

CHAPTER – 11
FACTORISATION

Exercise 11.1

Factorise the following (1 to 8) polynomials:

1.

(i) $8xy^3 + 12x^2y^2$

(ii) $15ax^3 - 9ax^2$

Solution:

(i) $8xy^3 + 12x^2y^2 = 4xy^2 (2y + 3x)$

(ii) $15ax^3 - 9ax^2 = 3ax^2 (5x - 3)$

2.

(i) $21py^2 - 56py$

(ii) $4x^3 - 6x^2$

Solution:

(i) $21py^2 - 56py = 7py (3y - 8)$

(ii) $4x^3 - 6x^2 = 2x^2 (2x - 3)$

3.

(i) $25abc^2 - 15a^2b^2c$

(ii) $x^2yz + xy^2z + xyz^2$

Solution:

(i) $25abc^2 - 15a^2b^2c = 5abc(5c - 3ab)$

(ii) $x^2yz + xy^2z + xyz^2 = xyz(x + y + z)$

4.

(i) $8x^3 - 6x^2 + 10x$

(ii) $14mn + 22m - 62p$

Solution:

(i) $8x^3 - 6x^2 + 10x = 2x(4x^2 - 3x + 5)$

(ii) $14mn + 22m - 62p = 2(7mn + 11m - 31p)$

5.

(i) $18p^2q^2 - 24pq^2 + 30p^2q$

(ii) $27a^3b^3 - 18a^2b^3 + 75a^3b^2$

Solution:

(i) $18p^2q^2 - 24pq^2 + 30p^2q$

$= 6pq(3pq - 4q + 5p)$

(ii) $27a^3b^3 - 18a^2b^3 + 75a^3b^2$

$= 3a^2b^2(9ab - 6b + 25a)$

6.

(i) $15a(2p - 3p) - 106(2p - 3q)$

(ii) $3a(x^2 + y^2) + 6b(x^2 + y^2)$

Solution:

$$(i) 15a(2p - 3q) - 10b(2p - 3q)$$

$$= (2p - 3q)(15a - 10b)$$

$$= (2p - 3q)(5)(3a - 2b)$$

$$= 5(2p - 3q)(3a - 2b)$$

$$(ii) 3a(x^2 + y^2) + 66(x^2 + y^2)$$

$$= (x^2 + y^2)(3a + 66)$$

$$= (x^2 + y^2)(3)(a + 22)$$

$$= 3(x^2 + y^2)(a + 22)$$

7.

$$(i) 6(x + 2y)^3 + 8(x + 2y)^2$$

$$(ii) 14(a - 3b)^3 - 21p(a - 3b)$$

Solution:

$$(i) 6(x + 2y)^3 + 8(x + 2y)^2$$

$$(x + 2y)^2 [6(x + 2y) + 8]$$

$$= (x + 2y)^2 [6x + 12y + 8]$$

$$= (x + 2y)^2 (2)(3x + 6y + 4)$$

$$= 2(x + 2y)^2 (3x + 6y + 4)$$

$$(ii) 14(a - 3b)^3 - 21p(a - 3b)$$

$$= 7[2(a - 3b)^3 - 3p(a - 3b)]$$

$$= 7 [(a - 3b) \{2 (a - 3b)^2 - 3p\}]$$

$$= 7 (a - 3b) [2 (a - 3b)^2 - 3p]$$

8. $10a (2p + q)^3 - 15b (2p + q)^2 + 35(2p + q)$

Solution:

$$10a (2p + q)^3 - 15b (2p + q)^2 + 35(2p + q)$$

$$= 5 [2a (2p + q)]^3 - 3b (2p + q)^2 + 7 (2p + q)$$

$$= 5(2p + q) [2a (2p + q)^2 - 3b(2p + q) + 7]$$

Exercise 11.2

Factorise the following (1 to 11) polynomials:

1.

(i) $x^2 + xy - x - y$

(ii) $y^2 - yz - 5y + 5z$

Solution:

(i) $x^2 + xy - x - y$

$$= x(x + y) - 1(x + y)$$

$$= (x + y)(x - 1)$$

(ii) $y^2 - yz - 5y + 5z$

$$= y(y - z) - 5(y - z)$$

$$= (y - z)(y - 5)$$

2.

(i) $5xy + 7y - 5y^2 - 7x$

(ii) $5p^2 - 8pq - 10p + 16q$

Solution:

(i) $5xy + 7y - 5y^2 - 7x$

$$= 5xy - 5y^2 + 7y - 7x$$

$$= 5y(x - y) - 7(x - y)$$

$$= (x - y)(5y - 7)$$

$$(ii) 5p^2 - 8pq - 10p + 16q$$

$$= 5p^2 - 10p - 8pq + 16q$$

$$= 5p(p - 2) - 8q(p - 2)$$

$$= (p - 2)(5p - 8q)$$

$$= (5p - 8q)(p - 2)$$

3.

$$(i) a^2b - ab^2 + 3a - 3b$$

$$(ii) x^3 - 3x^2 + x - 3$$

Solution:

$$(i) a^2b - ab^2 + 3a - 3b$$

$$= ab(a - b) + 3(a - b) = (a - b)(ab + 3)$$

$$(ii) x^3 - 3x^2 + x - 3$$

$$= x^2(x - 3) + 1(x - 3)$$

$$= (x - 3)(x^2 + 1)$$

4.

$$(i) 6xy^2 - 3xy - 10y + 5$$

$$(ii) 3ax - 6ay - 8by + 4bx$$

Solution:

$$(i) 6xy^2 - 3xy - 10y + 5$$

$$3xy(2y - 1) - 5(2y - 1)$$

$$= (2y - 1)(3xy - 5)$$

$$(ii) 3ax - 6ay - 8by + 4bx$$

$$= 3ax - 6ay + 4bx - 8by$$

$$= 3a(x - 2y) + 4b(x - 2y)$$

$$= (x - 2y)(3a + 4b)$$

5.

$$(i) x^2 + xy(1 + y) + y^3$$

$$(ii) y^2 - xy(1 - x) - x^3$$

Solution:

$$(i) x^2 + xy(1 + y) + y^3$$

$$= x^2 + xy + xy^2 + y^3$$

$$= x(x + y) + y^2(x + y)$$

$$= (x + y)(x + y^2)$$

$$(ii) y^2 - xy(1 - x) - x^3$$

$$= y^2 - xy + x^2y - x^3$$

$$= y(y - x) + x^2(y - x)$$

$$= (y - x)(y + x^2)$$

6.

(i) $ab^2 + (a - 1)b - 1$

(ii) $2a - 4b - xa + 2bx$

Solution:

(i) $ab^2 + (a - 1)b - 1$

$$= ab^2 + ab - b - 1$$

$$= ab(b + 1) - 1(b + 1)$$

$$= (b + 1)(ab - 1)$$

(ii) $2a - 4b - xa + 2bx$

$$= 2(a - 2b) - x(a - 2b)$$

$$= (a - 2b)(2 - x)$$

7.

(i) $5ph - 10qk + 2rph - 4qrk$

(ii) $x^2 - x(a + 2b) + 2a^2$

Solution:

(i) $5ph - 10qk + 2rph - 4qrk$

$$= 5(ph - 2qk) + 2r(ph - 2qk)$$

$$= (ph - 2qk)(5 + 2r)$$

(ii) $x^2 - x(a + 2b) + 2ab$

$$= x^2 - xa - 2bx + 2ab$$

$$= x(x - a) - 2b(x - a)$$

$$= (x - a)(x - 2b)$$

8.

(i) $ab(x^2 + y^2) - xy(a^2 + b^2)$

(ii) $(ax + by)^2 + (bx - ay)^2$

Solution:

(i) $ab(x^2 + y^2) - xy(a^2 + b^2)$

$$= abx^2 + aby^2 - a^2xy - b^2xy$$

$$= (abx^2 - b^2xy) + (aby^2 - a^2xy)$$

$$= bx(ax - by) - ay(ax - by)$$

$$= (ax - by)(bx - ay)$$

(ii) $(ax + by)^2 + (bx - ay)^2$

$$= (a^2x^2 + b^2y^2 + 2abxy) + (b^2x^2 + a^2y^2 - 2abxy)$$

$$= a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + a^2y^2 - 2abxy$$

$$= a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$$

$$= a^2x^2 + a^2y^2 + b^2x^2 + b^2y^2$$

$$= a^2(x^2 + y^2) + b^2(x^2 + y^2)$$

$$= (a^2 + b^2)(x^2 + y^2)$$

9.

(i) $a^3 + ab(1 - 2a) - 2b^2$

(ii) $3x^2y - 3xy + 12x - 12$

Solution:

(i) $a^3 + ab - 2a^2b - 2b^2$

$$= a^3 + ab - 2a^2b - 2b^2$$

$$= a(a^2 + b) - 2b(a^2 + b)$$

$$= (a^2 + b)(a - 2b)$$

(ii) $3x^2y - 3xy + 12x - 12$

$$= 3(x^2y - xy + 4x - 4)$$

$$= 3[xy(x - 1) + 4(x - 1)]$$

$$= 3(x - 1)(xy + 4)$$

10.

(i) $a^2b + ab^2 - abc - b^2c + axy + bxy$

(ii) $ax^2 - bx^2 + ay^2 - by^2 + az^2 - bz^2$

Solution:

(i) $a^2b + ab^2 - abc - b^2c + axy + bxy$

$$= ab(a + b) - bc(a + b) + xy(a + b)$$

$$= (a + b)(ab - bc + xy)$$

(ii) $ax^2 - bx^2 + ay^2 - by^2 + az^2 - bz^2$

$$= x^2 (a - b) + y^2 (a - b) + z^2 (a - b)$$

$$= (a - b)(x^2 + y^2 + z^2)$$

11.

$$\text{(i) } x - 1 - (x - 1)^2 + ax - a$$

$$\text{(ii) } ax + a^2x + aby + by - (ax + by)^2$$

Solution:

$$\text{(i) } x - 1 - (x - 1)^2 + ax - a$$

$$= (x - 1) - (x - 1)^2 + a(x - 1)$$

$$= (x - 1) [1 - (x - 1) + a]$$

$$= (x - 1) (1 - x + 1 + a)$$

$$= (x - 1) (2 - x + a)$$

$$\text{(ii) } ax + a^2x + aby + by - (ax + by)^2$$

$$= (ax + by) + (a^2x + aby) - (ax + by)^2$$

$$= (ax + by) + a(ax + by) - (ax + by)^2$$

$$= (ax + by) [1 + a - (ax + by)]$$

$$= (ax + by) (1 + a - ax - by)$$

Exercise 11.3

1. Factorise the following expressions using algebraic identities:

(i) $x^2 - 12x + 36$

(ii) $36p^2 - 60pq + 25q^2$

(iii) $9y^2 + 66xy + 121y^2$

(iv) $a^4 + 6a^2b^2 + 9b^4$

(v) $x^2 + \frac{1}{x^2} + 2$

(vi) $x^2 + x + \frac{1}{4}$

Solution:

Using $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$

(i) $y^2 - 12x + 36$

$$= (x)^2 - 2 \times x \times 6 + (6)^2$$

$$= (x - 6)^2$$

(ii) $36p^2 - 60pq + 25q^2$

$$= (6p)^2 - 2 \times 6p \times 5q + (5q)^2$$

$$= (6p - 5q)^2$$

(iii) $9x^2 + 66xy + 121y^2$

$$= (3x)^2 + 2 \times 3x \times 11y + (11y)^2$$

$$= (3x + 11y)^2$$

$$(iv) a^4 + 6a^2b^2 + 9b^4$$

$$= (a^2)^2 + 2 \times 2a^2 \times 3b^2 + (3b^2)^2$$

$$= (a^2 + 3b^2)^2$$

$$(v) x^2 + \frac{1}{x^2} + 2$$

$$= (x)^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2$$

$$= \left(x + \frac{1}{x}\right)^2$$

$$(vi) x^2 + x + \frac{1}{4}$$

$$= (x)^2 + 2 \times x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$$

$$= \left(x + \frac{1}{2}\right)^2$$

Factorise the following (2 to 13) expressions:

2.

(i) $4p^2 - 9$

(ii) $4x^2 - 169y^2$

Solution:

(i) $4p^2 - 9$

$$= (2p)^2 - (3)^2$$

$$= (2p + 3) (2p - 3)$$

$$(ii) 4x^2 - 169y^2$$

$$= (2x)^2 - (13y)^2$$

$$= (2x + 13y)(2x - 13y)$$

3.

$$(i) 9x^2y^2 - 25$$

$$(ii) 16x^2 - \frac{1}{144}$$

Solution:

$$(i) 9x^2y^2 - 25$$

$$= (3xy)^2 - (5)^2$$

$$= (3xy + 5)(3xy - 5)$$

$$(ii) 16x^2 - \frac{1}{144}$$

$$= (4x)^2 - \left(\frac{1}{12}\right)^2$$

$$= \left(4x + \frac{1}{12}\right)\left(4x - \frac{1}{12}\right)$$

4.

$$(i) 20x^2 - 45y^2$$

$$(ii) \frac{9}{16} - 25a^2b^2$$

Solution:

$$(i) 20x^2 - 45y^2$$

$$= 5(4x^2 - 9y^2)$$

$$= 5[(2x)^2 - (3y)^2]$$

$$= 5(2x + 3y)(2x - 3y)$$

$$\begin{aligned}
 & \text{(ii)} \frac{9}{16} - 25a^2b^2 \\
 &= \left(\frac{3}{4}\right)^2 - (5ab)^2 \\
 &= \left(\frac{3}{4} + 5ab\right) \left(\frac{3}{4} - 5ab\right)
 \end{aligned}$$

5.

$$\text{(i)} (2a + 3b)^2 - 16c^2$$

$$\text{(ii)} 1 - (b - c)^2$$

Solution:

$$\text{(i)} (2a + 3b)^2 - 16c^2$$

$$= (2a + 3b)^2 - (4c)^2$$

$$= (2a + 3b + 4c) (2a + 3b - 4c)$$

$$\text{(ii)} 1 - (b - c)^2$$

$$= (1)^2 - (b - c)^2$$

$$= [1 + b - c] [1 - (b - c)]$$

$$= (1 + b - c) (1 - b + c)$$

6.

$$\text{(i)} 9(x + y)^2 - x^2$$

$$\text{(ii)} (2m + 3n)^2 - (3m + 2n)^2$$

Solution:

$$\text{(i)} 9(x + y)^2 - x^2$$

$$= [3(x + y)]^2 - [x]^2$$

$$= [3(x + y) + x] [3(x + y) - x]$$

$$= (3x + 3y + x) (3x + 3y - x)$$

$$= (4x + 3y) (2x + 3y)$$

$$(ii) (2m + 3n)^2 - (3m + 2n)^2$$

$$= (4m^2 + 9n^2 + 12mn) - (9m^2 + 4n^2 + 12mn)$$

$$= 4m^2 + 9n^2 + 12mn - 9m^2 - 4n^2 - 12mn$$

$$= 4m^2 + 9n^2 - 9m^2 - 4n^2$$

$$= -5m^2 + 5n^2$$

$$= 5(n^2 - m^2)$$

$$= 5(m + n)(n - m)$$

7.

$$(i) 25(a + b)^2 - 16(a - b)^2$$

$$(ii) 9(3x + 2)^2 - 4(2x - 1)^2$$

Solution:

$$(i) 25(a + b)^2 - 16(a - b)^2$$

$$= [5(a + b)]^2 - [4(a - b)]^2$$

$$= (5a + 5b)^2 - (4a - 4b)^2$$

$$= [(5a + 5b)^2 + (4a - 4b)] [(5a + 5b) - (4a - 4b)]$$

$$= (5a + 5b + 4a - 4b) (5a + 5b - 4a + 4b)$$

$$= (9a + b) (a + 9b)$$

$$(ii) 9 (3x + 2)^2 - 4 (2x - 1)^2$$

$$= [3 (3x + 2)]^2 - [2 (2x - 1)]^2$$

$$= (9x + 6)^2 - (4x - 2)^2$$

$$= [(9x + 6) + (4x - 2)] [(9x + 6) - (4x - 2)]$$

$$= (9x + 6 + 4x - 2) (9x + 6 - 4x + 2)$$

$$= (13x + 4) (5x + 8)$$

$$8. (i) x^3 - 25x$$

$$(ii) 63p^2q^2 - 7$$

Solution:

$$(i) x^3 - 25x$$

$$= x (x^2 - 25) = x [(x)^2 - (5)^2]$$

$$= x (x + 5) (x - 5)$$

$$(ii) 63p^2q^2 - 7$$

$$= 7 (9p^2q^2 - 1)$$

$$= 7 [(3pq)^2 - (1)^2]$$

$$= 7 (3pq + 1) (3pq - 1)$$

9. (i) $32a^2b - 72b^3$

(ii) $9(a + b)^3 - 25(a + b)$

Solution:

(i) $32a^2b - 72b^3$

$$= 8b(4a^2 - 9b^2) \Rightarrow 8b[(2a)^2 - (3b)^2]$$

$$= 8b(2a + 3b)(2a - 3b)$$

(ii) $9(a + b)^3 - 25(a + b)$

$$= (a + b)[9(a + b)^2 - 25]$$

$$= (a + b)[\{3(a + b)\}^2 - (5)^2]$$

$$= (a + b)[(3a + 3b)^2 - (5)^2]$$

$$= (a + b)[(3a + 3b + 5)(3a + 3b - 5)]$$

$$= (a + b)(3a + 3b + 5)(3a + 3b - 5)$$

10.

(i) $x^2 - y^2 - 2y - 1$

(ii) $p^2 - 4pq + 4q^2 - r^2$

Solution:

(i) $x^2 - y^2 - 2y - 1$

$$= x^2 - (y^2 + 2y + 1)$$

$$= (x)^2 - (y + 1)^2$$

$$= [x + (y + 1)][x - (y + 1)]$$

$$= (x + y + 1)(x - y - 1)$$

(ii) $p^2 - 4pq + 4q^2 - r^2$

$$= (p)^2 - 2 \times p \times 2q + (2q)^2 - r^2 [\because (a - b)^2 = a^2 - 2ab + b^2]$$

$$= (p - 2q)^2 - (r)^2$$

$$= (p - 2q + r)(p - 2q - r) [\because a^2 - b^2 = (a + b)(a - b)]$$

11. (i) $9x^2 - y^2 + 4y - 4$

(ii) $4a^2 - 4b^2 + 4a + 1$

Solution:

(i) $9x^2 - y^2 + 4y - 4$

$$= 9x^2 - (y^2 - 4y + 4)$$

$$= 9x^2 - (y - 2)^2$$

$$= (3x)^2 (y - 2)^2$$

$$= [3x + (y - 2)] [3x - (y - 2)]$$

$$= (3x + y - 2) (3x - y + 2)$$

(ii) $4a^2 - 4b^2 + 4a + 1$

$$= (4a^2 + 4a + 1) - 4b^2$$

$$= (2a + 1)^2 - (2b)^2$$

$$= (2a + 2b + 1) (2a - 2b + 1)$$

12.

(i) $625 - p^4$

(ii) $5y^5 - 405y$

Solution:

(i) $625 - p^4$

$$= (25)^2 - (p^2)^2$$

$$= (25 + p^2) (25 - p^2)$$

$$= (25 + p^2) [(5)^2 - (p)^2]$$

$$= (25 + p^2) (5 + p) (5 - p)$$

(ii) $5y^5 - 405y$

$$= 5y(y^4 - 81)$$

$$= 5y [(y^2)^2 - (9)^2]$$

$$= 5y (y^2 + 9) (y^2 - 9)$$

$$= 5y (y^2 + 9) [(y)^2 - (3)^2]$$

$$= 5y (y^2 + 9) (y + 3) (y - 3)$$

13.

(i) $x^4 - y^4 + x^2 - y^2$

(ii) $64a^2 - 9b^2 + 42bc - 49c^2$

Solution:

(i) $x^4 - y^4 + x^2 - y^2$

$$= [(x^2)^2 - (y^2)^2] + (x^2 - y^2)$$

$$= (x^2 + y^2) (x^2 - y^2) + 1(x^2 - y^2)$$

$$[\text{Using, } a^2 - b^2 = (a + b) (a - b)]$$

$$= (x^2 - y^2) (x^2 + y^2 + 1)$$

$$= (x + y)(x - y)(x^2 + y^2 + 1)$$

$$(ii) 64a^2 - 9b^2 + 42bc - 49c^2$$

$$= 64a^2 - [9b^2 - 42bc + 49c^2]$$

$$= (8a)^2 - [(3b)^2 - 2 \times 3b \times 7c + (7c)^2]$$

$$[\because a^2 + b^2 - 2ab = (a - b)^2 \text{ and } a^2 - b^2 = (a + b)(a - b)]$$

$$= (8a)^2 - (3b - 7c)^2$$

$$= (8a + 3b - 7c) (8a - 3b + 7c)$$

Exercise 11.4

1.

(i) $x^2 + 3x + 2$,

(ii) $z^2 + 10z + 24$

Solution:

(i) $x^2 + 3x + 2$

$$= x^2 + 2x + x + 2$$

$$= x(x + 2) + 1(x + 2)$$

$$= (x + 2)(x + 1)$$

(ii) $z^2 + 10z + 24$

$$= z^2 + 6z + 4z + 24$$

$$= z(z + 6) + 4(z + 6)$$

$$= (z + 6)(z + 4)$$

2.

(i) $y^2 - 7y + 12$

(ii) $m^2 - 23m + 42$

Solution:

(i) $y^2 - 7y + 12$

$$= y^2 - 3y - 4y + 12$$

[Since, $12 = -3 \times (-4)$ and $-7 = -3 - 4$]

$$= y(y - 3) - 4(y - 3)$$

$$= (y - 3) (y - 4)$$

$$(ii) m^2 - 23m + 42$$

$$= m^2 - 2m - 21m + 42 \quad [\text{Since, } 42 = -2 \times (-21) \text{ and } -23 = -21 - 2]$$

$$= m(m - 2) - 21(m - 2)$$

$$= (m - 2) (m - 21)$$

3.

$$(i) y^2 - 5y - 24,$$

$$(ii) t^2 + 23t - 108$$

Solution:

$$(i) y^2 - 5y - 24$$

$$= y^2 - 8y + 3y - 24$$

$$= y(y - 8) + 3(y - 8)$$

$$= (y - 8) (y + 3)$$

$$(ii) t^2 + 23t - 108$$

$$= t^2 + 27t - 4t - 108$$

$$= t(t + 27) - 4(t + 27)$$

$$= (t + 27) (t - 4)$$

4.

(i) $3x^2 + 14x + 8$,

(ii) $3y^2 + 10y + 8$

Solution:

(i) $3x^2 + 14x + 8$

$$= 3x^2 + 12x + 2x + 8$$

$$= 3x(x + 4) + 2(x + 4)$$

$$= (x + 4)(3x + 2)$$

(ii) $3y^2 + 10y + 8$

$$= 3y^2 + 6y + 4y + 8$$

$$= 3y(y + 2) + 4(y + 2)$$

$$= (y + 2)(3y + 4)$$

5.

(i) $14x^2 - 23x + 8$,

(ii) $12x^2 - x - 35$

Solution:

(i) $14x^2 - 23x + 8$

$$= 14x^2 - 16x - 7x + 8$$

$$= 2x(7x - 8) - 1(7x - 8)$$

$$= (7x - 8)(2x - 1)$$

$$(ii) 12x^2 - x - 35$$

$$= 12x^2 - 21x + 20x - 35$$

$$= 3x(4x - 7) + 5(4x - 7)$$

$$= (4x - 7)(3x + 5)$$

6.

$$(i) 6x^2 + 11x - 10$$

$$(ii) 5 - 4x - 12x^2$$

Solution:

$$(i) 6x^2 + 11x - 10$$

$$= 6x^2 + 15x - 4x - 10$$

$$= 3x(2x + 5) - 2(2x + 5)$$

$$= (2x + 5)(3x - 2)$$

$$(ii) 5 - 4x - 12x^2$$

$$= 5 - 10x + 6x - 12x^2$$

$$= 5(1 - 2x) + 6x(1 - 2x)$$

$$= (1 - 2x)(5 + 6x)$$

7.

$$(i) 1 - 18y - 63y^2,$$

$$(ii) 3x^2 - 5xy - 12y^2$$

Solution:

$$(i) 1 - 18y - 63y^2$$

$$= 1 - 21y + 3y - 63y^2$$

$$= 1(1 - 21y) + 3y(1 - 21y)$$

$$= (1 - 21y)(1 + 3y)$$

$$(ii) 3x^2 - 5xy - 12y^2$$

$$= 3x^2 - 9xy + 4xy - 12y^2$$

$$= 3x(x - 3y) + 4y(x - 3y)$$

$$= (x - 3y)(3x + 4y)$$

8.

$$(i) x^2 - 3xy - 40y^2$$

$$(ii) 10p^2q^2 - 21pq + 9$$

Solution:

$$(i) x^2 - 3xy - 40y^2$$

$$= x^2 - 8xy + 5xy - 40y^2$$

$$= x(x - 8y) + 5y(x - 8y)$$

$$= (x - 8y)(x + 5y)$$

$$(ii) 10p^2q^2 - 21pq + 9$$

$$\begin{aligned}
&= 10p^2q^2 - 15pq - 6pq + 9 \\
&= 5pq (2pq - 3) - 3 (2pq - 3) \\
&= (2pq - 3) (5pq - 3)
\end{aligned}$$

9.

(i) $2a^2b^2 + ab - 45$

(ii) $x (12x + 7) - 10$

Solution:

(i) $2a^2b^2 + ab - 45$

$$= 2a^2b^2 + 10ab - 9ab - 45$$

$$= 2ab (ab + 5) - 9 (ab + 5)$$

$$= (ab + 5) (2ab - 9)$$

(ii) $x (12x + 7) - 10$

$$= 12x^2 + 7x - 10$$

$$= 12x^2 + 15x - 8x - 10$$

$$= 3x (4x + 5) - 2 (4x + 5)$$

$$= (4x + 5) (3x - 2)$$

10.

(i) $(a + b)^2 - 11(a + b) - 42$

(ii) $8 + 6(p + q) - 5(p + q)^2$

Solution:

(i) $(a + b)^2 - 11(a + b) - 42$

Let $(a + b) = x$, then we have

$$= x^2 - 11x - 42$$

$$= x^2 - 14x + 3x - 42 \quad [\because -42 = -14 \times 3 \text{ and } -11 = -14 + 3]$$

$$= x(x - 14) + 3(x - 14)$$

$$= (x - 14)(x + 3)$$

Substituting the value of x we get,

$$= (a + b - 14)(a + b + 3)$$

(ii) $8 + 6(p + q) - 5(p + q)^2$

Let $p + q = x$, then we have

$$= 8 + 6x - 5x^2$$

$$= -5x^2 + 6x + 8$$

$$= -(5x^2 - 6x - 8)$$

$$= 5x^2 - 10x + 4x - 8$$

$$[\because 5 \times (-8) = 40 \Rightarrow -40 = -10 \times 4 \text{ and } -6 = -10 + 4]$$

$$= (x - 2)(5x + 4)$$

Substituting the value of x, then

$$= -(p + q - 2) (5p + 5q + 4)$$

$$= (4 + 5p + 5q) (-p - q + 2)$$

$$= (4 + 5p + 5q) (2 - p - q)$$

11.

(i) $(x - 2y)^2 - 6(x - 2y) + 5$

(ii) $7 + 10(2x - 3y) - 8(2x - 3y)^2$

Solution:

(i) Let $x - 2y = z$

Then, $(x - 2y)^2 - 6(x - 2y) + 5$ becomes

$$= z^2 - 6z + 5$$

$$= z^2 - 5z - z + 5$$

$$= z(z - 5) - 1(z - 5)$$

$$= (z - 5)(z - 1)$$

Now, on substituting $z = x - 2y$, we get

$$= [(x - 2y) - 5] [(x - 2y) - 1]$$

$$= (x - 2y - 5) (x - 2y - 1)$$

(ii) $7 + 10(2x - 3y) - 8(2x - 3y)^2$

$$\text{Let } 2x - 3y = z$$

Then, $7 + 10(2x - 3y) - 8(2x - 3y)^2$ becomes

$$= 7 + 10z - 8z^2$$

$$= 7 + 14z - 4z - 8z^2$$

$$= 7(1 + 2z) - 4z(1 + 2z)$$

$$= (1 + 2z)(7 - 4z)$$

Now, on substituting $z = 2x - 3y$, we get

$$= [(1 + 2(2x - 3y))][7 - 4(2x - 3y)]$$

$$= (1 + 4x - 6y)(7 - 8x + 12y)$$

Exercise 11.5

Work out the following divisions:

(i) $(35x + 28) \div (5x + 4)$

(ii) $7p^2q^2(9r - 27) \div 63pq(r - 3)$

Solution:

(i) $(35x + 28) \div (5x + 4)$

$$\frac{7(5x+4)}{(5x+4)} = 7$$

(ii) $7p^2q^2(9r - 27) \div 63pq(r - 3)$

$$= \frac{7p^2q^2 \times 9(r-3)}{63pq(r-3)}$$

$$= p^{2-1} q^{2-1} \times 9 = 9pq$$

2. Divide as directed:

(i) $6(2x + 7) (5x - 3) \div 3(5x - 3)$

(ii) $33pq (p + 3) (2q - 5) \div 11p (2q - 5)$

Solution:

(i) $6(2x + 7) (5x - 3) \div 3(5x - 3)$

$$= \frac{6(2x+7)(5x-3)}{3(5x-3)}$$

$$= 2(2x + 7)$$

(ii) $33pq (p + 3) (2q - 5) \div 11p (2q - 5)$

$$= \frac{33pq(p+3)(2q-5)}{11p(2q-5)}$$

$$= 3q(p + 3)$$

3. Factorise the expression and divide them as directed:

(i) $(7x^2 - 63x) \div 7(x - 3)$

(ii) $(3p^2 + 17p + 10) \div (p + 5)$

(iii) $10xy(14y^2 + 43y - 21) \div 5x(7y - 3)$

(iv) $12pqr(6p^2 - 13pq + 6q^2) \div 6pq(2p - 3q)$

Solution:

(i) $(7x^2 - 63x) \div 7(x - 3)$

$$= \frac{7x(x^2-9)}{7(x-3)}$$

$$= \frac{7x[(x)^2-(3)^2]}{7(x-3)}$$

$$= \frac{7x(x+3)(x-3)}{7(x-3)}$$

$$= x(x + 3)$$

(ii) $(3p^2 + 17p + 10) \div (p + 5)$

$$= \frac{3p^2 + 17p + 10}{p + 5} \quad \left\{ \begin{array}{l} \because 3 \times 10 = 30 \\ \because 30 = 2 \times 15 \\ 17 = 2 + 15 \end{array} \right\}$$

$$= \frac{3p^2 + 2p+15p + 10}{p + 5}$$

$$= \frac{p(3p+2)+5(3p+2)}{p+5}$$

$$= \frac{(3p+2)(p+5)}{(p+5)}$$

$$= 3p + 2$$

$$(iii) 10xy(14y^2 + 43y - 21) \div 5x(7y - 3)$$

$$= \frac{10xy[14y^2 + 49y - 6y - 21]}{5x(7y - 3)} \quad \left\{ \begin{array}{l} \because -21 \times 14 = -294 \\ \therefore -294 = 49 \times (-6) \\ 43 = 49 - 6 \end{array} \right\}$$

$$= \frac{10xy[7y(2y+7) - 3(2y+7)]}{5x(7y-3)}$$

$$= \frac{10xy(2y+7)(7y-3)}{5x(7y-3)}$$

$$= 2x(2y + 7)$$

$$(iv) 12pqr(6p^2 - 13pq + 6q^2) \div 6pq(2p - 3q)$$

$$= \frac{12pqr[6p^2 - 9pq - 4pq + 6q^2]}{6pq(2p - 3q)} \quad \left\{ \begin{array}{l} \because 6 \times 6 = 36 \\ \therefore 36 = -9 \times (-4) \\ -13 = -9 - 4 \end{array} \right\}$$

$$= \frac{12pqr[3p(2p-3q) - 2q(2p-3q)]}{6pq(2p-3q)}$$

$$= \frac{12pqr(2p-3q)(3p-2q)}{6pq(2p-3q)}$$

$$= 2r(3p - 2q)$$

Mental Maths

Question 1: Fill in the blanks:

- (i) When an algebraic expression can be written as the product of two or more expressions then each of these expressions is called of the given expression.
- (ii) The process of finding two or more expressions whose product is the given expression is called
- (iii) HCF of two or more monomials = (HCF of their coefficients) $\times$ (HCF of their literal coefficients)
- (iv) HCB of literal coefficients = product of each common literal raised to the power.
- (v) To factorise the trinomial of the form $x^2 + px + q$, we need to find two integers a and b such that $a + b = \dots\dots\dots$ and $ab = \dots\dots\dots$
- (vi) To factorise the trinomial of the form $ax^2 + bx + c$, where a , b and c are integers, we split b into two parts such that of these parts is b and their is ac .

Solution:

- (i) When an algebraic expression can be written as the product of two or more expressions then each of these expressions is called factor of the given expression.
- (ii) The process of finding two or more expressions whose product is the given expression is called factorization.
- (iii) HCF of two or more monomials
= (HCF of their numerical coefficients) $\times$ (HCF of their literal coefficients)
- (iv) HCF of literal coefficients
= product of each common literal raised to the lowest power.
- (v) To factorise the trinomial of form $x^2 + px + q$,
we need to find two integers a and b such that $a + b = p$ and $ab = q$.
- (vi) To factorise the trinomial of the form $ax^2 + bx + c$,
where a , b and c are integers, we split b into two parts such that algebraic sum of these parts is b and their product is ac .

Question 2: State whether the following statements are true (T) or false (F):

- (i) Factorisation is the reverse process of multiplication.**
- (ii) HCF of two or more polynomials (with integral coefficients) is the smallest common factor of the given polynomials.**
- (iii) HCF of $6x^2y^2$ and $8xy^3$ is $2xy^2$.**
- (iv) Factorisation by grouping is possible only if the given polynomial contains an even number of terms.**
- (v) To factorise the trinomial of the form $ax^2 + bx + c$ where, a, b, c are integers we want to find two integers A and B such that $A + B = ac$ and $AB = b$**
- (vi) Factors of $4x^2 - 12x + 9$ are $(2x - 3)(2x - 3)$.**

Solution:

- (i) Factorisation is the reverse process of multiplication. True
 - (ii) HCF of two or more polynomials (with integral coefficients) is the smallest common factor of the given polynomials. False
 - (iii) HCF of $6x^2y^2$ and $8xy^2$ is $2xy^2$. True
 - (iv) Factorisation by grouping is possible only if the given polynomial contains an even number of terms. True
 - (v) To factorise the trinomial of the form $ax^2 + bx + c$ where, a, b, c are integers we want to find two integers A and B such that $A + B = ac$ and $AB = b$ False
- Correct :
 $A + B$ should be equal to bt . and $AB = ac$
- (vi) Factors of $4x^2 - 12x + 9$ are $(2x - 3)(2x - 3)$. True

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 14):

Question 3: H.C.F. of $6abc$, $24ab^2$, $12a^2b$ is

- (a) $6ab$**
- (b) $6ab^2$**
- (c) $6a^2b$**
- (d) $6abc$**

Solution:

H.C.F. of $6abc$, $24ab^2$, $12a^2b$

= H.C.F. of 6, 24, 12 $\times$ H.C.F. of abc , ab^2 , a^2b

= $6 \times a \times b = 6ab$ (a)

Question 4: Factors of $12a^2b + 15ab^2$ are

- (a) $3a(4ab + 5b^2)$**
- (b) $3ab(4a + 5b)$**
- (c) $3b(4a^2 + 5ab)$**
- (d) none of these**

Solution:

$12a^2b + 15ab^2 = 3ab(4a + 5b)$ (b)

Question 5: Factors of $6xy - 4y + 6 - 9x$ are

- (a) $(3y - 2)(2x - 3)$**
- (b) $(3x - 2)(2y - 3)$**
- (c) $(2y - 3)(2 - 3x)$**
- (d) none of these**

Solution:

$6xy - 4y + 6 - 9x$

= $6xy - 9x - 4y + 6$

= $3x(2y - 3) - 2(2y - 3)$

= $(2y - 3)(3x - 2)$

Question 6: Factors of $49p^3q - 36pq$ are

(a) $p(7p + 6q)(7p - 6q)$

(b) $q(7p - 6)(7p + 6)$

(c) $pq(7p + 6)(7p - 6)$

(d) none of these

Solution:

$$49p^2q - 36pq$$

$$= pq(49p^2 - 36)$$

$$= pq[(7p)^2 - (6)^2]$$

$$= pq(7p + 6)(7p - 6)$$

Question 7: Factors of $y(y - z) + 9(z - y)$ are

(a) $(y - z)(y + 9)$

(b) $(z - y)(y + 9)$

(c) $(y - z)(y - 9)$

(d) none of these

Solution:

$$y(y - z) + 9(z - y)$$

$$= y(y - z) - 9(y - z)$$

$$= (y - z)(y - 9) \text{ (c)}$$

Question 8: Factors of $(lm + l) + m + 1$ are

(a) $(lm + l)(m + l)$

(b) $(lm + m)(l + 1)$

(c) $l(m + 1)$

(d) $(l + 1)(m + 1)$

Solution:

Factors of $lm + l + m + 1$ are

$$l(m + 1) + 1(m + 1) = (m + 1)(l + 1) \text{ (d)}$$

Question 9: Factors of $z^2 - 4z - 12$ are

(a) $(z + 6)(z - 2)$

(b) $(z - 6)(z + 2)$

(c) $(z - 6)(z - 2)$

(d) $(z + 6)(z + 2)$

Solution:

Factors of $z^2 - 4z - 12$

$$\Rightarrow z^2 - 6z + 2z - 12$$

$$= z(z - 6) + 2(z - 6)$$

$$= (z - 6)(z + 2) \text{ (b)}$$

Question 10: Factors of $63a^2 - 112b^2$ are

(a) $63(a - 2b)(a + 2b)$

(b) $7(3a + 2b)(3a - 2b)$

(c) $7(3a + 4b)(3a - 4b)$

(d) none of these

Solution:

Factors of $63a^2 - 112b^2$ are

$$= 7(9a^2 - 16b^2)$$

$$= 7[(3a)^2 - (4b)^2]$$

$$= 7(3a + 4b)(3a - 4b) \text{ (c)}$$

Question 11: Factors of $p^4 - 81$ are

(a) $(p^2 - 9)(p^2 + 9)$

(b) $(p + 3)^2 (p - 3)^2$

(c) $(p + 3)(p - 3)(p^2 + 9)$

(d) none of these

Solution:

$$p^4 - 81 = (p^2)^2 - (9)^2$$

$$= (p^2 + 9)(p^2 - 9)$$

$$= (p^2 + 9) \{(p)^2 - (3)^2\}$$

$$= (p^2 + 9) (p + 3) (p - 3) (c)$$

Question 12: Factors of $3x^2 + 7x - 6$ are

- (a) $(3x - 2)(x + 3)$
- (b) $(3x + 2)(x - 3)$
- (c) $(3x - 2)(x - 3)$
- (d) $(3x + 2)(x + 3)$

Solution:

$$3x^2 + 7x - 6$$

$$= 3x^2 + 9x - 2x - 6$$

$$= 3x(x + 3) - 2(x + 3)$$

$$= (3x - 2)(x + 3) (a)$$

Question 13: Factors of $16x^2 + 40x + 25$ are

- (a) $(4x + 5)(4x + 5)$
- (b) $(4x + 5)(4x - 5)$
- (c) $(4x + 5)(4x + 8)$
- (d) none of these

Solution:

$$16x^2 + 40x + 25$$

$$= (4x)^2 + 2 \times 4x \times 5 + (5)^2$$

$$= (4x + 5)^2$$

$$= (4x + 5)(4x + 5) (a)$$

Question 14: Factors of $x^2 - 4xy + 4y^2$ are

- (a) $(x - 2y)(x + 2y)$
- (b) $(x - 2y)(x - 2y)$
- (c) $(x + 2y)(x + 2y)$
- (d) none of these

Solution:

$$\begin{aligned} & x^2 - 4xy + 4y^2 \\ &= (x)^2 - 2 \times x \times 2y + (2y)^2 = (x - 2y)^2 \\ &= (x - 2y)(x - 2y) \quad (b) \end{aligned}$$

Higher Order Thinking Skills (Hots)
Factorise the following

Question 1: $x^2 + \left(a + \frac{1}{a}\right)x + 1$

Solution:

$$\begin{aligned} & x^2 + \left(a + \frac{1}{a}\right)x + 1 \\ &= x^2 + ax + \frac{x}{a} + 1 \\ &= x(x + a) + \frac{1}{a}(x + a) \\ &= (x + a) \left(x + \frac{1}{a}\right) \end{aligned}$$

Question 2: $36a^4 - 97a^2b^2 + 36b^4$

Solution:

$$\begin{aligned} &= 36a^4 - 97a^2b^2 + 36b^4 \\ &= 36a^4 - 72a^2b^2 + 36b^4 - 25a^2b^2 \\ &= (6a^2)^2 - 2 \times 6a^2 \times 6b^2 + (6b^2)^2 - (5ab)^2 \\ &= (6a^2 - 6b^2)^2 - (5ab)^2 \\ &= (6a^2 - 6b^2 + 5ab)(6a^2 - 6b^2 - 5ab) \\ &= (6a^2 + 5ab - 6b^2)(6a^2 - 5ab - 6b^2) \\ &= [6a^2 + 9ab - 4ab - 6b^2] [6a^2 - 9ab + 4ab - 6b^2] \\ &= [3a(2a + 3b) - 2b(2a + 3b)] [3a(2a - 3b) + 2b(2a - 3b)] \\ &= (2a + 3b)(3a - 2b)(2a - 3b)(3a + 2b) \end{aligned}$$

Question 3: $2x^2 - \sqrt{3}x - 3$

Solution:

$$2x^2 - \sqrt{3}x - 3$$

$$= 2x^2 - 2\sqrt{3}x + \sqrt{3}x - 3$$

$$\{\because 2 \times (-3) = -6 \therefore -6 = -2\sqrt{3} \times \sqrt{3} - \sqrt{3} = -2\sqrt{3} + \sqrt{3}\}$$

$$= 2x(x - \sqrt{3}) + \sqrt{3}(x - \sqrt{3})$$

$$= (x - \sqrt{3})(2x + \sqrt{3})$$

Question 4: $y(y^2 - 2y) + 2(2y - y^2) - 2 + y$

Solution:

$$y(y^2 - 2y) + 2(2y - y^2) - 2 + y$$

$$= y^3 - 2y^2 + 4y - 2y^2 - 2 + y$$

$$= y^3 - 4y^2 + 5y - 2$$

$$= y^3 - 2y^2 + y - 2y^2 + 4y - 2$$

$$= y(y^2 - 2y + 1) - 2(y^2 - 2y + 1)$$

$$= (y^2 - 2y + 1)(y - 2)$$

$$= [(y)^2 - 2 \times y \times 1 + (1)^2](y - 2)$$

$$= (y - 1)^2(y - 2)$$

Check Your Progress

1. Find the HCF of the given polynomials:

(i) $14pq, 28p^2q^2$

(ii) $8abc, 24ab^2, 12a^2b$

Solution:

(i) $14pq, 28p^2q^2$

HCF of 14, 28 = 14

HCF of $14pq, 28p^2q^2 = 14pq$

(ii) $8abc, 24ab^2, 12a^2b$

HCF of 8, 24, 12 = 4

HCF of $8abc, 24ab^2, 12a^2b = 4ab$

2. Factorise the following:

(i) $10x^2 - 18x^3 + 14x^4$

(ii) $5x^2y + 10xyz + 15xy^2$

(iii) $p^2x^2 + c^2x^2 - ac^2 - ap^2$

(iv) $15(x + y)^2 - 5x - 5y$

(v) $(ax + by)^2 + (ay - bx)^2$

(vi) $ax + by + cx + bx + cy + ay$

(vii) $49x^2 - 70xy + 25y^2$

(viii) $4a^2 + 12ab + 9b^2$

(ix) $49p^2 - 36q^2$

(x) $100x^3 - 25xy^2$

(xi) $x^2 - 2xy + y^2 - z^2$

(xii) $x^8 - y^8$

(xiii) $12x^3 - 14x^2 - 10x$

(xiv) $p^2 - 10p + 21$

(xv) $2x^2 - x - 6$

(xvi) $6x^2 - 5xy - 6y^2$

(xvii) $x^2 + 2xy - 99y^2$

Solution:

(i) $10x^2 - 18x^3 + 14x^4$

HCF of 10, 18, 14 = 2

So, $10x^2 - 18x^3 + 14x^4$

$= 2x^2(5 - 9x + 7x^2)$

(ii) $5x^2y + 10xyz + 15xy^2$

HCF of 5, 10, 15 = 5

So, $5x^2y + 10xyz + 15xy^2$

$= 5xy(x + 2z + 3y)$

(iii) $p^2x^2 + c^2x^2 - ac^2 - ap$

$= p^2x^2 - ap^2 + c^2x^2 - ac^2$

$= p^2(x^2 - a) + c^2(x^2 - a)$

$= (x^2 - a)(p^2 + c^2)$

(iv) $15(x + y)^2 - 5x - 5y$

$= 15(x + y)^2 - 5(x + y)$

$$= 5(x + y) [3(x + y) - 1]$$

$$= 5(x + y) (3x + 3y - 1)$$

$$(v) (ax + by)^2 + (ay - bx)^2$$

On expanding, we have

$$= a^2x^2 + b^2y^2 + 2abxy + a^2y^2 + b^2x^2 - 2abxy$$

$$= a^2x^2 + a^2y^2 + b^2x^2 + b^2y^2$$

$$= a^2(x^2 + y^2) + b^2(x^2 + y^2)$$

$$= (x^2 + y^2) (a^2 + b^2)$$

$$(vi) ax + by + cx + bx + cy + ay$$

$$= ax + bx + cx + ay + by + cy \text{ [On grouping the like variables]}$$

$$= x(a + b + c) + y(a + b + c)$$

$$= (a + b + c) (x + y)$$

$$(vii) 49x^2 - 70xy + 25y^2$$

$$= (7x)^2 - 2 \times 7x \times 5y + (5y)^2 [\because (a - b)^2 = a^2 - 2ab + b^2]$$

$$= (7x - 5y)^2$$

$$(viii) 4a^2 + 12ab + 9b^2$$

$$= (2a)^2 + 2 \times 2a \times 3b + (3b)^2 [\because (a + b)^2 = a^2 + 2ab + b^2]$$

$$= (2a + 3b)^2$$

$$(ix) 49p^2 - 36q^2$$

$$= (7p)^2 - (6q)^2$$

$$= (7p + 6q) (7p - 6q) [\because a^2 - b^2 = (a + b) (a - b)]$$

$$(x) 100x^3 - 25xy^2$$

$$= 25x(x^2 - y^2) = 25x\{(x)^2 - (y)^2\}$$

$$= 25x(x + y) (x - y)$$

$$(xi) x^2 - 2xy + y^2 - z^2$$

$$= (x - y)^2 - (z)^2 [\because a^2 - 2ab + b^2 = (a - b)^2 \text{ and } a^2 - b^2 = (a + b) (a - b)]$$

$$= (x - y + z)(x - y - z)$$

$$(xii) x^8 - y^8$$

$$= (x^4)^2 - (y^4)^2 [\because a^2 - b^2 = (a + b)(a - b)]$$

$$= (x^4 + y^4) (x^4 - y^4)$$

$$= (x^4 + y^4) [(x^2)^2 - (y^2)^2]$$

$$= (x^4 + y^4) (x^2 + y^2) (x^2 - y^2)$$

$$= (x^4 + y^4 (x^2 + y^2) (x + y) (x - y)$$

$$(xiii) 12x^3 - 14x^2 - 10x$$

$$= 2x(6x^2 - 7x - 5) [\text{Now, as } 6 \times (-5) = -30 \Rightarrow -30 = -10 \times 3 \text{ and } -7 = -10$$

$$+ 3]$$

$$= 2x(6x^2 + 3x - 10x - 5)$$

$$= 2x\{3x(2x + 1) - 5(2x + 1)\}$$

$$= 2x(2x + 1) (3x - 5)$$

$$(xiv) p^2 - 10p + 21$$

$$= p^2 - 3p - 7p + 21 \text{ [Now, as } 21 = -3 \times (-7) \text{ and } -10 = -3 - 7]$$

$$= p(p - 3) - 7(p - 3)$$

$$= (p - 3)(p - 7)$$

$$(xv) 2x^2 - x - 6$$

$$= 2x^2 - 4x + 3x - 6 \text{ [Now, as } -6 \times 2 = -12 \Rightarrow -12 = -4 \times 3 \text{ and } -1 = -4 + 3]$$

$$= 2x(x - 2) + 3(x - 2)$$

$$= (x - 2)(2x + 3)$$

$$(xvi) 6x^2 - 5xy - 6y^2$$

$$= 6x^2 - 9xy + 4xy - 6y^2 \text{ [Now, as } 6 \times (-6) = -36 \Rightarrow -36 = -9 \times 4 \text{ and } -5 = -9 + 4]$$

$$= 3x(2x - 3y) + 2y(2x - 3y)$$

$$= (2x - 3y)(3x + 2y)$$

$$(xvii) x^2 + 2xy - 99y^2$$

$$= x^2 + 11xy - 9xy - 99y^2 \text{ [Now, as } -99 = -11 \times 9 \text{ and } -2 = -11 + 9 \text{ }]$$

$$= x(x + 11y) - 9y(x + 11y)$$

$$= (x + 11y)(x - 9y)$$

3. Divide as directed:

(i) $15(y + 3)(y^2 - 16) \div 5(y^2 - y - 12)$

(ii) $(3x^3 - 6x^2 - 24x) \div (x - 4)(x + 2)$

(iii) $(x^4 - 81) \div (x^3 + 3x^2 + 9x + 27)$

Solution:

(i) $15(y + 3)(y^2 - 16) \div 5(y^2 - y - 12)$

$$y^2 - 16 = (y)^2 - (4)^2$$

$$= (y + 4)(y - 4)$$

$$y^2 - y - 12 = y^2 - 4y + 3y - 12$$

$$= y(y - 4) + 3(y - 4)$$

$$= (y - 4)(y + 3)$$

Now,

$$\frac{15(y+3)(y^2-16)}{5(y^2-y-12)}$$

$$= \frac{15 \times (y+3)(y+4)(y-4)}{5(y-4)(y+3)}$$

$$= 3(y + 4)$$

(ii) $(3x^3 - 6x^2 - 24x) \div (x - 4)(x + 2)$

$$3x^3 - 6x^2 - 24x = 3x(x^2 - 2x - 8)$$

$$= 3x\{x^2 - 4x + 2x - 8\}$$

$$= 3x \{x(x-4) + 2(x-4)\}$$

$$= 3x(x-4)(x+2)$$

Now,

$$\begin{aligned} & \frac{3x^3 - 6x^2 - 24x}{(x-4)(x+2)} \\ &= \frac{3x(x-4)(x+2)}{(x-4)(x+2)} \\ &= 3x \end{aligned}$$

$$(iii) (x^4 - 81) \div (x^3 + 3x^2 + 9x + 27)$$

$$x^4 - 81 = (x^2)^2 - (9)^2 = (x^2 + 9)(x^2 - 9)$$

$$\begin{aligned} &= (x^2 + 9)[(x)^2 - (3)^2] \\ &= (x^2 + 9)(x+3)(x-3) \end{aligned}$$

And,

$$\begin{aligned} x^3 + 3x^2 + 9x + 27 &= (x)^2 + (x+3) + 9(x+3) \\ &= (x^2 + 9)(x+3) \end{aligned}$$

Now,

$$\begin{aligned} & \frac{x^4 - 81}{x^3 + 3x^2 + 9x + 27} \\ &= \frac{(x^2 + 9)(x+3)(x-3)}{(x^2 + 9)(x+3)} \\ &= (x-3) \end{aligned}$$