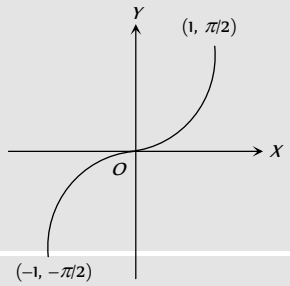
Chapter
12
प्रतिलोम त्रिकोणमितीय फलन

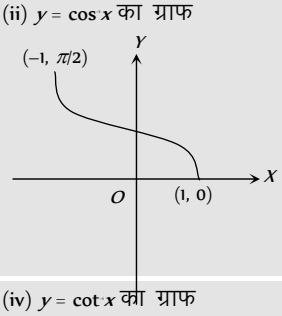
फलन $f: A \to B$ व्युत्क्रमणीय होगा यदि f एकैकी आच्छादक हो तथा यह इस प्रकार परिभाषित है, कि $f(x) = y \Rightarrow f^{-1}(y) = x$

प्रतिलोम त्रिकोणमितीय फलनों के ग्राफ
(Graphs of inverse trigonometric functions)

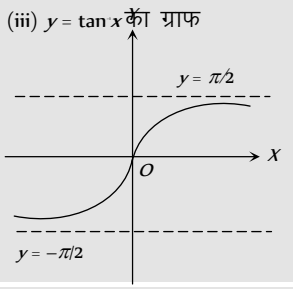
(i) $y = \sin x$ का ग्राफ



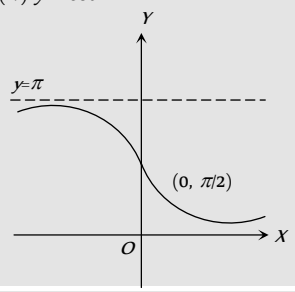
(ii) $y = \cos x$ का ग्राफ



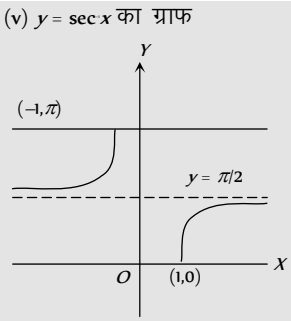
(iii) $y = \tan x$ का ग्राफ



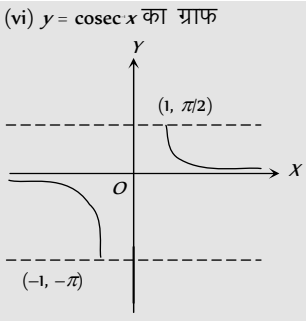
(iv) $y = \cot x$ का ग्राफ



(v) $y = \sec x$ का ग्राफ



(vi) $y = \operatorname{cosec} x$ का ग्राफ



प्रतिलोम फलनों के प्रान्त एवं परिसर
(Domain and range of inverse trigonometric functions)

फलन	डोमेन (D)	रेंज (R)
$\sin^{-1} x$	$-1 \le x \le 1$ या $[-1,1]$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ या $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$-1 \le x \le 1$ या $[-1,1]$	$0 \le \theta \le \pi$ या $[0, \pi]$
$\tan^{-1} x$	$-\infty < x < \infty$ अर्थात् $x \in R$ या $(-\infty, \infty)$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ या $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1} x$	$-\infty < x < \infty$ अर्थात् $x \in R$ या $(-\infty, \infty)$	$0 < \theta < \pi$ या $(0, \pi)$
$\sec^{-1} x$	$x \le -1, x \ge 1$ या $(-\infty, -1] \cup [1, \infty)$	$\theta \ne \frac{\pi}{2}, 0 \le \theta \le \pi$ या $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
$\operatorname{cosec}^{-1} x$	$x \le -1, x \ge 1$ या $(-\infty, -1] \cup [1, \infty)$	$\theta \ne 0, -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ या $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

प्रतिलोम त्रिकोणमितीय फलनों के गुणधर्म
(Graphs of inverse trigonometric functions)

(i) $\sin^{-1}(\sin\theta) = \theta$, जबकि $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$,

$\cos^{-1}(\cos\theta) = \theta$, जबकि $0 \le \theta \le \pi$

$\tan^{-1}(\tan\theta) = \theta$, जबकि $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$,

$\cot^{-1}(\cot\theta) = \theta$, जबकि $0 < \theta < \pi$

$\sec^{-1}(\sec\theta) = \theta$, जबकि $0 \le \theta < \frac{\pi}{2}$ या $\frac{\pi}{2} < \theta \le \pi$

$\operatorname{cosec}^{-1}(\operatorname{cosec}\theta) = \theta$, जबकि $-\frac{\pi}{2} \le \theta < 0$ या $0 < \theta \le \frac{\pi}{2}$

(2) $\sin(\sin^{-1} x) = x$, जबकि $-1 \le x \le 1$

$\cos(\cos^{-1} x) = x$, जबकि $-1 \le x \le 1$

$\tan(\tan^{-1} x) = x$, जबकि $-\infty < x < \infty$

$\cot(\cot^{-1} x) = x$, जबकि $-\infty < x < \infty$

$\sec(\sec^{-1} x) = x$, जबकि $-\infty < x \le -1$ या $1 \le x < \infty$

$\text{cosec}(\text{cosec}^{-1} x) = x$, जबकि $-\infty < x \le -1$ या $1 \le x < \infty$

(3) $\sin^{-1}(-x) = -\sin^{-1} x$, $\cos^{-1}(-x) = \pi - \cos^{-1} x$

$\tan^{-1}(-x) = -\tan^{-1} x$, $\cot^{-1}(-x) = \pi - \cot^{-1} x$

$\sec^{-1}(-x) = \pi - \sec^{-1} x$, $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x$

(4) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$, $x \in [-1, 1]$

$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$, $x \in R$

$\sec^{-1} x + \text{cosec}^{-1} x = \frac{\pi}{2}$, $x \in (-\infty, -1] \cup [1, \infty)$

(5) **वृत्तीय प्रतिलोम फलनों के मुख्य मान**

सारणी : 12.1

$x \ge 0$ के लिए मुख्य मान	$x < 0$ के लिए मुख्य मान
$0 \le \sin^{-1} x \le \frac{\pi}{2}$	$-\frac{\pi}{2} \le \sin^{-1} x < 0$
$0 \le \cos^{-1} x \le \frac{\pi}{2}$	$\frac{\pi}{2} < \cos^{-1} x \le \pi$
$0 \le \tan^{-1} x < \frac{\pi}{2}$	$-\frac{\pi}{2} < \tan^{-1} x < 0$
$0 < \cot^{-1} x \le \frac{\pi}{2}$	$\frac{\pi}{2} < \cot^{-1} x < \pi$
$0 \le \sec^{-1} x < \frac{\pi}{2}$	$\frac{\pi}{2} < \sec^{-1} x \le \pi$
$0 < \text{cosec}^{-1} x \le \frac{\pi}{2}$	$-\frac{\pi}{2} \le \text{cosec}^{-1} x < 0$

(6) **रूपान्तरणीय गुणधर्म** (Conversion property)

माना $\sin^{-1} x = y \Rightarrow x = \sin y$

$\Rightarrow \text{cosec}\, y = \left(\frac{1}{x}\right) \Rightarrow y = \text{cosec}^{-1}\left(\frac{1}{x}\right)$

$$\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$$

$$= \cot^{-1}\frac{\sqrt{1-x^2}}{x} = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \text{cosec}^{-1}\left(\frac{1}{x}\right)$$

$$\cos^{-1} x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$$

$$= \sec^{-1}\frac{1}{x} = \text{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

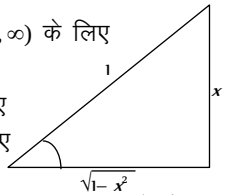
$$\tan^{-1} x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$$

$$= \cot^{-1}\left(\frac{1}{x}\right) = \sec^{-1}\sqrt{1+x^2} = \text{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right)$$

$\sin^{-1}\left(\frac{1}{x}\right) = \text{cosec}^{-1} x$, सभी $x \in (-\infty, -1] \cup [1, \infty)$ के लिए

$\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x$, सभी $x \in (-\infty, -1] \cup [1, \infty)$ के लिए

$$\tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1} x, & x > 0 \text{ के लिए} \\ -\pi + \cot^{-1} x, & x < 0 \text{ के लिए} \end{cases}$$



(7) **वृत्तीय प्रतिलोम फलनों का व्यापक हल :** हम जानते हैं कि यदि α न्यूनतम कोण हो जिसका sine, x है, तब x को $nx + (-1)^n \alpha$, जहाँ $n = 0, 1, 2, ...$ लिखते हैं। अतः $\sin^{-1} x$ का व्यापक मान $n\pi + (-1)^n \alpha$ है।

$\sin^{-1} x = n\pi + (-1)^n \alpha, -1 \le x \le 1$, यदि $\sin\alpha = x$ तथा $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$

इस प्रकार शेष वृत्तीय फलनों का व्यापक मान निम्न प्रकार से है :

$\cos^{-1} x = 2n\pi \pm \alpha, -1 \le x \le 1$	यदि $\cos\alpha = x$, $0 \le \alpha \le \pi$
$\tan^{-1} x = n\pi + \alpha$, $x \in R$;	यदि $\tan\alpha = x$, $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$
$\cot^{-1} x = n\pi + \alpha$, $x \in R$;	यदि $\cot\alpha = x$, $0 \le \alpha \le \pi$
$\sec^{-1} x = 2n\pi \ne \alpha$, $x \ge 1$ या $x \le -1$	यदि $\sec\alpha = x, 0 \le \alpha \le \pi$ तथा $\ne \frac{\pi}{2}$
$\text{cosec}^{-1} x = n\pi + (-1)^n \alpha$, $x \ge 1$ या $x \le -1$	यदि $\text{cosec}\ \alpha = x$, $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$ तथा $x \ne 0$

प्रतिलोम त्रिकोणमितीय फलनों के योग व अन्तर पर आधारित सूत्र (Formulae for sum and difference of inverse trigonometric function)

(1) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$;

यदि $x > 0, y > 0$ एवं $xy < 1$

(2) $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$;

यदि $x > 0, y > 0$ एवं $xy > 1$

(3) $\tan^{-1} x + \tan^{-1} y = -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$;

यदि $x < 0, y < 0$ एवं $xy > 1$

(4) $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$;

यदि $xy > -1$

(5) $\tan^{-1} x - \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right)$;

यदि $x > 0, y < 0$ एवं $xy < -1$

(6) $\tan^{-1} x - \tan^{-1} y = -\pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right)$;

यदि $x < 0, y > 0$ एवं $xy < -1$

(7) $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

(8) $\tan^{-1} x_1 + \tan^{-1} x_2 + \ldots\ldots\ldots + \tan^{-1} x_n$

$= \tan^{-1}\left[\frac{S_1 - S_3 + S_5 - \ldots\ldots\ldots .}{1 - S_2 + S_4 - S_6 + \ldots\ldots\ldots}\right]$

जहाँ $S_k ; x_1, x_2, \ldots\ldots\ldots, x_n$ के गुणनफलों का योग प्रदर्शित करता है।

(9) $\cot^{-1} x + \cot^{-1} y = \cot^{-1}\frac{xy-1}{y+x}$

(10) $\cot^{-1} x - \cot^{-1} y = \cot^{-1}\frac{xy+1}{y-x}$

(11) $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$;

यदि $-1 \le x, y \le 1$ एवं $x^2 + y^2 \le 1$ या यदि $xy < 0$ एवं $x^2 + y^2 > 1$

(12) $\sin^{-1} x + \sin^{-1} y = \pi - \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$,

यदि $0 < x, y \le 1$ एवं $x^2 + y^2 > 1$

(13) $\sin^{-1} x + \sin^{-1} y = -\pi - \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$,

यदि $-1 \le x; y < 0$ एवं $x^2 + y^2 > 1$

(14) $\sin^{-1} x - \sin^{-1} y = \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}$,

यदि $-1 \le x; y \le 1$ एवं $x^2 + y^2 \le 1$ यदि या $xy > 0$ एवं $x^2 + y^2 > 1$.

(15) $\sin^{-1} x - \sin^{-1} y = \pi - \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}$,

यदि $0 < x \le 1, -1 \le y < 0$ एवं $x^2 + y^2 > 1$.

(16) $\sin^{-1} x - \sin^{-1} y = -\pi - \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}$,

यदि $-1 \le x < 0, 0 < y \le 1$ एवं $x^2 + y^2 > 1$.

(17) $\cos^{-1} x + \cos^{-1} y = \cos^{-1}\{xy - \sqrt{1-x^2}.\sqrt{1-y^2}\}$,

यदि $-1 \le x, y \le 1$ एवं $x + y \ge 0$.

(18) $\cos^{-1} x + \cos^{-1} y = 2\pi - \cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}$,

यदि $-1 \le x, y \le 1$ एवं $x + y \le 0$

(19) $\cos^{-1} x - \cos^{-1} y = \cos^{-1}\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\}$,

यदि $-1 \le x, y \le 1$, एवं $x \le y$.

(20) $\cos^{-1} x - \cos^{-1} y = -\cos^{-1}\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\}$,

यदि $-1 \le y \le 0$, $0 < x \le 1$ एवं $x \ge y$.

अपवर्त्य कोणों के प्रतिलोम त्रिकोणमितीय अनुपात (Inverse trigonometric ratios of multiple angles)

(1) $2\sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$, यदि $-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

(2) $2\sin^{-1} x = \pi - \sin^{-1} 2x\sqrt{1-x^2}$, यदि $\frac{1}{\sqrt{2}} \le x \le 1$

(3) $2\sin^{-1} x = -\pi - \sin^{-1}(2x\sqrt{1-x^2})$, यदि $-1 \le x \le \frac{-1}{\sqrt{2}}$

(4) $3\sin^{-1} x = \sin^{-1}(3x - 4x^3)$, यदि $\frac{-1}{2} \le x \le \frac{1}{2}$

(5) $3\sin^{-1} x = \pi - \sin^{-1}(3x - 4x^3)$, यदि $\frac{1}{2} < x \le 1$

(6) $3\sin^{-1} x = -\pi - \sin^{-1}(3x - 4x^3)$, यदि $-1 \le x < -\frac{1}{2}$

(7) $2\cos^{-1} x = \cos^{-1}(2x^2 - 1)$, यदि $0 \le x \le 1$

(8) $2\cos^{-1} x = 2\pi - \cos^{-1}(2x^2 - 1)$, यदि $-1 \le x \le 0$

(9) $3\cos^{-1} x = \cos^{-1}(4x^3 - 3x)$, यदि $\frac{1}{2} \le x \le 1$

(10) $3\cos^{-1} x = 2\pi - \cos^{-1}(4x^3 - 3x)$, यदि $-\frac{1}{2} \le x \le \frac{1}{2}$

(11) $3\cos^{-1} x = 2\pi + \cos^{-1}(4x^3 - 3x)$, यदि $-1 \le x \le -\frac{1}{2}$

(12) $2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, यदि $-1 < x \le 1$

(13) $2\tan^{-1} x = \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, यदि $x > 1$

(14) $2\tan^{-1} x = -\pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, यदि $x < -1$

(15) $2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, यदि $-1 \le x \le 1$

(16) $2\tan^{-1} x = \pi - \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, यदि $x > 1$

(17) $2\tan^{-1} x = -\pi - \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, यदि $x < -1$

(18) $2\tan^{-1} x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, यदि $0 \le x < \infty$

(19) $2\tan^{-1} x = -\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, यदि $-\infty < x \le 0$

(20) $3\tan^{-1} x = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, यदि $\frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

(21) $3\tan^{-1} x = \pi + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, यदि $x > \frac{1}{\sqrt{3}}$

(22) $3\tan^{-1} x = -\pi + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, यदि $x < -\frac{1}{\sqrt{3}}$

(23) $\tan^{-1}\left[\frac{x}{\sqrt{a^2-x^2}}\right] = \sin^{-1}\frac{x}{a}$

(24) $\tan^{-1}\left[\frac{3a^2x - x^3}{a(a^2-3x^2)}\right] = 3\tan^{-1}\frac{x}{a}$

(25) $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$

(26) $\tan^{-1}\sqrt{\frac{1-x}{1+x}} = \frac{1}{2}\cos^{-1} x$

Tips & Tricks

- I चतुर्थांश सभी प्रतिलोम फलन में उभयनिष्ठ होता है।
- III चतुर्थांश का प्रतिलोम फलन में उपयोग नहीं किया जाता है।
- IV चतुर्थांश का उपयोग दक्षिणावर्त दिशा में होता है, अर्थात $-\frac{\pi}{2} \le y \le 0$.
- $\sin^{-1} x, \cos^{-1} x, \tan^{-1} x$ को क्रमशः arc $\sin x$, arc $\cos x$ एवं arc $\tan x$ भी लिखते हैं।
- यह ध्यान देना चाहिए यदि कुछ नहीं कहा गया है तब वृत्तीय प्रतिलोम फलनों का मुख्य मान ही लेते हैं।
- यदि $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$ हो, तो $xy + yz + zx = 1$.
- यदि $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$ हो, तो $x + y + z = xyz$.
- यदि $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{\pi}{2}$ हो, तो $x^2 + y^2 + z^2 + 2xyz = 1$.
- यदि $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$ हो, तो $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$.
- यदि $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$ हो, तो $xy + yz + zx = 3$.
- यदि $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ हो, तो $x^2 + y^2 + z^2 + 2xyz = 1$.
- यदि $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$ हो, तो $xy + yz + zx = 3$.
- यदि $\sin^{-1} x + \sin^{-1} y = \theta$ हो, तो $\cos^{-1} x + \cos^{-1} y = \pi - \theta$.
- यदि $\cos^{-1} x + \cos^{-1} y = \theta$ हो, तो $\sin^{-1} x + \sin^{-1} y = \pi - \theta$.
- यदि $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{2}$ हो, तो $xy = 1$.
- यदि $\cot^{-1} x + \cot^{-1} y = \frac{\pi}{2}$ हो, तो $xy = 1$.
- यदि $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \theta$ हो, तो $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\theta + \frac{y^2}{b^2} = \sin^2\theta$.

Ordinary Thinking

Objective Questions

प्रतिलोम त्रिकोणमितीय फलन

1. यदि $\cos^{-1}\left(\frac{1}{x}\right) = \theta$, तो $\tan\theta =$ [MNR 1978; MP PET 1989]

(a) $\frac{1}{\sqrt{x^2-1}}$ (b) $\sqrt{x^2+1}$

(c) $\sqrt{1-x^2}$ (d) $\sqrt{x^2-1}$

2. $\sin(\cot^{-1} x) =$ [MNR 1987; MP PET 2001; DCE 2002]

(a) $\sqrt{1+x^2}$ (b) x

(c) $(1+x^2)^{-3/2}$ (d) $(1+x^2)^{-1/2}$

3. $\cos\left(\sin^{-1}\frac{5}{13}\right) =$

(a) $\frac{12}{13}$ (b) $-\frac{12}{13}$

(c) $\frac{5}{12}$ (d) इनमें से कोई नहीं

4. $\cot^{-1}(-\sqrt{3}) =$

(a) $-\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$

(c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

5. $1 + \cot^2(\sin^{-1} x) =$

(a) $\frac{1}{2x}$ (b) x^2

(c) $\frac{1}{x^2}$ (d) $\frac{2}{x}$

6. यदि $\sin^{-1}\frac{1}{2} = \tan^{-1} x$, तो $x =$

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{\sqrt{2}}$ (d) इनमें से कोई नहीं

7. $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) =$

(a) $\tan^{-1} x$ (b) $\frac{1}{2}\tan^{-1} x$

(c) $2\tan^{-1} x$ (d) इनमें से कोई नहीं

8. $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}} =$

(a) $\frac{1}{a}\sin^{-1}\left(\frac{x}{a}\right)$ (b) $a\sin^{-1}\left(\frac{x}{a}\right)$

(c) $\sin^{-1}\left(\frac{x}{a}\right)$ (d) $\sin^{-1}\left(\frac{a}{x}\right)$

9. $\cos(\tan^{-1} x) =$ **[MP PET 1988; MNR 1981]**

(a) $\sqrt{1+x^2}$ (b) $\frac{1}{\sqrt{1+x^2}}$

(c) $1+x^2$ (d) इनमें से कोई नहीं

10. $\tan\left[\sec^{-1}\sqrt{1+x^2}\right] =$

(a) $\frac{1}{x}$ (b) x

(c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

11. $\sec^{-1}[\sec(-30^o)] =$ **[MP PET 1992]**

(a) -60^o (b) -30^o

(c) 30^o (d) 150^o

12. $\tan^{-1}\left[\frac{\cos x}{1+\sin x}\right] =$

(a) $\frac{\pi}{4}-\frac{x}{2}$ (b) $\frac{\pi}{4}+\frac{x}{2}$

(c) $\frac{x}{2}$ (d) $\frac{\pi}{4}-x$

13. $\tan^{-1}\frac{1}{\sqrt{x^2-1}} =$

(a) $\frac{\pi}{2}+\text{cosec}^{-1}x$ (b) $\frac{\pi}{2}+\sec^{-1}x$

(c) $\text{cosec}^{-1}x$ (d) $\sec^{-1}x$

14. $\sin^{-1}\left(-\frac{1}{2}\right)$ का मुख्य मान है

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) $-\frac{\pi}{3}$ (d) $-\frac{\pi}{6}$

15. $\sec^2(\tan^{-1}2)+\text{cosec}^2(\cot^{-1}3) =$ **[EAMCET 2001]**

(a) 5 (b) 13

(c) 15 (d) 6

16. $\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right] =$

(a) $\sin^{-1}x+\sin^{-1}\sqrt{x}$ (b) $\sin^{-1}x-\sin^{-1}\sqrt{x}$

(c) $\sin^{-1}\sqrt{x}-\sin^{-1}x$ (d) इनमें से कोई नहीं

17. यदि $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x$, तो $x =$

(a) 1 (b) $\sqrt{3}$

(c) $\frac{1}{\sqrt{3}}$ (d) इनमें से कोई नहीं

18. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) =$

(a) $\frac{7\pi}{6}$ (b) $\frac{5\pi}{6}$

(c) $\frac{\pi}{6}$ (d) इनमें से कोई नहीं

19. $\sin\cot^{-1}\tan\cos^{-1}x$ का मान है **[Bihar CEE 1974]**

(a) x (b) $\frac{\pi}{2}$

(c) 1 (d) इनमें से कोई नहीं

20. $\sin^{-1}\frac{\sqrt{x}}{\sqrt{x+a}} =$

(a) $\cos^{-1}\sqrt{\frac{x}{a}}$ (b) $\text{cosec}^{-1}\sqrt{\frac{x}{a}}$

(c) $\tan^{-1}\sqrt{\frac{x}{a}}$ (d) इनमें से कोई नहीं

21. यदि $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, तो $x =$

[MNR 1994; Kerala (Engg.) 2005]

(a) 1 (b) 0

(c) $\frac{4}{5}$ (d) $\frac{1}{5}$

22. यदि $\sin^{-1}x=\theta+\beta$ तथा $\sin^{-1}y=\theta-\beta$, तो $1+xy =$

(a) $\sin^2\theta+\sin^2\beta$ (b) $\sin^2\theta+\cos^2\beta$

(c) $\cos^2\theta+\cos^2\beta$ (d) $\cos^2\theta+\sin^2\beta$

23. यदि $\sin^{-1}\frac{1}{3}+\sin^{-1}\frac{2}{3}=\sin^{-1}x$, तो $x =$

[Roorkee 1995]

(a) 0 (b) $\frac{\sqrt{5}-4\sqrt{2}}{9}$

(c) $\frac{\sqrt{5}+4\sqrt{2}}{9}$ (d) $\frac{\pi}{2}$

24. $\tan(\cos^{-1}x) =$ **[IIT 1993]**

(a) $\frac{\sqrt{1-x^2}}{x}$ (b) $\frac{x}{1+x^2}$

(c) $\frac{\sqrt{1+x^2}}{x}$ (d) $\sqrt{1-x^2}$

25. $\sin^{-1}x$ का प्रान्त है **[Roorkee 1993]**

(a) $(-\pi,\pi)$ (b) $[-1,1]$

(c) $(0,2\pi)$ (d) $(-\infty,\infty)$

26. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ का मुख्य मान है **[Roorkee 1992]**

(a) $\frac{-2\pi}{3}$ (b) $\frac{-\pi}{3}$

(c) $\frac{4\pi}{3}$ (d) $\frac{5\pi}{3}$

27. $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right] =$ **[Karnataka CET 1994]**

(a) $\frac{25}{24}$ (b) $\frac{25}{7}$

(c) $\frac{24}{25}$ (d) इनमें से कोई नहीं

28. यदि $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$, तब $\sin^{-1}(\sin x)$ बराबर है

(a) x (b) $-x$
(c) $\pi + x$ (d) $\pi - x$

29. यदि $\pi \le x \le 2\pi$, तब $\cos^{-1}(\cos x)$ बराबर है

(a) x (b) $-x$
(c) $2\pi + x$ (d) $2\pi - x$

30. $\sin^{-1}(\sin 10)$ का मान है

(a) 10 (b) $10 - 3\pi$
(c) $3\pi - 10$ (d) इनमें से कोई नहीं

31. $\tan^{-1}\left(\frac{1-x}{1+x}\right)$, $0 \le x \le 1$, के अधिकतम व न्यूनतम मान हैं

(a) $0, \pi$ (b) $0, \frac{\pi}{4}$
(c) $-\frac{\pi}{4}, \frac{\pi}{4}$ (d) $\frac{\pi}{4}, \frac{\pi}{2}$

32. यदि x अधनात्मक सुग्राह (Permissible) मान रखता है, तब $\sin^{-1} x =$

(a) $\cos^{-1}\sqrt{1-x^2}$ (b) $-\cos^{-1}\sqrt{1-x^2}$
(c) $\cos^{-1}\sqrt{x^2-1}$ (d) $\pi - \cos^{-1}\sqrt{1-x^2}$

33. $\left[\sin\left(\tan^{-1}\frac{3}{4}\right)\right]^2 =$ **[EAMCET 1983]**

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$
(c) $\frac{9}{25}$ (d) $\frac{25}{9}$

34. $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ का मुख्य मान है **[MP PET 1996]**

(a) $\frac{5\pi}{3}$ (b) $-\frac{5\pi}{3}$
(c) $-\frac{\pi}{3}$ (d) $\frac{4\pi}{3}$

35. x का वह मान, जो समीकरण $\tan^{-1} x = \sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$ को संतुष्ट करता है, होगा **[Pb. CET 1999]**

(a) 3 (b) –3
(c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

36. $\sec(\text{cosec}^{-1} x) =$ **[Kurukshetra CEE 2001]**

(a) $\text{cosec}(\sec^{-1} x)$ (b) $\cot x$
(c) π (d) इनमें से कोई नहीं

37. $\tan^{-1} x$ का परिसर (रेंज) होगा **[DCE 2002]**

(a) $\left(\pi, \frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(c) $(-\pi, \pi)$ (d) $(0, \pi)$

38. यदि $\theta = \sin^{-1}[\sin(-600^o)]$, तब θ का एक संभावित मान होगा **[Kerala (Engg.) 2002]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$
(c) $\frac{2\pi}{3}$ (d) $\frac{-2\pi}{3}$

39. समीकरण $\sin^{-1} x = 2\tan^{-1} x$ का हल समुच्चय होगा **[AMU 2002]**

(a) {1, 2} (b) {–1, 2}
(c) {–1,1, 0} (d) {1, 1/2, 0}

40. $\cos(\tan^{-1}(\tan 2))$ का मान होगा **[AMU 2002]**

(a) $\frac{1}{\sqrt{5}}$ (b) $-\frac{1}{\sqrt{5}}$
(c) $\cos 2$ (d) $-\cos 2$

41. यदि $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{\pi}{2}$ तब $x^2 + y^2 + z^2 + 2xyz$ का मान है **[Pb. CET 2002]**

(a) 0 (b) 1
(c) 2 (d) 3

42. $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] =$

(a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{\sqrt{3}}{2}$
(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

43. $\sin[\cot^{-1}(\cos\tan^{-1} x)] =$

(a) $\frac{x}{\sqrt{x^2+2}}$ (b) $\frac{x}{\sqrt{x^2+1}}$
(c) $\frac{1}{\sqrt{x^2+2}}$ (d) $\sqrt{\frac{x^2+1}{x^2+2}}$

44. यदि $\sin[\cot^{-1}(x+1)] = \cos(\tan^{-1} x)$, तब $x =$ **[IIT Screening 2004]**

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$
(c) 0 (d) $\frac{9}{4}$

45. $\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{3}{5} =$

(a) $\tan^{-1}\frac{27}{11}$ (b) $\sin^{-1}\frac{11}{27}$
(c) $\cos^{-1}\frac{11}{27}$ (d) इनमें से कोई नहीं

46. $\sin^{-1} x + \sin^{-1}\frac{1}{x} + \cos^{-1} x + \cos^{-1}\frac{1}{x} =$

(a) π (b) $\frac{\pi}{2}$
(c) $\frac{3\pi}{2}$ (d) इनमें से कोई नहीं

47. $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2} =$

(a) 90^o (b) 60^o
(c) 45^o (d) $\tan^{-1} 2$

48. $\tan\left(90^o - \cot^{-1}\frac{1}{3}\right) =$

(a) 3 (b) $\frac{2}{3}$
(c) $\frac{1}{3}$ (d) $\frac{1}{\sqrt{10}}$

49. $\tan\left[\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right]=$

[IIT 1983; EAMCET 1988; MP PET 1990; MNR 1992]

(a) 6/17 (b) 17/6
(c) 7/16 (d) 16/7

50. $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$
(c) 0 (d) इनमें से कोई नहीं

51. $\cot^{-1}\frac{3}{4}+\sin^{-1}\frac{5}{13}=$

(a) $\sin^{-1}\frac{63}{65}$ (b) $\sin^{-1}\frac{12}{13}$
(c) $\sin^{-1}\frac{65}{68}$ (d) $\sin^{-1}\frac{5}{12}$

52. यदि $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$, तो [Roorkee 1994]

(a) $x^2+y^2+z^2+xyz=0$
(b) $x^2+y^2+z^2+2xyz=0$
(c) $x^2+y^2+z^2+xyz=1$
(d) $x^2+y^2+z^2+2xyz=1$

53. यदि $\tan^{-1}x-\tan^{-1}y=\tan^{-1}A$, तो $A=$ [MP PET 1988]

(a) $x-y$ (b) $x+y$
(c) $\frac{x-y}{1+xy}$ (d) $\frac{x+y}{1-xy}$

54. यदि $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$, तो

[Karnataka CET 1996]

(a) $x+y+z-xyz=0$ (b) $x+y+z+xyz=0$
(c) $xy+yz+zx+1=0$ (d) $xy+yz+zx-1=0$

55. यदि $\tan^{-1}\frac{x-1}{x+2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$, तो $x=$

(a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}$
(c) $\pm\sqrt{\frac{5}{2}}$ (d) $\pm\frac{1}{2}$

56. $\cos^{-1}\sqrt{1-x}+\sin^{-1}\sqrt{1-x}=$

(a) π (b) $\frac{\pi}{2}$
(c) 1 (d) 0

57. $\cos\left[2\cos^{-1}\frac{1}{5}+\sin^{-1}\frac{1}{5}\right]=$ [IIT 1981]

(a) $\frac{2\sqrt{6}}{5}$ (b) $-\frac{2\sqrt{6}}{5}$
(c) $\frac{1}{5}$ (d) $-\frac{1}{5}$

58. $\tan^{-1}\frac{a-b}{1+ab}+\tan^{-1}\frac{b-c}{1+bc}=$

(a) $\tan^{-1}a-\tan^{-1}b$ (b) $\tan^{-1}a-\tan^{-1}c$
(c) $\tan^{-1}b-\tan^{-1}c$ (d) $\tan^{-1}c-\tan^{-1}a$

59. यदि $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$, तो $x=$

[Roorkee 1978, 80; MNR 1986; Pb. CET 2001; Karnataka CET 2002]

(a) -1 (b) $\frac{1}{6}$
(c) $-1,\frac{1}{6}$ (d) इनमें से कोई नहीं

60. यदि $\cot^{-1}x+\tan^{-1}3=\frac{\pi}{2}$, तो $x=$

(a) 1/3 (b) 1/4
(c) 3 (d) 4

61. $2\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{24}{25}=$

(a) $\frac{\pi}{2}$ (b) $\frac{2\pi}{3}$
(c) $\frac{5\pi}{3}$ (d) इनमें से कोई नहीं

62. $\cos\left[\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}\right]=$

[MP PET 1991; MNR 1990]

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$
(c) $\frac{1}{2}$ (d) $\frac{\pi}{4}$

63. $\tan^{-1}x+\cot^{-1}(x+1)=$

(a) $\tan^{-1}(x^2+1)$ (b) $\tan^{-1}(x^2+x)$
(c) $\tan^{-1}(x+1)$ (d) $\tan^{-1}(x^2+x+1)$

64. $\cot^{-1}\frac{xy+1}{x-y}+\cot^{-1}\frac{yz+1}{y-z}+\cot^{-1}\frac{zx+1}{z-x}=$

(a) 0 (b) 1
(c) $\cot^{-1}x+\cot^{-1}y+\cot^{-1}z$ (d) इनमें से कोई नहीं

65. यदि $\tan^{-1}\frac{a+x}{a}+\tan^{-1}\frac{a-x}{a}=\frac{\pi}{6}$, तो $x^2=$

(a) $2\sqrt{3}a$ (b) $\sqrt{3}a$
(c) $2\sqrt{3}a^2$ (d) इनमें से कोई नहीं

66. यदि $\cos^{-1}\frac{3}{5}-\sin^{-1}\frac{4}{5}=\cos^{-1}x$, तो $x=$ [AMU 1978]

(a) 0 (b) 1
(c) –1 (d) 2

67. $\cot^{-1}3+\text{cosec}^{-1}\sqrt{5}=$

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

68. $\tan^{-1}\frac{1-x^2}{2x}+\cos^{-1}\frac{1-x^2}{1+x^2}=$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) π (d) 0

69. यदि $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$,तो $x=$

(a) $\pm\frac{1}{2}$ (b) $0,\frac{1}{2}$

(c) $0,-\frac{1}{2}$ (d) $0,\pm\frac{1}{2}$

70. यदि $\cos^{-1}x+\cos^{-1}y=2\pi$, तो $\sin^{-1}x+\sin^{-1}y=$

(a) π (b) $-\pi$

(c) $\frac{\pi}{2}$ (d) इनमें से कोई नहीं

71. $\sin^{-1}\frac{1}{\sqrt{5}}+\cot^{-1}3=$ **[MP PET 1993; Karnataka CET 1995]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

72. यदि $\cot^{-1}\alpha+\cot^{-1}\beta=\cot^{-1}x$, तो $x=$ **[MP PET 1992]**

(a) $\alpha+\beta$ (b) $\alpha-\beta$

(c) $\frac{1+\alpha\beta}{\alpha+\beta}$ (d) $\frac{\alpha\beta-1}{\alpha+\beta}$

73. यदि $\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\sin^{-1}\left(\frac{2b}{1+b^2}\right)=2\tan^{-1}x$, तो $x=$ **[MNR 1984; UPSEAT 1999; Pb. CET 2004]**

(a) $\frac{a-b}{1+ab}$ (b) $\frac{b}{1+ab}$

(c) $\frac{b}{1-ab}$ (d) $\frac{a+b}{1-ab}$

74. $\cos^{-1}\frac{1}{2}+2\sin^{-1}\frac{1}{2}=$ **[MP PET 1998; UPSEAT 2004]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

75. $\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{5}-\tan^{-1}\frac{8}{19}=$ **[AMU 1976, 77]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) इनमें से कोई नहीं

76. $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{70}+\tan^{-1}\frac{1}{99}=$ **[Roorkee 1981]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) इनमें से कोई नहीं

77. यदि $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$, तब $\cos^{-1}x+\cos^{-1}y=$ **[EAMCET 1994]**

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) π

78. $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)=$ **[EAMCET 1994]**

(a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$

(c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (d) $\tan^{-1}\left(\frac{1}{2}\right)$

79. $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)=$ **[EAMCET 1992]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{4}$ या $-\frac{3\pi}{4}$

80. यदि $\sin^{-1}\frac{x}{5}+\text{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$, तो $x=$ **[EAMCET 1983; Karnataka CET 2004]**

(a) 4 (b) 5

(c) 1 (d) 3

81. $2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)=$ **[EAMCET 1983]**

(a) $\tan^{-1}\left(\frac{49}{29}\right)$ (b) $\frac{\pi}{2}$

(c) 0 (d) $\frac{\pi}{4}$

82. $\cos^{-1}\left(\frac{15}{17}\right)+2\tan^{-1}\left(\frac{1}{5}\right)=$ **[EAMCET 1981]**

(a) $\frac{\pi}{2}$ (b) $\cos^{-1}\left(\frac{171}{221}\right)$

(c) $\frac{\pi}{4}$ (d) इनमें से कोई नहीं

83. $\sin^{-1}\left(\frac{3}{5}\right)+\tan^{-1}\left(\frac{1}{7}\right)=$ **[Karnataka CET 1994]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\cos^{-1}\left(\frac{4}{5}\right)$ (d) π

84. समीकरण $\tan^{-1}(1+x)+\tan^{-1}(1-x)=\frac{\pi}{2}$ का हल है **[Karnataka CET 1993]**

(a) $x=1$ (b) $x=-1$

(c) $x=0$ (d) $x=\pi$

85. यदि $x^2+y^2+z^2=r^2$, तब

$\tan^{-1}\left(\frac{xy}{zr}\right)+\tan^{-1}\left(\frac{yz}{xr}\right)+\tan\left(\frac{zx}{yr}\right)=$

(a) π (b) $\frac{\pi}{2}$

(c) 0 (d) इनमें से कोई नहीं

86. $(\sin^{-1}x)^3+(\cos^{-1}x)^3$ के अधिकतम व न्यूनतम मान हैं

(a) $-\frac{\pi}{2},\frac{\pi}{2}$ (b) $-\frac{\pi^3}{8},\frac{\pi^3}{8}$

(c) $\frac{7\pi^3}{8},\frac{\pi^3}{32}$ (d) इनमें से कोई नहीं

87. यदि $a<\frac{1}{32}$, $(\sin^{-1}x)^3+(\cos^{-1}x)^3=a\pi^3$ के हलों की संख्या है

(a) 0 (b) 1

(c) 2 (d) अनन्त

88. यदि $k\le\sin^{-1}x+\cos^{-1}x+\tan^{-1}x\le K$, तब

(a) $k=0,K=\pi$ (b) $k=0,K=\frac{\pi}{2}$

(c) $k=\frac{\pi}{2},K=\pi$ (d) इनमें से कोई नहीं

89. यदि $(\tan^{-1}x)^2+(\cot^{-1}x)^2=\frac{5\pi^2}{8}$, तब x बराबर है

(a) –1 (b) 1

(c) 0 (d) इनमें से कोई नहीं

90. यदि $\tan(x+y)=33$ तथा $x=\tan^{-1}3$, तब y होगा

(a) 0.3 (b) $\tan^{-1}(1.3)$

(c) $\tan^{-1}(0.3)$ (d) $\tan^{-1}\left(\frac{1}{18}\right)$

91. यदि $\tan^{-1}\frac{x-1}{x+1}+\tan^{-1}\frac{2x-1}{2x+1}=\tan^{-1}\frac{23}{36}$, तो $x=$

[ISM Dhanbad 1973]

(a) $\frac{3}{4},\frac{-3}{8}$ (b) $\frac{3}{4},\frac{3}{8}$

(c) $\frac{4}{3},\frac{3}{8}$ (d) इनमें से कोई नहीं

92. $\tan^{-1}\frac{c_1x-y}{c_1y+x}+\tan^{-1}\frac{c_2-c_1}{1+c_2c_1}+\tan^{-1}\frac{c_3-c_2}{1+c_3c_2}+...+\tan^{-1}\frac{1}{c_n}=$

(a) $\tan^{-1}\frac{y}{x}$ (b) $\tan^{-1}yx$

(c) $\tan^{-1}\frac{x}{y}$ (d) $\tan^{-1}(x-y)$

93. $\sin\left\{\sin^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{2}\right\}=$ [EAMCET 1985]

(a) 0 (b) –1

(c) 2 (d) 1

94. $\sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}=$ [ISM Dhanbad 1971]

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) इनमें से कोई नहीं

95. $\sin^{-1}x+\cos^{-1}x=$ [Pb. CET 1997; DCE 2002]

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) –1 (d) 1

96. $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=$

[MP PET 1997, 2003; UPSEAT 2000; Karnataka CET 2001; Pb. CET 2004]

(a) 0 (b) $\pi/4$

(c) $\pi/2$ (d) π

97. $\tan^{-1}\left(\frac{1}{11}\right)+\tan^{-1}\left(\frac{2}{12}\right)=$ [DCE 1999]

(a) $\tan^{-1}\left(\frac{33}{132}\right)$ (b) $\tan^{-1}\left(\frac{1}{2}\right)$

(c) $\tan^{-1}\left(\frac{132}{33}\right)$ (d) इनमें से कोई नहीं

98. यदि $\tan^{-1}x+2\cot^{-1}x=\frac{2\pi}{3}$, तो x का मान होगा

[Karnataka CET 1999]

(a) $\sqrt{2}$ (b) 3

(c) $\sqrt{3}$ (d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

99. यदि $\sin^{-1}x+\cot^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}$, तो x का मान होगा

[Roorkee 1999; Karnataka CET 1999]

(a) 0 (b) $\frac{1}{\sqrt{5}}$

(c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{3}}{2}$

100. यदि $\sin^{-1}a+\sin^{-1}b+\sin^{-1}c=\pi$, तब $a\sqrt{(1-a^2)}+b\sqrt{(1-b^2)}+c\sqrt{(1-c^2)}$ का मान होगा

[UPSEAT 1999]

(a) $2abc$ (b) abc

(c) $\frac{1}{2}abc$ (d) $\frac{1}{3}abc$

101. $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ का मान होगा

[Roorkee 2000]

(a) 0 (b) $\frac{\pi}{2}$

(c) $\frac{2\pi}{3}$ (d) $\frac{10\pi}{3}$

102. यदि $4\sin^{-1}x+\cos^{-1}x=\pi$, तब x का मान होगा

[UPSEAT 2001]

(a) 0 (b) $\frac{1}{2}$

(c) $-\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$

103. यदि $\sin^{-1}\frac{3}{5}+\cos^{-1}\left(\frac{12}{13}\right)=\sin^{-1}C$, तब C का मान होगा **[Pb. CET 1999]**

(a) $\frac{65}{56}$ (b) $\frac{24}{65}$

(c) $\frac{16}{65}$ (d) $\frac{56}{65}$

104. $\sin\left\{\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$ का मान होगा **[Kurukshetra CEE 2001]**

(a) 0 (b) 1

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

105. $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\sin^{-1}\left(\cos\frac{5\pi}{3}\right)$ का मान होगा **[UPSEAT 2003]**

(a) $\frac{\pi}{2}$ (b) $\frac{5\pi}{3}$

(c) $\frac{10\pi}{3}$ (d) 0

106. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)-\sin^{-1}\left(\frac{1}{2}\right)$ का मान होगा **[MP PET 2003]**

(a) 45° (b) 90°

(c) 15° (d) 30°

107. यदि $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi$, तब $xy+yz+zx$ का मान होगा **[Karnataka CET 2003]**

(a) 0 (b) 1

(c) 3 (d) –3

108. $\cos\left[\cos^{-1}\left(\frac{-1}{7}\right)+\sin^{-1}\left(\frac{-1}{7}\right)\right]=$ **[EAMCET 2003]**

(a) $-1/3$ (b) 0

(c) $1/3$ (d) $4/9$

109. $\tan\left[\sin^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)\right]$ का मान होगा **[AMU 2001]**

(a) $\frac{6}{17}$ (b) $\frac{6}{\sqrt{13}}$

(c) $\frac{\sqrt{13}}{5}$ (d) $\frac{17}{6}$

110. $\tan\left(\tan^{-1}\frac{1}{2}-\tan^{-1}\frac{1}{3}\right)$ का मान होगा **[AMU 2001]**

(a) $5/6$ (b) $7/6$

(c) $1/6$ (d) $1/7$

111. यदि $\cos^{-1}\sqrt{p}+\cos^{-1}\sqrt{1-p}+\cos^{-1}\sqrt{1-q}=\frac{3\pi}{4}$, तब q का मान होगा **[Karnataka CET 2002; Pb. CET 2000]**

(a) 1 (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

112. $\cot^{-1}[(\cos\alpha)^{1/2}]-\tan^{-1}[(\cos\alpha)^{1/2}]=x$, तब $\sin x$ का मान होगा **[AIEEE 2002]**

(a) $\tan^2\left(\frac{\alpha}{2}\right)$ (b) $\cot^2\left(\frac{\alpha}{2}\right)$

(c) $\tan\alpha$ (d) $\cot\left(\frac{\alpha}{2}\right)$

113. यदि $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$, तो $\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}=$ **[MP PET 1991]**

(a) 0 (b) 1

(c) $\frac{1}{xyz}$ (d) xyz

114. $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]=$

(a) $\frac{2a}{1+a^2}$ (b) $\frac{1-a^2}{1+a^2}$

(c) $\frac{2a}{1-a^2}$ (d) इनमें से कोई नहीं

115. यदि $A=\tan^{-1}x$, तो $\sin 2A=$ **[MNR 1988; UPSEAT 2000]**

(a) $\frac{2x}{\sqrt{1-x^2}}$ (b) $\frac{2x}{1-x^2}$

(c) $\frac{2x}{1+x^2}$ (d) इनमें से कोई नहीं

116. यदि $\cos(2\sin^{-1}x)=\frac{1}{9}$, तो x का मान होगा **[Roorkee 1975]**

(a) केवल $\frac{2}{3}$ (b) केवल $-\frac{2}{3}$

(c) $\frac{2}{3}, -\frac{2}{3}$ (d) न तो $\frac{2}{3}$ और न ही $-\frac{2}{3}$

117. यदि $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$, तो $x=$

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) इनमें से कोई नहीं

118. $\tan\left(2\cos^{-1}\frac{3}{5}\right)=$

(a) $\frac{7}{25}$ (b) $\frac{24}{25}$

(c) $-\frac{24}{7}$ (d) $\frac{8}{3}$

119. $\tan\left[2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right]=$ **[IIT 1984]**

(a) $\frac{17}{7}$ (b) $-\frac{17}{7}$

(c) $\frac{7}{17}$ (d) $-\frac{7}{17}$

120. यदि $2\cos^{-1}\sqrt{\frac{1+x}{2}}=\frac{\pi}{2}$, तो $x=$

(a) 1 (b) 0

(c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

121. $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right] =$ **[Roorkee 1986]**

(a) $\frac{3-\sqrt{5}}{2}$ (b) $\frac{3+\sqrt{5}}{2}$

(c) $\frac{2}{3-\sqrt{5}}$ (d) $\frac{2}{3+\sqrt{5}}$

122. $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) =$

(a) $\cot^{-1}\sqrt{x}$ (b) $\tan^{-1}\sqrt{x}$

(c) $\tan^{-1} x$ (d) $\cot^{-1} x$

123. $\sin\left(4\tan^{-1}\frac{1}{3}\right) =$

(a) $\frac{12}{25}$ (b) $\frac{24}{25}$

(c) $\frac{1}{5}$ (d) इनमें से कोई नहीं

124. $3\tan^{-1} a =$ **[MP PET 1993]**

(a) $\tan^{-1}\frac{3a+a^3}{1+3a^2}$ (b) $\tan^{-1}\frac{3a-a^3}{1+3a^2}$

(c) $\tan^{-1}\frac{3a+a^3}{1-3a^2}$ (d) $\tan^{-1}\frac{3a-a^3}{1-3a^2}$

125. $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239} =$ **[MNR 1995]**

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

126. यदि $3\sin^{-1}\frac{2x}{1-x^2}-4\cos^{-1}\frac{1-x^2}{1+x^2}+2\tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{3}$ तो $x =$

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) 1 (d) इनमें से कोई नहीं

127. $\sin\left(2\tan^{-1}\left(\frac{1}{3}\right)\right)+\cos(\tan^{-1}2\sqrt{2})$ का मान होगा **[AMU 1999]**

(a) $\frac{16}{15}$ (b) $\frac{14}{15}$

(c) $\frac{12}{15}$ (d) $\frac{11}{15}$

128. $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$ का मान होगा **[Karnataka CET 2003]**

(a) $\frac{1}{\sqrt{10}}$ (b) $-\frac{1}{\sqrt{10}}$

(c) $\frac{1}{10}$ (d) $-\frac{1}{10}$

129. $\cos^{-1}(\cos 12)-\sin^{-1}(\sin 14)$ का मान है **[J & K 2005]**

(a) -2 (b) $8\pi-26$

(c) $4\pi+2$ (d) इनमें से कोई नहीं

130. $\cos^{-1}\left(\frac{3+5\cos x}{5+3\cos x}\right)$ का मान है **[Kerala (Engg.) 2005]**

(a) $\tan^{-1}\left(\frac{1}{2}\tan\frac{x}{2}\right)$ (b) $2\tan^{-1}\left(2\tan\frac{x}{2}\right)$

(c) $\frac{1}{2}\tan^{-1}\left(2\tan\frac{x}{2}\right)$ (d) $2\tan^{-1}\left(\frac{1}{2}\tan\frac{x}{2}\right)$

(e) $\tan^{-1}\left(\tan\frac{x}{2}\right)$

131. यदि $\cos^{-1} x-\cos^{-1}\frac{y}{2}=\alpha$, हो तो $4x^2-4xy\cos\alpha+y^2 =$ **[AIEEE 2005]**

(a) $4\sin^2\alpha$ (b) $-4\sin^2\alpha$

(c) $2\sin 2\alpha$ (d) 4

132. यदि $\tan^{-1} x+\tan^{-1} y=\frac{\pi}{4}$ है, तो **[Karnataka CET 2005]**

(a) $x+y+xy=1$ (b) $x+y-xy=1$

(c) $x+y+xy+1=0$ (d) $x+y-xy+1=0$

133. यदि $\sin^{-1}(1-x)-2\sin^{-1} x=\frac{\pi}{2}$, तो x का मान है **[Orissa JEE 2005]**

(a) $\left(0,-\frac{1}{2}\right)$ (b) $\left(\frac{1}{2},0\right)$

(c) $\{0\}$ (d) $(-1,0)$

134. यदि ΔABC में $\angle A=90^o$ हो, तो

$\tan^{-1}\left(\frac{c}{a+b}\right)+\tan^{-1}\left(\frac{b}{a+c}\right) =$ **[Kerala (Engg.) 2005]**

(a) 0 (b) 1

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(e) $\frac{\pi}{8}$

135. $\sin^{-1} x-\sin^{-1} 2x=\pm\frac{\pi}{3}$ का हल है **[Karnataka CET 2005]**

(a) $\pm\frac{1}{3}$ (b) $\pm\frac{1}{4}$

(c) $\pm\frac{\sqrt{3}}{2}$ (d) $\pm\frac{1}{2}$

136. समीकरण $\cos^{-1} x+\cos^{-1} 2x+\pi=0$ के वास्तविक हलों की संख्या हैं **[Orissa JEE 2005]**

(a) 1 (b) 2

(c) 0 (d) ∞

137. $\sin\left[3\sin^{-1}\left(\frac{1}{5}\right)\right] =$ **[Kerala (Engg.) 2005]**

(a) $\frac{71}{125}$ (b) $\frac{74}{125}$

(c) $\frac{3}{5}$ (d) $\frac{1}{2}$

(e) $-\frac{3}{5}$

Critical Thinking

Objective Questions

1. $\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]=$ **[MNR 1986]**

(a) $\pi - x$ (b) $2\pi - x$

(c) $\frac{x}{2}$ (d) $\pi - \frac{x}{2}$

2. $\sin^{-1}\left[\sin\left(\frac{2\pi}{3}\right)\right]$ का मुख्य मान है **[IIT 1986]**

(a) $-\frac{2\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $\frac{4\pi}{3}$ (d) इनमें से कोई नहीं

3. यदि $\theta = \tan^{-1} a, \phi = \tan^{-1} b$ तथा $ab = -1$, तो $\theta - \phi =$

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) इनमें से कोई नहीं

4. यदि $\tan(\cos^{-1} x) = \sin\left(\cot^{-1}\frac{1}{2}\right)$, तो $x =$

(a) $\pm\frac{5}{3}$ (b) $\pm\frac{\sqrt{5}}{3}$

(c) $\pm\frac{5}{\sqrt{3}}$ (d) इनमें से कोई नहीं

5. यदि किसी $x \in (-1,1)$ के लिये $\sin^{-1} x = \frac{\pi}{5}$, तब $\cos^{-1} x =$ **[IIT 1992]**

(a) $\frac{3\pi}{10}$ (b) $\frac{5\pi}{10}$

(c) $\frac{7\pi}{10}$ (d) $\frac{9\pi}{10}$

6. यदि $\cos^{-1} p + \cos^{-1} q + \cos^{-1} r = \pi$ तो $p^2 + q^2 + r^2 + 2pqr =$ **[Karnataka CET 2004]**

(a) 3 (b) 1

(c) 2 (d) –1

7. $\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right]+\tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right]=$ **[MP PET 1999]**

(a) $\frac{2a}{b}$ (b) $\frac{2b}{a}$

(c) $\frac{a}{b}$ (d) $\frac{b}{a}$

8. $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]=$

(a) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1} x^2$ (b) $\frac{\pi}{4}+\cos^{-1} x^2$

(c) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1} x$ (d) $\frac{\pi}{4}-\frac{1}{2}\cos^{-1} x^2$

9. समीकरण $\sin^{-1} x - \cos^{-1} x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ रखता है

(a) कोई हल नहीं (b) अद्वितीय हल

(c) अनन्त हल (d) इनमें से कोई नहीं

10. $\sum_{m=1}^{n}\tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$ बराबर है

(a) $\tan^{-1}\left(\frac{n^2+n}{n^2+n+2}\right)$ (b) $\tan^{-1}\left(\frac{n^2-n}{n^2-n+2}\right)$

(c) $\tan^{-1}\left(\frac{n^2+n+2}{n^2+n}\right)$ (d) इनमें से कोई नहीं

11. $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ के वास्तविक हलों की संख्या है **[IIT 1999]**

(a) शून्य (b) एक

(c) दो (d) अनन्त

12. समीकरण $2\cos^{-1} x + \sin^{-1} x = \frac{11\pi}{6}$ के हल होंगे **[AMU 1999]**

(a) कोई हल नहीं (b) केवल एक हल

(c) दो हल (d) तीन हल

13. यदि $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, तब $x + y + z =$ **[Kerala (Engg.) 2002]**

(a) xyz (b) 0

(c) 1 (d) $2xyz$

14. $2\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right]=$ **[Dhanbad Engg. 1976]**

(a) $\cos^{-1}\left(\frac{a\cos\theta+b}{a+b\cos\theta}\right)$

(b) $\cos^{-1}\left(\frac{a+b\cos\theta}{a\cos\theta+b}\right)$

(c) $\cos^{-1}\left(\frac{a\cos\theta}{a+b\cos\theta}\right)$

(d) $\cos^{-1}\left(\frac{b\cos\theta}{a\cos\theta+b}\right)$

15. यदि $2\tan^{-1}(\cos x) = \tan^{-1}(\text{cosec}^2 x)$, तब x का मान होगा **[UPSEAT 2002]**

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

Answers

प्रतिलोम त्रिकोणमितीय फलन

1	d	2	d	3	a	4	b	5	c
6	b	7	b	8	c	9	b	10	b
11	c	12	a	13	c	14	d	15	c
16	b	17	c	18	b	19	a	20	c
21	d	22	b	23	c	24	a	25	b
26	b	27	d	28	d	29	d	30	c
31	b	32	b	33	c	34	c	35	a
36	a	37	b	38	a	39	c	40	c
41	b	42	c	43	d	44	a	45	a
46	a	47	d	48	c	49	b	50	d
51	a	52	d	53	c	54	d	55	c
56	b	57	b	58	b	59	b	60	c
61	a	62	a	63	d	64	a	65	c
66	b	67	b	68	b	69	d	70	b
71	b	72	d	73	d	74	d	75	a
76	c	77	b	78	a,d	79	c	80	d
81	d	82	d	83	a	84	c	85	b
86	c	87	a	88	a	89	a	90	c
91	d	92	c	93	d	94	a	95	b
96	b	97	d	98	c	99	b	100	a
101	a	102	b	103	d	104	b	105	a
106	d	107	c	108	b	109	d	110	d
111	d	112	a	113	b	114	c	115	c
116	c	117	b	118	c	119	d	120	b
121	a,d	122	b	123	b	124	d	125	d
126	b	127	b	128	a	129	a	130	d
131	a	132	b	133	c	134	c	135	d
136	c	137	a						

Critical Thinking Questions

1	d	2	d	3	c	4	b	5	a
6	b	7	b	8	a	9	b	10	a
11	c	12	a	13	a	14	a	15	d

Answers and Solutions

प्रतिलोम त्रिकोणमितीय फलन

1. (d) दिया गया है कि, $\cos^{-1}\left(\frac{1}{x}\right) = \theta \Rightarrow \cos\theta = \frac{1}{x}$

अब, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{1-(1/x)^2}}{1/x} = \sqrt{x^2-1}$.

2. (d) माना $\cot^{-1} x = \theta \Rightarrow x = \cot\theta$

अब $\text{cosec}\,\theta = \sqrt{1+\cot^2\theta} = \sqrt{1+x^2}$

$\therefore \sin\theta = \frac{1}{\text{cosec}\,\theta} = \frac{1}{\sqrt{1+x^2}} \Rightarrow \theta = \sin^{-1}\frac{1}{\sqrt{1+x^2}}$

$\therefore \sin(\cot^{-1} x) = \sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)$

$= \frac{1}{\sqrt{1+x^2}} = (1+x^2)^{-1/2}$.

3. (a) माना $\sin^{-1}\frac{5}{13} = x \Rightarrow \sin x = \frac{5}{13}$

$\Rightarrow \cos x = \sqrt{1-\frac{25}{169}} = \frac{12}{13}$

$\Rightarrow \cos\left(\sin^{-1}\frac{5}{13}\right) = \cos\left(\cos^{-1}\frac{12}{13}\right) = \frac{12}{13}$.

4. (b) $\cot^{-1}(-\sqrt{3}) = \pi - \cot^{-1}(\sqrt{3}) = \frac{5\pi}{6}$.

5. (c) माना $\sin^{-1} x = \theta \Rightarrow \sin\theta = x$

अब $1+\cot^2\theta = \text{cosec}^2\theta = \frac{1}{x^2}$

अतः $1+\cot^2(\sin^{-1} x) = \frac{1}{x^2}$.

6. (b) दिया गया है कि, $\sin^{-1}\frac{1}{2} = \tan^{-1} x$

$\Rightarrow \tan^{-1} x = \frac{\pi}{6} \Rightarrow x = \tan 30^o = \frac{1}{\sqrt{3}}$.

7. (b) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = \tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right]$

($x = \tan\theta$ रखने पर)

$= \tan^{-1}\left[\frac{\sec\theta-1}{\tan\theta}\right] = \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$

$= \tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right] = \tan^{-1}\tan\frac{\theta}{2} = \frac{\theta}{2} = \frac{1}{2}\tan^{-1} x$.

8. (c) $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}} = \tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$ ($x = a\sin\theta$ रखने पर)

$= \tan^{-1}(\tan\theta) = \theta = \sin^{-1}\left(\frac{x}{a}\right)$.

9. (b) माना $\theta = \tan^{-1} x \Rightarrow x = \tan\theta$

$\therefore \cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{1}{\sqrt{1+x^2}}$

अतः $\cos\theta = \cos(\tan^{-1} x) = \frac{1}{\sqrt{1+x^2}}$.

10. (b) $\tan\left(\sec^{-1}\sqrt{1+x^2}\right) = \tan\left(\sec^{-1}\sqrt{1+\tan^2\theta}\right)$

($x = \tan\theta$ रखने पर)

$= \tan(\sec^{-1}\sec\theta) = \tan\theta = x$.

11. (c) $\sec^{-1}[\sec(-30^o)] = \sec^{-1}(\sec 30^o) = 30^o$.

12. (a) $\tan^{-1}\left[\frac{\cos x}{1+\sin x}\right] = \tan^{-1}\left[\frac{\sin(\pi/2-x)}{1+\cos(\pi/2-x)}\right]$

$= \tan^{-1}\left[\frac{2\sin(\pi/4-x/2)\cos(\pi/4-x/2)}{2\cos^2(\pi/4-x/2)}\right]$

$= \tan^{-1}\tan\left(\frac{\pi}{4}-\frac{x}{2}\right) = \frac{\pi}{4}-\frac{x}{2}$.

13. (c) $\tan^{-1}\frac{1}{\sqrt{x^2-1}} = \tan^{-1}\frac{1}{\sqrt{\text{cosec}^2\theta-1}}$

($x = \text{cosec}\ \theta$ रखने पर)

$= \tan^{-1}\frac{1}{\cot\theta} = \theta = \text{cosec}^{-1} x$.

14. (d) $\sin^{-1}\left(-\frac{1}{2}\right)$ का मुख्य मान $= \sin^{-1}\sin(-30^o) = -\frac{\pi}{6}$.

15. (c) माना $\tan^{-1} 2 = \alpha \Rightarrow \tan\alpha = 2$

और $\cot^{-1} 3 = \beta \Rightarrow \cot\beta = 3$

$\sec^2(\tan^{-1} 2) + \text{cosec}^2(\cot^{-1} 3)$

$= \sec^2\alpha + \text{cosec}^2\alpha = 1+\tan^2\alpha + 1+\cot^2\alpha$

$= 2+(2)^2+(3)^2 = 15$.

16. (b) माना $x = \sin\theta$ तथा $\sqrt{x} = \sin\phi$

अतः $\sin^{-1}(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2})$

$= \sin^{-1}(\sin\theta\sqrt{1-\sin^2\phi} - \sin\phi\sqrt{1-\sin^2\theta})$

$= \sin^{-1}(\sin\theta\cos\phi - \sin\phi\cos\theta) = \sin^{-1}\sin(\theta-\phi)$

$= \theta-\phi = \sin^{-1}(x) - \sin^{-1}(\sqrt{x})$.

17. (c) हमें ज्ञात है कि, $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1} x$

$\Rightarrow \tan^{-1}\left[\frac{1-\tan\theta}{1+\tan\theta}\right] = \frac{1}{2}\theta$ ($x = \tan\theta$ रखने पर)

$\Rightarrow \tan^{-1}\left[\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right] = \frac{\theta}{2}$

$\Rightarrow \tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right) = \frac{\theta}{2} \Rightarrow \frac{\pi}{4}-\theta = \frac{\theta}{2}$

$\Rightarrow \theta = \frac{\pi}{6} = \tan^{-1} x \Rightarrow x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$.

18. (b) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left\{\cos\left(\pi+\frac{\pi}{6}\right)\right\}$

$= \cos^{-1}\left(-\cos\frac{\pi}{6}\right) = \pi - \cos^{-1}\cos\frac{\pi}{6} = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

19. (a) माना $\cos^{-1} x = \theta \Rightarrow x = \cos\theta \Rightarrow \sec\theta = \frac{1}{x}$

$\Rightarrow \tan\theta = \sqrt{\sec^2\theta - 1} = \sqrt{\frac{1}{x^2}-1} = \frac{1}{x}\sqrt{1-x^2}$

अब $\sin\cot^{-1}\tan\theta = \sin\cot^{-1}\left(\frac{1}{x}\sqrt{1-x^2}\right)$

साथ ही, $x = \sin\theta$ रखने पर,

$\sin\cot^{-1}\left(\frac{1}{x}\sqrt{1-x^2}\right) = \sin\cot^{-1}\left(\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}\right)$

$= \sin\cot^{-1}(\cot\theta) = \sin\theta = x$.

20. (c) $x = a\tan^2\theta$ रखने पर

$\sin^{-1}\frac{\sqrt{x}}{\sqrt{x+a}} = \sin^{-1}\frac{\sqrt{a}\sqrt{\tan^2\theta}}{\sqrt{a\tan^2\theta+a}} = \sin^{-1}\frac{\sqrt{a}\tan\theta}{\sqrt{a}\sec\theta}$

$= \sin^{-1}\sin\theta = \theta = \tan^{-1}\left(\sqrt{\frac{x}{a}}\right)$.

21. (d) $\sin^{-1}\frac{1}{5} + \cos^{-1} x = \frac{\pi}{2}$

$\therefore \sin^{-1}\frac{1}{5} = \frac{\pi}{2} - \cos^{-1} x = \sin^{-1} x$

$\therefore x = \frac{1}{5}$.

22. (b) स्पष्टतः $x = \sin(\theta+\beta)$ और $y = \sin(\theta-\beta)$

$\therefore 1+xy = 1+\sin(\theta+\beta)\sin(\theta-\beta)$

$= 1+\sin^2\theta - \sin^2\beta = \sin^2\theta + \cos^2\beta$.

23. (c) $\sin^{-1}\frac{1}{3} + \sin^{-1}\frac{2}{3}$

$= \sin^{-1}\left[\frac{1}{3}\sqrt{1-\frac{4}{9}} + \frac{2}{3}\sqrt{1-\frac{1}{9}}\right] = \sin^{-1}\left[\frac{\sqrt{5}+4\sqrt{2}}{9}\right]$

अतः $x = \frac{\sqrt{5}+4\sqrt{2}}{9}$.

24. (a) माना $\cos^{-1} x = \theta$ तब $x = \cos\theta$

$\Rightarrow \tan\theta = \sqrt{\sec^2\theta - 1} = \sqrt{\frac{1}{x^2}-1} = \sqrt{\frac{1-x^2}{x}}$

$\therefore \tan(\cos^{-1} x) = \tan\theta = \frac{\sqrt{1-x^2}}{x}$.

25. (b) यह स्पष्ट है।

26. (b) $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$.

27. (d) $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right] = \cot\left[\cot^{-1}\left(\frac{7}{24}\right)\right] = \frac{7}{24}$.

28. (d) हमें ज्ञात है कि, $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$

$\Rightarrow \frac{-\pi}{2} \le x - \pi \le \frac{\pi}{2} \Rightarrow \frac{-\pi}{2} \le \pi - x \le \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\{\sin(\pi - x)\} = \pi - x$.

29. (d) यह दिया गया है कि, $\pi \le x \le 2\pi$

$\Rightarrow -\pi \ge -x \ge -2\pi \Rightarrow \pi \ge 2\pi - x \ge 0$

$\Rightarrow \cos^{-1}\{\cos(2\pi - x)\} = 2\pi - x$.

30. (c) चूँकि $3\pi < 10 < 3\pi + \frac{\pi}{2} \Rightarrow 0 < 10 - 3\pi < \frac{\pi}{2}$

$\Rightarrow \frac{-\pi}{2} < 3\pi - 10 < 0 \Rightarrow \sin^{-1}\{\sin(3\pi - 10)\} = 3\pi - 10$.

31. (b) हमें ज्ञात है कि

$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1} 1 - \tan^{-1} x = \frac{\pi}{4} - \tan^{-1} x$

चूँकि $0 \le x \le 1 \Rightarrow 0 \le \tan^{-1} x \le \frac{\pi}{4}$

$\Rightarrow 0 \ge -\tan^{-1} x \ge \frac{-\pi}{4} \Rightarrow \frac{\pi}{4} \ge \frac{\pi}{4} - \tan^{-1} x \ge 0$

$\Rightarrow \frac{\pi}{4} \ge \tan^{-1}\left(\frac{1-x}{1+x}\right) \ge 0$.

32. (b) माना $\sin^{-1} x = y$ तब $x = \sin y$

चूँकि $-1 \le x \le 0$, अतः $\frac{-\pi}{2} \le \sin^{-1} x \le 0$

अतः $\frac{-\pi}{2} \le y \le 0$

हमें ज्ञात है कि, $\cos y = \sqrt{1 - \sin^2 y}$

$\Rightarrow \cos y = \sqrt{1 - x^2}$, $0 \le y \le \pi$ के लिए .....(i)

अब $-\frac{\pi}{2} \le y \le 0 \Rightarrow \frac{\pi}{2} \ge -y \ge 0$

$\Rightarrow \cos(-y) = \sqrt{1 - x^2}$ {(i) से}

$\Rightarrow -y = \cos^{-1}\sqrt{1 - x^2} \Rightarrow y = -\cos^{-1}\sqrt{1 - x^2}$.

33. (c) $\left[\sin\left(\tan^{-1}\frac{3}{4}\right)\right]^2 = \left[\sin\left(\sin^{-1}\frac{3}{5}\right)\right]^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$.

34. (c) $\sin^{-1}\left(\sin\frac{5\pi}{3}\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$.

35. (a) दिया गया है, $\tan^{-1} x = \sin^{-1}\left[\frac{3}{\sqrt{10}}\right]$

$\Rightarrow x = \tan\left\{\sin^{-1}\left[\frac{3}{\sqrt{10}}\right]\right\} = \tan\{\tan^{-1} 3\} \Rightarrow x = 3$.

36. (a) हम जानते है कि $\sec(\text{cosec}\, x) = \text{cosec}\,(\sec x)$

$= \frac{|x|}{\sqrt{x^2 - 1}}$, $|x| > 1$ के लिये।

37. (b) यह स्पष्ट है।

38. (a) $\theta = \sin^{-1}[\sin(-600^o)]$

$\Rightarrow \theta = \sin^{-1}[-(\sin 240^o)] = \sin^{-1}[-\sin(180^o + 60^o)]$

$\Rightarrow \theta = \sin^{-1}\sin 60^o = \sin^{-1}\left[\sin\left(\frac{\pi}{3}\right)\right] = \frac{\pi}{3}$.

39. (c) $\sin^{-1} x = 2\tan^{-1} x \Rightarrow \sin^{-1} x = \sin^{-1}\frac{2x}{1+x^2}$

$\Rightarrow \frac{2x}{1+x^2} = x \Rightarrow x^3 - x = 0$

$\Rightarrow x(x+1)(x-1) = 0 \Rightarrow x = \{-1, 1, 0\}$.

40. (c) $\cos[\tan^{-1}(\tan 2)] = \cos 2$.

41. (b) $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{\pi}{2}$

$\sin^{-1} x = \alpha$, $\sin^{-1} y = \beta$, $\sin^{-1} z = \gamma$ रखने पर

$\therefore \alpha + \beta + \gamma = \frac{\pi}{2}$, (दिया है)

या $\alpha + \beta = \frac{\pi}{2} - \gamma$ या $\cos(\alpha + \beta) = \cos\left(\frac{\pi}{2} - \gamma\right)$

$\cos\alpha\cos\beta - \sin\alpha\sin\beta = \sin\gamma$ .....(i)

ओर हमें ज्ञात है कि, $\sin\alpha = x \Rightarrow \cos\alpha = \sqrt{1 - x^2}$

इसी तरह $\cos\beta = \sqrt{1 - y^2}$

$\therefore$ समीकरण (i) से हमें $\sqrt{1-x^2}.\sqrt{1-y^2} = xy + z$ प्राप्त होता है।

दोनों पक्षों का वर्ग करने पर, $x^2 + y^2 + z^2 + 2xyz = 1$.

42. (c) $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] = \cos\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$= \cos\cos^{-1}\sqrt{1 - \frac{3}{4}} = \frac{1}{2}$.

43. (d) $\sin[\cot^{-1}(\cos\tan^{-1} x)]$

$= \sin\left[\cot^{-1}\left(\cos\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right]$

$= \sin\left[\cot^{-1}\frac{1}{\sqrt{1+x^2}}\right] = \sin\left[\sin^{-1}\sqrt{\frac{1+x^2}{2+x^2}}\right] = \sqrt{\frac{1+x^2}{2+x^2}}$.

44. (a) $\sin[\cot^{-1}(x+1)] = \sin\left(\sin^{-1}\frac{1}{\sqrt{x^2+2x+2}}\right)$

$= \frac{1}{\sqrt{x^2+2x+2}}$

$\cos(\tan^{-1} x) = \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right) = \frac{1}{\sqrt{1+x^2}}$

अतः $\frac{1}{\sqrt{x^2+2x+2}} = \frac{1}{\sqrt{1+x^2}}$

$\Rightarrow x^2 + 2x + 2 = 1 + x^2 \Rightarrow x = -\frac{1}{2}$.

45. (a) $\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{3}{5} = \tan^{-1}\left[\frac{\sqrt{\left(1-\frac{16}{25}\right)}}{\frac{4}{5}}\right] + \tan^{-1}\frac{3}{5}$

$\left[\text{चूँकि } \cos^{-1}x = \tan^{-1}\frac{\sqrt{(1-x^2)}}{x}\right]$

$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} = \tan^{-1}\left(\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{3}{4}.\frac{3}{5}}\right) = \tan^{-1}\left(\frac{27}{11}\right)$.

46. (a) $\sin^{-1}x + \sin^{-1}\frac{1}{x} + \cos^{-1}x + \cos^{-1}\frac{1}{x}$

$= \{\sin^{-1}(x) + \cos^{-1}(x)\} + \left\{\sin^{-1}\left(\frac{1}{x}\right) + \cos^{-1}\left(\frac{1}{x}\right)\right\}$

$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$.

47. (d) $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2} = \tan^{-1}\left(\frac{\frac{2}{3}}{1-\frac{1}{9}}\right) + \tan^{-1}\left(\frac{1}{2}\right)$

$= \tan^{-1}\left(\frac{\frac{2}{3}}{\frac{8}{9}}\right) + \tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{2}\right)$

$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{3}{4}}{1-\frac{1}{2}\times\frac{3}{4}}\right) = \tan^{-1}(2)$.

48. (c) $\tan\left(90^o - \cot^{-1}\frac{1}{3}\right) = \cot.\cot^{-1}\frac{1}{3} = \frac{1}{3}$.

49. (b) $\tan\left[\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right]$

$= \tan\left[\tan^{-1}\frac{\sqrt{\left(1-\frac{16}{25}\right)}}{\frac{4}{5}} + \tan^{-1}\frac{2}{3}\right]$

$= \tan\left[\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}.\frac{2}{3}}\right)\right] = \tan.\tan^{-1}\frac{17}{6} = \frac{17}{6}$.

50. (d) $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3$

$= \tan^{-1}1 + \pi + \tan^{-1}\left(\frac{5}{-5}\right) = \tan^{-1}1 + \pi - \tan^{-1}1 = \pi$.

51. (a) माना $\cot^{-1}\frac{3}{4} = \theta \Rightarrow \cot\theta = \frac{3}{4}$

तथा $\sin\theta = \frac{1}{\sqrt{1+\cot^2\theta}} = \frac{1}{\sqrt{1+(9/16)}} = \frac{4}{5}$

अतः $\cot^{-1}\frac{3}{4} + \sin^{-1}\frac{5}{13} = \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13}$

$= \sin^{-1}\left[\frac{4}{5}.\sqrt{1-\frac{25}{169}} + \frac{5}{13}.\sqrt{1-\frac{16}{25}}\right]$

$= \sin^{-1}\left[\frac{4}{5}.\frac{12}{13} + \frac{5}{13}.\frac{3}{5}\right] = \sin^{-1}\left[\frac{48+15}{65}\right] = \sin^{-1}\frac{63}{65}$.

52. (d) दिया गया है कि, $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$

$\Rightarrow \cos^{-1}(x) + \cos^{-1}(y) + \cos^{-1}(z) = \cos^{-1}(-1)$

$\Rightarrow \cos^{-1}(x) + \cos^{-1}(y) = \cos^{-1}(-1) - \cos^{-1}(z)$

$\Rightarrow \cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2}) = \cos^{-1}\{(-1)(z)\}$

$\Rightarrow xy - \sqrt{(1-x^2)(1-y^2)} = -z$

$\Rightarrow (xy+z) = \sqrt{(1-x^2)(1-y^2)}$

दोनों पक्षों का वर्ग करने पर, $x^2 + y^2 + z^2 + 2xyz = 1$.

ट्रिक : $x = y = z = \frac{1}{2}$ रखने पर,

$\cos^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2} = \pi$

स्पष्ट है कि *x, y, z* के इन मानों के लिये विकल्प (d) सही है।

53. (c) दिया गया है कि $\tan^{-1}x - \tan^{-1}y = \tan^{-1}A$

$\Rightarrow \tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}A$.

अतः $A = \frac{x-y}{1+xy}$.

54. (d) दिया गया है कि, $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-xz}\right] = \frac{\pi}{2}$

$\Rightarrow \left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right] = \tan\frac{\pi}{2} = \frac{1}{0}$

अतः $xy + yz + zx - 1 = 0$.

ट्रिक : $x = y = z = \frac{1}{\sqrt{3}}$,

अतः $\tan^{-1}\frac{1}{\sqrt{3}} + \tan^{-1}\frac{1}{\sqrt{3}} + \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{2}$

स्पष्ट है कि *x, y, z* के इन मानों के लिये विकल्प (d) सही है।

55. (c) हमें ज्ञात है कि $\tan^{-1}\frac{x-1}{x+2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left[\frac{\frac{x-1}{x+2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x+2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \frac{\pi}{4}$

$\Rightarrow \left[\frac{2x(x+2)}{x^2+4+4x-x^2+1}\right] = \tan\frac{\pi}{4}$

$\Rightarrow \frac{2x(x+2)}{4x+5} = \tan\frac{\pi}{4} = 1$

$\Rightarrow 2x^2 + 4x = 4x + 5 \Rightarrow x = \pm\sqrt{\frac{5}{2}}$.

56. (b) $\cos^{-1}\sqrt{1-x} + \sin^{-1}\sqrt{1-x} = \sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \frac{\pi}{2}$.

57. (b) $\cos\left(\cos^{-1}\frac{1}{5} + \sin^{-1}\frac{1}{5} + \cos^{-1}\frac{1}{5}\right) = \cos\left(\frac{\pi}{2} + \cos^{-1}\frac{1}{5}\right)$

$= -\sin\left(\cos^{-1}\frac{1}{5}\right) = -\sin\left(\sin^{-1}\sqrt{\frac{24}{25}}\right) = -\frac{2\sqrt{6}}{5}$.

58. (b) $\tan^{-1}\left(\frac{a-b}{1+ab}\right) + \tan^{-1}\left(\frac{b-c}{1+bc}\right)$

$= \tan^{-1}(a) - \tan^{-1}(b) + \tan^{-1}(b) - \tan^{-1}(c)$

$= \tan^{-1}(a) - \tan^{-1}(c)$.

59. (b) $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{2x+3x}{1-(2x)(3x)}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \tan^{-1}(1) \Rightarrow \frac{5x}{1-6x^2} = 1$

$\Rightarrow 1 - 6x^2 = 5x \Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow (x+1)\left(x - \frac{1}{6}\right) = 0 \Rightarrow x = -1, \frac{1}{6}$

किन्तु -1 संतुष्ट नहीं करता है।

ट्रिक : विकल्प की जाँच करें। स्पष्टतः $x = 1/6$ के लिये समीकरण संतुष्ट करता है किन्तु $x = -1$ के लिये नहीं।

60. (c) हमें ज्ञात हैं कि, $\cot^{-1}x + \tan^{-1}3 = \frac{\pi}{2}$

$\Rightarrow \cot^{-1}x + \tan^{-1}3 = \frac{\pi}{2} \Rightarrow \tan^{-1}\frac{1}{x} + \tan^{-1}3 = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{x}+3}{1-\frac{1}{x}.3}\right) = \tan^{-1}\left(\frac{1}{0}\right)$

$\Rightarrow \frac{3x+1}{x-3} = \frac{1}{0} \Rightarrow x = 3$

वैकल्पिक: जैसा कि हम जानते हैं कि

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$, अतः दिये गये प्रश्न के लिये $x = 3$ होना चाहिए।

61. (a) $2\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{24}{25} = \sin^{-1}2\times\frac{3}{5}\sqrt{1-\frac{9}{25}} + \cos^{-1}\frac{24}{25}$

$= \sin^{-1}\frac{24}{25} + \cos^{-1}\frac{24}{25} = \frac{\pi}{2}$.

62. (a) $\cos\left[\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}\right] = \cos\left[\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\times\frac{1}{2}}\right)\right]$

$= \cos\{\tan^{-1}(1)\} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$.

63. (d) $\tan^{-1}x + \cot^{-1}(x+1) = \tan^{-1}x + \tan^{-1}\left(\frac{1}{x+1}\right)$

$= \tan^{-1}\left[\frac{x+\frac{1}{x+1}}{1-\frac{x}{x+1}}\right] = \tan^{-1}(x^2 + x + 1)$.

64. (a) $\cot^{-1}\frac{xy+1}{x-y} + \cot^{-1}\frac{yz+1}{y-z} + \cot^{-1}\frac{zx+1}{z-x}$

$= \cot^{-1}y - \cot^{-1}x + \cot^{-1}z - \cot^{-1}y$

$+ \cot^{-1}x - \cot^{-1}z = 0$.

नोट : विद्यार्थी इसे सूत्र की तरह याद रखें।

65. (c) दिया गया समीकरण $\tan^{-1}\frac{a+x}{a} + \tan^{-1}\frac{a-x}{a} = \frac{\pi}{6}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{a+x}{a}+\frac{a-x}{a}}{1-\frac{a+x}{a}.\frac{a-x}{a}}\right) = \frac{\pi}{6}$

$\Rightarrow \frac{2a^2}{x^2} = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} \Rightarrow x^2 = 2\sqrt{3}a^2$.

66. (b) $\cos^{-1}\frac{3}{5} - \sin^{-1}\frac{4}{5} = \cos^{-1}x$

$\Rightarrow \cos^{-1}\frac{3}{5} - \cos^{-1}\sqrt{1-\frac{16}{25}} = \cos^{-1}x$

$\Rightarrow \cos^{-1}\frac{3}{5} - \cos^{-1}\frac{3}{5} = \cos^{-1}x$

$\Rightarrow \cos^{-1}x = 0 \Rightarrow x = 1$.

67. (b) $\cot^{-1}3 + \text{cosec}^{-1}\sqrt{5} = \cot^{-1}3 + \cot^{-1}2$

$= \cot^{-1}\left(\frac{3\times2-1}{3+2}\right) = \cot^{-1}(1) = \frac{\pi}{4}$.

68. (b) $x = \tan\theta$ रखने पर,

$\tan^{-1}\frac{1-x^2}{2x} + \cos^{-1}\frac{1-x^2}{1+x^2}$

$= \tan^{-1}\left(\frac{1-\tan^2\theta}{2\tan\theta}\right) + \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$= \tan^{-1}(\cot 2\theta) + \cos^{-1}(\cos 2\theta) = \frac{\pi}{2} - 2\theta + 2\theta = \frac{\pi}{2}$.

69. (d) $\tan^{-1}(x-1) + \tan^{-1}(x) + \tan^{-1}(x+1) = \tan^{-1}3x$

$\Rightarrow \tan^{-1}(x-1) + \tan^{-1}(x) = \tan^{-1}3x - \tan^{-1}(x+1)$

$\Rightarrow \tan^{-1}\left[\frac{(x-1)+x}{1-(x-1)(x)}\right] = \tan^{-1}\left[\frac{3x-(x+1)}{1+3x(x+1)}\right]$

$\Rightarrow \frac{2x-1}{1-x^2+x} = \frac{2x-1}{1+3x^2+3x}$

$\Rightarrow (1-x^2+x)(2x-1) = (1+3x^2+3x)(2x-1)$

हल करने के पश्चात् $x = 0, \pm\frac{1}{2}$.

70. (b) $\cos^{-1}x + \cos^{-1}y = 2\pi$

$\Rightarrow \frac{\pi}{2} - \sin^{-1}x + \frac{\pi}{2} - \sin^{-1}y = 2\pi$

$\Rightarrow \pi - (\sin^{-1} x + \sin^{-1} y) = 2\pi$

$\Rightarrow \sin^{-1} x + \sin^{-1} y = -\pi$

71. (b) $\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1} 3 = \cot^{-1}\left(\frac{\sqrt{1-\frac{1}{5}}}{\frac{1}{\sqrt{5}}}\right) + \cot^{-1} 3$

$= \cot^{-1}(2) + \cot^{-1}(3) = \cot^{-1}\left(\frac{2\times 3-1}{3+2}\right) = \cot^{-1}(1) = \frac{\pi}{4}$.

72. (d) दिया गया है कि, $\cot^{-1}\alpha + \cot^{-1}\beta = \cot^{-1} x$

$\Rightarrow \cot^{-1}\left(\frac{\alpha\beta-1}{\alpha+\beta}\right) = \cot^{-1} x \Rightarrow x = \frac{\alpha\beta-1}{\alpha+\beta}$.

73. (d) $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \sin^{-1}\left(\frac{2b}{1+b^2}\right) = 2\tan^{-1} x$

$a = \tan\theta$ और $b = \tan\phi$ रखने पर

अतः $\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) + \sin^{-1}\left(\frac{2\tan\phi}{1+\tan^2\phi}\right) = 2\tan^{-1} x$

$\Rightarrow \sin^{-1}\sin(2\theta) + \sin^{-1}\sin(2\phi) = 2\tan^{-1} x$

$\Rightarrow 2(\theta+\phi) = 2\tan^{-1} x$

अतः $x = \tan(\theta+\phi) \Rightarrow x = \frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$

इन मानों को प्रतिस्थापित करने पर, $x = \frac{a+b}{1-ab}$.

74. (d) $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2} = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{2\pi}{3}$.

75. (a) $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19}$

$= \tan^{-1}\left[\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{3}{4}\times\frac{3}{5}}\right] - \tan^{-1}\frac{8}{19} = \tan^{-1}\frac{27}{11} - \tan^{-1}\frac{8}{19}$

$= \tan^{-1}\left[\frac{\frac{27}{11}-\frac{8}{19}}{1+\frac{27}{11}\times\frac{8}{19}}\right] = \tan^{-1}\left(\frac{425}{425}\right) = \tan^{-1}(1) = \frac{\pi}{4}$.

76. (c) $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99}$

$= 2\tan^{-1}\left[\frac{\frac{2}{5}}{1-\frac{1}{25}}\right] - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99}$

$= 2\tan^{-1}\left(\frac{5}{12}\right) - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99}$

$= \tan^{-1}\left[\frac{\frac{5}{6}}{1-\frac{25}{144}}\right] - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99}$

$= \tan^{-1}\left(\frac{120}{119}\right) - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99}$

$= \tan^{-1}\left(\frac{120}{119}\right) + \tan^{-1}\left[\frac{\frac{1}{99}-\frac{1}{70}}{1+\frac{1}{99}.\frac{1}{70}}\right]$

$= \tan^{-1}\left(\frac{120}{119}\right) + \tan^{-1}\left(\frac{-29}{6931}\right)$

$= \tan^{-1}\frac{120}{119} - \tan^{-1}\frac{29}{6931} = \tan^{-1}\frac{120}{119} - \tan^{-1}\frac{1}{239}$

$= \tan^{-1}\left[\frac{\frac{120}{119}-\frac{1}{239}}{1+\frac{120}{119}\times\frac{1}{239}}\right] = \tan^{-1}(1) = \frac{\pi}{4}$.

77. (b) $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$

$\Rightarrow \frac{\pi}{2} - \cos^{-1} x + \frac{\pi}{2} - \cos^{-1} y = \frac{2\pi}{3}$

$\Rightarrow \cos^{-1} x + \cos^{-1} y = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$.

78. (a,d) $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \tan^{-1}\left(\frac{(1/4)+(2/9)}{1-(1/4)\times(2/9)}\right)$

$= \tan^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}.2\tan^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\tan^{-1}\frac{2(1/2)}{1-(1/4)}$

$= \frac{1}{2}\tan^{-1}\frac{4}{3} = \frac{1}{2}\cos^{-1}\frac{3}{5}$.

79. (c) $\tan^{-1}\frac{x}{y} - \tan^{-1}\left(\frac{x-y}{x+y}\right) = \tan^{-1}\frac{x}{y} - \tan^{-1}\left(\frac{1-y/x}{1+y/x}\right)$

$= \tan^{-1}\frac{x}{y} - \left(\tan^{-1}1 - \tan^{-1}\frac{y}{x}\right)$

$= \tan^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x} - \frac{\pi}{4}$

$= \tan^{-1}\frac{x}{y} + \cot^{-1}\frac{x}{y} - \frac{\pi}{4} = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$.

80. (d) $\sin^{-1}\frac{x}{5} + \text{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2} \Rightarrow \sin^{-1}\frac{x}{5} = \frac{\pi}{2} - \text{cosec}^{-1}\frac{5}{4}$

$\Rightarrow \sin^{-1}\frac{x}{5} = \frac{\pi}{2} - \sin^{-1}\frac{4}{5} \Rightarrow \sin^{-1}\frac{x}{5} = \cos^{-1}\frac{4}{5}$

$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \sin^{-1}\frac{3}{5} \Rightarrow x = 3$.

81. (d) $2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{2(1/3)}{1-(1/9)}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7} = \tan^{-1}\left(\frac{(3/4)+(1/7)}{1-(3/4)\times(1/7)}\right)$

$= \tan^{-1}\left(\frac{25}{25}\right) = \frac{\pi}{4}$.

82. (d) $\cos^{-1}\left(\frac{15}{17}\right) + 2\tan^{-1}\left(\frac{1}{5}\right)$

$= \cos^{-1}\left(\frac{15}{17}\right) + \cos^{-1}\left(\frac{1-1/25}{1+1/25}\right)$

$= \cos^{-1}\left(\frac{15}{17}\right) + \cos^{-1}\left(\frac{12}{13}\right)$

$$= \cos^{-1}\left(\frac{15}{17}\times\frac{12}{13}-\sqrt{1-\left(\frac{15}{17}\right)^2}\sqrt{1-\left(\frac{12}{13}\right)^2}\right)$$

$$= \cos^{-1}\left(\frac{140}{221}\right).$$

83. (a) $\sin^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{7}$

$$=\tan^{-1}\left(\frac{(3/4)+(1/7)}{1-(3/4)\times(1/7)}\right)=\tan^{-1}\left(\frac{25}{25}\right)=\tan^{-1}1=\frac{\pi}{4}.$$

84. (c) $\tan^{-1}(1+x)+\tan^{-1}(1-x)=\frac{\pi}{2}$

$$\Rightarrow \tan^{-1}(1+x)=\frac{\pi}{2}-\tan^{-1}(1-x)$$

$$\Rightarrow \tan^{-1}(1+x)=\cot^{-1}(1-x)$$

$$\Rightarrow \tan^{-1}(1+x)=\tan^{-1}\left(\frac{1}{1-x}\right)$$

$$\Rightarrow 1+x=\frac{1}{1-x}\Rightarrow 1-x^2=1\Rightarrow x=0.$$

85. (b) $\tan^{-1}\left(\frac{xy}{zr}\right)+\tan^{-1}\left(\frac{yz}{xr}\right)+\tan^{-1}\left(\frac{xz}{yr}\right)$

$$=\tan^{-1}\left[\frac{\frac{xy}{zr}+\frac{yz}{xr}+\frac{xz}{yr}-\frac{xyz}{r^3}}{1-\left(\frac{x^2+y^2+z^2}{r^2}\right)}\right]=\tan^{-1}\infty=\frac{\pi}{2}.$$

86. (c) हमें ज्ञात है कि, $(\sin^{-1}x)^3+(\cos^{-1}x)^3$

$$=(\sin^{-1}x+\cos^{-1}x)^3-3\sin^{-1}x\cos^{-1}x(\sin^{-1}x+\cos^{-1}x)$$

$$=\frac{\pi^3}{8}-3(\sin^{-1}x\cos^{-1}x)\frac{\pi}{2}$$

$$=\frac{\pi^3}{8}-\frac{3\pi}{2}\sin^{-1}x\left(\frac{\pi}{2}-\sin^{-1}x\right)$$

$$=\frac{\pi^3}{8}-\frac{3\pi^2}{4}\sin^{-1}x+\frac{3\pi}{2}(\sin^{-1}x)^2$$

$$=\frac{\pi^3}{8}+\frac{3\pi}{2}\left[(\sin^{-1}x)^2-\frac{\pi}{2}\sin^{-1}x\right]$$

$$=\frac{\pi^3}{8}+\frac{3\pi}{2}\left[\left(\sin^{-1}x-\frac{\pi}{4}\right)^2\right]-\frac{3\pi^3}{32}$$

$$=\frac{\pi^3}{32}+\frac{3\pi}{2}\left(\sin^{-1}x-\frac{\pi}{4}\right)^2$$

$\therefore$ न्यूनतम मान $\frac{\pi^3}{32}$ है और चूँकि $\left(\sin^{-1}x-\frac{\pi}{4}\right)^2\le\left(\frac{3\pi}{4}\right)^2$

$\therefore$ महत्तम मान $\frac{\pi^3}{32}+\frac{9\pi^2}{16}\times\frac{3\pi}{2}=\frac{7\pi^3}{8}$ है।

87. (a) पिछले हल द्वारा $\frac{\pi^3}{32}\le(\sin^{-1}x)^3+(\cos^{-1}x)^3\le\frac{7\pi^3}{8}$

यहाँ $a<\frac{1}{32}$. अतः हलों की संख्या शून्य है।

88. (a) हमें ज्ञात है कि,

$$\sin^{-1}x+\cos^{-1}x+\tan^{-1}x=\frac{\pi}{2}+\tan^{-1}x$$

चूँकि $\frac{-\pi}{2}\le\tan^{-1}x\le\frac{\pi}{2}\Rightarrow 0\le\frac{\pi}{2}+\tan^{-1}x\le\pi$

$\therefore\ K=\pi, k=0$.

89. (a) $(\tan^{-1}x)^2+(\cot^{-1}x)^2=\frac{5\pi^2}{8}$

$$\Rightarrow (\tan^{-1}x+\cot^{-1}x)^2-2\tan^{-1}x\left(\frac{\pi}{2}-\tan^{-1}x\right)=\frac{5\pi^2}{8}$$

$$\Rightarrow \frac{\pi^2}{4}-2\times\frac{\pi}{2}\tan^{-1}x+2(\tan^{-1}x)^2=\frac{5\pi^2}{8}$$

$$\Rightarrow 2(\tan^{-1}x)^2-\pi\tan^{-1}x-\frac{3\pi^2}{8}=0$$

$$\Rightarrow \tan^{-1}x=-\frac{\pi}{4},\frac{3\pi}{4}\Rightarrow \tan^{-1}x=-\frac{\pi}{4}\Rightarrow x=-1.$$

90. (c) $x+y=\tan^{-1}33\ \Rightarrow y=\tan^{-1}33-\tan^{-1}3$

$$=\tan^{-1}\frac{33-3}{1+99}=\tan^{-1}\frac{30}{100}\Rightarrow y=\tan^{-1}(0.3).$$

91. (d) $\frac{x-1}{x+1}+\frac{2x-1}{2x+1}=\frac{23}{36}\left[1-\frac{(x-1)(2x-1)}{(x+1)(2x+1)}\right]$

$$\Rightarrow 24x^2-23x-12=0\Rightarrow(3x-4)(8x+3)=0.$$

92. (c) $\tan^{-1}\left(\frac{c_1x-y}{c_1y+x}\right)+\tan^{-1}\left(\frac{c_2-c_1}{1+c_2c_1}\right)+\tan^{-1}\left(\frac{c_3-c_2}{1+c_3c_2}\right)+$

$$\ldots\ldots+\tan^{-1}\frac{1}{c_n}$$

$$=\tan^{-1}\left(\frac{\frac{x}{y}-\frac{1}{c_1}}{1+\frac{x}{y}.\frac{1}{c_1}}\right)+\tan^{-1}\left(\frac{\frac{1}{c_1}-\frac{1}{c_2}}{1+\frac{1}{c_1c_2}}\right)$$

$$+\tan^{-1}\left(\frac{\frac{1}{c_2}-\frac{1}{c_3}}{1+\frac{1}{c_2c_3}}\right)+\ldots\ldots+\tan^{-1}\frac{1}{c_n}$$

$$=\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{1}{c_1}+\tan^{-1}\frac{1}{c_1}-\tan^{-1}\frac{1}{c_2}+\tan^{-1}\frac{1}{c_2}$$

$$-\tan^{-1}\frac{1}{c_3}+\ldots+\tan^{-1}\frac{1}{c_{n-1}}-\tan^{-1}\frac{1}{c_n}+\tan^{-1}\frac{1}{c_n}$$

$$=\tan^{-1}\left(\frac{x}{y}\right).$$

93. (d) $\sin\left(\sin^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{2}\right)=\sin\frac{\pi}{2}=1.$

94. (a) $\sin^{-1}\frac{4}{5}=\tan^{-1}\frac{4}{3}, 2\tan^{-1}\frac{1}{3}=\tan^{-1}\frac{3}{4}=\cot^{-1}\frac{4}{3}$

और $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}.$

95. (b) यह गुणधर्म है।

96. (b) $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}\right)$

$=\tan^{-1}\left(\frac{5}{5}\right)=\tan^{-1}1=\frac{\pi}{4}$.

97. (d) $\tan^{-1}\left(\frac{1}{11}\right)+\tan^{-1}\left(\frac{2}{12}\right)$

$=\tan^{-1}\left(\frac{\frac{1}{11}+\frac{2}{12}}{1-\frac{1}{11}\times\frac{2}{12}}\right)=\tan^{-1}\left(\frac{12+22}{130}\right)$

$=\tan^{-1}\left(\frac{34}{130}\right)=\tan^{-1}\left(\frac{17}{65}\right)$.

98. (c) दिये गये समीकरण को निम्न प्रकार से लिखा जा सकता है

$\tan^{-1}x+\cot^{-1}x+\cot^{-1}x=\frac{2\pi}{3}$

$\Rightarrow \cot^{-1}x=\frac{2\pi}{3}-\frac{\pi}{2}=\frac{\pi}{6}\Rightarrow x=\sqrt{3}$.

99. (b) $\because \cot^{-1}\frac{1}{2}=\cos^{-1}\frac{1}{\sqrt{5}}$

अतः दिये गये समीकरण को निम्न प्रकार से लिखा जा सकता है

$\sin^{-1}x+\cos^{-1}\frac{1}{\sqrt{5}}=\frac{\pi}{2}$

$\Rightarrow x=\frac{1}{\sqrt{5}}$.

100. (a) माना $\sin^{-1}a=A$,

$\sin^{-1}b=B$,

$\sin^{-1}c=C$

$\therefore \sin A=a, \sin B=b, \sin C=c$

तथा $A+B+C=\pi$, तब

$\sin 2A+\sin 2B+\sin 2C$

$=4\sin A\sin B\sin C$ .....(i)

$\Rightarrow \sin A\cos A+\sin B\cos B+\sin C\cos C$

$=2\sin A\sin B\sin C$

$\Rightarrow \sin A\sqrt{(1-\sin^2 A)}+\sin B\sqrt{(1-\sin^2 B)}+\sin C\sqrt{1-\sin^2 C}$

$=2\sin A\sin B\sin C.$ .....(ii)

$\Rightarrow a\sqrt{(1-a^2)}+b\sqrt{(1-b^2)}+c\sqrt{(1-c)^2}=2abc$,

जबकि $\sin^{-1}a+\sin^{-1}b+\sin^{-1}c=\pi$.

101. (a) $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$

$=\cos^{-1}\left[\cos\left(2\pi-\frac{\pi}{3}\right)\right]+\sin^{-1}\left[\sin\left(2\pi-\frac{\pi}{3}\right)\right]$

$=\frac{\pi}{3}-\frac{\pi}{3}=0$.

102. (b) हम जानते हैं कि, $4\sin^{-1}x+\cos^{-1}x=\pi$

$\Rightarrow 3\sin^{-1}x+\sin^{-1}x+\cos^{-1}x=\pi$

$\Rightarrow 3\sin^{-1}x=\pi-\frac{\pi}{2}=\frac{\pi}{2}$

$\Rightarrow \sin^{-1}x=\pi/6\Rightarrow x=\sin\frac{\pi}{6}=\frac{1}{2}$.

103. (d) दिया गया है कि, $\sin^{-1}C=\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{12}{13}$

$\Rightarrow C=\sin\left(\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{12}{13}\right)$

$\sin(A+B)=\sin A\cos B+\cos A\sin B$ का प्रयोग करने पर,

$\Rightarrow C=\frac{3}{5}\times\frac{12}{13}+\sqrt{1-\frac{9}{25}}\sqrt{1-\frac{144}{169}}\Rightarrow C=\frac{56}{65}$.

104. (b) $\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$

$x=\tan\theta$ रखने पर,

$\sin\left[\tan^{-1}\left(\frac{1-\tan^2\theta}{2\tan\theta}\right)+\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)\right]$

$=\sin[\tan^{-1}(\cot 2\theta)+\cos^{-1}(\cos 2\theta)]$

$=\sin[\tan^{-1}\tan(\pi/2-2\theta)+\cos^{-1}\cos 2\theta]$

$=\sin\frac{\pi}{2}=1$.

105. (a) $\cos^{-1}\left[\cos\frac{5\pi}{3}\right]+\sin^{-1}\left[\frac{\cos 5\pi}{3}\right]=\frac{\pi}{2}$

$\left(\because \sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}\right)$.

106. (d) $\sin^{-1}\left[\frac{\sqrt{3}}{2}\right]-\sin^{-1}\left[\frac{1}{2}\right]=60^o-30^o=30^o$.

107. (c) दिया गया है, $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi$

$\because 0\le\cos^{-1}x\le\pi$

$\therefore 0\le\cos^{-1}y\le\pi$ और $0\le\cos^{-1}z\le\pi$

यहाँ $\cos^{-1}x=\cos^{-1}y=\cos^{-1}z=\pi$

$\Rightarrow x = y = z = \cos\pi = -1$

$\therefore\ xy + yz + zx = (-1)(-1) + (-1)(-1) + (-1)(-1)$

$= 1 + 1 + 1 = 3$.

108. (b) $\cos\left\{\cos^{-1}\left(\frac{-1}{7}\right) + \sin^{-1}\left(\frac{-1}{7}\right)\right\} = \cos\frac{\pi}{2} = 0$.

109. (d) $\tan\left[\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)\right]$

$= \tan\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right) = \tan\left(\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right)$

$= \tan\left[\tan^{-1}\frac{17}{12}\times\frac{12}{6}\right] = \frac{17}{6}$.

110. (d) $\tan\left[\tan^{-1}\frac{1}{2} - \tan^{-1}\frac{1}{3}\right] = \tan\left[\tan^{-1}\frac{\frac{1}{2}-\frac{1}{3}}{1+\frac{1}{6}}\right]$

$= \tan\tan^{-1}\left(\frac{1}{6}\times\frac{6}{7}\right) = \frac{1}{7}$.

111. (d) माना $\alpha = \cos^{-1}\sqrt{p};\ \beta = \cos^{-1}\sqrt{1-p}$

और $\gamma = \cos^{-1}\sqrt{1-q}$ or $\cos\alpha = \sqrt{p};\ \cos\beta = \sqrt{1-p}$

और $\cos\gamma = \sqrt{1-q}$.

अतः $\sin\alpha = \sqrt{1-p},\ \sin\beta = \sqrt{p}$ तथा $\sin\gamma = \sqrt{q}$.

दी गयी समीकरण को लिखा जा सकता है :

$\alpha + \beta + \gamma = \frac{3\pi}{4}$ या $\alpha + \beta = \frac{3\pi}{4} - \gamma$

या $\cos(\alpha+\beta) = \cos\left(\frac{3\pi}{4} - \gamma\right)$

$\Rightarrow \cos\alpha\ \cos\beta - \sin\alpha\sin\beta =$

$$\cos\left\{\pi - \left(\frac{\pi}{4} + \gamma\right)\right\} = -\cos\left(\frac{\pi}{4} + \gamma\right)$$

$\Rightarrow \sqrt{p}\sqrt{1-p} - \sqrt{1-p}\sqrt{p} = -\left(\frac{1}{\sqrt{2}}\sqrt{1-q} - \frac{1}{\sqrt{2}}\cdot\sqrt{q}\right)$

$\Rightarrow 0 = \sqrt{1-q} - \sqrt{q} \Rightarrow 1 - q = q \Rightarrow q = \frac{1}{2}$.

112. (a) $\tan^{-1}\left[\frac{1}{\sqrt{\cos\alpha}}\right] - \tan^{-1}\left[\sqrt{\cos\alpha}\right] = x$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{1}{\sqrt{\cos\alpha}} - \sqrt{\cos\alpha}}{1 + \frac{\sqrt{\cos\alpha}}{\sqrt{\cos\alpha}}}\right] = x \Rightarrow \tan x = \frac{1-\cos\alpha}{2\sqrt{\cos\alpha}}$$

$$\therefore \sin x = \frac{1-\cos\alpha}{1+\cos\alpha} = \frac{2\sin^2\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}} = \tan^2\left(\frac{\alpha}{2}\right).$$

113. (b) $\tan^{-1}(x) + \tan^{-1}(y) + \tan^{-1}(z) = \pi$

$\Rightarrow \tan^{-1}x + \tan^{-1}y = \pi - \tan^{-1}z$

$\Rightarrow \frac{x+y}{1-xy} = -z \Rightarrow x + y = -z + xyz$

$\Rightarrow x + y + z = xyz$

xyz से भाग देने पर,

$\frac{1}{yz} + \frac{1}{xz} + \frac{1}{xy} = 1$.

नोट : विद्यार्थी इस प्रश्न को सूत्र की तरह याद रखें।

114. (c) $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$

$= \tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)\right]$

(माना $a = \tan\theta$)

$= \tan\left[\frac{1}{2}\sin^{-1}(\sin 2\theta) + \frac{1}{2}\cos^{-1}(\cos 2\theta)\right]$

$= \tan(2\theta) = \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2a}{1-a^2}$

ट्रिक : $a = 0$ रखने पर $\tan(0+0) = 0$ जो कि विकल्प (a) व (c) द्वारा दिया जाता है। साथ ही $a = 1$ रखने पर, $\tan\left(\frac{\pi}{4} + \frac{\pi}{4}\right) = \infty$ जो कि विकल्प (c) द्वारा दिया जाता है।

115. (c) दिया गया है कि $A = \tan^{-1}x$

अब $x = \tan A \Rightarrow \sin 2A = \frac{2\tan A}{1+\tan^2 A} = \frac{2x}{1+x^2}$.

116. (c) $\cos(2\sin^{-1}x) = \frac{1}{9}$

$\Rightarrow \cos(\sin^{-1}2x\sqrt{1-x^2}) = \frac{1}{9}$

$\Rightarrow \cos(\cos^{-1}\sqrt{1-4x^2+4x^4}) = \frac{1}{9}$

$\Rightarrow 1 - 2x^2 = \frac{1}{9} \Rightarrow 2x^2 = 1 - \frac{1}{9} = \frac{8}{9}$

$\Rightarrow x^2 = \frac{4}{9} \Rightarrow x = \pm\frac{2}{3}$.

117. (b) हमें ज्ञात है कि, $2\tan^{-1}(\cos x) = \tan^{-1}(2\text{cosec}\, x)$

$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\,\text{cosec}\ x)$

$\frac{2\cos x}{\sin^2 x} = 2\text{cosec}\ x \Rightarrow 2\cos x = 2\sin x$

या $\sin x = \cos x$

$\Rightarrow x = \frac{\pi}{4}$.

118. (c) $\tan\left(2\cos^{-1}\frac{3}{5}\right) = \tan\left[\cos^{-1}\left(2.\frac{9}{25}-1\right)\right]$

$\{\because 2\cos^{-1}x = \cos^{-1}(2x^2-1)\}$

$= \tan\cos^{-1}\left(-\frac{7}{25}\right) = \tan\tan^{-1}\left[\sqrt{\frac{1-\frac{49}{625}}{-\frac{7}{25}}}\right] = -\frac{24}{7}$

अतः $\tan\left(2\cos^{-1}\frac{3}{5}\right) = -\frac{24}{7}$.

119. (d) $\tan\left[2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right] = \tan\left[\tan^{-1}\frac{\frac{2}{5}}{1-\frac{1}{25}} - \tan^{-1}(1)\right]$

$= \tan\left[\tan^{-1}\frac{5}{12} - \tan^{-1}(1)\right] = \tan\tan^{-1}\left(\frac{\frac{5}{12}-1}{1+\frac{5}{12}}\right) = -\frac{7}{17}$.

120. (b) दी गयी समीकरण है, $2\cos^{-1}\sqrt{\left(\frac{1+x}{2}\right)} = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}\sqrt{\left(\frac{1+x}{2}\right)} = \frac{\pi}{4} \Rightarrow \cos\frac{\pi}{4} = \frac{\sqrt{1+x}}{\sqrt{2}}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\sqrt{1+x}}{\sqrt{2}} \Rightarrow 1 = \sqrt{1+x} \Rightarrow x = 0$.

121. (a, d) $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$

माना $\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3} = \theta \Rightarrow \cos 2\theta = \frac{\sqrt{5}}{3}$

किन्तु $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} \Rightarrow \frac{\sqrt{5}}{3} = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\Rightarrow \sqrt{5}+\sqrt{5}\tan^2\theta = 3-3\tan^2\theta$

$\Rightarrow (\sqrt{5}+3)\tan^2\theta = 3-\sqrt{5} \Rightarrow \tan^2\theta = \frac{3-\sqrt{5}}{3+\sqrt{5}}$

$\Rightarrow \tan^2\theta = \frac{(3-\sqrt{5})^2}{4} \Rightarrow \tan\theta = \frac{3-\sqrt{5}}{2}$

परिमेयीकरण करने पर,

$\Rightarrow \tan\theta = \frac{3-\sqrt{5}}{2}\times\frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{2}{3+\sqrt{5}}$.

122. (b) माना $x = \tan^2\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$

अब, $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

$= \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$= \frac{1}{2}\cos^{-1}\cos 2\theta = \frac{2\theta}{2} = \theta = \tan^{-1}\sqrt{x}$.

123. (b) $\sin\left(4\tan^{-1}\frac{1}{3}\right) = \sin\left[2\tan^{-1}\left(\frac{2/3}{1-(1/9)}\right)\right]$

$= \sin\left[2\tan^{-1}\frac{3}{4}\right] = \sin\sin^{-1}\left(\frac{2\times(3/4)}{1+(9/16)}\right)$

$= \frac{3}{2}\times\frac{16}{25} = \frac{24}{25}$ $\left(\because 2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}\right)$

124. (d) यह स्पष्ट है।

125. (d) चूँकि $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$

$\therefore\ 4\tan^{-1}\frac{1}{5} = 2\left[2\tan^{-1}\frac{1}{5}\right] = 2\tan^{-1}\frac{\frac{2}{5}}{1-\frac{1}{25}}$

$= 2\tan^{-1}\frac{10}{24} = \tan^{-1}\frac{\frac{20}{24}}{1-\frac{100}{576}} = \tan^{-1}\frac{120}{119}$

$4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239} = \tan^{-1}\frac{120}{119} - \tan^{-1}\frac{1}{239}$

$= \tan^{-1}\frac{\frac{120}{119}-\frac{1}{239}}{1+\frac{120}{119}.\frac{1}{239}} = \tan^{-1}\frac{(120\times239)-119}{(119\times239)+120}$

$\Rightarrow \tan^{-1}1 = \frac{\pi}{4}$.

126. (b) $3\sin^{-1}\frac{2x}{1+x^2} - 4\cos^{-1}\frac{1-x^2}{1+x^2} + 2\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$

$x = \tan\theta$ रखने पर,

$3\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) - 4\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$+ 2\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = \frac{\pi}{3}$

$\Rightarrow 3\sin^{-1}(\sin 2\theta) - 4\cos^{-1}(\cos 2\theta)$

$+ 2\tan^{-1}(\tan 2\theta) = \frac{\pi}{3}$

$\Rightarrow 3(2\theta) - 4(2\theta) + 2(2\theta) = \frac{\pi}{3} \Rightarrow 6\theta - 8\theta + 4\theta = \frac{\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{6} \Rightarrow \tan^{-1} x = \frac{\pi}{6} \Rightarrow x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

127. (b) $\sin\left[2\tan^{-1}\left(\frac{1}{3}\right)\right] + \cos[\tan^{-1}(2\sqrt{2})]$

$= \sin\left[\tan^{-1}\frac{2/3}{1-1/9}\right] + \cos[\tan^{-1}(2\sqrt{2})]$

$= \sin[\tan^{-1} 3/4] + \cos[\tan^{-1} 2\sqrt{2}]$

$= \frac{3}{5} + \frac{1}{3} = \frac{14}{15}$.

128. (a) माना $\cos^{-1}\frac{4}{5} = x \Rightarrow \cos x = \frac{4}{5}$ .....(i)

अब $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right) = \sin\left(\frac{x}{2}\right)$ .....(ii)

समीकरण (i) से, $\cos x = \frac{4}{5} \Rightarrow 1 - 2\sin^2\frac{x}{2} = \frac{4}{5}$

$\Rightarrow 2\sin^2\frac{x}{2} = 1 - \frac{4}{5} = \frac{1}{5} \Rightarrow \sin\frac{x}{2} = \sqrt{\frac{1}{10}}$.

129. (a) $\cos^{-1}(\cos 12) - \sin^{-1}(\sin 14) \Rightarrow 12 - 14 = -2$.

130. (d) $x = \frac{\pi}{2}$ लेने पर $\cos x = 0$

$\cos^{-1}\left(\frac{3+5\cos x}{5+3\cos x}\right) = \cos^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{4}{3}\right)$

$x = \frac{\pi}{2}$, $2\tan^{-1}\left(\frac{1}{2}\tan\frac{x}{2}\right)$ में रखने पर

$2\tan^{-1}\left(\frac{1}{2}\tan\frac{\pi}{4}\right)$

$= 2\tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}\left(\frac{2.\frac{1}{2}}{1-\frac{1}{4}}\right) = \tan^{-1}\left(\frac{4}{3}\right)$.

131. (a) यदि $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \theta$

तब $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\theta + \frac{y^2}{b^2} = \sin^2\theta$

यहाँ $\cos^{-1}\frac{x}{1} + \cos^{-1}\frac{y}{2} = \alpha$;

$\therefore \frac{x^2}{1} - \frac{2xy}{2}\cos\alpha + \frac{y^2}{4} = \sin^2\alpha$

$x^2 - xy\cos\alpha + \frac{y^2}{4} = \sin^2\alpha$

$4x^2 - 4xy\cos\alpha + y^2 = 4\sin^2\alpha$.

132. (b) $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$; $\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan^{-1} 1$

$\frac{x+y}{1-xy} = 1$; $x + y + xy = 1$.

133. (c) $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}(1-x) = \left(\frac{\pi}{2} - 2\sin^{-1} x\right)$

$\Rightarrow 1 - x = \sin\left(\frac{\pi}{2} - 2\sin^{-1} x\right)$

$\Rightarrow 1 - x = \sin\frac{\pi}{2}\cos(2\sin^{-1} x) - \cos\frac{\pi}{2}\sin(2\sin^{-1} x)$

$\Rightarrow 1 - x = \cos(2\sin^{-1} x)^2$

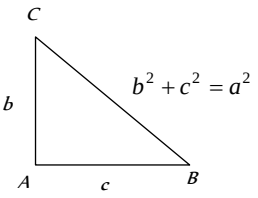
$\Rightarrow 1 - x = \cos\cos^{-1}(1-2x) \Rightarrow 2x^2 - x = 0$

$x = 0$, $x = 1/2$ जोकि दिये गये समीकरण को संतुष्ट नहीं करता हैं।

$\therefore x = 0$ एकमात्र हल है।

134. (c) $\angle A = 90^o$

$\tan^{-1}\left(\frac{c}{a+b}\right) + \tan^{-1}\left(\frac{b}{a+c}\right)$



$= \tan^{-1}\left[\frac{\frac{c}{a+b} + \frac{b}{a+c}}{1 - \left(\frac{c}{a+b}\right)\left(\frac{b}{a+c}\right)}\right]$

$= \tan^{-1}\left[\frac{ca + c^2 + ab + b^2}{a^2 + ab + ca + bc - bc}\right]$

$= \tan^{-1}\left[\frac{a^2 + ab + ca}{a^2 + ab + ca}\right] = \tan^{-1}(1) = \frac{\pi}{4}$.

135. (d) $\sin^{-1} 2x = \sin^{-1} x - \sin^{-1}\frac{\sqrt{3}}{2}$

$\sin^{-1} 2x = \sin^{-1}\left(x\sqrt{\left(1-\frac{3}{4}\right)} - \frac{\sqrt{3}}{2}\sqrt{1-x^2}\right)$

$2x = \left(\frac{x}{2} - \frac{\sqrt{3}}{2}\sqrt{1-x^2}\right)$

$\frac{\sqrt{3}}{2}\sqrt{1-x^2} = \frac{x}{2} - 2x = \frac{-3x}{2}$

$\frac{3(1-x^2)}{4} = \frac{9x^2}{4}$

$\Rightarrow 3 - 3x^2 = 9x^2 \Rightarrow x^2 = \frac{1}{4} \Rightarrow x = \pm\frac{1}{2}$.

136. (c) $\cos^{-1} x + \cos^{-1}(2x) = -\pi \Rightarrow \cos^{-1} 2x = -\pi - \cos^{-1} x$

$\Rightarrow 2x = \cos(\pi + \cos^{-1} x)$

$\Rightarrow 2x = \cos\pi(\cos\cos^{-1} x) - \sin\pi\sin(\cos^{-1} x)$

$2x = -x \Rightarrow x = 0$

किन्तु $x = 0$ दिये गये समीकरण को सन्तुष्ट नहीं करता है।

अतः कोई हल विद्यमान नहीं है।

137. (a) $\sin\left[3\sin^{-1}\frac{1}{5}\right] = \sin\left[\sin^{-1}\left\{3\left(\frac{1}{5}\right)-4\left(\frac{1}{5}\right)^3\right\}\right]$

$= \sin\left[\sin^{-1}\left\{\frac{3}{5}-\frac{4}{125}\right\}\right] = \sin\left[\sin^{-1}\left(\frac{75-4}{125}\right)\right]$

$= \sin\left[\sin^{-1}\frac{71}{125}\right] = \frac{71}{125}$.

Critical Thinking Questions

1. (d) $\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$

$= \cot^{-1}\left[\frac{(\sqrt{1-\sin x}+\sqrt{1+\sin x})}{(\sqrt{1-\sin x}-\sqrt{1+\sin x})}\cdot\frac{(\sqrt{1-\sin x}+\sqrt{1+\sin x})}{(\sqrt{1-\sin x}+\sqrt{1+\sin x})}\right]$

$= \cot^{-1}\left[\frac{(1-\sin x)+(1+\sin x)+2\sqrt{1-\sin^2 x}}{(1-\sin x)-(1+\sin x)}\right]$

$= \cot^{-1}\left[\frac{2(1+\cos x)}{-2\sin x}\right] = \cot^{-1}\left[-\frac{2\cos^2(x/2)}{2\sin(x/2)\cos(x/2)}\right]$

$= \cot^{-1}\left(-\cot\frac{x}{2}\right) = \cot^{-1}\left[\cot\left(\pi-\frac{x}{2}\right)\right] = \pi-\frac{x}{2}$.

ट्रिक : $x = \frac{\pi}{4}$ रखने पर व्यंजक

$\cot^{-1}\left[\frac{\sqrt{\sqrt{2}-1}+\sqrt{\sqrt{2}+1}}{\sqrt{\sqrt{2}-1}-\sqrt{\sqrt{2}+1}}\right]$ होगा।

$= \cot^{-1}\left[\frac{\sqrt{2}-1+\sqrt{2}+1+2\sqrt{2-1}}{\sqrt{2}-1-\sqrt{2}-1}\right]$

$= \cot^{-1}\left[\frac{2\sqrt{2}+2}{-2}\right] = \cot^{-1}(-1-\sqrt{2}) = 157.5°$.

2. (d) $\sin^{-1}\left[\sin\left(\pi-\frac{2\pi}{3}\right)\right]$ का मुख्य मान $= \sin^{-1}\sin\left(\frac{\pi}{3}\right) = \frac{\pi}{3}$.

3. (c) दिया गया है कि $\theta = \tan^{-1}a$ और $\phi = \tan^{-1}b$ और $ab = -1$.

$\Rightarrow \tan\theta\tan\phi = -1 \Rightarrow \tan\theta = -\cot\phi \Rightarrow \theta-\phi = \frac{\pi}{2}$.

4. (b) दिया गया है कि $\tan\{\cos^{-1}(x)\} = \sin\left(\cot^{-1}\frac{1}{2}\right)$

माना $\cot^{-1}\frac{1}{2} = \phi \Rightarrow \frac{1}{2} = \cot\phi$

$\Rightarrow \sin\phi = \frac{1}{\sqrt{1+\cot^2\phi}} = \frac{2}{\sqrt{5}}$

माना $\cos^{-1}x = \theta \Rightarrow \sec\theta = \frac{1}{x}$

$\Rightarrow \tan\theta = \sqrt{\sec^2\theta-1}$

$\Rightarrow \tan\theta = \sqrt{\frac{1}{x^2}-1}$

$\Rightarrow \tan\theta = \frac{\sqrt{1-x^2}}{x}$

अतः $\tan\{\cos^{-1}(x)\} = \sin\left(\cot^{-1}\frac{1}{2}\right)$

$\Rightarrow \tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right) = \sin\left(\sin^{-1}\frac{2}{\sqrt{5}}\right)$

$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}} \Rightarrow \sqrt{(1-x^2)5} = 2x$

दोनों पक्षों का वर्ग करने पर, $x = \pm\frac{\sqrt{5}}{3}$.

5. (a) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}x = \frac{\pi}{2}-\sin^{-1}x = \frac{\pi}{2}-\frac{\pi}{5} = \frac{3\pi}{10}$.

6. (b) **ट्रिक :** दिये गये प्रतिबंधानुसार $p = q = r = \frac{1}{2}$ रखने पर,

$p^2+q^2+r^2+2pqr$

$= \left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+2.\frac{1}{2}.\frac{1}{2}.\frac{1}{2}$

$= \frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{2}{8} = 1$.

7. (b) $\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right]+\tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right]$

माना $\frac{1}{2}\cos^{-1}\frac{a}{b} = \theta \Rightarrow \cos 2\theta = \frac{a}{b}$

अतः $\tan\left[\frac{\pi}{4}+\theta\right]+\tan\left[\frac{\pi}{4}-\theta\right]$

$= \frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta} = \frac{(1+\tan\theta)^2+(1-\tan\theta)^2}{(1-\tan^2\theta)}$

$= \frac{1+\tan^2\theta+2\tan\theta+1+\tan^2\theta-2\tan\theta}{(1+\tan^2\theta)}$

$= \frac{2(1+\tan^2\theta)}{1-\tan^2\theta} = 2\sec 2\theta = \frac{2}{\cos 2\theta}$

$= \frac{2}{a/b} = \frac{2b}{a}$.

8. (a) $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$

$= \tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right]$

$(x^2 = \cos 2\theta$ रखने पर $\Rightarrow \theta = \frac{1}{2}\cos^{-1} x^2)$

$= \tan^{-1}\left[\frac{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}\right]$

$= \tan^{-1}\left[\frac{1+\tan\theta}{1-\tan\theta}\right] = \tan^{-1}\left[\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}\right]$

$= \tan^{-1}\tan\left(\frac{\pi}{4}+\theta\right) = \frac{\pi}{4}+\theta = \frac{\pi}{4}+\frac{1}{2}\cos^{-1} x^2$.

9. (b) हमें ज्ञात है कि, $\sin^{-1} x - \cos^{-1} x = \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$

किन्तु $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

$\therefore \sin^{-1} x = \frac{\pi}{3}$ और $\cos^{-1} x = \frac{\pi}{6}$

$\Rightarrow x = \frac{\sqrt{3}}{2}$ अद्वितीय हल है।

10. (a) $\sum_{m=1}^{n}\tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$

$= \sum_{m=1}^{n}\tan^{-1}\left(\frac{2m}{1+(m^2+m+1)(m^2-m+1)}\right)$

$= \sum_{m=1}^{n}\tan^{-1}\left(\frac{(m^2+m+1)-(m^2-m+1)}{1+(m^2+m+1)(m^2-m+1)}\right)$

$= \sum_{m=1}^{n}[\tan^{-1}(m^2+m+1)-\tan^{-1}(m^2-m+1)]$

$= (\tan^{-1}3-\tan^{-1}1)+(\tan^{-1}7-\tan^{-1}3)+$

$(\tan^{-1}13-\tan^{-1}7)+\ldots\ldots+[\tan^{-1}(n^2+n+1)$

$-\tan^{-1}(n^2-n+1)]$

$= \tan^{-1}(n^2+n+1)-\tan^{-1}1 = \tan^{-1}\left(\frac{n^2+n}{2+n^2+n}\right)$.

11. (c) $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$

$\tan^{-1}\sqrt{x(x+1)}$ परिभाषित है जब

$x(x+1) \geq 0$ .....(i)

$\sin^{-1}\sqrt{x^2+x+1}$ परिभाषित है जब

$0 \leq x(x+1)+1 \leq 1$ या $0 \leq x(x+1) \leq 0$ .....(ii)

(i) और (ii) से, $x(x+1) = 0$

या $x = 0$ और -1

अतः हलों की संख्या 2 है।

12. (a) दिया गया समीकरण $2\cos^{-1} x + \sin^{-1} x = \frac{11\pi}{6}$ है।

$\Rightarrow \cos^{-1} x + (\cos^{-1} x + \sin^{-1} x) = \frac{11\pi}{6}$

$\Rightarrow \cos^{-1} x + \frac{\pi}{2} = \frac{11\pi}{6}$

$\Rightarrow \cos^{-1} x = 4\pi/3$

जोकि संभव नहीं हैं क्योंकि $\cos^{-1} x \in [0, \pi]$

13. (a) $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$

$\Rightarrow \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right] = \pi$

$\Rightarrow x+y+z-xyz = 0$

$\Rightarrow x+y+z = xyz$.

14. (a) $2\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right] = \cos^{-1}\left[\frac{1-\left(\frac{a-b}{a+b}\right)\tan^2\frac{\theta}{2}}{1+\left(\frac{a-b}{a+b}\right)\tan^2\frac{\theta}{2}}\right]$

$\left(\because 2\tan^{-1} x = \cos^{-1}\frac{1-x^2}{1+x^2}\right)$

$= \cos^{-1}\left[\frac{(a+b)-(a-b)\tan^2\frac{\theta}{2}}{(a+b)+(a-b)\tan^2\frac{\theta}{2}}\right]$

$= \cos^{-1}\left[\frac{a\left(1-\tan^2\frac{\theta}{2}\right)+b\left(1+\tan^2\frac{\theta}{2}\right)}{a\left(1+\tan^2\frac{\theta}{2}\right)+b\left(1-\tan^2\frac{\theta}{2}\right)}\right]$

$$= \cos^{-1}\left[\frac{\frac{a\left(1-\tan^2\frac{\theta}{2}\right)}{1+\tan^2\frac{\theta}{2}}+b}{a+b\left(\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right)}\right] = \cos^{-1}\left[\frac{a\cos\theta+b}{a+b\cos\theta}\right].$$

15. (d) $2\tan^{-1}(\cos x) = \tan^{-1}(\operatorname{cosec}^2 x)$

$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(\frac{1}{\sin^2 x}\right)$$

$$\Rightarrow \frac{2\cos x}{\sin^2 x} = \frac{1}{\sin^2 x} \Rightarrow 2\cos x = 1$$

$$\Rightarrow x = \frac{\pi}{3}.$$

प्रतिलोम त्रिकोणमितीय फलन

SET Self Evaluation Test - 12

1. यदि $\theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x, x \geq 0$, तब वह न्यूनतम अंतराल, जिसमें θ स्थित होगा, है **[Orissa JEE 2003]**

(a) $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{4}$ (b) $0 < \theta < \pi$

(c) $-\frac{\pi}{4} \leq \theta \leq 0$ (d) $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$

2. यदि $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, तब

$x^{100} + y^{100} + z^{100} - \frac{9}{x^{101} + y^{101} + z^{101}}$ का मान है

(a) 0 (b) 3

(c) – 3 (d) 9

3. यदि $x + \frac{1}{x} = 2$, तब $\sin^{-1} x$ का मुख्य मान है

(a) $\pi / 4$ (b) $\pi / 2$

(c) π (d) $3\pi / 2$

4. यदि $\tan(\cos^{-1} x) = \sin\left[\cot^{-1}\left(\frac{1}{2}\right)\right]$, तब x (केवल मुख्य मान लेने पर) का मान होगा **[IIT 1991; AMU 2001]**

(a) $\frac{1}{\sqrt{5}}$ (b) $\frac{2}{\sqrt{5}}$

(c) $\frac{3}{\sqrt{5}}$ (d) $\frac{\sqrt{5}}{3}$

5. यदि ΔABC में $A = \tan^{-1} 2$ और $B = \tan^{-1} 3$, तो कोण C का मान है

(a) $\pi / 2$ (b) $\pi / 3$

(c) $\pi / 4$ (d) इनमें से कोई नहीं

6. यदि a, b, c धनात्मक वास्तविक संख्यायें हैं, एवं

$\theta = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$,

तो $\tan\theta$ = **[IIT 1981]**

(a) 0 (b) 1

(c) $\frac{a+b+c}{abc}$ (d) इनमें से कोई नहीं

7. यदि $\cos^{-1}\left(\frac{x}{a}\right) + \cos^{-1}\left(\frac{y}{b}\right) = \alpha$, तो $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} =$ **[MP PET 1995; UPSEAT 1999]**

(a) $\sin^2\alpha$ (b) $\cos^2\alpha$

(c) $\tan^2\alpha$ (d) $\cot^2\alpha$

8. यदि x_1, x_2, x_3, x_4 समीकरण

$x^4 - x^3 \sin 2\beta + x^2 \cos 2\beta - x\cos\beta - \sin\beta = 0$ के मूल हैं, तब $\tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4 =$

(a) β (b) $\frac{\pi}{2} - \beta$

(c) $\pi - \beta$ (d) $-\beta$

9. यदि $\tan^{-1} x + \cos^{-1}\frac{y}{\sqrt{(1+y^2)}} = \sin^{-1}\frac{3}{\sqrt{10}}$ तथा दोनों x तथा y धनात्मक व पूर्णांक हैं, तब x तथा y के मान हैं **[Roorkee 1993]**

(a) (1, 2) तथा (2, 7) (b) (1, 2) तथा (1, 7)

(c) (1, 7) तथा (2, 7) (d) (1, 7) तथा (2, 1)

10. यदि $a_1, a_2, a_3 ....., a_n$ समान्तर श्रेणी में हैं, तथा सार्वनुपात d है तो

$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + ... + \tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right] =$

(a) $\frac{(n-1)d}{a_1 + a_n}$ (b) $\frac{(n-1)d}{1+a_1a_n}$

(c) $\frac{nd}{1+a_1a_n}$ (d) $\frac{a_n - a_1}{a_n + a_1}$

11. यदि प्रतिलोम फलन के केवल मुख्य मानों का उपयोग किया जावे तो $\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{(17)}}\right)$ का मान होगा **[IIT 1994]**

(a) $\sqrt{29/3}$ (b) $29/3$

(c) $\sqrt{3/29}$ (d) $3/29$

12. यदि $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - ....\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - ...\right) = \frac{\pi}{2}$

जबकि $0 < |x| < \sqrt{2}$, तब x का मान होगा **[IIT Screening 2001]**

(a) $\frac{1}{2}$ (b) 1

(c) $-\frac{1}{2}$ (d) – 1

13. यदि $\sec^{-1} x = \text{cosec}^{-1} y$, तब $\cos^{-1}\frac{1}{x} + \cos^{-1}\frac{1}{y} =$ **[Orissa JEE 2002]**

(a) π (b) $\frac{\pi}{4}$

(c) $\frac{-\pi}{2}$ (d) $\frac{\pi}{2}$

14. $\sin(2\sin^{-1} 0.8) =$ **[MNR 1980]**

(a) 0.96 (b) 0.48

(c) 0.64 (d) इनमें से कोई नहीं

15. यदि $\sin^{-1}\frac{2a}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2} = \tan^{-1}\frac{2x}{1-x^2}$, तो $x =$ **[EAMCET 1981]**

(a) a (b) b

(c) $\frac{a+b}{1-ab}$ (d) $\frac{a-b}{1+ab}$

Answers and Solutions

(SET - 12)

1. (b) $\because\ \theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x = \frac{\pi}{2} - \tan^{-1} x$

हम जानते हैं कि, $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$

$\Rightarrow\ \frac{\pi}{2} > -\tan^{-1} x > -\frac{\pi}{2}$

$\therefore\ 0 < \frac{\pi}{2} - \tan^{-1} x < \pi$.

2. (a) जैसा कि हम जानते हैं कि $\sin^{-1} x$, $\frac{\pi}{2}$ से बड़ा नहीं हो सकता है।

अतः $\sin^{-1} x = \sin^{-1} y = \sin^{-1} z = \frac{\pi}{2}$

अतः $x = y = z = 1$

इन मानों को व्यंजक में रखने पर, $1+1+1-\frac{9}{1+1+1} = 0$.

3. (b) $x + \frac{1}{x} = 2 \Rightarrow x = 1$

अतः $\sin^{-1} x$ का मुख्य मान $\frac{\pi}{2}$ है।

4. (d) $\cot^{-1}\left(\frac{1}{2}\right) = \theta \Rightarrow \cot\theta = \frac{1}{2}$ रखने पर,

$\therefore\ \sin\theta = \frac{2}{\sqrt{5}}.\ \cos^{-1} x = \phi$ रखने पर, $\therefore\ x = \cos\phi$

साथ ही, $\tan\phi = \frac{2}{\sqrt{5}}$, $\quad \therefore x = \cos\phi = \frac{\sqrt{5}}{3}$.

5. (c) दिया गया है कि, $\angle A = \tan^{-1} 2, \angle B = \tan^{-1} 3$

हम जानते हैं कि, $\angle A + \angle B + \angle C = \pi$

$\Rightarrow \tan^{-1} 2 + \tan^{-1} 3 + \angle C = \pi$

$\Rightarrow \tan^{-1}\left(\frac{2+3}{1-2\times 3}\right) + \angle C = \pi \Rightarrow \tan^{-1}(-1) + \angle C = \pi$

$\Rightarrow \frac{3\pi}{4} + \angle C = \pi \Rightarrow \angle C = \frac{\pi}{4}$.

6. (a) $\theta = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}}$

$+ \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$

माना $s^2 = \frac{a+b+c}{abc}$

अतः $\theta = \tan^{-1}\sqrt{a^2 s^2} + \tan^{-1}\sqrt{b^2 s^2} + \tan^{-1}\sqrt{c^2 s^2}$

$= \tan^{-1}(as) + \tan^{-1}(bs) + \tan^{-1}(cs)$

$= \tan^{-1}\left[\frac{as + bs + cs - abcs^3}{1 - abs^2 - acs^2 - bcs^2}\right]$

अतः $\tan\theta = \left[s\frac{(a+b+c) - abcs^2}{1-(ab+bc+ca)s^2}\right]$

$= \left[\frac{s[(a+b+c)-(a+b+c)]}{1-s^2(ab+bc+ca)}\right] = 0$,

[चूँकि $s^2 abc = (a+b+c)$

ट्रिक : यह एक सर्वसमिका है अतः यह *a, b, c* के किसी भी मान के लिए सत्य होगी। माना $a = b = c = 1$ तब $\theta = \tan^{-1}\sqrt{3} + \tan^{-1}\sqrt{3} + \tan^{-1}\sqrt{3} = \pi \Rightarrow \tan\theta = 0$.

7. (a) हमें ज्ञात है कि, $\cos^{-1}\left[\frac{x}{a}.\frac{y}{b} - \sqrt{\left(1-\frac{x^2}{a^2}\right)}\sqrt{\left(1-\frac{y^2}{b^2}\right)}\right] = \alpha$

$\Rightarrow\ \frac{xy}{ab} - \sqrt{\left(1-\frac{x^2}{a^2}\right)}\sqrt{\left(1-\frac{y^2}{b^2}\right)} = \cos\alpha$

$\therefore\ \left(\frac{xy}{ab} - \cos\alpha\right)^2 = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{x^2y^2}{a^2b^2}$

$\Rightarrow\ \frac{x^2y^2}{a^2b^2} + \cos^2\alpha - \frac{2xy}{ab}\cos\alpha = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{x^2y^2}{a^2b^2}$

$\Rightarrow\ \frac{x^2}{y^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} = 1 - \cos^2\alpha = \sin^2\alpha$.

8. (b) हमें ज्ञात है कि,

$\Sigma x_1 = \sin 2\beta, \Sigma x_1 x_2 = \cos 2\beta, \Sigma x_1 x_2 x_3 = \cos\beta$

तथा $x_1 x_2 x_2 x_4 = -\sin\beta$

$\therefore\ \tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4$

$= \tan^{-1}\left(\frac{\Sigma x_1 - \Sigma x_1 x_2 x_3}{1 - \Sigma x_1 x_2 + x_1 x_2 x_3 x_4}\right)$

$= \tan^{-1}\left(\frac{\sin 2\beta - \cos\beta}{1 - \cos 2\beta - \sin\beta}\right)$

$= \tan^{-1}\left(\frac{(2\sin\beta - 1)\cos\beta}{\sin\beta(2\sin\beta - 1)}\right) = \tan^{-1}(\cot\beta)$

$= \tan^{-1}\left[\tan\left(\frac{\pi}{2} - \beta\right)\right] = \frac{\pi}{2} - \beta$.

9. (a) $\tan^{-1} x + \tan^{-1}\frac{1}{y} = \tan^{-1} 3$

$\therefore\ xy + 1 = 3(y - x)$

अतः (1, 2) तथा (2,7) चूँकि x, y धनात्मक व पूर्णांक हैं।

10. (b) हमें ज्ञात है कि, $\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + ...$

$$..... + \tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)$$

$$= \tan^{-1}\left(\frac{a_2 - a_1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{a_3 - a_2}{1+a_2a_3}\right) + ...$$

$$..... + \tan^{-1}\left(\frac{a_n - a_{n-1}}{1+a_{n-1}a_n}\right)$$

$$= (\tan^{-1} a_2 - \tan^{-1} a_1) + (\tan^{-1} a_3 - \tan^{-1} a_2) + ...$$

$$..... + (\tan^{-1} a_n - \tan^{-1} a_{n-1})$$

$$= \tan^{-1} a_n - \tan^{-1} a_1 = \tan^{-1}\left(\frac{a_n - a_1}{1+a_na_1}\right)$$

$$= \tan^{-1}\left(\frac{(n-1)d}{1+a_1a_n}\right).$$

11. (d) $\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right)$

$$= \tan(\tan^{-1} 7 - \tan^{-1} 4) = \tan\left[\tan^{-1}\left(\frac{7-4}{1+28}\right)\right] = \frac{3}{29}.$$

12. (b) हम जानते हैं कि, $\sin^{-1} y + \cos^{-1} y = \frac{\pi}{2}, |y| \le 1$

$\therefore$ प्रश्नानुसार,

$$x - \frac{x^2}{2} + \frac{x^3}{4} - ... = x^2 - \frac{x^4}{2} + \frac{x^6}{4} ...$$

$$\Rightarrow \frac{x}{1+\frac{x}{2}} = \frac{x^2}{1+\frac{x^2}{2}}, \qquad (\because 0 < |x| < \sqrt{2})$$

$$\Rightarrow \frac{x}{2+x} = \frac{x^2}{2+x^2} \Rightarrow 2x + x^3 = 2x^2 + x^3 \Rightarrow x = x^2$$

$\therefore x - x^2 = 0 \Rightarrow x(1-x) = 0 \Rightarrow x = 0$ तथा $x = 1$,

किन्तु $x \neq 0$. अतः $x = 1$.

13. (d) दिया गया है, $\sec^{-1} x = \text{cosec}^{-1} y$

$$\cos^{-1}\left(\frac{1}{x}\right) = \sin^{-1}\left(\frac{1}{y}\right) \Rightarrow \cos^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{1}{y}\right)$$

$$\Rightarrow \cos^{-1}\left(\frac{1}{x}\right) + \cos^{-1}\left(\frac{1}{y}\right) = \frac{\pi}{2}.$$

14. (a) $\sin(2\sin^{-1} 0.8) = \sin\left(2\sin^{-1}\frac{4}{5}\right)$

$$= \sin\left(\sin^{-1} 2.\frac{4}{5}.\sqrt{1-\frac{16}{25}}\right) = \frac{8}{5}.\frac{3}{5} = \frac{24}{25} = 0.96.$$

15. (d) $a = \tan\theta, b = \tan\phi$ तथा $x = \tan\psi$ रखने पर

$\sin^{-1}(\sin 2\theta) - \cos^{-1}(\cos 2\phi) = \tan^{-1}(\tan 2\psi)$

$\Rightarrow 2\theta - 2\phi = 2\psi \Rightarrow \theta - \phi = \psi$

दोनों पक्षों का 'tan' लेने पर $\tan(\theta - \phi) = \tan\psi$

$$\Rightarrow \frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi} = \tan\psi$$

इन मानों को प्रतिस्थापित करने पर, $\frac{a-b}{1+ab} = x$.

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