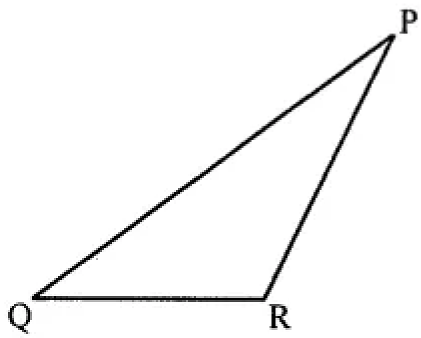
Chapter 11

Triangles and its Properties

Exercise 11.1

Question 1.
In the adjoining figure:



(i) Name the vertex opposite to side PQ.
(ii) Name the side opposite vertex Q.
(iii) Name the angle opposite to side QR.
(iv) Name the side opposite to ∠R.
Solution:

In the given figure,

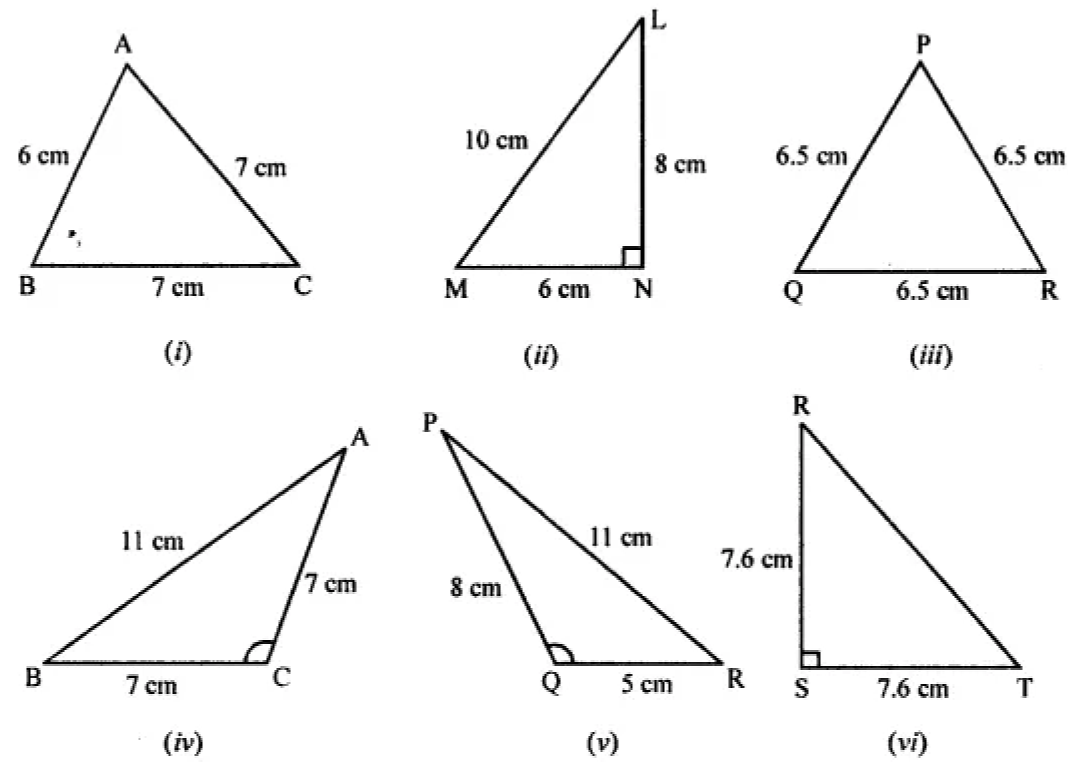
(i) Vertex opposite to side PQ is R.

(ii) The side opposite to the vertex Q is PR.

(iii) Angle opposite to the side QR is ∠R.

(iv) the side opposite to ∠R is PQ.

Question 2.
Look at the figures given below and classify each of the triangles according to its (a) sides (b) angles
(You may judge the nature of the angle by observation):



Solution:

(i) Two sides are equal. So it is an isosceles triangle.

(ii) Three sides are unequal and one angle is 90°,

so it is a right-angled triangle end also it is a scalene triangle also.

(iii) Three sides are equal. So, it is an equilateral triangle.

(iv) Two sides are equal. So, it is an isosceles triangle.

It's one angle is obtuse, therefore it obtuse angled triangle also.

(v) Three sides are not equal. So, it is a scalene triangle.

It's one angle is obtuse. So, it is an obtuse angled triangle also.

(vi) Its two sides are equal and one angle is a right angle.

So it is a right-angled isosceles triangle.

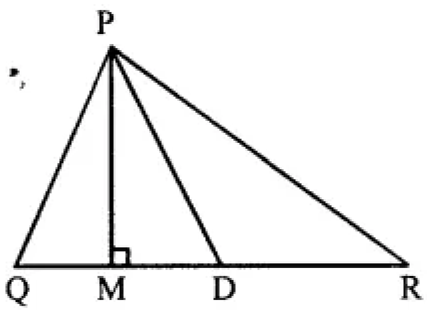
Question 3.

In the given ΔPQR, if D is the mid-point of $\overline{QR}$, then

(i) $\overline{PM}$ is ……..

(ii) $\overline{PD}$ is ………

Is QM = MR?



Solution:

If the given figure, in ΔPQR

D is mid-point of $\bar{QR}$, then

(i) PM is an altitude.

(ii) PQ is the median.

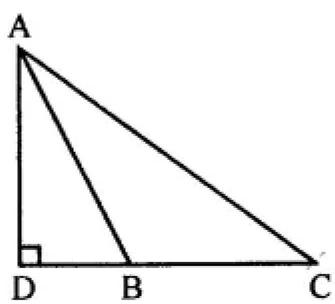
No, QM ≠ MR

Question 4.
Will an altitude always lie in the interior of a triangle? If no, draw a rough sketch to show such a case.
Solution:

No, it is not necessary, it may lie outside the triangle also.

Here is given a rough sketch of the case AD is the altitude of ΔABC.

Draw from A to the side CB (produced).

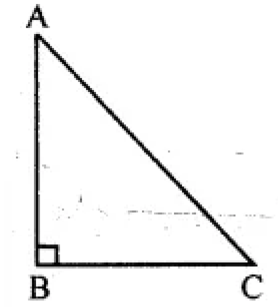


Question 5.
Can you think of a triangle in which two altitudes of the triangle is its sides? If yes, draw a rough sketch to show such a case.

Solution:

Yes, it is a right-angled triangle.

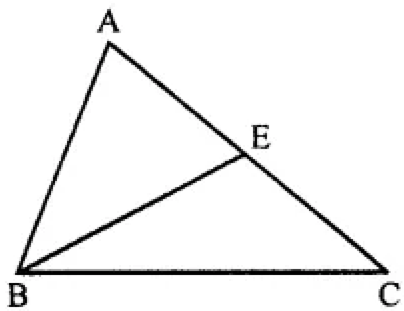
Here, AB ⊥ BC and BC ⊥ AB



Question 6.
Draw rough sketches for the following:
(i) In ΔABC, BE is a median of the triangle.
(ii) In ΔPQR, PQ and PR are altitudes of the triangle.
(iii) In ΔXYZ, YL is an altitude in the exterior of the triangle.
Solution:

(i) In ΔABC, BE is the median of the triangle.

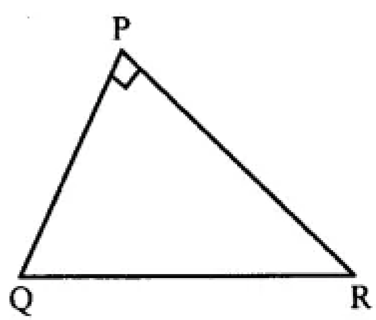
E is the mid-point of AC. So, BE is the median.



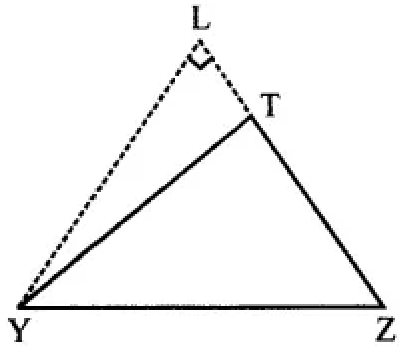
(ii) In ΔPQR, PQ and PR are the altitudes of the triangle.

In ΔPQR, ∠P = 90°

So, PQ and PR are the altitudes.



(iii) In ΔXYZ, YL is the altitude in the exterior of the triangle YL is altitude on ZX (produced).



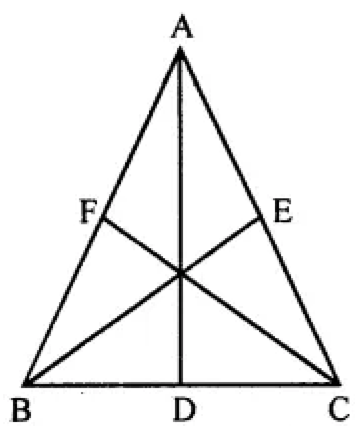
Question 7.
Take an equilateral triangle and draw its medians and altitudes and check that the medians and altitudes are the same.
Solution:

ΔABC is an equilateral triangle.

AD, BE and CF are altitudes of the triangle.

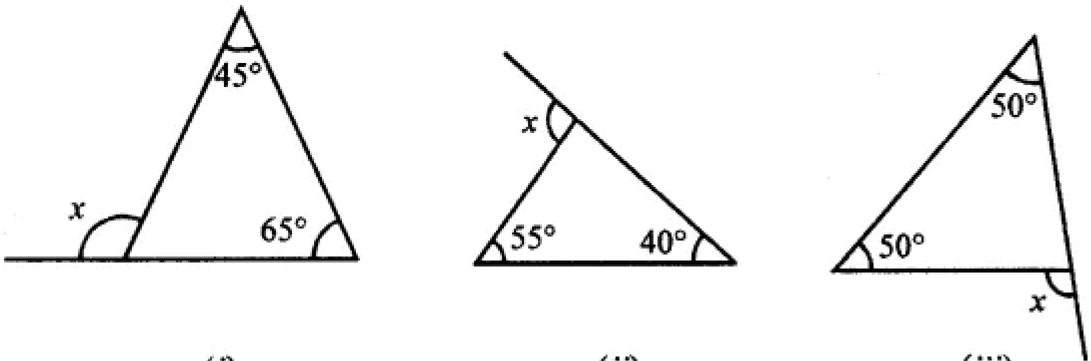
The altitudes of an equilateral triangle divide the sides into two equal parts.

So, altitudes are also the medians of the triangle.



Exercise 11.2

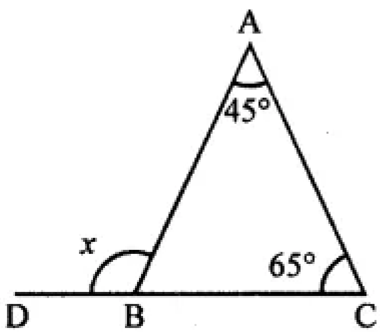
Question 1.
Find the value of the unknown exterior angle x in each of the following diagrams:



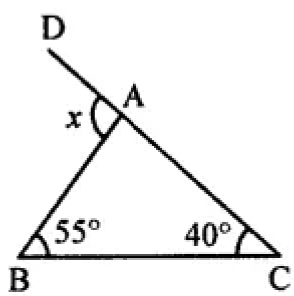
(i) (ii) (iii)

Solution:

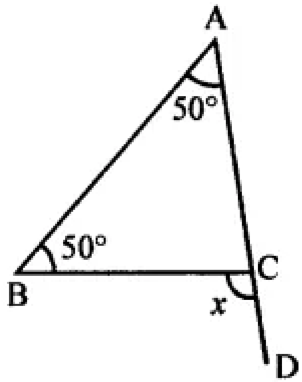
We know that the exterior angle of a triangle
is equal to the sum of its interior opposite angles.
Therefore,
(i) Ext. $\angle x = 45° + 65° = 110°$



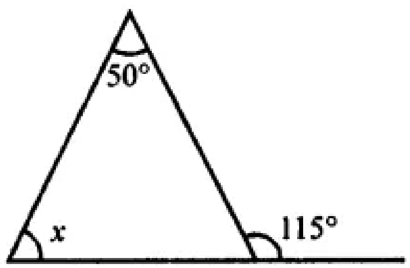
(ii) Ext. $\angle x = 55° + 40° = 95°$



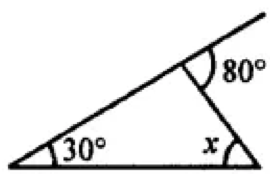
(iii) Ext. $\angle x = 50° + 50° = 100°$



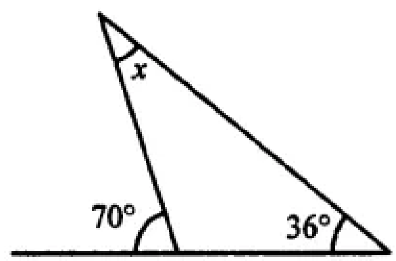
Question 2.
Find the value of the unknown interior angle x in each of the following diagrams:



(*i*)



(*ii*)



(*iii*)

Solution:

We know that the exterior angle of a triangle

is equal to the sum of its interior opposite angles.

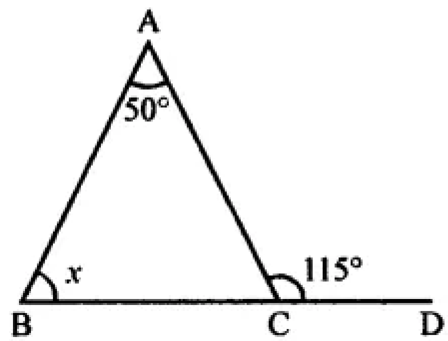
Therefore,

(i) In the given triangle,

Ext. $\angle 115° = x + 50°$

$\Rightarrow x = 115° - 50° = 65°$

$\Rightarrow x = 65°$

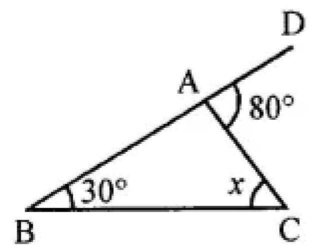


(ii) In given triangle,

Ext. ∠80° = 30° + x

⇒ x = 80° − 30° = 50°

⇒ x = 50°

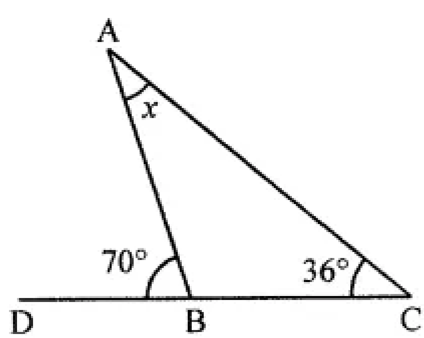


(iii) In given triangle,

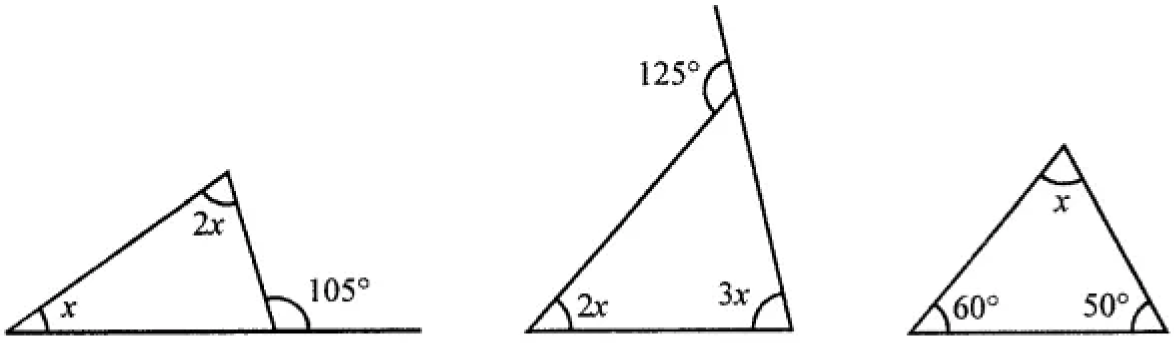
Ext. ∠70° = x + 36°

⇒ x = 70° − 36° = 34°

⇒ x = 34°



Question 3.
Find the value of x in each of the following diagrams:



(i) (ii) (iii)

Solution:

We know that the exterior angle of a triangle

is equal to the sum of its interior opposite angles.

Therefore,

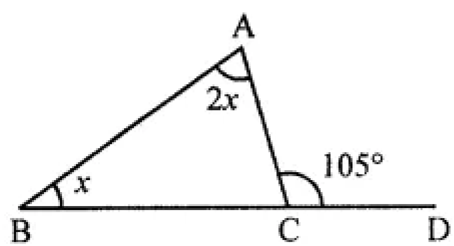
(i) In a given triangle,

Ext. ∠105° = 2x + x

⇒ 3x = 105°

⇒ x = 35°

x = 35°



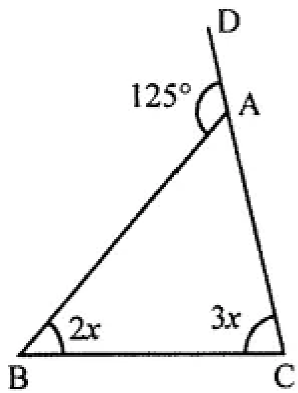
(ii) In given triangle,

Ext. ∠125° = 2x + 3x

⇒ 5x = 125°

⇒ x = 25°

x = 25°



(iii) In given triangle,

$\angle A + \angle B + \angle C = 180°$

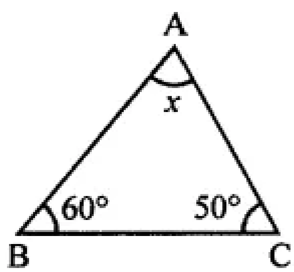
(Sum of angles of a triangle)

$\Rightarrow x + 60° + 50° = 180°$

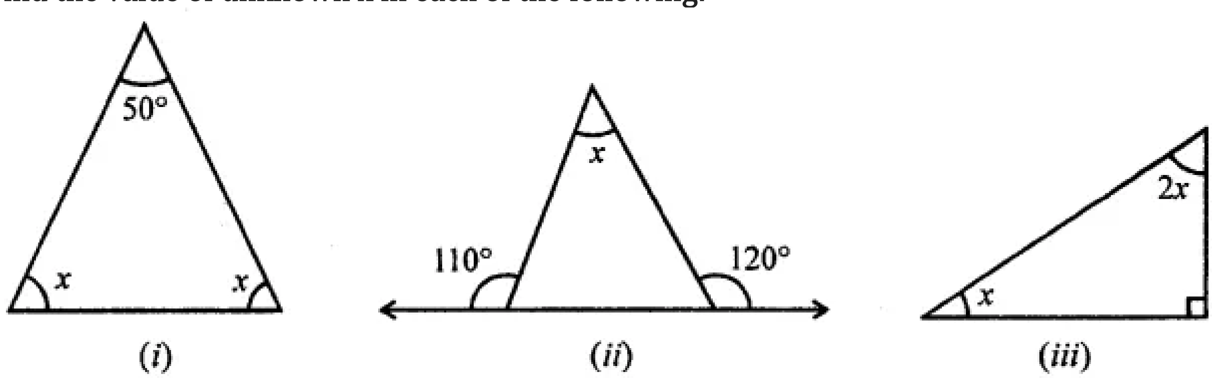
$\Rightarrow x + 110° = 180°$

$\Rightarrow x = 180° - 110° = 70°$

$x = 70°$



Question 4.
Find the value of unknown x in each of the following:



Solution:

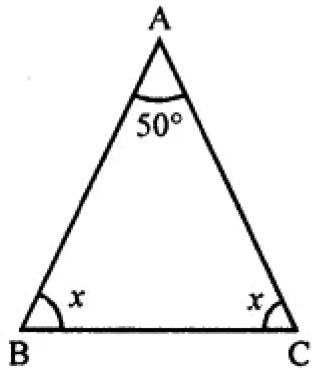
(i) In given triangle = Let ΔABC

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 50° + x + x = 180°$

$\Rightarrow 2x = 180° - 50° = 130°$

Hence, $x = 65°$



(ii) In the given figure,

Let the name of Δ be ABC

$\angle ABC + \angle ABE = 180°$

$\Rightarrow \angle ABC + 110° = 180°$

$\Rightarrow \angle ABC = 180° - 110° = 70°$

Similarly,

$\angle ACB + \angle ACD = 180°$

$\Rightarrow \angle ACB + 120° = 180°$

$\Rightarrow \angle ACB = 180° - 120° = 60°$

Now in ΔABC

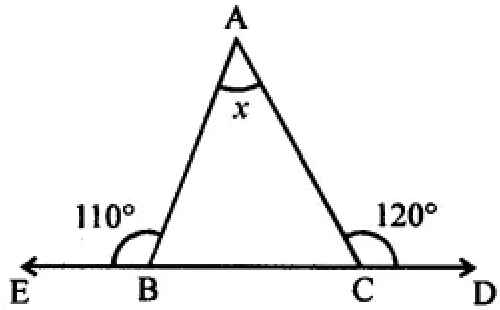
$\angle BAC + \angle ABC + \angle ACB = 180°$

$\Rightarrow x + 70° + 60° = 180°$

$\Rightarrow x + 130° = 180°$

$\Rightarrow x = 180° - 130° = 50°$

$\Rightarrow x = 50°$



(iii) Let the given triangle be named as ΔABC,

where $\angle C = 90°$

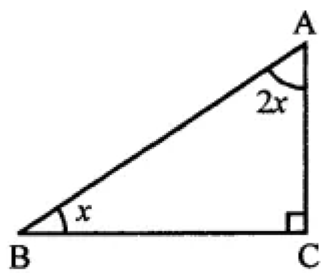
In ΔABC,

$\angle A + \angle B + \angle C = 180°$ (Sum of angles of a triangle)

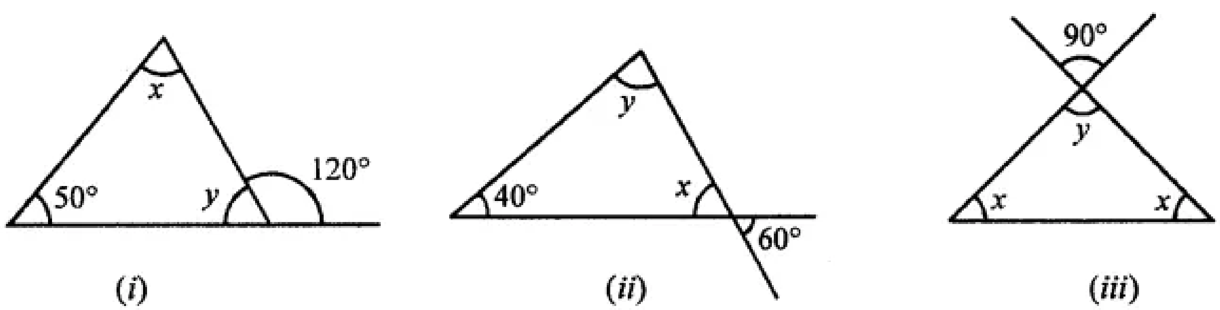
$\Rightarrow 2x + x + 90° = 180°$

$\Rightarrow 3x = 180° - 90° = 90°$

$\Rightarrow x = 30°$



Question 5.
Find the values of x and y in each of the following diagrams:



Solution:

We know that an exterior angle of a triangle

is equal to the sum of its interior opposite angle.

Therefore,

(i) Let the Δ's name = ΔABC

In ΔABC

Ext. ∠ACD = ∠A + ∠B

$120° = x + 50°$

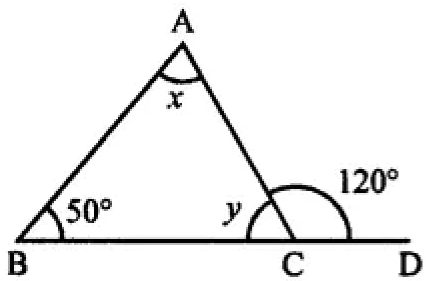
$\Rightarrow x = 120° - 50° = 70°$

But ∠ACD + ∠ABC = 180° (Linear pair)

$120° + y = 180°$

$\Rightarrow y = 180° - 120° = 60°$

$x = 70°, y = 60°$



(ii) In the given figure,

∠ACB = ∠DCE (Vertically opposite angles)

$x = 60°$

But ∠A + ∠B + ∠ACB = 180°

(Sum of angles of a triangle)

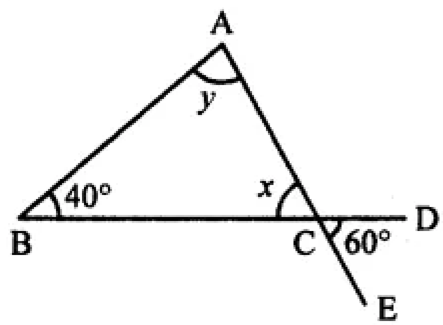
$\Rightarrow y + 40° + x = 180°$

$\Rightarrow y + 40° + 60° = 180°$

$\Rightarrow y + 100° = 180$

$\Rightarrow y = 180° - 100° = 80°$

Hence, $x = 60°, y = 80°$



(iii) In the given figure,

∠BAC = ∠EAF (Vertically opposite angles)

y = 90°

In ΔABC,

∠A + ∠B + ∠C = 180° (Sum of angles of a triangle)

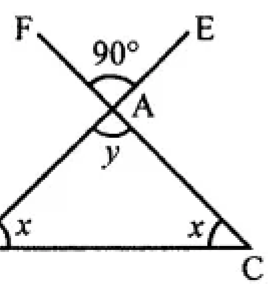
⇒ y + x + x = 180°

⇒ 90° + 2x = 180°

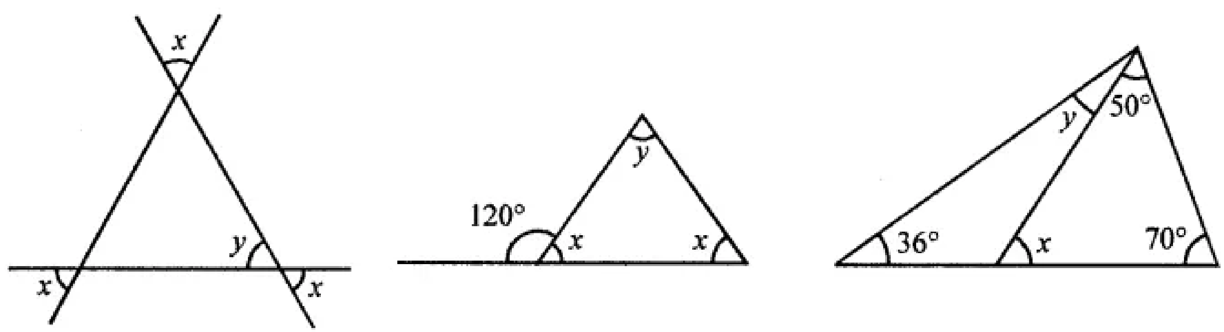
⇒ 2x = 180° – 90° = 90°

⇒ x = 45°

Hence, x = 45°



Question 6.
Find the values of x and y in each of the following diagrams:



(i) (ii) (iii)

Solution:

(i) In the given figure,

In ΔABC,

$x = y$ (Vertically opposite angles)

Similarly,

$\angle BAC = y$, $\angle ABC = y$, $\angle BCA = y$

But $\angle BAC + \angle ABC + \angle BCA = 180°$

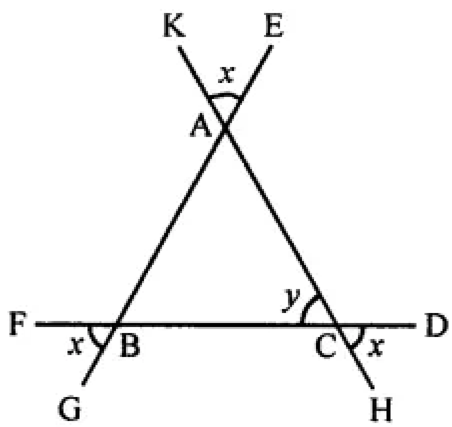
(Angles of a triangle)

$\Rightarrow y + y + y = 180°$

$\Rightarrow 3y = 180°$

$\Rightarrow y = 60°$

$x = 60°$, $y = 60°$



(ii) In ΔABC,

$\angle ABC + \angle ABD = 180°$

$\Rightarrow x + 125° = 180°$

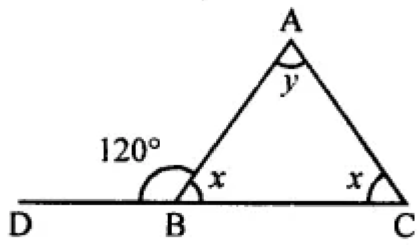
$\Rightarrow x = 180° - 125° = 55°$

and Ext. $\angle ABD = x + y$

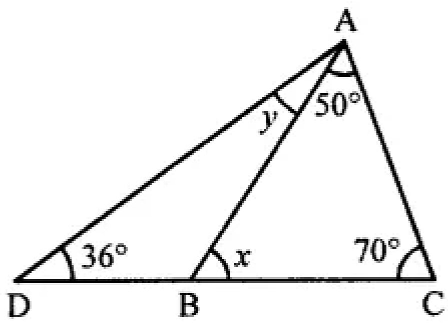
$\Rightarrow 125° = 55° + y$

$\Rightarrow y = 125° - 55° = 70°$

$x = 55°$, $y = 70°$



(iii) In ΔABC,



Ext. ∠ABD = ∠A + ∠B = 50° + 70° = 120°

But ∠ABC + ∠ABD = 180° (Linear pair)

⇒ x + ∠ABD = 180°

⇒ x + 120° = 180°

⇒ x = 180° − 120° = 60°

But in ΔABD

Ext. ∠ABC = ∠D + ∠DAB

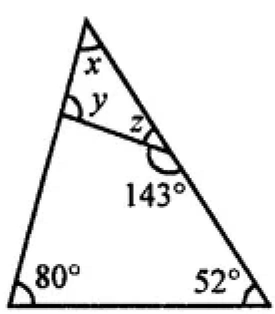
⇒ x = y + 30°

⇒ 60° = y + 30°

⇒ y = 60° − 30° = 30°

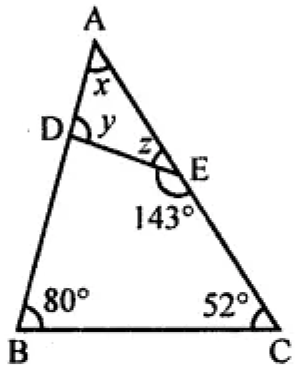
x = 60°, y = 30°

Question 7.
In the adjoining figure, find the size of each lettered angle.



Solution:

In the given figure,



In ΔABC

∠A + ∠B + ∠C = 180° (Sum of angles of a triangle)

⇒ x + 80° + 52° = 180°

⇒ x + 132° = 180°

⇒ x = 180° − 132° = 48°

∠DEC + ∠DEA = 180° (Linear pair)

⇒ 143° + z = 180°

⇒ z = 180° − 143° = 37°

Now in ΔADE

Ext. ∠DEC = ∠A + ∠ADE

⇒ 143° = x + y

⇒ 143° = 48° + y

⇒ y = 143° − 48° = 95°

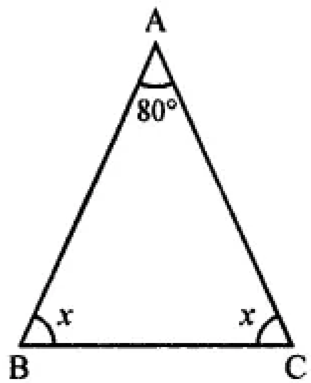
x = 48°, y = 95°, z = 37°

Question 8.
One of the angles of a triangle measures 80° and the other two angles are equal. Find the measure of each of the equal angles.

Solution:

One angle of an $\Delta ABC = 80°$

Let $\angle A = 80°$ and the other two angles are equal



Let $\angle B = \angle C = x$

In ΔABC,

$\angle A + \angle B + \angle C = 180°$ (Sum of angles of a triangle)

$\Rightarrow 80° + x + x = 180°$

$\Rightarrow 2x = 180° - 80° = 100°$

$\Rightarrow x = 50°$

$\angle A = 80°, \angle B = 50°, \angle C = 50°$

Question 9.
If one angle of a triangle is 60° and the other two angles are in the ratio 2 : 3, find these angles.
Solution:

One angle of a triangle = 60°

Other two angles are in the ratio 2 : 3

Sum of other two angles = $180° - 60° = 120°$

Let one of other two angles = $2x$

Then third angle = $3x$

$2x + 3x = 120°$

$\Rightarrow 5x = 120°$

$\Rightarrow x = 24$

Other two angles are $2x = 2 \times 24 = 48°$

and $3x = 3 \times 24 = 72°$

Other two angles of the triangle are $48°$, $72°$

Question 10.
If the angles of a triangle are in the ratio 1 : 2 : 3, find the angles. Classify the triangle in two different ways.
Solution:

Sum of angles of a triangle = $180°$

Ratio in the angles of a triangle = $1 : 2 : 3$

Let first angle = x

Second angle = $2x$

Third angle = $3x$

$x + 2x + 3x = 180°$

$\Rightarrow 6x = 180°$

$\Rightarrow x = 30°$

$\therefore$ First angle = $30°$

Second angle = $30° \times 2 = 60°$

and third angle = $30° \times 3 = 90°$

$\because$ One angles is $90°$

$\therefore$ It is a right angled triangle

$\because$ Sides an different

$\therefore$ It is a scalene triangle.

Question 11.
Can a triangle have three angles whose measures are
(i) 65°, 74°, 39°?
(ii) 13 right angle, 1 right angle, 60°?

Solution:

We know that sum of angles of a triangle = 180°

(i) Angles are 65°, 74°, 39°

Sum of angles = 65° + 74° + 39° = 178°

178° ≠ 180°

There three angles can not be of triangle

(ii) $\frac{1}{3}$ right angle = $\frac{1}{2}$ × 90° = 30°

1 right angle = 90°

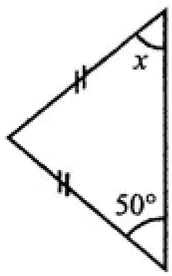
Third angle = 60°

Sum of angles = 30° + 90° + 60° = 180°

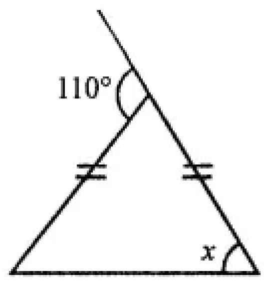
These angles are of a triangle.

Exercise 11.3

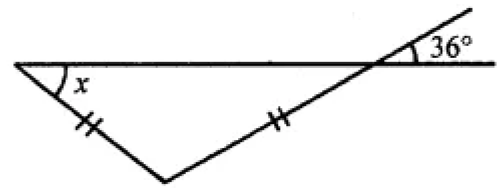
Question 1.
Find the value of x in each of the following figures:



(i)



(ii)



(iii)

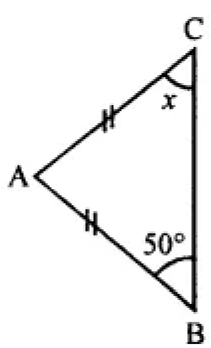
Solution:

(i) In ΔABC,

AB = AC

∠B = ∠C (Angles opposite to equal sides)

x = 50°



(ii) In ΔABC,

AB = AC

∠B = ∠C (Angles opposite to equal sides)

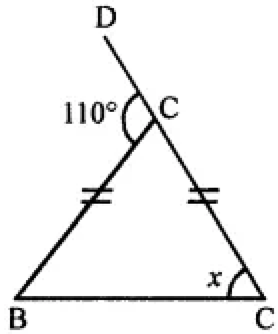
∠B = ∠C = x

and Ext. ∠BAD = ∠B + ∠C = x + x

110° = ∠B + ∠C

110° = x + x = 2x

x = 55°



(iii) In the given figure,

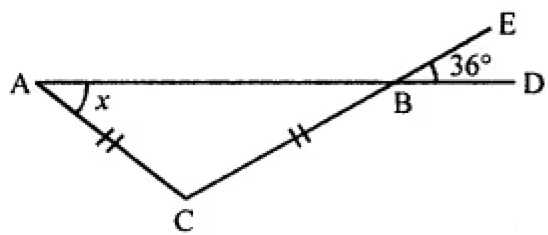
CA = CB

∠A = ∠ABC = x

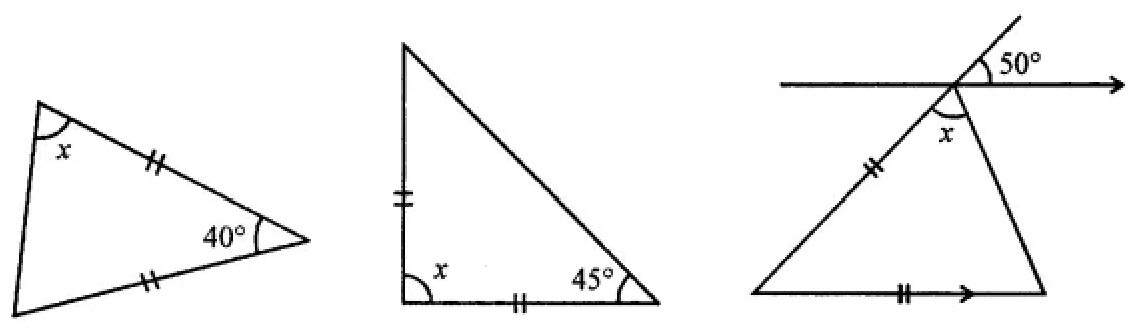
∠EBD = 36°

∠ABC = ∠EBD = 36° (Vertically opposite angles)

x = 36°



Question 2.
Find the value of x in each of the following figures:



(i) (ii) (iii)

Solution:

(i) In ΔABC,

$AC = BC$, $\angle C = 40°$

$\angle A = \angle B$ (Angles opposite to equal sides)

$\angle A = \angle B = x$

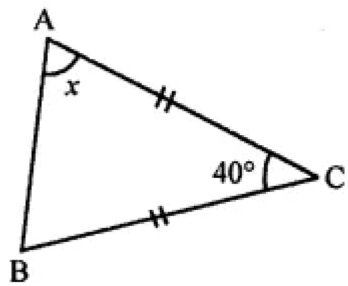
But $A + B + C = 180°$ (Angles of a triangle)

$\Rightarrow x + x + 40° = 180°$

$\Rightarrow 2x = 180° - 40° = 140°$

$\Rightarrow x = 70°$

$x = 70°$



(ii) In ΔABC,

$\angle B = 45°$

$AC = BC$

$\angle A = \angle B = 45°$

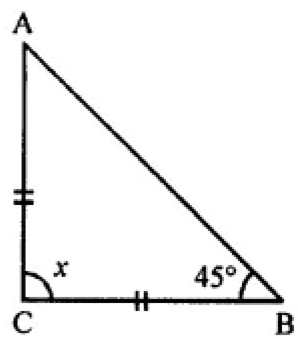
But $\angle A + \angle B + \angle C = 180°$

$\Rightarrow 45° + 45° + x° = 180°$

$\Rightarrow x + 90° = 180°$

$\Rightarrow x = 180° - 90° = 90°$

$x = 90°$



(iii) In the given figure, EF || BC

In ΔABC, Ext. $\angle DAF = 50°$

$AB = BC$

$\angle A = \angle C = x$

EF || BC

$\angle DAF = \angle ABC = 50°$

Now in ΔABC

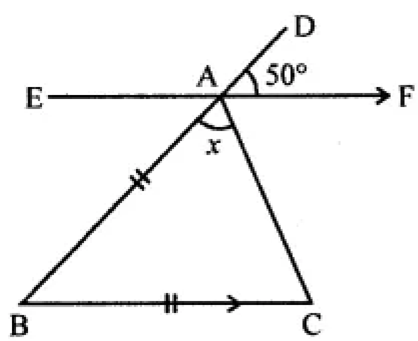
$\angle BAC + \angle ABC + \angle BCA = 180°$

$\Rightarrow x + 50° + x = 180°$

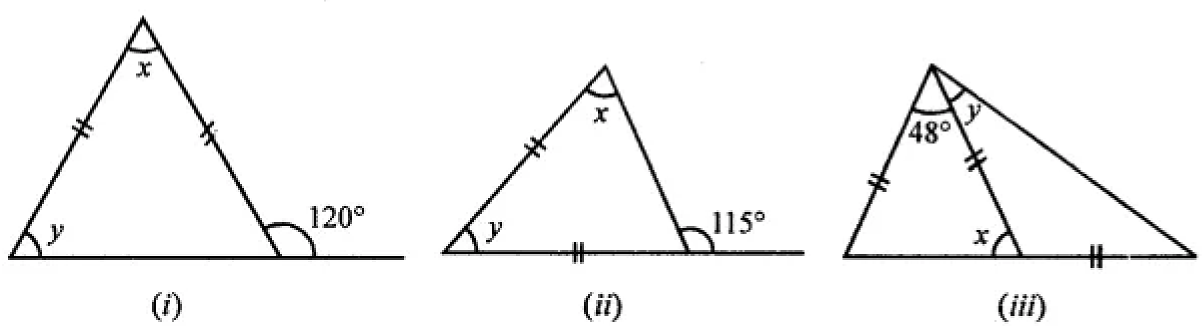
$\Rightarrow 2x = 180° - 50° = 130°$

$\Rightarrow x = 65°$

$x = 65°$



Question 3.
Find the values of x and y in each of the following figures:



Solution:

(i) In the given figure of ΔABC

AB = AC

∠ABC = ∠ACB = y

But, Ext. ∠ACD + ∠ACB = 120° (Linear pair)

∠ACB = 180° − 120° = 60°

y = 60°

Now in ΔABC,

∠A + ∠B + ∠ACB = 180° (Angles of a triangle)

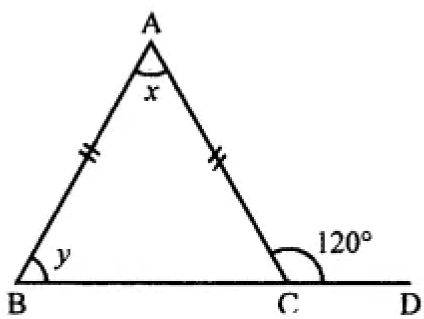
⇒ x + y + y = 180°

⇒ x + 60° + 60° = 180°

⇒ x + 120° = 180°

⇒ x = 180° − 120° = 60°

Here, x = 60°, y = 60°



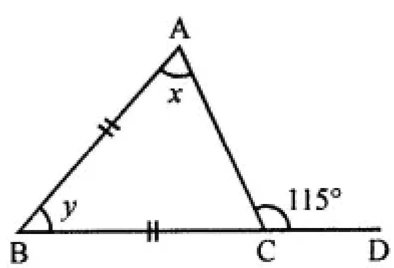
(ii) Here, ∠A = ∠ACB

∠ACB = x (∵ angles opposite to equal sides)

⇒ x + 115° = 180°

⇒ x = 180° − 115° = 65

∠ACB = ∠A = 65°



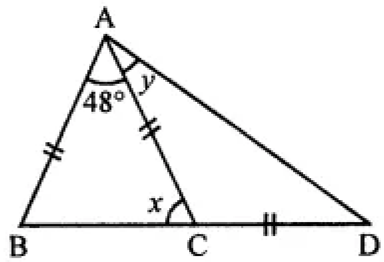
Now, $\angle A + \angle B + \angle ACB = 180°$ (Sum of $\angle$s of a Δ)

$65° + 65° + \angle y = 180°$

$130° + \angle y = 180°$

$\angle y = 180° - 130° = 50°$

(iii) In ΔABC,



$\angle ABC = \angle ACB$ ($\angle$s opposite to equal sides)

$\angle ABC = x = \angle ACB$

Now, In ΔABC

$48° + x + x = 180°$ (Sum of $\angle$s of a Δ)

$\Rightarrow 2x = 180° - 48°$

$\Rightarrow 2x = 132°$

$\Rightarrow x = 66°$

In ΔACD,

$\angle CAD = \angle CDA$ ($\angle$s opposite to equal sides)

$\angle CAD = y = \angle CDA$

Now, $x + \angle ACD = 180°$ (Linear pair $\angle$s)

$66° + \angle ACD = 180°$

$\angle ACD = 180° - 66° = 114°$

Now, in ΔACD

$y + y + 114° = 180°$ (Sum of $\angle$s of a Δ)

$\Rightarrow 2y + 114° = 180°$

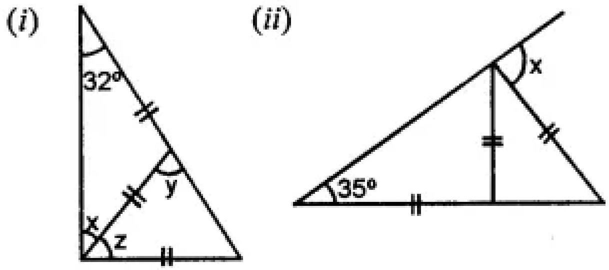
$\Rightarrow 2y = 180° - 114°$

$\Rightarrow 2y = 66°$

$\Rightarrow y = 33°$

Hence, $x = 66°$ and $y = 33°$

Question 4.
Calculate the size of each lettered angle in the following figures:



Solution:

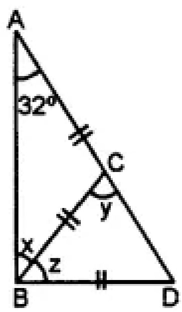
(i) In ΔABC,

AC = BC (Given)

∠ABC = ∠BAC (∠s opposite to equal sides)

∠ABC = 32°

⇒ x = 32°



Now, y = 32° + 32° = 64°

(∵ Exterior angle = Sum of two opposite interior ∠s)

In ΔBCD,

BC = BD

∠BDC = ∠BCD (∠s opposite to equal sides)

∠BDC = y = ∠BCD

∠BDC = 64° = ∠BCD

Now, z + 64° + 64° = 180° (Sum of ∠s of a Δ)

⇒ z + 128°= 180°

⇒ z = 180° − 128° = 52°

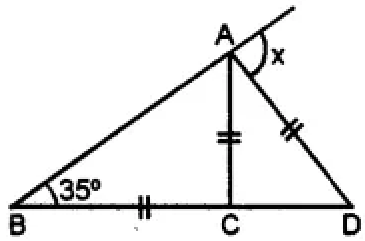
Hence, x = 32°, y = 64°, z = 52°

(iii) In ΔABC,

$AC = BC$ (Given)

$\angle ABC = \angle BAC$ ($\angle$s opposite to equal sides)

$\angle ABC = 35° = \angle BAC$



Now, $\angle ACD = \angle ABC + \angle BAC$ (Exterior angle = Sum)

$= 35° + 35° = 70°$.

In ΔACD,

$AC = AD$ (Given)

$\angle ADC = \angle ACD$ ($\angle$s opposite to equal sides)

$\angle ADC = 70°$

Now, $\angle x = \angle ABD + \angle ADB$

(Exterior angle = Sum of two opposite interior $\angle$s)

$= 35° + 70° = 105°$

Hence, $\angle x = 105°$

Question 5.
If the angles of a triangle are in the ratio 1 : 2 : 1, find all the angles of the triangle. Classify the triangle in two different ways.
Solution:

Ratio in the angles of a triangle are 1 : 2 : 1

Sum of angles of a triangle = 180°

Let first angle = x

Then second = 2x

and third angle x

$x + 2x + x = 180°$

$\Rightarrow 4x = 180°$

$\Rightarrow x = 45°$

Angles are 45°, 45° × 2 = 90° and 45°

Two angles are equal

Their opposite sides are also equal

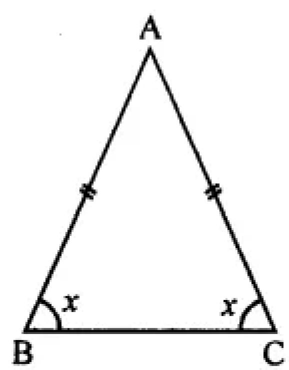
It is an isosceles triangle

It's one angle is 90°

It is a right-angled triangle.

Question 6.
In an isosceles triangle, a base angle is four times its vertical angle. Find all the angles of the triangle.
Solution:

In an isosceles triangle ABC, AB = AC



Base angles are equal

Let ∠B = ∠C = x

$$\angle A = \frac{x}{4}$$

$$\therefore\ x + x + \frac{x}{4} = 180° \Rightarrow \frac{4x + 4x + x}{4} = 180°$$

$$\Rightarrow \frac{9x}{4} = 180° \Rightarrow x = 180° \times \frac{4}{9} = 80°$$

$$\text{Vertical angle} = \frac{x}{4} = \frac{80°}{4} = 20°$$

Angles of the triangle will be 80°, 80°, 20°

Exercise 11.4

Question 1.
Is it possible to have a triangle with the following sides?
(i) 2 cm, 3 cm, 5 cm
(ii) 2.5 cm, 4.5 cm, 8 cm
(iii) 10.2 cm, 5.8 cm, 4.5 cm
(iv) 3.4 cm, 4.7 cm, 6.2 cm
Solution:

(i) 2 cm, 3 cm, 5 cm

We know that in a triangle,

the sum of any two sides is greater than its third side.

Now, 2 cm, 3 cm, 5 cm

It is not possible to draw the a triangle.

(ii) 2.5 cm, 4.5 cm, 8 cm

2.5 + 4.5 cm = 7 cm < 8 cm

It is also not possible to draw the triangle.

(iii) 10.2 cm, 5.8 cm, 4.5 cm

5.8 + 4.5 = 10.3 > 10.2 cm

It is possible to draw the triangle.

(iv) 3.4 cm, 4.7 cm, 6.2 cm

3.4 + 4.7 = 8.1 > 6.2 cm

It is possible to draw the triangle.

Question 2.
If the lengths of two sides of a triangle are 7 cm and 10 cm, then what can be the length of the third side?
Solution:

Length of two sides of a triangle is 7 cm and 10 cm.

In order to draw a triangle,

the third side must be less than the sum of these two sides in

7 cm + 10.2 cm = 17.2 cm

Question 3.

We know that in a triangle, the sum of lengths of any two sides is greater than the length of the third side. Is the sum of any angles of a triangle also greater than the third angle? If no, draw a rough sketch to show such a case.

Solution:

In a triangle, the sum of any two sides must be

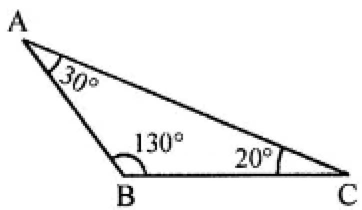
greater than its third side but in case of its angles.

It is not necessary that the sum of any two angles be

more than its third angle.

Such as in this triangle sum of any two angles is less than its third angle.

Such as $30° + 20° = 50° < 130°$

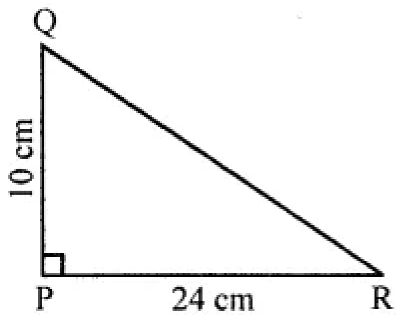


Exercise 11.5

Question 1.
PQR is a triangle, right-angled at P. If PQ = 10 cm and PR = 24 cm, find QR.
Solution:

In ΔPQR, $\angle P = 90°$

$PQ = 10$ cm, $PR = 24$ cm



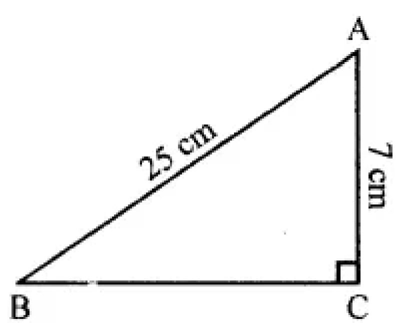
Using Pythagoras Theorem,

$QR^2 = PQ^2 + PR^2 = 10^2 + 24^2 = 100 + 576 = 676 = (26)^2$

$QR = 26$ cm

Question 2.
ABC is a triangle, right-angled at C. If AB = 25 cm and AC = 7 cm, find BC.
Solution:

In ΔABC, $\angle C = 90°$



$AB = 25$ cm and $AC = 7$ cm

$AB^2 = AC^2 + BC^2$

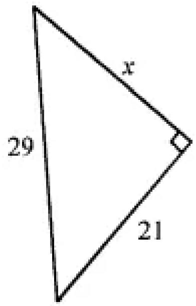
$\Rightarrow (25)^2 = (7)^2 + BC^2$

$\Rightarrow 625 = 49 + BC^2$

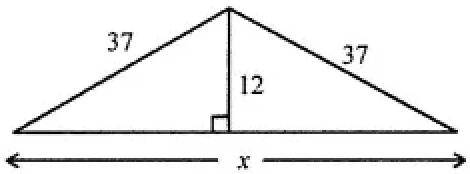
$\Rightarrow BC^2 = 625 - 49 = 576 = (24)^2$

$\Rightarrow BC = 24$ cm

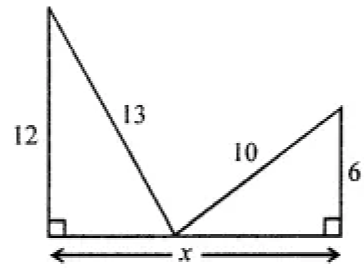
Question 3.
Find the value of x in each of the following figures. All measurements are in centimeters.



(i)



(ii)



(iii)

Solution:

(i) In ΔABC, $\angle C = 90°$

$AB = 29$, $BC = 21$, $AC = x$

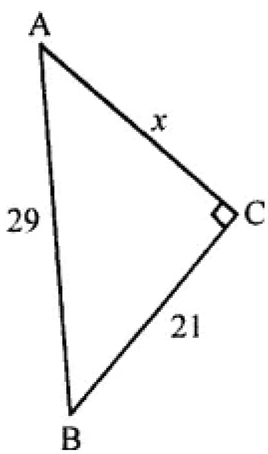
$AB^2 = AC^2 + BC^2$ (Using Pythagoras theorem)

$\Rightarrow (29)^2 = (21)^2 + x^2$

$\Rightarrow 841 = 441 + x^2$

$\Rightarrow x^2 = 841 - 441 = 400 = (20)^2$

$x = 20$



(ii) In ΔABC, $AD \perp BC$

$AB = 37$, $AC = 37$

In right ΔABD

$AB^2 = AD^2 + BD^2$ (Pythagoras Theorem)

$\Rightarrow (37)^2 = (12)^2 + BD^2$

$\Rightarrow 1369 = 144 + BD^2$

$\Rightarrow BD^2 = 1369 - 144$

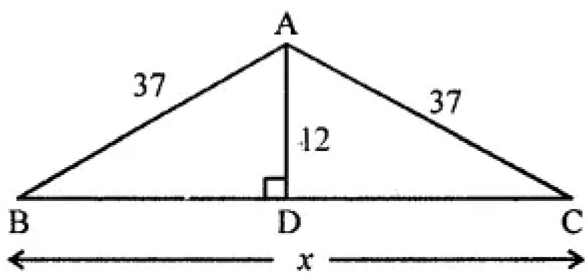
$\Rightarrow BD^2 = 1225 = (35)^2$

$\Rightarrow BD = 35$ cm

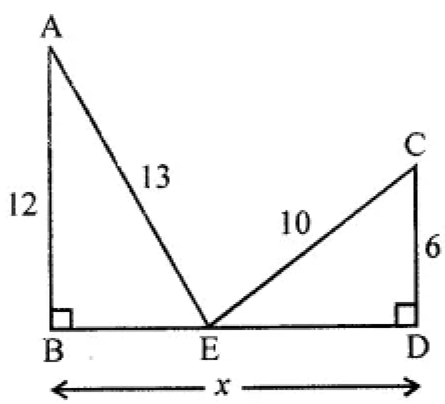
But AD bisects BC at D

$BC = 2 \times BD$

$\Rightarrow x = 2 \times 35 = 70$ cm



(iii) In right ΔABE, $\angle B = 90°$



$AE^2 = AB^2 + BE^2$ (Pythagoras Theorem)

$\Rightarrow (13)^2 = 12^2 + BE^2$

$\Rightarrow 169 = 144 + BE^2$

$\Rightarrow BE^2 = 169 - 144 = 25 = (5)^2$

$\Rightarrow BE = 5$ cm

Similarly in right ΔCDE, $\angle D = 90°$

$CE^2 = CD^2 + ED^2$

$\Rightarrow 10^2 = 6^2 + ED^2$

$\Rightarrow 100 = 36 + ED^2$

$\Rightarrow ED^2 = 100 - 36 = 64 = (8)^2$

$\Rightarrow ED = 8$ cm

Now, $BD = x = BE + ED = 5 + 8 = 13$ cm

Question 4.
Which of the following can be the sides of a right-angled triangle?
(i) 4 cm, 5 cm, 7 cm
(ii) 1.5 cm, 2 cm, 2.5 cm
(iii) 7 cm, 5.6 cm, 4.2 cm
Solution:

(i) Sides are : 4 cm, 5 cm, 7 cm

$(\text{Longest side})^2 = (7)^2 = 49$

Sum of squares of other two sides

$= 4^2 + 5^2$

$= 16 + 25$

$= 41$

$49 \neq 41$

These are not the sides of the right triangle.

(ii) Sides are : 1.5 cm, 2 cm, 2.5 cm

$(\text{Longest side})^2 = (2.5)^2 = 6.25$

Sum of the squares of the other two sides

$= 1.5^2 + 2^2$

$= 2.25 + 4$

$= 6.25$

$6.25 = 6.25$

There are the sides of a right triangle

and right angle is opposite to the side 2.5 cm

(iii) Sides are : 7 cm, 5.6 cm, 4.2 cm

$(\text{Longest side})^2 = 7^2 = 49$

Sum of the squares of the other two sides

$= (5.6)^2 + (4.2)^2$

$= 31.36 + 17.64$

$= 49$

$49 = 49$

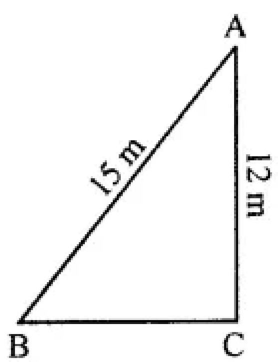
These are the sides of a right triangle

Right angle is opposite to the side 5.6 cm

Question 5.
A 15 m long ladder reaches a window 12 m high from the ground on placing it against a wall. How far is the foot of the ladder from the wall?
Solution:

Let length of ladder AB = 15 m

and height of wind AC = 12 m



BC is the distance from wall to the foot of ladder

In right ΔABC, $\angle C = 90°$

$AB^2 = AC^2 + BC^2$ (Pythagoras Theorem)

$\Rightarrow (15)^2 = 12^2 + BC^2$

$\Rightarrow 225 = 144 + BC^2$

$\Rightarrow BC^2 = 225 - 144 = 81 = (9)^2$

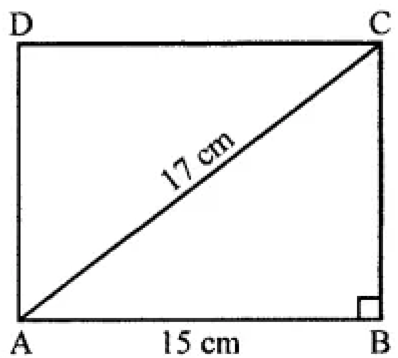
$\Rightarrow BC = 9$ m

Distance of the foot of the ladder and the wall = 9 m

Question 6.
Find the area and the perimeter of the rectangle whose length is 15 cm and the length of one diagonal is 17 cm.
Solution:

Length of a rectangle = 15 cm

and length of its one diagonal = 17 cm



In right ΔABC

$AC^2 = AB^2 + BC^2$ (Pythagoras Theorem)

$\Rightarrow 17^2 = 15^2 + BC^2$

$\Rightarrow 289 = 225 + BC^2$

$\Rightarrow BC^2 = 289 - 225 = 64 = (8)^2$

Breadth = 8 cm

Area = Length × Breadth = 15 × 8 = 120 cm^2

and perimeter = 2(Length + Breadth)

= 2(15 + 8)

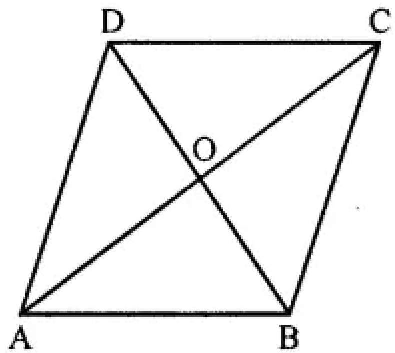
= 2 × 23

= 46 cm

Question 7.
If the diagonals of a rhombus measure 10 cm and 24 cm, find its perimeter.

Solution:

Length of diagonals of a rhombus are 10 cm and 24 cm



The diagonals of a rhombus bisect each other at right angles.

O is the mid-point of AC and BD

AO = OC = $\frac{24}{2}$ = 12 cm

and BO = OD = $\frac{10}{2}$ = 5 cm

Now in right ΔAOB

$AB^2 = AO^2 + BO^2$ (Pythagoras Theorem)

$\Rightarrow AB^2 = 12^2 + 5^2$

$\Rightarrow AB^2 = 144 + 25 = 169 = (13)^2$

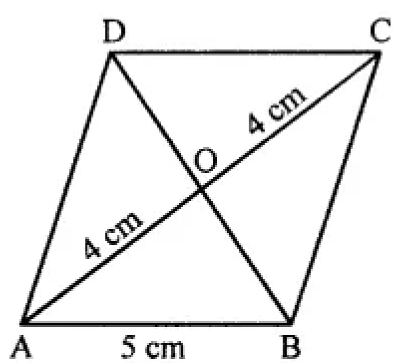
$\Rightarrow$ AB = 13 cm

Perimeter of rhombus = 4 × Side = 4 × 13 = 52 cm

Question 8.
The side of a rhombus is 5 cm. If the length of one diagonal of the rhombus is 8 cm, then find the length of the other diagonal.
Solution:

Side of a rhombus = 5 cm

Length of one diagonal (AC) = 8 cm



The diagonal of a rhombus bisect each other at right angles.

AO = OC = 4 cm, BO = OD

In right ΔAOB,

$AB^2 = AO^2 + BO^2$

$5^2 = 4^2 + BO^2$

$25 = 16 + BO^2$

$BO^2 = 25 - 16 = 9 = (3)^2$

BO = 3 cm

and diagonal BD = $2 \times 3 = 6$ cm

Objective Type Questions

Question 1.
Fill in the blanks:
(i) A triangle has at least ………… acute angles.
(ii) A triangle cannot have more than ……… right angle.
(iii) A triangle cannot have more than ………… obtuse angle.
(iv) In every triangle, the sum of (interior) angles of a triangle = ……….. right angles.
(v) In every triangle, an exterior angle + adjacent interior angle = ………… degrees.
(vi) In every triangle, an exterior angle = sum of the …….. interior opposite angles.
(vii) In a right-angled triangle, if one of the acute angles measures 25° then the measure of the other acute angle is …………
Solution:
(i) A triangle has at least two acute angles.
(ii) A triangle cannot have more than one right angle.
(iii) A triangle cannot have more than one obtuse angle.
(iv) In every triangle, the sum of (interior) angles of a triangle = two right angles.
(v) In every triangle, an exterior angle + adjacent interior angle = 180° degrees.
(vi) In every triangle, an exterior angle = sum of the two interior opposite angles.
(vii) In a right-angled triangle, if one of the acute angles measures 25°
then the measure of the other acute angle is 65°.

Question 2.
State whether the following statements are true (T) or false (F):
(i) A triangle can have all three angles with a measure greater than 60°.
(ii) A triangle can have all three angles with a measure of less than 60°.
(iii) If an exterior angle of a triangle is a right angle, then each of its interior opposite angles is acute.
(iv) In a right-angled triangle, the sum of two acute angles is 90°.
(v) If all the three sides of a triangle are equal, then it is called a scalene triangle.
(vi) Every equilateral triangle is an isosceles triangle.
(vii) Every isosceles triangle must be an equilateral triangle.
(viii) Each acute angle of an isosceles right-angled triangle measures 60°.
(ix) A median of a triangle always lies inside the triangle.
(x) An altitude of a triangle always lies outside the triangle.
(xii) In a triangle, the sum of squares of two sides is equal to the square of the third side.
Solution:

(i) A triangle can have all three angles with a measure greater than 60°. (False)
Correct:
The sum of three angles = 180°
(ii) A triangle can have all three angles with a measure of less than 60°. (False)
Correct:
The sum of three angles = 180°
(iii) If an exterior angle of a triangle is a right angle,
then each of its interior opposite angles is acute. (True)
(iv) In a right-angled triangle, the sum of two acute angles is 90°. (True)
(v) If all the three sides of a triangle are equal,
then it is called a scalene triangle. (False)
Correct:
It is an equilateral triangle.
(vi) Every equilateral triangle is an isosceles triangle. (True)
(vii) Every isosceles triangle must be an equilateral triangle. (False)
Correct:
An isosceles triangle has any two sides equal
but an equilateral triangle has three sides equal.
(viii) Each acute angle of an isosceles right-angled triangle measures 60°. (False)
Correct:
Each acute angle will be of 45°, not 60°.
(ix) A median of a triangle always lies inside the triangle. (True)
(x) An altitude of a triangle always lies outside the triangle. (False)
Correct:
Not always, but in some cases only.
(xii) In a triangle, the sum of squares of two sides is
equal to the square of the third side. (False)
Correct:
Only in right triangle, the sum of squares on two sides is
equal to the square on the hypotenuse.

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 17):
Question 3.
A triangle formed by the sides of lengths 4.5 cm, 6 cm, and 4.5 cm is
(a) scalene
(b) isosceles
(c) equilateral
(d) none of these
Solution:

A triangle formed by the sides of lengths
4.5 cm, 6 cm, and 4.5 cm is isosceles. (b)

Question 4.
The number of medians in a triangle is
(a) 1
(b) 2
(c) 3
(d) 4
Solution:

The number of medians in a triangle is 3. (c)

Question 5.
An exterior angle of a triangle is 125°. If one of the two interior opposite angles is 55° then the other interior opposite angle is
(a) 70°
(b) 55°
(c) 60°
(d) 80°
Solution:

An exterior angle of a triangle is 125°.
If one of the two interior opposite angles is 55°
then the other interior opposite angle is $125° - 55° = 70°$ (a)

Question 6.
In a ΔABC, if $\angle A = 40°$ and $\angle B = 55°$ then $\angle C$ is
(a) 75°
(b) 80°
(c) 95°
(d) 85°

Solution:

In a ΔABC, if $\angle A = 40°$ and $\angle B = 55°$

then $\angle C$ is $180° - (40° + 55°) = 180° - 95° = 85°$ (d)

Question 7.
If the angles of a triangle are 35°, 35°, and 110°, then it is
(a) an isosceles triangle
(b) an equilateral triangle
(c) a scalene triangle
(d) right-angled triangle
Solution:

If the angles of a triangle are 35°, 35°, and 110°,

then it is an isosceles triangle. (a)

Question 8.
A triangle whose two angles measure 30° and 120° is
(a) scale
(b) isosceles
(c) equilateral
(d) none of these
Solution:

A triangle whose two angles measure

30° and 120° is an isosceles triangle. (b)

Question 9.
A triangle can have two
(a) right angles
(b) obtuse angles
(c) acute angles
(d) straight angles
Solution:

A triangle can have two acute angles. (c)

Question 10.
A triangle whose angles measure 35°, 35° and 90° is
(a) acute-angled
(b) right-angled
(c) obtuse-angled
(d) isosceles

Solution:

A triangle whose angles measure
35°, 35° and 90° is right-angled. (b)

Question 11.
A triangle is not possible whose angles measure
(a) 40°, 65°, 75°
(b) 50°, 56°, 74°
(c) 72°, 63°, 45°
(d) 67°, 42°, 81°
Solution:

A triangle is not possible whose angles measure 67°, 42°, 81°. (d)
(Sum is more than 180°)

Question 12.
If in an isosceles triangle, each of the base angles is 40°, then the triangle is
(a) right-angled triangle
(b) acute-angled triangle
(c) obtuse-angled triangle
(d) isosceles right-angled triangle
Solution:

If in an isosceles triangle, each of the base angles is 40°,
then the triangle is an obtuse angled triangle. (c)

Question 13.
A triangle is not possible with sides of lengths (in cm)
(a) 6, 4, 10
(b) 5, 3, 7
(c) 7, 8, 9
(d) 3.6, 5.4, 8
Solution:

A triangle is not possible with sides of lengths (in cm) 6, 4, 10. (a)
(Sum of two sides must be greater than its third side)

Question 14.
Which of the following can be the length of the third side of a triangle whose two sides measure 18 cm and 14 cm?
(a) 32 cm
(b) 3 cm

(c) 4 cm
(d) 5 cm
Solution:

The length of the third side of a triangle
whose two sides measure 18 cm and 14 cm = 5 cm (d)
($\because$ The sum of any two sides must be greater than its third side)

Question 15.
In a right-angled triangle, the lengths of two leg's are 6 cm and 8 cm. The length of the hypotenuse is
(a) 14 cm
(b) 10 cm
(c) 11 cm
(d) 12 cm
Solution:

In a right-angled triangle,
the lengths of two legs are 6 cm and 8 cm.
The length of the hypotenuse is 10 cm. (b)

($\because$ Hypotenuse

$$= \sqrt{\text{Sum of squares on the two legs}} \;)$$

$$= \sqrt{(6)^2 + (8)^2} = \sqrt{36 + 64}$$

$$= \sqrt{100} = 10 \text{ cm}$$

Question 16.
If the dimensions of a rectangle are 15 m and 8 m, then the length of a diagonal is
(a) 7 m
(b) 23 m
(c) 17 m
(d) 20 m
Solution:

If the dimensions of a rectangle are 15 m and 8 m,
then the length of a diagonal is 17 m.
($\because 17^2 = 8^2 + 15^2$) (c)

Question 17.
If p, q, and r are the lengths of the three sides of a triangle, then which of the following statements is correct?
(a) $p + q = r$
(b) $p + q < r$ (c) $p + q > r$
(d) $p - q > r$
Solution:

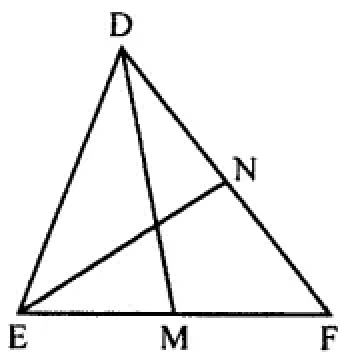
$p + q > r$ (c)

(∵ Sum of any two sides of a triangle is greater than its third side)

Higher Order Thinking Skills (HOTS)

Question 1.
In ΔDEF, DM and EN are two medians. Prove that $3(DF + EF) > 2(DM + EN)$.
Solution:

In ΔDEF, DM and EN are two medians.



To prove : $3(DF + EF) > 2(DM + EN)$

Proof: In ΔDMF

$DF + MF > DM$

(Sum of any two sides is greater than its third side)

$\Rightarrow DF = \frac{1}{2} EF > DM$ ......(i)

(∵ M is mid-point of EF)

Similarly in ΔEFN,

$EF + NF > EN$

$\Rightarrow EF + \frac{1}{2} DF > EN$ ....(ii)

Adding (i) and (ii),

$(FD + \frac{1}{2} EF + EF + \frac{1}{2} DF) > (DM + EN)$

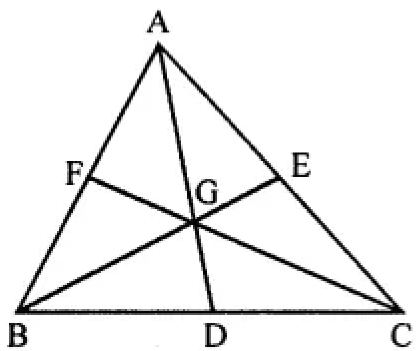
$\Rightarrow (\frac{3}{2} FD + \frac{3}{2} EF) > (DM + EN)$

$\Rightarrow \frac{3}{2} (FD + EF) > (DM) + EN)$

$\Rightarrow 3(FD + EF) > 2(DM + EN)$

Question 2.
In a ΔABC, medians AD, BE and CF intersect each other at point G. Prove that 3(AB + BC + CA) > 2(AD + BE + CF).
Solution:

In ΔABC, medians AD, BE and CF intersect each other at G.



To prove :

3(AB + BC + CA) > 2(AD + BE + CF)

Proof: In ΔABC, D, E and F are the midpoints of

BC, CA and AB respectively

In ΔABD,

Sum of any two sides is greater than its third side

AB + BD > AD

$\Rightarrow AB + \frac{1}{2} RC > AD$ ......(i)

Similarly in ΔBCE

$BC + CE > BE$

$\Rightarrow BC + \frac{1}{2} AC > BE$ ......(ii)

and in ΔCAF,

$CA + AF > CF$

$\Rightarrow CA + \frac{1}{2} AB > CF$ ......(iii)

Adding (i), (ii) and (iii),

$AB + \frac{1}{2} BC + BC + \frac{1}{2} AC + AC + \frac{1}{2} AB > AD + BE + CF$

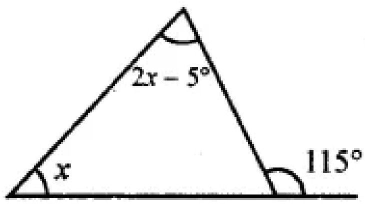
$\Rightarrow \frac{3}{2} AB + \frac{3}{2} BC + \frac{3}{2} CA > AD + BE + CF$

$\Rightarrow \frac{3}{2} (AB + BC + CA) > (AD + BE + CF)$

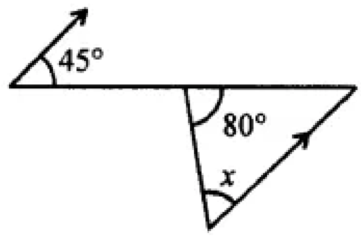
$\Rightarrow 3(AB + BC + CA) > 2(AD + BE + CF)$

<u>Check Your Progress</u>

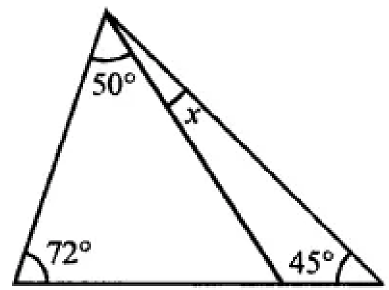
Question 1.
Find the value of x in each of the following diagrams:



(i)



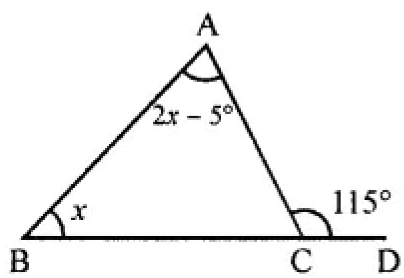
(ii)



(iii)

Solution:

(i) In the given figure,



Ext. angle of triangle = Sum of its interior opposite angles.

⇒ 115° = 2x − 50 + x

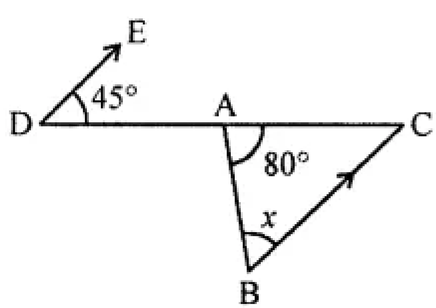
⇒ 3x − 50 = 115°

⇒ 3x = 115° + 5° = 120°

⇒ x = 40°

x = 40°

(ii) In the given figure,



∠1 = 45° (Alternate angles)

Now in triangle,

80° + x + ∠1 = 180° (Sum of angles of a triangle)

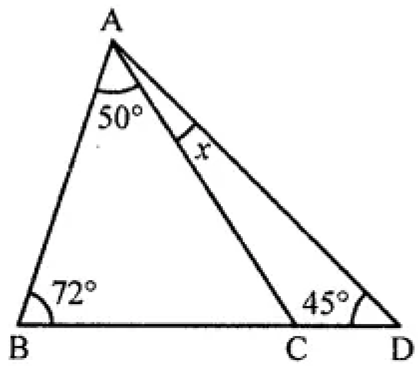
⇒ 80°+ x + 45° = 180°

⇒ x + 125°= 180°

⇒ x = 180° − 125° = 55°

(iii) In ΔABC

∠BAC + ∠ABC + ∠ACB = 180° (Sum of angles of a triangle)



⇒ 50° + 72° + ∠ACB = 180°

⇒ 120° + ∠ACB = 180°

⇒ ∠ACB = 180°- 122° = 58°

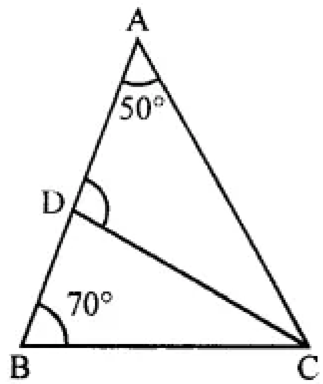
Now in ΔACD

Ext. ∠ACB = ∠CAB + ∠CDA

⇒ 58° = x + 45°

⇒ x = 58° − 45° = 13°

Question 2.
In the given figure, ∠B = 70° and ∠A = 50°. If the bisector of ∠C meets AB in D, then find ∠ADC.



Solution:

In the given figure,

$\angle A = 50°$, $\angle B = 70°$, $\angle ADC = x$

CD is the bisector of $\angle C$

In ΔABC,

$\angle A + \angle B + \angle ACB = 180°$ (Angles of a triangle)

$\Rightarrow 50° + 70° + \angle ACB = 180°$

$\Rightarrow 120° + \angle ACB = 180°$

$\Rightarrow \angle ACB = 180° - 120° = 60°$

But CD is the bisector of $\angle C$

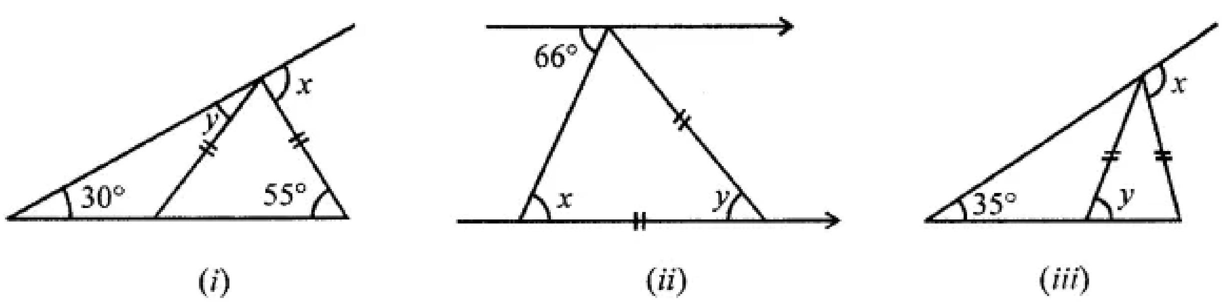
$\angle DCB = \frac{60}{2} = 30°$

Now in ΔBCD,

Ext. $\angle ADC = \angle B + \angle DCB$ (Interior opposite angles)

$= 70° + 30° = 100°$

Question 3.
Find the values of x and y in each of the following figures:



Solution:

(i) In the given figure, AC = AE

Ext. $\angle DAC = \angle B + \angle C$ (Interior opposite angles)

$\Rightarrow x = 30° + 55° = 85°$

In ΔAEC,

AC = AE

$\angle AEC = \angle ACE = 55°$

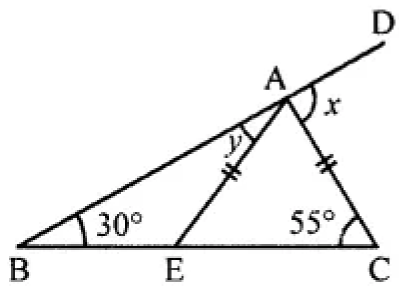
In ΔABE,

Now, Ext. $\angle AEC = y + 30°$

$\Rightarrow 55° = y + 30°$

$\Rightarrow y = 55° - 30° = 25°$

$x = 85°$ and $y = 25°$



(ii) In the given figure,

PQ || EF

In ΔABC,

AC = BC

$\angle PAB = 66°$

$\angle ABC = \angle PAB$ (Alternate angles)

$x = 66°$

AC = BC

$\angle BAC = \angle ABC = 66°$

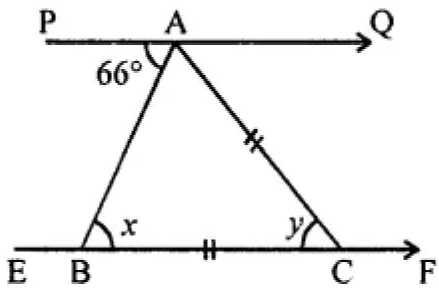
But $\angle ABC + \angle ACB + \angle BAC = 180°$ (Angles of a triangle)

$\Rightarrow 66° + y + 66° = 180°$

$\Rightarrow 132° + y = 180°$

$\Rightarrow y = 180° - 132° = 48°$

$x = y = 48°$



(iii) In the given figure,

BD = AD = AC

∠B = 35°

In ΔABD

AD = BD

∠BAD = ∠ABD = 35°

Now, Ext. ∠ADC = B + BAD

y = 35° + 35° = 70°

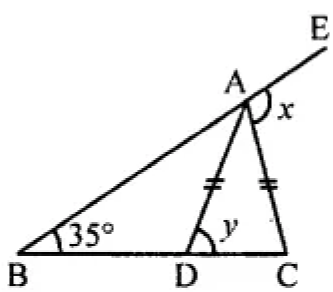
In ΔADC,

AD = AC

∠ACD = ∠ADC = y = 70°

and Ext. ∠EAC = ∠ABC + ∠ACB = 35° + 70° = 105°

Hence, x = 105°, y = 70°



Question 4.
If the two acute angles of a right-angled triangle are in the ratio 7 : 8, find these angles.

Solution:

In a right-angled triangle.

Sum of two acute angles = 90°

Ratio in two angles = 7 : 8

First angle = $\frac{90}{7+8} \times 7$

= $\frac{90}{15} \times 7$ = 42°

and second angle = $\frac{90}{15} \times 8$ = 48°

Question 5.
If the angles of a triangle are (3x)°, (2x – 7)° and (4x – 11)°, then find the value of x.
Solution:

Angles of a triangle are (3x)°, (2x − 7)° and (4x − 11)°

But sum of three angles of a triangle = 180°

3x + 2x − 7 + 4x − 11° = 180°

⇒ 9x − 18° = 180°

⇒ 9x = 180° + 18° = 198°

⇒ x = 22°

Question 6.
In an isosceles triangle, the vertical angle is 15° greater than each of its base angles. Find all the angles of the triangle.
Solution:

In an isosceles triangle,

Vertical angle = 15° greater than each base angles

Let each base angle = x

Then vertical angle = x + 15°

Now sum of angles of a triangle = 180°

x + 15° + x + x = 180°

⇒ 3x + 15° = 180°

⇒ 3x = 180° − 15° = 165°

⇒ x = 55°

Each base angle = 55°

and vertical angle = 55° + 15° = 70°

Question 7.
Can a triangle have three sides whose lengths are
(i) 4.5 cm, 3.8 cm, 7.2 cm?
(ii) 3.2 cm, 5.3 cm, 9.4 cm?
Solution:

(?) Sides are 4.5 cm, 3.8 cm, 7.2 cm

Sum of two sides = 4.5 + 3.8 = 8.3 cm

8.3 > 7.2 cm

The triangle can have there sides.

(ii) Sides are 3.2 cm, 5.3 cm, 9.4 cm

Sum of sides 3.2 and 5.3 cm = 3.2 + 5.3 = 8.5 cm

8.5 cm < 9.4 cm

The triangle does not have there sides.

Question 8.
If the lengths of two sides of a triangle are 5 cm and 12 cm, then what can be the length of the third side?
Solution:

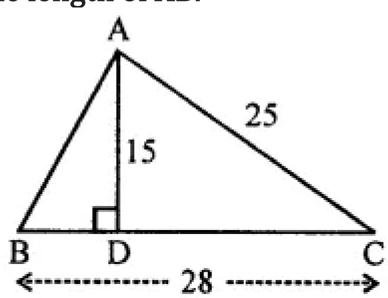
Length of two sides of a triangle is 5 cm and 12 cm

Sum of there two sides = 5 + 12 = 17 cm

and difference = 12 − 5 = 7 cm

The third side will be greater than 7 cm but less than 17 cm.

Question 9.
In the given figure, all measurements are in centimeters. If AD is perpendicular to BC, find the length of AB.



Solution:

In the given figure,

ABC is a triangle in which AC = 25 cm, BC = 28 cm

AD ⊥ BC and AD = 15 cm

To find the length of AB

In right ΔADC

$AC^2 = AD^2 + DC^2$ (Pythagoras Theorem)

$\Rightarrow (25)^2 = 15^2 + DC^2$

$\Rightarrow 625 = 225 + DC^2$

$\Rightarrow DC^2 = 625 - 225 = 400 = (20)^2$

$\Rightarrow$ DC = 20 cm

But BC = 28 cm

BD = 28 − 20 = 8 cm

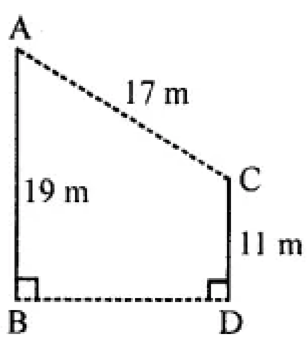
Now in right ΔADB

$AB^2 = AD^2 + BD^2$

$= 15^2 + 8^2 = 225 + 64 = 289 = (17)^2$

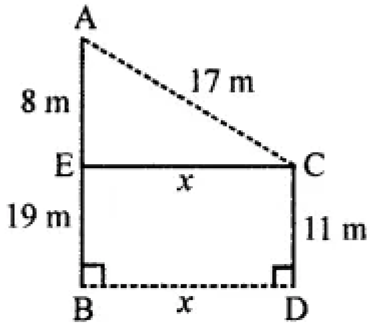
$\Rightarrow$ AB = 17 cm

Question 10.
In the given figure, AB and CD are two vertical poles of height 19 m and 11 m respectively. If the shortest distance between their tops is 17 m, find how far apart they are?



Solution:

In the given figure,



Pole AB = 19 m, Pole CD = 11 m

Distance between their tops AC = 17 cm

Draw EC || BD, then

Let BD = CE = x

EB = CD = 11 m

AE = AB − EB = 19 − 11 = 8m

Now in right ΔAEC,

$AC^2 = AE^2 + EC^2$

$\Rightarrow (17)^2 = (8)^2 + x^2$

$\Rightarrow 289 = 64 + x^2$

$\Rightarrow x^2 = 289 - 64 = 225 = (15)^2$

$\Rightarrow x = 15$

BD = EC = x = 15m