

Chapter 4

Exponents and powers

Exercise 4.1

Question 1.

Fill in the blanks:

- (i) In the expression 3^7 , base =and exponent =
- (ii) In the expression $(-7)^5$ base = and exponent =
- (iii) In the expression $\left(\frac{2}{5}\right)^{11}$, base = and exponent =
- (iv) if base is 6 and exponent is 8, then exponential form =

Solution :

- (i) In the expression 3^7 , base = 3 and exponent = 7
- (ii) In the expression $(-7)^5$ base = -7 and exponent = 5 .
- (iii) In the expression $\left(\frac{2}{5}\right)^{11}$, base = $\frac{2}{5}$ and exponent = 11.
- (iv) if base is 6 and exponent is 8, then exponential form = 6^8

Question 2.

Find the value of the following :

- (i) 2^6
- (ii) 5^5
- (iii) $(-6)^4$

$$\text{(iv)} \left(\frac{2}{3}\right)^4$$

$$\text{(v)} \left(\frac{-2}{3}\right)^5$$

$$\text{(vi)} (-2)^9$$

Solution:

$$\text{(i)} 2^6$$

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= 64$$

$$\text{(ii)} 5^5$$

$$= 5 \times 5 \times 5 \times 5 \times 5$$

$$= 3125$$

$$\text{(iii)} (-6)^4$$

$$= (-6) \times (-6) \times (-6) \times (-6)$$

$$= 1296$$

$$\text{(iv)} \left(\frac{2}{3}\right)^4$$

$$= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$$

$$= \frac{16}{81}$$

$$\text{(v)} \left(\frac{-2}{3}\right)^5$$

$$= \left(-\frac{2}{5}\right) \times \left(-\frac{2}{5}\right) \times \left(-\frac{2}{5}\right) \times \left(-\frac{2}{5}\right) \times \left(-\frac{2}{5}\right)$$

$$= \frac{(-2) \times (-2) \times (-2) \times (-2) \times (-2)}{5 \times 5 \times 5 \times 5}$$

$$= \frac{-32}{3125}$$

$$(vi) (-2)^9$$

$$= (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2)$$

$$= -512$$

Question 3.

Express the following in the exponential form:

$$(i) 6 \times 6 \times 6 \times 6 \times 6$$

$$(ii) t \times t \times t$$

$$(iii) 2 \times 2 \times a \times a \times a \times a$$

$$(iv) a \times a \times a \times c \times c \times c \times c \times d$$

Solution:

$$(i) 6 \times 6 \times 6 \times 6 \times 6$$

$$= 6^5$$

$$(ii) t \times t \times t$$

$$= t^3$$

$$(iii) 2 \times 2 \times a \times a \times a \times a$$

$$= 2^2 a^4$$

$$(iv) a \times a \times a \times c \times c \times c \times c \times d$$

$$= a^3 c^4 d$$

Question 4.

Simplify the following :

(i) 7×10^3

(ii) $2^5 \times 9$

(iii) $3^3 \times 10^4$

Solution:

(i) 7×10^3

$$= 7 \times 10 \times 10 \times 10$$

$$= 7 \times 1000$$

$$= 7000$$

(ii) $2^5 \times 9$

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 9$$

$$= 32 \times 9$$

$$= 288$$

(iii) $3^3 \times 10^4$

$$= 3 \times 3 \times 3 \times 10 \times 10 \times 10$$

$$= 27 \times 10000$$

$$= 270000$$

Question 5.

Simplify the following:

(i) $(-3) \times (-2)^3$

(ii) $(-3)^2 \times (-5)^2$

(iii) $(-2)^3 \times (-10)^4$

(iv) $(-1)^9$

(v) $25^2 \times (-1)^{31}$

(vi) $4^2 \times 3^3 \times (-1)^{122}$

Solution :

(i) $(-3) \times (-2)^3$

$$= (-3) \times (-2) \times (-2) \times (-2)$$

$$= 24$$

(ii) $(-3)^2 \times (-5)^2$

$$= (-3) \times (-3) \times (-5) \times (-5)$$

$$= 9 \times 25$$

$$= 225$$

(iii) $(-2)^3 \times (-10)^4$

$$= (-2) \times (-2) \times (-2) \times 10 \times 10 \times 10 \times 10$$

$$= -8 \times 10000$$

$$= -80000$$

$$(iv) (-1)^9 = -1 \text{ (9 is Odd)}$$

$$(v) 25^2 \times (-1)^{31} = 25 \times 25 \times (-1) \text{ (31 is odd)}$$

$$= 625 \times (-1)$$

$$= -625$$

$$(vi) 4^2 \times 3^3 \times (-1)^{122}$$

$$= 4 \times 4 \times 3 \times 3 \times (1)$$

$$= 144 \times 1$$

$$= 144 \text{ (122 is even)}$$

Question 6.

Identify the greater number in each of the following :

$$(i) 4^3 \text{ or } 3^4$$

$$(ii) 7^3 \text{ or } 3^7$$

$$(iii) 4^5 \text{ or } 5^4$$

$$(iv) 2^{10} \text{ or } 10^2$$

Solution :

$$(i) 4^3 \text{ or } 3^4$$

$$4^3 = 4 \times 4 \times 4 = 64$$

$$3^4 = 3 \times 3 \times 3 \times 3 = 81$$

3^4 is greater

(ii) 7^3 or 3^3

$$7^3 = 7 \times 7 \times 7 = 343$$

$$3^7 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 2187$$

3^7 is greater

(iii) 4^5 or 5^4

$$4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024$$

$$5^4 = 5 \times 5 \times 5 \times 5 = 625$$

4^5 is greater

(iv) 2^{10} or 10^2

$$2^{10} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1024$$

$$10^2 = 10 \times 10 = 100$$

2^{10} is greater

Question 7.

Write the following numbers powers of 2:

(i) 8

(ii) 128

(iii) 1024

Solution

$$(i) 8 = 2 \times 2 \times 2 = 2^3$$

(iii) $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$$(iii) 1024 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{10}$$

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Question 8.

To what power (-2) should raised to get 16 ?

Solution:

$$16 = (-2) \times (-2) \times (-2) \times (-2) = (-2)^4$$

Power = 4

Question 9.

Write the following numbers as powers of (-3) :

(i) 9

(ii) -27

(iii) 81

Solution :

(i) $9 = (-3) \times (-3) = (-3)^2$

(ii) $-27 = (-3) \times (-3) \times (-3) = (-3)^3$

(iii) $81 = (-3) \times (-3) \times (-3) \times (-3) = (-3)^4$

Question 10 .

Find the value of x in each of the following :

(i) $7^x = 343$

(ii) $3^x = 729$

(iii) $(-8)^x = -1024$

(iv) $(-4)^x = -1024$

(v) $\left(\frac{2}{5}\right)^x = \frac{32}{3125}$

(vi) $\left(\frac{-3}{4}\right)^x = -\frac{243}{1024}$

Solution :

(i) $7^x = 343$

$$7^3 = 343$$

7		343
7		49
7		7
		1

$$X = 3$$

(ii) $3^x = 729$

3		729
3		243
3		81
3		27
3		9
3		3
		1

$$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$$

$$X = 6$$

$$(iii) (-8)^x = -1024$$

8	512
8	64
8	8
	1

$$(-8) \times (-8) \times (-8) = (-8)^3$$

$$X = 3$$

$$(iv) (-4)^x = 1024$$

4	1024
4	256
4	64
4	16
4	4
	1

$$= (-4) \times (-4) \times (-4) \times (-4) \times (-4) = (-4)^5$$

$$X = 5$$

$$(v) \left(\frac{2}{5}\right)^x = \frac{32}{3125}$$

5	3125
5	625
5	125
5	25
5	5
	1

2	32
2	16
2	8
2	4
2	2
	1

$$= \frac{2 \times 2 \times 2 \times 2 \times 2}{5 \times 5 \times 5 \times 5 \times 5}$$

$$= \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$$

$$= \left(\frac{2}{5}\right)^5$$

$$X = 5$$

$$(vi) \left(-\frac{3}{4}\right)^x = -\frac{243}{1024}$$

4	1024
4	256
4	64
4	16
4	4
	1

3	243
3	81
3	27
3	9
3	3
	1

$$= \frac{(-3) \times (-3) \times (-3) \times (-3)}{4 \times 4 \times 4 \times 4}$$

$$= \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) = \left(\frac{-3}{4}\right)^5$$

$$X = 5$$

Question 11. Write the prime factorization of the following numbers in the exponential form:

(i) 72

(ii) 360

(iii) 405

(iv) 540

(v) 2280

(vi) 3600

(vii) 4725

(viii) 8400

Solution :

(i) $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

2	72
2	36
2	18
3	9
3	3
	1

$$(ii) 360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 2^3 \times 3^2 \times 5$$

2	360
2	180
2	90
3	45
3	15
5	5
	1

$$405 = 3 \times 3 \times 3 \times 3 \times 5 = 3^4 \times 5^1$$

3	405
3	135
3	45
3	15
5	5
	1

$$(iv) 540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^2 \times 3^3 \times 5^1$$

2	540
2	270
3	135
3	45
3	15
5	5
	1

$$(v) 2280 = 2 \times 2 \times 2 \times 3 \times 5 \times 19 = 2^3 \times 3^1 \times 5^1 \times 19^1$$

2	2280
2	1140
2	570
2	285
5	95
19	19
	1

$$(vi) 3600 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 2^4 \times 3^2 \times 5^2$$

2	3600
2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

$$(vii) 4725 = 3 \times 3 \times 3 \times 5 \times 5 \times 7 = 3^3 \times 5^2 \times 7$$

3	4725
3	1575
3	525
5	175
5	35
7	7
	1

$$(viii) 8400 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 7 = 2^4 \times 3^1 \times 5^2 \times 7^1$$

2	8400
2	4200
2	2100
2	1050
3	525
5	175
5	35
7	7
	1

Exercise 4.2

Question 1:

Using laws of exponents, simplify and write the following in the exponential form:

(i) $2^7 \times 2^4$

(ii) $P^5 \times P^3$

(iii) $(-7)^5 \times (-7)^{11}$

(iv) $\left(\frac{3}{5}\right)^6 \div \left(\frac{3}{5}\right)^2$

(v) $(-6)^7 \div (-6)^3$

(vi) $7^x \times 7^3$

Solution :

Using $a^m \times a^n = a^{m+n}$ and $a^m \div a^n$

(i) $2^7 \times 2^4$

$$= 2^{7+4}$$

$$= 2^{11} \quad \{ a^m \times a^n = a^{m+n} \}$$

(ii) $P^5 \times P^3$

$$= P^{5+3}$$

$$= P^8$$

(iii) $(-7)^5 \times (-7)^{11}$

$$= (-7)^{5+11}$$

$$= (-7)^{16}$$

$$\begin{aligned} \text{(iv)} \quad & \left(\frac{3}{5}\right)^6 \div \left(\frac{3}{5}\right)^2 \\ &= \left(\frac{3}{5}\right)^{6-2} \\ &= \left(\frac{3}{5}\right)^4 \{ a^m \div a^n \} \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad & (-6)^7 \div (-6)^3 \\ &= (-6)^{7-3} \\ &= (-6)^4 \end{aligned}$$

$$\begin{aligned} \text{(vi)} \quad & 7^x \times 7^x \\ &= 7^{x+3} \{ a^m \times a^n = a^{m+n} \} \end{aligned}$$

Question 2.

Simplify and write the following in the exponential form:

$$\text{(i)} \quad 5^3 \times 5^7 \times 5^{12}$$

$$\text{(ii)} \quad a^5 \times a^3 \times a^7$$

$$\text{(iii)} \quad (7^{12} \times 7^3) \div 7^4$$

Solution :

$$\begin{aligned} \text{(i)} \quad & 5^3 \times 5^7 \times 5^{12} \\ &= 5^{3+7+12} \end{aligned}$$

$$= a^{22}$$

$$(ii) a^5 \times a^3 \times a^7$$

$$= a^{5+3+7}$$

$$= a^{15}$$

$$(iii) (7^{12} \times 7^3) \div 7^4$$

$$= 7^{12+3} \div 7^4$$

$$= 7^{15-4}$$

$$= 7^{11}$$

Question 3:

Simplify and write the following in the exponential form:

$$(i) (2^2)^{100}$$

$$(ii) ((-7)^6)^5$$

$$(iii) (3^2)^5 \times (3^4)^7$$

Solution :

Using $(a^m)^n = a^{mn}$

$$(i) (2^2)^{100}$$

$$= 2^2 \times 100$$

$$= 2^{200}$$

$$(ii) [(-7)^6]^5$$

$$= (-7)^{30}$$

$$(iii) (3^2)^5 \times (3^4)^7$$

$$= 3^{2 \times 5} \times 3^{4 \times 7}$$

$$= 3^{10} \times 3^{28}$$

$$= 3^{38}$$

Question 4:

Simplify and write in the exponential form:

$$(i) \frac{a^3 \times a^2}{(a^3)^2}$$

$$(ii) (2^3)^4 \div 2^5$$

$$(iii) [(6^2)^3 \div 6^3] \times 6^5$$

Solution :

$$(i) \frac{a^3 \times a^2}{(a^3)^2}$$

$$= \frac{a^{3+2}}{a^{3 \times 2}}$$

$$= \frac{a^5}{a^6}$$

$$= a^{5-6}$$

$$= a^{-1}$$

$$(ii) \quad (2^3)^4 \div 2^5$$

$$= 2^{3 \times 4} \div 2^5$$

$$= 2^{12} \div 2^5$$

$$= 2^{12-5}$$

$$= 2^7$$

$$(iii) \quad [(6^2)^3 \div 6^3] \times 6^5$$

$$= [(6^{2 \times 3} \div 6^3] \times 6^5$$

$$= [(6^6 \div 6^3] \times 6^5$$

$$= 6^{6-3} \times 6^5$$

$$= 6^3 \times 6^5$$

$$= 6^{3+5}$$

$$= 6^8$$

Question 5:

Simplify and write in the exponential form:

$$(i) \quad 5^4 \times 8^4$$

$$(ii) \quad (-3)^6 \times (-5)^6$$

$$(iii) \quad \left(\frac{3}{10}\right)^5 \times \left(\frac{2}{15}\right)^5$$

Solution :

$$(i) \quad 5^4 \times 8^4 = (5 \times 8)^4 = 40^4 \quad \{a^m \times a^n = a^{m+n}\}$$

$$(ii) \quad (-3)^6 \times (-5)^6 = [(-3) \times (-5)]^6 = (15)^6 = 15^6$$

$$\begin{aligned}
 \text{(iii)} \quad \left(\frac{3}{10}\right)^5 \times \left(\frac{2}{15}\right)^5 &= \left(\frac{3}{10} \times \frac{2}{15}\right)^5 \\
 &= \left(\frac{1}{25}\right)^5 = \left(\frac{1}{5 \times 5}\right)^5 = \left(\frac{1}{5^2}\right)^5 \\
 &= \left(\frac{1}{5}\right)^{5 \times 2} = \left(\frac{1}{5}\right)^{10}
 \end{aligned}$$

Question 6:

Simplify and express each of the following in the exponential form:

$$\text{(i)} \quad \frac{2^4 \times 2 \times 7^3 \times 7^6}{2^3 \times 7^4}$$

$$\text{(ii)} \quad \frac{(3^2)^3 \times (-2)^5}{(-2)^3}$$

$$\text{(iii)} \quad \frac{2^8 \times a^5}{4^3 \times a^3}$$

$$\text{(iv)} \quad \frac{3 \times 7^2 \times 11^8}{21 \times 11^3}$$

$$\text{(v)} \quad (2^0 + 3^0)4^0$$

$$\text{(vi)} \quad 3^0 \times 4^0 \times 5^0$$

Solution :

$$\text{(i)} \quad \frac{2^4 \times 2 \times 7^3 \times 7^6}{2^3 \times 7^4}$$

$$= \frac{2^{4+1} \times 7^{3+6}}{2^3 \times 7^4}$$

$$= \frac{2^5 \times 7^9}{2^3 \times 7^4}$$

$$= 2^{5-3} \times 7^{9-4}$$

$$= 2^2 \times 7^5$$

$$\begin{aligned}
 & \text{(ii)} \quad \frac{(3^2)^3 \times (-2)^5}{(-2)^3} \\
 &= \frac{3^{2 \times 3} \times (-2)^5}{(-2)^3} \\
 &= \frac{3^6 \times (-2)^5}{(-2)^3} \\
 &= 3^6 \times (-2)^{5-3} \\
 &= 3^6 \times (-2)^2 \\
 &= 3^6 \times 2^2
 \end{aligned}$$

$$\begin{aligned}
 & \text{(iii)} \quad \frac{2^8 \times a^5}{4^3 \times a^3} \\
 &= \frac{2^8 \times a^5}{(2^2)^3 \times a^3} \\
 &= \frac{2^8 \times a^5}{2^6 \times a^3} \\
 &= 2^{8-6} \times a^{5-3} \\
 &= 2^2 \times a^2 \\
 &= (2a)^2
 \end{aligned}$$

$$\begin{aligned}
 & \text{(iv)} \quad \frac{3 \times 7^2 \times 11^8}{21 \times 11^3} \\
 &= \frac{3^1 \times 7^2 \times 11^8}{3^1 \times 7^1 \times 11^3} \\
 &= 3^{1-1} \times 7^{2-1} \times 11^{8-3} \\
 &= 3^0 \times 7^1 \times 11^5
 \end{aligned}$$

$$= 1 \times 7 \times 11^5$$

$$= 7 \times 11^5 \quad \{\because a^0 = 1\}$$

$$(v) (2^0 + 3^0)4^0$$

$$= (1 + 1)1$$

$$= 2 \times 1 = 2 \quad \{\because a^0 = 1\}$$

$$(vi) 3^0 \times 4^0 \times 5^0$$

$$= 1 \times 1 \times 1 = 1 \quad \{\because a^0 = 1\}$$

Question 7:

Express each of the following rational numbers in the exponential form:

$$(i) \frac{25}{64}$$

$$(ii) -\frac{125}{216}$$

$$(iii) -\frac{343}{729}$$

Solution :

$$(i) \frac{25}{64}$$

$$= \frac{5 \times 5}{8 \times 8}$$

$$= \frac{5^2}{8^2}$$

$$= \left(\frac{5}{8}\right)^2$$

$$\text{(ii)} -\frac{125}{216}$$

$$= \frac{(-5) \times (-5) \times (-5)}{6 \times 6 \times 6}$$

$$= \frac{-5}{6} \times \frac{-5}{6} \times \frac{-5}{6} = \left(\frac{-5}{6}\right)^3$$

$$\text{(iii)} -\frac{343}{729}$$

$$= \frac{(-7) \times (-7) \times (-7)}{9 \times 9 \times 9}$$

$$= \frac{-7}{9} \times \frac{-7}{9} \times \frac{-7}{9}$$

$$= \left(\frac{-7}{9}\right)^3$$

Question 8:

Simplify the following :

$$\text{(i)} \frac{(2^5)^2 \times 7^3}{8^3 \times 7}$$

$$\text{(ii)} \frac{25 \times 5^2 \times t^8}{10^3 \times t^4}$$

$$\text{(iii)} \frac{3^5 \times 10^5 \times 25}{5^7 \times 5^5}$$

$$\text{(iv)} \left(-\frac{3}{5}\right)^{-3}$$

Solution :

$$\begin{aligned}
 \text{(i)} \quad & \frac{(2^5)^2 \times 7^3}{8^3 \times 7} \\
 &= \frac{2^{5 \times 2} \times 7^3}{(2^3)^3 \times 7} \\
 &= \frac{2^{10} \times 7^3}{2^9 \times 7} \\
 &= 2^{10-9} \times 7^{3-1} \\
 &= 2^1 \times 7^2 \\
 &= 2 \times 49 \\
 &= 98
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & \frac{25 \times 5^2 \times t^8}{10^3 \times t^4} \\
 &= \frac{5^2 \times 5^2 \times t^8}{(2 \times 5)^3 \times t^4} \\
 &= \frac{5^2 \times 5^2 \times t^8}{2^3 \times 5^3 \times t^4} \\
 &= \frac{5^{2+2-3} \times t^{8-4}}{2^3} \\
 &= \frac{5^1 \times t^4}{2^3} \\
 &= \frac{5t^4}{8}
 \end{aligned}$$

$$\text{(iii)} \quad \frac{3^5 \times 10^5 \times 25}{5^7 \times 6^5}$$

$$\begin{aligned}
&= \frac{3^5 \times (2 \times 5)^5 \times (5 \times 5)}{5^7 \times (2 \times 3)^5} \\
&= \frac{3^5 \times 2^5 \times 5^5 \times 5^2}{5^7 \times 2^5 \times 3^5} \\
&= 2^{5-5} \times 3^{5-5} \times 5^{5+2-7} \\
&= 2^0 \times 3^0 \times 5^0 \\
&= 1 \times 1 \times 1 \quad \{a^0 = 1\} \\
&= 1
\end{aligned}$$

$$(iv) \left(-\frac{3}{5}\right)^{-3} = \left(\frac{-5}{3}\right)^3 \quad \left\{ \because \left(\frac{1}{a}\right)^{-1} = a^1 \right\}$$

$$\begin{aligned}
&\frac{(-5) \times (-5) \times (-5)}{3 \times 3 \times 3} \\
&= \frac{-125}{27}
\end{aligned}$$

Question 9:

Simplify the following:

$$\begin{aligned}
(i) &\left(\frac{-1}{2}\right)^5 \times 2^6 \times \left(\frac{3}{4}\right)^3 \\
(ii) &\left[\left(\frac{-3}{4}\right)^3 - \left(\frac{-5}{2}\right)^3\right] \times \left(-\frac{2}{3}\right)^4
\end{aligned}$$

Solution :

$$\begin{aligned}
(i) &\left(\frac{-1}{2}\right)^5 \times 2^6 \times \left(\frac{3}{4}\right)^3 \\
&= \frac{(-1)^5}{2^5} \times 2^6 \times \frac{3^3}{4^3}
\end{aligned}$$

$$\begin{aligned}
&= \frac{(-1)^3}{2^5} \times 2^6 \times \frac{3^3}{(2^2)^3} \\
&= \frac{(-1)^3}{2^5} \times 2^6 \times \frac{3^3}{2^6} \\
&= \frac{-1 \times 3}{2^5} \times \frac{-1 \times 27}{32} \times \frac{-27}{32}
\end{aligned}$$

$$\begin{aligned}
\text{(ii)} & \left[\left(\frac{-3}{4} \right)^3 - \left(\frac{-5}{2} \right)^3 \right] \times \left(-\frac{2}{3} \right)^4 \\
&= \left[\frac{(-3)^3}{4^3} - \frac{(-5)^3}{2^3} \right] \times \frac{(-2)^4}{3^4} \\
&= \left[\frac{-27}{64} - \frac{-125}{8} \right] \times \frac{16}{81} \\
&= \frac{-27+1000}{64} \\
&= \frac{973}{64} \times \frac{16}{81} \\
&= \frac{973}{324} \\
&= 3 \frac{1}{324}
\end{aligned}$$

Question 10 :

Simplify the following :

$$\text{(i)} \left(\frac{3}{2} \right)^{-1} \div \left(\frac{-2}{5} \right)^{-1}$$

$$\text{(ii)} \left[\left\{ \left(\frac{-1}{4} \right)^2 \right\}^{-1} \right]^{-2}$$

Solution :

$$\begin{aligned}
 \text{(i)} \quad & \left(\frac{3}{2}\right)^{-1} \div \left(\frac{-2}{5}\right)^{-1} \\
 &= \left(\frac{2}{3}\right)^1 \div \left(\frac{5}{-2}\right)^1 \\
 &= \frac{2}{3} \times \frac{-2}{5} = \frac{-4}{15}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & \left[\left\{ \left(\frac{-1}{4}\right)^2 \right\}^{-1} \right]^{-2} \\
 &= \left(\frac{-1}{4}\right)^{2 \times (-1) \times (-2)} \\
 &= \left(\frac{-1}{4}\right)^4 \\
 &= \frac{-1}{4} \times \frac{-1}{4} \times \frac{-1}{4} \times \frac{-1}{4} \\
 &= \frac{1}{256}
 \end{aligned}$$

Question 11:

Simplify:

$$\left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} + \left(\frac{1}{5}\right)^{-2} - \left(\frac{1}{6}\right)^{-2}$$

Solution :

$$\begin{aligned}
 & \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} + \left(\frac{1}{5}\right)^{-2} - \left(\frac{1}{6}\right)^{-2} \\
 &= (3)^2 + (4)^2 + (5)^2 - (6)^2 \\
 &= 9 + 16 + 25 - 36 \\
 &= 50 - 36 = 14
 \end{aligned}$$

Question 12:

Express each of the following as a product of prime factors in the exponential form:

(i) 108×192

(ii) 729×64

(iii) 384×147

Solution:

(i) 108×192

2	108
2	54
3	27
3	9
3	3
	1

2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

$$= (2 \times 2 \times 3 \times 3 \times 3) \times (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3)$$

$$= 2^2 \times 3^3 \times 2^6 \times 3^1$$

$$= 2^{2+6} \times 3^{3+1} = 2^8 \times 3^4$$

(ii) 729×64

3	729
3	243
3	81
3	27
3	9
3	3
	1

2	64
2	32
2	16
2	8
2	4
2	2
	1

$$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= 3^6 \times 2^6$$

(iii) 384×147

2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1
3	147

$$\begin{array}{r|l}
 7 & 49 \\
 \hline
 7 & 7 \\
 \hline
 & 1
 \end{array}$$

$$= (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3) \times (3 \times 7 \times 7)$$

$$= 2^7 \times 3 \times 3 \times 7^2$$

$$= 2^7 \times 3^2 \times 7^2$$

Question 13:

Simplify and write the following in the exponential form :

(i) $3^3 \times 2^2 + 2^2 \times 5^0$

(ii) $9^2 + 11^2 - 2^2 \times 3 \times 17^0$

Solution :

(i) $3^3 \times 2^2 + 2^2 \times 5^0$

$$= 3 \times 3 \times 3 \times 2 \times 2 + 2 \times 2 \times 1$$

$$= 108 + 4 = 112$$

$$= 2 \times 7 \times 2 \times 2 \times 7 = 2^4 \times 7^1$$

$$\begin{array}{r|l}
 2 & 112 \\
 \hline
 2 & 56 \\
 \hline
 2 & 28 \\
 \hline
 2 & 14 \\
 \hline
 7 & 7 \\
 \hline
 & 1
 \end{array}$$

(ii) $9^2 + 11^2 - 2^2 \times 3 \times 17^0$

$$= 9 \times 9 + 11 \times 11 - 2 \times 2 \times 3 \times 1$$

$$= 81 + 121 - 12 = 202 - 12 = 190$$

$$= 2 \times 5 \times 19 = 2^1 \times 5^1 \times 19^1$$

2	190
5	95
19	19
	1

Question 14 :

(i) By what number should we multiply 3^4 so that the product is 3^7 ?

(ii) By what number should be multiple $(-6)^{-1}$ so that the product is 10^{-1} ?

Solution :

(i) Product = 3^7

Required number = $3^7 \div 3^4$

$$= 3^{7-4}$$

$$= 3^3$$

$$= 27$$

(ii) Product = 10^{-1}

Required number = $10^{-1} \div (-6)^{-1}$

$$= \frac{1}{10} \div \frac{1}{-6}$$

$$= \frac{1}{10} \times \frac{-6}{1}$$

$$= \frac{-3}{5}$$

Question 15

If

$$\left(\frac{12}{13}\right)^4 \times \left(\frac{13}{12}\right)^{-8} = \left(\frac{12}{13}\right)^{2x}, \text{ then find the value of } x.$$

Solution :

$$\left(\frac{12}{13}\right)^4 \times \left(\frac{13}{12}\right)^{-8} = \left(\frac{12}{13}\right)^{2x}$$

$$\Rightarrow \left(\frac{12}{13}\right)^4 \times \left(\frac{12}{13}\right)^8 = \left(\frac{12}{13}\right)^{2x}$$

$$\Rightarrow \left(\frac{12}{13}\right)^{4+8} = \left(\frac{12}{13}\right)^{2x} \Rightarrow \left(\frac{12}{13}\right)^{12} = \left(\frac{12}{13}\right)^{2x}$$

$$\therefore 2x = 12 \Rightarrow x = \frac{12}{2} = 6$$

Hence, $x = 6$

Exercise 4.3

Question 1:

Write the following numbers in the standard form (or scientific notation) :

(i) 530.7

(ii) 3908.78

(iii) 39087.8

(iv) 2.35

(v) 3,43,000

(vi) 70,00,000

(vii) 3,18,65,00,000

(viii) 893,000,000

(ix) 70,040,000,000

Solution:

(i) 530.7

$$= 5.307 \times 10^2$$

(ii) 3908.78

$$= 3.90878 \times 10^3$$

(iii) 39087.8

$$= 3.90878 \times 10^4$$

(iv) 2.35

$$= 2.35 \times 10^0 \quad \{a^0 = 1\}$$

(v) 3,43,000

$$= 3.43000 \times 10^5$$

(vi) 70,00,000

$$= 7.0 \times 10^6$$

(vii) 3,18,65,00,000

$$= 3.186500000 \times 10^9$$

(viii) 893,000,000

$$= 8.93 \times 10^8$$

(ix) 70,040,000,000

$$= 7.004 \times 10^{10}$$

Question 2:

Write the following numbers in usual decimal notation:

(i) 4.7×10^3

(ii) 1.205×10^5

(iii) 1.234×10^6

(iv) 4.87×10^7

(v) 6.05×10^8

(vi) 9.083×10^{11}

Solution :

(i) $4.7 \times 10^3 = 4700$

(ii) $1.205 \times 10^5 = 120500$

(iii) $1.234 \times 10^6 = 1234000$

(iv) $4.87 \times 10^7 = 48700000$

(v) $6.05 \times 10^8 = 605000000$

(vi) $9.083 \times 10^{11} = 908300000000$

Question 3:

Express the numbers appearing in the following statements in scientific notation (or standard form) :

(i) The distance between the earth and the moon is 384,000,000 m.

(ii) The diameter of the sun is 1,400,000m .

(iii) The universe is estimated to be about 12,0000,000 years old.

(iv) In a galaxy, there are on an average 100,000,000 stars.

(v) The distance of the sun from the centre of milky way galaxy is estimated to be 300,000,000,000,000,000 km.

(vi) The astronomical distance of unit is a light year (a light year is a distance travelled by the light in one year.) A light year is about 9,467,500,000,000 kilometres .

Solution :

- (i) The distance between the earth and the moon is 384,000,000 m. = 3.84×10^8 m
- (ii) The diameter of the sun is 1,400,000m . = 1.4×10^8 m
- (iii) The universe is estimated to be about 12,000,000 years old = 1.2×10^{10}
- (iv) In a galaxy, there are on an average 100,000,000 stars. = 1.4×10^8 m.
- (v) The distance of the sun from the centre of milky way galaxy is estimated to be 300,000,000,000,000,000 km. = 3.0×10^{17}
- (vi) The astronomical distance of unit is a light year (a light year is a distance travelled by the light in one year.) A light year is about 9,467,500,000,000 kilometres . = 9.4675×10^{12} km

Question 4:

Compare the following numbers:

- (i) 4.3×10^{14} ; 3.01×10^{17}
- (ii) 1.439×10^{12} ; 1.4335×10^{12}

Solution :

- (i) 4.3×10^{14} ; 3.01×10^{17}
 $3.01 \times 10^{17} > 4.3 \times 10^{14}$ {17 > 14}

- (ii) 1.439×10^{12} ; 1.4335×10^{12}
 $1.439 > 1.4335$
 $1.439 \times 10^{12} > 1.4335 \times 10^{12}$

Question 5:

Write the following numbers in the expended Exponential form:

(i) 279404

(ii) 3006194

(iii) 28061906

Solution :

(i) 279404

$$= 2 \times 10^5 + 7 \times 10^4 + 9 \times 10^3 + 4 \times 10^2 + 4 \times 10^0$$

(ii) 3006194

$$= 3 \times 10^6 + 6 \times 10^3 + 1 \times 10^2 + 9 \times 10^1 + 4 \times 10^0$$

(iii) 28061906

$$= 2 \times 10^7 + 8 \times 10^6 + 6 \times 10^5 + 1 \times 10^3 + 9 \times 10^2 + 6 \times 10^0$$

Question 6:

Find the number from each of the expended from:

(i) $3 \times 10^4 + 7 \times 10^2 + 5 \times 10^0$

(ii) $4 \times 10^5 + 5 \times 10^3 + 3 \times 10^2 + 2 \times 10^0$

(iii) $8 \times 10^7 + 3 \times 10^4 + 7 \times 10^3 + 5 \times 10^2 + 8 \times 10^1$

Solution :

(i) $3 \times 10^4 + 7 \times 10^2 + 5 \times 10^0$

$$= 30705$$

$$\begin{aligned} \text{(ii)} \quad & 4 \times 10^5 + 5 \times 10^3 + 3 \times 10^2 + 2 \times 10^0 \\ &= 400000 + 5000 + 300 + 2 \times 1 \\ &= 405302 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad & 8 \times 10^7 + 3 \times 10^4 + 7 \times 10^3 + 5 \times 10^2 + 8 \times 10^1 \\ &= 80000000 + 30000 + 7000 + 500 + 80 \\ &= 80037580 \end{aligned}$$

Objective type Questions

Mental maths

Question 1:

Fill in the blanks:

- (i) In the expression $(-5)^9$, exponent = and base =
- (ii) If the base is $\frac{-3}{4}$ and exponent is 5, then exponential form is.....
- (iii) The expression (x^2y^2) in the simplest form is
- (iv) If $(100)^0 = 10^n$, then the value of n is
- (v) $\left(-\frac{1}{2}\right)^0 + (-2)^0 = \dots\dots\dots$
- (vi) $(-3)^2 \times (-1)^{2017} = \dots\dots\dots$
- (vii) $(-3)^8 \div (-3)^5 = (-3) \dots\dots\dots$
- (viii) $\left(\frac{5}{13}\right)^{10} \div \left[\left(\frac{5}{13}\right)^3\right]^2 = \left(\frac{5}{13}\right) \dots\dots\dots$
- (ix) $35070000 = 3.507 \times 10 \dots\dots\dots$
- (x) If $(-2)^n = 128$, Then n =

Solution :

- (i) In the expression $(-5)^9$, exponent = 9 and base = -5
- (ii) If the base is $\frac{-3}{4}$ and exponent is 5, then exponential form is
 $\left(-\frac{3}{4}\right)^5$
- (iii) The expression (x^2y^2) in the simplest form is $(x^2)^3 \times (y^2)^3 = x^6y^6$

(iv) If $(100)^0 = 10^n$, then the value of n is 0 .

$$\{ (10^2)^0 = 10^n = 10^0 \}$$

$$N = 0$$

$$(v) \left(-\frac{1}{2}\right)^0 + (-2)^0 = 1+1 = 2$$

$$(vi) (-3)^2 \times (-1)^{2017} = (-3) \times (-3) \times (-1) \quad \{ 2017 \text{ is odd } \}$$

$$9 \times (-1) = -9$$

$$(vii) (-3)^8 \div (-3)^5 = (-3)^{8-5} = (-3)^3$$

$$(viii) \left(\frac{5}{13}\right)^{10} \div \left[\left(\frac{5}{13}\right)^3\right]^2 = \left(\frac{5}{13}\right)^{10} \div \left(\frac{5}{13}\right)^6 = \left(\frac{5}{13}\right)^{10-6} = \left(\frac{5}{13}\right)^4$$

$$(ix) 35070000 = 3.507 \times 10^7$$

$$(x) \text{ If } (-2)^n = 128, \text{ Then } n = (-2)^7$$

$$; n = 7$$

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Question 2:

State whether the following statements are true (T) or false (F)

(i) If a is a rational number then $a^m \times a^n = a^{m \times n}$

(ii) $2^3 \times 3^2 = 6^5$.

(iii) The value of $(-2)^{-3}$ is $\frac{-1}{8}$.

(iv) The value of the expression $2^9 \times 2^{91} - 2^{19} \times 2^{81}$ is 1.

(v) $3^0 = (1000)^0$

(vi) $5^6 \div (-2)^6 = \frac{-5}{2}$

(vii) $5^0 \times 3^0 = 8^0$

(viii) $\frac{2^3}{7} < \left(\frac{2}{7}\right)^3$

(ix) $(10 + 10)^4 = 10^4 + 10^4$.

(x) $x^0 \times x^0 = x^0 \div x^0$, Where x is a non – zero rational number .

(xi) 4^9 is greater than 16^3 .

(xii) $x^m + x^m = x^{2m}$, Where x is a non-zero rational number and m is a positive integer.

(xiii) $\left(\frac{4}{3}\right)^5 \times \left(\frac{5}{7}\right)^5 = \left(\frac{4}{3} + \frac{5}{7}\right)^5$

Solution:

(i) If a is a rational number the $a^m \times a^n = a^{m \times n}$ **(False)**

Correct:

$\{\because \text{It is true for fraction}\}$

$$a^m \times a^n = a^{m+n}$$

(ii) $2^3 \times 3^2 = 6^5$. **(False)**

Correct:

$$\{\because 2^3 \times 3^2 \neq 6^5\}$$

(iii) The value of $(-2)^{-3}$ is $-\frac{1}{8}$. **(True)**

$$(-2)^{-3} = \frac{1}{(-2)^3} = \frac{1}{-8} = -\frac{1}{8}$$

(iv) The value of the expression $2^9 \times 2^{91} - 2^{19} \times 2^{81}$ is 1.
(False)

Correct:

$$2^{9+91} - 2^{19+81} \Rightarrow 2^{100} - 2^{100} = 0$$

(v) $3^0 = (1000)^0$. **(True)**

{As $3^0 = 1$ and $(1000)^0 = 1$ }

(vi) $5^6 \div (-2)^6 = -\frac{5}{2}$ **(False)**

Correct:

$$5^6 \div (-2)^6$$

$$= \frac{5^6}{(-2)^6} = \left(\frac{5}{-2}\right)^6$$

(vii) $5^0 \times 3^0 = 8^0$. **True**

$$1 \times 1 = 1$$

$$\{5^0 = 1, \quad 3^0 = 1, \quad 8^0\}$$

(viii) $\frac{2^3}{7} < \left(\frac{2}{7}\right)^3$ **(False)**

Correct:

$$\frac{8}{7} < \frac{8}{343} \quad \left(\because \frac{8}{7} > \frac{8}{343}\right)$$

(ix) $(10 + 10)^4 = 10^4 + 10^4$ **(False)**

Correct:

$$\because (10 + 10)^4 = 20^4 \text{ But } 10^4 + 10^4 = 2 \times 10^4$$

(x) $x^0 \times x^0 = x^0 \div x^0$ **(True)**

$$1 \times 1 = 1 \div 1 \Rightarrow 1 = 1$$

(xi) 4^9 is greater than 16^3 .

$$4^9 \text{ And } 16^3 = (4^2)^3 = 4^6$$

$$\therefore 4^9 > 4^6$$

(xii) $x^m + x^m = x^{2m}$ (True)

Correct:

$$\text{As } x^m + x^m = 2 \times x^m \text{ and } x^{2m} = (x^2)^m$$

(xiii) $\left(\frac{4}{5}\right)^5 \times \left(\frac{5}{7}\right)^5 = \left(\frac{4}{5} + \frac{5}{7}\right)^5$. (False)

Correct:

$$\left(\frac{4}{5}\right)^5 \times \left(\frac{5}{7}\right)^5 \neq \left(\frac{4}{5} + \frac{5}{7}\right)^5$$

Multiple choice Questions

Choose the correct answer from the given four options (3 to 8)

Question 3:

$a \times a \times a \times b \times b \times b$ is equal to

(a) a^3b^3

(b) a^2b^3

(c) $(ab)^3$

(d) a^6b^6

Solution:

$$a \times a \times a \times b \times b \times b = a^3b^3 = (ab)^3 \text{ (c)}$$

Question 4:

$(-2)^3 \times (-3)^2$ is equal to

(a) 6^5

(b) $(-6)^6$

(c) 72

(d) -72

Solution:

$$(-2)^3 \times (-3)^2 = -8 \times 9 = -72$$

Question 5:

The expression $(Pqr)^3$ is equal to

(i) P^3qr

(b) pq^3r

(c) pqr^3

(d) $p^3q^3r^3$

Solution :

$(pqr)^3$ (d)

Question 6:

$\left(-\frac{3}{2}\right)^{-1}$ is equal to

(a) $\frac{2}{3}$

(b) $-\frac{2}{3}$

(c) $\frac{3}{2}$

(d) $\frac{4}{9}$

Solution :

$$\left(-\frac{3}{2}\right)^{-1} = -\frac{2}{3}$$

Question 7:

$\left(-\frac{3}{4}\right)^5$ is equal to

(a) $\frac{81}{256}$

(b) $-\frac{81}{256}$

(c) $-\frac{243}{1024}$

(d) $\frac{243}{1024}$

Solution :

$$\begin{aligned} &\left(-\frac{3}{4}\right)^5 \\ &= \frac{(-3) \times (-3) \times (-3) \times (-3) \times (-3)}{4 \times 4 \times 4 \times 4 \times 4} \\ &= -\frac{243}{1024} \quad \text{(c)} \end{aligned}$$

Question 8:

The value of $(5^{30} \times 5^{20}) \div (5^5)^9$ in the exponential form is

(a) 5^{-5}

(b) 5^5

(c) 5^{50}

(d) 5^{95}

Solution:

$$\begin{aligned} & (5^{30} \times 5^{20}) \div (5^5)^9 \\ &= 5^{30+20} \div 5^{5 \times 9} \\ &= 5^{50} \div 5^{45} \\ &= 5^{50-45} \\ &= 5^5 \text{ (b)} \end{aligned}$$

Question 9:

The law $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ does not hold when

(a) $a = 3$, $b = 2$

(b) $a = -2$, $b = 3$

(c) $n = 0$

(d) $b = 0$

Solution:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$b = 0 \text{ as } \left(\frac{a}{0}\right)^n = \frac{a^n}{0^n}$$

Division by 0 is not admissible. (d)

Question 10:

The value of the expression $\frac{-1^{101} \times 8^5}{4^7}$ is equal to

(a) 2

(b) -2

(c) $\frac{1}{16}$

(d) $-\frac{1}{16}$

Solution:

$$\begin{aligned} & \frac{(-1)^{101} \times (8)^5}{(4)^7} \\ &= \frac{(-1)^{101} \times (2^3)^5}{(2^2)^7} = \frac{-1 \times 2^{3 \times 5}}{2^{2 \times 7}} \\ &= \frac{-1 \times 2^{15}}{2^{14}} = -1 \times 2^{15-14} \\ &= -1 \times 2 \\ &= -2 \end{aligned}$$

Question 11:

The value of $\frac{10^{22} + 10^{20}}{10^{20}}$ is

(a) 10

(b) 101

(c) 10^{22}

(d) 10^{42}

Solution:

$$\begin{aligned} & \frac{10^{22} + 10^{20}}{10^{20}} \\ &= \frac{10^{20} \times 10^2 + 10^{20}}{10^{20}} \\ &= \frac{10^{20}(10^2 + 1)}{10^{20}} = 10^2 + 1 \\ &= 100 + 1 \\ &= 101 \end{aligned}$$

Question 12:

The value of $5^{-1} - 6^{-1}$ is

(a) 130

(b) -130

(c) 30

(d) -30

Solution:

$$\begin{aligned} & 5^{-1} - 6^{-1} \\ &= \frac{1}{5} - \frac{1}{6} \\ &= \frac{6-5}{30} \\ &= \frac{1}{30} \end{aligned}$$

Question 13:

The value of $(6^{-1} - 8^{-1})^{-1}$ is

(a) -12

(b) -2

(c) 124

(d) 24

Solution:

$$(6^{-1} - 8^{-1})^{-1}$$

$$= \left(\frac{1}{6} - \frac{1}{8}\right)^{-1}$$

$$= \left(\frac{4-3}{24}\right)^{-1}$$

$$= \left(\frac{1}{24}\right)^{-1}$$

$$= 24$$

Question 14:

The value of $\left[\left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}\right] \div \left(\frac{1}{5}\right)^{-2}$ is

(a) 0

(b) -1

(c) 1

(d) $\frac{7}{5}$

Solution:

$$\begin{aligned}& \left[\left(\frac{1}{3} \right)^{-2} + \left(\frac{1}{4} \right)^{-2} \right] \div \left(\frac{1}{5} \right)^{-2} \\&= (3^2 + 4^2 \div 5^2) \\&= (9 + 16) \div 25 \\&= 25 \div 25 \\&= 1\end{aligned}$$

Question 15:

If $2^3 + 1^3 = 3^x$, then the value of x is

- (a) 0
- (b) 1
- (c) 2
- (d) 3

Solution:

$$\begin{aligned}2^3 + 1^3 &= 3^x \\ \Rightarrow 8 + 1 &= 3^x \\ \Rightarrow 9 &= 3^x \\ \Rightarrow 3^2 &= 3^x \\ \Rightarrow x &= 2 \text{ (c)}\end{aligned}$$

Question 16:

The standard form of 751.65 is

- (a) 7.5165×10^2

(b) 75.175×10^1

(c) 7.5165×10^4

(d) 7.51×10^2

Solution:

Standard form of $751.65 = 7.5165 \times 10^2$ (a)

Question: 17

The usual form of 5.658×10^5 is

(a) 5658

(b) 56580

(c) 565800

(d) 5658000

Solution:

$5.658 \times 10^5 = 565800$ (c)

Question 18:

Which of the following numbers is in the standard form?

(a) 26.57×10^4

(b) 2.657×10^4

(c) 265.7×10^3

(d) 0.2657×10^6

Solution:

2.657×10^5 is in standard form . (b)

Value Based Questions

Question 1:

Typhoid is caused by bacteria Salmonella typhi. The size of salmonella typhi is about 0.0000000005 mm. Express it in standard form. Vinay is suffering from typhoid, his doctor advised him to take healthy food and avoid eating food or drinking beverages from street vendors. Why should we eat healthy food and why should we not eat food from street vendors?

Solution:

Size of salmonella typhi = 0.0000000005 mm

Standard form = 5.0×10^{-10}

We should take healthy food for our healthy body we should not eat or drink from street vendors as it may be unhygienic.

Higher order thinking skills (HOTS)

Question 1:

if $\left(\frac{5}{12}\right)^8 \times \left(\frac{12}{5}\right)^{-4} = \left(\frac{125}{1728}\right)^x$ find x.

Solution:

$$\left(\frac{5}{12}\right)^8 \times \left(\frac{12}{5}\right)^{-4} = \left(\frac{125}{1728}\right)^x$$

$$\left(\frac{5}{12}\right)^8 \times \left(\frac{5}{12}\right)^4 = \left(\frac{5^3}{12^3}\right)^x$$

$$\left(\frac{5}{12}\right)^{8+4} = \left[\left(\frac{5}{12}\right)^3\right]^x$$

$$\left(\frac{5}{12}\right)^{12} = \left(\frac{5}{12}\right)^{3x}$$

Comparing, we get

$$\therefore 3x = 12 \Rightarrow x = \frac{12}{3} = 4$$

Hence, $x = 4$

Question 2:

Prove that $\left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b \times \left(\frac{x^a}{x^b}\right)^c = 1$.

Solution:

$$\begin{aligned} \text{L.H.S.} & \cdot \left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b \times \left(\frac{x^a}{x^b}\right)^c \\ &= (x^{b-c})^a \times (x^{c-a})^b \times (x^{a-b})^c \\ &= x^{a(b-c)} \times x^{b(c-a)} \times x^{c(a-b)} \\ &= x^{ab-ac} \times x^{bc-ab} \times x^{ac-bc} \\ &= x^{ab-ac+bc-ab+ac-bc} = x^0 = 1 = \text{R.H.S.} \end{aligned}$$

Check your progress

Question 1:

Find the value of each of the following :

(i) $(-3)^3 \times 5^2$

(ii) $(-1)^{501} \times [(27)^4 \div (9)^5]$

(iii) $\left(-3\frac{1}{2}\right)^3$

Solution:

(i) $(-3)^3 \times 5^2$

$$= (-3) \times (-3) \times (-3) \times 5 \times 5$$

$$= -27 \times 25$$

$$= -675$$

(ii) $(-1)^{501} \times [(27)^4 \div (9)^5]$

$$= -1 \times [(3^3)^4 \div (3^2)^5]$$

$\{\because 501 \text{ is an odd numbers}\}$

$$= -1 \times [3^{12} \div 3^{10}]$$

$$= -1 \times [3^{12-10}]$$

$$= -1(3^2)$$

$$= -1 \times 3 \times 3$$

$$= -9$$

$$\text{(iii)} \left(-3\frac{1}{2}\right)^3 = \left(\frac{-7}{2}\right)^3$$

$$= \frac{-7}{2} \times \frac{-7}{2} \times \frac{-7}{2}$$

$$= \frac{-343}{8}$$

$$= -42\frac{7}{8}$$

Question 2:

Simplify the following:

$$\text{(i)} \frac{7^3 \times 11^4 \times 13^0}{7^2 \times 11^2}$$

$$\text{(ii)} \frac{(-2)^3 \times (3x)^2 \times (-xy^3)}{3x^2y}$$

$$\text{(iii)} \frac{[(-5)^3]^4 \times 8^2}{4^3 \times (25)^5}$$

Solution:

$$\text{(i)} \frac{7^3 \times 11^4 \times 13^0}{7^2 \times 11^2}$$

$$= 7^{3-2} \times 11^{4-2} \times 13^0$$

$$= 7^1 \times 11^2 \times 1$$

$$= 7 \times 11 \times 11$$

$$= 847$$

(ii)

$$\frac{(-2)^3 \times (3x)^2 \times (-xy^3)}{3x^2y}$$

$$= \frac{(-2) \times (-2) \times (-2) \times 3^2 \times x^2 \times (-x) \times y^3}{3 \times x^2 \times y}$$

$$= \frac{-8 \times 9x^2 \times (-1)x \times y^3}{3x^2 \times y}$$

$$= \frac{72}{3}xy^3 - 1$$

$$= 24xy^2$$

(iii)

$$\frac{[(-5)^3]^4 \times 8^2}{4^3 \times (25)^5}$$

$$= \frac{(-5)^{3 \times 4} \times (2^3)^2}{(2^2)^3 \times (5^2)^5}$$

$$= \frac{(-5)^{12} \times 2^6}{2^6 \times 5^{10}}$$

$$= \frac{(-1)^{12} (5)^{12} \times 2^6}{2^6 \times 5^{10}}$$

$$= 1 \times 5^{12-10} \times 2^{6-6}$$

$$= 5^2 \times 2^0$$

$$= 25 \times 1$$

$$= 25$$

Question 3:

Simplify and write the following in exponential form:

$$(i) \frac{(-3)^5 \times 8^3 \times 2^5}{3^2 \times 4^4}$$

$$(ii) \frac{9^8 \times (x^2)^5}{(27)^4 \times (x^3)^2}$$

$$(iii) \frac{3^2 \times 7^8 \times 13^6}{21^2 \times 91^3}$$

Solution :

$$(i) \frac{(-3)^5 \times 8^3 \times 2^5}{3^2 \times 4^4}$$

$$= \frac{(-1)^5 \times 3^5 \times (2^3)^3 \times 2^5}{3^2 \times (2^2)^4}$$

$$= \frac{-1 \times 3^5 \times 2^9 \times 2^5}{3^2 \times 2^8}$$

$$= -1[3^{5-2} \times 2^{9+5-8}]$$

$$= -1[3^3 \times 2^6]$$

$$= (-3)^3 \times 2^6$$

$$= (-27 \times 64)$$

$$= -1728$$

$$\begin{aligned}
\text{(ii)} \quad & \frac{9^8 \times (x^2)^5}{(27)^4 \times (x^3)^2} \\
&= \frac{(3^2)^8 \times x^{2 \times 5}}{(3^3)^4 \times x^{3 \times 2}} \\
&= \frac{3^{16} \times x^{10}}{3^{12} \times x^6} \\
&= 3^{16-12} \times x^{10-6} \\
&= 3^4 \times x^4 \\
&= (3x)^4
\end{aligned}$$

$$\begin{aligned}
\text{(iii)} \quad & \frac{3^2 \times 7^8 \times 13^6}{21^2 \times 91^3} = \frac{3^2 \times 7^8 \times 13^6}{(3 \times 7)^2 \times (7 \times 13)^3} \\
&= \frac{3^2 \times 7^8 \times 13^6}{3^2 \times 7^2 \times 7^3 \times 13^2} \\
&= 3^{2-2} \times 7^{8-2-3} \times 13^{6-3} \\
&= 3^0 \times 7^3 \times 13^3 = 1 \times 7^3 \times 13^3 \\
&= (7 \times 13)^3 = (91)^3
\end{aligned}$$

Question4:

If $\left(-\frac{3}{5}\right)^x = -\frac{27}{125}$ then find the value of x.

Solution :

$$\left(-\frac{3}{5}\right)^x = -\frac{27}{125}$$

$$= \left(\frac{-3}{5}\right)^n = \left(\frac{-3}{5}\right)^3$$

Comparing, we get

$$x = 3$$

Question 5:

Write the prime factorisation of the following numbers in the exponential form:

(i) 24000

(ii) 12600

(iii) 14157

Solution :

(i) 24000

2	24000
2	1200
2	6000
2	3000
2	1500
2	750
3	375
5	125
5	25
5	5
	1

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 5$$

$$= 2^6 \times 3^1 \times 5^3$$

(ii) 12600

2	12600
2	6300
2	3150
3	1575
3	525
5	175
5	35
7	7
	1

$$\begin{aligned} &= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 \\ &= 2^3 \times 3^2 \times 5^2 \times 7^1 \end{aligned}$$

(iii) 14157

3	14157
3	4789
11	1573
11	143
13	13
	1

$$\begin{aligned} &= 3 \times 3 \times 11 \times 11 \times 13 \\ &= 3^2 \times 11^2 \times 13^1 \end{aligned}$$

Question 6:

Express the numbers appearing in the following statements in scientific notation:

(i) The earth has 1,353,000,000 cubic km of water.

(ii) The population of India was about 1,027,000,000 in march, 2001.

(iii) 60,230,000,000,000,000,000 molecules are contained in a drop of water.

Solution :

(i) The earth has 1,353,000,000 cubic km of water.

$$= 1.353 \times 10^9 \text{ cubic km}$$

(ii) The population of India was about 1,027,000,000 in march, 2001.

$$= 1.027 \times 10^9$$

(iii) 60,230,000,000,000,000,000 molecules are contained in a drop of water.

$$6.023 \times 10^{22} \text{ molecules}$$

Question 7:

Compare the following numbers:

(i) 5.976×10^{24} ; 8.689×10^{23}

(ii) 3.7662×10^{17} ; 3.7671×10^{17}

Solution:

(i) 5.976×10^{24} ; 8.689×10^{23}

Here in 5.976×10^{24} , 10^2 is greater than in 8.689×10^{23}

$$5.976 \times 10^{24} > 8.689 \times 10^{23}$$

(ii) 3.7662×10^{17} ; 3.7671×10^{17}

Here 10^{17} is multiplied in both

and $76671 > 7662$

$$3.76671 \times 10^{17} > 3.7662 \times 10^{17}$$