

Chapter 12

Congruence of Triangles

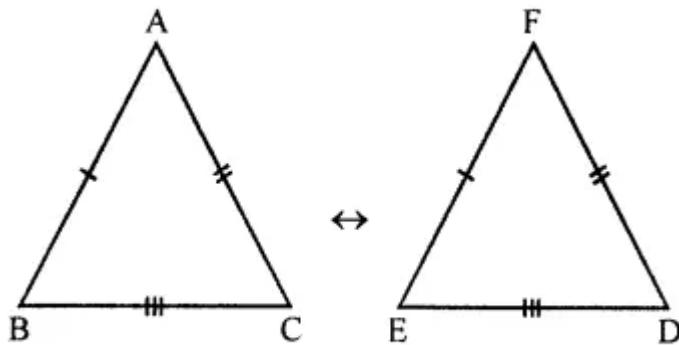
Exercise 12.1

Question 1.

If ΔABC and ΔDEF are congruent under the correspondence $ABC \leftrightarrow FED$, write all the corresponding congruent parts of the triangles.

Solution:

ΔABC and ΔDEF are congruent
under the correspondence, $ABC \leftrightarrow FED$



$\angle A \leftrightarrow \angle F, \angle B \leftrightarrow \angle E, \angle C \leftrightarrow \angle D$
 $AB \leftrightarrow FE, BC \leftrightarrow ED$ and $AC \leftrightarrow FD$

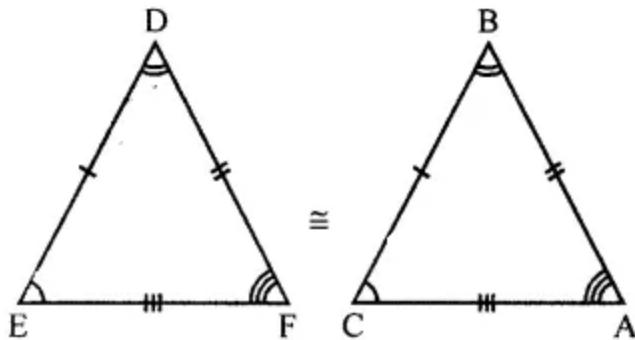
Question 2.

If $\Delta DEF = \Delta BCA$, then write the part(s) of ΔBCA that correspond to

- (i) $\angle E$
- (ii) EF^-
- (iii) $\angle F$
- (iv) DF^-

Solution:

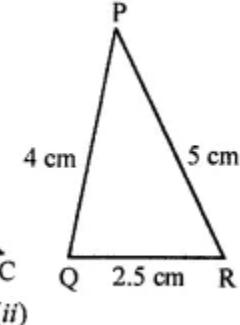
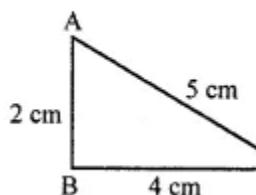
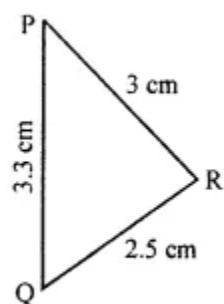
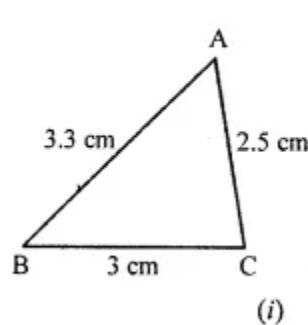
If $\triangle DEF \cong \triangle BCA$, then



- (i) $\angle E \leftrightarrow \angle C$
- (ii) $EF \leftrightarrow CA$
- (iii) $\angle F \leftrightarrow \angle A$
- (iv) $DF \leftrightarrow BA$

Question 3.

In the figure given below, the lengths of the sides of the triangles are indicated. By using SSS congruency rule, state which pairs of triangles are congruent. In the case of congruent triangles, write the result in symbolic form:



Solution:

- (i) In the given figure,
In $\triangle ABC$ and $\triangle PQR$
 $AB \leftrightarrow PQ$, $BC \leftrightarrow PR$, and $AC \leftrightarrow QR$
 \triangle s are congruent
 $\triangle ABC = \triangle PQR$

(ii) In the given figure,

In $\triangle ABC$ and $\triangle PQR$

$AC \leftrightarrow PR$, $BC \leftrightarrow PQ$

But $AB \neq QR$

\triangle s are not congruent.

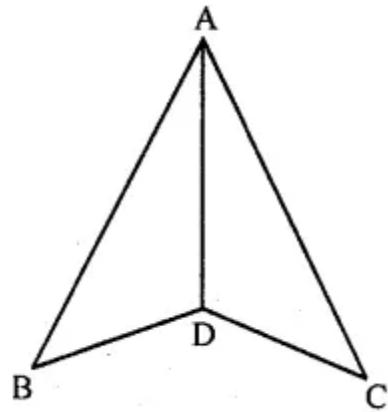
Question 4.

In the given figure, $AB = 5 \text{ cm}$, $AC = 5 \text{ cm}$, $BD = 2.5 \text{ cm}$ and $CD = 2.5 \text{ cm}$

(i) State the three pairs of equal parts in $\triangle ADB$ and $\triangle ADC$

(ii) Is $\triangle ADB = \triangle ADC$? Give reasons.

(iii) Is $\angle B = \angle C$? Why?



Solution:

In the given figure,

$AB = 5 \text{ cm}$, $AC = 5 \text{ cm}$, $BD = 2.5 \text{ cm}$

and $CD = 2.5 \text{ cm}$

In $\triangle ABD$ and $\triangle ACD$

(i) $AB = AC = 5 \text{ cm}$

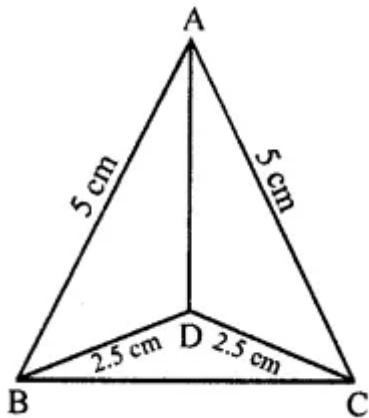
$BD = CD = 2.5 \text{ cm}$

$AD = AD$ (Common Side)

$\triangle ABD = \triangle ACD$

(ii) $\triangle ADB = \triangle ADC$ (SSS axiom)

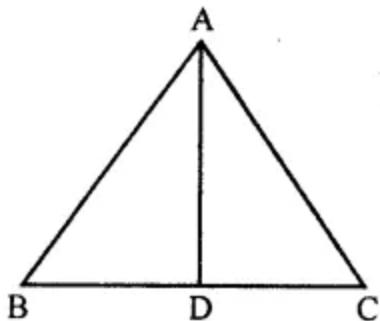
$\angle B = \angle C$ (c.p.c.t.)



Question 5.

In the given figure, $AB = AC$ and D is the mid-point of BC .

- State the three pairs of equal parts in $\triangle ADB$ and $\triangle ADC$.
- Is $\triangle ADB = \triangle ADC$? Give reasons.
- Is $\angle B = \angle C$? Why?



Solution:

(i) In $\triangle ABC$,

$$AB = AC$$

D is the mid-point of BC

$$BD = DC$$

Now in $\triangle ADB$ and $\triangle ADC$

$$AB = AC \text{ (Given)}$$

$$AD = AD \text{ (Common)}$$

$$BD = DC \text{ (D is mid-point of BC)}$$

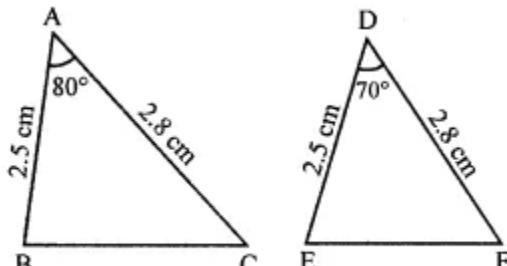
(ii) $\triangle ADB = \triangle ADC$ (SSS axiom)

(iii) $\angle B = \angle C$ (c.p.c.t.)

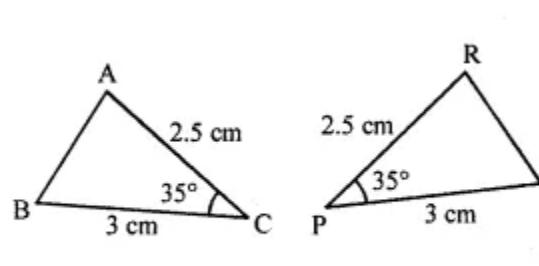
Question 6.

In the figure given below, the measures of some parts of the triangles are indicated. By

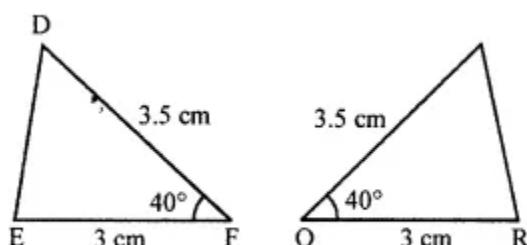
using SAS rule of congruency, state which pairs of triangles are congruent. In the case of congruent triangles, write the result in symbolic form.



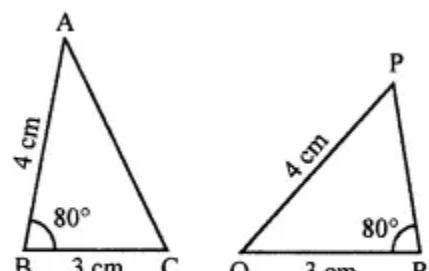
(i)



(ii)



(iii)



(iv)

Solution:

(i) In $\triangle ABC$ and $\triangle DEF$

$AB = DE$ (Each = 2.5 cm)

$AC = DF$ (Each = 2.8 cm)

$\angle A \neq \angle D$ (Have different measure)

$\triangle ABC$ is not congruent to $\triangle DEF$

(ii) In $\triangle ABC$ and $\triangle RPQ$

$AC = RP$ (Each = 2.5 cm)

$CB = PQ$ (Each = 3 cm)

$\angle C = \angle P$ (Each = 35°)

$\triangle ACB$ and $\triangle RPQ$ are congruent (SAS axiom)

(iii) In $\triangle DEF$ and $\triangle PQR$

$FD = QP$ (Each = 3.5 cm)

$FE = QR$ (Each = 3 cm)

$\angle F = \angle Q$ (Each 40°)

$\triangle DEF$ and $\triangle PQR$ are congruent

(iv) In $\triangle ABC$ and $\triangle PRQ$

$AB = PQ$ (Each = 4 cm)

$BC = QR$ (Each = 3 cm)

But included angles B and $\angle Q$ are not equal

$\triangle ABC$ and $\triangle PRQ$ are not congruent.

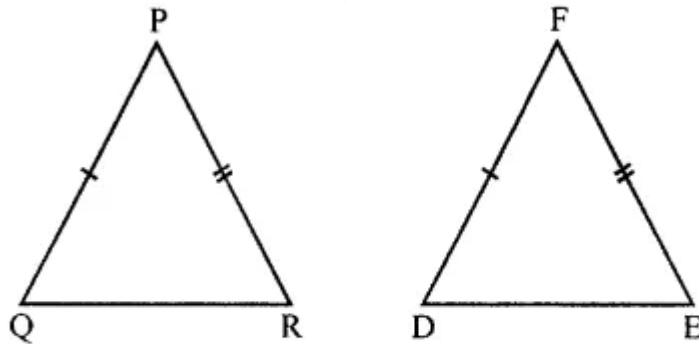
Question 7.

By applying SAS congruence rule, you want to establish that $\triangle PQR = \triangle FED$. If is given that

$PQ = EF$ and $RP = DF$. What additional information is needed to establish the congruence?

Solution:

In $\triangle PQR$ and $\triangle FED$



$PQ = FE$

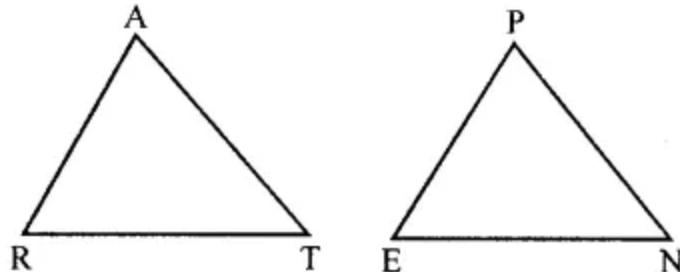
$RP = DF$

Their included angles $\angle P$ must be equal to $\angle F$ for congruency.

Hence, $\angle P = \angle F$

Question 8.

You want to show that $\triangle ART = \triangle PEN$



(a) If you have to use SSS criterion, then you need to show

(i) $AR = \dots$

(ii) $RT = \dots$

(iii) $AT = \dots$

(b) If it is given that $\angle T = \angle N$ and you are to use the SAS criterion, you need to have

(i) $RT = \dots$ and (ii) $PN = \dots$

Solution:

(a) In $\triangle ART$ and $\triangle PEN$

For SSS criterion

$AR = DE$

$RT = EN$ and $AT = PN$

$\triangle ART = \triangle PEN$

(b) $\angle T = \angle N$ (Given)

In $\triangle ART$ and $\triangle PEN$

If $RT = EN$

$AT = PN$ and $\angle T = \angle N$

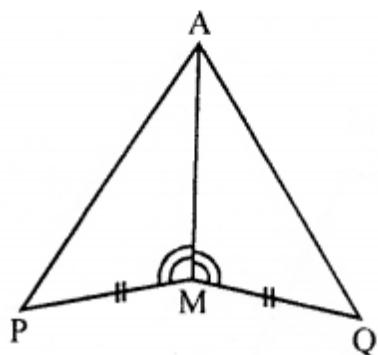
Then $\triangle ART = \triangle PEN$ (SAS criterion)

Question 9.

You have to show that $\triangle AMP = \triangle AMQ$.

In the following proof, supply the missing reasons.

Steps	Reasons
(i) $PM = QM$	(i)
(ii) $\angle PMA = \angle QMA$	(ii)
(iii) $AM = AM$	(iii)
(iv) $\triangle AMP = \triangle AMQ$	(iv)



Solution:

In order to show that,

$$\Delta AMP = \Delta AMQ$$

$$PM = QM \text{ (Given)}$$

$$\angle PMA = \angle QMA \text{ (Given)}$$

$$AM = AM \text{ (Common)}$$

$$\Delta AMP = \Delta AMQ \text{ (SAS criterion)}$$

Question 10.Solution:

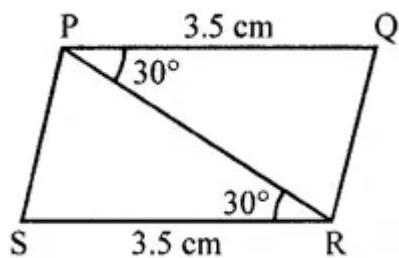
In the given figure:

(i) State three pairs of equal parts in ΔPSR and ΔRQP .

(ii) Is $\Delta PSR = \Delta RQP$? Give reasons

(iii) Is $PS = RQ$? Why?

(iv) Is $\angle S = \angle Q$? Why?



Solution:

In ΔPSR and ΔRQP

$$SR = PQ \text{ (each} = 3.5 \text{ cm)}$$

$$PR = PR \text{ (Common side)}$$

$$\angle SRP = \angle RPQ \text{ (Each} = 30^\circ)$$

$$\Delta PSR = \Delta RQP \text{ (SAS criterion)}$$

$$PS = RQ \text{ (c.p.c.t.)}$$

$$\angle S = \angle Q \text{ (c.p.c.t.)}$$

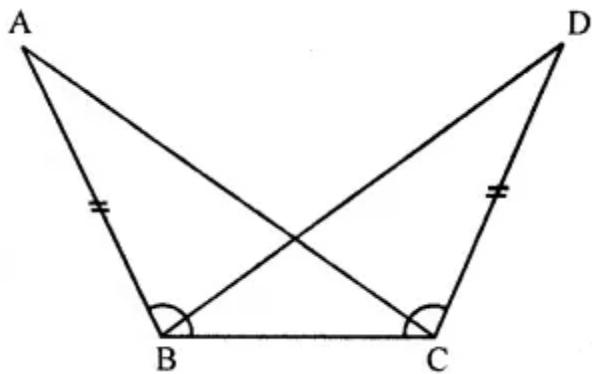
Question 11.

In the given figure, $AB = DC$ and $\angle ABC = \angle DCB$.

(i) State three pairs of equal parts in ΔABC and ΔDCB .

(ii) Is $\Delta ABC = \Delta DCB$? Give reasons.

(iii) Is $AC = DB$? Why?



Solution:

In $\triangle ABC$ and $\triangle DBC$

$AB = DC$ (Given)

$\angle ABC = \angle DCB$ (Given)

$BC = BC$ (Common)

$\triangle ABC \cong \triangle DCB$ (SAS criterion)

$AC = DB$ (c.p.c.t.)

Question 12.

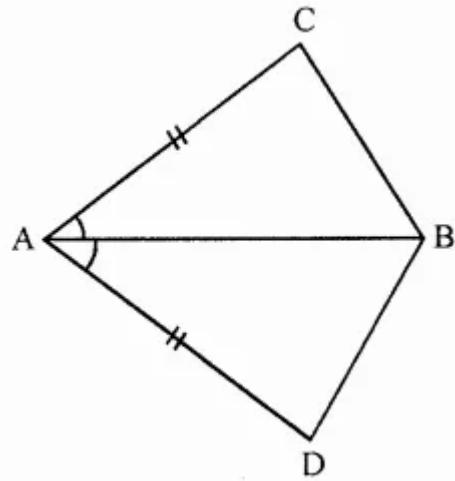
In the quadrilateral, $AC = AD$, and AB bisects $\angle CAD$.

(i) State three pairs of equal parts in $\triangle ABC$ and $\triangle ABD$.

(ii) Is $\triangle ABC \cong \triangle ABD$? Give reasons.

(iii) Is $BC = BD$? Why?

(iv) Is $\angle C = \angle D$? Why?



Solution:

In quadrilateral ACBD,

$AC = AD$, AB bisects $\angle CAD$

Now in $\triangle ABC$ and $\triangle ABD$

$AC = AD$ (Given)

$\angle CAB = \angle DAB$ (Given)

(AB bisects $\angle CAD$)

$AB = AB$ (Common)

$\triangle ABC \cong \triangle ABD$ (SAS criterion)

$BC = BD$ (c.p.c.t.)

$\angle C = \angle D$ (c.p.c.t.)

Exercise 12.2

Question 1.

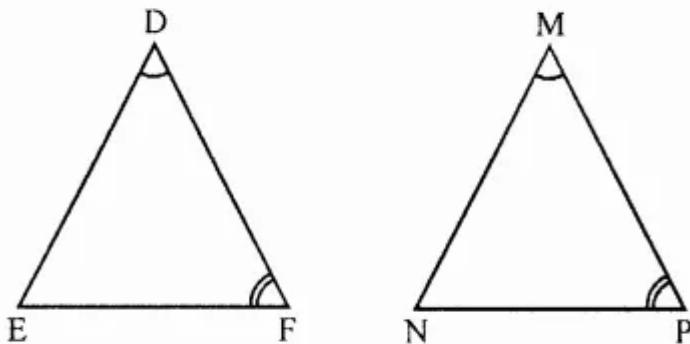
You want to establish $\Delta DEF = \Delta MNP$, using ASA rule of congruence. You are given that $\angle D = \angle M$ and $\angle F = \angle P$. What additional information is needed to establish the congruence?

Solution:

In ΔDEF and ΔMNP

$\angle D = \angle M$ (Given)

$\angle F = \angle P$ (Given)



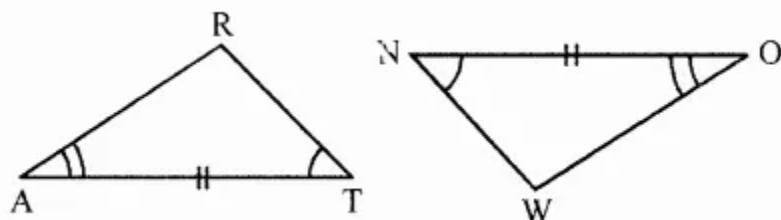
For using ASA rule, we need

$DF = MP$ (Included side)

Then $\Delta DEF = \Delta MNP$

Question 2.

In the given figure, two triangles are congruent. The corresponding parts are marked. We can write $\Delta RAT \cong \Delta$?



Solution:

In the given figure,

$\triangle ART$ and $\triangle NOW$

$AT = NO$ (Given)

$\angle A = \angle O$ (Given)

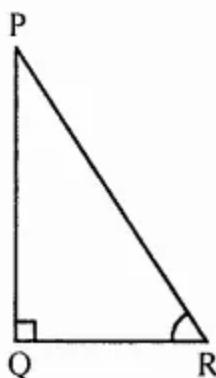
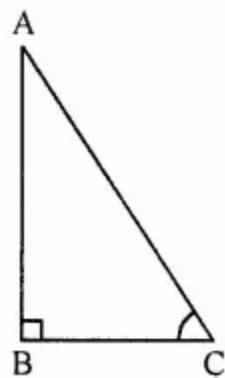
$\angle T = \angle N$ (Given)

$\triangle ART \cong \triangle NOW$ (ASA criterion)

Question 3.

If $\triangle ABC$ and $\triangle PQR$ are to be congruent, name one additional pair of corresponding parts.

What criterion did you use?



Solution:

In $\triangle ABC$ and $\triangle PQR$

$\angle B = \angle Q$

$\angle C = \angle R$

Now we need $BC = QR$

$\triangle ABC \cong \triangle PQR$ (ASA criterion)

Question 4.

Given below are measurements of some parts of two triangles. Examine whether the two triangles are congruent or not, by ASA congruence rule. In the case of congruence, write its in symbolic form.

ΔDEF	ΔPQR
(i) $\angle D = 60^\circ$, $\angle F = 80^\circ$, $DF = 5 \text{ cm}$	(i) $\angle Q = 60^\circ$, $\angle R = 80^\circ$, $QR = 5 \text{ cm}$
(ii) $\angle D = 60^\circ$, $\angle F = 80^\circ$, $DF = 6 \text{ cm}$	(ii) $\angle Q = 60^\circ$, $\angle R = 80^\circ$, $QR = 5 \text{ cm}$
(iii) $\angle E = 80^\circ$, $\angle F = 30^\circ$, $EF = 5 \text{ cm}$	(iii) $\angle P = 80^\circ$, $PQ = 5 \text{ cm}$, $\angle R = 30^\circ$

Solution:

In ΔDEF and ΔPQR

(i) $\angle D = 60^\circ$, $\angle F = 80^\circ$, $DF = 5 \text{ cm}$

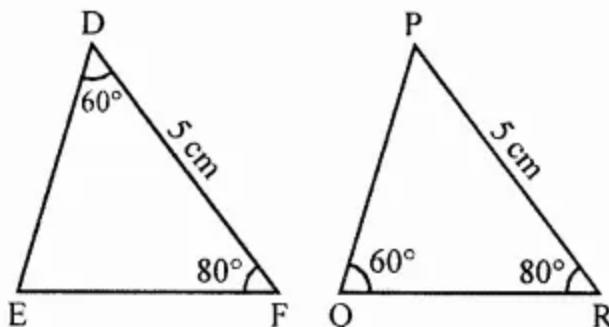
$\angle Q = 60^\circ$, $\angle R = 80^\circ$, $QR = 5 \text{ cm}$

$\angle D = \angle Q$ (Each 60°)

$\angle F = \angle R$ (Each 80°)

Included side $DF = QR$

$\Delta DEF \cong \Delta PQR$ (ASA criterion)



(ii) In ΔDEF and ΔPQR

$\angle D = 60^\circ$, $\angle F = 80^\circ$, $DF = 6 \text{ cm}$

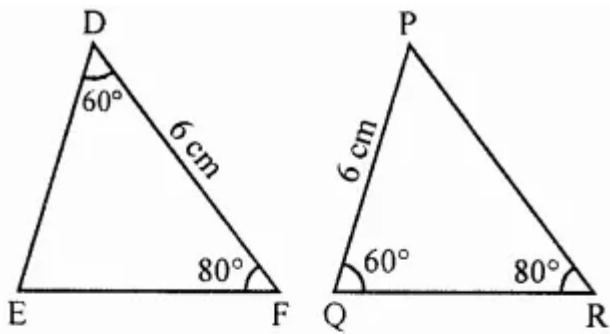
$\angle Q = 60^\circ$, $\angle R = 80^\circ$ and $QP = 6 \text{ cm}$

Here, $\angle D = \angle Q$ (Each 80°)

$\angle F = \angle R$ (Each 80°)

But included side $DF \neq QR$

ΔDEF and ΔPQR are not congruent.



(iii) In $\triangle DEF$ and $\triangle PQR$

$\angle E = 80^\circ$, $\angle F = 30^\circ$, $EF = 5 \text{ cm}$

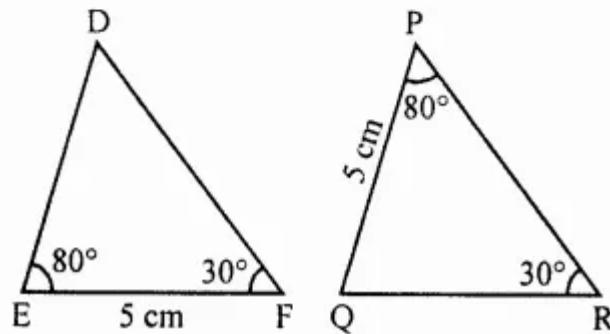
$\angle P = 80^\circ$, $PQ = 5 \text{ cm}$, $\angle R = 30^\circ$

Here, $\angle E = \angle P$ (Each $= 80^\circ$)

$\angle F = \angle R$ (Each $= 30^\circ$)

But included sides are not equal.

$\triangle DEF$ and $\triangle PQR$ are not congruent.



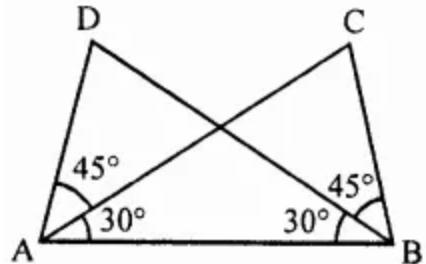
Question 5.

In the adjoining figure, measures of some parts are indicated.

(i) State three pairs of equal parts in triangles ABC and ABD.

(ii) Is $\triangle ABC \cong \triangle BAD$? Give reasons.

(iii) Is $BC = AD$? Why?



Solution:

In the given figure,

$$\angle DAC = 45^\circ, \angle CAB = 30^\circ, \angle CBD = 45^\circ \text{ and } \angle DBA = 30^\circ$$

Now in $\triangle ABC$ and $\triangle BAD$,

$$\angle DAC + \angle CAB = 45^\circ + 30^\circ = 75^\circ$$

$$\text{and } \angle CBD + \angle DBA = 45^\circ + 30^\circ = 75^\circ$$

$$\angle DAB = \angle CBA$$

Now in $\triangle ABC$ and $\triangle DAB$

$$AB = AB \text{ (Common)}$$

$$\angle CBA = \angle DAB \text{ (Proved)}$$

$$\angle CAB = \angle DBA \text{ (Each } = 30^\circ\text{)}$$

$$\triangle ABC \cong \triangle DAB \text{ (ASA criterion)}$$

$$\text{Yes, } BC = AD \text{ (c.p.c.t.)}$$

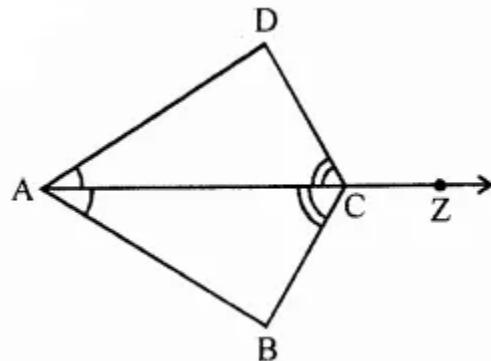
Question 6.

In the adjoining figure, ray AZ bisects $\angle DAB$ as well as $\angle DCB$.

(i) State the three pairs of equal parts in triangles BAC and DAC .

(ii) Is $\triangle BAC \cong \triangle DAC$? Give reasons.

(iii) Is $CD = CB$? Give reasons.



Solution:

In the given figure

$$\angle DAC = \angle BAC$$

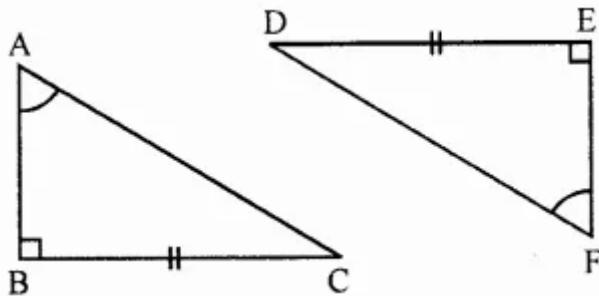
$$\angle DCA = \angle BCA$$

Now in $\triangle BAC$ and $\triangle DAC$

$$AC = AC \text{ (Common)}$$

$\angle BAC = \angle DAC$ (Given)
 $\angle BCA = \angle DCA$ (Given)
 $\triangle BAC = \triangle DAC$ (ASA criterion)
 Yes, $AB = AD$ (c.p.c.t.)
 Yes, $CD = CB$ (c.p.c.t.)

Question 7.
Explain why $\triangle ABC = \triangle FED$?



Solution:

In $\triangle ABC$ and $\triangle FED$
 $BC = DE$
 $\angle B = \angle E$ (Each = 90°)
 $\angle A = \angle F$
 $\angle C = 90^\circ - \angle A$ and $\angle D = 90^\circ - \angle F$
 But $\angle A = \angle F$ (Given)
 $\angle C = \angle D$
 Now in $\triangle ABC$ and $\triangle DEF$
 $BC = DE$ (Given)
 $\angle B = \angle E$ (Given 90°)
 $\angle C = \angle D$ (Proved)
 $\triangle ABC = \triangle DEF$ (ASA criterion)

Question 8.
Given below are the measurements of some parts of triangles. Examine whether the two triangles are congruent or not, using RHS congruence rule. In the case of congruent triangles, write the result in symbolic form

ΔABC	ΔPQR
(i) $\angle B = 90^\circ$, $AC = 8 \text{ cm}$, $AB = 3 \text{ cm}$	(i) $\angle P = 90^\circ$, $PR = 3 \text{ cm}$, $QR = 8 \text{ cm}$
(ii) $\angle A = 90^\circ$, $AC = 5 \text{ cm}$, $BC = 9 \text{ cm}$	(ii) $\angle Q = 90^\circ$, $PR = 8 \text{ cm}$, $PQ = 5 \text{ cm}$

Solution:

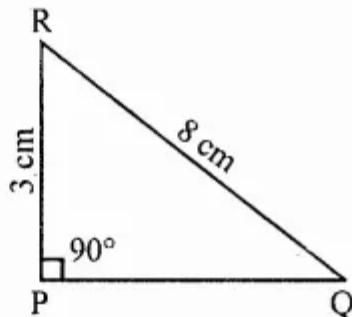
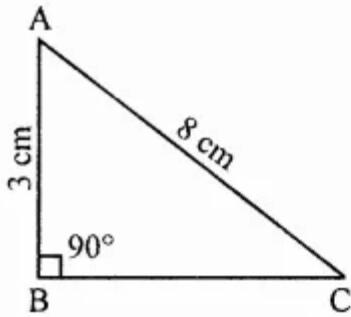
We are given the measurement of some parts of the triangles.

We have to examine whether the two triangles are congruent or not using RHS congruency rule.

In ΔABC and ΔPQR

(i) $\angle B = 90^\circ$, $AC = 8 \text{ cm}$, $AB = 3 \text{ cm}$

$\angle P = 90^\circ$, $PR = 3 \text{ cm}$, $QR = 8 \text{ cm}$



We see that in two Δ s ABC and RPQ

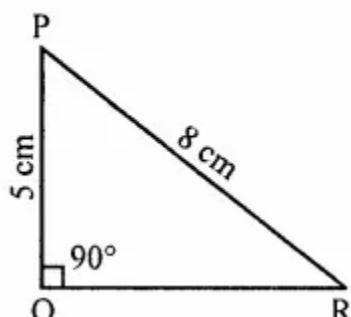
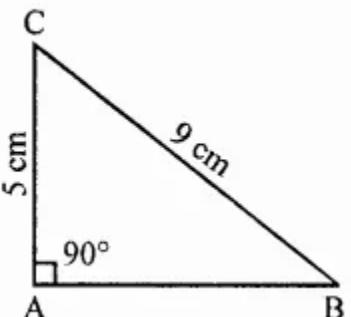
$\angle B = \angle P$ (Each = 90°)

Side $AB = RP$ (Each = 3 cm)

Hypotenuse $AC = RQ$

$\Delta ABC \cong \Delta RPQ$ (RHS criterion)

(ii) In ΔABC and ΔPQR



$\angle A = \angle Q$ (Each = 90°)

Side $AC = QP$ (Each = 5 cm)

But hypotenuse BC and PR are not equal to each other.

Triangles are not congruent.

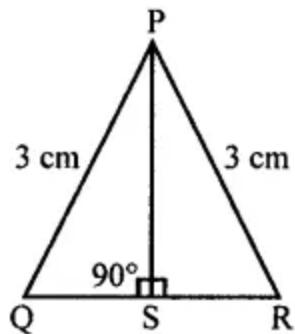
Question 9.

In the given figure, measurements of some parts are given.

(i) State the three pairs of equal parts in $\triangle PQS$ and $\triangle PRS$.

(ii) Is $\triangle PQS = \triangle PRS$? Give reasons.

(iii) Is S mid-point of QR ? Why?



Solution:

In the given figure,

$PQ = 3 \text{ cm}$, $PR = 3 \text{ cm}$

$PS \perp QR$

(i) Now in right $\triangle PQS$ and $\triangle PRS$ right angles at S . ($\because PS \perp QR$)

side $PS = PS$ (Common)

Hypotenuse $PQ = PR$ (Each = 3 cm)

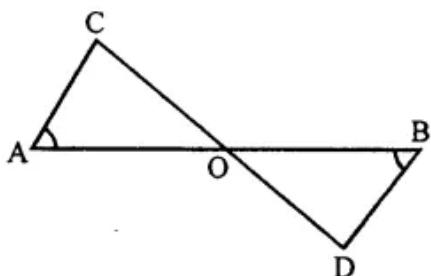
(ii) $\triangle PQS = \triangle PRS$ (RHS criterion)

(iii) $QS = SR$ (c.p.c.t.)

S is the mid point of QR

Question 10.

In the given figure, O is mid-point of AB^- and $\angle A = \angle B$. Show that $\triangle AOC = \triangle BOD$.



Solution:

In the given figure,

O is the mid-point of AB

$AO = OB$

Now in $\triangle AOC$ and $\triangle BOD$

$AO = OB$ ($\because O$ is mid-point of AB)

$\angle A = \angle B$ (Given)

$\angle AOC = \angle BOD$ (Vertically opposite angles)

$\triangle AOC \cong \triangle BOD$ (ASA criterion)

Objective Type Questions

Question 1.

Fill in the blanks:

- (i) Two line segments are congruent if
- (ii) Among two congruent angles, one has a measure of 63° ; the measure of the other angle is
- (iii) When we write $\angle A = \angle B$, we actually mean
- (iv) The side included between $\angle M$ and $\angle N$ of ΔMNP is
- (v) The side QR of ΔPQR is included between angles
- (vi) If two triangles ABC and PQR are congruent under the correspondence $A \leftrightarrow R$, $B \leftrightarrow P$ and $C \leftrightarrow Q$, then in symbolic form it can be written as $\Delta ABC = \Delta PQR$
- (vii) If $\Delta DEF = \Delta SRT$, then the correspondence between vertices is

Solution:

- (i) Two line segments are congruent if they are of the same length.
- (ii) Among two congruent angles, one has a measure of 63° ;
the measure of the other angle is 63° .
- (iii) When we write $\angle A = \angle B$, we actually mean $m\angle A = m\angle B$.
- (iv) The side included between $\angle M$ and $\angle N$ of ΔMNP is MN.
- (v) The side QR of ΔPQR is included between angles $\angle Q$ and $\angle R$.
- (vi) If two triangles ABC and PQR are congruent
under the correspondence $A \leftrightarrow R$, $B \leftrightarrow P$ and $C \leftrightarrow Q$,
then in symbolic form it can be written as $\Delta ABC = \Delta PQR$.
- (vii) If $\Delta DEF = \Delta SRT$, then the correspondence between vertices is
 $D \leftrightarrow S$, $E \leftrightarrow R$ and $F \leftrightarrow T$.

Question 2.

State whether the following statements are true (T) or false (F):

- (i) All circles are congruent.
- (ii) Circles having equal radii are congruent.
- (iii) Two congruent triangles have equal areas and equal perimeters.
- (iv) Two triangles having equal areas are congruent.
- (v) Two squares having equal areas are congruent.
- (vi) Two rectangles having equal areas are congruent.
- (vii) All acute angles are congruent.
- (viii) All right angles are congruent.
- (ix) Two figures are congruent if they have the same shape.
- (x) A two rupee coin is congruent to a five rupee coin.
- (xi) All equilateral triangles are congruent.
- (xii) Two equilateral triangles having equal perimeters are congruent.

(xii) If two legs of one right triangle are equal to two legs of another right angle triangle, then the two triangles are congruent by SAS rule.

(xiv) If three angles of two triangles are equal, then triangles are congruent.

(xv) If two sides and one angle of one triangle are equal to two sides and one angle of another triangle, then the triangle are congruent.

Solution:

(i) All circles are congruent. (False)

Correct:

As if all circles have equal radii otherwise not.

(ii) Circles having equal radii are congruent. (True)

(iii) Two congruent triangles have equal areas and equal perimeters. (True)

(iv) Two triangles having equal areas are congruent. (False)

Correct:

As they may have different sides and angles.

(v) Two squares having equal areas are congruent. (True)

(vi) Two rectangles having equal areas are congruent. (False)

Correct:

As their side can be different.

(vii) All acute angles are congruent. (False)

Correct:

As acute angles have different measures.

(viii) All right angles are congruent. (True)

(ix) Two figures are congruent if they have the same shape. (False)

Correct:

As the same shapes have different measures.

(x) A two rupee coin is congruent to a five rupee coin. (False)

Correct:

As they have different size.

(xi) All equilateral triangles are congruent. (False)

Correct:

As they have different sides in length.

(xii) Two equilateral triangles having equal perimeters are congruent. (True)

(xiii) If two legs of one right triangle are equal to

two legs of another right angle triangle,

then the two triangles are congruent by SAS rule. (True)

(xiv) If three angles of two triangles are equal,

then triangles are congruent. (False)

Correct:

They can be similar to each other.

(xv) If two sides and one angle of one triangle are equal to two sides

and one angle of another triangle, then the triangle is congruent. (False)

Correct:

If the angles are included, they can be congruent.

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 14):

Question 3.

Which one of the following is not a standard criterion of congruency of two triangles?

(a) SSS

(b) SSA

(c) SAS

(d) ASA

Solution:

The axiom SSA is not a standard criterion

of congruency of triangles. (b)

Question 4.

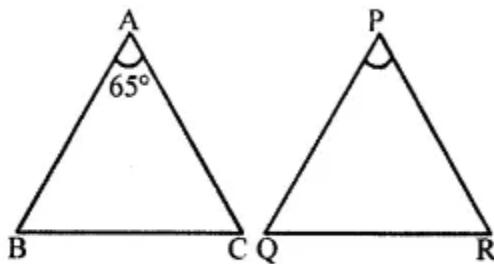
If $\Delta ABC = \Delta PQR$ and $\angle CAB = 65^\circ$, then $\angle RPQ$ is

(a) 65°

(b) 75°
 (c) 90°
 (d) 115°

Solution:

$$\Delta ABC = \Delta PQR$$



$$\angle CAB = 65^\circ$$

$$\angle RPQ = 65^\circ \text{ (corresponding angles) (a)}$$

Question 5.

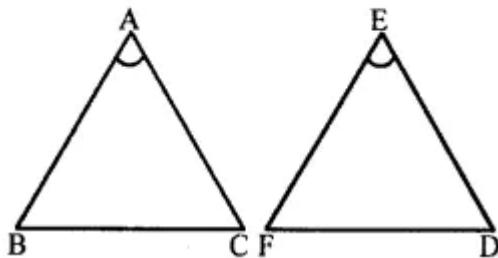
If $\Delta ABC = \Delta EFD$, then the correct statement is

(a) $\angle A = \angle D$
 (b) $\angle A = \angle F$
 (c) $\angle A = \angle E$
 (d) $\angle B = \angle E$

Solution:

$$\Delta ABC = \Delta EFD$$

$$\text{Then } \angle A = \angle E \text{ (c)}$$



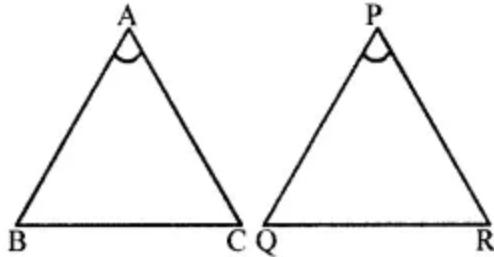
Question 6.

If $\Delta ABC = \Delta PQR$, then the correct statement is

(a) $AB = QR$
 (b) $AB = PR$
 (c) $BC = PR$
 (d) $AC = PR$

Solution:

$$\Delta ABC = \Delta PQR$$



$$\text{Then } AB = PQ$$

$$AC = PR \text{ (d)}$$

Question 7.

If $\angle D = \angle P$, $\angle E = \angle Q$ and $DE = PQ$, then $\Delta DEF = \Delta PQR$, by the congruence rule

- (a) SAS
- (b) ASA
- (c) SSS
- (d) RHS

Solution:

$$\text{In } \Delta DEF = \Delta PQR$$

$$\angle D = \angle P, \angle E = \angle Q$$

$$DE = PQ$$

$$\Delta DEF = \Delta PQR \text{ (ASA axiom) (b)}$$

Question 8.

In ΔABC and ΔPQR , $BC = QR$ and $\angle C = \angle R$. To establish $\Delta ABC = \Delta PQR$ by SAS congruence rule, the additional information required is

- (a) $AC = PR$
- (b) $AB = PR$
- (c) $CA = PQ$
- (d) $AB = PQ$

Solution:

If $\Delta ABC = \Delta PQR$ by SAS

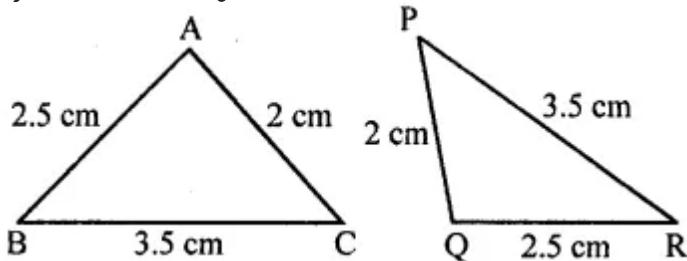
$BC = QR$ and $\angle C = \angle R$, then $AC = PR$ (a)

Question 9.

In the given figure, the lengths of the sides of two triangles are given. The correct statement is

- (a) $\Delta ABC = \Delta PQR$

(b) $\Delta ABC = \Delta QRP$
 (c) $\Delta ABC = \Delta QPR$
 (d) $\Delta ABC = \Delta RPQ$



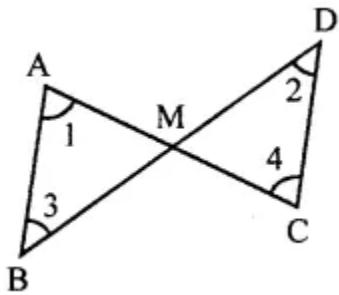
Solution:

Correct statement is $\Delta ABC = \Delta QRP$. (b)

Question 10.

In the given figure, M is the mid-point of both AC and BD. Then

(a) $\angle 1 = \angle 2$
 (b) $\angle 1 = \angle 4$
 (c) $\angle 2 = \angle 4$
 (d) $\angle 1 = \angle 3$



Solution:

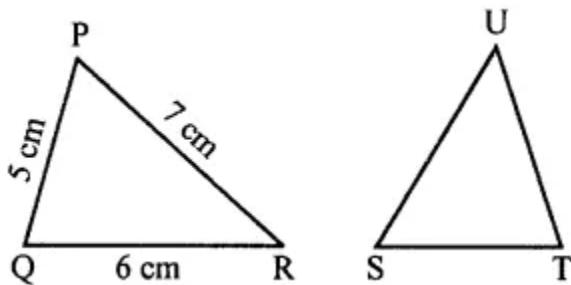
In the given figure,

M is mid-point of AC and BD both then $\angle 1 = \angle 4$. (b)

Question 11.

In the given figure, $\Delta PQR = \Delta STU$. What is the length of TU?

(a) 5 cm
 (b) 6 cm
 (c) 7 cm
 (d) cannot be determined



Solution:

In the given figure,

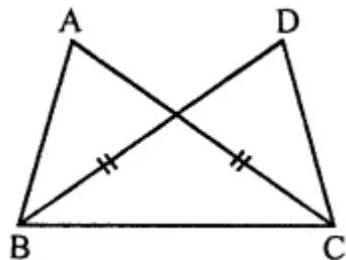
$$\Delta PQR = \Delta STU$$

$$TU = QR = 6 \text{ cm}$$

Question 12.

In the given figure, ΔABC and ΔDBC are on the same base BC. If $AB = DC$ and $AC = DB$, then which of the following statement is correct?

- (a) $\Delta ABC = \Delta DBC$
- (b) $\Delta ABC = \Delta CBD$
- (c) $\Delta ABC = \Delta DCB$
- (d) $\Delta ABC = \Delta BCD$



Solution:

In the given figure,

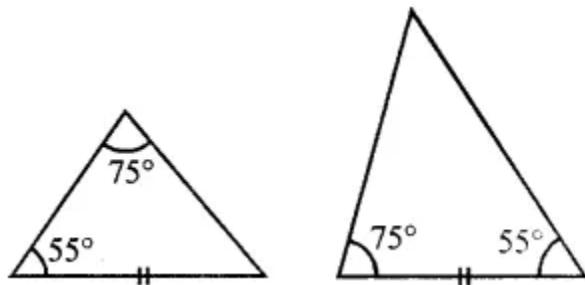
$$AB = DC, AC = DB$$

$$\text{Then, } \Delta ABC = \Delta DCB \text{ (c)}$$

Question 13.

The two triangles shown in the given figure are:

- (a) congruent by AAS rule
- (b) congruent by ASA rule
- (c) congruent by SAS rule
- (d) not congruent.



Solution:

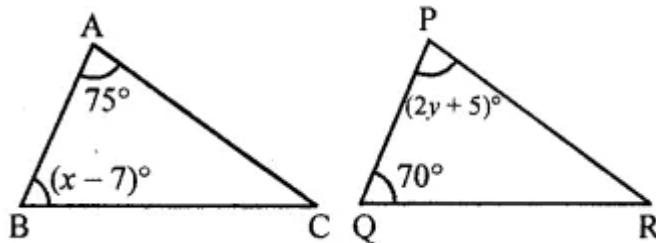
In the given two triangles are not congruent.

In first triangle, AAS are given while in second ASA are given. (d)

Question 14.

In the given figure, $\Delta ABC = \Delta PQR$. The values of x and y are:

- (a) $x = 63, y = 35$
- (b) $x = 77, y = 35$
- (c) $x = 35, y = 77$
- (d) $x = 63, y = 40$



Solution:

In the given figure,

$$\Delta ABC = \Delta PQR$$

$$\angle A = \angle P \text{ and } \angle B = \angle Q$$

$$\text{Now } x - 7 = 70^\circ$$

$$\Rightarrow x = 70^\circ + 7 = 77^\circ$$

$$\text{and } 2y + 5 = 75$$

$$\Rightarrow 2y = 75^\circ - 5 = 70^\circ$$

$$\Rightarrow y = 35^\circ$$

$$x = 77^\circ, y = 35^\circ \text{ (b)}$$

Higher Order Thinking Skills (HOTS)

Question 1.

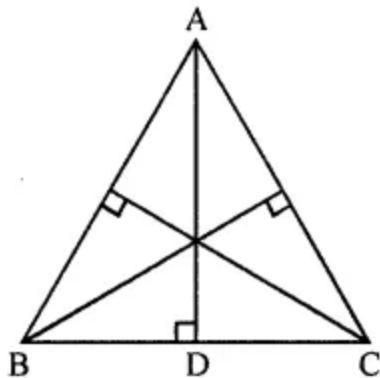
If all the three altitudes of a triangle are equal, then prove that it is an equilateral triangle.

Solution:

Given: In $\triangle ABC$,

AD, BE and CF are altitudes of the triangle

and $AD = BE = CF$.



To prove: $\triangle ABC$ is an equilateral.

Proof: In $\triangle ABD$ and $\triangle CFB$

$$AD = CF \text{ (Given)}$$

$$\angle D = \angle F \text{ (Each } = 90^\circ\text{)}$$

$$\angle B = \angle B \text{ (Common)}$$

$$\Delta ABD \cong \Delta CFB \text{ (AAS criterion)}$$

Similarly in Δ BEC and Δ ADC

$$BE = AD \text{ (Given)}$$

$\angle C = \angle C$ (Common)

$$\angle E = \angle D \text{ (Each } = 90^\circ\text{)}$$

$$\Delta BEC \cong \Delta ADC \text{ (AAS c)}$$

$$BC \equiv AC \quad \text{(ii)}$$

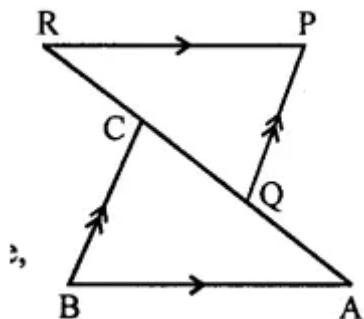
From (i) and (ii)

$$AB = BC = AC$$

$\triangle ABC$ is an equilateral triangle

Question 2.

In the given fig., if $BA \parallel RP$, $QP \parallel BC$ and $AQ = CR$, then prove that $\Delta ABC = \Delta RPQ$.



Solution:

In the given figure, $BA \parallel RP$

$QP \parallel BC$ and $AQ = CR$

To prove : $\Delta ABC = \Delta RPQ$

Proof: $AQ = CR$

Adding CQ to both sides

$AQ + CQ = CR + CQ$

$\Rightarrow AC = RQ$

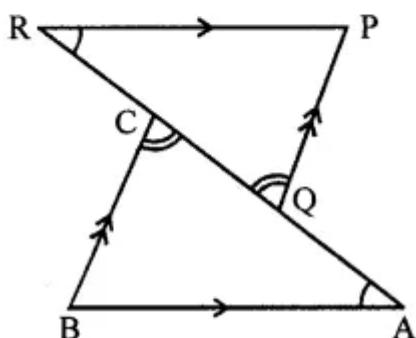
Now in ΔABC and ΔRPQ

$\angle A = \angle R$ (Alternate angles)

$\angle C = \angle Q$ (Alternate angles)

$AC = RQ$ (Proved)

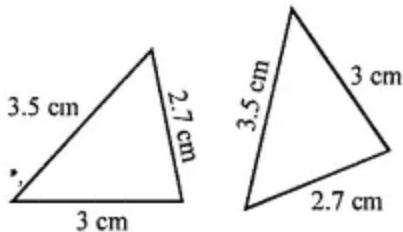
$\Delta ABC = \Delta RPQ$ (ASA criterion)



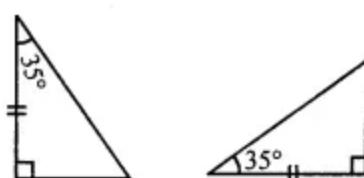
Check Your Progress

Question 1.

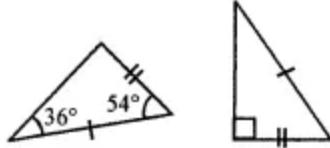
State, giving reasons, whether the following pairs of triangles are congruent or not:



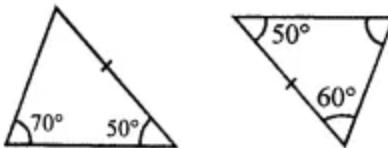
(i)



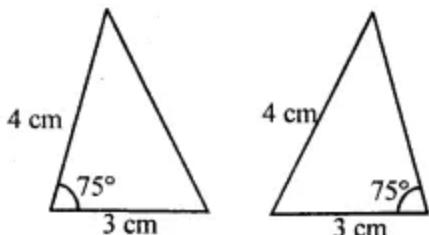
(ii)



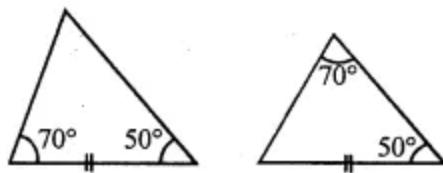
(iii)



(iv)



(v)



(vi)

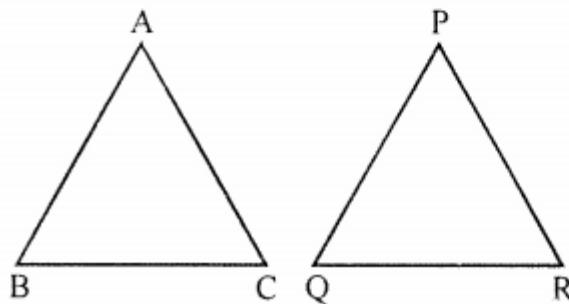
Solution:

- (i) In the given figure, using the SSS criterion triangles are congruent.
- (ii) Triangles are congruent for the criterion ASA criterion.
- (iii) Triangles are congruent for the criterion RHS.
- (iv) In the first triangle, third angle = $180^\circ - (70^\circ + 50^\circ) = 180^\circ - 120^\circ = 60^\circ$
Now triangles are congruent for ASA criterion.
- (v) Not congruent as included angles of the given two sides are not equal.
- (vi) Not congruent as the included sides are different.

Question 2.

Given below are measurements of some parts of two triangles. Examine whether the two triangles are congruent or not. In case of congruence, give reasons and write in symbolic form:

ΔABC	ΔPQR
(i) $AB = 4 \text{ cm}$, $BC = 5 \text{ cm}$, $\angle B = 70^\circ$	(i) $QR = 4 \text{ cm}$, $RP = 5 \text{ cm}$, $\angle R = 70^\circ$
(ii) $AB = 4 \text{ cm}$, $BC = 5 \text{ cm}$, $\angle B = 80^\circ$	(ii) $PQ = 4 \text{ cm}$, $RP = 5 \text{ cm}$, $\angle R = 80^\circ$
(iii) $BC = 6 \text{ cm}$, $\angle A = 90^\circ$, $\angle C = 50^\circ$	(iii) $QR = 6 \text{ cm}$, $\angle R = 50^\circ$, $\angle Q = 40^\circ$
(iv) $AB = 5 \text{ cm}$, $\angle A = 90^\circ$, $BC = 8 \text{ cm}$	(iv) $PR = 5 \text{ cm}$, $\angle P = 90^\circ$, $QR = 8 \text{ cm}$



Solution:

(i) In ΔABC and ΔPQR

$$AB = QR = 4 \text{ cm}$$

$$BC = RP = 5 \text{ cm}$$

$$\angle B = \angle R = 70^\circ$$

$\Delta ABC \cong \Delta PQR$ (SAS criterion)

(ii) In ΔABC and ΔPQR

$$AB = PQ = 4 \text{ cm}$$

$BC = RP = 5 \text{ cm}$ not corresponding sides

$\angle B = \angle R = 80^\circ$ not corresponding angles

Triangles are not congruent.

(iii) $BC = QR = 6 \text{ cm}$

$$\angle A = \angle P = 90^\circ$$
 (Third angle)

$$\angle C = \angle R = 50^\circ$$

Triangles are congruent for ASA criterion.

(iv) $AB = PR = 5 \text{ cm}$ (Side)

$\angle A = \angle P = 90^\circ$

$BC = QR = 8 \text{ cm}$ (Hypotenuse)

Triangles are congruent for RHS criterion.

Question 3.

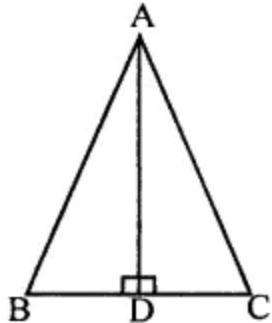
In the given figure, ABC is an isosceles triangle with $AB = AC$ and AD is one of its altitudes.

(i) State the three pairs of equal parts in ΔADB and ΔADC .

(zz) Is $\Delta ADB = \Delta ADC$? Give reasons.

(iii) Is $\angle B = \angle C$? Why?

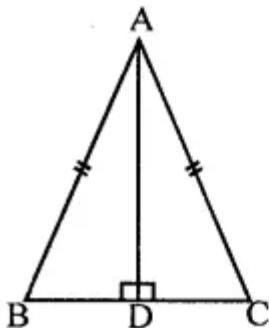
(iv) Is $BD = DC$? Why?



Solution:

ΔABC is an isosceles triangle with $AB = AC$

and AD is one of the altitudes.



(i) In ΔADB and ΔADC

Side $AD = AD$ (Common)

Hypotenuse, $AB = AC$ (Given)

$\angle ADB = \angle ADC = 90^\circ$ ($\because AB \perp BC$)

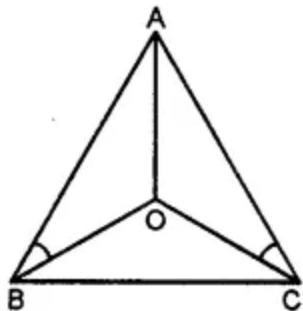
$\triangle ADB \cong \triangle ADC$

$\angle B = \angle C$ (c.p.c.t)

and $BD = CD$ (c.p.c.t)

Question 4.

In the given figure, OA bisects $\angle A$ and $\angle ABO = \angle OCA$. Prove that $OB = OC$.



Solution:

In $\triangle OAB$ and $\triangle OAC$

$\angle OAB = \angle OAC$ ($\because OA$ bisects $\angle A$)

$\angle ABO = \angle ACO$ (Given)

$OA = OA$ (common)

$\triangle OAB \cong \triangle OAC$ (AAS congruence rule)

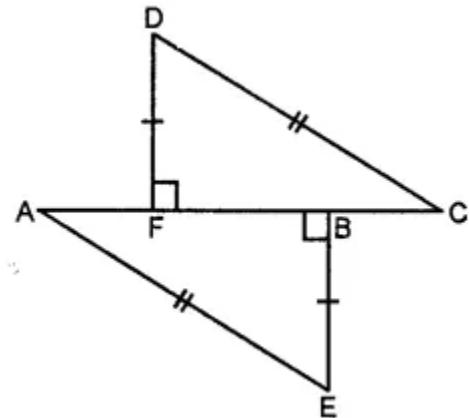
$OB = OC$ (Corresponding parts of congruent As)

Question 5.

In the given figure, prove that

(i) $AB = FC$

(ii) $AF = BC$.



Solution:

In $\triangle ABE$ and $\triangle DFC$

$\angle B = \angle F$ (each 90°)

$AE = DC$ (Given)

$BE = DF$ (Given)

$\triangle ABE \cong \triangle DFC$ (RHS congruence rule)

(i) $AB = FC$ (Corresponding parts of congruent \triangle s)

(ii) As $AB = FC$ (Proved above)

$$\Rightarrow AF + FB = FB + BC$$

$$\Rightarrow AF + FB - FB = BC$$

$$\Rightarrow AF = BC$$