{1}{4}}}$$

Taking LCM

$$= \sqrt{\frac{4 - \frac{3}{4}}{4 - \frac{1}{4}}}$$

$$= \sqrt{\frac{\frac{1}{4}}{\frac{3}{4}}}$$

$$= \sqrt{\frac{1}{4} \times \frac{4}{3}}$$

So we get

$$= \sqrt{\frac{1}{3}}$$

$$= \frac{1}{\sqrt{3}}$$

$$(ii) \frac{\sin 45^\circ \cos 45^\circ \cos 60^\circ}{\sin 60^\circ \cos 30^\circ \tan 45^\circ}$$

Substituting the values

$$= \frac{\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \frac{1}{2}}{\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \times 1}$$

By further calculation

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{3}{4} \times 1}$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}}$$

$$= \frac{1}{3}$$

7. if $\theta = 30^\circ$, verify that

(i) $\sin 2\theta = 2\sin\theta \cos\theta$

(ii) $\cos 2\theta = 2\cos^2\theta - 1$

(iii) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

(iv) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

Solution

It is given that $\theta = 30^\circ$

$$(i) \sin 2\theta = 2 \sin \theta \cos \theta$$

Consider

$$\text{LHS} = \sin 2\theta$$

Substituting the value of θ

$$= \sin 2 \times 30^\circ$$

$$= \sin 60^\circ$$

$$= \frac{\sqrt{3}}{2}$$

Therefore LHS = RHS

$$(ii) \cos 2\theta = 2\cos^2 \theta - 1$$

Consider

$$\text{LHS} = \cos 2\theta$$

Substituting the value of θ

$$= \cos 2 \times 30^\circ$$

$$= \cos 60^\circ$$

$$= \frac{1}{2}$$

$$\text{LHS} = 2 \cos^2 \theta - 1$$

Substituting the value of θ

$$= 2 \cos^2 30^\circ - 1$$

So we get

$$= 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1$$

$$= 2 \times \frac{3}{4} - 1$$

$$= \frac{3}{2} - 1$$

$$= \frac{3-2}{2}$$

$$= \frac{1}{2}$$

Therefore , LHS = RHS

$$(iii) \sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

Consider

$$\text{LHS} = \sin 3\theta$$

Substituting the value of θ

$$\sin 3 \times 30^\circ$$

$$\sin 90^\circ$$

$$1$$

$$\text{RHS} = 3 \sin\theta - 4\sin^3\theta$$

Substituting the value of θ

$$= 3 \sin 30^\circ - 4 \sin^3 30^\circ$$

So we get

$$= 3 \times \frac{1}{2} - 4 \times \left(\frac{1}{2}\right)^3$$

$$= \frac{3}{2} - 4 \times \frac{1}{8}$$

$$= \frac{3}{2} - \frac{1}{2}$$

Taking LCM

$$= \frac{3-1}{2}$$

$$= \frac{2}{2}$$

$$= 1$$

Therefore , LHS = RHS

$$(iv) \cos 3\theta = 4\cos^3 \theta - 3 \cos \theta$$

Consider

$$\text{LHS} = \cos 3\theta$$

Substituting the value of θ

$$= \cos 3 \times 30^\circ$$

$$= \cos 90^\circ$$

$$= 0$$

$$\text{RHS} = 4\cos^3 \theta - 3\cos \theta$$

Substituting the value of θ

$$= 4\cos^3 30^\circ - 3\cos 30^\circ$$

So we get

$$= 4 \times \left(\frac{\sqrt{3}}{2}\right)^3 - 3 \times \left(\frac{\sqrt{3}}{2}\right)$$

By further calculation

$$= 4 \times \frac{3\sqrt{3}}{8} - \frac{3\sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$$

$$= 0$$

Therefore LHS = RHS

8. if $\theta = 30^\circ$, find the ratio $2 \sin \theta : \sin 2 \theta$

Solution

It is given that $\theta = 30^\circ$

We know that

$$2 \sin \theta : \sin 2 \theta = 2 \sin 30^\circ : \sin 2 \times 30^\circ$$

So we get

$$= 2 \sin 30^\circ : \sin 60^\circ$$

$$= \frac{2 \sin 30^\circ}{\sin 60^\circ}$$

Substituting the values

$$= \frac{2 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

By further simplification

$$\begin{aligned} &= \frac{\frac{1}{\sqrt{3}}}{2} \\ &= \frac{1 \times 2}{\sqrt{3}} \\ &= \frac{2}{\sqrt{3}} \\ &= 2 : \sqrt{3} \end{aligned}$$

Therefore $2 \sin \theta : \sin 2 \theta = 2 : \sqrt{3}$

9. by means of an example , show that $\sin (A+B) \neq \sin A + \sin B$.

Solution

Consider $A = 30^\circ$ and $B = 60^\circ$

$$\text{LHS} = \sin(A+B)$$

Substituting the values of A and B

$$= \sin(30^\circ + 60^\circ)$$

$$= \sin 90^\circ$$

$$= 1$$

$$\text{RHS} = \sin A + \sin B$$

Substituting the values

$$= \sin 30^\circ + \sin 60^\circ$$

So we get

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$= \frac{1+\sqrt{3}}{2}$$

Therefore $\text{LHS} \neq \text{RHS}$ i.e. $\sin (A+B) \neq \sin A + \sin B$.

10. if $A = 60^\circ$ and $B = 30^\circ$, verify that

(i) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

(ii) $\cos (A+B) = \cos A \cos B - \sin A \sin B$

(iii) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

(iv) $\tan (A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Solution

It is given that $A = 60^\circ$ and $B = 30^\circ$

(i) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

Here

$$\text{LHS} = \sin(A+B)$$

Substituting the values of A and B

$$= \sin (60^\circ + 30^\circ)$$

$$= \sin 90^\circ$$

$$= 1$$

$$\text{RHS} = \sin A \cos B + \cos A \sin B$$

Substituting the values of A and B

$$= \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

So we get

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

By further calculation

$$= \frac{3}{4} + \frac{1}{4}$$

$$= \frac{4}{4}$$

$$= 1$$

Therefore , LHS = RHS

$$(ii) \cos (A+B) = \cos A \cos B - \sin A \sin B$$

Here

$$\text{LHS} = \cos(A+B)$$

Substituting the value of A and B

$$= \cos(60^\circ + 30^\circ)$$

$$= \cos 90^\circ$$

$$= 0$$

$$\text{RHS} = \cos A \cos B - \sin A \sin B$$

Substituting the value of A and B

$$= \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$$

So we get

$$= \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

$$= 0$$

Therefore, LHS = RHS

$$(iii) \sin(A-B) = \sin A \cos B - \cos A \sin B$$

Here

$$\text{LHS} = \sin(A-B)$$

Substituting the values of A and B

$$= \sin(60^\circ - 30^\circ)$$

$$= \sin 30^\circ$$

$$= \frac{1}{2}$$

$$\text{RHS} = \sin A \cos B - \cos A \sin B$$

Substituting the values of A and B.

$$= \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$$

So we get

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} - \frac{1}{4}$$

$$= \frac{3-1}{4}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

Therefore, LHS = RHS

$$(iv) \tan (A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Here

$$\text{LHS} = \tan (A-B)$$

Substituting the values of A and B

$$= \tan(60^\circ - 30^\circ)$$

$$= \tan 30^\circ$$

$$= \frac{1}{\sqrt{3}}$$

$$\text{RHS} = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Substituting the values of A and B

$$= \frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$$

So we get

$$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}}$$

By further simplification

$$= \frac{3-1}{\frac{\sqrt{3}}{1+1}}$$

$$= \frac{2}{\frac{\sqrt{3}}{2}}$$

We get

$$= \frac{2}{\sqrt{3}} \times \frac{1}{2}$$

$$\frac{1}{\sqrt{3}}$$

Therefore , LHS = RHS

11. (i) if 2θ is an acute angle and $2 \sin 2 \theta = \sqrt{3}$, find the value of θ .

(ii) if $20^\circ + x$ is an acute angle and $\cos (20^\circ + x) = \sin 60^\circ$, then find the value of x .

(iii) if $3 \sin^2 \theta = 2\frac{1}{4}$ and θ is less than 90° , find the value of θ .

Solution

(i) it is given that

2θ is an acute angle

$$2\sin 2 \theta = \sqrt{3}$$

It can be written as

$$\sin 2 \theta = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

By comparing

$$2 \theta = 60^\circ$$

So we get

$$\theta = \frac{60^\circ}{2} = 30^\circ$$

Therefore $\theta = 30^\circ$

(ii) it is given that

$20^\circ + x$ is an acute angle

$$\begin{aligned}\cos(20^\circ + x) &= \sin 60^\circ = \cos(90^\circ - 60^\circ) \\ &= \cos 30^\circ\end{aligned}$$

By comparing

$$20^\circ + x = 30^\circ$$

$$x = 30^\circ - 20^\circ = 10^\circ$$

Therefore, $x = 10^\circ$

(iii) it is given that

$$3\sin^2 \theta = 2\frac{1}{4}$$

θ is less than 90°

We can write it as

$$\sin^2 \theta = \frac{9}{4 \times 3} = \frac{3}{4}$$

So we get

$$\sin \theta = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

By comparing

$$\theta = 60^\circ$$

Therefore, $\theta = 60^\circ$

12. if θ is an acute angle and $\sin \theta = \cos \theta$, find the value of θ and hence, find the value of $2 \tan^2 \theta + \sin^2 \theta - 1$

Solution

It is given that

$$\sin \theta = \cos \theta$$

We can write it as

$$\frac{\sin \theta}{\cos \theta} = 1$$

$$\tan \theta = 1$$

we know that $\tan 45^\circ = 1$

$$\tan \theta = \tan 45^\circ$$

so we get

$$\theta = 45^\circ$$

We know that

$$2 \tan^2 \theta + \sin^2 \theta - 1 = 2 \tan^2 45^\circ + \sin^2 45^\circ - 1$$

Substituting the values

$$= 2(1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 1$$

By further calculation

$$= 2 \times 1 \times 1 + \frac{1}{2} - 1$$

$$= 2 + \frac{1}{2} - 1$$

$$= \frac{5}{2} - 1$$

Taking LCM

$$= \frac{5-2}{2}$$

$$= \frac{3}{2}$$

$$\text{Therefore, } 2\tan^2 \theta + \sin^2 \theta - 1 = \frac{3}{2}$$

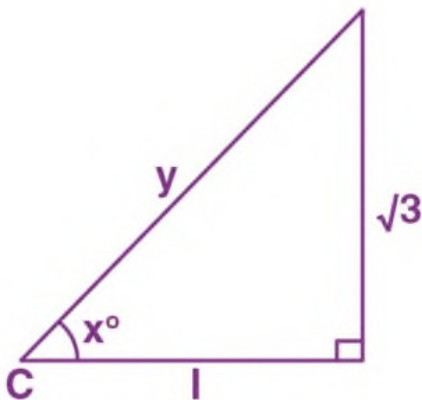
13. from the adjoining figure, find

(i) $\tan x^\circ$

(ii) x

(iii) $\cos x^\circ$

(iv) use $\sin x^\circ$ to find y .



Solution

$$(i) \tan x^\circ = \frac{\text{perpendicular}}{\text{base}}$$

It can be written as

$$= \frac{AB}{BC}$$

$$= \frac{\sqrt{3}}{1}$$

$$= \sqrt{3}$$

$$(ii) \tan x^\circ = \sqrt{3}$$

We know that $\tan 60^\circ = \sqrt{3}$

$$\tan x^\circ = \tan 60^\circ$$

$$x = 60$$

(iii) we know that

$$\cos x^\circ = \cos 60^\circ$$

So we get

$$\cos x^\circ = \frac{1}{2}$$

$$(iv) \sin x^\circ = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{AC}$$

Substituting $x = 60$ from (ii)

$$\sin 60^\circ = \frac{\sqrt{3}}{y}$$

$$\text{We know that } \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{y}$$

$$\text{We know that } \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{y}$$

By further calculation

$$Y = \frac{\sqrt{3} \times 2}{\sqrt{3}}$$

$$Y = 2 \times \frac{1}{1} = 2$$

Therefore, $y = 2$

14. if 3θ is an acute angle , solve the following equation for θ :

$$(i) \ 2 \sin 3\theta = \sqrt{3}$$

$$(ii) \ \tan 3\theta = 1$$

Solution

$$(i) 2 \sin 3\theta = \sqrt{3}$$

It can be written as

$$\sin 3\theta = \frac{\sqrt{3}}{2}$$

$$\text{We know that } \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\sin 3\theta = \sin 60^\circ$$

$$3\theta = 60^\circ$$

So we get

$$\theta = \frac{60}{3} = 20^\circ$$

$$(ii) \tan 3\theta = 1$$

$$\text{We know that } \tan 45^\circ = 1$$

$$\tan 3\theta = \tan 45^\circ$$

so we get

$$3\theta = 45^\circ$$

$$\theta = 15^\circ$$

15. if $\tan 3x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$, find the value of x .

Solution

We know that

$$\tan 3x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$$

substituting the values

$$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2}$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

We know that

$$\tan 3x = \tan 45^\circ$$

By comparing

$$3x = 45^\circ$$

$$x = \frac{45}{3} = 15^\circ$$

Therefore, the value of x is 15°

16. if $4 \cos^2 x^\circ - 1 = 0$ and $0 \leq x \leq 90$, find

(i) x

(ii) $\sin^2 x^\circ + \cos^2 x^\circ$

(iii) $\cos^2 x^\circ - \sin^2 x^\circ$

Solution :

It is given that

$$4\cos^2 x^\circ - 1 = 0$$

$$4\cos^2 x^\circ = 1$$

It can be written as

$$\cos^2 x^\circ = \frac{1}{4}$$

$$\cos x^\circ = \pm \frac{\sqrt{1}}{4}$$

$$\cos x^\circ = +\frac{\sqrt{1}}{4} \quad [0 \leq x \leq 90^\circ, \text{ then } \cos x^\circ \text{ is positive}]$$

$$\cos x^\circ = \frac{1}{2}$$

$$\text{We know that } \cos 60^\circ = \frac{1}{2}$$

$$\cos x^\circ = \cos 60^\circ$$

By comparing

$$X = 60$$

$$(ii) \sin^2 x^\circ + \cos^2 x^\circ = \sin^2 60^\circ + \cos^2 60^\circ$$

Substituting the values

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

By further calculation

$$= \frac{3}{4} + \frac{1}{4}$$

$$= 3 + \frac{1}{4}$$

$$= \frac{4}{4}$$

$$= 1$$

Therefore , $\sin^2 x^\circ + \cos^2 x^\circ = 1$

$$(iii) \cos^2 x^\circ - \sin^2 x^\circ = \cos^2 60^\circ - \sin^2 60^\circ$$

Substituting the values

$$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

By further calculation

$$= \frac{1}{4} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} - \frac{3}{4}$$

$$= \frac{1-3}{4}$$

$$= -\frac{2}{4}$$

$$= -\frac{1}{2}$$

$$\text{Therefore, } \cos^2 x^\circ - \sin^2 x^\circ = -\frac{1}{2}$$

17 . (i) if $\sec \theta = \operatorname{cosec} \theta$ and $0^\circ \leq \theta \leq 90^\circ$, find the value of θ

(ii) if $\tan \theta = \cot \theta$ and $0^\circ \leq \theta \leq 90^\circ$, find the value of θ

Solution

(i) it is given that

$$\sec \theta = \operatorname{cosec} \theta$$

We know that

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\operatorname{Cosec} \theta = \frac{1}{\sin \theta}$$

So we get

$$\frac{1}{\cos \theta} = \frac{1}{\sin \theta}$$

$$\frac{\sin \theta}{\cos \theta} = 1$$

$$\tan \theta = 1$$

here $\tan 45^\circ = 1$

$$\tan \theta = \tan 45^\circ$$

$$\theta = 45^\circ$$

(ii) it is given that

$$\tan \theta = \cot \theta$$

$$\text{we know that } \cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{1}{\tan \theta}$$

so we get

$$\tan^2 \theta = 1$$

$$\tan \theta = \pm \sqrt{1}$$

$$\tan \theta = +1 \quad [0^\circ \leq \theta \leq 90^\circ, \tan \theta \text{ is positive}]$$

$$\tan \theta = \tan 45^\circ$$

by comparing

$$\theta = 45^\circ$$

18. if $\sin 3x = 1$ and $0^\circ \leq 3x \leq 90^\circ$, find the values of

(i) $\sin x$

(ii) $\cos 2x$

(iii) $\tan^2 x - \sec^2 x$.

Solution

It is given that

$$\sin 3x = 1$$

We know that $\sin 90^\circ = 1$

$$\sin 3x = \sin 90^\circ$$

By comparing

$$3x = 90$$

$$x = \frac{90}{3}$$

$$x = 30^\circ$$

$$(i) \sin x = \sin 30^\circ = \frac{1}{2}$$

$$(ii) \cos 2x = \cos 2 \times 30 = \cos 60^\circ = \frac{1}{2}$$

$$(iii) \tan^2 x - \sec^2 x = \tan^2 30^\circ - \sec^2 30^\circ$$

Substituting the values

$$= \left(\frac{1}{\sqrt{3}}\right)^2 - \left(\frac{2}{\sqrt{3}}\right)^2$$

By further calculation

$$= \frac{1}{3} - \frac{4}{3}$$

$$= \frac{1-4}{3}$$

$$= -\frac{3}{3}$$

$$= -1$$

Therefore, $\tan^2 x - \sec^2 x = -1$

19 . if $3\tan^2 \theta - 1 = 0$, find $\cos 2 \theta$, given that θ is acute .

Solution

It is given that

$$3\tan^2 \theta - 1 = 0$$

We can write it as

$$3 \tan^2 \theta = 1$$

$$\tan^2 \theta = \frac{1}{3}$$

$$\tan \theta = \frac{1}{\sqrt{3}} \text{ [} \theta \text{ is acute so } \tan \theta \text{ is positive]}$$

$$\theta = 30^\circ$$

So we get

$$\cos 2 \theta = \cos 2 \times 30^\circ = \cos 60^\circ = \frac{1}{2}$$

20 . if $\sin x + \cos y = 1$, $x = 30^\circ$ and y is acute angle , find the value of y .

Solution

It is given that

$$\sin x + \cos y = 1$$

$$X = 30^\circ$$

Substituting the values

$$\sin 30^\circ + \cos y = 1$$

$$\frac{1}{2} + \cos y = 1$$

It can be written as

$$\cos y = 1 - \frac{1}{2}$$

Taking LCM

$$\cos y = \frac{2-1}{2} = \frac{1}{2}$$

$$\text{We know that } \cos 60^\circ = \frac{1}{2}$$

$$\cos y = \cos 60^\circ$$

So we get

$$Y = 60^\circ$$

**21. if $\sin(A+B) = \frac{\sqrt{3}}{2} = \cos(A-B)$, $0^\circ < A+B \leq 90^\circ$ ($A > B$),
find the values of A and B.**

Solution

It is given that

$$\sin(A+B) = \frac{\sqrt{3}}{2} = \cos(A-B)$$

Consider

$$\sin(A+B) = \frac{\sqrt{3}}{2}$$

$$\text{We know that } \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\sin(A+B) = \sin 60^\circ$$

$$A+B = 60^\circ \dots\dots(1)$$

Similarly

$$\cos(A-B) = \frac{\sqrt{3}}{2}$$

$$\text{We know that } \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\cos(A-B) = \cos 30^\circ$$

$$A-B = 30^\circ \dots\dots(2)$$

By adding both the equations

$$A+B + A-B = 60^\circ + 30^\circ$$

So we get

$$2A = 90^\circ$$

$$A = \frac{90^\circ}{2} = 45^\circ$$

Now substitute the value of A in equation (1)

$$45^\circ + B = 60^\circ$$

By further calculation

$$B = 60^\circ - 45^\circ = 15^\circ$$

Therefore , $A = 45^\circ$ and $B = 15^\circ$

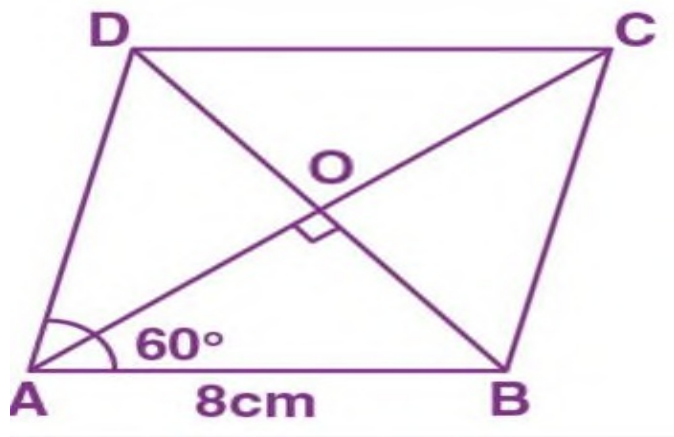
22. if the length of each side of a rhombus is 8 cm and its one angle is 60° , then find the lengths of the diagonals of the rhombus.

Solution

It is given that

Each side of a rhombus = 8 cm

One angle = 60°



We know that the diagonals bisect the opposite angles

$$\angle OAB = \frac{60^\circ}{2} = 30^\circ$$

In right $\angle AOB$

$$\sin 30^\circ = \frac{OB}{AB}$$

So we get

$$\frac{1}{2} = \frac{OB}{8}$$

By further calculation

$$OB = \frac{8}{2} = 4 \text{ cm}$$

$$BD = 2OB = 2 \times 4 = 8 \text{ cm}$$

$$\cos 30^\circ = \frac{AO}{AB}$$

Substituting the values

$$\frac{\sqrt{3}}{2} = \frac{AO}{8}$$

By further calculation

$$AO = 8 \frac{\sqrt{3}}{2} = 4\sqrt{3}$$

Here

$$AC = 4\sqrt{3} \times 2 = 8\sqrt{3} \text{ cm}$$

Therefore , the length of the diagonals of the rhombus are 8cm and $8\sqrt{3}$ cm .

23. in the right angled triangle ABC , $\angle C = 90^\circ$ and $\angle B = 60^\circ$. if AC = 6cm , find the lengths of the sides BC and AB

Solution

In the right – angled triangle ABC , $\angle C = 90^\circ$ and $\angle B = 60^\circ$

$$AC = 6 \text{ cm}$$

We know that

$$\tan B = \frac{AC}{BC}$$

substituting the values

$$\tan 60^\circ = \frac{6}{bc}$$

so we get

$$\sqrt{3} = \frac{6}{BC}$$

$$BC = \frac{6}{\sqrt{3}}$$

It can be written as

$$= \frac{6\sqrt{3}}{\sqrt{3}+\sqrt{3}}$$

$$= \frac{6\sqrt{3}}{3}$$

$$= 2\sqrt{3} \text{ cm}$$

$$\sin 60^\circ = \frac{AC}{AB}$$

Substituting the values

$$\frac{\sqrt{3}}{2} = \frac{6}{AB}$$

By further calculation

$$AB = \frac{6 \times 2}{\sqrt{3}}$$

So we get

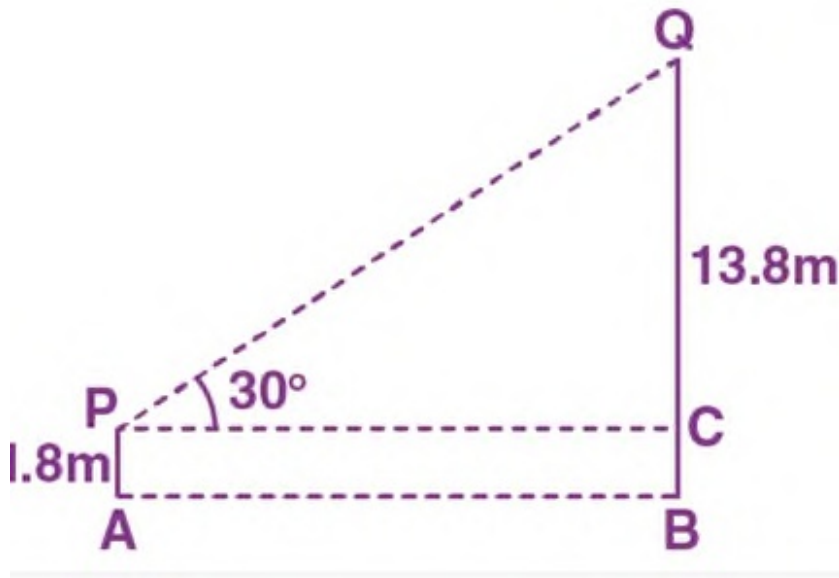
$$AB = \frac{12 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$= 12 \frac{\sqrt{3}}{3}$$

$$= 4\sqrt{3} \text{ cm}$$

Therefore the lengths of the sides $BC = 2\sqrt{3} \text{ cm}$ and $AB = 4\sqrt{3} \text{ cm}$.

24. in the adjoining figure , AP is a man of height 1.8 m and BQ is a building 13.8 m high. If the man sees the top of building by focusing his binoculars at an angle of 30° to the horizontal, find the distance of the man from the building.



Solution

It is given that

Height of man $AP = 1.8 \text{ m}$

Height of building $BQ = 13.8 \text{ m}$

Angle of elevation from the top of the building to the man $= 30^\circ$

Consider AB as the distance of the man from the building

$AB = x$ then $PC = x$

$QC = 13.8 - 1.8 = 12 \text{ m}$

In right $\triangle PQC$

$$\tan \theta = \frac{QC}{PC}$$

substituting the values

$$\tan 30^\circ = \frac{12}{x}$$

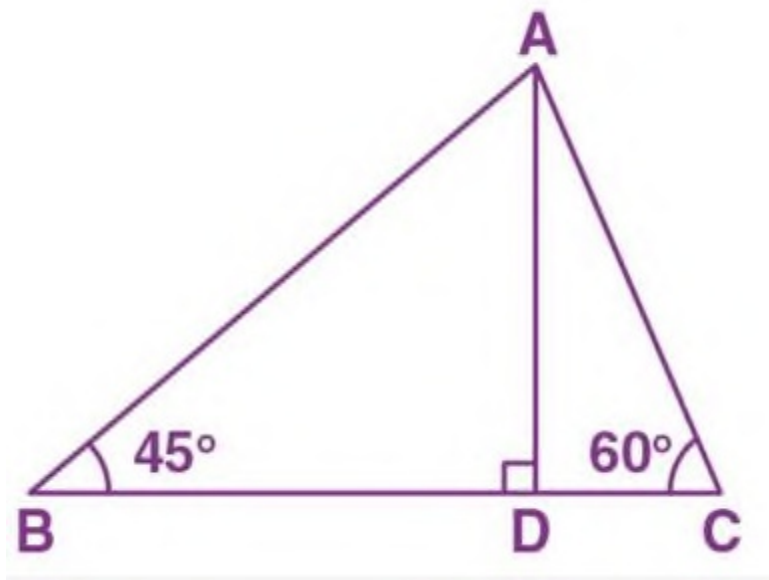
by further calculation

$$\frac{1}{\sqrt{3}} = \frac{12}{x}$$

$$x = 12\sqrt{3} \text{ m}$$

therefore , the distance of the man from the building is $12\sqrt{3}$ m.

25. In the adjoining figure, ABC is a triangle in which $\angle B = 45^\circ$ and $\angle C = 60^\circ$. if $AD \perp BC$ and $BC = 8$ m , find the length of the altitude AD .



Solution

In triangle ABC

$$\angle B = 45^\circ \text{ and } \angle C = 60^\circ$$

$$AD \perp BC \text{ and } BC = 8\text{m}$$

In right $\triangle ABC$

$$\tan 45^\circ = \frac{AD}{BD}$$

so we get

$$1 = \frac{AD}{BD}$$

$$AD = BD$$

In right $\triangle ACD$

$$\tan 60^\circ = \frac{AD}{DC}$$

so we get

$$\sqrt{3} = \frac{AD}{DC}$$

$$DC = \frac{AD}{\sqrt{3}}$$

Here

$$BD + DC = AD + \frac{AD}{\sqrt{3}}$$

Taking LCM

$$BC = \frac{\sqrt{3}AD + AD}{\sqrt{3}}$$

$$8 = \left[AD \frac{\sqrt{3}+1}{\sqrt{3}} \right]$$

By further calculation

$$AD = \frac{8\sqrt{3}}{\sqrt{3}+1}$$

It can be written as

$$= \frac{8\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

So we get

$$= \frac{8(3-\sqrt{3})}{3-1}$$

$$= \frac{8(3-\sqrt{3})}{2}$$

$$= 4(3 - \sqrt{3})\text{m}$$

Therefore , the length of the altitude AD is $(3 - \sqrt{3})$ m

Exercise 18.2

Without using trigonometric tables , evaluate the following (1 to 5):

1. (i) $\frac{\cos 18^\circ}{\sin 72^\circ}$

(ii) $\frac{\tan 41^\circ}{\cot 49^\circ}$

(iii) $\frac{\operatorname{cosec} 17^\circ 30'}{\sec 7^\circ 30'}$

Solution

(i) $\frac{\cos 18^\circ}{\sin 72^\circ}$

It can be written as

$$= \frac{\cos 18^\circ}{\cos 18^\circ}$$

$$= 1$$

(ii) $\frac{\tan 41^\circ}{\cot 49^\circ}$

It can be written as

$$= \frac{\tan 41^\circ}{\cot(90^\circ - 41^\circ)}$$

So we get

$$= \frac{\tan 41^\circ}{\tan 41^\circ}$$

$$= 1$$

$$(iii) \frac{\operatorname{cosec} 17^{\circ} 30'}{\sec 7^{\circ} 30'}$$

It can be written as

$$= \frac{\operatorname{cosec} 17^{\circ} 30'}{\sec(90^{\circ} - 17^{\circ} 30')}$$

So we get

$$= \frac{\operatorname{cosec} 17^{\circ} 30'}{\operatorname{cosec}(17^{\circ} 30')}$$

$$= 1$$

$$2. (i) \frac{\cot 40^{\circ}}{\tan 50^{\circ}} - \frac{1}{2} \left(\frac{\cos 35^{\circ}}{\sin 55^{\circ}} \right)$$

$$(ii) \left(\frac{\sin 49^{\circ}}{\cos 41^{\circ}} \right)^2 + \left(\frac{\cos 41^{\circ}}{\sin 49^{\circ}} \right)^2$$

$$(iii) \frac{\sin 72^{\circ}}{\cos 18^{\circ}} - \frac{\sec 32^{\circ}}{\operatorname{cosec} 58^{\circ}}$$

$$(iv) \frac{\cos 75^{\circ}}{\sin 15^{\circ}} + \frac{\sin 12^{\circ}}{\cos 78^{\circ}} - \frac{\cos 18^{\circ}}{\sin 72^{\circ}}$$

$$(v) \frac{\sin 25^{\circ}}{\sec 65^{\circ}} + \frac{\cos 25^{\circ}}{\operatorname{cosec} 65^{\circ}}$$

Solution

$$(i) \frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2} \left(\frac{\cos 35^\circ}{\sin 55^\circ} \right)$$

It can be written as

$$= \frac{\cot 40^\circ}{\tan(90^\circ - 40^\circ)} - \frac{1}{2} \left(\frac{\cos 35^\circ}{\sin 90^\circ - 35^\circ} \right)$$

By further calculation

$$= \frac{\cot 40^\circ}{\cot 40^\circ} - \frac{1}{2} \frac{\cos 35^\circ}{\cos 35^\circ}$$

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

$$(ii) \left(\frac{\sin 49^\circ}{\cos 41^\circ} \right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ} \right)^2$$

It can be written as

$$= \left(\frac{\sin 49^\circ}{\cos 90^\circ - 49^\circ} \right)^2 + \left(\frac{\cos 41^\circ}{\sin 90^\circ - 41^\circ} \right)^2$$

By further calculation

$$= \left(\frac{\sin 49^\circ}{\sin 49^\circ} \right)^2 + \left(\frac{\cos 41^\circ}{\cos 41^\circ} \right)^2$$

So we get

$$= 1^2 + 1^2$$

$$= 1 + 1$$

$$= 2$$

$$(iii) \frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$$

It can be written as

$$= \frac{\sin 72^\circ}{\cos(90^\circ - 72^\circ)} - \frac{\sec 32^\circ}{\operatorname{cosec}(90^\circ - 32^\circ)}$$

By further calculation

$$= \frac{\sin 72^\circ}{\sin(72^\circ)} - \frac{\sec 32^\circ}{\sec(32^\circ)}$$

$$= 1 - 1$$

$$= 0$$

$$(iv) \frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\cos 18^\circ}{\sin 72^\circ}$$

It can be written as

$$= \frac{\cos 75^\circ}{\sin(90^\circ - 75^\circ)} + \frac{\sin 12^\circ}{\sin 12^\circ} - \frac{\cos 18^\circ}{\cos 18^\circ}$$

So we get

$$= 1 + 1 - 1$$

$$=1$$

$$(v) \frac{\sin 25^\circ}{\sec 65^\circ} + \frac{\cos 25^\circ}{\operatorname{cosec} 65^\circ}$$

It can be written as

$$= (\sin 25^\circ \times \cos 65^\circ) + (\cos 25^\circ \times \sin 65^\circ)$$

By further calculation

$$= [\sin 25^\circ \times \cos(90^\circ - 25^\circ)] + [\cos 25^\circ \times \sin(90^\circ - 25^\circ)]$$

So we get

$$= (\sin 25^\circ \times \sin 25^\circ) + (\cos 25^\circ \times \cos 25^\circ)$$

$$= \sin^2 25^\circ + \cos^2 25^\circ$$

$$= 1$$

$$3. (i) \sin 62^\circ - \cos 28^\circ$$

$$(ii) \operatorname{cosec} 35^\circ - \sec 55^\circ$$

Solution

$$(i) \sin 62^\circ - \cos 28^\circ$$

It can be written as

$$= \sin(90^\circ - 28^\circ) - \cos 28^\circ$$

So we get

$$= \cos 28^\circ - \cos 28^\circ$$

$$= 0$$

$$(ii) \operatorname{cosec} 35^{\circ} - \sec 55^{\circ}$$

It can be written as

$$= \operatorname{cosec} 35^{\circ} - \sec(90^{\circ} - 35^{\circ})$$

$$= 0$$

$$4. (i) \frac{\cos^2 26^{\circ} + \cos 64^{\circ} \sin 26^{\circ} + \tan 36^{\circ}}{\cot 54^{\circ}}$$

$$(ii) \frac{\sec 17^{\circ}}{\operatorname{cosec} 73^{\circ}} + \frac{\tan 68^{\circ}}{\cot 22^{\circ} + \cos^2 44^{\circ} + \cos^2 46^{\circ}}$$

Solution

$$(i) \frac{\cos^2 26^{\circ} + \cos 64^{\circ} \sin 26^{\circ} + \tan 36^{\circ}}{\cot 54^{\circ}}$$

It can be written as

$$= \frac{\cos^2 26^{\circ} + \cos(90^{\circ} - 26^{\circ}) \sin 26^{\circ} + \tan 36^{\circ}}{\cot(90^{\circ} - 36^{\circ})}$$

We know that

$$\cos(90^{\circ} - \theta) = \sin \theta$$

$$\cot(90^{\circ} - \theta) = \tan \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

So we get

$$= \frac{\cos^2 26^\circ + \sin^2 26^\circ + \tan 36^\circ}{\tan 36^\circ}$$

$$= 1 + 1$$

$$= 2$$

$$(ii) \frac{\sec 17^\circ}{\operatorname{cosec} 73^\circ} + \frac{\tan 68^\circ}{\cot 22^\circ + \cos^2 44^\circ + \cos^2 46^\circ}$$

It can be written as

$$= \frac{\sec(90^\circ - 73^\circ)}{\operatorname{cosec} 73^\circ} + \frac{\tan(90^\circ - 22^\circ)}{\cot 22^\circ + \cos^2(90^\circ - 46^\circ) + \cos^2 46^\circ}$$

We know that $\sin^2 \theta + \cos^2 \theta = 1$

So we get

$$= 1 + 1 + 1$$

$$= 3$$

$$5. (i) \frac{\cos 65^\circ}{\sin 25^\circ} + \frac{\cos 32^\circ}{\sin 58^\circ} - \sin 28^\circ \sec 62^\circ + \operatorname{cosec}^2 30^\circ$$

$$(ii) \frac{\sec 29^\circ}{\operatorname{cosec} 61^\circ} + 2 \cot 8^\circ \cot 17^\circ \cot 45^\circ \cot 73^\circ \cot 82^\circ - 3(\sin^2 38^\circ + \sin^2 52^\circ)$$

Solution

$$(i) \frac{\cos 65^\circ}{\sin 25^\circ} + \frac{\cos 32^\circ}{\sin 58^\circ} - \sin 28^\circ \sec 62^\circ + \operatorname{cosec}^2 30^\circ$$

It can be written as

$$= \frac{\cos 65^\circ}{\sin(90^\circ - 65^\circ)} + \frac{\cos 32^\circ}{\sin(90^\circ - 32^\circ)} - \sin 28^\circ \sec(90^\circ - 28^\circ) + \operatorname{cosec}^2 30^\circ$$

By further calculation

$$= \frac{\cos 65^\circ}{\cos 65^\circ} + \frac{\cos 32^\circ}{\cos 32^\circ} - \sin 28^\circ \operatorname{cosec} 28^\circ + \operatorname{cosec}^2 30^\circ$$

We know that $\operatorname{cosec} 30^\circ = 2$

$$= 1 + 1 - 1 + 4$$

$$= 5$$

$$(ii) \frac{\sec 29^\circ}{\operatorname{cosec} 61^\circ} + 2 \cot 8^\circ \cot 17^\circ \cot 45^\circ \cot 73^\circ \cot 82^\circ - 3(\sin^2 38^\circ + \sin^2 52^\circ)$$

It can be written as

$$= \frac{\sec 29^\circ}{\operatorname{cosec}(90^\circ - 29^\circ)} + 2 \cot 8^\circ \cot 17^\circ \cot 45^\circ \cot(90^\circ - 17^\circ) \cot(90^\circ - 8^\circ) - 3(\sin^2 38^\circ + \sin^2(90^\circ - 38^\circ))$$

By Further calculation

$$= \frac{\sec 29^\circ}{\sec 29^\circ} + 2 \cot 8^\circ \cot 17^\circ \times 1 \times \tan 17^\circ \cot 45^\circ \cot(90^\circ - 17^\circ) \cot(90^\circ - 8^\circ) - 3(\sin^2 38^\circ + \sin^2(90^\circ - 38^\circ))$$

By further calculation

$$= \frac{\sec 29^\circ}{\sec 29^\circ} + 2 \cot 8^\circ \cot 17^\circ \times 1 \times \tan 17^\circ \tan 8^\circ - 3(\sin^2 38^\circ + \cos^2 38^\circ)$$

So we get

$$= 1 + 2 \cot 8^\circ \tan 8^\circ \cot 17^\circ \tan 17^\circ \times 1 - 3 \times 1$$

We know that

$$\operatorname{Cosec}(90^\circ - \theta) = \sec \theta$$

$$\operatorname{Cot}(90^\circ - \theta) = \tan \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

Here

$$= 1 + 2 \times 1 \times 1 \times 1 - 3$$

$$= 1 + 2 - 3$$

$$= 0$$

6. express each of the following in terms of trigonometric ratios of angle between 0° to 45°

(i) $\tan 81^\circ + \cos 72^\circ$

(ii) $\cot 49^\circ + \operatorname{cosec} 87^\circ$

Solution

(i) $\tan 81^\circ + \cos 72^\circ$

It can be written as

$$= \tan(90^\circ - 9^\circ) + \cos(90^\circ - 18^\circ)$$

So we get

$$= \cot 9^\circ + \sin 18^\circ$$

(ii) $\cot 49^\circ + \operatorname{cosec} 87^\circ$

It can be written as

$$= \cot(90^\circ - 41^\circ) + \operatorname{cosec}(90^\circ - 3^\circ)$$

So we get

$$= \tan 41^\circ + \sec 3^\circ$$

Without using trigonometric table , prove that(7 to 11) :

7. (i) $\sin^2 28^\circ - \cos^2 62^\circ = 0$

(ii) $\cos^2 25^\circ + \cos^2 65^\circ = 1$

(iii) $\operatorname{cosec}^2 67^\circ - \tan^2 23^\circ = 1$

(iv) $\sec^2 22^\circ - \cot^2 68^\circ = 1$

Solution

$$(i) \sin^2 28^\circ - \cos^2 62^\circ = 0$$

Consider

$$\text{LHS} = \sin^2 28^\circ - \cos^2 62^\circ$$

It can be written as

$$= \sin^2 28^\circ - \cos^2 (90^\circ - 28^\circ)$$

So we get

$$= \sin^2 28^\circ - \sin^2 28^\circ$$

$$= 0$$

$$= \text{RHS}$$

$$(ii) \cos^2 25^\circ + \cos^2 65^\circ = 1$$

Consider

$$\text{LHS} = \cos^2 25^\circ + \cos^2 65^\circ$$

It can be written as

$$= \cos^2 25^\circ + \cos^2 (90^\circ - 25^\circ)$$

We know that $\sin^2 \theta + \cos^2 \theta = 1$

So we get

$$= \cos^2 25^\circ + \sin^2 25^\circ$$

$$= 1$$

$$(iii) \operatorname{cosec}^2 67^\circ - \tan^2 23^\circ = 1$$

Consider

$$\text{LHS} = \operatorname{cosec}^2 67^\circ - \tan^2 23^\circ$$

It can be written as

$$= \operatorname{cosec}^2 67^\circ - \tan^2 (90^\circ - 67^\circ)$$

$$\text{We know that } \operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

So we get

$$= \operatorname{cosec}^2 67^\circ - \cot^2 67^\circ$$

$$= 1$$

$$(iv) \sec^2 22^\circ - \cot^2 68^\circ = 1$$

Consider

$$\text{LHS} = \sec^2 22^\circ - \cot^2 68^\circ$$

It can be written as

$$= \sec^2 22^\circ - \cot^2 (90^\circ - 22^\circ)$$

$$\text{We know that } \sec^2 \theta - \tan^2 \theta = 1$$

So we get

$$= \sec^2 22^\circ - \tan^2 22^\circ$$

$$= 1$$

8. (i) $\sin 63^\circ \cos 27^\circ + \cos 63^\circ \sin 27^\circ = 1$

(ii) $\sec 31^\circ \sin 59^\circ + \cos 31^\circ \operatorname{cosec} 59^\circ = 2$

Solution

(i) $\sin 63^\circ \cos 27^\circ + \cos 63^\circ \sin 27^\circ = 1$

Consider

$$\text{LHS} = \sin 63^\circ \cos 27^\circ + \cos 63^\circ \sin 27^\circ$$

It can be written as

$$= \sin 63^\circ \cos(90^\circ - 63^\circ) + \cos 63^\circ \sin(90^\circ - 63^\circ)$$

$$= \sin 63^\circ \sin 63^\circ + \cos 63^\circ \cos 63^\circ$$

$$\text{We know that } \sin^2 \theta + \cos^2 \theta = 1$$

So we get

$$= \sin^2 63^\circ + \cos^2 63^\circ$$

$$= 1$$

(ii) $\sec 31^\circ \sin 59^\circ + \cos 31^\circ \operatorname{cosec} 59^\circ = 2$

Consider

$$\text{LHS} = \sec 31^\circ \sin 59^\circ + \cos 31^\circ \operatorname{cosec} 59^\circ$$

It can be written as

$$= \frac{1}{\cos 31^\circ} \times \sin 59^\circ + \cos 31^\circ \times \frac{1}{\sin 59^\circ}$$

By further calculation

$$= \frac{\sin 59^\circ}{\cos(90^\circ - 59^\circ)} + \frac{\cos 31^\circ}{\sin(90^\circ - 31^\circ)}$$

So we get

$$= \frac{\sin 59^\circ}{\sin 59^\circ} + \frac{\cos 31^\circ}{\cos 31^\circ}$$

$$= 1 + 1$$

$$= 2$$

$$= \text{RHS}$$

$$\mathbf{9. (i) \sec 70^\circ \sin 20^\circ - \cos 20^\circ \operatorname{cosec} 70^\circ = 0}$$

$$\mathbf{(ii) \sin^2 20^\circ + \sin^2 70^\circ - \tan^2 45^\circ = 0}$$

Solution

$$(i) \sec 70^\circ \sin 20^\circ - \cos 20^\circ \operatorname{cosec} 70^\circ = 0$$

Consider

$$\text{LHS} = \sec 70^\circ \sin 20^\circ - \cos 20^\circ \operatorname{cosec} 70^\circ$$

By further simplification

$$= \frac{\sin 20^\circ}{\cos(90^\circ - 20^\circ)} - \frac{\cos 20^\circ}{\sin(90^\circ - 20^\circ)}$$

So we get

$$= \frac{\sin 20^\circ}{\sin 20^\circ} - \frac{\cos 20^\circ}{\cos 20^\circ}$$

$$= 1 - 1$$

$$= 0$$

= RHS

$$(ii) \sin^2 20^\circ + \sin^2 70^\circ - \tan^2 45^\circ = 0$$

Consider

$$\text{LHS} = \sin^2 20^\circ + \sin^2 70^\circ - \tan^2 45^\circ$$

It can be written as

$$= \sin^2 20^\circ + \sin^2 (90^\circ - 20^\circ) - \tan^2 45^\circ$$

By further calculation

$$= \sin^2 20^\circ + \cos^2 20^\circ - \tan^2 45^\circ$$

We know that $\sin^2 \theta + \cos^2 \theta = 1$ and $\tan 45^\circ = 1$

So we get

$$= 1 - 1$$

$$= 0$$

= RHS

$$10. (i) \frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2 = 0$$

$$(ii) \frac{\sin 50^\circ}{\cos 40^\circ} + \frac{\operatorname{cosec} 40^\circ}{\sec 50^\circ} - 4 \cos 50^\circ \operatorname{cosec} 40^\circ + 2 = 0$$

Solution

$$(i) \frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2 = 0$$

Consider

$$\text{LHS} = \frac{\cot 54^\circ}{\tan(90^\circ - 54^\circ)} + \frac{\tan 20^\circ}{\cot(90^\circ - 20^\circ)} - 2$$

By further calculation

$$\frac{\cot 54^\circ}{\cot 54^\circ} + \frac{\tan 20^\circ}{\tan 20^\circ} - 2$$

So we get

$$= 1 + 1 - 2$$

$$= 0$$

$$= \text{RHS}$$

$$(ii) \frac{\sin 50^\circ}{\cos 40^\circ} + \frac{\operatorname{cosec} 40^\circ}{\sec 50^\circ} - 4 \cos 50^\circ \operatorname{cosec} 40^\circ + 2 = 0$$

Consider

$$\text{LHS} = \frac{\sin 50^\circ}{\cos 40^\circ} + \frac{\operatorname{cosec} 40^\circ}{\sec 50^\circ} - 4 \cos 50^\circ \operatorname{cosec} 40^\circ + 2$$

It can be written as

$$= \frac{\sin 50^\circ}{\cos(90^\circ - 50^\circ)} + \frac{\operatorname{cosec} 40^\circ}{\sec(90^\circ - 40^\circ)} - 4 \cos 50^\circ \operatorname{cosec}(90^\circ - 50^\circ) + 2$$

By further calculation

$$= \frac{\sin 50^\circ}{\sin 50^\circ} + \frac{\operatorname{cosec} 40^\circ}{\operatorname{cosec} 40^\circ} - \cos 50^\circ \sec 50^\circ + 2$$

So we get

$$= \frac{1+1-4 \cos 50^\circ}{\cos 50^\circ+2}$$

$$= 1 + 1 - 4 + 2$$

$$= 4 - 4$$

$$= 0$$

= RHS

$$11. (i) \frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ = 0$$

$$(ii) \frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ = 2$$

Solution

$$(i) \frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ = 0$$

Consider

$$\text{LHS} = \frac{\cos 70^\circ}{\sin(90^\circ - 70^\circ)} + \frac{\cos 59^\circ}{\sin(90^\circ - 59^\circ)} - 8\sin^2 30^\circ = 0$$

$$\text{We know that } \sin 30^\circ = \frac{1}{2}$$

$$= \frac{\cos 70^\circ}{\cos 70^\circ} + \frac{\cos 59^\circ}{\cos 59^\circ} - 8\left(\frac{1}{2}\right)^2$$

By further calculation

$$= 1 + 1 - 8 \times \frac{1}{4}$$

So we get

$$= 2 - 2$$

$$= 0$$

= RHS

$$(ii) \frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ = 2$$

Consider

$$\text{LHS} = \frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ$$

It can written as

$$= \frac{\cos 80^\circ}{\sin(90^\circ - 80^\circ)} + \frac{\cos 59^\circ}{\sin 31^\circ}$$

By further simplification

$$= \frac{\cos 80^\circ}{\cos 80^\circ} + \frac{\cos 59^\circ}{\sin(90^\circ - 59^\circ)}$$

So we get

$$= 1 + \frac{\cos 59^\circ}{\cos 59^\circ}$$

$$= 1 + 1$$

$$= 2$$

$$= \text{RHS}$$

12. without using Trigonometrical tables , evaluate :

$$(i) 2 \left(\frac{\tan 35^\circ}{\cot 55^\circ} \right)^2 + \left(\frac{\cot 55^\circ}{\tan 35^\circ} \right) - 3 \left(\frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ} \right)$$

$$(ii) \frac{\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ}{\operatorname{cosec}^2 10^\circ - \tan^2 80^\circ}$$

$$(iii) \sin^2 34^\circ + \sin^2 56^\circ + 2 \tan 18^\circ \tan 72^\circ - \cot^2 30^\circ .$$

Solution

$$(i) 2 \left(\frac{\tan 35^\circ}{\cot 55^\circ} \right)^2 + \left(\frac{\cot 55^\circ}{\tan 35^\circ} \right) - 3 \left(\frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ} \right)$$

It can be written as

$$= 2 \left(\frac{\tan(90^\circ - 55^\circ)}{\cot 55^\circ} \right)^2 + \left(\frac{\cot(90^\circ - 35^\circ)}{\tan 35^\circ} \right) - 3 \left(\frac{\sec(90^\circ - 50^\circ)}{\operatorname{cosec} 50^\circ} \right)$$

By further calculation

$$= 2 \left(\frac{\cot 55^\circ}{\cot 55^\circ} \right)^2 + \left(\frac{\tan 35^\circ}{\tan 35^\circ} \right) - 3 \left(\frac{\operatorname{cosec} 50^\circ}{\operatorname{cosec} 50^\circ} \right)$$

So we get

$$= 2 + 1 - 3$$

$$= 0$$

$$(ii) \frac{\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ}{\operatorname{cosec}^2 10^\circ - \tan^2 80^\circ}$$

It can be written as

$$= \frac{\sin 35^\circ \cos(90^\circ - 35^\circ) + \cos 35^\circ \sin(90^\circ - 35^\circ)}{\operatorname{cosec}^2 10^\circ - \tan^2(90^\circ - 10^\circ)}$$

By further calculation

$$= \frac{\sin 35^\circ \cdot \sin 35^\circ + \cos 35^\circ \cdot \cos 35^\circ}{\operatorname{cosec}^2 10^\circ - \cot^2 10^\circ}$$

So we get

$$= \frac{\sin^2 35^\circ + \cos^2 35^\circ}{\operatorname{cosec}^2 10^\circ - \cot^2 10^\circ}$$

We know that $\sin^2\theta + \cos^2\theta = 1$ and $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$$= \frac{1}{1}$$

$$= 1$$

$$(iii) \sin^2 34^\circ + \sin^2 56^\circ + 2\tan 18^\circ \tan 72^\circ - \cot^2 30^\circ.$$

It can be written as

$$= \sin^2 34^\circ + [\sin(90^\circ - 34^\circ)]^2 + 2\tan 18^\circ \tan(90^\circ - 18^\circ) - \cot^2 30^\circ$$

By further simplification

$$= \sin^2 34^\circ + \cos^2 34^\circ + 2\tan 18^\circ \cot 18^\circ - (\sqrt{3})^2$$

So we get

$$= 1 + 2\tan 18^\circ \times \frac{1}{\tan 18^\circ} - 3$$

$$= 1 + 2 - 3$$

$$= 0$$

13 . prove the following

$$(i) \frac{\cos\theta}{\sin(90^\circ - \theta)} + \frac{\sin\theta}{\cos(90^\circ - \theta)} = 2$$

$$(ii) \cos\theta \sin(90^\circ - \theta) + \sin\theta \cos(90^\circ - \theta) = 1$$

$$(iii) \frac{\tan\theta}{\tan(90^\circ - \theta)} + \frac{\sin(90^\circ - \theta)}{\cos\theta} = \sec^2\theta$$

Solution

$$(i) \frac{\cos \theta}{\sin(90^\circ - \theta)} + \frac{\sin \theta}{\cos(90^\circ - \theta)} = 2$$

We know that

$$\text{LHS} = \frac{\cos \theta}{\sin(90^\circ - \theta)} + \frac{\sin \theta}{\cos(90^\circ - \theta)}$$

So we get

$$= \frac{\cos \theta}{\cos \theta} + \frac{\sin \theta}{\sin \theta}$$

$$= 1 + 1$$

$$= 2$$

$$= \text{RHS}$$

$$(ii) \cos \theta \sin(90^\circ - \theta) + \sin \theta \cos(90^\circ - \theta) = 1$$

Consider

$$\text{LHS} = \cos \theta \sin(90^\circ - \theta) + \sin \theta \cos(90^\circ - \theta)$$

It can be written as

$$= \cos \theta \cdot \cos \theta + \sin \theta \cdot \sin \theta$$

So we get

$$= \cos^2 \theta + \sin^2 \theta$$

$$= 1$$

$$= \text{RHS}$$

$$(iii) \frac{\tan \theta}{\tan(90^\circ - \theta)} + \frac{\sin(90^\circ - \theta)}{\cos \theta} = \sec^2 \theta$$

Consider

$$\text{LHS} = \frac{\tan \theta}{\tan(90^\circ - \theta)} + \frac{\sin(90^\circ - \theta)}{\cos \theta}$$

By further calculation

$$= \frac{\tan \theta}{\cot \theta} + \frac{\cos \theta}{\cos \theta}$$

So we get

$$= \tan \theta \times \tan \theta + 1$$

$$= \tan^2 \theta + 1$$

$$= \sec^2 \theta$$

$$= \text{RHS}$$

14. prove the following

$$(i) \frac{\cos(90^\circ - A) \sin(90^\circ - A)}{\tan(90^\circ - A)} = 1 - \cos^2 A$$

$$(ii) \frac{\sin(90^\circ - A)}{\csc(90^\circ - A)} + \frac{\cos(90^\circ - A)}{\sec(90^\circ - A)} = 1$$

Solution

$$(i) \frac{\cos(90^\circ - A) \sin(90^\circ - A)}{\tan(90^\circ - A)} = 1 - \cos^2 A$$

Consider

$$\text{LHS} = \frac{\cos(90^\circ - A) \sin(90^\circ - A)}{\tan(90^\circ - A)}$$

It can be written as

$$= \frac{\sin A \cos A}{\cot A}$$

So we get

$$= \frac{(\sin A \cos A \times \sin A)}{\cos A}$$

$$= \sin^2 A$$

$$= 1 - \cos^2 A$$

$$= \text{RHS}$$

$$(ii) \frac{\sin(90^\circ - A)}{\operatorname{cosec}(90^\circ - A)} + \frac{\cos(90^\circ - A)}{\sec(90^\circ - A)} = 1$$

Consider

$$\text{LHS} = \frac{\sin(90^\circ - A)}{\operatorname{cosec}(90^\circ - A)} + \frac{\cos(90^\circ - A)}{\sec(90^\circ - A)}$$

It can be written as

$$= \frac{\cos A}{\sec A} + \frac{\sin A}{\operatorname{cosec} A}$$

So we get

$$= \cos A \times \cos A + \sin A \times \sin A$$

$$= \cos^2 A + \sin^2 A$$

$$= 1$$

$$= \text{RHS}$$

15. simply the following

$$(i) \frac{\cos \theta}{\sin(90^\circ - \theta)} + \frac{\cos(90^\circ - \theta)}{\sec(90^\circ - \theta)} - 3\tan^2 30^\circ$$

$$(ii) \frac{\operatorname{cosec}(90^\circ - \theta) \sin(90^\circ - \theta) \cot(90^\circ - \theta)}{\cos(90^\circ - \theta) \sec(90^\circ - \theta) \tan \theta} + \frac{\cot \theta}{\tan(90^\circ - \theta)}$$

Solution :

$$(i) \frac{\cos \theta}{\sin(90^\circ - \theta)} + \frac{\cos(90^\circ - \theta)}{\sec(90^\circ - \theta)} - 3\tan^2 30^\circ$$

It can be written as

$$= \frac{\cos \theta}{\cos \theta} + \frac{\sin \theta}{\operatorname{cosec} \theta} - 3\tan^2 30^\circ$$

By further calculation

$$= 1 + \sin \theta \times \sin \theta - 3 \left(\frac{1}{\sqrt{3}} \right)^2$$

So we get

$$= \sin^2 \theta + 1 - 3 \times \frac{1}{3}$$

$$= \sin^2 \theta + 1 - 1$$

$$= \sin^2 \theta$$

$$(ii) \frac{\operatorname{cosec}(90^\circ - \theta) \sin(90^\circ - \theta) \cot(90^\circ - \theta)}{\cos(90^\circ - \theta) \sec(90^\circ - \theta) \tan \theta} + \frac{\cot \theta}{\tan(90^\circ - \theta)}$$

It can be written as

$$= \frac{\sec \theta \cos \theta \tan \theta}{\sin \theta \operatorname{cosec} \theta \tan \theta} + \frac{\cot \theta}{\cot \theta}$$

So we get

$$= \frac{\sec \theta \cos \theta}{\sin \theta \operatorname{cosec} \theta + 1}$$

$$= \frac{1}{1} + 1$$

$$= 1 + 1$$

$$= 2$$

16. show that

$$\frac{\cos^2(45^\circ + \theta) + \cos^2(45^\circ - \theta)}{\tan(60^\circ + \theta) \tan(30^\circ - \theta)} = 1$$

Solution

$$\text{LHS} = \frac{\cos^2(45^\circ + \theta) + \cos^2(45^\circ - \theta)}{\tan(60^\circ + \theta) \tan(30^\circ - \theta)}$$

It can be written as

$$= \frac{\cos^2(45^\circ + \theta) + \cos^2[90^\circ - (45^\circ + \theta)]}{\tan(60^\circ + \theta) \tan[90^\circ - (60^\circ + \theta)]}$$

By further calculation

$$= \frac{\cos^2(45^\circ + \theta) + \sin^2(45^\circ + \theta)}{\tan(60^\circ + \theta) \cot(60^\circ + \theta)}$$

We know that $\cos(90^\circ - \theta) = \sin \theta$, $\tan(90^\circ - \theta) = \cot \theta$ and $\tan \theta$

$$\cot \theta = 1$$

So we get

$$= \frac{1}{1}$$

$$= 1$$

$$= \text{RHS}$$

17. find the value of A if

(i) $\sin 3A = \cos (A - 6^\circ)$, where $3A$ and $A - 6^\circ$ are acute angles

(ii) $\tan 2A = \cot(A - 18^\circ)$, where $2A$ and $A - 18^\circ$ are acute angles

(iii) if $\sec 2A = \operatorname{cosec} (A - 27^\circ)$ where $2A$ is an acute angle , find the measure of $\angle A$.

Solution

(i) $\sin 3A = \cos (A - 6^\circ)$, where $3A$ and $A - 6^\circ$ are acute angles

It is given that

$$\sin 3A = \cos(A - 6^\circ)$$

We know that $\cos (90^\circ - \theta) = \sin \theta$

$$\cos (90^\circ - 3A) = \cos(A - 6^\circ)$$

By comparing both

$$90^\circ - 3A = A - 6^\circ$$

By further calculation

$$90^\circ + 6^\circ = A + 3A$$

$$96^\circ = 4A$$

So we get

$$A = \frac{96^\circ}{4} = 24^\circ$$

Hence the value of A is 24° .

(ii) $\tan 2A = \cot(A - 18^\circ)$,

We know that $\cot(90^\circ - \theta) = \tan \theta$

$$\cot(90^\circ - 2A) = \cot(A - 18^\circ)$$

By comparing both

$$90^\circ - 2A = A - 18^\circ$$

By further calculation

$$90^\circ + 18^\circ = A + 2A$$

So we get

$$3A = 108^\circ$$

$$A = \frac{108^\circ}{3} = 36^\circ$$

Hence, the value of A is 36°

(iii) if $\sec 2A = \operatorname{cosec}(A - 27^\circ)$

We know that $\operatorname{cosec}(90^\circ - \theta) = \sec \theta$

$$\operatorname{Cosec}(90^\circ - 2A) = \cos(A - 27^\circ)$$

By comparing both

$$90^\circ - 2A = A - 27^\circ$$

By further calculation

$$90^\circ + 27^\circ = A + 2A$$

So we get

$$3A = 117^\circ$$

$$A = \frac{117^\circ}{3} = 39^\circ$$

Hence the value of $A = 39^\circ$.

18 . find the value of $\theta (0^\circ < \theta < 90^\circ)$ if

(i) $\cos 63^\circ \sec(90^\circ - \theta) = 1$

(ii) $\tan 35^\circ \cot(90^\circ - \theta) = 1$

Solution

(i) $\cos 63^\circ \sec(90^\circ - \theta) = 1$

It can be written as

$$\cos 63^\circ = \frac{1}{\sec(90^\circ - \theta)} = 1$$

We know that $\frac{1}{\sec \theta} = \cos \theta$

$$\cos 63^\circ = \cos (90^\circ - \theta)$$

By comparing both

$$90^\circ - \theta = 63^\circ$$

By further calculation

$$\theta = 90^\circ - 63^\circ = 27^\circ$$

(ii) $\tan 35^\circ \cot(90^\circ - \theta) = 1$

It can be written as

$$\tan 35^\circ = \frac{1}{\cot(90^\circ - \theta)}$$

we know that $\frac{1}{\cot \theta} = \cos \theta$

$$\tan 35^\circ = \tan(90^\circ - \theta)$$

by comparing both

$$35^\circ = 90^\circ - \theta$$

By further calculation

$$\theta = 90^\circ - 35^\circ = 55^\circ$$

19. If A,B and C are the interior angles of a ΔABC , show that

$$(i) \cos \frac{A+B}{2} = \sin \frac{C}{2}$$

$$(ii) \tan \frac{C+A}{2} = \cot \frac{B}{2}$$

Solution :

A,B and C are the interior angles of a ΔABC

It can be written as

$$\angle A + \angle B + \angle C = 180^\circ$$

Dividing both sides by 2

$$\frac{\angle A + \angle B + \angle C}{2} = \frac{180^\circ}{2}$$

$$\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = 90^\circ$$

$$(i) \cos \frac{A+B}{2} = \sin \frac{C}{2}$$

We can write it as

$$\frac{A+B}{2} = 90^\circ - \frac{C}{2}$$

We know that

$$\cos \left(90^\circ - \frac{C}{2} \right) = \sin \frac{C}{2}$$

$$\text{Here } \cos (90^\circ -) = \sin \theta$$

$$\sin \frac{C}{2} = \sin \frac{C}{2}$$

$$(ii) \tan \frac{C+A}{2} = \cot \frac{B}{2}$$

$$\text{We know that } \frac{A+C}{2} = 90^\circ - \frac{B}{2}$$

$$= \tan \left(90^\circ - \frac{B}{2} \right)$$

So we get

$$= \cot \frac{B}{2}$$

$$= \text{RHS}$$

Chapter test

1. find the values of :

(i) $\sin^2 60^\circ - \cos^2 45^\circ + 3\tan^2 30^\circ$

(ii) $\frac{2\cos^2 45^\circ + 3\tan^2 30^\circ}{\sqrt{3}\cos 30^\circ + \sin 30^\circ}$

(iii) $\sec 30^\circ \tan 60^\circ + \sin 45^\circ \operatorname{cosec} 45^\circ + \cos 30^\circ \cot 60^\circ$

Solution

(i) $\sin^2 60^\circ - \cos^2 45^\circ + 3\tan^2 30^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 + 3\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= \frac{3}{4} - \frac{1}{2} + 3 \times \frac{1}{3} = \frac{3}{4} - \frac{1}{2} + \frac{1}{1}$$

$$= \frac{3-2+4}{4} = \frac{7-2}{4} = \frac{5}{4} = 1\frac{1}{4}$$

Therefore, $\sin^2 60^\circ - \cos^2 45^\circ + 3\tan^2 30^\circ = 1\frac{1}{4}$

$$(ii) \frac{2\cos^2 45^\circ + 3\tan^2 30^\circ}{\sqrt{3}\cos 30^\circ + \sin 30^\circ}$$

$$= \frac{2\left(\frac{1}{\sqrt{2}}\right)^2 + 3 \times \left(\frac{1}{\sqrt{3}}\right)^2}{\sqrt{3} \times \left(\sqrt{\frac{3}{2}}\right) + \frac{1}{2}}$$

$$= \frac{2 \times \frac{1}{2} + 3 \times \frac{1}{3}}{\frac{\sqrt{3}}{2} + \frac{1}{2}} = \frac{1+1}{\frac{\sqrt{3}+1}{2}} = \frac{2}{\frac{\sqrt{3}+1}{2}} = \frac{2}{1} = 1$$

Hence ,

$$\frac{2\cos^2 45^\circ + 3\tan^2 30^\circ}{\sqrt{3}\cos 30^\circ + \sin 30^\circ} = 1$$

$$(iii) \sec 30^\circ \tan 60^\circ + \sin 45^\circ \operatorname{cosec} 45^\circ + \cos 30^\circ \cot 60^\circ$$

$$= \frac{2}{\sqrt{3}} \times \sqrt{3} + \frac{1}{\sqrt{2}} \times \sqrt{2} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{2}{1} + \frac{1}{1} + \frac{1}{2}$$

$$= 2 + 1 + \frac{1}{2} = 3 + \frac{1}{2} = \frac{6+1}{2}$$

$$= \frac{7}{2} = 3\frac{1}{2}$$

$$\text{Thus , } \sec 30^\circ \tan 60^\circ + \sin 45^\circ \operatorname{cosec} 45^\circ + \cos 30^\circ \cot 60^\circ = 3\frac{1}{2}$$

2. taking $A = 30^\circ$, verify that

(i) $\cos^4 A - \sin^4 A = \cos 2A$

(ii) $4\cos A \cos (60^\circ - A) \cos (60^\circ + A) = \cos 3A$.

Solution

(i) $\cos^4 A - \sin^4 A = \cos 2A$

Let's take $A = 30^\circ$

So, we have

$$\text{LHS} = \cos^4 A - \sin^4 A = \cos^4 30^\circ - \sin^4 30^\circ$$

$$= \left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{1}{2}\right)^4$$

$$= \frac{\sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3}}{2 \times 2 \times 2 \times 2} - \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{9}{16} - \frac{1}{16}$$

$$= \frac{9-1}{16} = \frac{8}{16} = \frac{1}{2}$$

Now,

$$\text{R.H.S} = \cos 2A = \cos 2(30^\circ) = \frac{1}{2}$$

Therefore, $\text{LHS} = \text{RHS}$ hence verified

$$(ii) 4 \cos A \cos(60^\circ - A) \cos(60^\circ + A) = \cos 3A$$

Let's take $A = 30^\circ$

$$\text{LHS} = 4 \cos A \cos(60^\circ - A) \cos(60^\circ + A)$$

$$= 4 \cos 30^\circ \cos(60^\circ - 30^\circ) \cos(60^\circ + 30^\circ)$$

$$= 4 \cos 30^\circ \cos 30^\circ \cos 90^\circ$$

$$= 4 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \times 0$$

$$= 0$$

Now

$$\text{LHS} = \cos 3A$$

$$\text{R.H.S} = \cos 3A$$

$$= \cos(3 \times 30^\circ) = \cos 90^\circ = 0$$

Hence, L.H.S = R.H.S hence verified

$$3. \text{ if } A = 45^\circ \text{ and } B = 30^\circ, \text{ verify that } \frac{\sin A}{(\cos A + \sin A + \sin B)} = \frac{2}{3}$$

Solution

Taking

$$\text{L.H.S} \frac{\sin A}{(\cos A + \sin A + \sin B)}$$

$$= \frac{\sin 45^\circ}{(\cos 45^\circ + \sin 45^\circ + \sin 30^\circ)}$$

$$= \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{4}}$$

$$= \frac{\frac{\sqrt{2}}{2}}{\frac{2\sqrt{2} + \sqrt{2}}{4}} = \frac{\frac{\sqrt{2}}{2}}{\frac{3\sqrt{2}}{4}}$$

$$= \frac{\sqrt{2}}{2} \times \frac{4}{3\sqrt{2}} = \frac{2}{3} = R.H.S$$

Hence verified

4. taking $A = 60^\circ$ and $B = 30^\circ$, verify that

$$(i) \frac{\sin a + b}{\cos A \cos B} = \tan a + \tan b$$

$$(ii) \frac{\sin(A-B)}{\sin A \sin B} = \cot B - \cot A$$

Solution

(i) here $A = 60^\circ$ and $B = 30^\circ$

$$L.H.S = \frac{\sin a + b}{\cos A \cos B} = \frac{\sin(60^\circ + 30^\circ)}{\cos 60^\circ \cos 30^\circ}$$

$$= \frac{\sin 90^\circ}{\cos 60^\circ \cos 30^\circ} = \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{\frac{\sqrt{3}}{4}} = \frac{4}{\sqrt{3}}$$

Now,

$$\text{R.H.S} = \tan A + \tan B$$

$$= \tan 60^\circ + \tan 30^\circ$$

$$= \sqrt{3} + \frac{1}{\sqrt{3}}$$

$$= \frac{3+1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$$

$$\therefore \text{L.H.S} = \text{R.H.S}$$

$$\text{(ii) } A = 60^\circ, B = 30^\circ$$

$$\text{L.H.S} = \frac{\sin(A-B)}{\sin A \sin B} = \frac{\sin(60^\circ-30^\circ)}{\sin 60^\circ \sin 30^\circ}$$

$$= \frac{\sin 30^\circ}{\sin 60^\circ \sin 30^\circ} = \frac{1}{\sin 60^\circ} = \frac{\frac{1}{\sqrt{3}}}{\frac{1}{2}}$$

$$= \frac{2}{\sqrt{3}}$$

$$\text{R.H.S} = \cot B - \cot A$$

$$\cot 30^\circ - \cot 60^\circ$$

$$= \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{3-1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\text{L.H.S} = \text{R.H.S}$$

5. if $\sqrt{2} \tan 2\theta = \sqrt{6}$ and $\theta^\circ < 2\theta < 90^\circ$, find the value of $\sin \theta + \sqrt{3} \cos \theta - 2 \tan^2 \theta$.

Solution

Given ,

$$\sqrt{2} \tan 2\theta = \sqrt{6}$$

$$\tan 2\theta = \frac{\sqrt{6}}{\sqrt{2}}$$

$$= \sqrt{3}$$

$$= \tan 60^\circ$$

$$= 2\theta = 60^\circ$$

$$= \theta = 30^\circ$$

Now ,

$$\sin \theta + \sqrt{3} \cos \theta - 2 \tan^2 \theta$$

$$= \sin 30^\circ + \sqrt{3} \cos 30^\circ - 2 \tan^2 30^\circ$$

$$= \frac{1}{2} + \sqrt{3} \times \frac{\sqrt{3}}{2} - 2 \left(\frac{1}{\sqrt{3}} \right)^2$$

$$= \frac{1}{2} + \frac{3}{2} - \frac{2}{3}$$

$$\begin{aligned}
&= \frac{4}{2} - \frac{2}{3} \\
&= \frac{12-4}{6} \\
&= \frac{8}{6} \\
&= \frac{4}{3}
\end{aligned}$$

6. if 3θ is an acute angle , solve the following equation for θ :

(i) (cosec 3θ - 2) (cot 2θ -1) = 0

(ii) (tan θ -1) (cosec 3θ -1) = 0

Solution

(i) (cosec 3θ - 2) (cot 2θ -1) = 0

Now , either

$$\text{Cosec } 3\theta - 2 \text{ or } \cot 2\theta - 1 = 0$$

$$= \text{cosec } 3\theta = 2 \text{ or } \cot 2\theta = 1$$

So ,

$$\text{Cosec } 3\theta = \text{cosec } 3\theta^\circ \text{ or } \cot 2\theta = \cot 45^\circ$$

$$= 3\theta = 30^\circ \text{ or } 2\theta = 45^\circ$$

$$\text{Thus, } \theta = 30^\circ \text{ or } 45^\circ$$

$$(ii) (\tan \theta - 1)(\operatorname{cosec} 3\theta - 1) = 0$$

Now, either

$$\tan \theta - 1 = 0 \text{ or } \operatorname{cosec} 3\theta - 1 = 0$$

$$= \tan \theta = 1 \text{ or } \operatorname{cosec} 3\theta = 1$$

So ,

$$\tan \theta = \tan 45^\circ \text{ or } \operatorname{cosec} 3\theta = \operatorname{cosec} 90^\circ$$

$$= \theta = 45^\circ \text{ or } 3\theta = 90^\circ \text{ i.e. } \theta = 30^\circ$$

Thus $\theta = 45^\circ$ or 30° .

7. if $\tan (A+B) = \sqrt{3}$ and $\tan (A- B)=1$ and $A, B (B < A)$ are acute angles, find the values of A and B.

Solution

$$\text{Given , } \tan (A+B) = \sqrt{3}$$

$$\text{So, } \tan (A +B) = \tan 60^\circ [\text{ since , } \tan 60^\circ = \sqrt{3}]$$

$$= A + B = 60^\circ \text{(i)}$$

Also given

$$\tan (A - B) = 1$$

$$\text{so , } \tan (A - B) = \tan 45^\circ [\tan 45^\circ = 1]$$

$$= A - B = 45^\circ \text{(ii)}$$

From equation (1) and (2) we get

$$A + B = 60^\circ$$

$$A - B = 45^\circ$$

$$2A = 105^\circ$$

$$A = 52\frac{1}{2}^\circ$$

Now on substituting the value of A in equation (i) , we get

$$52\frac{1}{2}^\circ + B = 60^\circ$$

$$B = 60^\circ - 52\frac{1}{2}^\circ = 7\frac{1}{2}^\circ$$

Therefore , the value of $A = 52\frac{1}{2}^\circ$ and $B = 7\frac{1}{2}^\circ$

8. without using Trigonometrical tables , evaluate the following :

(i) $\sin^2 28^\circ + \sin^2 62^\circ - \tan^2 45^\circ$

(ii) $2\frac{\cos 27^\circ}{\sin 63^\circ} + \frac{\tan 27^\circ}{\cot 63^\circ} + \cos 0^\circ$

(iii) $\cos 18^\circ \sin 72^\circ + \sin 18^\circ \cos 72^\circ$

(iv) $5 \sin 50^\circ \sec 40^\circ - 3 \cos 59^\circ \operatorname{cosec} 31^\circ$

Solution

$$\begin{aligned} & \text{(i) } \sin^2 28^\circ + \sin^2 62^\circ - \tan^2 45^\circ \\ &= \sin^2 28^\circ + \sin^2 (90^\circ - 28^\circ) - \tan^2 45^\circ \\ &= \sin^2 28^\circ + \cos^2 28^\circ - \tan^2 45^\circ \\ &= 1 - (1)^2 \quad (\because \sin^2 \theta + \cos^2 \theta = 1 \text{ and } \tan 45^\circ = 1) \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

$$\begin{aligned} & \text{(ii) } 2 \frac{\cos 27^\circ}{\sin 63^\circ} + \frac{\tan 27^\circ}{\cot 63^\circ} + \cos 0^\circ \\ &= 2 \frac{\cos 27^\circ}{\sin(90^\circ - 27^\circ)} + \frac{\tan 27^\circ}{\cot(90^\circ - 27^\circ)} + \cos 0^\circ \\ &= 2 \frac{\cos 27^\circ}{\cos 27^\circ} + \frac{\tan 27^\circ}{\tan 27^\circ} + 1 \quad (\because \cos 0^\circ = 1) \\ &= 2 \times 1 + 1 + 1 \\ &= 2 + 1 + 1 \\ &= 4 \end{aligned}$$

$$\begin{aligned}
& \text{(iii) } \cos 18^\circ \sin 72^\circ + \sin 18^\circ \cos 72^\circ \\
&= \cos(90^\circ - 12^\circ) \sin 72^\circ + \sin(90^\circ - 12^\circ) \cos 12^\circ \\
&= \sin 72^\circ \cdot \sin 12^\circ + \cos 12^\circ \cos 12^\circ \\
&= \sin^2 12^\circ + \cos^2 12^\circ \\
&= 1 \quad (\because \sin^2 \theta + \cos^2 \theta = 1)
\end{aligned}$$

$$\begin{aligned}
& \text{(iv) } 5 \sin 50^\circ \sec 40^\circ - 3 \cos 59^\circ \operatorname{cosec} 31^\circ \\
&= 5 \frac{\sin 50^\circ}{\cos 40^\circ} - 3 \frac{\cos 59^\circ}{\sin 31^\circ} \\
&= 5 \frac{\sin 50^\circ}{\cos(90^\circ - 50^\circ)} - 3 \frac{\cos 59^\circ}{\sin(90^\circ - 59^\circ)} \\
&= 5 \frac{\sin 50^\circ}{\sin 50^\circ} - 3 \frac{\cos 59^\circ}{\cos 59^\circ} = 5 \times 1 - 3 \times 1 \\
&= 5 - 3 \\
&= 2
\end{aligned}$$

9. prove that :

$$\frac{\cos(90^\circ - \theta) \sec(90^\circ - \theta) \tan \theta}{\operatorname{cosec}(90^\circ - \theta) \sin(90^\circ - \theta) \cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot \theta} = 2$$

Solution

$$\text{LHS} = \frac{\cos(90^\circ - \theta) \sec(90^\circ - \theta) \tan \theta}{\operatorname{cosec}(90^\circ - \theta) \sin(90^\circ - \theta) \cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot \theta}$$

$$= \frac{\sin\theta \operatorname{cosec}\theta \tan\theta}{\sec\theta \cos\theta \tan\theta} + \frac{\cot\theta}{\cot\theta}$$

$$= \frac{1 \times \tan\theta}{1 \times \tan\theta} + 1 = 1 + 1 = 2 = \text{R.H.S}$$

Thus , L.H.S = R.H.S

Hence proved

10. when $0^\circ < A < 90^\circ$, solve the following equations:

(i) $\sin 3A = \cos 2A$

(ii) $\tan 5A = \cot A$

Solution :

(i) $\sin 3A = \cos 2A$

$$= \sin 3A = \sin(90^\circ - 2A)$$

So,

$$3A = 90^\circ - 2A$$

$$3A + 2A = 90^\circ$$

$$5A = 90^\circ$$

$$\therefore A = \frac{90^\circ}{5} = 18^\circ$$

(ii) $\tan 5A = \cot A$

$$= \tan 5A = \tan (90^\circ - A)$$

So ,

$$5A = 90^\circ - A$$

$$5A + A = 90^\circ$$

$$6A = 90^\circ$$

$$\therefore A = \frac{90^\circ}{6} = 15^\circ$$

11. find the value of θ if

(i) $\sin (\theta + 36^\circ) = \cos \theta$, where θ and $\theta + 36^\circ$ are acute angles.

(ii) $\sec 4\theta = \operatorname{cosec} (\theta - 20^\circ)$, where 4θ and $\theta - 20^\circ$ are acute angles.

Solution

(i) given , θ and $(\theta + 36^\circ)$ are acute angles.

And,

$$\sin(\theta + 36^\circ) = \cos \theta = \sin(90^\circ - \theta) \text{ [As, } \sin(90^\circ - \theta) = \cos \theta \text{]}$$

On comparing ,we get

$$\theta + 36^\circ = 90^\circ - \theta$$

$$\theta + \theta = 90^\circ - 36^\circ$$

$$2\theta = 54^\circ$$

$$\theta = \frac{54^\circ}{2}$$

$$\theta = 27^\circ$$

(ii) given θ and $(\theta - 20^\circ)$ are acute angles

And ,

$$\sec 4\theta = \operatorname{cosec}(\theta - 20^\circ)$$

$$\operatorname{Cosec}(90^\circ - 4\theta) = \operatorname{cosec}(\theta - 20^\circ) \text{ [since , } \operatorname{cosec}(90^\circ - \theta) = \sec \theta \text{]}$$

On comparing , we get

$$90^\circ - 4\theta = \theta - 20^\circ$$

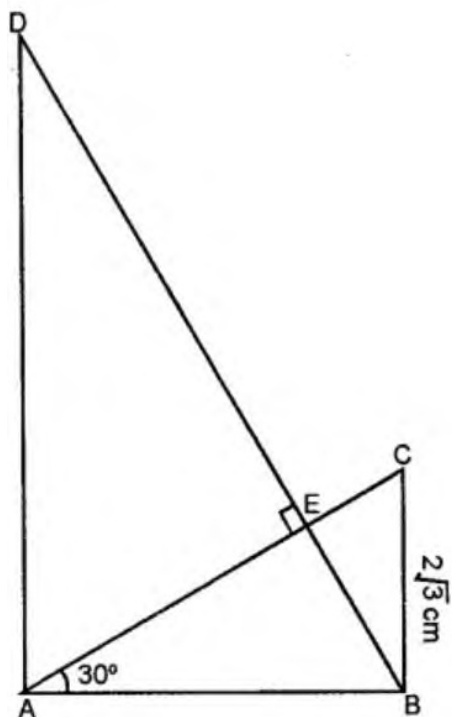
$$90^\circ + 20^\circ = \theta + 4\theta$$

$$5\theta = 110^\circ$$

$$\theta = \frac{110^\circ}{5}$$

$$\therefore \theta = 22^\circ$$

12. in the adjoining figure , ABC is right angled triangle at B and ABD is right angled triangle at A. If $BD \perp AC$ and $BC = 2\sqrt{3}$ cm , find the length of AD .



Solution

Given ΔABC and ΔABD are right angled triangles in which $\angle A = 90^\circ$ and $\angle B = 90^\circ$

And ,

$BC = 2\sqrt{3}$ cm . AC and BD intersect each other at E at right angle
and $\angle CAB = 30^\circ$

Now in right ΔABC we have

$$\tan \theta = \frac{BC}{AB}$$

$$= \tan 30^\circ = \frac{2\sqrt{3}}{AB}$$

$$= \frac{1}{\sqrt{3}} = \frac{2\sqrt{3}}{AB}$$

$$AB = 2\sqrt{3} \times \sqrt{3} = 2 \times 3 = 6 \text{ cm}$$

In ΔABE , $\angle EAB = 30^\circ$ and $\angle EAB = 90^\circ$

Hence,

$$\angle ABE \text{ or } \angle ABD = 180^\circ - 90^\circ - 30^\circ$$

$$= 60^\circ$$

Now in right ΔABD , we have

$$\tan 60^\circ = \frac{AD}{AB}$$

$$= \sqrt{3} = \frac{AD}{6}$$

Thus $AD = 6\sqrt{3} \text{ cm}$.