

CHAPTER – 12
LINEAR EQUATIONS AND INEQUALITIES IN ONE VARIABLE

Exercise 12.1

Solve the following equations:

1.

(i) $5x - 3 = 3x - 5$

(ii) $3x - 7 = 3(5 - x)$

Solution:

(i) $5x - 3 = 3x - 5$

$$5x - 3x = -5 + 3$$

$$2x = -2$$

We get,

$$x = -\frac{2}{2}$$

$$x = -1$$

(ii) $3x - 7 = 3(5 - x)$

$$3x - 7 = 15 - 3x$$

$$3x + 3x = 15 + 7$$

We get,

$$6x = 22$$

$$x = \frac{22}{6}$$

$$x = \frac{11}{3}$$

2.

(i) $4(2x + 1) = 3(x - 1) + 7$

(ii) $3(2p - 1) = 5 - (3p - 2)$

Solution:

(i) $4(2x + 1) = 3(x - 1) + 7$

$$8x + 4 = 3x - 3 + 7$$

$$8x + 4 = 3x + 4$$

On further calculation, we get,

$$8x - 3x = 4 - 4$$

We get,

$$x = 0$$

(ii) $3(2p - 1) = 5 - (3p - 2)$

$$6p - 3 = 5 - 3p + 2$$

$$6p + 3p = 5 + 3 + 2$$

We get,

$$9p = 10$$

$$p = \frac{10}{9}$$

$$p = 1\frac{1}{9}$$

3.

(i) $5y - 2\{y - 3(y - 5)\} = 6$

(ii) $0.3(6 - x) = 0.4(x + 8)$

Solution:

$$(i) 5y - 2 \{y - 3(y - 5)\} = 6$$

$$5y - 2 (y - 3y + 15) = 6$$

$$5y - 2 (- 2y + 15) = 6$$

On simplification, we get,

$$5y + 4y - 30 = 6$$

$$5y + 4y = 6 + 30$$

$$9y = 36$$

$$y = \frac{36}{9}$$

We get,

$$y = 4$$

$$(ii) 0.3 (6 - x) = 0.4 (x + 8)$$

$$1.8 - 0.3x = 0.4x + 3.2$$

$$- 0.3x - 0.4x = 3.2 - 1.8$$

We get,

$$- 0.7x = 1.4$$

$$x = - \left(\frac{1.4}{0.7} \right)$$

$$x = - \left(\frac{14}{7} \right)$$

$$x = - 2$$

4.

$$(i) \frac{(x - 1)}{3} = \left\{ \frac{(x + 2)}{6} \right\} + 3$$

$$(ii) \frac{(x+7)}{3} = 1 + \left\{ \frac{(3x-2)}{5} \right\}$$

Solution:

$$(i) \left\{ \frac{(x-1)}{3} \right\} = \left\{ \frac{(x+2)}{6} \right\} + 3$$

$$\left\{ \frac{x-1}{3} \right\} = \left\{ \frac{x+2}{6} \right\} = 3$$

On further calculation, we get,

$$\frac{\{2(x-1) - 1(x+2)\}}{6} = 3$$

$$\frac{(2x-2-1x-2)}{6} = 3$$

$$\frac{(x-4)}{6} = 3$$

We get,

$$x-4 = 6 \times 3$$

$$x-4 = 18$$

$$x = 18 + 4$$

We get,

$$x = 22$$

$$(ii) \left\{ \frac{(x+7)}{3} \right\} = 1 + \left\{ \frac{(3x-2)}{5} \right\}$$

$$\left\{ \frac{(x+7)}{3} \right\} - \left\{ \frac{(3x-2)}{5} \right\} = 1$$

On further calculation, we get,

$$\frac{\{5(x+7) - 3(3x-2)\}}{15} = 1$$

$$\frac{\{(5x + 35) - (9x - 6)\}}{15} = 1$$

$$\frac{(5x - 9x + 35 + 6)}{15} = 1$$

$$\frac{(-4x + 41)}{15} = 1$$

$$-4x + 41 = 15$$

$$-4x = 15 - 41$$

$$-4x = -26$$

$$x = \frac{-26}{-4}$$

We get,

$$x = \frac{13}{2}$$

$$x = 6\frac{1}{2}$$

5.

$$(i) \left\{ \frac{(y+1)}{3} \right\} - \left\{ \frac{(y-1)}{2} \right\} = \frac{(1+2y)}{3}$$

$$(ii) \left(\frac{p}{3} \right) + \left(\frac{p}{4} \right) = 55 - \left\{ \frac{(p+40)}{5} \right\}$$

Solution:

$$(i) \left\{ \frac{(y+1)}{3} \right\} - \left\{ \frac{(y-1)}{2} \right\} = \frac{(1+2y)}{3}$$

$$\frac{\{2(y+1) - 3(y-1)\}}{6} = \frac{(1+2y)}{3}$$

$$\frac{(2y+2-3y+3)}{6} = \frac{(1+2y)}{3}$$

We get,

$$\frac{(-y + 5)}{6} = \frac{(1 + 2y)}{3}$$

$$3(-y + 5) = 6(1 + 2y)$$

$$-3y + 15 = 6 + 12y$$

$$-3y - 12y = 6 - 15$$

$$-15y = -9$$

$$y = \frac{-9}{-15}$$

We get,

$$y = \frac{3}{5}$$

$$(ii) \left(\frac{p}{3}\right) + \left(\frac{p}{4}\right) = 55 - \left\{\frac{(p + 40)}{5}\right\}$$

$$\left(\frac{p}{3}\right) + \left(\frac{p}{4}\right) + \left\{\frac{(p + 40)}{5}\right\} = 55$$

Here, L.C.M. of 3, 4, 5 is 60

We get,

$$\frac{\{20p + 15p + 12(p + 40)\}}{60} = 55$$

$$\frac{(20p + 15p + 12p + 480)}{60} = 55$$

$$\frac{(47p + 480)}{6} = 55$$

$$47p + 480 = 55 \times 60$$

$$47p + 480 = 3300$$

$$47p = 3300 - 480$$

$$47p = 2820$$

$$p = \frac{2820}{47}$$

We get,

$$p = 60$$

6.

$$(i) \ n - \left\{ \frac{(n-1)}{2} \right\} = 1 - \left\{ \frac{(n-2)}{3} \right\}$$

$$(ii) \ \left\{ \frac{(3t-2)}{3} \right\} + \left\{ \frac{(2t+3)}{2} \right\} = t + \left(\frac{7}{6} \right)$$

Solution:

$$(i) \ n - \left\{ \frac{(n-1)}{2} \right\} = 1 - \left\{ \frac{(n-2)}{3} \right\}$$

$$\frac{(2n - n + 1)}{2} = \frac{(3 - n + 2)}{3}$$

$$\frac{(n + 1)}{2} = \frac{(5 - n)}{3}$$

$$3(n + 1) = 2(5 - n)$$

$$3n + 3 = 10 - 2n$$

$$3n + 2n = 10 - 3$$

$$5n = 7$$

$$n = \frac{7}{5}$$

We get,

$$n = 1\frac{2}{5}$$

$$(ii) \ \left\{ \frac{(3t-2)}{3} \right\} + \left\{ \frac{(2t+3)}{2} \right\} = t + \left(\frac{7}{6} \right)$$

$$\frac{\{2(3t-2)+3(2t+3)\}}{6} = \frac{(6t+7)}{6}$$

$$(6t-4) + (6t+9) = 6t+7$$

On further calculation, we get,

$$6t + 6t + 9 - 4 = 6t + 7$$

$$12t + 5 = 6t + 7$$

$$12t - 6t = 7 - 5$$

$$6t = 2$$

$$t = \frac{2}{6}$$

We get,

$$t = \frac{1}{3}$$

7.

$$(i) 4(3x+2) - 5(6x-1) = 2(x-8) - 6(7x-4)$$

$$(ii) 3(5x+7) + 5(2x-11) = 3(8x-5) - 15$$

Solution:

$$(i) 4(3x+2) - 5(6x-1) = 2(x-8) - 6(7x-4)$$

$$12x + 8 - 30x + 5 = 2x - 16 - 42x + 24$$

On further calculation, we get,

$$-18x + 13 = -40x + 8$$

$$-18x + 40x = 8 - 13$$

We get,

$$22x = -5$$

$$x = \frac{-5}{22}$$

$$(ii) 3(5x + 7) + 5(2x - 11) = 3(8x - 5) - 15$$

$$15x + 21 + 10x - 55 = 24x - 15 - 15$$

On further calculation, we get,

$$25x - 34 = 24x - 30$$

$$25x - 24x = -30 + 34$$

We get,

$$x = 4$$

8.

$$(i) \frac{(3-2x)}{(2x+5)} = -\left(\frac{3}{11}\right)$$

$$(ii) \frac{(5p+2)}{(8-2p)} = \frac{7}{6}$$

Solution:

$$(i) \frac{(3-2x)}{(2x+5)} = -\left(\frac{3}{11}\right)$$

$$11(3 - 2x) = -3(2x + 5)$$

On simplification, we get,

$$33 - 22x = -6x - 15$$

$$-22x + 6x = -15 - 33$$

$$-16x = -48$$

$$x = \frac{48}{16}$$

We get,

$$x = 3$$

$$(ii) \frac{(5p+2)}{(8-2p)} = \frac{7}{6}$$

$$6 (5p + 2) = 7 (8 - 2p)$$

On further calculation, we get,

$$30p + 12 = 56 - 14p$$

$$30p + 14p = 56 - 12$$

$$44p = 44$$

$$p = \frac{44}{44}$$

We get,

$$p = 1$$

9.

$$(i) \frac{5}{x} = \frac{7}{x-4}$$

$$(ii) \frac{4}{2x+3} = \frac{5}{x+4}$$

Solution:

$$(i) \frac{5}{x} = \frac{7}{(x-4)}$$

$$5 (x - 4) = 7x$$

$$5x - 20 = 7x$$

We get,

$$5x - 7x = 20$$

$$- 2x = 20$$

$$x = \frac{20}{-2}$$

$$x = \left(\frac{-20}{2} \right)$$

We get,

$$x = -10$$

$$(ii) \frac{4}{2x+3} = \frac{5}{x+4}$$

$$4(x+4) = 5(2x+3)$$

$$4x + 16 = 10x + 15$$

On further calculation, we get,

$$4x - 10x = 15 - 16$$

$$-6x = -1$$

$$x = \frac{-1}{-6}$$

We get,

$$x = \frac{1}{6}$$

10.

$$(i) \left\{ \frac{(2x+5)}{2} \right\} - \left\{ \frac{5x}{(x-1)} \right\} = x$$

$$(ii) \frac{1}{5} \left\{ \left(\frac{1}{3x} \right) - 5 \right\} = \frac{1}{3} \left\{ 3 - \left(\frac{1}{x} \right) \right\}$$

Solution:

$$(i) \left\{ \frac{(2x+5)}{2} \right\} - \left\{ \frac{5x}{(x-1)} \right\} = x$$

$$\frac{\{(2x+5)(x-1) - (5x)(2)\}}{\{2(x-1)\}} = x$$

On further calculation, we get,

$$\frac{\{2x(x-1)+5(x-1)-10x\}}{(2x-2)} = x$$

$$\frac{(2x^2-2x+5x-5-10x)}{(2x-2)} = x$$

$$\frac{(2x^2-7x-5)}{(2x-2)} = x$$

$$2x^2 - 7x - 5 = x(2x - 2)$$

$$2x^2 - 7x - 5 = 2x^2 - 2x$$

$$-7x - 5 = -2x$$

$$-7x + 2x = 5$$

$$-5x = 5$$

$$x = \frac{5}{-5}$$

We get,

$$x = -1$$

$$(ii) \frac{1}{5} \left\{ \left(\frac{1}{3x} \right) - 5 \right\} = \frac{1}{3} \left\{ 3 - \left(\frac{1}{x} \right) \right\}$$

$$\frac{1}{5} \left[\frac{\{1 - 5(3x)\}}{3x} \right] = \frac{1}{3} \left[\frac{\{(3x - 1)\}}{x} \right]$$

On further calculation, we get,

$$\frac{1}{5} \left\{ \frac{(1 - 15x)}{3x} \right\} = \frac{1}{3} \left\{ \frac{(3x - 1)}{x} \right\}$$

We get,

$$\frac{(1-15x)}{15x} = \frac{(3x-1)}{3x}$$

$$3x(1 - 15x) = 15x(3x - 1)$$

$$3(1 - 15x) = 15(3x - 1)$$

$$3 - 45x = 45x - 15$$

$$-45x - 45x = -15 - 3$$

$$-90x = -18$$

$$x = \frac{18}{90}$$

We get,

$$x = \frac{1}{5}$$

11.

$$(i) \left\{ \frac{(2x-3)}{(2x-1)} \right\} = \left\{ \frac{(3x-1)}{(3x+1)} \right\}$$

$$(ii) \left\{ \frac{(2y+3)}{(3y+2)} \right\} = \left\{ \frac{(4y+5)}{(6y+7)} \right\}$$

Solution:

$$(i) \left\{ \frac{(2x-3)}{(2x-1)} \right\} = \left\{ \frac{(3x-1)}{(3x+1)} \right\}$$

$$(2x-3)(3x+1) = (3x-1)(2x-1)$$

On further calculation, we get,

$$6x^2 + 2x - 9x - 3 = 6x^2 - 3x - 2x + 1$$

$$6x^2 - 7x - 3 = 6x^2 - 5x + 1$$

$$6x^2 - 7x - 6x^2 + 5x = 1 + 3$$

$$-7x + 5x = 4$$

$$-2x = 4$$

We get,

$$x = \frac{4}{-2}$$

$$x = -2$$

$$(ii) \left\{ \frac{(2y+3)}{(3y+2)} \right\} = \left\{ \frac{(4y+5)}{(6y+7)} \right\}$$

$$(2y+3)(6y+7) = (4y+5)(3y+2)$$

$$12y^2 + 14y + 18y + 21 = 12y^2 + 8y + 15y + 10$$

On simplification, we get,

$$32y + 21 = 23y + 10$$

$$32y - 23y = 10 - 21$$

$$9y = -11$$

We get,

$$y = \frac{-11}{9}$$

12. If $x = p + 1$, find the value of p from the equation $\left(\frac{1}{2}\right)(5x - 30) - \left(\frac{1}{3}\right)(1 + 7p) = \frac{1}{4}$

Solution:

Given

$$x = p + 1 \quad \dots\dots\dots (1)$$

$$\left(\frac{1}{2}\right)(5x - 30) - \left(\frac{1}{3}\right)(1 + 7p) = \frac{1}{4} \quad \dots\dots\dots (2)$$

Substituting the value of x from (1) in (2), we get,

$$\left(\frac{1}{2}\right)\{5(p+1) - 30\} - \left(\frac{1}{3}\right)(1 + 7p) = \frac{1}{4}$$

$$\frac{1}{2}(5p + 5 - 30) - \frac{1}{3}(1 + 7p) = \frac{1}{4}$$

On further calculation, we get,

$$\frac{1}{2}(5p - 25) - \frac{1}{3}(1 + 7p) = \frac{1}{4}$$

$$\frac{(5p - 25)}{2} - \frac{(1 + 7p)}{3} = \frac{1}{4}$$

$$\frac{\{3(5p - 25) - 2(1 + 7p)\}}{6} = \frac{1}{4}$$

$$\frac{(15p - 75 - 2 - 14p)}{6} = \frac{1}{4}$$

$$\frac{(p - 77)}{6} = \frac{1}{4}$$

On simplification, we get,

$$4(p - 77) = 6(1)$$

$$4p - 308 = 6$$

$$4p = 6 + 308$$

$$4p = 314$$

$$p = \frac{314}{4}$$

We get,

$$p = \frac{157}{2}$$

$$p = 78\frac{1}{2}$$

13: Solve $\left\{\frac{(x+3)}{3}\right\} - \left\{\frac{(x-2)}{2}\right\} = 1$, Hence find p if $\left(\frac{1}{x}\right) + P = 1$

Solution:

$$\left\{\frac{(x+3)}{3}\right\} - \left\{\frac{(x-2)}{2}\right\} = 1$$

$$\frac{\{2(x+3) - 3(x-2)\}}{6} = 1$$

$$\frac{(2x + 6 - 3x + 6)}{6} = 1$$

On further calculation, we get,

$$\frac{(-x + 12)}{6} = 1$$

$$-x + 12 = 1 \times 6$$

$$-x + 12 = 6$$

$$-x = 6 - 12$$

$$-x = -6$$

We get,

$$x = 6$$

Now,

$$\left(\frac{1}{x}\right) + P = 1$$

Substituting $x = 6$, we get,

$$\left(\frac{1}{6}\right) + P = 1$$

$$\frac{(1 + 6P)}{6} = 1$$

$$(1 + 6P) = 6$$

$$6P = 6 - 1$$

$$6P = 5$$

$$P = \frac{5}{6}$$

Exercise 12.2

1. Three more than twice a number is equal to four less than the number. Find the number.

Solution:

Let the number be x

Twice the number = $2x$

As per the given statement,

$$3 + 2x = x - 4$$

$$3 + 2x + 4 = x$$

We get,

$$7 = x - 2x$$

$$7 = -x$$

$$-x = 7$$

We get,

$$x = -7$$

Therefore, the required number is -7

2. When four consecutive integers are added, the sum is 46. Find the integers.

Solution:

Let the first integer be x , then the next three consecutive integers will be,

$(x + 1)$, $(x + 2)$ and $(x + 3)$

According to the problem,

$$x + (x + 1) + (x + 2) + (x + 3) = 46$$

$$x + x + 1 + x + 2 + x + 3 = 46$$

On further calculation, we get,

$$4x + 6 = 46$$

$$4x = 46 - 6$$

$$4x = 40$$

$$x = \frac{40}{4}$$

We get,

$$x = 10$$

Therefore, four consecutive integers are 10, $(10 + 1)$, $(10 + 2)$ and $(10 + 3)$ i.e. 10, 11, 12 and 13

3. Manjula thinks a number and subtracts $\frac{7}{3}$ from it. She multiplies the result by 6. The result now obtained is 2 less than twice the same number she thought of. What is the number?

Solution:

Let a number thought by Manjula be x

According to the statement,

$$\left(x - \frac{7}{3}\right) \times 6 = 2x - 2$$

$$6x - 14 = 2x - 2$$

On further calculation, we get,

$$6x - 2x = -2 + 14$$

$$4x = 12$$

$$x = \frac{12}{4}$$

We get,

$$x = 3$$

Therefore, the required number is 3

4. A positive number is 7 times another number. If 15 is added to both the numbers, then one of the new numbers becomes $\left(\frac{5}{2}\right)$ times the other new number. What are the numbers?

Solution:

Let the required number be x

Then the other number = $\frac{x}{7}$

According to the condition,

$$x + 15 = \frac{5}{2} \left\{ \left(\frac{x}{7} \right) + 15 \right\}$$

$$2(x + 15) = \left(\frac{5x}{7} \right) + (5 \times 15)$$

$$2x + 30 = \left(\frac{5x}{7} \right) + 75$$

On simplification, we get,

$$2x - \left(\frac{5}{7} \right) x = 75 - 30$$

$$\left\{ \frac{(14-5)}{7} \right\} x = 45$$

$$\frac{9x}{7} = 45$$

$$x = 45 \times \frac{7}{9}$$

We get,

$$x = 35$$

Hence,

$$\text{One number} = 35$$

$$\text{Other number} = \frac{35}{7} = 5$$

Therefore, the numbers are 35 and 5

5. When three consecutive even integers are added, the sum is zero. Find the integers.

Solution:

Let the first even integer = x

Then next two consecutive even integers = $(x + 2)$ and $(x + 4)$

According to the given statement,

$$x + (x + 2) + (x + 4) = 0$$

$$x + x + 2 + x + 4 = 0$$

$$3x + 6 = 0$$

$$3x = -6$$

$$x = \frac{-6}{3}$$

We get,

$$x = -2$$

Therefore, three consecutive integers are -2, $(-2 + 2)$ and $(-2 + 4)$ i.e. -2, 0 and 2

6. Find two consecutive odd integers such that two-fifth of the smaller exceeds two-ninth of the greater by 4.

Solution:

Let the first odd integer = x

Then next consecutive odd integers = $(x + 2)$

According to the given problem,

$$\left(\frac{2}{5}\right)(x) = \left(\frac{2}{9}\right)(x + 2) + 4$$

$$\frac{2x}{5} = \frac{\{2(x + 2)\}}{9} + 4$$

On further calculation, we get,

$$\left(\frac{2x}{5}\right) - \frac{\{2(x + 2)\}}{9} = 4$$

$$\frac{\{18x - 10(x + 2)\}}{45} = 4$$

$$\frac{(18x - 10x - 20)}{45} = 4$$

$$\frac{(8x - 20)}{45} = 4$$

$$8x - 20 = 4 \times 45$$

$$8x - 20 = 180$$

$$8x = 180 + 20$$

$$8x = 200$$

$$x = \frac{200}{8}$$

We get,

$$x = 25$$

So, two consecutive odd integers are $x = 25$ and

$$(x + 2) = (25 + 2) = 27$$

Therefore, two consecutive odd integers are 25 and 27

7. The denominator of a fraction is 1 more than twice its numerator. If the numerator and denominator are both increased by 5, it becomes $\left(\frac{3}{5}\right)$. Find the original fraction.

Solution:

Let the numerator of the original fraction = x

Then, its denominator = $2x + 1$

Hence,

$$\text{The fraction} = \frac{x}{(2x+1)}$$

According to the given problem,

$$\frac{(x + 5)}{\{(2x + 1) + 5\}} = \frac{3}{5}$$

$$\frac{(x + 5)}{(2x + 1 + 5)} = \frac{3}{5}$$

On further calculation, we get,

$$5(x + 5) = 3(2x + 6)$$

$$5x + 25 = 6x + 18$$

$$5x - 6x = 18 - 25$$

$$-x = -7$$

We get,

$$x = 7$$

Now,

$$\text{Original fraction} = \frac{x}{2x + 1}$$

$$= \frac{7}{\{2(7) + 1\}}$$

$$= \frac{7}{14 + 1}$$

We get,

$$= \frac{7}{15}$$

Therefore, the original fraction is $\frac{7}{15}$

8. Find two positive numbers in the ratio 2: 5 such that their difference is 15.

Solution:

Let the two numbers be $2x$ and $5x$

Because the ratio of these two numbers $= \frac{2x}{5x}$

$$= \frac{2}{5}$$

$$= 2: 5$$

According to the given problem,

$$5x - 2x = 15$$

$$3x = 15$$

$$x = \frac{15}{3}$$

We get,

$$x = 5$$

So, the numbers are $2x = 2 \times 5 = 10$ and $5x = 5 \times 5 = 25$

Therefore the required numbers are 10 and 25

9. What number should be added to each of the numbers 12, 22, 42 and 72 so that the resulting numbers may be in proportion?

Solution:

Let x be the required number

According to the given problem,

$(12 + x)$, $(22 + x)$, $(42 + x)$ and $(72 + x)$ are in proportion

So,

$$\frac{(12 + x)}{(22 + x)} = \frac{(42 + x)}{(72 + x)}$$

On cross multiplication, we get,

$$(12 + x)(72 + x) = (42 + x)(22 + x)$$

On simplification, we get,

$$12(72 + x) + x(72 + x) = 42(22 + x) + x(22 + x)$$

$$864 + 12x + 72x + x^2 = 924 + 42x + 22x + x^2$$

We get,

$$864 + 84x + x^2 = 924 + 64x + x^2$$

$$864 + 84x + x^2 - 924 - 64x - x^2 = 0$$

$$864 + 84x - 64x - 924 = 0$$

$$84x - 64x = 924 - 864$$

$$20x = 60$$

We get,

$$x = \frac{60}{20}$$

$$x = 3$$

Therefore, the required number is 3

10. The digits of a two-digit number differ by 3. If the digits are interchanged and the resulting number is added to the original number, we get 143. What can be the original number?

Solution:

Let one's digit of a two-digit number be x

Given that the difference between both the digits is 3,

Then ten's digit = $x + 3$

Hence, the number = $x + 10(x + 3)$

$$= x + 10x + 30$$

$$= 11x + 30$$

By interchanging the digits, we get,

One's digit of a new number = $x + 3$ and

Ten's digit of a new number = x

Hence,

$$\text{Number} = x + 3 + 10x = 11x + 3$$

According to the condition,

$$11x + 30 + 11x + 3 = 143$$

$$22x + 33 = 143$$

$$22x = 143 - 33$$

We get,

$$22x = 110$$

$$x = \frac{110}{22}$$

$$x = 5$$

Therefore, original number = $11x + 30$

$$= 11 \times 5 + 30$$

$$= 55 + 30$$

We get,

$$= 85$$

Hence, the original number is 85

11. Sum of the digits of a two-digit number is 11. When we interchange the digits, it is found that the resulting new number is greater than the original number by 63. Find the two-digit number.

Solution:

Given

Sum of the digits of a two-digit numbers = 11

Let unit's digit of a 2-digit number be x

Then ten's digit will be $11 - x$

So, number = $x + 10(11 - x)$

$$= x + 110 - 10x$$

$$= 110 - 9x$$

Now, by interchanging the digits, we get,

One's digit of a new number = $11 - x$

And ten's digit will be = x

Hence, number = $11 - x + 10x$

$$= 11 + 9x$$

According to the condition,

$$11 + 9x - (110 - 9x) = 63$$

$$11 + 9x - 110 + 9x = 63$$

$$18x = 63 - 11 + 110$$

$$18x = 162$$

We get,

$$x = \frac{162}{18}$$

$$x = 9$$

So, original number = $110 - 9x$

$$= 110 - 9 \times 9$$

$$= 110 - 81$$

$$= 29$$

Therefore, the original number is 29

12. Ritu is now four times as old as his brother Raju. In 4 years' time, her age will be twice of Raju's age. What are their present ages?

Solution:

Let the age of Raju be x years

Then the age of Ritu will be $= 4 \times x$ years

In 4 years' time,

Age of Raju = $(x + 4)$ years

Age of Ritu = $(4x + 4)$ years

According to the given problem,

$$4x + 4 = 2(x + 4)$$

$$4x + 4 = 2x + 8$$

$$4x - 2x = 8 - 4$$

$$2x = 4$$

We get,

$$x = \frac{4}{2}$$

$$x = 2$$

Therefore, Raju's age = 2 years and

Ritu's age = $4 \times 2 = 8$ years

13. A father is 7 times as old as his son. Two years ago, the father was 13 times as old as his son. How old are they now?

Solution:

Let the present age of son be x years

Then, age of his father will be $7x$ years

Two years ago, age of son = $(x - 2)$ years

Two years ago, age of his father = $(7x - 2)$ years

According to the given problem,

$$7x - 2 = 13(x - 2)$$

$$7x - 2 = 13x - 26$$

$$7x - 13x = -26 + 2$$

$$-6x = -24$$

$$x = \frac{-24}{-6}$$

We get,

$$x = 4$$

Therefore, age of son = 4 years and

Age of his father = $7x = 7 \times 4 = 28$ years

14. The ages of Sona and Sonali are in the ratio 5: 3. Five years hence, the ration of their ages will be 10: 7. Find their present ages.

Solution:

Given

Ratio of ages of Sona and Sonali = 5: 3

Let us consider the present age of Sona and Sonali be $5x$ and $3x$ years respectively

Five years hence,

The age of Sona = $5x + 5$ and

The age of Sonali = $3x + 5$

According to the given problem,

$$\frac{(5x + 5)}{(3x + 5)} = \frac{10}{7}$$

On cross multiplication, we get,

$$7(5x + 5) = 10(3x + 5)$$

$$35x + 35 = 30x + 50$$

$$35x - 30x = 50 - 35$$

$$5x = 15$$

$$x = \frac{15}{5}$$

We get,

$$x = 3$$

Now,

Present age of Sona = $5x = 5 \times 3 = 15$ years and

Present age of Sonali = $3x = 3 \times 3 = 9$ years

Therefore, the present age of Sona and Sonali is 15 years and 9 years

15. An employee works in a company on a contract of 30 days on the condition that he will receive Rs 200 for each day he works and he will be fined Rs 20 for each day if he is absent. If he receives Rs 3800 in all, for how many days did he remain absent?

Solution:

Given

Period of contract = 30 days

If an employee works a day, he will get = Rs 200

If he is absent, he will be fined = Rs 20 per day

At the end of contract period, he gets = Rs 3800

Let an employee remain absent for x days

Then, number of days, he worked = $(30 - x)$ days

According to the given condition,

$$(30 - x) \times 200 - x \times 20 = 3800$$

$$6000 - 200x - 20x = 3800$$

$$6000 - 220x = 3800$$

$$220x = 6000 - 3800$$

$$220x = 2200$$

$$x = \frac{2200}{220}$$

We get,

$$x = 10$$

Therefore, an employee remained absent for 10 days.

16. I have a total of Rs 300 in coins of denomination Rs 1, Rs 2 and Rs 5. The number of coins is 3 times the number of Rs 5 coins. The total number of coins is 160. How many coins of each denomination are with me?

Solution:

Given

Amount of coins = Rs 300

Total number of coins = 160

Let the number of coins of Rs 5 = x

Then number of coins of Rs 2 = $3x$

And number of coins of Rs 1 = $160 - (x + 3x) = 160 - 4x$

According to the given problem,

$$(160 - 4x) \times 1 + 3x \times 2 + x \times 5 = 300$$

$$160 - 4x + 6x + 5x = 300$$

We get,

$$160 + 7x = 300$$

$$7x = 300 - 160$$

$$7x = 140$$

$$x = \frac{140}{7}$$

We get,

$$x = 20$$

Hence, 5 Rs coins = 20

2 Rs coins = $3x = 3 \times 20 = 60$ and

1 Rs coins = $160 - 60 - 20 = 80$

17. A local bus is carrying 40 passengers, some with Rs 5 tickets and the remaining with Rs 7.50 tickets. If the total receipts from these passengers are Rs 230, find the number of passengers with Rs 5 tickets.

Solution:

Let the number of passengers with Rs 5 tickets = x

Then, the number of passengers with Rs 7.50 tickets = $(40 - x)$

According to the given statement,

$$5 \times x + (40 - x) \times 7.50 = 230$$

On simplification, we get,

$$5x + 300 - 7.5x = 230$$

$$5x - 7.5x = 230 - 300$$

$$-2.5x = -70$$

$$x = \frac{-70}{-2.5}$$

We get,

$$x = 28$$

Therefore, the number of passengers with Rs 5 tickets = 28

18. On a school picnic, a group of students agree to pay equally for the use of a full boat and pay Rs 10 each. If there had been 3 more students in the group, each would have paid Rs 2 less. How many students were there in the group?

Solution:

Let the number of students in a group be x

If there are 3 more students in a group, then,

The total number of students in a group = $x + 3$

According to the given problem,

$$10 \times x = (x + 3) \times (10 - 2)$$

On simplification, we get,

$$10x = (x + 3) \times 8$$

$$10x = 8(x + 3)$$

$$10x = 8x + 24$$

$$10x - 8x = 24$$

$$2x = 24$$

$$x = \frac{24}{2}$$

We get,

$$x = 12$$

Therefore, the total number of students in the group = 12

19. Half of a herd of deer are grazing in the field and three-fourths of the remaining are playing nearby. The rest 9 are drinking water from the pond. Find the number of deer in the herd.

Solution:

Let the number of deer in the herd = x

Number of deer grazing in the field = $\frac{x}{2}$

Remaining = $x - \left(\frac{x}{2}\right) = \frac{x}{2}$

Given that the $\left(\frac{3}{4}\right)$ of the remaining deer are playing

= $\left(\frac{3}{4}\right) \times \left(\frac{1}{2}\right) x$

We get,

= $\left(\frac{3}{8}\right) x$

Hence,

Rest of deer = $\left(\frac{x}{2}\right) - \left(\frac{3}{8}\right) x$

= $\left(\frac{1}{8}\right) x$

Therefore,

$\left(\frac{1}{8}\right) x = 9$

$x = 9 \times 8$

We get,

$x = 72$

Hence, total number of deer in the herd = 72

20. Sakshi takes some flowers in a basket and visits three temples one by one. At each temple, she offers one-half of the flowers from the basket. If she is left with 6 flowers at the end, find the number of flowers she had in the beginning.

Solution:

Let the total number of flowers in the basket = x

Flowers offered in first temple = $\frac{x}{2}$

Remaining flowers = $x - \left(\frac{x}{2}\right)$

$$= \left(\frac{x}{2}\right)$$

Flowers offered in the second temple

$$\left(\frac{x}{2}\right) \times \left(\frac{1}{2}\right) = \frac{x}{4}$$

Remaining flowers = $\left(\frac{x}{2}\right) - \left(\frac{x}{4}\right)$

$$= \left(\frac{x}{4}\right)$$

Flowers offered in the third temple = $\left(\frac{x}{4}\right) \times \left(\frac{1}{2}\right)$

$$= \left(\frac{x}{8}\right)$$

Remaining flowers = $\left(\frac{x}{4}\right) - \left(\frac{x}{8}\right)$

$$= \left(\frac{x}{8}\right)$$

Hence,

$$\left(\frac{x}{8}\right) = 6$$

We get,

$$x = 6 \times 8$$

$$x = 48$$

Therefore, number of flowers she had in the beginning = 48

21. Two supplementary angles differ by 50° . Find the measure of each angle.

Solution:

Let the measure of angle be x

Then, its supplementary angle = $180^\circ - x$

According to the given problem,

$$x - (180^\circ - x) = 50^\circ$$

On simplification, we get,

$$x - 180^\circ + x = 50^\circ$$

$$2x - 180^\circ = 50^\circ$$

$$2x = 180^\circ + 50^\circ$$

$$2x = 230^\circ$$

$$x = \frac{230^\circ}{2}$$

We get,

$$x = 115^\circ$$

Measurement of each angle = $x = 115^\circ$ and

$$(180^\circ - x) = 180^\circ - 115^\circ$$

$$= 65^\circ$$

Therefore, the measurement of each angel is 115° and 65°

22. If the angles of a triangle are in the ratio 5: 6: 7, find the angles.

Solution:

Let the angles of a triangle are $5x$, $6x$ and $7x$

We know that,

$$5x + 6x + 7x = 180^0$$

$$18x = 180^0$$

$$x = \frac{180^0}{18}$$

We get,

$$x = 10^0$$

Now, the angles of a triangle are,

$$5x = 5 \times 10^0 = 50^0$$

$$6x = 6 \times 10^0 = 60^0$$

$$7x = 7 \times 10^0 = 70^0$$

Therefore, the angles of a triangle are 50^0 , 60^0 and 70^0

23. Two equal sides of an isosceles triangle are $3x - 1$ and $2x + 2$ units. The third side is $2x$ units. Find x and the perimeter of the triangle.

Solution:

Given

Two equal sides of an isosceles triangle are $3x - 1$ and $2x + 2$

That is, $3x - 1 = 2x + 2$

$$3x - 2x = 2 + 1$$

$$x = 3$$

Given that,

Third side of triangle = $2x$

$$= 2 \times 3$$

$$= 6 \text{ units}$$

Equal sides of a triangle = $3x - 1$

$$= 3 \times 3 - 1$$

$$= 9 - 1$$

$$= 8 \text{ units}$$

Perimeter of the triangle = $8 + 8 + 6$

$$= 22 \text{ units}$$

Therefore, the perimeter of the triangle = 22 units

24. If each side of a triangle is increased by 4 cm, the ratio of the perimeters of the new triangle and the given triangle is 7: 5. Find the perimeter of the given triangle.

Solution:

Let the perimeter of original triangle be x cm

If each side of a triangle is increased by 4, then,

The perimeter will be = $x + 4 \times 3 = (x + 12)$ cm

Now,

Ratio of perimeter of new triangle and given triangle = 7: 5

$$\frac{(x + 12)}{x} = \frac{7}{5}$$

On cross multiplication, we get,

$$5(x + 12) = 7x$$

$$5x + 60 = 7x$$

$$7x - 5x = 60$$

$$2x = 60$$

$$x = \frac{60}{2}$$

We get,

$$x = 30$$

Therefore, the perimeter of the given triangle is 30 cm

25. The length of a rectangle is 5 cm less than twice its breadth. If the length is decreased by 3 cm and breadth increased by 2 cm, the perimeter of the resulting rectangle is 72 cm. find the area of the original rectangle.

Solution:

Let the breadth of the original rectangle be x cm

Then, length of the original rectangle will be $(2x - 5)$ cm

If the length is decreased by 3 cm, then,

$$\text{New length} = \{(2x - 5) - 3\} = (2x - 8) \text{ cm}$$

If breadth is increased by 2 cm, then,

$$\text{New breadth} = (x + 2) \text{ cm}$$

$$\text{New perimeter} = 2 (\text{new length} + \text{new breadth})$$

$$= 2 \{(2x - 8) + (x + 2)\}$$

$$= 2 (2x - 8 + x + 2)$$

$$= 2 (3x - 6)$$

We get,

$$= 6x - 12$$

According to the given problem,

$$6x - 12 = 72$$

$$6x = 72 + 12$$

$$6x = 84$$

$$x = \frac{84}{6}$$

We get,

$$x = 14$$

Breadth of the original rectangle = 14 cm and

Length of the original rectangle = $(2x - 5)$

$$= 2 \times 14 - 5$$

$$= 28 - 5$$

$$= 23 \text{ cm}$$

Area of original rectangle = Length $\times$ Breadth

$$= (23 \times 14) \text{ cm}^2$$

$$= 322 \text{ cm}^2$$

Therefore, area of the original rectangle is 322 cm^2

26. A rectangle is 10 cm long and 8 cm wide. When each side of the rectangle is increased by x cm, its perimeter is doubled. Find the equation in x and hence find the area of the new rectangle.

Solution:

Given

Length of rectangle (l) = 10 cm and

Breadth of the rectangle = 8 cm

Perimeter = 2 (Length + Breadth)

$$= 2 (10 + 8) \text{ cm}$$

$$= 2 \times 18$$

$$= 36 \text{ cm}$$

If each side of the rectangle is increased by x cm, then,

Perimeter = 2 (10 + x + 8 + x)

$$= 2 (18 + 2x)$$

$$= (36 + 4x) \text{ cm}$$

According to the given condition,

$$36 + 4x = 2 (36)$$

$$36 + 4x = 72$$

$$4x = 72 - 36$$

$$4x = 36$$

$$x = \frac{36}{4}$$

We get,

$$x = 9$$

Hence,

Length of new rectangle = l + x = 10 + 9 = 19 cm and

Breadth of new rectangle = b + x = 8 + 9 = 17 cm

$$\text{Area} = \text{Length} \times \text{Breadth} = 19 \times 17 \text{ cm}^2 = 323 \text{ cm}^2$$

27. A steamer travels 90 km downstream in the same time as it takes to travel 60 km upstream. If the speed of the steamer is 5 km/ hr, find the speed of the steamer in still water.

Solution:

Let the speed of the steamer = x km/ h

The speed downstream = $(x + 5)$ km/h and

The speed upstream = $(x - 5)$ km/ h

According to the given problem,

$$\frac{90}{x+5} = \frac{60}{x-5}$$

On cross multiplication, we get,

$$90(x - 5) = 60(x + 5)$$

$$90x - 450 = 60x + 300$$

$$90x - 60x = 300 + 450$$

We get,

$$30x = 750$$

$$x = \frac{750}{30}$$

$$x = 25$$

Therefore, the speed of the steamer in still water is 25 km/ h

28. A steamer goes downstream and covers the distance between two ports in 5 hours while it covers the same distance upstream in 6 hours. If the speed of the stream is 1 km/h, find the speed of the steamer in still water and the distance between two ports.

Solution:

Given

Speed of the stream in still water = 1 km/h

Let speed of the streamer = x km/h

Speed downstream = $(x + 1)$ km/h

Speed upstream = $(x - 1)$ km/h

According to the given condition,

$$(x + 1) \times 5 = (x - 1) \times 6$$

$$5x + 5 = 6x - 6$$

Calculating further, we get,

$$6x - 5x = 5 + 6$$

$$x = 11$$

Hence,

Speed of streamer in still water is 11 km/h and

Distance between two ports = $(11 + 1) \times 5 = 60$ km/h

29. Distance between two places A and B is 350 km. Two cars start simultaneously from A and B towards each other and the distance between them after 4 hours is 62 km. If the speed of one car is 8 km/h less than the speed of other cars, find the speed of each car.

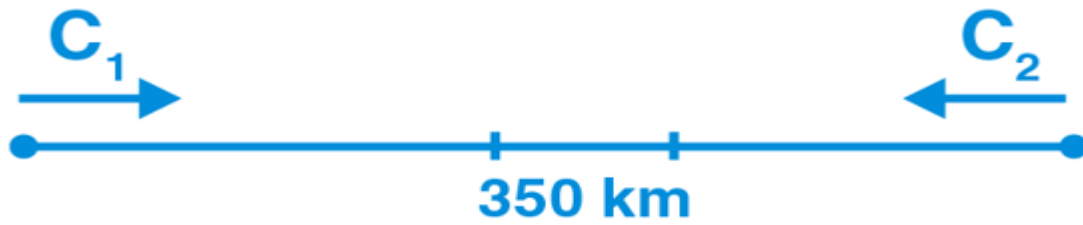
Solution:

Given

Distance between two places A and B = 350 km

Let the speed of car $C_1 = x$ km/h and

Speed of car $C_2 = (x - 8)$ km/h



After 4 hours, the distance between two cars is 62 km

Hence,

$$x \times 4 + (x - 8) \times 4 = 350 - 62$$

$$4x + 4x - 32 = 288$$

$$8x = 288 + 32$$

$$8x = 320$$

$$x = 320 / 8$$

We get,

$$x = 40$$

Therefore, speed of car $C_1 = 40$ km/h

Speed of car $C_2 = (x - 8) = (40 - 8) = 32$ km/h

Exercise 12.3

1. If the replacement set = $\{-7, -5, -3, -1, 1, 3\}$, find the solution set of:

(i) $x > -2$

(ii) $x < -2$

(iii) $x > 2$

(iv) $-5 < x \leq 5$

(v) $-8 < x < 1$

(vi) $0 \leq x \leq 4$

Solution:

Given

Replacement set = $\{-7, -5, -3, -1, 1, 3\}$

The solution set for the given replacement set is as follows:

(i) Solution set of $x > -2$ is $\{-1, 0, 1, 3\}$

(ii) Solution set of $x < -2$ is $\{-7, -5, -3\}$

(iii) Solution set of $x > 2$ is $\{3\}$

(iv) Solution set of $-5 < x \leq 5$ is $\{-3, -1, 0, 1, 3\}$

(v) Solution set of $-8 < x < 1$ is $\{-7, -5, -3, -1, 0\}$

(vi) Solution set of $0 \leq x \leq 4$ is $\{0, 1, 3\}$

2. Represent the solution of the following inequalities graphically:

(i) $x \leq 4, x \in \mathbb{N}$

(ii) $x < 5, x \in \mathbb{W}$

(iii) $-3 \leq x < 3, x \in \mathbb{I}$

Solution:

(i) Given

$$x \leq 4, x \in \mathbb{N}$$

The solution set = $\{1, 2, 3, 4\}$

These four numbers are shown indicating with thick dots on the number line given below



(ii) Given

$$x < 5, x \in \mathbb{W}$$

The solution set = $\{0, 1, 2, 3, 4\}$

These five numbers are shown indicating with thick dots on the number line given below



(iii) Given

$$-3 \leq x < 3, x \in \mathbb{I}$$

The solution set = $\{-3, -2, -1, 0, 1, 2\}$

These six numbers are shown indicating with thick dots on the number line given below



3. If the replacement set is $\{-6, -4, -2, 0, 2, 4, 6\}$; then represent the solution set of the inequality $-4 \leq x < 4$ graphically.

Solution:

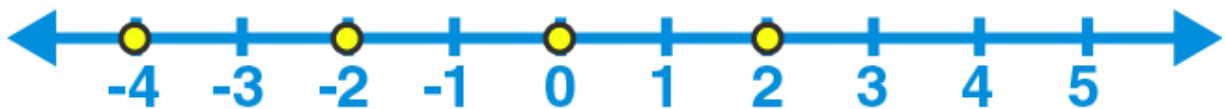
Given

Replacement set = $\{-6, -4, -2, 0, 2, 4, 6\}$ and

Inequality = $-4 \leq x < 4$

Solution set = $\{-4, -2, 0, 2\}$

Graphically representation of solution set is as follows:



4. Find the solution set of the inequality $x < 4$ if the replacement set is

(i) $\{1, 2, 3 \dots 10\}$

(ii) $\{-1, 0, 1, 2, 5, 8\}$

(iii) $\{-5, 10\}$

(iv) $\{5, 6, 7, 8, 9, 10\}$

Solution:

(i) Given

Inequality = $x < 4$

Replacement set = $\{1, 2, 3 \dots 10\}$

Therefore, solution set = $\{1, 2, 3\}$

(ii) Given

Inequality = $x < 4$

Replacement set = $\{-1, 0, 1, 2, 5, 8\}$

Therefore, solution set = $\{-1, 0, 1, 2\}$

(iii) Given

Inequality = $x < 4$

Replacement set = $\{-5, 10\}$

Therefore, solution set = $\{-5\}$

(iv) Given

Inequality = $x < 4$

Replacement set = $\{5, 6, 7, 8, 9, 10\}$

Therefore, solution set = ϕ

5. If the replacement set = $\{-6, -3, 0, 3, 6, 9, 12\}$, find the truth set of the following:

(i) $2x - 3 > 7$

(ii) $3x + 8 \leq 2$

(iii) $-3 < 1 - 2x$

Solution:

Given

Replacement set = $\{-6, -3, 0, 3, 6, 9, 12\}$

(i) $2x - 3 > 7$

$$2x > 7 + 3$$

$$2x > 10$$

$$x > 10 / 2$$

We get,

$$x > 5$$

Therefore, solution set = $\{6, 9, 12\}$

$$(ii) 3x + 8 \leq 2$$

$$3x \leq 2 - 8$$

$$3x \leq -6$$

$$x \leq -6 / 3$$

We get,

$$x \leq -2$$

Therefore, solution set = $\{-6, -3\}$

$$(iii) -3 < 1 - 2x$$

$$2x - 3 < 1$$

$$2x < 1 + 3$$

$$2x < 4$$

$$x < \frac{4}{2}$$

We get,

$$x < 2$$

Therefore, solution set = $\{-6, -3, 0\}$

6. Solve the following inequations:

(i) $4x + 1 < 17, x \in \mathbf{N}$

(ii) $4x + 1 \leq 17, x \in \mathbf{W}$

(iii) $4 > 3x - 11, x \in \mathbf{N}$

(iv) $-17 \leq 9x - 8, x \in \mathbf{Z}$

Solution:

(i) $4x + 1 < 17$

$$4x < 17 - 1$$

$$4x < 16$$

$$x < \frac{16}{4}$$

We get,

$$x < 4$$

As $x \in \mathbf{N}$

Hence, solution set = $\{1, 2, 3\}$

(ii) $4x + 1 \leq 17$

$$4x \leq 17 - 1$$

$$4x \leq 16$$

$$x \leq \frac{16}{4}$$

We get,

$$x \leq 4$$

As $x \in \mathbf{W}$

Hence, solution set = $\{0, 1, 2, 3, 4\}$

(iii) $4 > 3x - 11$

$$4 + 11 > 3x$$

$$15 > 3x$$

$$\frac{15}{3} > x$$

We get,

$$5 > x$$

$$x < 5$$

As $x \in \mathbb{N}$

Hence, solution set = $\{1, 2, 3, 4\}$

$$\text{(iv) } -17 \leq 9x - 8$$

$$-17 + 8 \leq 9x$$

$$-9 \leq 9x$$

$$\frac{-9}{9} \leq x$$

$$-1 \leq x$$

$$x \geq -1$$

As $x \in \mathbb{Z}$

Hence, solution set = $\{-1, 0, 1, 2 \dots\}$

7. Solve the following inequations:

$$\text{(i) } \left\{ \frac{(2y - 1)}{5} \right\} \leq 2, y \in \mathbb{N}$$

$$\text{(ii) } \left\{ \frac{(2y + 1)}{3} \right\} + 1 \leq 3, y \in \mathbb{W}$$

$$\text{(iii) } \left(\frac{2}{3} \right) p + 5 < 9, p \in \mathbb{W}$$

$$(iv) - 2(p + 3) > 5, p \in \mathbb{I}$$

Solution:

$$(i) \left\{ \frac{(2y - 1)}{5} \right\} \leq 2$$

$$2y - 1 \leq 2 \times 5$$

$$2y - 1 \leq 10$$

$$2y \leq 10 + 1$$

$$2y \leq 11$$

We get,

$$y \leq \frac{11}{2}$$

As $y \in \mathbb{N}$,

Therefore, solution set = $\{1, 2, 3, 4, 5\}$

$$(ii) \left\{ \frac{(2y + 1)}{3} \right\} + 1 \leq 3$$

$$\left\{ \frac{(2y + 1 + 3)}{3} \right\} \leq 3$$

$$\left\{ \frac{(2y + 4)}{3} \right\} \leq 3$$

$$(2y + 4) \leq 3 \times 3$$

$$2y + 4 \leq 9$$

$$2y \leq 9 - 4$$

$$2y \leq 5$$

We get,

$$y \leq \frac{5}{2}$$

As $y \in \mathbb{N}$,

Therefore, solution set = $\{0, 1, 2\}$

$$\text{(iii) } \left(\frac{2}{3}\right)p + 5 < 9$$

$$\left(\frac{2}{3}\right)p < 9 - 5$$

$$\left(\frac{2}{3}\right)p < 4$$

$$2p < 4 \times 3$$

$$2p < 12$$

We get,

$$p < \frac{12}{2}$$

$$p < 6$$

As $p \in W$,

Therefore, solution set = $\{0, 1, 2, 3, 4, 5\}$

$$\text{(iv) } -2(p + 3) > 5$$

$$-2p - 6 > 5$$

$$-2p > 5 + 6$$

$$-2p > 11$$

We get,

$$p > \left(\frac{11}{-2}\right)$$

$$p > \frac{-11}{2}$$

As $p \in I$,

Therefore, solution set = $\{\dots -8, -7, -6\}$

8. Solve the following inequations:

(i) $2x - 3 < x + 2, x \in \mathbb{N}$

(ii) $3 - x \leq 5 - 3x, x \in \mathbb{W}$

(iii) $3(x - 2) < 2(x - 1), x \in \mathbb{W}$

(iv) $\left(\frac{3}{2}\right) - \left(\frac{x}{2}\right) > -1, x \in \mathbb{N}$

Solution:

(i) $2x - 3 < x + 2$

$$2x - x < 2 + 3$$

We get,

$$x < 5$$

As $x \in \mathbb{N}$,

Hence, solution set = $\{1, 2, 3, 4\}$

(ii) $3 - x \leq 5 - 3x$

$$3x - x \leq 5 - 3$$

$$2x \leq 2$$

We get,

$$x \leq 1$$

As $x \in \mathbb{W}$,

Hence, solution set = $\{0, 1\}$

(iii) $3(x - 2) < 2(x - 1)$

$$3x - 6 < 2x - 2$$

$$3x - 2x < -2 + 6$$

We get,

$$x < 4$$

As $x \in W$,

Hence, solution set = $\{0, 1, 2, 3\}$

$$(iv) \left(\frac{3}{2}\right) - \left(\frac{x}{2}\right) > -1$$

$$\left(\frac{3}{2}\right) + 1 > \left(\frac{x}{2}\right)$$

$$\left\{\frac{(3+2)}{2}\right\} > \left(\frac{x}{2}\right)$$

$$\left(\frac{5}{2}\right) > \left(\frac{x}{2}\right)$$

So,

$$5 > x$$

$$x < 5$$

As $x \in N$,

Hence, solution set = $\{1, 2, 3, 4\}$

9. If the replacement set is $\{-3, -2, -1, 0, 1, 2, 3\}$, solve the inequation $\left\{\frac{(3x-1)}{2}\right\} < 2$. Represent its solution on the number line.

Solution:

Given

Replacement set = $\{-3, -2, -1, 0, 1, 2, 3\}$ and

$$\text{Inequation} = \left\{\frac{(3x-1)}{2}\right\} < 2$$

$$3x - 1 < 2 \times 2$$

$$3x - 1 < 4$$

$$3x < 4 + 1$$

$$3x < 5$$

We get,

$$x < \frac{5}{3}$$

Therefore, solution set = $\{\dots -3, -2, -1, 0, 1\}$

Graphical representation of this solution set is as follows:



10. Solve $\left(\frac{x}{3}\right) + \left(\frac{1}{4}\right) < \left(\frac{x}{6}\right) + \left(\frac{1}{2}\right)$, $x \in \mathbb{W}$. Also represent its solution on the number line.

Solution:

Given

$$\left(\frac{x}{3}\right) + \left(\frac{1}{4}\right) < \left(\frac{x}{6}\right) + \left(\frac{1}{2}\right)$$

$$\left(\frac{x}{3}\right) - \left(\frac{x}{6}\right) < \left(\frac{1}{2}\right) - \left(\frac{1}{4}\right)$$

$$\frac{(2x - x)}{6} < \frac{(2 - 1)}{4}$$

$$\frac{x}{6} < \frac{1}{4}$$

$$x < \frac{6}{4}$$

We get,

$$x < \frac{3}{2}$$

As $x \in W$,

Hence, solution set = $\{0, 1\}$

Graphical representation of this solution set is as follows:



11. Solve the following inequations and graph their solutions on a number line

(i) $-4 \leq 4x < 14, x \in N$

(ii) $-1 < \left(\frac{x}{2}\right) + 1 \leq 3, x \in I$

Solution:

(i) Given

$$-4 \leq 4x < 14$$

Dividing by 4, we get,

$$\left(\frac{-4}{4}\right) \leq \left(\frac{4}{4}x\right) < \left(\frac{14}{4}\right)$$

$$-1 \leq x < \frac{7}{2}$$

As $x \in N$,

Therefore, solution set = $\{1, 2, 3\}$

The graphical representation for this solution set is as follows:



(ii) Given

$$-1 < \left(\frac{x}{2}\right) + 1 \leq 3$$

By subtracting -1 , we get,

$$-1 - 1 < \left\{\left(\frac{x}{2}\right) + 1\right\} - 1 \leq 3 - 1$$

$$-2 < \left(\frac{x}{2}\right) \leq -2$$

Multiplying by 2, we get,

$$-2 \times 2 < \left(\frac{x}{2}\right) \times 2 \leq -2 \times 2$$

$$-4 < x \leq -4$$

As $x \in \mathbb{I}$,

Therefore, solution set = $\{-3, -2, -1, 0, 1, 2, 3, 4\}$

The graphical representation for this solution set is as follows:



Mental Maths

Question 1: Fill in the blanks:

- (i) An equation of the type $ax + b = 0$ where $a \neq 0$ is called a in variable x .
- (ii) Any value of the variable which satisfies the equation is called a of the equation.
- (iii) The process of finding all the solutions of an equation is called
- (iv) We can add the to both sides of an equation.
- (v) We can divide both sides of an equation by the same number.
- (vi) The solution set of the inequality $3x \leq 10$, $x \in \mathbf{N}$ is

Solution:

- (i) An equation of the type $ax + b = 0$ where $a \neq 0$ is called a linear equation in variable x .
- (ii) Any value of the variable which satisfies the equation is called a solution of the equation.
- (iii) The process of finding all the solutions of an equation is called solving the equation.
- (iv) We can add the same number to both sides of an equation.
- (v) We can divide both sides of an equation by the same non-zero number.
- (vi) The solution set of the inequality $3x \leq 10$, $x \in \mathbf{N}$ is $(1, 2, 3)$.

Question 2: State whether the following statements are true (T) or false (F):

- (i) An equation is a statement that two expressions are equal.
- (ii) A term may be transposed from-one side of the equation to the other side, but its sign will not change.
- (iii) We cannot subtract the same number from both sides of an

equation.

(iv) $3x + 2 = 4(x + 7) + 9$ is a linear equation in variable x .

(v) $x = 1$ is the solution of equation $4(x + 5) = 24$.

Solution:

(i) An equation is a statement that two expressions are equal. True.

(ii) A term may be transposed from one side of the equation to the other side, but its sign will not change. False

Correct:

The sign will change.

(iii) We cannot subtract the same number from both sides of an equation. False

Correct:

We can subtract.

(iv) $3x + 2 = 4(x + 7) + 9$ is a linear equation in variable x . True.

(v) $x = 1$ is the solution of equation $4(x + 5) = 24$. True

$$4(1 + 5) = 24 \Rightarrow 4 \times 6 = 24$$

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 16):

Question 3: Which of the following is not a linear equation in one variable?

(a) $3x + 2 = 0$

(b) $2y - 4 = y$

(c) $x + 2y = 7$

(d) $2(x - 3) + 7 = 0$

Solution:

$x + 2y = 7$ is not a linear equation in one variable as there are two variables x and y . (c)

Question 4: The solution of the equation $\frac{2}{3}x+1=\frac{15}{9}$ is

(a) 1

(b) $\frac{3}{2}$

(c) 2

(d) $\frac{2}{3}$

Solution:

The solution of the equation

$$\frac{2}{3}x + 1 = \frac{15}{9}$$

$$\Rightarrow \frac{2}{3}x = \frac{15}{9} - 1 = \frac{15-9}{9} = \frac{6}{9}$$

$$x = \frac{6}{9} \times \frac{3}{2} = 1$$

$$x = 1 \quad (a)$$

Question 5: The solution of the equation $4z + 3 = 6 + 2z$ is

(a) 1

(b) $\frac{3}{2}$

(c) 2

(d) 3

Solution:

Solution of equation $4z + 3 = 6 + 2z$

$$\Rightarrow 4z - 2z = 6 - 3 \Rightarrow 2z = 3 \Rightarrow z = \frac{3}{2} \quad (b)$$

Question 6: The solution of the equation $\frac{3x}{5} + 1 = \frac{4x}{15}$ is +7 is

(a) 12

(b) 14

(c) 16

(d) 18

Solution:

$$\frac{3x}{5} + 1 = \frac{4x}{15} + 7$$

$$\frac{3x}{5} - \frac{4x}{15} = 7 - 1$$

$$= \frac{9x-4x}{15} = 6 \Rightarrow \frac{5x}{15} = 6$$

$$x = \frac{6 \times 15}{5} = 18 \Rightarrow x = 18 \quad (d)$$

Question 7: The solution of the equation $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$ is

(a) 2.7

(b) 1.8

(c) 2.9

(d) 1.7

Solution:

The solution of the equation

$$\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$$

$$\Rightarrow \frac{x}{2} - \frac{x}{3} = \frac{1}{4} + \frac{1}{5}$$

$$\Rightarrow \frac{3-2}{6}x = \frac{5+4}{20} \Rightarrow \frac{1}{6}x = \frac{9}{20}$$

$$x = \frac{9}{20} \times \frac{6}{1} = \frac{27}{10} = 2.7 \quad (a)$$

Question 8: The solution of the $\frac{8x-3}{3x} = 2$ is

(a) $\frac{1}{2}$

(b) $\frac{2}{3}$

(c) $\frac{3}{2}$

(d) $\frac{1}{3}$

Solution:

$$\frac{8x-3}{3x} = 2$$

$$8x - 3 = 6x \Rightarrow 8x - 6x = 3$$

$$\Rightarrow 2x = 3 \Rightarrow x = \frac{3}{2} \quad (\text{c})$$

Question 9: If we subtract $\frac{1}{2}$ from a number and multiply the result by $\frac{1}{2}$, we get $\frac{1}{8}$, then the number is

(a) $\frac{1}{2}$

(b) $\frac{3}{4}$

(c) $\frac{1}{4}$

(d) none of these

Solution:

Let number be x , then

$$\left(x - \frac{1}{2}\right) \frac{1}{2} = \frac{1}{8}$$

$$\Rightarrow x - \frac{1}{2} = \frac{1}{8} \times \frac{2}{1} = \frac{1}{4}$$

$$\Rightarrow x = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

$$\therefore x = \frac{3}{4} \quad (\text{b})$$

Question 10: Fifteen years from now Ravi's age will be four times his present age. What is Ravi's present age?

(a) 4 years

(b) 5 years

(c) 6 years

(d) 3 years

Solution:

Let present age of Ravi = x years

After 15 years, his age will be = $(x + 5)$ years

$$\therefore x + 15 = 4x$$

$$\Rightarrow 15 = 4x - x = 3x$$

$$\Rightarrow x = \frac{15}{3} = 5$$

$\therefore$ His present age = 5 years (b)

Question 11: If the sum of three consecutive integers is 51, then the largest integer is

(a) 16

(b) 17

(c) 18

(d) 19

Solution:

Let first integers = x

Then next two integers = $x + 1, x + 2$

$$\therefore x + x + 1 + x + 2 = 51$$

$$\Rightarrow 3x + 3 = 51$$

$$\Rightarrow 3x = 51 - 3 = 48$$

$$\Rightarrow x = \frac{48}{3} = 16$$

$\therefore$ First integer = 16 and other two integer = 17, 18

Largest integers = 18 (c)

Question 12: If the perimeter of a rectangle is 13 cm and its Width is $2\frac{3}{4}$ cm, then its length is

(a) $2\frac{3}{4}$ cm

(b) $3\frac{3}{4}$ cm

(c) $4\frac{3}{4}$ cm

(d) $5\frac{3}{4}$ cm

Solution:

Perimeter of a rectangle = 13 cm

$$\text{Width} = 2\frac{3}{4} = \frac{11}{4} \text{ cm}$$

Let length = x cm

$$\therefore \text{Perimeter} = 2(\text{Length} + \text{Breadth})$$

$$\Rightarrow 13 = 2\left(x + \frac{11}{4}\right)$$

$$\Rightarrow \frac{13}{2} = x + \frac{11}{4}$$

$$\Rightarrow x = \frac{13}{2} - \frac{11}{4} = \frac{26-11}{4} = \frac{15}{4}$$

$$\therefore \text{Length} = \frac{15}{4} = 3\frac{3}{4} \text{ cm} \quad (\text{b})$$

Question 13: What should be added to twice the rational number $\frac{-7}{3}$ to get $\frac{3}{7}$?

(a) $\frac{58}{21}$

(b) $\frac{29}{21}$

(c) $\frac{89}{21}$

(d) $\frac{107}{21}$

Solution:

Let x be added

According to the condition,

$$\therefore \frac{-7}{3} \times 2 + x = \frac{3}{7}$$

$$\frac{-14}{3} + x = \frac{3}{7} \Rightarrow x = \frac{3}{7} + \frac{14}{3}$$

$$\therefore x = \frac{9+98}{21} = \frac{107}{21} \quad (\text{d})$$

Question 14: Sum of digits of a two digit number is 8. If the number obtained by reversing the digits is 18 more than the original number, then the original number is

- (a) 35**
- (b) 53**
- (c) 26**
- (d) 62**

Solution:

Sum of digits of a two digit number = 8

Let unit digit = x

Then tens digit = $8 - x$

$$\therefore \text{Number} = x + 10(8 - x) = x + 80 - 10x = 80 - 9x$$

By reversing the digits,

Unit digit = $8 - x$

and tens digit = x

$$\therefore \text{Number} = 8 - x + 10x = 8 + 9x$$

$$\therefore 8 + 9x = 80 - 9x + 18$$

$$\Rightarrow 9x + 9x = 80 + 18 - 8$$

$$\Rightarrow 18x = 90$$

$$\Rightarrow x = \frac{90}{18} = 5$$

$$\therefore \text{Number} = 80 - 9x = 80 - 9 \times 5 = 80 - 45 = 35 \text{ (a)}$$

Question 15: Arjun is twice as old as Shriya. If five years ago his age was three times Shriya's age, then Arjun's present age is

- (a) 10 years**
- (b) 15 years**
- (c) 20 years**
- (d) 25 years**

Solution:

Let Shriya's age = x years

Then Arjun's age = $2x$

5 years ago,

Age of Shriya was = $(x - 5)$ years

and age of Arjun's = $(2x - 5)$ years

$$\therefore 2x - 5 = 3(x - 5)$$

$$\Rightarrow 2x - 5 = 3x - 15$$

$$\Rightarrow 3x - 2x = 15 - 5 = 10$$

$$\Rightarrow x = 10$$

$$\therefore \text{Arjun's present age} = 2x = 2 \times 10 = 20 \text{ years (c)}$$

Question 16: If the replacement set is $\{-5, -3, -1, 0, 1, 3\}$, then the solution set of the inequation $-3 < x < 3$ is

(a) $\{-2, -1, 0, 1, 2\}$

(b) $\{-1, 0, 1, 2\}$

(c) $\{-3, -1, 0, 1, 3\}$

(d) $\{-1, 0, 1\}$

Solution:

Replacement set = $\{-5, -3, -1, 0, 1, 3\}$

$$-3 < x < 3$$

$\therefore x = \{-2, -1, 0, 1, 2\}$ from the replacement set,

Solution set $x = \{-1, 0, 1\}$ (d)

Value Based Questions

Question 1: Seema is habitual of saving her pocket money. She collected some 50 paise and 25 paise coins in her piggy bank. If she collected ₹25 and number of 50 paise coins is double the number of 25 paise coins. How many coins of each type did she collect? What values are being promoted? Is saving a good habit?

Solution:

Seema has 50 paise and 25 paise coins in his piggy bank.

Total amount = ₹25

Let 25 paise coins = x

Then 50 paise coins = $2x$

$$\frac{x \times 25}{100} + \frac{2x \times 25}{100} = 25$$

$$\frac{x}{4} + x = 25 \Rightarrow \frac{x+4x}{4} = 25$$

$$\Rightarrow \frac{5x}{4} = 25 \Rightarrow x = \frac{25 \times 4}{5} = 20$$

$\therefore$ 25-paise coins = 20 and 50 paise coins = $20 \times 2 = 40$

This is a good habit to save some money, we can solve any financial problem with its help at any time.

Question 2: Ramesh gave one-fourth of his property to his two sons in equal shares and rest to his wife Sunita. Sunita gave one-third of her share to an orphanage. If the amount given by Sunita to the orphanage was ₹20000, find the total value of the Ramesh's property and the amount each person got? What value is shown by the Sunita?

Solution:

Let Ramesh's property = x

$\frac{1}{4}$ th part of property was given to two sons equally = $\frac{x}{4}$

So, each son's share = $\frac{x}{4} \times \frac{1}{2} = \frac{x}{8}$

Rest to his wife Sunita = $x - \frac{1}{4}x = \frac{3}{4}x$

Sunita gave one third of her share to orphanages

Property given to orphanage $\frac{1}{3}$ of $\frac{3}{4}x = \frac{1}{4}x$

$\therefore \frac{1}{4}x = ₹20000$

$\therefore$ Total value of Ramesh property = ₹20000 $\times \frac{1}{4}$ = ₹80000

Each son will get = ₹ $\frac{x}{8}$ $\times$ 80000 = ₹ 10000

and wife will get = ₹80000 $\times \frac{3}{4}$ = ₹60000

Sunita done a good deed to help the orphanage
where needy person are living and they need your help and support.

Higher Order Thinking Skills (Hots)

Question 1: A man covers a distance of 24 km in $3\frac{1}{2}$ hours partly on foot at the speed of 4.5 km/h and partly on bicycle at the speed of 10 km/h. Find the distance covered on foot.

Solution:

Total distance = 24 km

Time taken = $3\frac{1}{2}$ hours = $\frac{7}{2}$ hours

Let a man travels x km on foot at the speed of 4.5 km
and (24 – x) km on bicycle at the speed of 10 km/hr

$$\therefore \frac{x}{4.5} + \frac{24-x}{10} = \frac{7}{2}$$

$$\Rightarrow \frac{2x}{9} + \frac{24-x}{10} = \frac{7}{2}$$

$$\Rightarrow \frac{20x+216-9x}{90} = \frac{7}{2}$$

$$\Rightarrow 11x + 216 = \frac{7}{2} \times 90$$

$$\Rightarrow 11x = 315 - 216 = 99$$

$$x = \frac{99}{11} = 9$$

$\therefore$ He travelled 9 km on foot.

Question 2: The perimeter of a rectangle is 240 cm. If its length is decreased by 10% and breadth is increased by 20% we get the same perimeter. Find the original length and breadth of the rectangle.

Solution:

Perimeter of a rectangle = 240 cm

$$\therefore \text{Length} + \text{breadth} = \frac{240}{2} = 120 \text{ cm}$$

Let length = x cm

Then breadth = $(120 - x)$ cm

By decreasing length by 10%
and increasing breadth by 20%, we get

$$\begin{aligned} \text{Length} &= x \times \frac{(100-10)}{100} \\ &= \frac{x \times 9}{100} = \frac{9}{10} x \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{And breadth} &= (120 - x) \times \frac{(100+20)}{100} \\ &= \frac{120}{100} (120 - x) = \frac{12}{10} (120 - x) \end{aligned}$$

According to the condition,

$$\text{Length} + \text{Breadth} = 120$$

$$\Rightarrow \frac{9}{10} x = \frac{(120-x) \times 12}{10} = 120$$

$$\Rightarrow 9x + 1440 - 12x = 1200$$

$$\Rightarrow -3x = 1200 - 1440 = -240$$

$$\Rightarrow 3x = 240 \Rightarrow x = \frac{240}{3} = 80$$

$$\therefore \text{Length} = 80 \text{ cm and breadth} = 120 - 80 = 40 \text{ cm}$$

Question 3: A person preparing a medicine wants to convert 15% alcohol solution into 32% alcohol solution. Find how much pure

alcohol he should mix in 400 mL of 15% alcohol solution to obtain required solution?

Solution:

15% of alcohol mixture = 400 mL

$$\therefore \text{Alcohol} = \frac{15}{100} \times 400 = 60 \text{ mL}$$

and other solution = $400 - 60 = 340 \text{ mL}$

In new mixture alcohol = 32%

Other solution = $100 - 32 = 68\%$

In 86 mL, alcohol = 32

and in 340 mL, alcohol will be = $\frac{32 \times 340}{68} = 160 \text{ mL}$

Already alcohol = 60 mL

$\therefore$ More alcohol required = $160 - 60 = 100 \text{ mL}$

Question 4: Rahul covers a distance from P to Q on bicycle at 10 km/h and returns back at 9 km/h. Anuj covers the distance from P to Q and Q to P both at 12 km/h. On calculating we find that Anuj took 10 minutes less than Rahul. Find the distance between P and Q.

Solution:

Let distance between P and Q = $x \text{ km}$

Speed of Rahul from P to Q = 10 km/h

and back Q to P = 9 km/h

$$\therefore \text{Time taken both ways} = \left(\frac{x}{10} + \frac{x}{9} \right) \text{ hour}$$

Speed of Anuj both ways = 12 km/h

$$\therefore \text{Time taken} = \frac{2x}{12} = \frac{x}{6} \text{ hours}$$

According to the condition,

$$\Rightarrow \left(\frac{x}{10} + \frac{x}{9} \right) - \frac{x}{6} = \frac{10}{60} = \frac{1}{6}$$

$$\Rightarrow \frac{x}{10} + \frac{x}{9} - \frac{x}{6} = \frac{1}{6}$$

$$\Rightarrow \frac{9x+10x-15x}{90} = \frac{1}{6}$$

$$4x = \frac{1}{6} \times 90$$

$$\Rightarrow 4x = 15$$

$$x = \frac{15}{4} = 3.75$$

Distance between P and Q = 3.75 km.

Question 5: Solve:

$$\frac{x-b-c}{a} + \frac{x-c-a}{b} + \frac{x-a-b}{c} = 3$$

Solution:

$$\frac{x-b-c}{a} + \frac{x-c-a}{b} + \frac{x-a-b}{c} = 3$$

Subtracting 3 from both sides, we get

$$\Rightarrow \frac{x-b-c}{a} - 1 + \frac{x-c-a}{b} - 1 + \frac{x-a-b}{c} - 1 = 0$$

$$\Rightarrow \frac{x-b-c-a}{a} + \frac{x-c-a-b}{b} + \frac{x-a-b-c}{c} = 0$$

$$\Rightarrow (x - a - b - c) \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) = 0$$

$$\Rightarrow x - a - b - c = 0$$

$$\Rightarrow x = a + b + c$$

Check Your Progress

Question 1: Solve the following equations:

- i. $\frac{3}{5}x + 2 = \frac{2}{3}x$
- ii. $\frac{x}{2} - \frac{3x}{4} + \frac{x}{6} = \frac{-2}{3}$
- iii. $5x - 3(4x - 3) = 2(4 - 5x) + 10$
- iv. $\frac{3x+2}{5x+4} = \frac{3}{4}$
- v. $\frac{2x-3}{5x-4} = \frac{2x+1}{5x+20}$
- vi. $\frac{2x+5}{2} - \frac{5x}{x-1} = x$

Solution:

i. $\frac{3}{5}x + 2 = \frac{2}{3}x$

$$\Rightarrow \frac{2}{3}x - \frac{3}{5}x = 2$$

$$\frac{10x-9x}{15} = 2 \Rightarrow \frac{x}{15} = 2 \Rightarrow x = 2 \times 15$$

$$\therefore x = 30$$

ii. $\frac{x}{2} - \frac{3x}{4} + \frac{x}{6} = \frac{-2}{3}$

$$\frac{6x-9x+2x}{12} = \frac{-2}{3}$$

$$\frac{-x}{12} = \frac{-2}{3} \Rightarrow -3x = -24$$

$$\Rightarrow x = \frac{24}{3} = 8$$

$$\Rightarrow x = 8$$

iii. $5x - 3(4x - 3) = 2(4 - 5x) + 10$

$$\Rightarrow 5x - 12x + 9 = 8 - 10x + 10$$

$$\Rightarrow 5x - 12x + 10x = 8 + 10 - 9$$

$$\Rightarrow 3x = 9$$

$$\Rightarrow x = \frac{9}{3} = 3$$

$$\therefore x = 3$$

$$\text{iv. } \frac{3x+2}{5x+4} = \frac{3}{4}$$

By cross multiplication

$$12x + 8 = 15x + 12$$

$$\Rightarrow 12x - 15x = 12 - 8$$

$$\Rightarrow -3x = 4$$

$$\Rightarrow x = \frac{4}{-3} = \frac{-4}{3}$$

$$\therefore x = \frac{-4}{3}$$

$$\text{v. } \frac{2x-3}{5x-4} = \frac{2x+1}{5x+20}$$

By cross multiplication

$$(2x - 3)(5x + 20) = (2x + 1)(5x - 4)$$

$$\Rightarrow 10x^2 + 40x - 15x - 60 = 10x^2 - 8x + 5x - 4$$

$$\Rightarrow 25x - 60 = -3x - 4$$

$$\Rightarrow 25x + 3x = -4 + 60$$

$$\Rightarrow 28x = 56$$

$$\Rightarrow x = \frac{56}{28} = 2$$

$$\therefore x = 2$$

$$\text{vi. } \frac{2x+5}{2} - \frac{5x}{x-1} = x$$

$$\frac{2x+5}{2} - \frac{5x}{x-1} = x$$

$$\Rightarrow \frac{(2x+5)(x-1)-5x \times 2}{2(x-1)} = x$$

$$\Rightarrow \frac{2x(x-1)+5(x-1)-10x}{2x-2} = x$$

$$\Rightarrow \frac{2x^2-2x+5x-5-10x}{2x-2} = x$$

$$\begin{aligned}
&\Rightarrow \frac{2x^2-7x-5}{2x-2} = x \\
&\Rightarrow 2x^2 - 7x - 5 = x(2x - 2) \\
&\Rightarrow 2x^2 - 7x - 5 = 2x^2 - 2x \\
&\Rightarrow 2x^2 - 7x - 2x^2 - 5 + 2x = 0 \\
&\Rightarrow -7x - 5 + 2x = 0 \\
&\Rightarrow -5x - 5 = 0 \\
&\Rightarrow -5x = 5 \\
&\Rightarrow x = \frac{5}{-5} \\
&\Rightarrow x = -1
\end{aligned}$$

Question 2: The sum of three consecutive multiples of 11 is 363. Find these multiples.

Solution:

Let first multiple of 11 = $11x$

Then second multiple = $11x + 11$

and third multiple = $11x + 22$

$$\therefore 11x + 11x + 11 + 11x + 22 = 363$$

$$\Rightarrow 33x + 33 = 363$$

$$\Rightarrow x + 1 = 11$$

$$\Rightarrow x = 11 - 1 = 10$$

$$\therefore \text{Multiples are } 11 \times 10 = 110$$

$$110 + 11 = 121$$

$$110 + 22 = 132$$

Hence number are 110, 121, 132

Question 3: Sum of two numbers is 95. If one exceeds the other by 15, find the numbers.

Solution:

Sum of two numbers = 95

Let first number = x

Then second number = $95 - x$

According to the condition,

$$x - (95 - x) = 15$$

$$\Rightarrow x - 95 + x = 15$$

$$\Rightarrow 2x = 15 + 95 = 110$$

$$\Rightarrow x = \frac{110}{2} = 55$$

$\therefore$ First number = 55

and second = $95 - 55 = 40$

Hence numbers are 55, 40

Question 4: One-half of a number is equal to one-third of its succeeding number. Find the first number.

Solution:

Let, the first number be x

According to given problem,

$$\frac{x}{2} = \frac{1}{3}(x + 1)$$

$$\Rightarrow 3x = 2(x + 1)$$

$$\Rightarrow 3x = 2x + 2$$

$$\Rightarrow 3x - 2x = 2 \quad x = 2$$

Hence, the first number be 2.

Question 5: The numerator of a rational number is 8 less than its denominator. If the numerator is increased by 2 and denominator is decreased by 1, the number obtained is $\frac{1}{2}$. Find the number.

Solution:

Let denominator of a rational number = x

Then its numerator = $x - 8$

$$\text{and fraction} = \frac{x-8}{x}$$

According to the condition,

$$\frac{x-8+2}{x-1} = \frac{1}{2} \Rightarrow \frac{x-6}{x-1} = \frac{1}{2}$$

$$\Rightarrow (x-6) \times 2 = x-1$$

$$\Rightarrow 2x - 12 = x - 1$$

$$\Rightarrow 2x - x = 12 - 1$$

$$\Rightarrow x = 11$$

$$\therefore \text{Fraction} = \frac{11-8}{11} = \frac{3}{11}$$

Question 6: The present ages of Rohit and Mayank are in the ratio 11: 8. 8 years later the sum of their ages will be 54 years. What are their present ages?

Solution:

Ratio in the present ages of Rohit and Mayank = 11 : 8

Let Rohit's age = $11x$ years

and Mayank's age = $8x$ years

8 years later,

Rohit's age = $11x + 8$

and Mayank's age = $8x + 8$

According to the condition,

$$\Rightarrow 11x + 8 + 8x + 8 = 54$$

$$\Rightarrow 19x = 54 - 8 - 8 = 54 - 16 = 38$$

$$x = \frac{38}{19}$$

$\therefore$ Rohit's present age = $11 \times 2 = 22$ years

and Mayank's age = $8 \times 2 = 16$ years

Question 7: A father's age is 3 times the sum of ages of his two sons. Five years later he will be twice the sum of ages of his two sons. Find the present age of the father.

Solution:

Let sum of ages of two sons = x years

Then father's age = $3x$ years

5 years later

Sum of ages of two sons = $x + 5 + 5 = x + 10$ years

and father's age = $(3x + 5)$ years

According to the condition,

$$3x + 5 = 2(x + 10)$$

$$\Rightarrow 3x + 5 = 2x + 20$$

$$\Rightarrow 3x - 2x = 20 - 5$$

$$\Rightarrow x = 15$$

$$\therefore \text{Father's age} = 3 \times 15 = 45 \text{ years}$$

Question 8: The digits of a two-digit number differ by 7. If the digits are interchanged and the resulting number is added to the original number we get 121. Find the original number.

Solution:

Let unit's digit = x

Then ten's digit = $x - 7$

$$\text{Number} = x + 10(x - 7) = x + 10x - 70 = 11x - 70$$

After interchanged the digits,

$$\text{Unit's digit} = x - 7$$

$$\text{and ten's digit} = x$$

$$\therefore \text{Number} = x - 7 + 10x = 11x - 7$$

According to the condition,

$$11x - 70 + 11x - 7 = 121$$

$$\Rightarrow 22x - 77 = 121$$

$$\Rightarrow 22x = 121 + 77 = 198$$

$$x = \frac{198}{22} = 9$$

$$\therefore \text{Original number} = 11x - 70 = 11 \times 9 - 70 = 99 - 70 = 29$$

Hence number is 29 or 92

Question 9: The ten's digit of a two-digit number exceeds its unit's digit by 5. When digits are reversed, the new number added to the original number becomes 99. Find the original number.

Solution:

Let, the digit at unit's place = x

And, digit at ten's place = $x + 5$

$$\text{Number} = 10 \times (x + 5) + 1 \times x$$

$$= 10(x + 5) + x$$

$$= 10x + 50 + x$$

$$= 11x + 50$$

$$\text{Reversing the number} = 1 \times (x + 5) + 10 \times x$$

$$= x + 5 + 10x = 11x + 5$$

According to given problem,

$$(11x + 50) + (11x + 5) = 99$$

$$\Rightarrow 11x + 50 + 11x + 5 = 99$$

$$\Rightarrow 22x + 55 = 99$$

$$\Rightarrow 22x = 99 - 55$$

$$\Rightarrow 22x = 44$$

$$\Rightarrow x = \frac{44}{22}$$

$$\Rightarrow x = 2$$

$$\text{Hence, the number} = 11 \times 2 + 50 = 22 + 50 = 72$$

Question 10: Sonia went to a bank with ₹2,00,000. She asked the cashier to give her ₹500 and ₹2000 currency notes in return. She got

250 currency notes in all. Find the number of each kind of currency notes.

Solution:

Total amount = ₹2,00,000

and total number of currency notes = 250

Let 500 rupees notes = x

Then 2000's rupee notes = 250 - x

According to the condition,

$$x \times 500 + (250 - x) \times 2000 = 2,00,000$$

$$\Rightarrow 500x + 5,00,000 - 2000x = 2,00,000$$

$$\Rightarrow -1500x = 2,00,000 - 5,00,000$$

$$\Rightarrow -1500x = -3,00,000$$

$$\Rightarrow x = \frac{-3,00,000}{-1500} = 200$$

$\therefore$ 500 rupees notes = 200

and 2000 rupees notes = 250 - 200 = 50

Question 11: Ajay covers a distance of 240 km in $4\frac{1}{4}$ hours. Some part of the journey was covered at the speed of 45 km/h and the remaining at 60 km/h. Find the distance covered by him at the rate of 60 km/h.

Solution:

Let the distance covered by Ajay at the rate of 60 km/hr = x km.

Then, remaining distance covered by Ajay at the rate of 45 km/hr = (240 - x) km.

$\therefore$ Time taken by Ajay to cover the distance at the rate of 60 km/hr. = $\frac{x}{60}$

$$\text{hr [time} = \frac{\text{Distance}}{\text{time}}$$

Also, time taken by Ajay to cover the distance at the rate of 45 km/hr = $\frac{(240-x)}{45}$ hr

According to given problem,
Total time to cover the distance

$$= 4\frac{1}{4} \text{ hr} = \frac{17}{4}$$

$$\Rightarrow \frac{x}{60} + \frac{240-x}{45} = \frac{17}{4}$$

$$\Rightarrow \frac{3x+4(240-x)}{180} = \frac{17}{4}$$

$$\Rightarrow \frac{3x+960-4x}{180} = \frac{17}{4}$$

$$\Rightarrow \frac{-x+960}{180} = \frac{17}{4} \Rightarrow -x + 960 = \frac{17}{4} \times 180$$

$$\Rightarrow -x + 960 = 17 \times 45 \Rightarrow -x + 960 = 765$$

$$\Rightarrow -x = 765 - 960 \Rightarrow -x = -195$$

$$\Rightarrow x = 195$$

Hence, the distance covered by Ajay at the rate of 60 km/hr = 195 km.

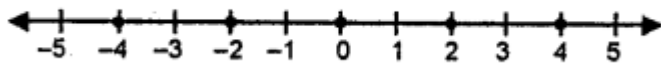
Question 12: If $x \in \{\text{even integers}\}$, represent the solution set of the inequality $-5 \leq x < 5$ on a number line.

Solution:

Given inequality $-5 \leq x < 5$

As $x \in \{\text{even integers}\}$ then, solution is $\{-4, -2, 0, 2, 4\}$

Graphical representation on the number line



Question 13: Solve the following inequality and graph its solution on a number line:

$$\frac{-1}{4} \leq \frac{1}{2} - \frac{x}{3} < 2, x \in \mathbb{I}$$

Solution:

$$\frac{-1}{4} \leq \frac{1}{2} - \frac{x}{3} < 2, x \in \mathbb{I}$$

$$\Rightarrow \frac{-1}{2} - \frac{1}{4} \leq \frac{-1}{2} + \left(\frac{1}{2} - \frac{x}{3}\right) < \frac{-1}{2} + 2 \quad \left[\text{Adding } \left(\frac{-1}{2}\right)\right]$$

$$\Rightarrow \frac{-2-1}{4} \leq \frac{-x}{3} < \frac{-1+4}{2} \Rightarrow \frac{-3}{4} \leq \frac{-x}{3} < \frac{3}{2}$$

$$\Rightarrow 3 \times \left(\frac{-3}{4}\right) \leq 3 \times \frac{-x}{3} < \frac{3}{2} \times 3$$

$$\Rightarrow \frac{-9}{4} \leq -x < \frac{9}{2} \Rightarrow \frac{9}{4} \geq x > -\frac{9}{2}$$

$$\Rightarrow \frac{-9}{2} \leq x < \frac{9}{4}$$

As $x \in \mathbb{I}$, then its solution set is $\{-4, -3, -2, -1, 0, 1, 2\}$

Graphical representation is

