

10. Circles

Exercise 10.1

1. Question

Fill in the blanks:

- (i) The common point of a tangent and the circle is called..... .
- (ii) A circle may have parallel tangents.
- (iii) A tangent to a circle intersects it in points(s).
- (iv) A line intersecting a circle in two points is called a
- (v) The angle between tangent at a point on a circle and the radius through the point is

Answer

- (i) The tangent at any point of a circle is perpendicular to the radius through the point of contact.
- (ii) A circle may have two parallel tangents
- (iii) A tangent to a circle intersects it in one point.
- (iv) Secant is a line intersecting a circle in two points
- (v) The angle between tangent at a point on a circle and the radius through the point is 90°

2. Question

How many tangents can a circle have?

Answer

A circle can have infinite tangents.

3. Question

O is the centre of a circle of radius 8 cm. The tangent at a point A on the circle cuts a line through O at B such that AB = 15 cm. Find OB.

Answer

$$OB^2 = 8^2 + 15^2$$

$$OB = \sqrt{64 + 225}$$

$$OB = 17\text{cm}$$

4. Question

If the tangent at a point P to a circle with centre O cuts a line through O at Q such that PQ = 24 cm and OQ = 25 cm. Find the radius of the circle.

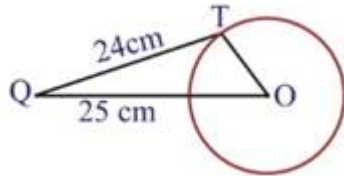
Answer

since QT is a tangent to the circle at T and OT is radius,

Therefore OT perpendicular QT

It is given that OQ=25 cm and QT= 24 cm

By Pythagoras theorem we have



$$OB^2 = OA^2 + AB^2$$

$$OQ^2 = QT^2 + OT^2$$

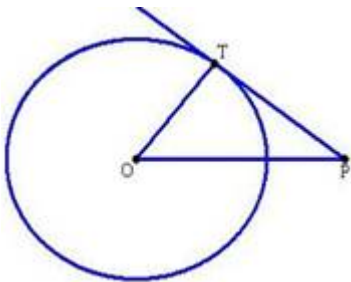
$$OT^2 = 25^2 - 24^2$$

$$= 49 \times 1 = 49$$

$$OT = 7$$

Exercise 10.2**1. Question**

If PT is a tangent at T to a circle whose centre is O and OP = 17 cm, OT = 8 cm, find the length of the tangent segment PT.

Answer

Given that O is the center of the circle and OP =17 cm and the radius of the circle OT=8cm.

We need to find the length of the segment PT.

The line PT is the tangent line to the circle at the point T, the line through the centre is perpendicular to PT.

$$OP^2 = OT^2 + PT^2$$

$$17^2 = 8^2 + PT^2$$

$$PT^2 = 289 - 64$$

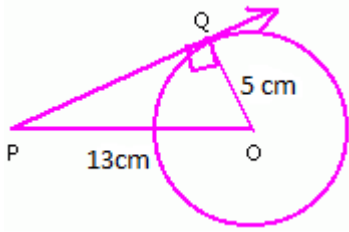
$$= 225$$

$$PT^2 = \sqrt{225} = 15$$

2. Question

Find the length of a tangent drawn to a circle with radius 5 cm, from a point 13 cm from the centre of the circle.

Answer



Given: PQ is a tangent to the circle intersect at $OP=13\text{cm}$ and $OQ=5\text{ cm}$

Proof: In right triangle OQP

$$PQ = \sqrt{OP^2 - OQ^2} = \sqrt{169 - 25} = 12\text{cm}$$

Therefore the length of the tangent from the point is 12 cm

3. Question

A point P is 26 cm away from the centre O of a circle and the length PT of the tangent drawn from P to the circle is 10 cm. Find the radius of the circle.

Answer

Let PQ be a tangent to the circle from point P and OQ be the radius at the point of contact.

$$\therefore \angle OQP = 90^\circ$$

$$\Rightarrow OP^2 = OQ^2 + PQ^2$$

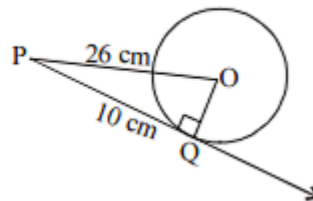
($\therefore$ using Pythagoras theorem.)

$$\Rightarrow OQ^2 = OP^2 - PQ^2 = 26^2 - 10^2$$

$$= (26 + 10)(26 - 10) = 36 \times 16$$

$$\Rightarrow OQ = 6 \times 4 = 24$$

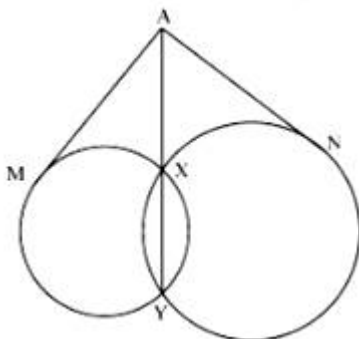
$$\therefore \text{Radius of the circle} = 24\text{ cm.}$$



4. Question

If from any point on the common chord of two intersecting circles, tangents be drawn to the circles, prove that they are equal.

Answer



Let the two circles intersect at points X and Y, XY is the common chord.

Suppose A is a point on their common chord and AM and AN be the tangent drawn from A to the circle

AM is the tangent and AXY is a secant.

$$AM^2 = AX \times AY \quad \dots\dots\dots(i)$$

AN is the tangent and AXY is the secant.

$$AN^2 = AX \times AY \quad \dots\dots\dots(ii)$$

Therefore, from equations (i) and (ii), we get,

$$AM = AN.$$

5. Question

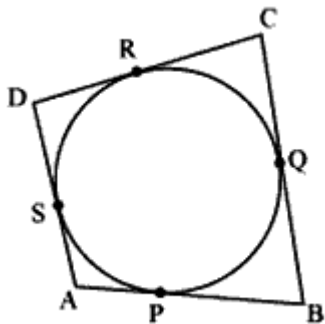
If the sides of a quadrilateral touch a circle, prove that the sum of a pair of opposite sides is equal to the sum of the other pair.

Answer

Given: the sides of a quadrilateral touch a circle

To prove: the sum of a pair of opposite sides is equal to the sum of the other pair.

Proof:

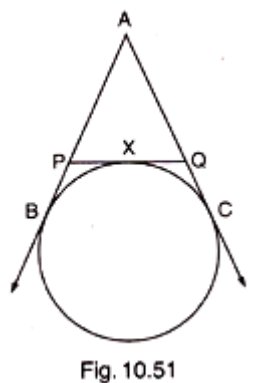


From the theorem which states that the lengths of the two tangents drawn from an external point to a circle are equal. From points A the tangents drawn are AP and AS, $AP = AS$ (1) From points B the tangents drawn are BP and BQ, $BP = BQ$ (2) From points D the tangents drawn are DR and DS, $DR = DS$ (3) From points C the tangents drawn are CR and CQ, $CR = CQ$ (4) Add 1, 2, 3 and 4 to get $AP + BP + DR + CR = AS + BQ + DS + CQ$ $(AP + BP) + (DR + CR) = (AS + DS) + (BQ + CQ)$ $AB + DC = AD + BC$

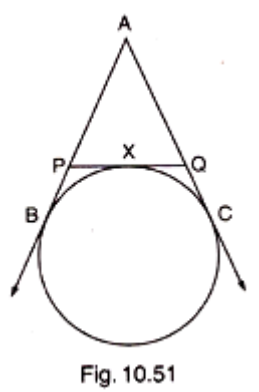
Hence proved

6. Question

If AB, AC, PQ are tangents in Fig. 10.51 and $AB = 5$ cm, find the perimeter of $\triangle APQ$.



Answer



Given: AB and AC are tangent to the circle with centre O

PQ is tangent to the circle at X which intersect AB and AC in P and Q

To find : Perimeter of triangle APQ

Proof:

$$AB = AC$$

$$QC = QX$$

$$PB = PX$$

$$AB = AC = 5cm$$

$$Perimeter\ of\ \triangle APQ = AQ + QP + AP$$

$$= AC + AB = 5 + 5 = 10cm$$

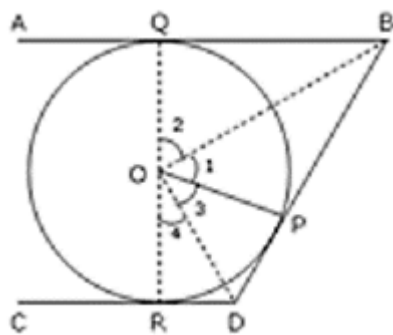
7. Question

Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.

Answer

Three tangent AB, CD and BD of a circle such as AB and CD are two parallel tangent BD intercept an angle BOD at the centre.

To Prove: $\angle BOD = 90^\circ$



Construction: Join OQ and OR

Proof: $OP \perp BD$

In right angle $\triangle OQB$ and OPB .

$\triangle OQB \cong OPB$.

Since $OQ = OP$, $\angle Q = \angle P$ and OB is common

So $\angle 1 = \angle 2$

Similarly in $\triangle ORD$ and OPD we have $\angle 3 = \angle 4$

$\angle BOD = \angle 1 + \angle 3$

$$= \frac{1}{2}(\angle 1 + \angle 2 + \angle 3 + \angle 4) = \frac{1}{2}(180^\circ) = 90^\circ$$

8. Question

In Fig. 10.52, PQ is tangent at a point R of the circle with centre O. If $\angle TRQ = 30^\circ$, find $m\angle PRS$.

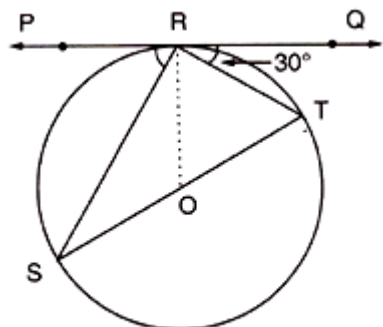


Fig. 10.52

Answer

Given that, $\angle TRQ = 30^\circ$

Since, ST is a diameter and angle in a semi-circle is a right angle

Therefore, $\angle SRT = 90^\circ$

Now,

$$\angle TRQ + \angle SRT + \angle PRS = 180^\circ$$

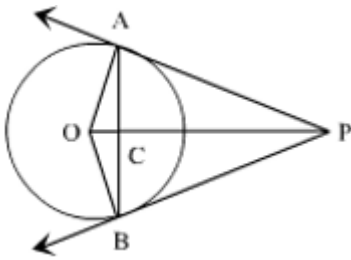
$$30^\circ + 90^\circ + \angle PRS = 180^\circ$$

$$\angle PRS = 60^\circ$$

9. Question

If PA and PB are tangents from an outside point P. such that PA = 10 cm and $\angle APB = 60^\circ$. Find the length of chord AB.

Answer



Given: PA and PB are tangent of a circle PA= 10 cm and angle APB= 60°

Let O be the center of the given circle and C be the point of intersection of OP and AB

In triangle PAC and triangle PBC

PA = PB (tangent from an external point are equal)

$\angle APC = \angle BPC$ (tangent from an external point are equally inclined to the segment joining center to the point)

PC = PC (common)

$\triangle PAC \cong \triangle PBC$ (by SAS)

$AC = BC$

$\angle APB = \angle APC + \angle BPC$

$$\angle APC = \frac{1}{2} \angle APB = \frac{1}{2} \times 60^\circ = 30^\circ$$

$$\angle APC + \angle BCP = 180^\circ = 90^\circ$$

In right triangle ACP

$$\sin 30^\circ = \frac{AC}{AP}$$

$$\frac{1}{2} = \frac{AC}{10} = 5 \text{ cm}$$

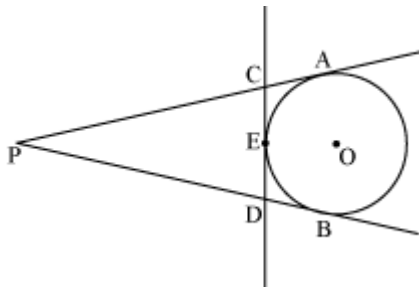
$AC = BC$

$$AB = 5 + 5 = 10 \text{ cm}$$

10. Question

From an external point P, tangents PA are drawn to a circle with centre O. If CD is the tangent to the circle at a point E and PA = 14 cm, find the perimeter of $\triangle PCD$.

Answer



Given: PA and PB are tangent to the circle with centre O

CD is tangent to the circle at E which intersect PA and PB in C and D

To find : Perimeter of triangle PCD

Proof:

$$PA = PB$$

$$CA = DE$$

$$DB = DE$$

$$PA = PB = 14\text{cm}$$

$$\text{Perimeter of } \triangle PCD = PC + CD + PD$$

$$= PA + PB = 14 + 14 = 28\text{cm}$$

11. Question

In Fig. 10.53, ABC is a right triangle right-angled at B such that $BC = 6\text{ cm}$ and $AB = 8\text{ cm}$. Find the radius of its incircle.

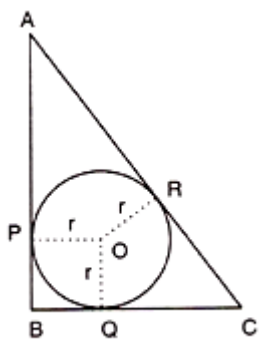
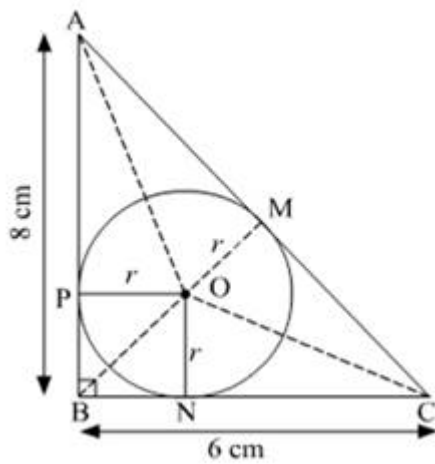


Fig. 10.53

Answer

Let ABC be the right angled triangle such that angle $B=90^\circ$, $BC=6\text{cm}$, $AB= 8\text{cm}$. Let O be the centre and r be the radius of the in circle.



AB, BC and CA are tangent to the circle at P, N and M

$OP=ON=OM=r$ (radius of the circle)

$$\text{Area of } \triangle ABC = \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$$

By Pythagoras theorem,

$$CA^2 = AB^2 + BC^2$$

$$CA^2 = 8^2 + 6^2$$

$$CA = 10 \text{ cm}$$

$$\text{Area of } \triangle ABC = \text{Area of } \triangle OAB + \text{Area of } \triangle OBC + \text{Area of } \triangle OCA$$

$$24 = \frac{1}{2}r \times AB + \frac{1}{2}r \times BC + \frac{1}{2}r \times CA$$

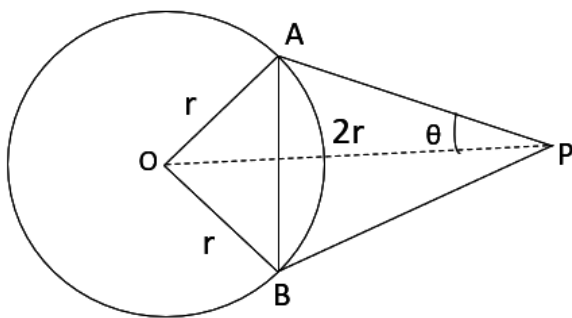
$$r = \frac{2 \times 24}{AB + BC + CA}$$

$$r = \frac{48}{8 + 6 + 10} = \frac{48}{24} = 2 \text{ cm}$$

12. Question

From a point P, two tangents PA and PB are drawn to a circle with centre O. If OP = diameter of the circle, show that $\triangle APB$ is equilateral.

Answer



AP is the tangent to the circle,

According to the theorem which states that tangent to a circle is perpendicular to the

radius through the point of contact.

$$\Rightarrow OA \perp AP$$

$$\angle OAP = 90^\circ$$

$$\text{Also } OB \perp BP$$

$$\Rightarrow \angle OBP = 90^\circ$$

In $\triangle OAP$ $\sin \theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$

$$\sin \angle OPA = \frac{r}{2r} = \frac{1}{2}$$

As

$$\sin 30^\circ = \frac{1}{2}$$

$$\Rightarrow \angle OPA = 30^\circ$$

Similarly $\angle OPB = 30^\circ$ Now $\angle APB = \angle OPA + \angle OPB$

$$= 30^\circ + 30^\circ$$

$$= 60^\circ \dots (1)$$

In $\triangle PAB$, As PA and PB are drawn from external point P , By theorem which states that the lengths of the two tangents drawn from external point to a circle are equal

$$\therefore PA = PB \text{ Also } \angle PAB = \angle PBA \dots (2)$$

As $\angle PAB + \angle PBA + \angle APB = 180^\circ$ (sum of angles of triangle)

$$\angle PAB + \angle PBA = 180^\circ - \angle APB$$

$$\angle PAB + \angle PBA = 180^\circ - 60^\circ$$

$$\Rightarrow 2\angle PAB = 120^\circ$$

$$\Rightarrow \angle PAB = 60^\circ \dots (3)$$

From 1 and 2 and 3, $\angle PAB = \angle PBA = \angle APB = 60^\circ$ Hence $\triangle PAB$ is an equilateral triangle.

13. Question

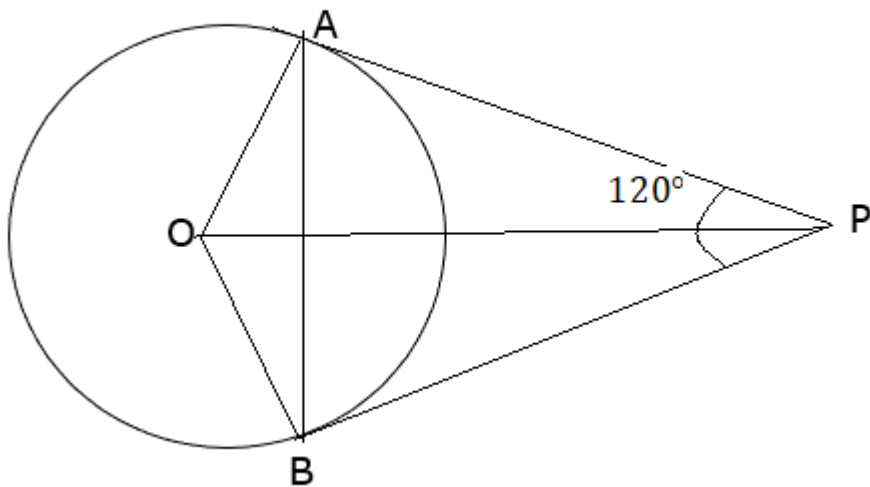
Two tangent segments PA and PB are drawn to a circle with centre O such that $\angle APB = 120^\circ$. Prove that $OP = 2 AP$.

Answer

Given: Two tangent segments PA and PB are drawn to a circle with centre O such that $\angle APB = 120^\circ$.

To prove: $OP = 2 AP$

Proof: Construct the figure according to the conditions given.



Here In triangle OAP and OBP, $PA = PB$ (Length of Tangents from external point are equal)

$OA = OB$ (Radii of same circle)

$OP = OP$ (common)

$\Delta OAP \sim \Delta OBP$ (By SSS criterion)

$\angle OPA = \angle OPB = 60^\circ$.

In Triangle OAP , $\angle OAP = 90^\circ$ (By theorem which states that tangent to a circle is perpendicular to the radius through the point of contact) We know in a right angle triangle

$$\sin \theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\Rightarrow \sin 60^\circ = AP / OP , \text{ i.e } 1/2 = AP / OP$$

$$\text{So, } OP = 2 AP$$

Hence proved.

14. Question

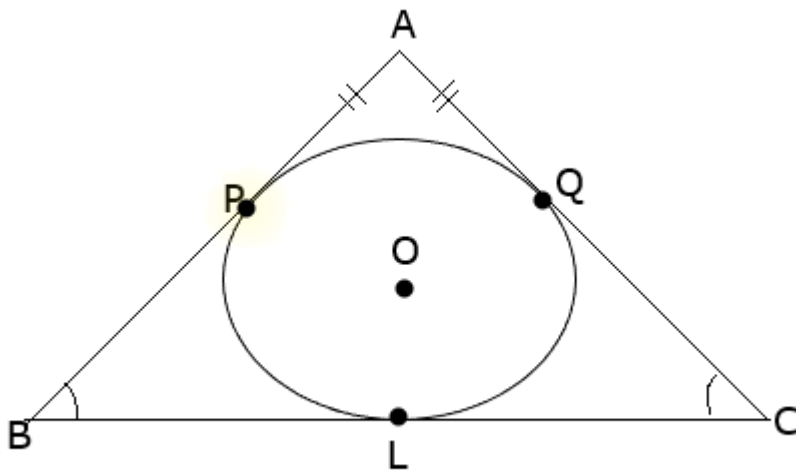
If ΔABC is isosceles with $AB = AC$ and $C(O, r)$ is the incircle of the ΔABC touching BC at L , prove that L bisect BC .

Answer

Given: If ΔABC is isosceles with $AB = AC$ and $C(O, r)$ is the incircle of the ΔABC touching BC at L .

To prove: L bisect BC .

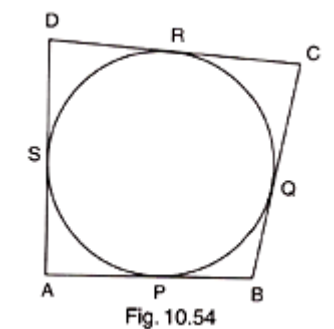
Proof: Construct the figure according to given condition.



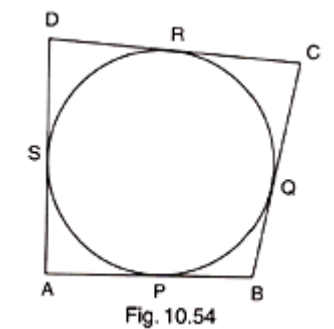
$AB = AC$ (given) From the theorem which states that the lengths of two tangents drawn from external point to a circle are equal. (1) As tangents AP and AQ are drawn from the external point A. $AP = AQ$ Also, $AB = AC \Rightarrow AP + PB = AQ + QC \Rightarrow AP + PB = AP + QC \Rightarrow PB = QC$ From (1) as tangents BP and BL are drawn from external point B, And tangents CQ and CL are drawn from external point C. $\Rightarrow BP = BL$ (3) $CQ = CL$ (4) As we have proved $PB = QC$ From 3 and 4 $BL = CL \Rightarrow L$ bisects BC. Hence proved.

15. Question

In Fig. 10.54, a circle touches all the four sides of a quadrilateral ABCD with $AB = 6$ cm, $BC = 7$ cm and $CD = 4$ cm. Find AD.



Answer



Here,

$$AP = AS$$

$$\text{Let, } AP = AS = X$$

$$\text{Simillarly, } BP = BQ$$

$$CQ = CR$$

$$RD = DS$$

$$\text{Sin ce, } AP = X$$

$$\Rightarrow BP = AB - AP = 6 - X$$

$$\text{Now, } BP = BQ = 6 - X$$

$$\begin{aligned} CQ = BC - BQ &= 7 - (6 - X) \\ &= 1 + X \end{aligned}$$

$$\text{now, } CQ = CR = 1 + X$$

$$\begin{aligned} RD = CD - CR &= 4 - (1 + X) \\ &= 3 - X \end{aligned}$$

$$RD = DS = 3 - X$$

$$AD = AS + SD$$

$$X + 3 - X = 3$$

$$AD = 3\text{cm}$$

Here,

$$AP = AS$$

$$\text{Let, } AP = AS = X$$

$$\text{Simillarly, } BP = BQ$$

$$CQ = CR$$

$$RD = DS$$

$$\text{Sin ce, } AP = X$$

$$\Rightarrow BP = AB - AP = 6 - X$$

$$\text{Now, } BP = BQ = 6 - X$$

$$\begin{aligned} CQ = BC - BQ &= 7 - (6 - X) \\ &= 1 + X \end{aligned}$$

$$\text{now, } CQ = CR = 1 + X$$

$$\begin{aligned} RD = CD - CR &= 4 - (1 + X) \\ &= 3 - X \end{aligned}$$

$$RD = DS = 3 - X$$

$$AD = AS + SD$$

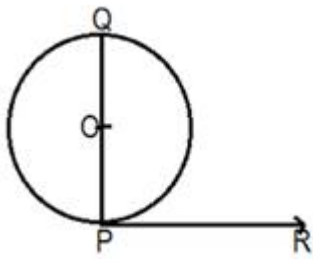
$$X + 3 - X = 3$$

$$AD = 3\text{cm}$$

16. Question

Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

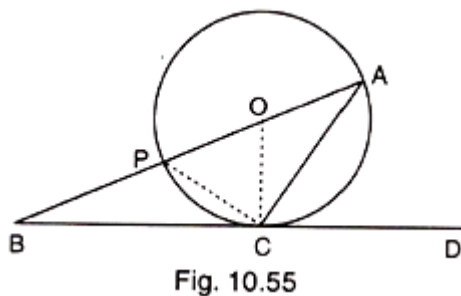
Answer



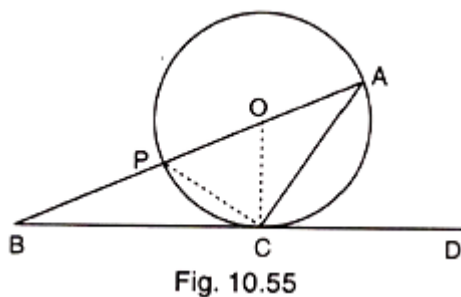
Draw a circle with centre O, draw a tangent PR touching circle at P. Draw QP perpendicular to RP at a point P, QP lies in the circle. Now, $\angle OPR = 90^\circ$. Also, $\angle QPR = 90^\circ$. Therefore, $\angle OPR = \angle QPR$. This is possible only when O lies on QP. Hence, it is proved that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

17. Question

In fig. 10.55, O is the centre of the circle and BCD is tangent to it at C. Prove that $\angle BAC + \angle ACD = 90^\circ$.



Answer



Given: In the above figure, O is the centre of the circle and BCD is tangent to it at C. To prove: $\angle BAC + \angle ACD = 90^\circ$
Proof:

In $\triangle OAC$

$OA = OC$ [radii of same circle]

$\Rightarrow \angle OCA = \angle OAC$ [angles opposite to equal sides are equal]

$\Rightarrow \angle OCA = \angle BAC$ [1]

Also,

$OC \perp BD$ [Tangent at any point on a circle is perpendicular to the radius through point of contact]

$\Rightarrow \angle OCD = 90^\circ$

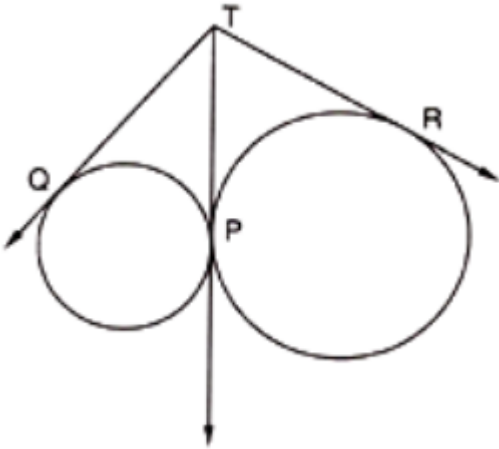
$\Rightarrow \angle OCA + \angle ACD = 90^\circ$

$$\Rightarrow \angle BAC + \angle ACD = 90^\circ \quad [\text{From 1}]$$

Hence Proved

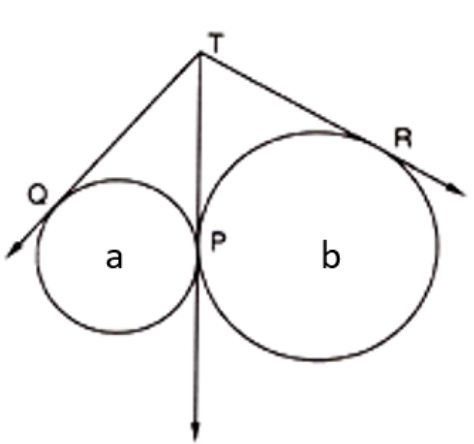
18. Question

Two circles touch externally at a point P. From a point T on the tangent at P, tangents TQ and TR are drawn to the circles with points of contact Q and R respectively. Prove that $TQ = TR$



Answer

Let us label two circles as 'a' and 'b'



As TQ and TP are tangents to circle a, And TP and TR are tangents to circle b. By theorem which states that the lengths of the two tangents drawn from external point to a circle are equal.

$TQ = TP \dots (1)$ $TP = TR \dots (2)$ From 1 and 2, $TQ = TR$ Hence proved

19. Question

In Fig 10.57, a circle is inscribed in a quadrilateral ABCD in which $\angle B = 90^\circ$. If $AD = 23$ cm, $AB = 29$ cm and $DS = 5$ cm, find the radius r of the circle.

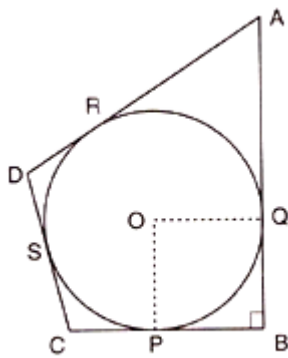
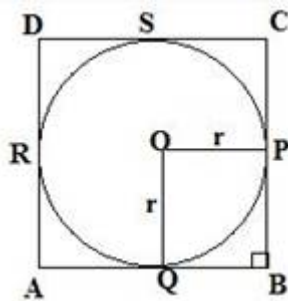


Fig. 10.57

Answer



Given: ABCD is a quadrilateral in which

$$\angle B = 90^\circ$$

$$AD = 23\text{ cm}, DS = 5\text{ cm and } AB = 29\text{ cm}$$

Let the radius of the incircle be r cm.

$$AD = DS = 5\text{ cm (tangent from an external point)}$$

$$\text{Since } AD = 23\text{ cm}$$

So,

$$AR + RD = AD$$

$$AR + 5 = 23\text{ cm}$$

$$AR = 18\text{ cm} \text{ -----(i)}$$

$$\text{and } AQ = AR$$

$$\text{since } AR = 18\text{ cm}$$

$$\text{So, } AQ + QB = AB$$

Now OP and OQ are radius of the circle. So from tangent P and Q

$$\angle OPB = \angle OQB = 90^\circ$$

OPBQ is a square

$$OP = QB$$

$$\text{Radius of the circle} = 11\text{ cm}$$

20. Question

In Fig. 10.58, there are two concentric circles with centre O of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP.

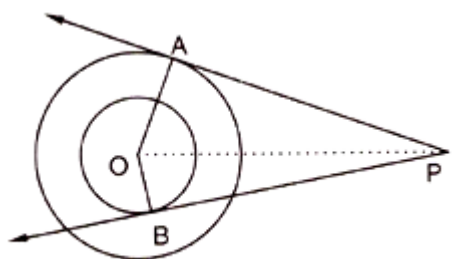
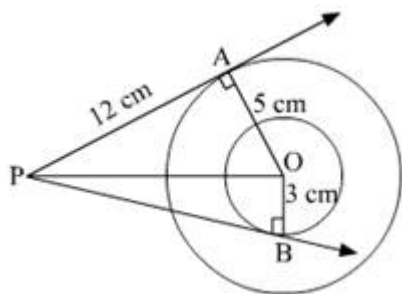


Fig. 10.58

Answer



PA and PB are the tangents drawn from the external point P to outer and inner circle respectively

$$\angle OAP = 90^\circ \quad \angle OBP = 90^\circ$$

Given $OA = 5 \text{ cm}$, $OB = 3 \text{ cm}$ and $AP = 12 \text{ cm}$

In $\triangle OAP$

$$OP^2 = (12 \text{ cm})^2 + (5 \text{ cm})^2 = 169 \text{ cm}^2$$

$$OP = 13 \text{ cm}$$

In $\triangle OBP$,

$$PB^2 = OP^2 - OB^2$$

$$PB^2 = (13 \text{ cm})^2 - (3 \text{ cm})^2 = 160 \text{ cm}^2$$

$$PB = 4\sqrt{10} \text{ cm}$$

Thus the length of $PB = 4\sqrt{10} \text{ cm}$

21. Question

In Fig. 10.59, AB is a chord of length 16 cm of a circle of radius 10 cm. The tangents at A and B intersect at a point P. Find the length of PA.

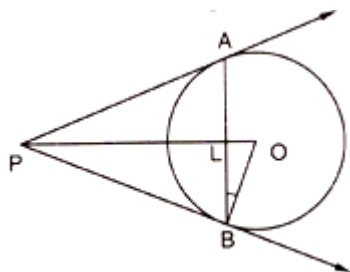
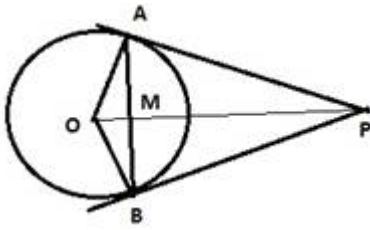


Fig. 10.59

Answer



$$OA = 10 \text{ cm}$$

As we know Perpendicular from centre to the chord bisects the chord.

$$\text{So } AM = MB = 8 \text{ cm}$$

Using Pythagoras theorem in triangle AOM

$$OM = \sqrt{10^2 - 8^2} = 6 \text{ cm}$$

$$\tan \angle AOM = \frac{8}{6} = \frac{4}{3}$$

Now in $\triangle OAP$

$$\tan \angle AOM = \frac{PA}{OA}$$

$$PA = \frac{40}{3}$$

22. Question

In Fig. 10.60, PA and PB are tangents from an external point P to a circle with centre O. LN touches the circle at A. Prove that $PL + LM = PN + MN$.

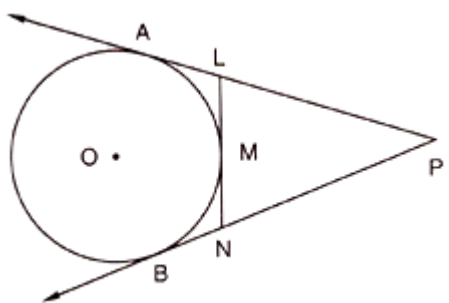


Fig. 10.60

Answer

$$PA = PB \text{ ----- (i)}$$

As tangent drawn from external points to a circle are equal in length

$$PL + AL = PN + BN \text{ ----- (ii)}$$

PLA and PNB are the two tangents which are equal

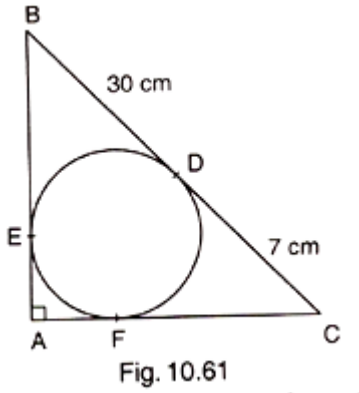
$$AL = ML \text{ and } BN = MN \text{ ----- (iii)}$$

From (ii) and (iii)

$$PL = ML$$

23. Question

In Fig. 10.61, BDC is a tangent to the given circle at point D such that BD = 30 cm and CD = 7 cm. The other tangents BE and CF are drawn respectively from B and C to the circle and meet when produced at A making BAC a right angle triangle. Calculate (i) AF (ii) radius of the circle.



Answer

Given : AB, BC and AC are tangents to the circle at E, D and F.

BD = 30 cm and DC = 7 cm and $\angle BAC = 90^\circ$

Recall that tangents drawn from an exterior point to a circle are equal in length

Hence BE = BD = 30 cm

Also FC = DC = 7 cm

Let AE = AF = x $\rightarrow$ (1)

Then AB = BE + AE = (30 + x)

AC = AF + FC = (7 + x)

BC = BD + DC = 30 + 7 = 37 cm

Consider right ΔABC , by Pythagoras theorem we have

$$BC^2 = AB^2 + AC^2$$

$$\Rightarrow (37)^2 = (30 + x)^2 + (7 + x)^2$$

$$\Rightarrow 1369 = 900 + 60x + x^2 + 49 + 14x + x^2$$

$$\Rightarrow 2x^2 + 74x + 949 - 1369 = 0$$

$$\Rightarrow 2x^2 + 74x - 420 = 0$$

$$\Rightarrow x^2 + 37x - 210 = 0$$

$$\Rightarrow x^2 + 42x - 5x - 210 = 0$$

$$\Rightarrow x(x + 42) - 5(x + 42) = 0$$

$$\Rightarrow (x - 5)(x + 42) = 0$$

$$\Rightarrow (x - 5) = 0 \text{ or } (x + 42) = 0$$

$$\Rightarrow x = 5 \text{ or } x = -42$$

$$\Rightarrow x = 5 \text{ [Since } x \text{ cannot be negative]}$$

$$\therefore AF = 5 \text{ cm [From (1)]}$$

$$\text{Therefore } AB = 30 + x = 30 + 5 = 35 \text{ cm}$$

$$AC = 7 + x = 7 + 5 = 12 \text{ cm}$$

Let 'O' be the centre of the circle and 'r' the radius of the circle.

Join point O, F; points O, D and points O, E.

From the figure,

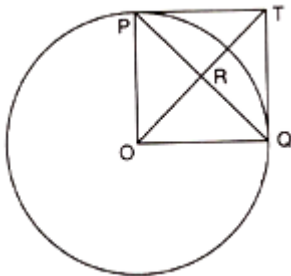
$$\text{Area of } (\triangle ABC) = \text{Area } (\triangle AOB) + \text{Area } (\triangle BOC) + \text{Area } (\triangle AOC)$$

$$\therefore r = 5$$

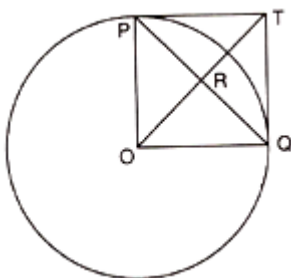
Thus the radius of the circle is 5 cm

24. Question

In Fig. 10.62, $PO \perp OQ$. The tangents to the circle at P and Q intersect at a point T. Prove that PQ and OT are right bisectors of each other.



Answer



To prove: PQ and OT are the right bisectors.

Proof: To prove PQ and OT are the right bisectors,

We need to prove $\angle PRT = \angle TRQ = \angle QRO = \angle ORP = 90^\circ$

As it is given that $PO \perp OQ$,

$$\Rightarrow \angle POQ = 90^\circ$$

In ΔPOT and ΔOQT

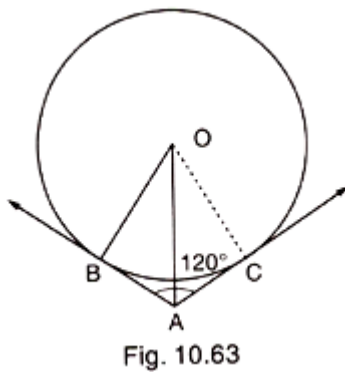
$OP=OQ$ (Radius) $\angle OPT = \angle OQT = 90^\circ$ (Tangent to a circle at a point is perpendicular to the radius through the point of contact) $OT=OT$ (common)

$\therefore \Delta POT \cong \Delta OQT$ Thus $PT=OQ$ (BY C.P.C.T) (1) Now in ΔPRT and ΔORQ $\angle TPR = \angle OQR$ (alternate angles) $\angle PTO = \angle TOQ$ (alternate angles) $PT=OQ$ (from (1)) $\therefore \Delta PRT \cong \Delta ORQ$ Thus $TQ = OP$ (By C.P.C.T) Hence $PT=TQ=OQ=OP$ Thus it is a square, $\Rightarrow$ The diagonals bisect at 90° .

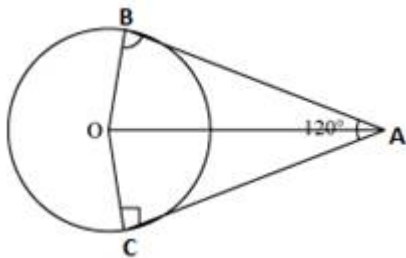
Hence proved

25. Question

In Fig. 10.63, two tangents AB and AC are drawn to a circle with centre O such that $\angle BAC = 120^\circ$. Prove that $OA = 2AB$.



Answer



It can be clearly show that OA bisects angle CAB ,

$$\angle OBC = \angle OBD = 60^\circ$$

In ΔOAB

$$\angle OAB = 60^\circ, \angle OBA = 90^\circ$$

$$\angle BOA + \angle OAB + \angle OBA = 180^\circ$$

$$\angle BOA = 180^\circ - 150^\circ = 30^\circ$$

$$\sin(\angle BOA) = \frac{AB}{AO}$$

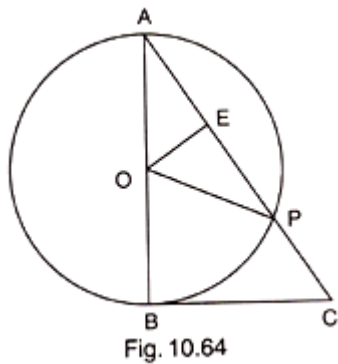
$$\sin 30^\circ = \frac{AB}{AO}$$

$$\frac{1}{2} = \frac{AB}{AO}$$

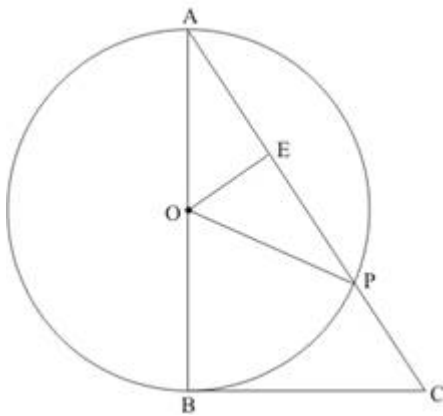
$$AO = 2AB$$

26. Question

In Fig. 10.64, BC is a tangent to the circle O. OE bisects AP. Prove that $\triangle AEO \sim \triangle ABC$.



Answer



Triangle AOP is an isosceles triangle because $OA = OP$ as they are the radius of the circle. We know that radius of the circle is always perpendicular to the tangent at the point of contact.

Here OB is the radius and BC is the tangent and B is the point of contact, Therefore

$$\angle ABC = 90^\circ$$

Also from the property of isosceles triangle we have found that

$$\angle OEA = 90^\circ$$

Therefore,

$$\angle ABC = \angle OEA$$

$\angle A$ is common angle to both triangle

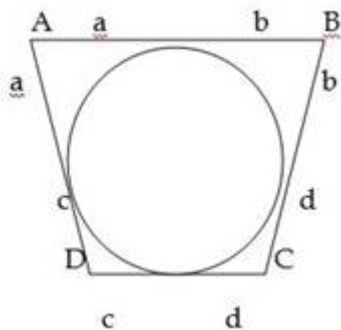
Therefore, from AA postulates of similar triangle

$$\triangle AOE \sim \triangle ABC$$

27. Question

The lengths of three consecutive sides of a quadrilateral circumscribing a circle are 4 cm, 5 cm and 7 cm respectively. Determine the length of the fourth side.

Answer



Given

$$AB = 7, BC = 5, CD = 4$$

Length of the tangent drawn from an external point to the circle are equal

Therefore

$$AB + CD = BC + AD$$

$$7 + 4 = 5 + AD$$

$$AD = 11 - 5 = 6$$

28. Question

In Fig. 10.65, common tangents PQ and RS to two circles intersect at A. Prove that $PQ = RS$.

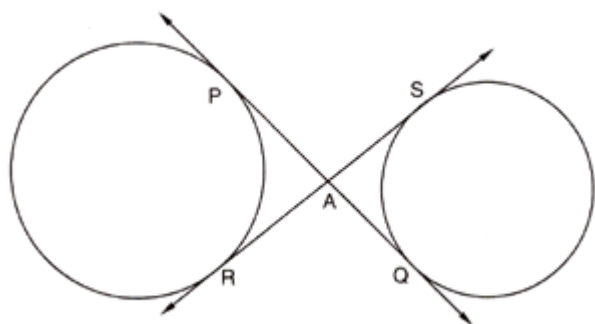
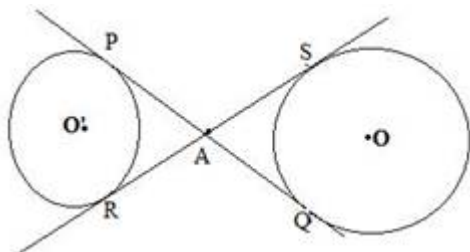


Fig. 10.65

Answer



Given: PQ and RS are the two common tangent to the two circle

To Proof: A is the point of intersection of PQ and RS

We know that , length of two tangent drawn from an exterior point to a circle are equal.

Therefore

$$PA = RA \text{----- (i)}$$

$$QA = SA \text{ ----- (ii)}$$

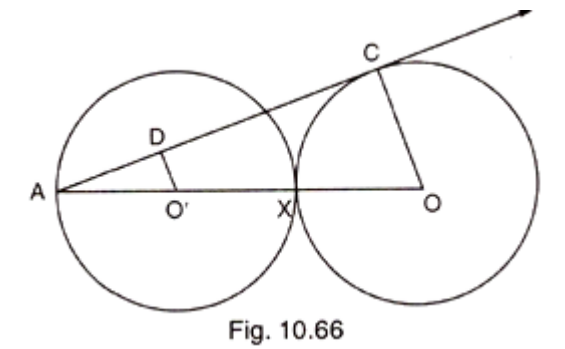
Adding two equations we get

$$PA + QA = RA + SA$$

$$PQ = RS \text{ (proved)}$$

29. Question

Equal circles with centres O and O' touch each other at X. OO' produced to meet a circle with centre O', at A. AC is tangent to the circle whose centre is O. O'D is perpendicular to AC. Find the value of $\frac{DO'}{CO}$.



Answer

We know that $\angle ADO' = 90^\circ$ (since O'D is perpendicular to AC)

As we know radius is perpendicular to the tangent.

So, $OC \perp AC$

$$\Rightarrow \angle ACO = 90^\circ$$

In $\triangle ADO'$ and $\triangle ACO$,

$$\angle ADO' = \angle ACO \text{ (each } 90^\circ)$$

$$\angle DAO = \angle CAO \text{ (common)}$$

By AA criteria,

$$\triangle ADO' \sim \triangle ACO$$

As we know corresponding sides of a triangle are in ratio. $\frac{AO'}{AO} = \frac{DO'}{CO}$

$$AO = AO' + O'X + OX$$

As radii of two circles are equal.

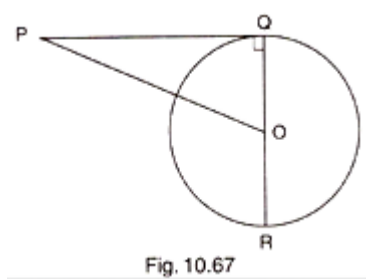
$$\Rightarrow AO = AO' + AO' + AO'$$

$$= 3 AO'$$

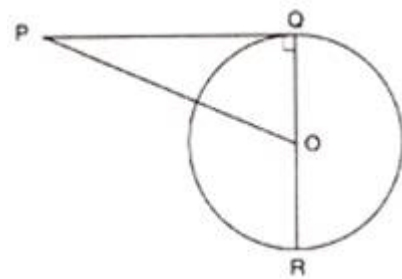
$$\frac{AO'}{AO} = \frac{AO'}{3AO} = \frac{1}{3} \quad \frac{DO'}{CO} = \frac{AO'}{3AO} = \frac{1}{3} \quad \Rightarrow \frac{DO'}{CO} = \frac{1}{3}$$

30. Question

In Fig. 10.67, $OQ:PQ=3:4$ and perimeter of $\triangle POQ = 60$ cm. Determine PQ, OR and OP.



Answer



Given that $OQ:PQ=3:4$

Let ratio coefficient $=x$, so

$OQ=3x$ and $PQ=4x$

We know that a tangent to a circle is perpendicular to the radius at the point of tangency

So

$$\angle OQP = 90^\circ$$

Then applying Pythagoras theorem in triangle POQ

$$OP^2 = OQ^2 + PQ^2$$

$$OP^2 = (3x)^2 + (4x)^2$$

$$OP^2 = 9x^2 + 16x^2$$

$$OP^2 = 25x^2$$

$$OP = 5x$$

Perimeter of a triangle POQ is $= 60$ cm, So

$$3x + 4x + 5x = 60$$

$$12x = 60$$

$$x = 5$$

So,

$$OQ = 3x = 15 \text{ cm}$$

$$PQ = 4x = 20 \text{ cm}$$

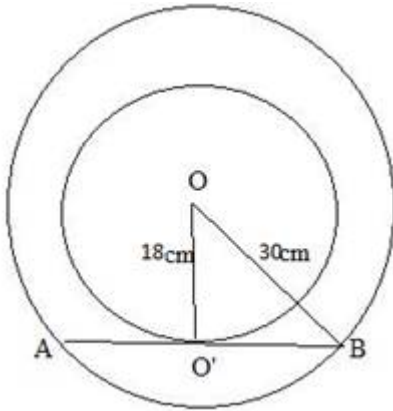
$$OP = 5x = 25 \text{ cm}$$

$$QR = 2(OQ) = 2 \times 15 = 30 \text{ cm}$$

31. Question

Two concentric circles are of diameters 30 cm and 18 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer



In the diagram AB is the chord touching the smaller circle. We have the right angled triangle OO'B

By Pythagoras theorem

$$O'B = \sqrt{(30)^2 + (18)^2} = 24 \text{ cm}$$

Now since the chord of the larger circle which touches the smaller circle is bisected at the point of contact

We have

$$AB = 2 \times 24 = 48 \text{ cm}$$

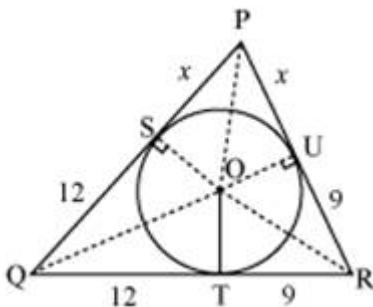
So ans is 18 cm.

32. Question

A triangle PQR is drawn to circumscribe a circle of radius 8 cm such that the segments QT and TR, into which QR is divided by the point of contact T, are of lengths 14 cm and 16 cm respectively. If area of $\triangle PQR$ is 336 cm^2 , find the sides PQ and PR.

Answer

Let PQ and PR touch the circle at points S and U respectively. Join O with P, Q, R, S and U



We have $OS = OT = OU = 6 \text{ cm}$

$$QT = 12 \text{ cm and } TR = 9 \text{ cm}$$

$$QR = QT + TR = 12 \text{ cm} + 9 \text{ cm} = 21 \text{ cm}$$

Now $QT = QS = 12 \text{ cm}$ (tangent from the same point)

$$TR = RU = 9 \text{ cm}$$

$$\text{Let } PS = PU = x \text{ cm}$$

$$\text{Then } PQ = PS + SQ = (12 + x) \text{ cm and } PR = PU + RU = (9 + x) \text{ cm}$$

It is clear that

$$\text{ar}(\triangle OQR) + \text{ar}(\triangle OPR) + \text{ar}(\triangle OPQ) = \text{ar}(\triangle PQR)$$

$$\frac{1}{2} \times QR \times OT + \frac{1}{2} \times PR \times OU + \frac{1}{2} \times PQ \times OS = 189 \text{ cm}^2$$

$$\frac{1}{2} \times (12 + x) \times 6 + \frac{1}{2} \times (9 + x) \times 6 + \frac{1}{2} \times 21 \times 6 = 189$$

$$\frac{1}{2} \times 6(12 + x + 9 + x + 21) = 189$$

$$3(42 + 2x) = 189$$

$$42 + 2x = 63$$

$$x = \frac{21}{2} = 10.5$$

$$\text{Thus } PQ = (12 + 10.5) \text{ cm} = 22.5 \text{ cm and } PR = (9 + 10.5) \text{ cm} = 19.5 \text{ cm}$$

33. Question

In Fig. 10.68, a $\triangle ABC$ is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC are of lengths 8 cm and 6 cm respectively. Find the lengths of sides AB and AC, when area of $\triangle ABC$ is 84 cm^2 .

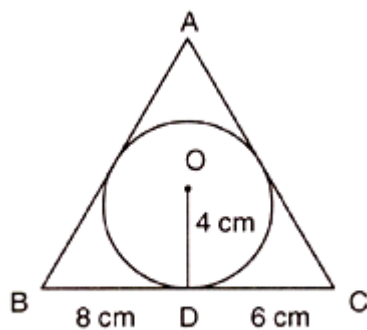
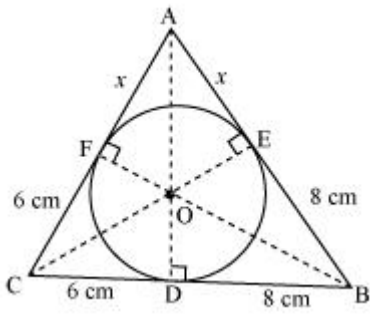


Fig. 10.68

Answer



Firstly consider that the given circle will touch the sides AB and AC of the triangle at a point E and F respectively.

Let $AF = x$

Now in triangle ABC

$CF = CD = 6\text{ cm}$

(Tangent drawn from an external point to a circle are equal. Here tangent is drawn from external point C)

$BE = BD = 8\text{ cm}$ (Tangent drawn from an external point to a circle are equal. Here tangent is drawn from external point B)

$AE = AF = x$

Now $AB = AE + EB = x + 8$

Also $BC = BD + DC = 8 + 6 = 14$ and $CA = CF + FA = 6 + x$

Now we get all side of the triangle and its area can be find by using heron's formula

$$\text{Semi-perimeter} = s = \frac{28+2x}{2} = 14+x$$

$$\begin{aligned}\text{Area of } \triangle ABC &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{(14+x)\{(14+x)-14\}\{(14+x)(6+x)\}\{(14+x)-(8+x)\}} \\ &= 4\sqrt{3(14x+x^2)}\end{aligned}$$

$$\text{Ar of } \triangle OBC = \frac{1}{2} \times 4 \times 14 = 28$$

$$\text{Ar of } \triangle OCA = \frac{1}{2} \times 4 \times (6+x) = 12+2x$$

$$\text{Ar of } \triangle OAB = \frac{1}{2} \times 4 \times (8+x) = 16+2x$$

$$\text{Area of } \triangle ABC = \text{Ar of } \triangle OBC + \text{Ar of } \triangle OCA + \text{Ar of } \triangle OAB$$

$$4\sqrt{3(14x+x^2)} = 28+12+2x+16+2x$$

$$\sqrt{3(14x+x^2)} = 14+x$$

Squaring both side and solving we get

$$x(x+14) - 7(x+14) = 0$$

$$\text{or } x = -14 \text{ and } 7$$

$$x = -14 \text{ is not possible}$$

$$\text{so } x = 7$$

$$\text{hence } AB = 7+8 = 15 \text{ cm}$$

$$CA = 6+7 = 13 \text{ cm}$$

34. Question

In Fig. 10.69, AB is a diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58^\circ$, find $\angle ATQ$.

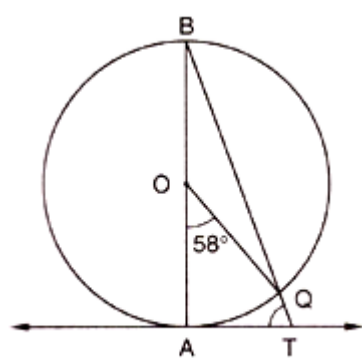


Fig. 10.69

Answer

$$\begin{aligned}\angle ABQ &= \frac{1}{2} \angle AOQ \\ &= \frac{1}{2} \times 58 = 29^\circ\end{aligned}$$

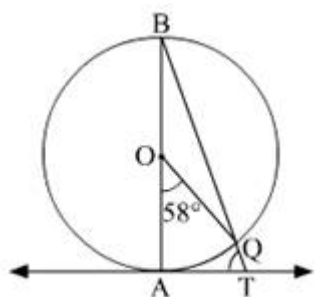
$$\angle A = 90^\circ$$

So

$$\angle BAT + \angle ABT + \angle ATQ = 180$$

$$\angle ATQ = 180 - 90 + 29 = 61^\circ$$

$$\angle ATQ = 61^\circ$$



35. Question

In Fig. 10.70, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30^\circ$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$.

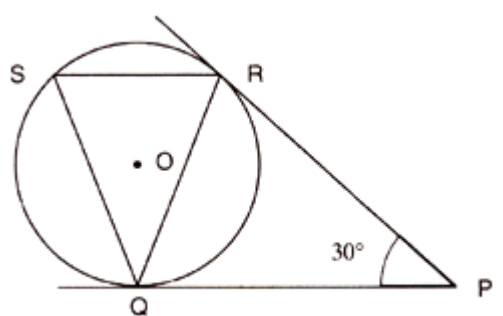


Fig. 10.70

Answer

As we know that the tangents drawn from an external point to a circle are equal.

Therefore, $PQ = PR$

Also, from the figure, PQR is an isosceles triangle, because $PQ = PR$

Therefore, $\angle RQP = \angle QRP$ (Because the corresponding angles of the equal sides of the isosceles triangle are equal) And, from the angle sum property of a triangle,

$$\angle RQP + \angle QRP + \angle RPQ = 180^\circ$$

$$\angle RQP + \angle RQP + \angle RPQ = 180^\circ$$

$$2\angle RQP + \angle RPQ = 180^\circ$$

$$2\angle RQP + 30^\circ = 180^\circ$$

$$2\angle RQP = 180^\circ - 30^\circ$$

$$2\angle RQP = 150^\circ$$

$$\angle RQP = 150^\circ/2$$

$$\text{Therefore, } \angle RQP = 75^\circ$$

SR || QP and QR is a transversal

$$\therefore \angle SRQ = \angle PQR \quad \dots[\text{Alternate interior angle}]$$

$$\therefore \angle SRQ = 75^\circ$$

$$\Rightarrow \angle ORP = 90^\circ \dots[\text{Tangent is Perpendicular to the radius through the point of contact}]$$

$$\angle ORP = \angle ORQ + \angle QRP$$

$$\Rightarrow 90^\circ = \angle ORQ + 75^\circ$$

$$\Rightarrow \angle ORQ = 15^\circ$$

$$\text{Similarly, } \angle RQO = 15^\circ$$

In ΔQOR ,

$$\angle QOR + \angle QRO + \angle OQR = 180^\circ$$

$$\Rightarrow \angle QOR + 15^\circ + 15^\circ = 180^\circ$$

$$\Rightarrow \angle QOR = 150^\circ$$

$$\Rightarrow \angle QSR = \angle QOR/2$$

$$\Rightarrow \angle QSR = 150^\circ/2 = 75^\circ$$

In ΔRSQ ,

$$\angle RSQ + \angle QRS + \angle RQS = 180^\circ$$

$$\Rightarrow 75^\circ + 75^\circ + \angle RQS = 180^\circ$$

$$\angle RQS = 30^\circ$$

CCE - Formative Assessment

1. Question

In Fig. 10.72, PA and PB are tangents to the circle drawn from an external point P. CD are a third tangent touching the circle at Q. If PB = 10 cm and CQ = 2 cm, what is the length PC?

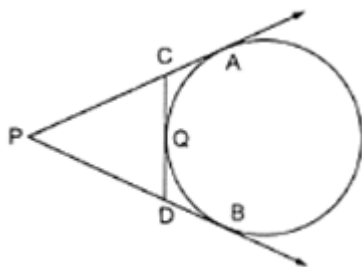


Fig. 10.72

Answer

Given:

$$PB = 10 \text{ cm}$$

$$CQ = 2 \text{ cm}$$

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Using the above property,

$$PA = PB = 10 \text{ cm (tangent from P)}$$

And,

$$CA = CQ = 10 \text{ cm (tangent from C)}$$

Now,

$$PC = PA - CA$$

$$= 10 \text{ cm} - 2 \text{ cm}$$

$$= 8 \text{ cm}$$

Hence, $PC = 8 \text{ cm}$

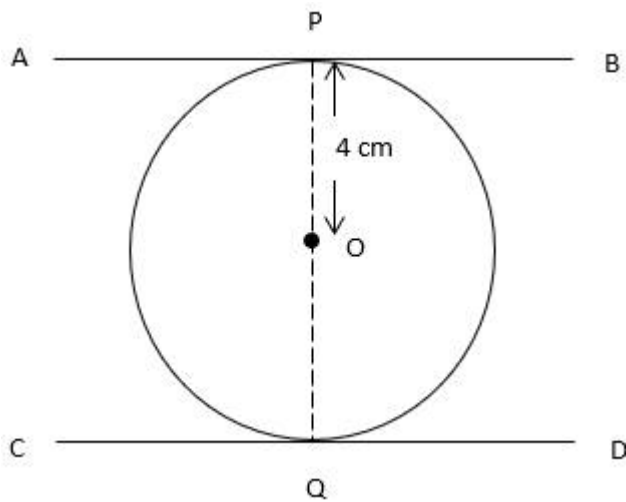
2. Question

What is the distance between two parallel tangents of a circle of radius 4 cm?

Answer

Given:

$$\text{Radius of circle (say PO)} = 4 \text{ cm}$$



Let $AB \parallel CD$ be two tangents which meet the circle at P and Q respectively. And, O be the center of circle.

Property: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, we can say that distance between two parallel tangents of a circle is equal to its diameter.

Therefore,

$$PQ = 2 \times PO$$

$$= 2 \times 4 \text{ cm}$$

$$= 8 \text{ cm}$$

Hence, Distance between tangents = 8 cm

3. Question

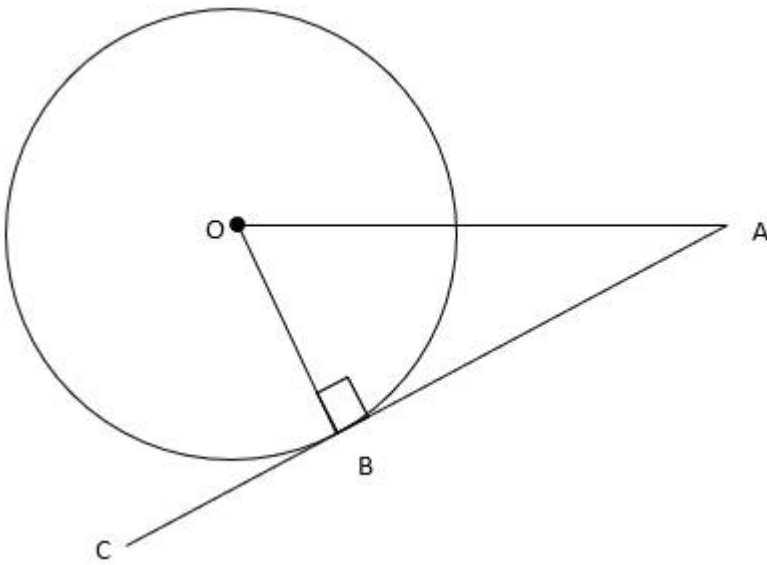
The length of tangent from a point A at a distance of 5 cm from the center of the circle is 4 cm. What is the radius of the circle?

Answer

Given:

$$OA \text{ (say)} = 5 \text{ cm}$$

$$AB = 4 \text{ cm}$$



Let AC be the tangent which meets the circle at the point B and O be the center of circle.

Property: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, $\triangle AOB$ is right-angled at $\angle OBA$.

Therefore, by Pythagoras Theorem,

$$AB^2 + OB^2 = AO^2$$

$$\Rightarrow OB^2 = AO^2 - AB^2$$

$$\Rightarrow OB = \sqrt{AO^2 - AB^2}$$

$$\Rightarrow OB = \sqrt{5^2 - 4^2}$$

$$\Rightarrow OB = \sqrt{25 - 16}$$

$$\Rightarrow OB = \sqrt{9}$$

$$\Rightarrow OB = 3 \text{ cm}$$

Hence, Radius = 3 cm

4. Question

Two tangents TP and TQ are drawn from an external point T to a circle with center O as shown in Fig. 10.73. If they are inclined to each other at an angle of 100° , then what is the value of $\angle POQ$?

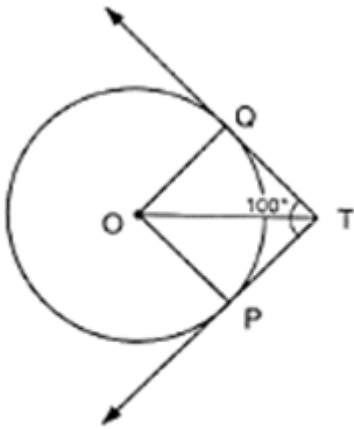


Fig. 10.73

Answer

Given:

$$\angle QTP = 100^\circ$$

Property 1: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: Sum of all angles of a quadrilateral = 360° .

By property 1,

$$\angle OPT = 90^\circ \text{ and } \angle OQT = 90^\circ$$

And,

By property 2,

$$\angle QTP + \angle OPT + \angle OQT + \angle POQ = 360^\circ$$

$$\Rightarrow \angle POQ = 360^\circ - \angle QTP + \angle OPT + \angle OQT$$

$$\Rightarrow \angle POQ = 360^\circ - 100^\circ + 90^\circ + 90^\circ$$

$$\Rightarrow \angle POQ = 360^\circ - 280^\circ$$

$$\Rightarrow \angle POQ = 80^\circ$$

Hence, $\angle POQ = 80^\circ$

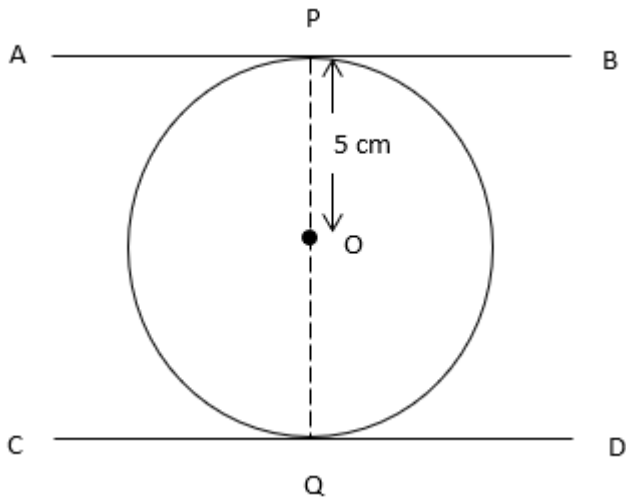
5. Question

What the distance between two parallel tangents to a circle of radius 5 cm?

Answer

Given:

Radius of circle (say PO) = 5 cm



Let $AB \parallel CD$ be two tangents which meet the circle at P and Q respectively. And, O be the center of circle.

Property: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, we can say that distance between two parallel tangents of a circle is equal to its diameter.

Therefore,

$$PQ = 2 \times PO$$

$$= 2 \times 5 \text{ cm}$$

$$= 10 \text{ cm}$$

Hence, Distance between tangents = 10 cm

6. Question

In Q. No. 1, if $PB = 10 \text{ cm}$, what is the perimeter of $\triangle PCD$?

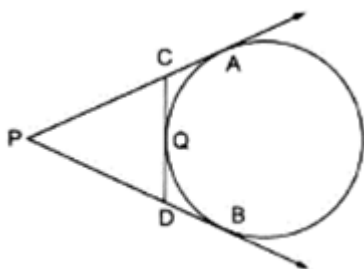


Fig. 10.72

Answer

Given:

$$PB = 10 \text{ cm}$$

$$CQ = 2 \text{ cm}$$

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Using the above property,

$$PA = PB = 10 \text{ cm (tangent from P)}$$

$$DB = DQ = 10 \text{ cm (tangent from D)}$$

And,

$$CA = CQ = 10 \text{ cm (tangent from C)}$$

Now,

$$\text{Perimeter of } \triangle PCD = PC + CD + DP$$

$$= PC + CQ + QD + DP$$

$$= PC + CA + DB + PD [\because CA = CQ \text{ and } DB = DQ]$$

$$= PA + PB [\because PA = PC + CA \text{ and } PB = PD + BD]$$

$$= 10 \text{ cm} + 10 \text{ cm}$$

$$= 20 \text{ cm}$$

$$\text{Hence, Perimeter of } \triangle PCD = 20 \text{ cm}$$

7. Question

In Fig. 10.74, CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm and BC = 7 cm, then find the length of BR.

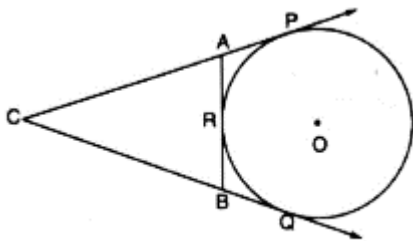


Fig. 10.74

Answer

Given:

$$CP = 11 \text{ cm}$$

$$BC = 7 \text{ cm}$$

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Using the above property,

$$CP = CQ = 11 \text{ cm (tangent from C)}$$

$$BQ = BR \text{ (tangent from B)}$$

And,

$$AP = AR \text{ (tangent from A)}$$

Now,

$$BR = BQ = CQ - CB$$

$$= 11 \text{ cm} - 7 \text{ cm}$$

$$= 4 \text{ cm}$$

Hence, $BR = 4 \text{ cm}$

8. Question

In Fig. 10.75, $\triangle ABC$ is circumscribing a circle. Find the length of BC .

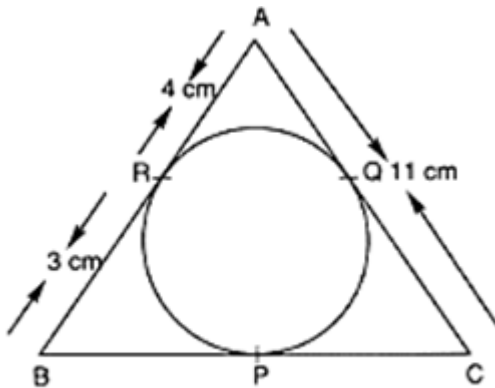


Fig. 10.75

Answer

Given:

$$AR = 4 \text{ cm}$$

$$BR = 3 \text{ cm}$$

$$AC = 11 \text{ cm}$$

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Using the above property,

$$AR = AQ = 4 \text{ cm (tangent from A)}$$

$$BR = BP \text{ (tangent from B)}$$

And,

$$CP = CQ \text{ (tangent from C)}$$

Also,

$$CQ = CA - AQ = 11 \text{ cm} - 4 \text{ cm} = 7 \text{ cm}$$

Now,

$$BC = BP + PC$$

$$= BR + CQ [\because BR = BP \text{ and } CP = CQ = 7 \text{ cm}]$$

$$= 3 \text{ cm} + 7 \text{ cm}$$

$$= 10 \text{ cm}$$

Hence, $BC = 10 \text{ cm}$

9. Question

In Fig. 10.76, CP and CQ are tangents from an external point C to a circle with centre O. AB is another tangent which touches the circle at R. If $CP = 11 \text{ cm}$ and $BR = 4 \text{ cm}$, find the length of BC.

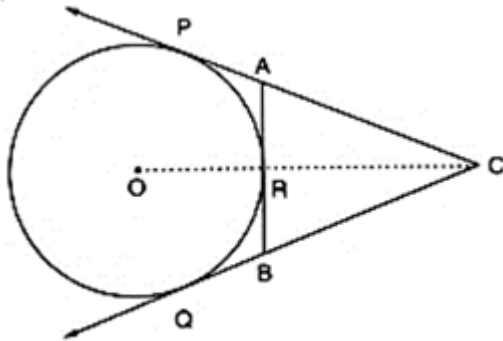


Fig. 10.76

[Hint: We have, $CP = 11 \text{ cm}$

$$\therefore CP = CQ = 11 \text{ cm}$$

Now, $BR = BQ$ [Tangents drawn from B]

$$\Rightarrow BQ = 4 \text{ cm}$$

$$\therefore BC = CQ - BQ = (11 - 4)\text{cm} = 7 \text{ cm}$$

Answer

Given:

$$BR = 4 \text{ cm}$$

$$CP = 11 \text{ cm}$$

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Using the above property,

$$BR = BQ = 4 \text{ cm (tangent from B)}$$

And,

$$CP = CQ = 11 \text{ cm (tangent from C)}$$

Now,

$$BC = CQ - BQ$$

$$= 11 \text{ cm} - 4 \text{ cm}$$

$$= 7 \text{ cm}$$

Hence, $BC = 7 \text{ cm}$

10. Question

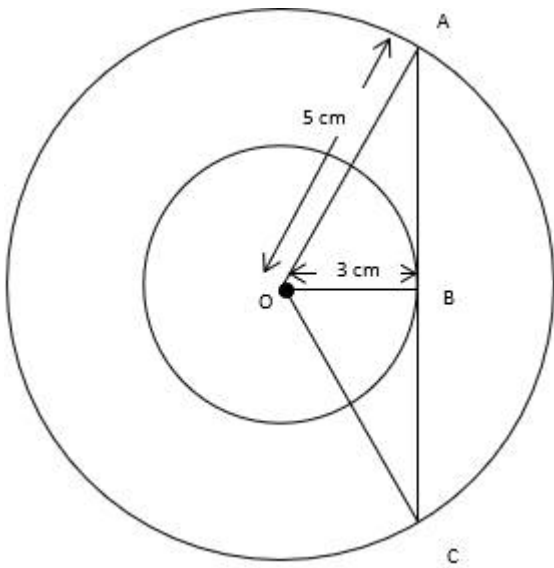
Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer

Given:

$$AO \text{ (say)} = CO \text{ (say)} = 5 \text{ cm}$$

$$BO \text{ (say)} = 3 \text{ cm}$$



Let AC be the tangent which meets the circle at the point B and O be the center of circle.

Property: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔAOB is right-angled at $\angle OBA$ and ΔCOB is right-angled at $\angle OBC$.

Therefore,

By Pythagoras Theorem in ΔAOB ,

$$AB^2 + OB^2 = AO^2$$

$$\Rightarrow AB^2 = AO^2 - OB^2$$

$$\Rightarrow AB = \sqrt{AO^2 - OB^2}$$

$$\Rightarrow AB = \sqrt{5^2 - 3^2}$$

$$\Rightarrow AB = \sqrt{25 - 9}$$

$$\Rightarrow AB = \sqrt{16}$$

$$\Rightarrow AB = 4 \text{ cm}$$

Similarly,

By Pythagoras Theorem in $\triangle COB$,

$$AB^2 + OB^2 = CO^2$$

$$\Rightarrow CB^2 = CO^2 - OB^2$$

$$\Rightarrow CB = \sqrt{CO^2 - OB^2}$$

$$\Rightarrow CB = \sqrt{5^2 - 3^2}$$

$$\Rightarrow CB = \sqrt{25 - 9}$$

$$\Rightarrow CB = \sqrt{16}$$

$$\Rightarrow CB = 4 \text{ cm}$$

Now,

$$AC = AB + BC$$

$$= 4 \text{ cm} + 4 \text{ cm}$$

$$= 8 \text{ cm}$$

Hence, Length of chord = 8 cm

11. Question

In Fig. 10.77, PA and PB are tangents to the circle with centre O such that $\angle APB = 50^\circ$. Write the measure of $\angle OAB$

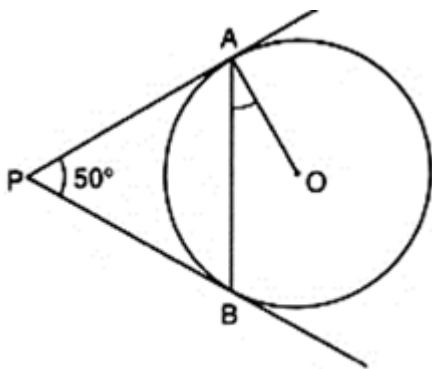


Fig. 10.77

Answer

Given:

$$\angle APB = 50^\circ$$

Property 1: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Property 2: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 3: Sum of all angles of a triangle = 180° .

By property 1,

$$AP = BP \text{ (tangent from P)}$$

$$\text{Therefore, } \angle PAB = \angle PBA$$

Now,

By property 3 in $\triangle PAB$,

$$\angle PAB + \angle PBA + \angle APB = 180^\circ$$

$$\Rightarrow \angle PAB + \angle PBA = 180^\circ - \angle APB$$

$$\Rightarrow \angle PAB + \angle PBA = 180^\circ - 50^\circ$$

$$\Rightarrow \angle PAB + \angle PBA = 130^\circ$$

$$\Rightarrow \angle PAB = \angle PBA = \frac{130^\circ}{2} = 65^\circ$$

By property 2,

$$\angle PAO = 90^\circ$$

Now,

$$\angle PAO = \angle PAB + \angle OAB$$

$$\Rightarrow \angle OAB = \angle PAO - \angle PAB$$

$$\Rightarrow \angle OAB = 90^\circ - 65^\circ = 25^\circ$$

$$\text{Hence, } \angle OAB = 25^\circ$$

1. Question

A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q such that OQ = 12 cm. Length PQ is cm

A. 12 cm

B. 13 cm

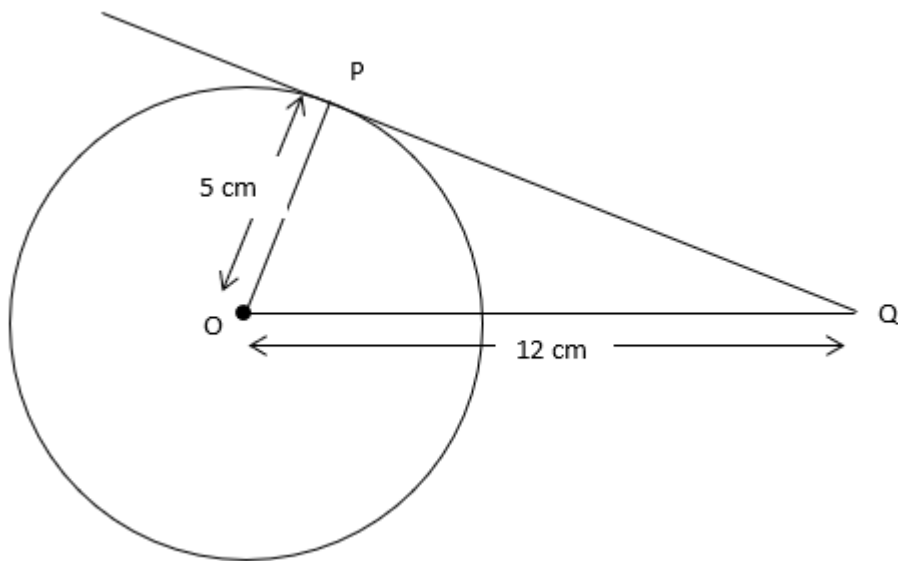
C. 8.5 cm

D. $\sqrt{119}$ cm

Answer

Given:

$$OQ = 12 \text{ cm}$$



Property: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔPOQ is right-angled at $\angle OPQ$.

Therefore,

By Pythagoras Theorem in ΔPOQ ,

$$OP^2 + PQ^2 = OQ^2$$

$$\Rightarrow PQ^2 = OQ^2 - OP^2$$

$$\Rightarrow PQ = \sqrt{(OQ^2 - OP^2)}$$

$$\Rightarrow PQ = \sqrt{(12^2 - 5^2)}$$

$$\Rightarrow PQ = \sqrt{(144 - 25)}$$

$$\Rightarrow PQ = \sqrt{119} \text{ cm}$$

$$\text{Hence, } PQ = \sqrt{119} \text{ cm}$$

2. Question

From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

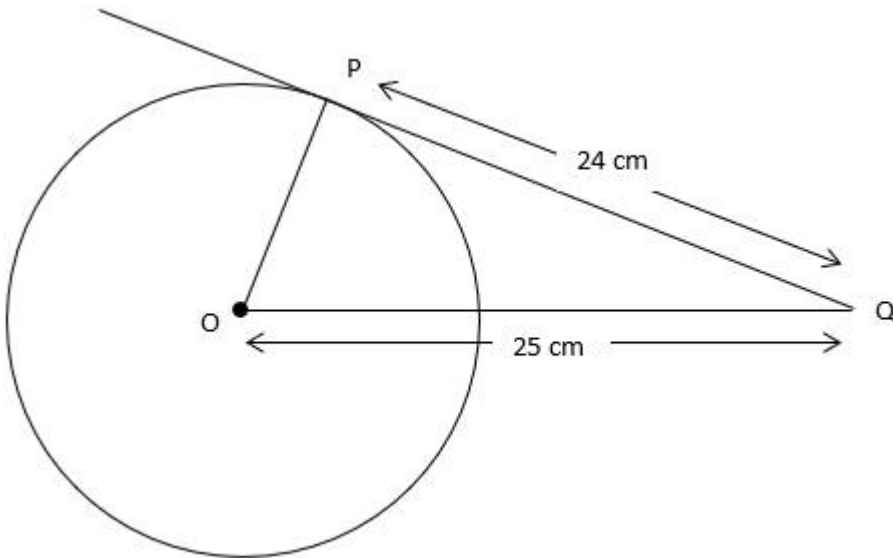
- A. 7 cm
- B. 12 cm
- C. 15 cm
- D. 24.5 cm

Answer

Given:

$$OQ = 25 \text{ cm}$$

$$PQ = 24 \text{ cm}$$



Property: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔPOQ is right-angled at $\angle OPQ$.

Therefore,

By Pythagoras Theorem in ΔPOQ ,

$$OP^2 + PQ^2 = OQ^2$$

$$\Rightarrow OP^2 = OQ^2 - PQ^2$$

$$\Rightarrow OP = \sqrt{OQ^2 - PQ^2}$$

$$\Rightarrow OP = \sqrt{25^2 - 24^2}$$

$$\Rightarrow OP = \sqrt{625 - 576}$$

$$\Rightarrow OP = \sqrt{49} \text{ cm}$$

$$\Rightarrow OP = 7 \text{ cm}$$

Hence, $OP = 7 \text{ cm}$

3. Question

The length of the tangent from a point A at a circle, of radius 3 cm, is 4 cm. The distance of A from the centre of the circle is

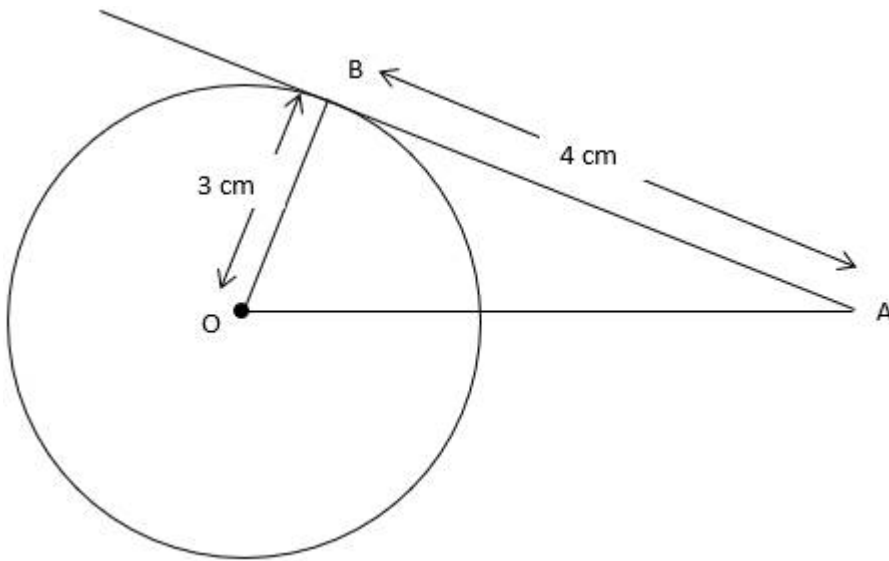
- A. $\sqrt{7} \text{ cm}$
- B. 7 cm
- C. 5 cm
- D. 25 cm

Answer

Given:

AB (say) = 4 cm

Radius (OB) = 3 cm



Property: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, $\triangle AOB$ is right-angled at $\angle ABO$.

Therefore,

By Pythagoras Theorem in $\triangle POQ$,

$$OA^2 = OB^2 + BA^2$$

$$\Rightarrow OA = \sqrt{OB^2 + BA^2}$$

$$\Rightarrow OA = \sqrt{3^2 + 4^2}$$

$$\Rightarrow OA = \sqrt{9 + 16}$$

$$\Rightarrow OA = \sqrt{25} \text{ cm}$$

$$\Rightarrow OA = 5 \text{ cm}$$

Hence, distance of A from center = 5 cm

4. Question

If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80° then $\angle POA$ is equal to

A. 50°

B. 60°

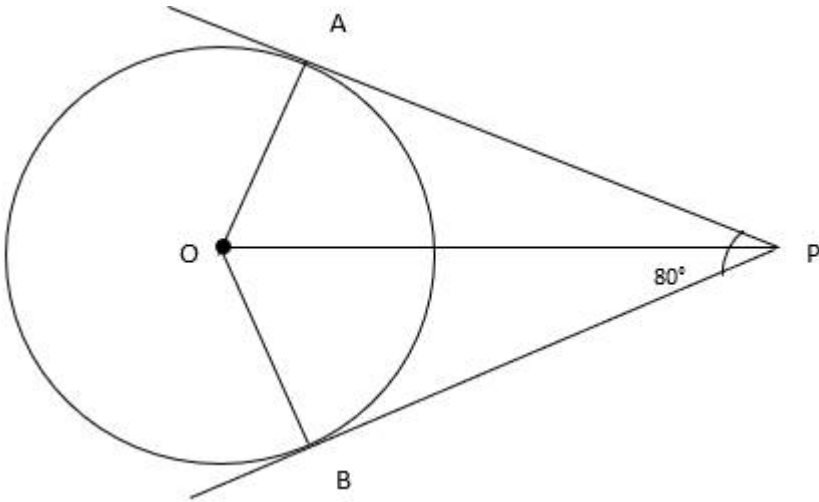
C. 70°

D. 80°

Answer

Given:

$$\angle APB = 80^\circ$$



Property 1: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: Sum of all angles of a quadrilateral = 360° .

Property 3: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By property 1,

$$\angle PAO = 90^\circ$$

$$\angle PBO = 90^\circ$$

By property 2,

$$\angle APB + \angle PAO + \angle PBO + \angle AOB = 360^\circ$$

$$\Rightarrow \angle AOB = 360^\circ - \angle APB + \angle PAO + \angle PBO$$

$$\Rightarrow \angle AOB = 360^\circ - (80^\circ + 90^\circ + 90^\circ)$$

$$\Rightarrow \angle AOB = 360^\circ - 260^\circ$$

$$\Rightarrow \angle AOB = 100^\circ$$

Now, in $\triangle POA$ and $\triangle POB$

$$OA = OB \quad [\because \text{radius of circle}]$$

$$PA = PB \quad [\text{By property 3 (tangent from P)}]$$

$$OP = OP \quad [\because \text{common}]$$

$\therefore$ By SSS congruency,

$$\triangle POA \cong \triangle POB$$

Hence, by CPCTC

$$\angle POA = \angle POB$$

Now,

$$\angle AOB = 100^\circ$$

$$\Rightarrow \angle POA + \angle POB = 100^\circ [\because \angle AOB = \angle POA + \angle POB]$$

$$\Rightarrow \angle POA + \angle POA = 100^\circ [\because \angle POA = \angle POB]$$

$$\Rightarrow 2\angle POA = 100^\circ$$

$$\Rightarrow \angle POA = \frac{100^\circ}{2}$$

$$\Rightarrow \angle POA = 50^\circ$$

Hence, $\angle POA = 50^\circ$

5. Question

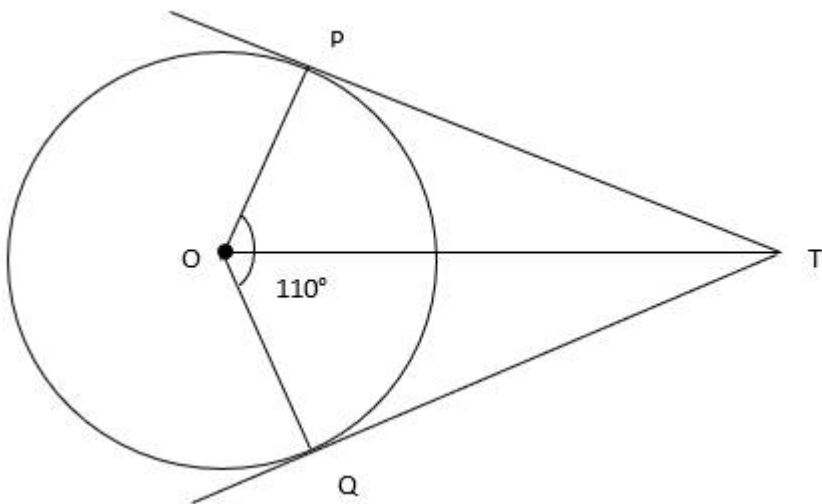
If TP and TQ are two tangents to a circle with centre O so that $\angle POQ = 110^\circ$, then, $\angle PTQ$ is equal to

- A. 60°
- B. 70°
- C. 80°
- D. 90°

Answer

Given:

$$\angle POQ = 110^\circ$$



Property 1: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: Sum of all angles of a quadrilateral = 360° .

By property 1,

$$\angle TPO = 90^\circ$$

$$\angle TQO = 90^\circ$$

By property 2,

$$\angle POQ + \angle TPO + \angle TQO + \angle PTQ = 360^\circ$$

$$\Rightarrow \angle PTQ = 360^\circ - \angle POQ + \angle TPO + \angle TQO$$

$$\Rightarrow \angle PTQ = 360^\circ - (110^\circ + 90^\circ + 90^\circ)$$

$$\Rightarrow \angle PTQ = 360^\circ - 290^\circ$$

$$\Rightarrow \angle PTQ = 70^\circ$$

Hence, $\angle PTQ = 70^\circ$

6. Question

PQ is a tangent to a circle with centre O at the point P. If ΔOPQ is an isosceles triangle, then $\angle OQP$ is equal to

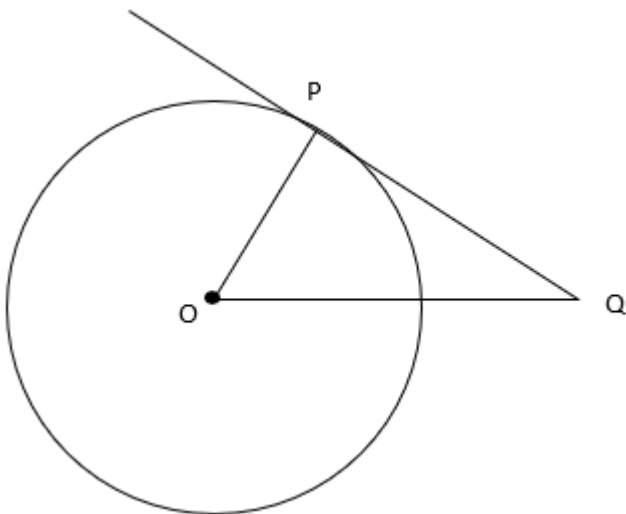
A. 30°

B. 45°

C. 60°

D. 90°

Answer



Property 1: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: Sum of all angles of a triangle = 180° .

By property 1, ΔPOQ is right-angled at $\angle OPQ$ (i.e., $\angle OPQ = 90^\circ$).

$\therefore \Delta POQ$ is an isosceles triangle

$$\therefore \angle POQ = \angle OQP$$

By property 2,

$$\angle POQ + \angle OQP + \angle QPO = 180^\circ$$

$$\Rightarrow \angle POQ + \angle OQP = 180^\circ - \angle QPO$$

$$\Rightarrow \angle POQ + \angle OQP = 180^\circ - 90^\circ$$

$$\Rightarrow \angle POQ + \angle OQP = 180^\circ - 90^\circ$$

$$\Rightarrow \angle POQ + \angle OQP = 90^\circ$$

$$\Rightarrow \angle OQP + \angle OQP = 90^\circ [\because \angle POQ = \angle OQP]$$

$$\Rightarrow 2\angle OQP = 90^\circ$$

$$\Rightarrow \angle OQP = \frac{90^\circ}{2} \Rightarrow \angle OQP = 45^\circ$$

Hence, $\angle OQP = 45^\circ$

7. Question

Two equal circles touch each other externally at C and AB is a common tangent to the circles. Then, $\angle ACB =$

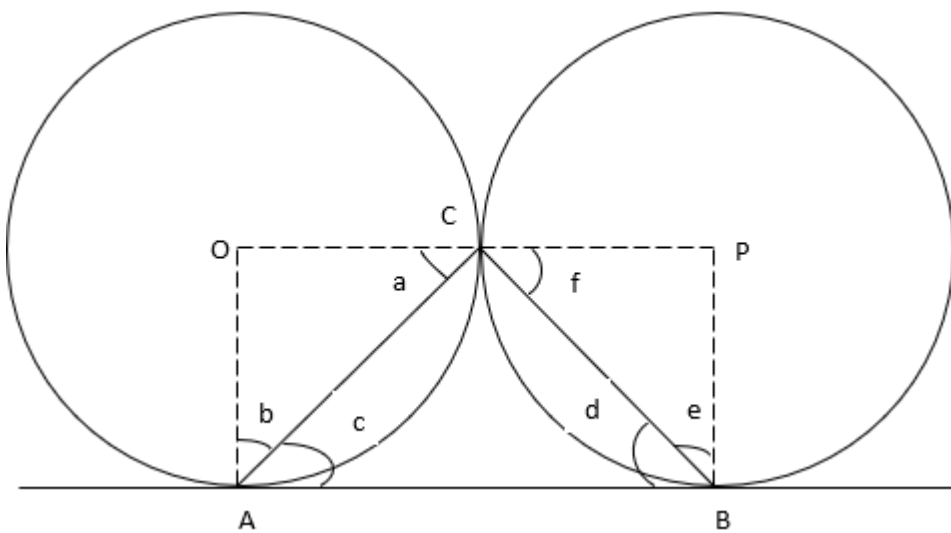
A. 60°

B. 45°

C. 30°

D. 90°

Answer



Property 1: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: Sum of all angles of a straight line = 180° .

Property 3: Sum of all angles of a triangle = 180° .

By property 1, ΔOAB is right-angled at $\angle OAB$ (i.e., $\angle OAB = 90^\circ$) and ΔPBA is right-angled at $\angle PBA$ (i.e., $\angle PBA = 90^\circ$)

Clearly,

$$\angle b + \angle c = \angle OAB$$

$$\Rightarrow \angle b + \angle c = 90^\circ$$

$$\Rightarrow \angle b = 90^\circ - \angle c$$

Similarly,

$$\angle d + \angle e = \angle PBA$$

$$\Rightarrow \angle d + \angle e = 90^\circ$$

$$\Rightarrow \angle e = 90^\circ - \angle d$$

Now,

$$\angle a = \angle b = 90^\circ - \angle c \quad [\because OA = OC \text{ (Radius)}]$$

And,

$$\angle e = \angle f = 90^\circ - \angle d \quad [\because PB = PC \text{ (Radius)}]$$

By property 2,

$$\angle a + \angle f + \angle ACB = 180^\circ$$

$$\Rightarrow \angle ACB = 180^\circ - \angle a - \angle f$$

$$\Rightarrow \angle ACB = 180^\circ - (90^\circ - \angle c) - (90^\circ - \angle d)$$

$$\Rightarrow \angle ACB = 180^\circ - 90^\circ + \angle c - 90^\circ + \angle d$$

$$\Rightarrow \angle ACB = \angle c + \angle d$$

Now, in ΔACB

By property 3,

$$\angle ACB + \angle c + \angle d = 180^\circ$$

$$\Rightarrow \angle ACB + \angle ACB = 180^\circ \quad [\because \angle ACB = \angle c + \angle d]$$

$$\Rightarrow 2\angle ACB = 180^\circ$$

$$\Rightarrow \angle ACB = \frac{180^\circ}{2}$$

$$\Rightarrow \angle ACB = 90^\circ$$

Hence, $\angle ACB = 90^\circ$

8. Question

ABC is a right angled triangle, right angled at B such that $BC = 6$ cm and $AB = 8$ cm. A circle with centre O is inscribed in $\triangle ABC$. The radius of the circle is

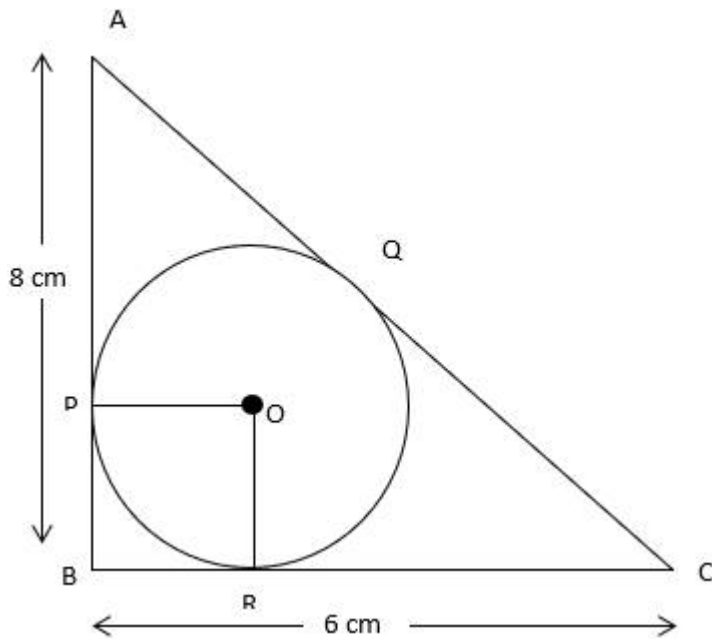
- A. 1 cm
- B. 2 cm
- C. 3 cm
- D. 4 cm

Answer

Given:

$BC = 6$ cm

$AB = 8$ cm



Property 1: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Property 2: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 3: Sum of all angles of a quadrilateral = 360° .

By property 1,

$AP = AQ$ (Tangent from A)

$BP = BR$ (Tangent from B)

$CR = CQ$ (Tangent from C)

$\therefore$ ABC is a right-angled triangle, $\therefore$ by Pythagoras Theorem

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = 8^2 + 6^2$$

$$\Rightarrow AC^2 = 64 + 36$$

$$\Rightarrow AC^2 = 100$$

$$\Rightarrow AC = \sqrt{100}$$

$$\Rightarrow AC = 10 \text{ cm}$$

Clearly,

$$AQ + QC = AC = 10 \text{ cm}$$

$$\Rightarrow AP + RC = 10 \text{ cm } [\because AQ = AP \text{ and } QC = RC]$$

Also,

$$AB + BC = 8 \text{ cm} + 6 \text{ cm} = 14 \text{ cm}$$

$$\Rightarrow AP + PB + BR + RC = 14 \text{ cm } [\because AB = AP + PB \text{ and } BC = BR + RC]$$

$$\Rightarrow AP + RC + PB + BR = 14 \text{ cm}$$

$$\Rightarrow 10 \text{ cm} + BR + BR = 14 \text{ cm } [\because AP + RC = 10 \text{ cm and } PB = BR]$$

$$\Rightarrow 10 \text{ cm} + 2BR = 14 \text{ cm}$$

$$\Rightarrow 2BR = 14 \text{ cm} - 10 \text{ cm} = 4 \text{ cm}$$

$$\Rightarrow BR = \frac{4}{2} \text{ cm}$$

$$\Rightarrow BR = 2 \text{ cm}$$

Now,

$$\angle BPO = 90^\circ \text{ [By property 3]}$$

$$\angle BRO = 90^\circ \text{ [By property 3]}$$

$$\angle PBM = 90^\circ \text{ [Given]}$$

Now by property 2,

$$\angle BPO + \angle BRO + \angle PBM + \angle ROP = 360^\circ$$

$$\Rightarrow \angle ROP = 360^\circ - \angle BPO + \angle BRO + \angle PBM$$

$$\Rightarrow \angle ROP = 360^\circ - (90^\circ + 90^\circ + 90^\circ)$$

$$\Rightarrow \angle ROP = 360^\circ - 270^\circ$$

$$\Rightarrow \angle ROP = 90^\circ$$

Now, $\because \angle ROP = 90^\circ$ and $BP = BR$ which are adjacent sides

$\therefore$ Quadrilateral PBRO is a square

$$\Rightarrow PO = BR = 2 \text{ cm}$$

Hence, Radius = 2 cm

9. Question

PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that $\angle POR = 120^\circ$, then $\angle OPQ$ is

A. 60°

B. 45°

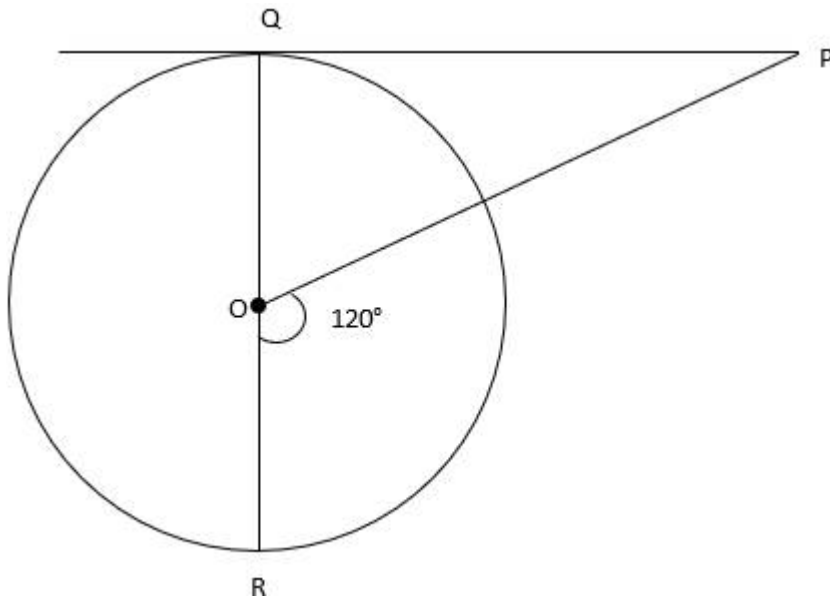
C. 30°

D. 90°

Answer

Given:

$$\angle POR = 120^\circ$$



Property 1: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: Sum of all angles of a straight line = 180° .

Property 3: Sum of all angles of a triangle = 180° .

By property 1,

$$\angle PQO = 90^\circ$$

By property 2,

$$\angle POQ + \angle POR = 180^\circ$$

$$\Rightarrow \angle POQ + 120^\circ = 180^\circ$$

$$\Rightarrow \angle POQ = 180^\circ - 120^\circ$$

$$\Rightarrow \angle POQ = 60^\circ$$

Now by property 3 in $\triangle OPQ$,

$$\angle POQ + \angle PQO + \angle OPQ = 180^\circ$$

$$\Rightarrow \angle OPQ = 180^\circ - \angle POQ + \angle PQO$$

$$\Rightarrow \angle OPQ = 180^\circ - (60^\circ + 90^\circ)$$

$$\Rightarrow \angle OPQ = 180^\circ - 150^\circ$$

$$\Rightarrow \angle OPQ = 30^\circ$$

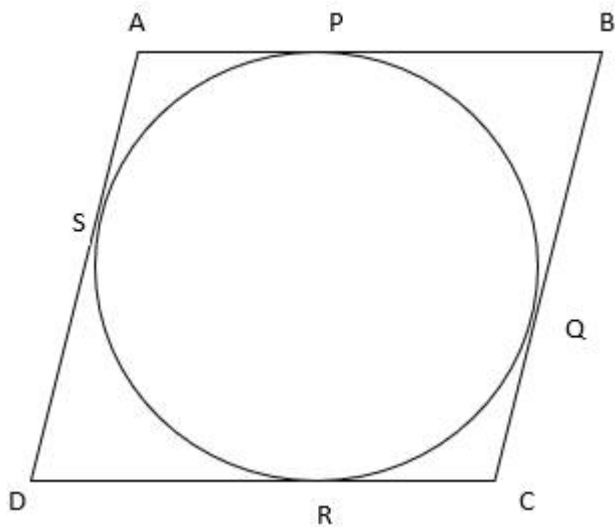
Hence, $\angle OPQ = 30^\circ$

10. Question

If four sides of a quadrilateral ABCD are tangential to a circle, then

- A. $AC + AD = BD + CD$
- B. $AB + CD = BC + AD$
- C. $AB + CD = AC + BC$
- D. $AC + AD = BC + DB$

Answer



Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

$$AP = AS \text{ (tangent from A)}$$

$$BP = BQ \text{ (tangent from B)}$$

$$CR = CQ \text{ (tangent from C)}$$

$$DR = DS \text{ (tangent from D)}$$

Now we add above 4 equations,

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$\Rightarrow AB + CD = AD + BC$$

$$[\because AP + BP = AB$$

$$CR + DR = CD$$

$$AS + DS = AD$$

$$BQ + CQ = BC]$$

Hence, the right option is $AB + CD = AD + BC$

11. Question

The length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm is

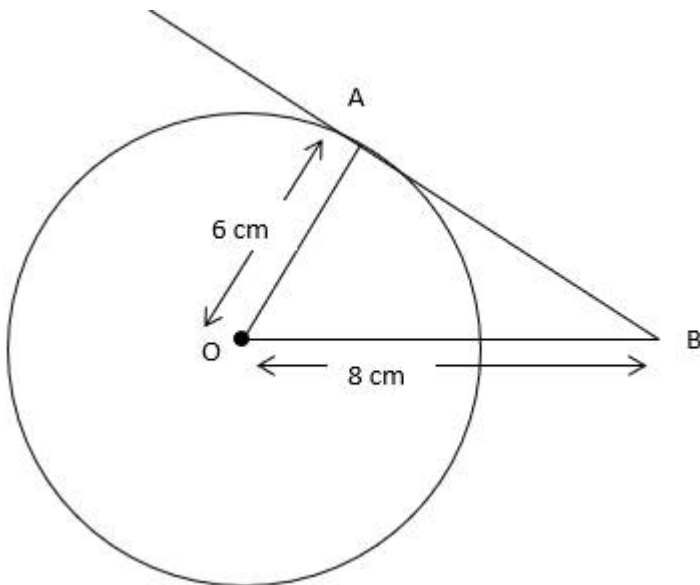
- A. $\sqrt{7}$ cm
- B. $2\sqrt{7}$ cm
- C. 10 cm
- D. 5 cm

Answer

Given:

$$OA = 6 \text{ cm}$$

$$OB = 8 \text{ cm}$$



Property: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, $\triangle AOB$ is right-angled at $\angle OAB$ (i.e., $\angle OAB = 90^\circ$).

Therefore by Pythagoras theorem,

$$OA^2 + AB^2 = OB^2$$

$$\Rightarrow AB^2 = OB^2 - OA^2$$

$$\Rightarrow AB^2 = 8^2 - 6^2$$

$$\Rightarrow AB^2 = 64 - 36$$

$$\Rightarrow AB^2 = 28$$

$$\Rightarrow AB = \sqrt{28}$$

$$\Rightarrow AB = 2\sqrt{7}$$

Hence, length of tangent is $2\sqrt{7}$ cm.

12. Question

AB and CD are two common tangents to circles which touch each other at C. If D lies on AB such that $CD = 4$ cm, then AB is equal to

- A. 4 cm
- B. 6 cm
- C. 8 cm
- D. 12 cm

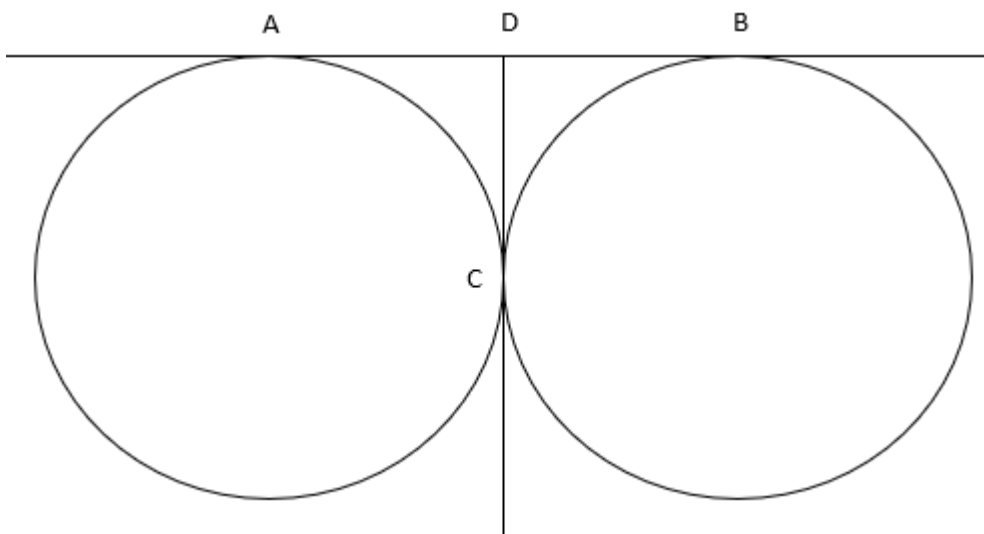
Answer

Given:

AB and CD are two common tangents to circles which touch each other at C. If D lies on AB such that $CD = 4$ cm

To find: length of AB

Solution:



Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Since c is the external point to the circles and two common tangents touch each other at c.

By the above property,

$$AD = BD = CD = 4 \text{ cm (tangent from D)}$$

Now clearly,

$$AB = AD + BD$$

$$\Rightarrow AB = AD + BD$$

$$\Rightarrow AB = 4 \text{ cm} + 4 \text{ cm}$$

$$\Rightarrow AB = 8 \text{ cm}$$

Hence, $AB = 8 \text{ cm}$

13. Question

In Fig. 10.78, if AD, AE and BC are tangents to the circle at D, E and F respectively. Then,

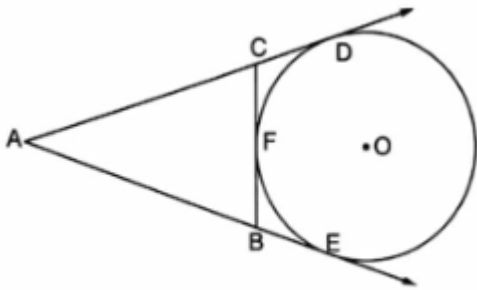


Fig. 10.78

- A. $AD = AB + BC + CA$
- B. $2AD = AB + BC + CA$
- C. $3AD = AB + BC + CA$
- D. $4AD = AB + BC + CA$

Answer

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

$$AE = AD \text{ (tangent from A)}$$

$$AB = AC \text{ (tangent from A)}$$

$$CD = CF \text{ (tangent from C)}$$

$$BF = BE \text{ (tangent from B)}$$

Now adding the above equations,

$$AB + BC + CA = AB + BF + FC + CA$$

$$\Rightarrow AB + BC + CA = AB + BE + CD + CA$$

$$\Rightarrow AB + BC + CA = AE + AD [\because AE = AB + BE \text{ and } AD = AC + CD]$$

$$\Rightarrow AB + BC + CA = AD + AD [\because AD = AE]$$

$$\Rightarrow AB + BC + CA = 2AD$$

$$\text{Hence, } 2AD = AB + BC + CA$$

14. Question

In Fig. 10.79, RQ is a tangent to the circle with centre O. If SQ = 6 cm and QR = 4 cm, then OR =

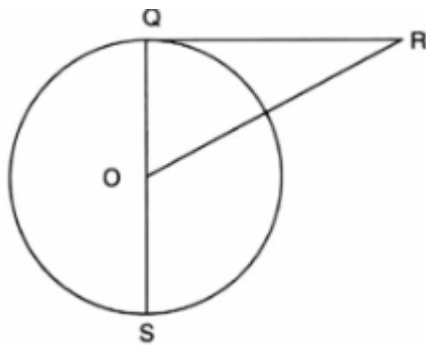


Fig. 10.79

A. 8 cm

B. 3 cm

C. 2.5 cm

D. 5 cm

Answer

Given:

$$SQ = 6 \text{ cm}$$

$$QR = 4 \text{ cm}$$

Property: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, $\triangle ROQ$ is right-angled at $\angle OQR$ (i.e., $\angle OQR = 90^\circ$).

$$\text{Diameter } QS = 6 \text{ cm}$$

$$\text{Radius} = \frac{\text{Diameter}}{2}$$

$$\Rightarrow \text{Radius} = \frac{6 \text{ cm}}{2}$$

$$\Rightarrow \text{Radius (OQ)} = 3 \text{ cm}$$

Now by Pythagoras theorem,

$$OR^2 = OQ^2 + QR^2$$

$$\Rightarrow OR^2 = 3^2 + 4^2$$

$$\Rightarrow OR^2 = 9 + 16$$

$$\Rightarrow OR^2 = 25$$

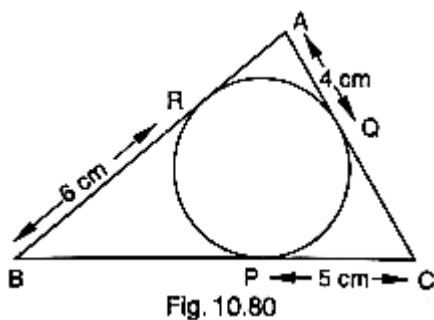
$$\Rightarrow OR = \sqrt{25}$$

$$\Rightarrow OR = 5 \text{ cm}$$

Hence, $OR = 5 \text{ cm}$.

15. Question

In Fig. 10.80, the perimeter of $\triangle ABC$ is



A. 30 cm

B. 60 cm

C. 45 cm

D. 15 cm

Answer

Given:

$$AQ = 4 \text{ cm}$$

$$BR = 6 \text{ cm}$$

$$PC = 5 \text{ cm}$$

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

$$AR = AQ = 4 \text{ cm (tangent from A)}$$

$$BR = BP = 6 \text{ cm (tangent from B)}$$

$$CP = CQ = 5 \text{ cm (tangent from C)}$$

Now,

$$\text{Perimeter of } \triangle ABC = AB + BC + CA$$

$$\Rightarrow \text{Perimeter of } \triangle ABC = AR + RB + BP + PC + CQ + QA$$

$$[\because AB = AR + RB]$$

$$BC = BP + PC$$

$$CA = CQ + QA]$$

$$\Rightarrow \text{Perimeter of } \triangle ABC = 4 \text{ cm} + 6 \text{ cm} + 6 \text{ cm} + 5 \text{ cm} + 5 \text{ cm} + 4 \text{ cm}$$

$$\Rightarrow \text{Perimeter of } \triangle ABC = 30 \text{ cm}$$

Hence, Perimeter of $\triangle ABC = 30 \text{ cm}$

16. Question

In Fig. 10.81, AP is a tangent to the circle with centre O such that $OP = 4 \text{ cm}$ and $\angle OPA = 30^\circ$. Then, $AP =$

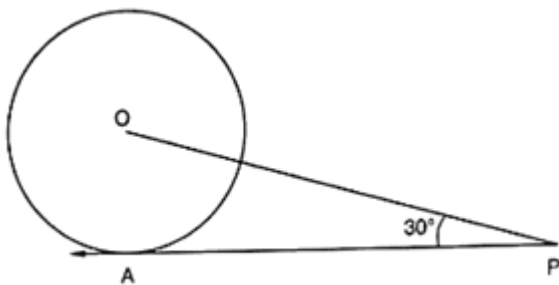


Fig. 10.81

A. $2\sqrt{2} \text{ cm}$

B. 2 cm

C. $2\sqrt{3} \text{ cm}$

D. $3\sqrt{2} \text{ cm}$

Answer

Given:

$$OP = 4 \text{ cm}$$

$$\angle OPA = 30^\circ$$

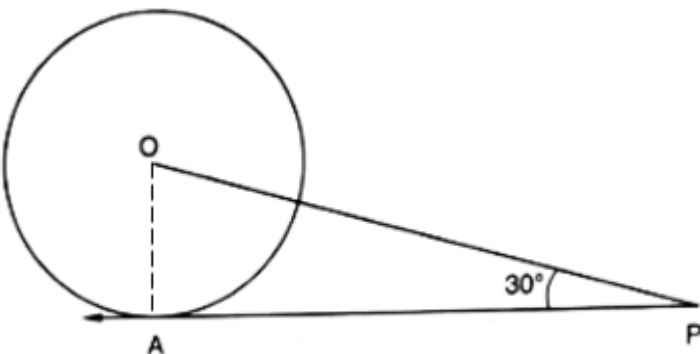


Fig. 10.81

Property: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔPOA is right-angled at $\angle OAP$ (i.e., $\angle OAP = 90^\circ$).

Now we know that,

$$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

Therefore,

$$\cos \angle P = \frac{AP}{OP}$$

$$\Rightarrow \cos 30^\circ = \frac{AP}{4}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AP}{4}$$

$$\Rightarrow AP = \frac{4 \times \sqrt{3}}{2}$$

$$\Rightarrow AP = 2\sqrt{3} \text{ cm}$$

Hence, $AP = 2\sqrt{3} \text{ cm}$

17. Question

AP and PQ are tangents drawn from a point A to a circle with centre O and radius 9 cm. If $OA = 15$ cm, then $AP + AQ =$

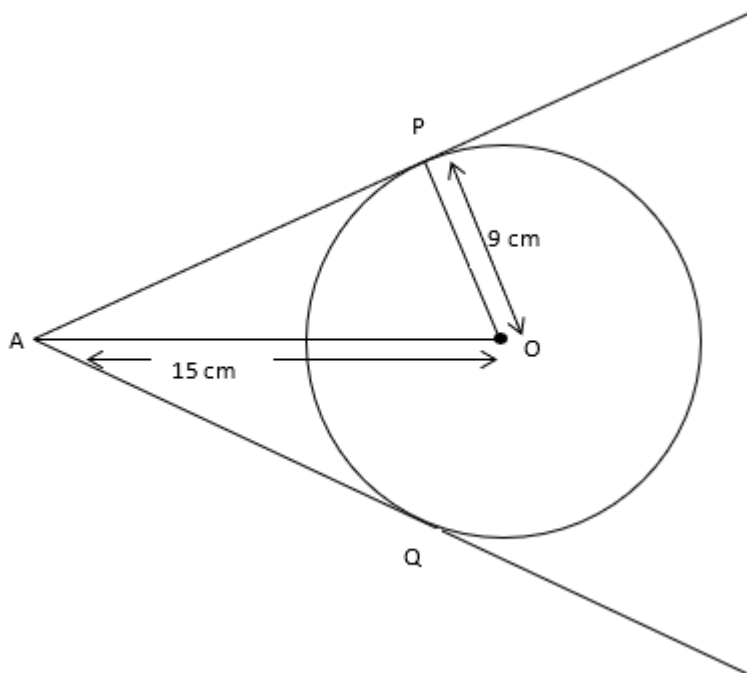
- A. 12 cm
- B. 18 cm
- C. 24cm
- D. 36 cm

Answer

Given:

Radius = 9 cm

$OA = 15 \text{ cm}$



Property 1: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

$$AP = AQ \text{ (tangent from A)}$$

Property 2: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔPOA is right-angled at $\angle OPA$ (i.e., $\angle OPA = 90^\circ$).

Therefore by Pythagoras theorem,

$$AP^2 + PO^2 = AO^2$$

$$\Rightarrow AP^2 = AO^2 - PO^2$$

$$\Rightarrow AP^2 = 15^2 - 9^2$$

$$\Rightarrow AP^2 = 225 - 81$$

$$\Rightarrow AP^2 = 144$$

$$\Rightarrow AP = \sqrt{144}$$

$$\Rightarrow AP = 12$$

$$AP + AQ = 12 \text{ cm} + 12 \text{ cm} = 24 \text{ cm}$$

Hence, $AP + AQ = 24 \text{ cm}$

18. Question

At one end of a diameter PQ of a circle of radius 5 cm, tangent XPY is drawn to the circle. The length of chord AB parallel to XY and at a distance of 8 cm from P is

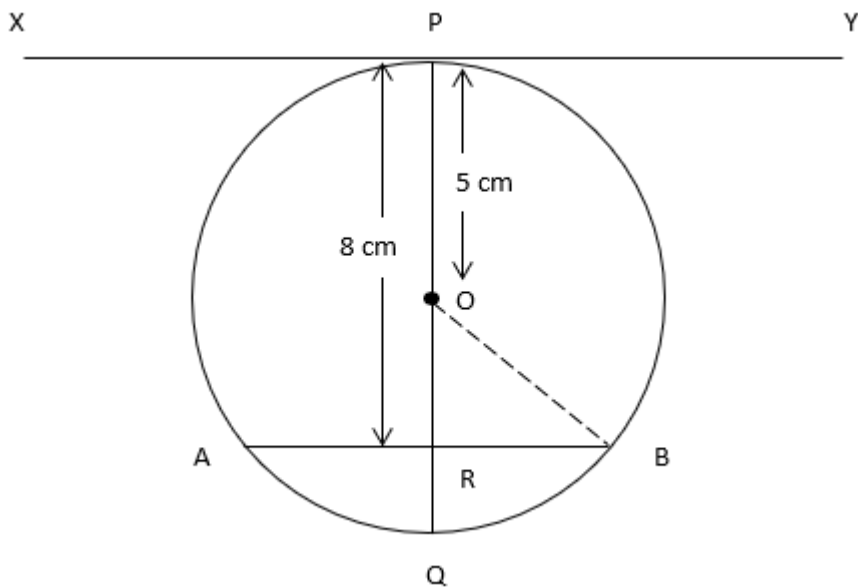
- A. 5 cm
- B. 6 cm
- C. 7 cm
- D. 8 cm

Answer

Given:

Radius = $OP = 5$ cm

Distance of AB and XY = 8 cm



$\therefore$ Distance of AB and XY = 8 cm

And AB is parallel to XY

$\therefore PR = 8$ cm

Join OB

Now,

$OB = OP = 5$ cm [radius]

Also,

$OR = PR - PO$

$\Rightarrow OR = 8 \text{ cm} - 5 \text{ cm}$

$\Rightarrow OR = 3 \text{ cm}$

$\therefore$ By Pythagoras theorem in $\triangle ORB$,

$$OB^2 = OR^2 + RB^2$$

$$\Rightarrow 5^2 = 3^2 + RB^2$$

$$\Rightarrow RB^2 = 5^2 - 3^2$$

$$\Rightarrow RB^2 = 25 - 9$$

$$\Rightarrow RB^2 = 16$$

$$\Rightarrow RB = 4$$

Now,

$$AB = AR + RB$$

$$\Rightarrow AB = 2RB$$

$$\Rightarrow AB = 2 \times 4$$

$$\Rightarrow AB = 8 \text{ cm}$$

Hence, Length of chord = 8 cm

19. Question

If PT is tangent drawn from a point P to a circle touching it at T and O is the centre of the circle, then $\angle OPT + \angle POT =$

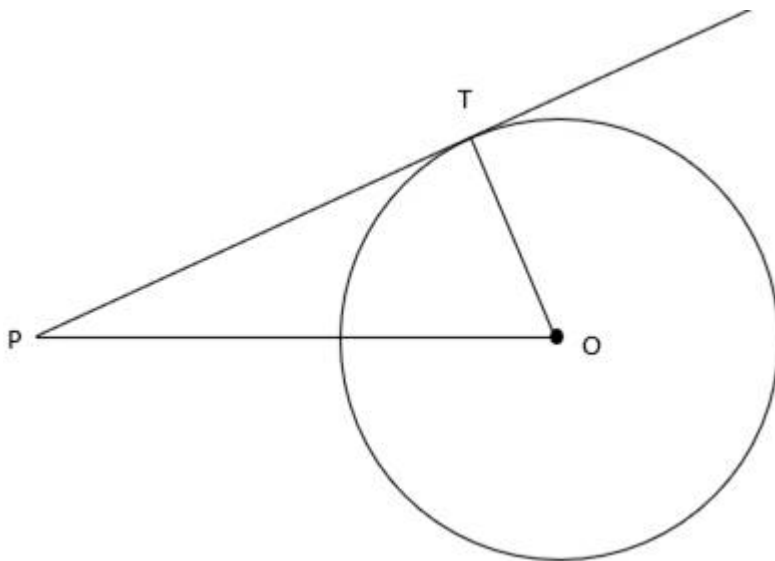
A. 30°

B. 60°

C. 90°

D. 180°

Answer



Property 1: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: Sum of all angles of a triangle = 180°

By property 1, $\triangle PTO$ is right-angled at $\angle OTP$ (i.e., $\angle OTP = 90^\circ$).

By property 2,

$$\angle OTP + \angle POT + \angle TPO = 180^\circ$$

$$\Rightarrow 90^\circ + \angle POT + \angle TPO = 180^\circ$$

$$\Rightarrow \angle POT + \angle TPO = 180^\circ - 90^\circ$$

$$\Rightarrow \angle POT + \angle TPO = 90^\circ$$

Hence, $\angle POT + \angle TPO = 90^\circ$

20. Question

In the adjacent figure, if $AB = 12$ cm, $BC = 8$ cm and $AC = 10$ cm, then $AD =$

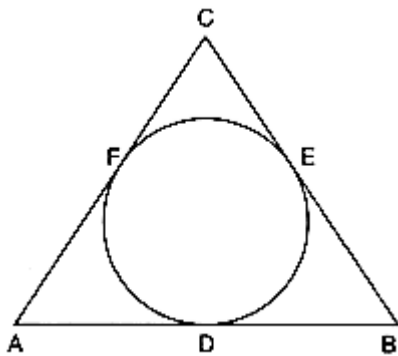


Fig. 10.82

A. 5 cm

B. 4 cm

C. 6 cm

D. 7 cm

Answer

Given:

$$AB = 12 \text{ cm}$$

$$BC = 8 \text{ cm}$$

$$AC = 10 \text{ cm}$$

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

$$AD = AF \text{ (tangent from A)}$$

$$BD = BE \text{ (tangent from B)}$$

$$CF = CE \text{ (tangent from C)}$$

Clearly,

$$AB = AD + DB = 12 \text{ cm}$$

$$BC = BE + EC = 8 \text{ cm}$$

$$AC = AF + FC = 10 \text{ cm}$$

Now,

$$AB - BC = 12 \text{ cm} - 8 \text{ cm}$$

$$\Rightarrow (AD + DB) - (BE + EC) = 12 \text{ cm} - 8 \text{ cm}$$

$$\Rightarrow AD + DB - BE - EC = 12 \text{ cm} - 8 \text{ cm}$$

$$\Rightarrow AD + BE - BE - CF = 12 \text{ cm} - 8 \text{ cm} [\because DB = BE \text{ and } CF = CE]$$

$$\Rightarrow AD - CF = 12 \text{ cm} - 8 \text{ cm}$$

$$\Rightarrow AD - (10 \text{ cm} - AF) = 12 \text{ cm} - 8 \text{ cm} [\because AF + FC = 10 \text{ cm} \Rightarrow FC = 10 \text{ cm} - AF]$$

$$\Rightarrow AD - (10 \text{ cm} - AF) = 4 \text{ cm}$$

$$\Rightarrow AD - 10 \text{ cm} + AF = 4 \text{ cm}$$

$$\Rightarrow AD + AD = 4 \text{ cm} + 10 \text{ cm} [\because AD = AF]$$

$$\Rightarrow 2AD = 14 \text{ cm}$$

$$\Rightarrow AD = \frac{14 \text{ cm}}{2}$$

$$\Rightarrow AD = 7 \text{ cm}$$

Hence, $AD = 7 \text{ cm}$

21. Question

In Fig. 10.83, if $AP = PB$, then

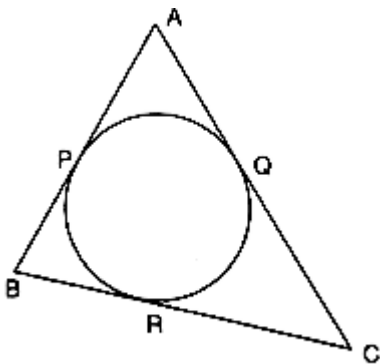


Fig. 10.83

A. $AC = AB$

B. $AC = BC$

C. $AQ = QC$

D. $AB = BC$

Answer

Given:

$$AP = PB$$

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

$$AP = AQ \text{ (tangent from A)}$$

$$BR = BP \text{ (tangent from B)}$$

$$CQ = CR \text{ (tangent from C)}$$

Clearly,

$$AP = BP = BR$$

$$AQ = AP = BR$$

Now,

$$AQ + QC = BR + RC$$

$$\Rightarrow AC = BC \text{ [}\because AC = AQ + QC \text{ and } BC = BR + RC\text{]}$$

Hence, $AC = BC$

22. Question

In Fig. 10.84, if $AP = 10$ cm, then $BP =$

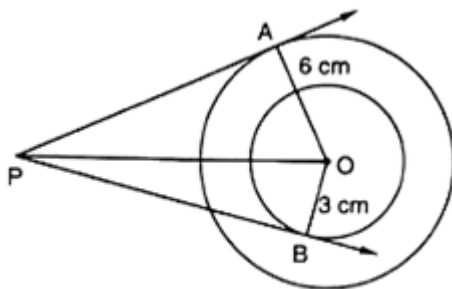


Fig. 10.84

A. $\sqrt{109}$ cm

B. $\sqrt{127}$ cm

C. $\sqrt{119}$ cm

D. $\sqrt{109}$ cm

Answer

Given:

$$AP = 10 \text{ cm}$$

$$OA = 6 \text{ cm}$$

$$OB = 3 \text{ cm}$$

Property : The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, $\triangle PAO$ is right-angled at $\angle PAO$ (i.e., $\angle PAO = 90^\circ$) and $\triangle PBO$ is right-angled at $\angle PBO$ (i.e., $\angle PBO = 90^\circ$).

Therefore by Pythagoras theorem in $\triangle PAO$,

$$OP^2 = OA^2 + AP^2$$

$$\Rightarrow OP^2 = 6^2 + 10^2$$

$$\Rightarrow OP^2 = 36 + 100$$

$$\Rightarrow OP = \sqrt{136}$$

Now by Pythagoras theorem in $\triangle PBO$,

$$OP^2 = OB^2 + BP^2$$

$$BP^2 = OP^2 - OB^2$$

$$\Rightarrow BP^2 = (\sqrt{136})^2 - 3^2$$

$$\Rightarrow BP^2 = 136 - 9$$

$$\Rightarrow BP = \sqrt{127}$$

Hence, $BP = \sqrt{127} \text{ cm}$

23. Question

In Fig. 10.85, if PR is tangent to the circle at P and Q is the centre of the circle, then $\angle POQ =$

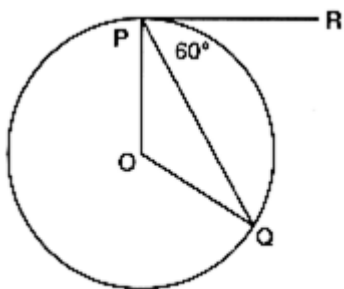


Fig. 10.85

A. 110°

B. 100°

C. 120°

D. 90°

Answer

Given:

$$\angle RPQ = 60^\circ$$

Property 1: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: Sum of all angles of a triangle = 180° .

By property 1, $\triangle OPR$ is right-angled at $\angle OPR$ (i.e., $\angle OPR = 90^\circ$).

$$OP = OQ [\because \text{radius of circle}]$$

$$\therefore \angle OPQ = \angle OQP = 30^\circ$$

Now by property 2,

$$\angle OPQ + \angle OQP + \angle POQ = 180^\circ$$

$$\Rightarrow 30^\circ + 30^\circ + \angle POQ = 180^\circ$$

$$\Rightarrow 60^\circ + \angle POQ = 180^\circ$$

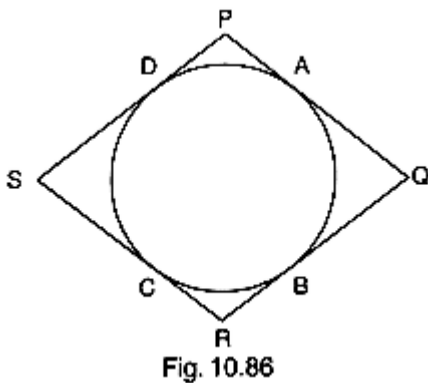
$$\Rightarrow \angle POQ = 180^\circ - 60^\circ$$

$$\Rightarrow \angle POQ = 120^\circ$$

$$\text{Hence, } \Rightarrow \angle POQ = 120^\circ$$

24. Question

In Fig. 10.86, if quadrilateral PQRS circumscribes a circle, then $PD + QB =$



- A. PQ
- B. QR
- C. PR
- D. PS

Answer

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

$$PD = PA \text{ (tangent from P)}$$

$$QB = QA \text{ (tangent from Q)}$$

$$RC = RB \text{ (tangent from R)}$$

$$SC = SD \text{ (tangent from S)}$$

Now,

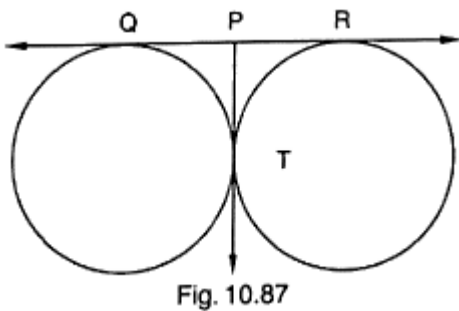
$$PD + QB = PA + QA$$

$$\Rightarrow PD + QB = PQ [\because PQ = PA + QA]$$

$$\text{Hence, } PD + QB = PQ$$

25. Question

In Fig. 10.87, two equal circles touch each other at T, if $QP = 4.5$ cm, then $QR =$



A. 9 cm

B. 18 cm

C. 15 cm

D. 13.5 cm

Answer

Given:

$$QP = 4.5 \text{ cm}$$

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

$$PQ = PT = PR = 4.5 \text{ cm (tangent from P)}$$

Now,

$$QR = PQ + PR$$

$$QR = PQ + PQ [\because PQ = PR]$$

$$QR = 2PQ$$

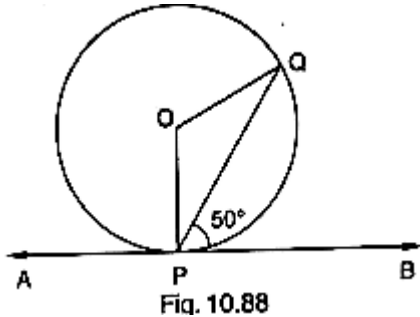
$$QR = 2 \times 4.5 \text{ cm}$$

$$QR = 9 \text{ cm}$$

Hence, $QR = 9 \text{ cm}$

26. Question

In Fig. 10.88, APB is a tangent to a circle with centre O at point P. If $\angle QPB = 50^\circ$, then the measure of $\angle POQ$ is



A. 100°

B. 120°

C. 140°

D. 150°

Answer

Given:

$$\angle QPB = 50^\circ$$

Property 1: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: Sum of all angles of a triangle = 180° .

By property 1, $\triangle OPB$ is right-angled at $\angle OPB$ (i.e., $\angle OPB = 90^\circ$).

$$\angle OPQ = \angle OPB - \angle QPB$$

$$\Rightarrow \angle OPQ = 90^\circ - 50^\circ = 40^\circ$$

And,

$$\angle OPQ = \angle OQP [\because OP = OQ \text{ (radius of circle)}]$$

Now by property 2,

$$\angle OPQ + \angle OQP + \angle POQ = 180^\circ$$

$$\Rightarrow 40^\circ + 40^\circ + \angle POQ = 180^\circ$$

$$\Rightarrow 80^\circ + \angle POQ = 180^\circ$$

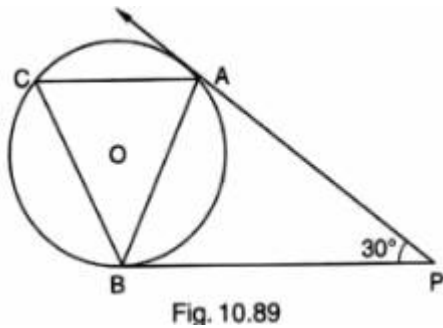
$$\Rightarrow \angle POQ = 180^\circ - 80^\circ$$

$$\Rightarrow \angle POQ = 100^\circ$$

Hence, $\Rightarrow \angle POQ = 100^\circ$

27. Question

In Fig. 10.89, if tangents PA and PB are drawn to a circle such that $\angle APS = 30^\circ$ and chord AC is drawn parallel to the tangent PB, then $\angle ABC =$



- A. 60°
- B. 90°
- C. 30°
- D. None of these

Answer

Given:

$$\angle APB = 30^\circ$$

Property 1: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Property 2: Sum of all angles of a triangle = 180°

By property 1,

$$PA = PB \text{ (tangent from P)}$$

And,

$$\angle PAB = \angle PBA \text{ } [\because PA = PB]$$

By property 2,

$$\angle PAB + \angle PBA + \angle APB = 180^\circ$$

$$\Rightarrow \angle PAB + \angle PBA + 30^\circ = 180^\circ$$

$$\Rightarrow \angle PAB + \angle PBA = 180^\circ - 30^\circ$$

$$\Rightarrow \angle PAB + \angle PBA = 150^\circ$$

$$\Rightarrow \angle PBA + \angle PBA = 150^\circ \text{ } [\because \angle PAB = \angle PBA]$$

$$\Rightarrow 2\angle PBA = 150^\circ$$

$$\Rightarrow \angle PBA = \frac{150^\circ}{2}$$

$$\Rightarrow \angle PBA = 75^\circ$$

Now,

$$\angle PBA = \angle CAB = 75^\circ \text{ [Alternate angles]}$$

$$\angle PBA = \angle ACB = 75^\circ \text{ [Alternate segment theorem]}$$

Again by property 2,

$$\angle CAB + \angle ACB + \angle CBA = 180^\circ$$

$$\Rightarrow 75^\circ + 75^\circ + \angle CBA = 180^\circ$$

$$\Rightarrow 150^\circ + \angle CBA = 180^\circ$$

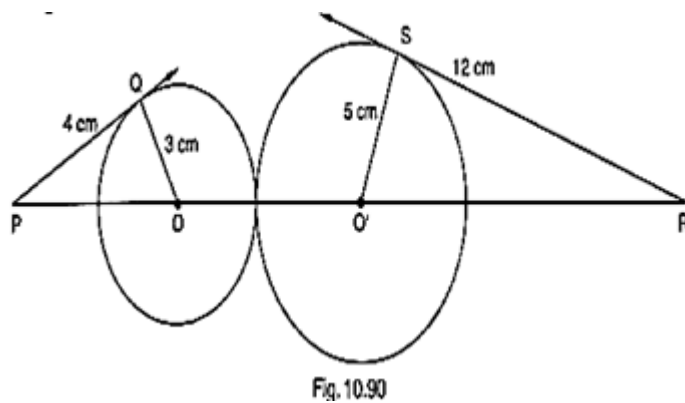
$$\Rightarrow \angle CBA = 180^\circ - 150^\circ$$

$$\Rightarrow \angle CBA = 30^\circ$$

Hence, $\angle CBA = 30^\circ$

28. Question

In Fig. 10.90, PR =



- A. 20 cm
- B. 26 cm
- C. 24 cm
- D. 28 cm

Answer

Given:

$$QP = 4 \text{ cm}$$

$$OQ = 3 \text{ cm}$$

$$SR = 12 \text{ cm}$$

$$SO' = 5 \text{ cm}$$

Property: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔOPQ is right-angled at $\angle OQP$ (i.e., $\angle OQP = 90^\circ$) and $\Delta O'SR$ is right-angled at $\angle O'SR$ (i.e., $\angle O'SR = 90^\circ$).

By Pythagoras theorem in ΔOPQ ,

$$OP^2 = QP^2 + OQ^2$$

$$\Rightarrow OP^2 = 4^2 + 3^2$$

$$\Rightarrow OP^2 = 16 + 9$$

$$\Rightarrow OP^2 = 25$$

$$\Rightarrow OP = \sqrt{25}$$

$$\Rightarrow OP = 5 \text{ cm}$$

By Pythagoras theorem in $\Delta O'SR$,

$$O'R^2 = SR^2 + O'S^2$$

$$\Rightarrow O'R^2 = 12^2 + 5^2$$

$$\Rightarrow O'R^2 = 144 + 25$$

$$\Rightarrow O'R^2 = 169$$

$$\Rightarrow O'R = \sqrt{169}$$

$$\Rightarrow O'R = 13 \text{ cm}$$

Now,

$$PR = PO + ON + NO' + O'R$$

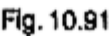
$$\Rightarrow PR = 5 \text{ cm} + 3 \text{ cm} + 5 \text{ cm} + 13 \text{ cm}$$

$$\Rightarrow PR = 26 \text{ cm}$$

Hence, $PR = 26 \text{ cm}$

29. Question

Two circles of same radii r and centres O and O' touch each other at P as shown in Fig. 10.91. If OO' is produced to meet the circle $C(O', r)$ at A and AT is a tangent to the circle $C(O, r)$ such that $O'Q \perp AT$. Then $AO:AO' =$



- ## Answer

Now,

$$\frac{AO}{AO'} = \frac{3r}{r} = 3$$

30. Question

Fig. 10.92

- A. 4 cm

- B. 6 cm
- C. 8 cm
- D. 10 cm

Answer

Given:

$$OA = 5 \text{ cm}$$

$$OQ = 3 \text{ cm}$$

Property 1: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By property 1, $\triangle OAQ$ is right-angled at $\angle OQA$ (i.e., $\angle OQA = 90^\circ$).

By Pythagoras theorem in $\triangle OAQ$,

$$OA^2 = QA^2 + OQ^2$$

$$\Rightarrow QA^2 = OA^2 - OQ^2$$

$$\Rightarrow QA^2 = 5^2 - 3^2$$

$$\Rightarrow QA^2 = 25 - 9$$

$$\Rightarrow QA^2 = 16$$

$$\Rightarrow QA = \sqrt{16}$$

$$\Rightarrow QA = 4 \text{ cm}$$

By property 2,

$$BQ = BP \text{ (tangent from B)}$$

And,

$$AQ = BQ = 4 \text{ cm } [\because Q \text{ is midpoint of } AB]$$

$$PB = PC = 4 \text{ cm } [\because P \text{ is midpoint of } BC]$$

Now,

$$BC = BP + PC$$

$$\Rightarrow BC = 4 \text{ cm} + 4 \text{ cm}$$

$$\Rightarrow BC = 8 \text{ cm}$$

Hence, $BC = 8 \text{ cm}$

31. Question

In Fig. 10.93, there are two concentric circles with centre O. PR and PQS are tangents to the inner circle from point P lying on the outer circle. If PR = 7.5 cm, then PS is equal to

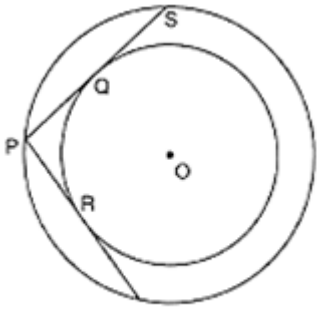


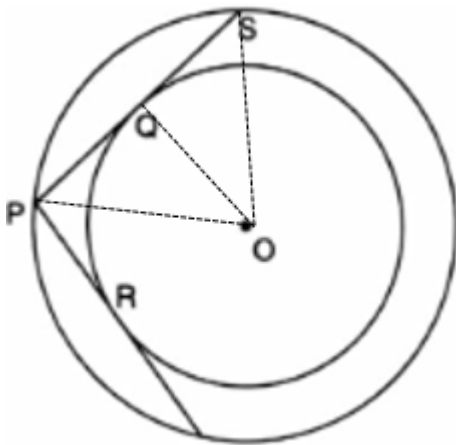
Fig. 10.93

- A. 10 cm
- B. 12 cm
- C. 15 cm
- D. 18 cm

Answer

Given:

$$PR = 7.5 \text{ cm}$$



Property 1: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By property 1, ΔOSQ is right-angled at $\angle OQS$ (i.e., $\angle OQS = 90^\circ$) and ΔOPQ is right-angled at $\angle OQP$ (i.e., $\angle OQP = 90^\circ$).

$$\therefore OQ \perp PS$$

$$\therefore PO = OS \text{ [radius of circle]}$$

$$\therefore \Delta POS \text{ is an isosceles triangle}$$

Now,

$\therefore \triangle POS$ is an isosceles triangle and OQ is perpendicular to its base

$\therefore OQ$ bisects PS

i.e., $PQ = QS$

By property 2,

$PR = PQ = 7.5$ cm (tangent from P)

Now,

$PS = PQ + QS$

$\Rightarrow PS = PQ + PQ$ [$\because PQ = QS$]

$\Rightarrow PS = 7.5$ cm + 7.5 cm

$\Rightarrow PS = 15$ cm

Hence, $PS = 15$ cm

32. Question

In Fig. 10.94, if $AB = 8$ cm and $PE = 3$ cm, then $AE =$

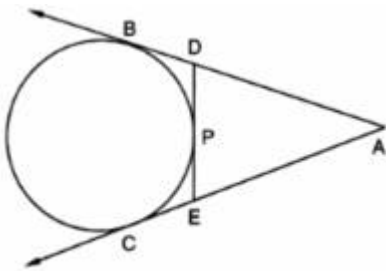


Fig. 10.94

A. 11 cm

B. 7 cm

C. 5 cm

D. 3 cm

Answer

Given:

$AB = 8$ cm

$PE = 3$ cm

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By above property,

$AB = AC = 8$ cm (tangent from A)

$PE = CE = 3$ cm (tangent from E)

Now,

$$AE = AC - CE$$

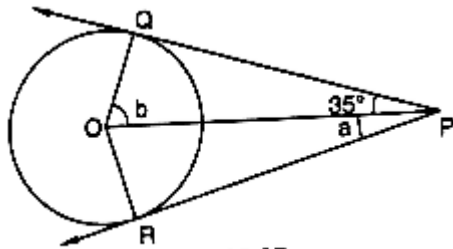
$$\Rightarrow AE = 8 \text{ cm} - 3 \text{ cm}$$

$$\Rightarrow AE = 5 \text{ cm}$$

Hence, $AE = 5 \text{ cm}$

33. Question

In Fig. 10.95, PQ and PR are tangents drawn from P to a circle with centre O. If $\angle OPQ = 35^\circ$, then



A. $a = 30^\circ$, $b = 60^\circ$

B. $a = 35^\circ$, $b = 55^\circ$

C. $a = 40^\circ$, $b = 50^\circ$

D. $a = 45^\circ$, $b = 45^\circ$

Answer

Given:

$$\angle OPQ = 35^\circ$$

Property 1: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Property 2: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 3: The sum of all angles of a triangle = 180° .

By property 1,

$$QP = QR \text{ (tangent from Q)}$$

By property 2, ΔOPQ is right-angled at $\angle OQP$ (i.e., $\angle OQP = 90^\circ$) and ΔORP is right-angled at $\angle ORP$ (i.e., $\angle ORP = 90^\circ$).

$$\therefore OQ \perp QP$$

$$OR \perp RP$$

Now,

$$\angle OQP = \angle ORP = 90^\circ \text{ [Property 1]}$$

$$QP = QR \text{ [Property 2]}$$

$$OP = OP \text{ [Given]}$$

$$\therefore \triangle OPQ \cong \triangle OPR \text{ By SAS}$$

$$\text{Hence, } \angle OPQ = \angle OPR = 35^\circ \text{ By CPCTC}$$

$$\text{i.e. } \angle a = 35^\circ$$

By property 3,

$$\angle OQP + \angle OPQ + \angle QOP = 180^\circ$$

$$\Rightarrow 90^\circ + 35^\circ + \angle QOP = 180^\circ$$

$$\Rightarrow 125^\circ + \angle QOP = 180^\circ$$

$$\Rightarrow \angle QOP = 180^\circ - 125^\circ$$

$$\Rightarrow \angle QOP = 55^\circ$$

$$\text{i.e. } \angle b = 55^\circ$$

$$\text{Hence, } \angle a = 35^\circ \text{ and } \angle b = 55^\circ$$

34. Question

In Fig. 10.96, if TP and TQ are tangents drawn from an external point T to a circle with centre O such that $\angle TQP = 60^\circ$, then $\angle OPQ =$

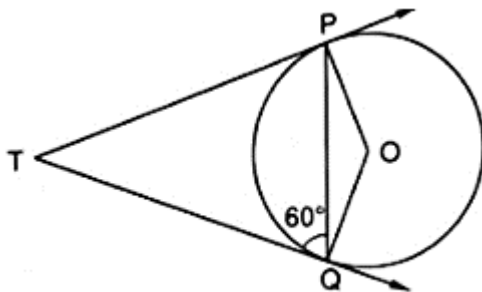


Fig. 10.96

A. 25°

B. 30°

C. 40°

D. 60°

Answer

Given:

$$\angle TQP = 60^\circ$$

Property 1: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Property 2: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By property 1,

$$TP = TQ \text{ (tangent from T)}$$

$$\Rightarrow \angle TPQ = \angle TQP = 60^\circ$$

By property 2, $\triangle OPT$ is right-angled at $\angle OPT$ (i.e., $\angle OPT = 90^\circ$) and $\triangle OQT$ is right-angled at $\angle OQT$ (i.e., $\angle OQT = 90^\circ$).

Now,

$$\angle OPQ = \angle OPT - \angle TPQ$$

$$\Rightarrow \angle OPQ = 90^\circ - 60^\circ$$

$$\Rightarrow \angle OPQ = 30^\circ$$

$$\text{Hence, } \angle OPQ = 30^\circ$$

35. Question

In Fig. 10.97, the sides AB, BC and CA of triangle ABC, touch a circle at P, Q and R respectively. If PA = 4 cm, BP = 3 cm and AC = 11cm, then length of BC is

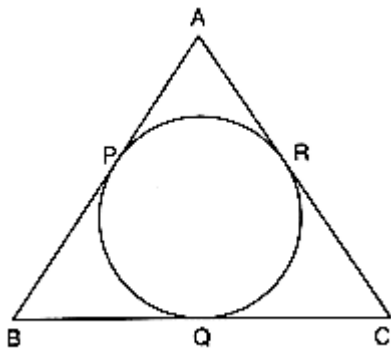


Fig. 10.97

A. 11 cm

B. 10 cm

C. 14 cm

D. 15 cm

Answer

Given:

$$PA = 4 \text{ cm}$$

$$BP = 3 \text{ cm}$$

$$AC = 11 \text{ cm}$$

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By above property,

$$AP = AR = 4 \text{ cm (tangent from A)}$$

$$BP = BQ = 3 \text{ cm (tangent from B)}$$

$$QC = RC \text{ (tangent from C)}$$

Clearly,

$$RC = AC - AR$$

$$\Rightarrow RC = 11 \text{ cm} - 4 \text{ cm}$$

$$\Rightarrow RC = 7 \text{ cm}$$

Now,

$$BC = BQ + QC$$

$$\Rightarrow BC = BQ + RC [\because QC = RC]$$

$$\Rightarrow BC = 3 \text{ cm} + 7 \text{ cm}$$

$$\Rightarrow BC = 10 \text{ cm}$$

Hence, $BC = 10 \text{ cm}$

36. Question

In Fig. 10.98, a circle touches the side DF of $\triangle EDF$ at H and touches ED and EF produced at K and M respectively. If $EK = 9 \text{ cm}$, then the perimeter of $\triangle EDF$ is

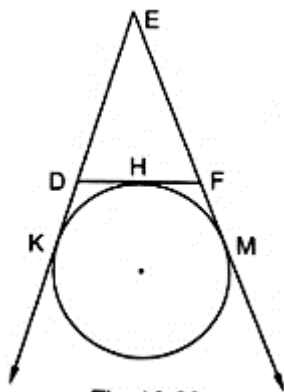


Fig. 10.98

- A. 18 cm
- B. 13.5 cm
- C. 12 cm
- D. 9 cm

Answer

Given:

$$EK = 9 \text{ cm}$$

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By above property,

$$EM = EK = 9 \text{ cm (tangent from E)}$$

$$DK = DH \text{ (tangent from D)}$$

$$FM = FH \text{ (tangent from F)}$$

Now,

$$\text{Perimeter of } \triangle EDF = ED + DF + FE$$

$$\Rightarrow \text{Perimeter of } \triangle EDF = (EK - KD) + (DH + HF) + (EM - MF)$$

$$[\because ED = EK - KD]$$

$$DF = DH + HF$$

$$FE = EM - MF]$$

$$\Rightarrow \text{Perimeter of } \triangle EDF = EK - KD + DH + HF + EM - MF$$

$$\Rightarrow \text{Perimeter of } \triangle EDF = EK - DH + DH + HF + EM - HF [\because DK = DH \text{ and } FM = FH]$$

$$\Rightarrow \text{Perimeter of } \triangle EDF = EK + EM$$

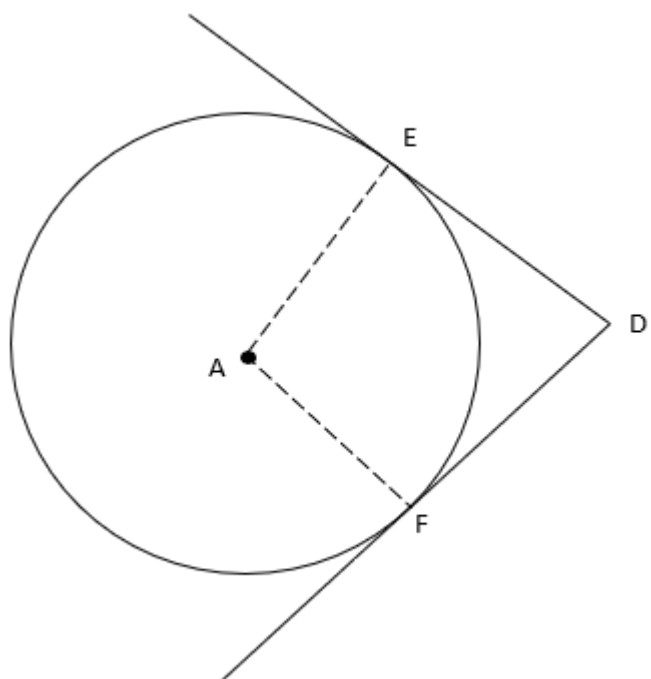
$$\Rightarrow \text{Perimeter of } \triangle EDF = 9 \text{ cm} + 9 \text{ cm}$$

$$\Rightarrow \text{Perimeter of } \triangle EDF = 18 \text{ cm}$$

Hence, Perimeter of $\triangle EDF = 18 \text{ cm}$

37. Question

In Fig, DE and DF are tangents from an external point D to a circle with centre A. If $DE = 5 \text{ cm}$ and $DE \perp DF$, then the radius of the circle is



A. 3 cm

B. 5 cm

C. 4 cm

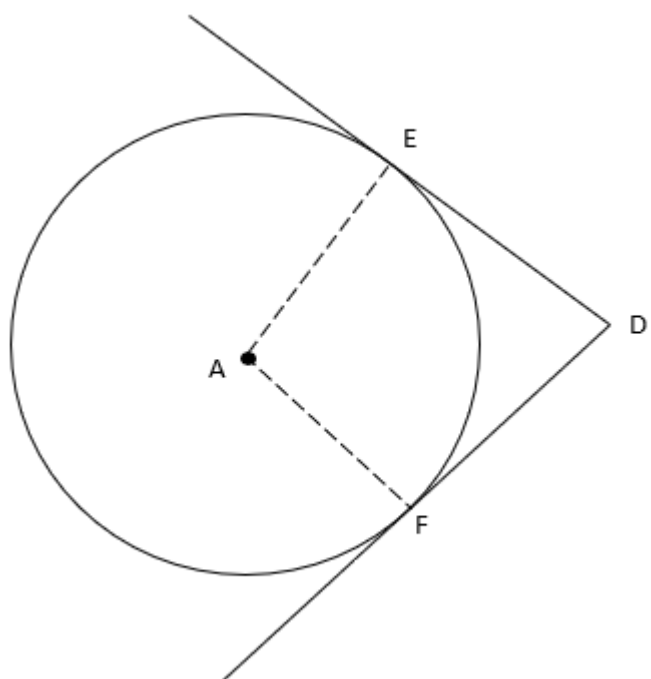
D. 6 cm

Answer

Given:

$DE = 5 \text{ cm}$

$DE \perp DF$



Join AE and AF

Property 1: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Property 2: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 3: Sum of all angles of a quadrilateral = 360° .

By property 1,

$$EF = ED = 5 \text{ cm (tangent from E)}$$

And,

$$AE = AF \text{ [radius]}$$

By property 2, $\angle AED = 90^\circ$ and $\angle AFD = 90^\circ$.

Also,

$$\angle EDF = 90^\circ [\because ED \perp EF]$$

By property 3,

$$\angle AED + \angle AFD + \angle EDF + \angle EAF = 360^\circ$$

$$\Rightarrow 90^\circ + 90^\circ + 90^\circ + \angle EAF = 360^\circ$$

$$\Rightarrow \angle EAF = 360^\circ - (90^\circ + 90^\circ + 90^\circ)$$

$$\Rightarrow \angle EAF = 360^\circ - 270^\circ$$

$$\Rightarrow \angle EAF = 90^\circ$$

$\therefore$ All angles are equal and adjacent sides are equal $\therefore$ AEDF is a square.

Hence, all sides are equal

$$\Rightarrow AE = AF = ED = EF = 5 \text{ cm}$$

Hence, Radius of circle = 5 cm

38. Question

In Fig. 10.100, a circle with centre O is inscribed in a quadrilateral ABCD such that, it touches sides BC, AB, AD and CD at points P, Q, R and S respectively. If $AB = 29 \text{ cm}$, $AD = 23 \text{ cm}$, $\angle B = 90^\circ$ and $DS = 5 \text{ cm}$, then the radius of the circle (in cm) is

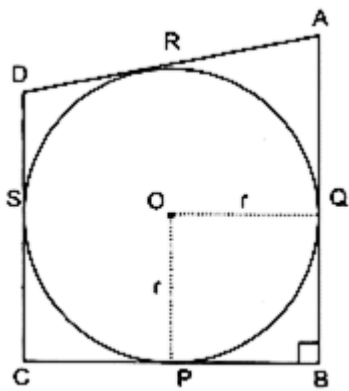


Fig. 10.100

A. 11

B. 18

C. 6

D. 15

Answer

Given:

$AB = 29$ cm

$AD = 23$ cm

$\angle B = 90^\circ$

$DS = 5$ cm

Property 1: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Property 2: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 3: Sum of all angles of a quadrilateral = 360° .

By property 1,

$BP = BQ$ (tangent from B)

$DS = DR = 5$ cm (tangent from D)

$AR = AQ$ (tangent from A)

Also,

$OQ = OP$ (radius)

By property 2, $\triangle OQB$ is right-angled at $\angle OQB$ (i.e., $\angle OQB = 90^\circ$) and $\triangle OPB$ is right-angled at $\angle OPB$ (i.e., $\angle OPB = 90^\circ$).

Now by property 3,

$$\angle PBC + \angle BQO + \angle QOP + \angle OPB = 360^\circ$$

$$\Rightarrow 90^\circ + 90^\circ + \angle QOP + 90^\circ = 360^\circ$$

$$\Rightarrow 270^\circ + \angle QOP = 360^\circ$$

$$\Rightarrow \angle QOP = 360^\circ - 270^\circ$$

$$\Rightarrow \angle QOP = 90^\circ$$

$\therefore$ adjacent sides (i.e., $BP = BQ$ and $OQ = OP$) are equal and all angles are 90°

$\therefore$ quadrilateral $OPBQ$ is a square

Now,

$$AD = 23 \text{ cm}$$

$$\Rightarrow AR + RD = 23 \text{ cm} [\because AD = AR + RD]$$

$$\Rightarrow AR + 5 \text{ cm} = 23 \text{ cm}$$

$$\Rightarrow AR = 23 \text{ cm} - 5 \text{ cm}$$

$$\Rightarrow AR = 18 \text{ cm}$$

$$\Rightarrow AQ = AR = 18 \text{ cm}$$

Now,

$$AB = 29 \text{ cm}$$

$$\Rightarrow AQ + QB = 29 \text{ cm} [\because AD = AR + RD]$$

$$\Rightarrow 18 \text{ cm} + QB = 29 \text{ cm}$$

$$\Rightarrow QB = 29 \text{ cm} - 18 \text{ cm}$$

$$\Rightarrow QB = 11 \text{ cm}$$

$\therefore$ $OPBQ$ is a square

$$\therefore OP = BQ = 11 \text{ cm}$$

Hence, radius = 11 cm

39. Question

In a right triangle ABC , right angled at B , $BC = 12 \text{ cm}$ and $AB = 5 \text{ cm}$. The radius of the circle inscribed in the triangle (in cm) is

A. 4

B. 3

C. 2

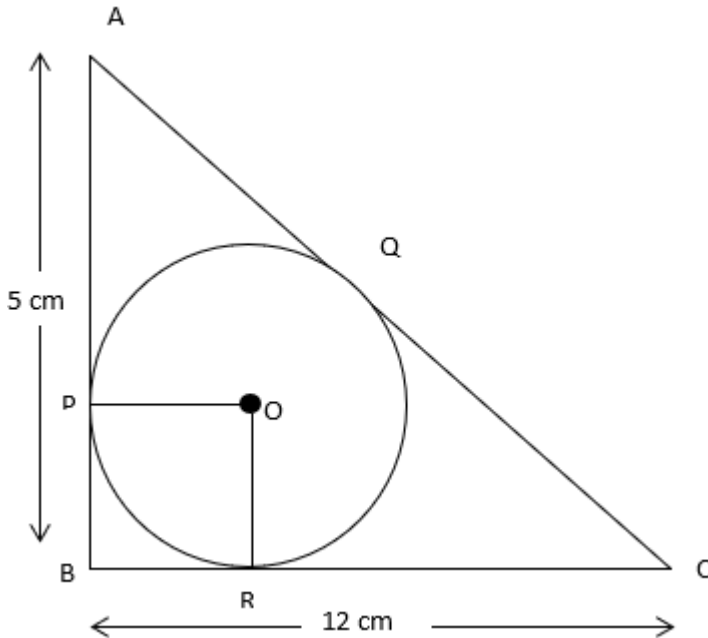
D. 1

Answer

Given:

BC = 12 cm

AB = 5 cm



Property 1: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Property 2: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 3: Sum of all angles of a quadrilateral = 360° .

By property 1,

$$AP = AQ \text{ (Tangent from A)}$$

$$BP = BR \text{ (Tangent from B)}$$

$$CR = CQ \text{ (Tangent from C)}$$

$\because$ ABC is a right-angled triangle, $\therefore$ by Pythagoras Theorem

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = 5^2 + 12^2$$

$$\Rightarrow AC^2 = 25 + 144$$

$$\Rightarrow AC^2 = 169$$

$$\Rightarrow AC = \sqrt{169}$$

$$\Rightarrow AC = 13 \text{ cm}$$

Clearly,

$$AQ + QC = AC = 13 \text{ cm}$$

$$\Rightarrow AP + RC = 13 \text{ cm } [\because AQ = AP \text{ and } QC = RC]$$

Also,

$$AB + BC = 5 \text{ cm} + 12 \text{ cm} = 17 \text{ cm}$$

$$\Rightarrow AP + PB + BR + RC = 17 \text{ cm} [\because AB = AP + PB \text{ and } BC = BR + RC]$$

$$\Rightarrow AP + RC + PB + BR = 17 \text{ cm}$$

$$\Rightarrow 13 \text{ cm} + BR + BR = 17 \text{ cm} [\because AP + RC = 10 \text{ cm and } PB = BR]$$

$$\Rightarrow 13 \text{ cm} + 2BR = 17 \text{ cm}$$

$$\Rightarrow 2BR = 17 \text{ cm} - 13 \text{ cm} = 4 \text{ cm}$$

$$\Rightarrow BR = \frac{4}{2} \text{ cm}$$

$$\Rightarrow BR = 2 \text{ cm}$$

Now,

$$\angle BPO = 90^\circ [\text{By property 2}]$$

$$\angle BRO = 90^\circ [\text{By property 2}]$$

$$\angle PBM = 90^\circ [\text{Given}]$$

Now by property 3,

$$\angle BPO + \angle BRO + \angle PBM + \angle ROP = 360^\circ$$

$$\Rightarrow \angle ROP = 360^\circ - (\angle BPO + \angle BRO + \angle PBM)$$

$$\Rightarrow \angle ROP = 360^\circ - (90^\circ + 90^\circ + 90^\circ)$$

$$\Rightarrow \angle ROP = 360^\circ - 270^\circ$$

$$\Rightarrow \angle ROP = 90^\circ$$

Now, $\because \angle ROP = 90^\circ$ and $BP = BR$ which are adjacent sides

$\therefore$ Quadrilateral PBRO is a square

$$\Rightarrow PO = BR = 2 \text{ cm}$$

Hence, Radius = 2 cm

40. Question

Two circles touch each other externally at P. AB is a common tangent to the circle touching them at A and B. The value of $\angle APB$ is

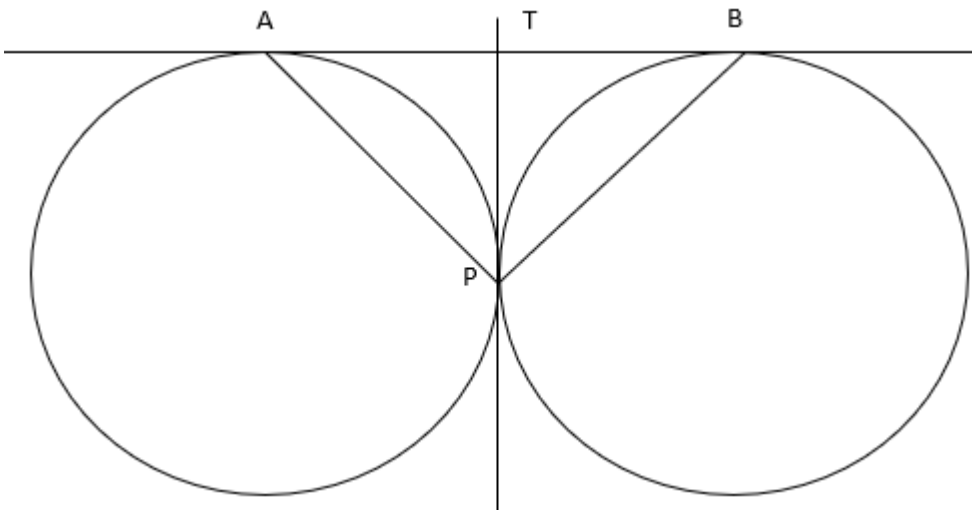
A. 30°

B. 45°

C. 60°

D. 90°

Answer



Draw a tangent from a point T on B to P.

Property 1: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Property 2: Sum of all angles of a triangle = 180° .

By property 1,

$$TA = TP \text{ (tangent from T)}$$

$$TB = TP \text{ (tangent from T)}$$

Now in $\triangle ATP$,

$$TA = TP$$

$$\therefore \angle APT = \angle PAT$$

And in $\triangle BTP$,

$$TB = TP$$

$$\therefore \angle BPT = \angle PBT$$

By property 2,

$$\angle APB + \angle PBA + \angle PAB = 180^\circ$$

$$\Rightarrow \angle APB + \angle PBT + \angle PAT = 180^\circ$$

$$\Rightarrow \angle APB + \angle BPT + \angle APT = 180^\circ [\because \angle APT = \angle PAT \text{ and } \angle BPT = \angle PBT]$$

$$\Rightarrow \angle APB + \angle APB = 180^\circ [\because \angle APB = \angle BPT + \angle APT]$$

$$\Rightarrow 2\angle APB = 180^\circ$$

$$\Rightarrow \angle APB = \frac{180^\circ}{2}$$

$$\Rightarrow \angle APB = 90^\circ$$

Hence, $\angle APB = 90^\circ$

41. Question

In Fig. 10.101, PQ and PR are two tangents to a circle with centre O. If $\angle QPR = 46^\circ$, then $\angle QOR$ equals

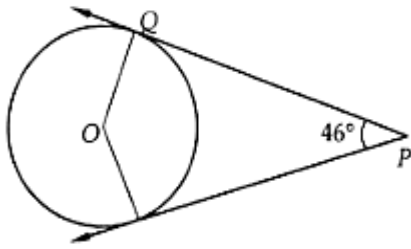


Fig. 10.101

- A. 67°
- B. 134°
- C. 44°
- D. 46°

Answer

Given:

$$\angle QPR = 46^\circ$$

Property 1: The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: Sum of all angles of a quadrilateral = 360° .

By property 1, $\triangle OQP$ is right-angled at $\angle OQP$ (i.e., $\angle OQP = 90^\circ$) and $\triangle ORP$ is right-angled at $\angle ORP$ (i.e., $\angle ORP = 90^\circ$).

Now by property 2,

$$\angle OQP + \angle ORP + \angle QOR + \angle QPR = 360^\circ$$

$$\Rightarrow \angle QOR = 360^\circ - (\angle OQP + \angle ORP + \angle QPR)$$

$$\Rightarrow \angle QOR = 360^\circ - (90^\circ + 90^\circ + 46^\circ)$$

$$\Rightarrow \angle QOR = 360^\circ - 226^\circ$$

$$\Rightarrow \angle QOR = 134^\circ$$

Hence, $\angle QOR = 134^\circ$

42. Question

In Fig. 10.102, QR is a common tangent to the given circles touching externally at the point T. The tangent at T meets QR at P. If $PT = 3.8$ cm, then the length of QR (in cm) is

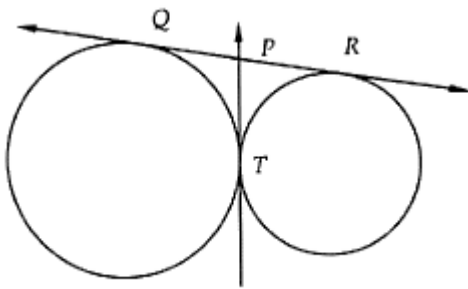


Fig. 10.102

A. 3.8

B. 7.6

C. 5.7

D. 1.9

Answer

Given:

$$PT = 3.8 \text{ cm}$$

Property 1: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By property 1,

$$PQ = PT \text{ (Tangent from P)}$$

$$PR = PT \text{ (Tangent from P)}$$

Now,

$$QR = PQ + PR$$

$$\Rightarrow QR = PT + PT$$

$$\Rightarrow QR = 3.8 \text{ cm} + 3.8 \text{ cm}$$

$$\Rightarrow QR = 7.6 \text{ cm}$$

Hence, $QR = 7.6 \text{ cm}$

43. Question

In Fig. 10.103, a quadrilateral ABCD is drawn to circumscribe a circle such that its sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively. If $AB = x \text{ cm}$, $BC = 7 \text{ cm}$, $CR = 3 \text{ cm}$ and $AS = 5 \text{ cm}$, then $x =$

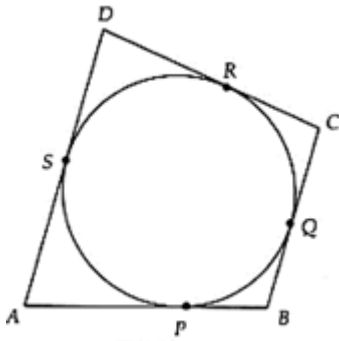


Fig. 10.103

- A. 10
- B. 9
- C. 8
- D. 7

Answer

Given:

$$AB = x \text{ cm}$$

$$BC = 7 \text{ cm}$$

$$CR = 3 \text{ cm}$$

$$AS = 5 \text{ cm}$$

Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

$$AP = AS \text{ (tangent from A)}$$

$$BP = BQ \text{ (tangent from B)}$$

$$CR = CQ \text{ (tangent from C)}$$

$$DR = DS \text{ (tangent from D)}$$

Clearly,

$$QB = CB - CQ$$

$$\Rightarrow QB = CB - CR [\because CQ = CR]$$

$$\Rightarrow QB = 7 \text{ cm} - 3 \text{ cm}$$

$$\Rightarrow QB = 4 \text{ cm}$$

Now,

$$AB = AP + PB$$

$$\Rightarrow AB = AS + QB$$

$$\Rightarrow AB = 5 \text{ cm} + 4 \text{ cm}$$

$$\Rightarrow AB = 9 \text{ cm}$$

$$\Rightarrow AB = x = 9 \text{ cm}$$

Hence, $x = 9 \text{ cm}$