Chapter

23

अनिश्चित समाकलन

परिभाषा (Definition)

यदि $\phi'(x) = f(x)$ हो, तब मूल फलन $\phi(x)$ को $f(x)$ का पूर्वग या प्रतिअवकलज कहते हैं।

माना $\phi(x)$, फलन $f(x)$ का पूर्वग है तथा c कोई अचर है,

तब $\frac{d}{dx}(\phi(x)+c) = f(x) \Rightarrow \int f(x)\,dx = \phi(x)+c$

$\Rightarrow$ $\phi(x)+c$ को भी $f(x)$ का प्रतिअवकलज कहते हैं, जहाँ c एक स्वेच्छ अचर है जिसके भिन्न–भिन्न मान हो सकते हैं।

समाकलन के कुछ मानक सूत्र

(Fundamental integration formulae)

(1) (i) $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$

(ii) $\int (ax+b)^n dx = \frac{1}{a}.\frac{(ax+b)^{n+1}}{n+1} + c,\ n \neq -1$

(2) (i) $\int \frac{1}{x} dx = \log|x| + c$

(ii) $\int \frac{1}{ax+b} dx = \frac{1}{a}(\log|ax+b|) + c$

(3) $\int e^x dx = e^x + c$

(4) $\int a^x dx = \frac{a^x}{\log_e a} + c$

(5) $\int \sin x\,dx = -\cos x + c$

(6) $\int \cos x\,dx = \sin x + c$

(7) $\int \sec^2 x\,dx = \tan x + c$

(8) $\int \text{cosec}^2 x\,dx = -\cot x + c$

(9) $\int \sec x \tan x\,dx = \sec x + c$

(10) $\int \text{cosec } x \cot x\,dx = -\text{cosec } x + c$

(11) $\int \tan x\,dx = -\log|\cos x| + c = \log|\sec x| + c$

(12) $\int \cot x\,dx = \log|\sin x| + c = -\log|\text{cosec } x| + c$

(13) $\int \sec x\,dx = \log|\sec x + \tan x| + c = \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + c$

(14) $\int \text{cosec } x\,dx = \log|\text{cosec } x - \cot x| + c = \log\tan\frac{x}{2} + c$

(15) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + c = -\cos^{-1}x + c$

(16) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c = -\cos^{-1}\frac{x}{a} + c$

(17) $\int \frac{dx}{1+x^2} = \tan^{-1}x + c = -\cot^{-1}x + c$

(18) $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c = \frac{-1}{a}\cot^{-1}\frac{x}{a} + c$

(19) $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}x + c = -\text{cosec}^{-1}x + c$

(20) $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + c = \frac{-1}{a}\text{cosec}^{-1}\frac{x}{a} + c$

समाकलन के मानक सूत्र में यदि x की जगह $ax+b$ आता है तब वही सूत्र उपयोग होगा लेकिन उसमें x के गुणांक से या $ax+b$ के अवकलन अर्थात् a से भाग देते हैं।

सामान्यतया यदि $\int f(x)dx = \phi(x) + c$, तब

$$\int f(ax+b)dx = \frac{1}{a}\phi(ax+b) + c$$

$$\int \sin(ax+b)dx = \frac{-1}{a}\cos(ax+b) + c,$$

$\int \sec(ax+b)dx = \frac{1}{a}\log|\sec(ax+b)+\tan(ax+b)| + c$ इत्यादि।

कुछ महत्वपूर्ण परिणाम

(i) $\int \frac{1}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c = \frac{-1}{a}\coth^{-1}\frac{x}{a} + c$, जब $x > a$

(ii) $\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c = \frac{1}{a}\tanh^{-1}\frac{x}{a} + c$, जब $x < a$

(iii) $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\{|x+\sqrt{x^2-a^2}|\} + c = \cos h^{-1}\left(\frac{x}{a}\right) + c$

(iv) $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\{|x+\sqrt{x^2+a^2}|\} + c = \sinh^{-1}\left(\frac{x}{a}\right) + c$

(v) $\int \sqrt{a^2-x^2}dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\sin^{-1}\left(\frac{x}{a}\right) + c$

(vi) $\int \sqrt{x^2-a^2}dx = \frac{1}{2}x\sqrt{x^2-a^2} - \frac{1}{2}a^2\log\{x+\sqrt{x^2-a^2}\} + c$

$= \frac{1}{2}x\sqrt{x^2-a^2} - \frac{1}{2}a^2\cosh^{-1}\left(\frac{x}{a}\right) + c$

(vii) $\int \sqrt{x^2+a^2}dx = \frac{1}{2}x\sqrt{x^2+a^2} + \frac{1}{2}a^2\log\{x+\sqrt{x^2+a^2}\} + c$

$= \frac{1}{2}x\sqrt{x^2+a^2} + \frac{1}{2}a^2\sinh^{-1}\left(\frac{x}{a}\right)$

प्रतिस्थापन द्वारा समाकलन (Integration by substitution)

(1) $\int f[\phi(x)]\phi'(x)dx$ **रूप के फलन का समाकलनः** माना $\phi(x) = t$, $\phi'(x)dx = dt$, तब समाकलन मानक रूप $\int f(t)dt$ में बदल जाता है।

(2) $I = \int f(x).f'(x).dx$ **रूप के फलन का समाकलनः** यहाँ दो घटक है जिनमें एक घटक, दूसरे का अवकलन है। इसमें $f(x) = t$ रखने पर समाकलन मानक रूप में बदल जाता है।

(3) $f(ax+b)$ **रूप के फलन का समाकलन :** इसमें $ax+b = t$ रखने पर समाकलन मानक रूप में बदल जाता है। यदि $\int f(x)dx = \phi(x)$ हो, तो $\int f(ax+b)dx = \frac{1}{a}\phi(ax+b) + c$

(4) $\frac{f'(x)}{f(x)}$ **रूप के फलन का समाकलन :** इसमें $f(x) = t$ रखने पर, $f'(x)dx = dt$ हो जाता है। अतः $\int \frac{f'(x)}{f(x)}dx = \log[f(x)] + c$

(5) $[f(x)]^n f'(x)$ **रूप के फलन का समाकलन :** इसमें $f(x) = t$ रखने पर, $f'(x)dx = dt$ $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + c$, $[n \neq -1]$

(6) $\frac{f'(x)}{\sqrt{f(x)}}$ इस तरह के समाकलन में $f(x) = t$ रखने पर, $\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + c$

(7) **मानक प्रतिस्थापन**

सारणी : 23.1

	समाकलन रूप	प्रतिस्थापन
(i)	$\sqrt{a^2-x^2}$, $\frac{1}{\sqrt{a^2-x^2}}$, a^2-x^2	$x = a\sin\theta$, या $x = a\cos\theta$
(ii)	$\sqrt{x^2+a^2}$, $\frac{1}{\sqrt{x^2+a^2}}$, x^2+a^2	$x = a\tan\theta$ या $x = a\sinh\theta$
(iii)	$\sqrt{x^2-a^2}$, $\frac{1}{\sqrt{x^2-a^2}}$, x^2-a^2	$x = a\sec\theta$ या $x = a\cosh\theta$
(iv)	$\sqrt{\frac{x}{a+x}}$, $\sqrt{\frac{a+x}{x}}$, $\sqrt{x(a+x)}$, $\frac{1}{\sqrt{x(a+x)}}$	$x = a\tan^2\theta$
(v)	$\sqrt{\frac{x}{a-x}}$, $\sqrt{\frac{a-x}{x}}$, $\sqrt{x(a-x)}$, $\frac{1}{\sqrt{x(a-x)}}$	$x = a\sin^2\theta$
(vi)	$\sqrt{\frac{x}{x-a}}$, $\sqrt{\frac{x-a}{x}}$, $\sqrt{x(x-a)}$, $\frac{1}{\sqrt{x(x-a)}}$	$x = a\sec^2\theta$
(vii)	$\sqrt{\frac{a-x}{a+x}}$, $\sqrt{\frac{a+x}{a-x}}$	$x = a\cos 2\theta$
(viii)	$\sqrt{\frac{x-\alpha}{\beta-x}}$, $\sqrt{(x-\alpha)(\beta-x)}$, $(\beta > \alpha)$	$x = \alpha\cos^2\theta + \beta\sin^2\theta$

खण्डशः समाकलन (Integration by parts)

(1) **जब समाकल्य में एक से अधिक फलन हो :** इस प्रकार के समाकलनों को खण्डशः समाकलन द्वारा हल करते हैं।

हम जानते है $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

$\Rightarrow d(uv) = udv + vdu \Rightarrow \int d(uv) = \int udv + \int vdu$

यदि u तथा v, x के दो फलन हो, तो $\int \underset{I}{u}\,\underset{II}{v}\,dx = u\int v\,dx - \int\{\frac{du}{dx}.\int vdx\}dx$ अर्थात् दो फलनों के गुणनफल का समाकलन = (प्रथम फलन) × (द्वितीय फलन का समाकलन) – {(प्रथम फलन का अवकलन) × (द्वितीय फलन का समाकलन)} का समाकलन

खण्डशः समाकलन में प्रथम फलन व द्वितीय फलन का उचित चयन करने से समाकलन शीघ्र व आसानी से हो जाता है अतः प्रथम फलन का चयन करने में निम्न बिन्दुओं का ध्यान रखते हैं :

(i) सामान्यतः प्रतिलोम व लघुगणक फलन को प्रथम फलन लेते हैं, जैसे $\log|x|$, $\sin^{-1}x, \cos^{-1}x, \tan^{-1}x$ .....इत्यादि।

(ii) यदि प्रतिलोम त्रिकोणमितीय एवं लघुगणक फलन के साथ अन्य कोई फलन विद्यमान नहीं हो, तो 1 को द्वितीय फलन लेकर खण्डशः समाकलन करते हैं। जैसे $\int \sin^{-1}x\,dx, \int \log x\,dx$, इत्यादि में।

(iii) यदि दोनों फलनों का समाकलन आसानी से हो सके, तब प्रथम फलन का चयन इस प्रकार करते हैं, कि इसके अवकलज से प्राप्त फलन का समाकलन आसानी से हो सके।

सामान्यतः प्रथम फलन का चयन **"ILATE"** नियम के अनुसार किया जाता है।

(2) $\int e^x\{f(x)+f'(x)\}dx$ **रूप का समाकलनः** यदि $\int e^x\{f(x)+f'(x)\}dx$, रूप का समाकलन हो तब इस समाकलन को दो समाकल्यों में विभक्त करते हैं। इनमें से एक समाकलन को खण्डशः समाकलन द्वारा हल करते हैं तथा दूसरे समाकलन को रोककर रखते हैं।

(i) $\int e^x[f(x)+f'(x)]dx = e^x f(x)+c$

(ii) $\int e^{mx}[mf(x)+f'(x)]dx = e^{mx}f(x)+c$

(iii) $\int e^{mx}\left[f(x)+\frac{f'(x)}{m}\right]dx = \frac{e^{mx}f(x)}{m}+c$

(3) $\int [x\ f'(x)+f(x)]\,dx$ **रूप का समाकलन :**

$\int[x\,f'(x)+f(x)]dx$ रूप के समाकलन को हल करने के लिए हम समाकल्य को दो भागों में विभक्त करते हैं। प्रथम भाग को खण्डशः समाकलन द्वारा हल करते हैं तथा दूसरे भाग को रोककर रखते हैं। अतः इस प्रकार, $\int[x\,f'(x)+f(x)]dx = x\,f(x)+c$

(4) $\int e^{ax}\sin bx dx$, $\int e^{ax}\cos bx\,dx$ **रूप का समाकलन**

$$\int e^{ax}\sin bx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)+c$$
$$= \frac{e^{ax}}{\sqrt{a^2+b^2}}\sin(bx-\tan^{-1}\frac{b}{a})+c$$

$$\int e^{ax}.\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)+c$$
$$= \frac{e^{ax}}{\sqrt{a^2+b^2}}\cos\left(bx-\tan^{-1}\frac{b}{a}\right)+c$$

$$\int e^{ax}.\sin(bx+c)dx = \frac{e^{ax}}{a^2+b^2}[a\sin(bx+c)-b\cos(bx+c)]+k$$
$$= \frac{e^{ax}}{\sqrt{a^2+b^2}}\sin\left[(bx+c)-\tan^{-1}\left(\frac{b}{a}\right)\right]+k$$

$$\int e^{ax}.\cos(bx+c)dx = \frac{e^{ax}}{a^2+b^2}[a\cos(bx+c)-b\sin(bx+c)]+k$$
$$= \frac{e^{ax}}{\sqrt{a^2+b^2}}\cos\left[(bx+c)-\tan^{-1}\left(\frac{b}{a}\right)\right]+k$$

मानक परिणामों के उपयोग से विभिन्न रूपों के समाकलन ज्ञात करना (Evaluation of the various forms of integrals by use of standard results)

(1) $\int\frac{dx}{ax^2+bx+c}$ **रूप का समाकलन**

कार्य विधि : $\int\frac{dx}{ax^2+bx+c} = \frac{1}{a}\int\frac{dx}{x^2+\frac{b}{a}x+\frac{c}{a}}$

$= \frac{1}{a}\int\frac{dx}{\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4c}}$, जो कि $\int\frac{dx}{X^2-A^2}, \int\frac{dx}{X^2+A^2}$ या $\int\frac{dx}{A^2-X^2}$ रूप में होगा।

(2) $\int\frac{dx}{\sqrt{ax^2+bx+c}}$ **रूप का समाकलनः** इसे निम्न रूप में परिवर्तित कर सकते हैं $\int\frac{dx}{\sqrt{a^2-x^2}}, \int\frac{dx}{\sqrt{x^2-a^2}}$ या $\int\frac{dx}{\sqrt{x^2+a^2}}$.

(3) $\int\sqrt{ax^2+bx+c}\,dx$ **रूप का समाकलनः** इसे निम्न रूप में परिवतर्तित कर सकते हैं $\int\sqrt{a^2-x^2}\,dx, \int\sqrt{x^2-a^2}\,dx$ या $\int\sqrt{a^2+x^2}\,dx$.

(4) $\int\frac{(px+q)dx}{ax^2+bx+c}$, $\int\frac{(px+q)dx}{\sqrt{ax^2+bx+c}}$, $\int(px+q)\sqrt{ax^2+bx+c}\,dx$ **:**

इस प्रकार के समाकलनों को हल करने के लिए

$$px+q = A\left\{\frac{d}{dx}(ax^2+bx+c)\right\}+B$$

दोनों पक्षों से x के गुणांक व अचर पद की तुलना करके A व B के मान ज्ञात करते हैं।

इस प्रकार समाकलन को दो भागों में विभक्त कर हल करते हैं।

(5) $\int\frac{x^2+1}{x^4+kx^2+1}dx, \int\frac{x^2-1}{x^4+kx^2+1}dx, \int\frac{dx}{x^4+kx^2+1}$ **जहाँ** $k\in R$ **रूप का समाकलन**

कार्य विधि : (i) इस प्रकार के समाकलन का मान ज्ञात करने के लिए अंश व हर में x^2 से भाग देते हैं।

(ii) $x+\frac{1}{x}=t$ या $x-\frac{1}{x}=t$ रखते हैं।

(6) $\int\frac{x^2+a^2}{x^4+kx^2+a^4}dx, \int\frac{x^2-a^2}{x^4+kx^2+a^4}dx$, जहाँ k एक अचर, ऋणात्मक या शून्य है।

इस प्रकार के समाकलनों को ज्ञात करने के लिए अंश व हर में x^2 से भाग देते हैं तथा $x-\frac{a^2}{x}=t$ तथा $x+\frac{a^2}{x}=t$ रखते हैं।

(7) **अपरिमेय फलनों के लिए उचित प्रतिस्थापन**

(i) $\int\frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}, \int\sqrt{\left(\frac{x-\alpha}{\beta-x}\right)}dx$

$\int\sqrt{(x-\alpha)(\beta-x)}\,dx$, $x=\alpha\cos^2\theta+\beta\sin^2\theta$ रखकर

(ii) $\int\frac{dx}{(px+q)\sqrt{(ax+b)}}$, $ax+b=t^2$ रखकर

(iii) $\int\frac{dx}{(px+q)\sqrt{ax^2+bx+c}}$, $px+q=\frac{1}{t}$ रखकर

(iv) $\int\frac{dx}{(px^2+r)\sqrt{(ax^2+c)}}$,प्रथम में $x=\frac{1}{t}$ रखकर तथा $a+ct^2=z^2$.

(8) $\int\frac{dx}{P\sqrt{Q}}$ **रूप का समाकलन (जहाँ** P **और** Q, x **में रेखीय या वर्ग समीकरण हों) :**

(i) जब Q रेखीय तथा P रेखीय या वर्ग समीकरण हों, तब $Q = t^2$ रखते हैं।

(ii) जब P रेखीय तथा Q वर्ग समीकरण हों, तब $P = \frac{1}{t}$ रखते हैं।

(iii) जब P और Q दोनों वर्ग समीकरण हों, तब $x = \frac{1}{t}$ रखते हैं।

$\int \frac{dx}{a + b\cos x}$ और $\int \frac{dx}{a + b\sin x}$ रूप का समाकलन

इस प्रकार के समाकलन को निम्न प्रकार से ज्ञात कर सकते है:

(1) $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$ तथा $\sin x = \frac{2\tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$ रखते हैं।

(2) अंश में $1 + \tan^2 \frac{x}{2}$ के स्थान पर $\sec^2 \frac{x}{2}$ रखते हैं।

(3) $\tan \frac{x}{2} = t$ तब $\frac{1}{2}\sec^2 \frac{x}{2} dx = dt$ रखते हैं।

(4) अब प्राप्त समाकलन को, जो $\int \frac{dt}{at^2 + bt + c}$ के रूप में होगा, पहले दी गई विधि से हल करते हैं।

(i) $\int \frac{dx}{a + b\cos x}$

स्थिति I : जब $a > b$, तब

$$\int \frac{dx}{a + b\cos x} = \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1}\left(\sqrt{\frac{a-b}{a+b}} \tan \frac{x}{2}\right) + c$$

स्थिति II : जब $a < b$, तब

$$\int \frac{dx}{a + b\cos x} = \frac{1}{\sqrt{b^2 - a^2}} \log\left|\frac{\sqrt{b-a}\tan \frac{x}{2} + \sqrt{b+a}}{\sqrt{b-a}\tan \frac{x}{2} - \sqrt{b+a}}\right| + c$$

स्थिति III : जब $a = b$, तब $\int \frac{dx}{a + b\cos x} = \frac{1}{a}\tan \frac{x}{2} + c$.

(ii) $\int \frac{dx}{a + b\sin x}$

स्थिति I : जब $a^2 > b^2$ या $a > 0$ तथा $a > b$, तब

$$\int \frac{dx}{a + b\sin x} = \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1}\left[\frac{a\tan \frac{x}{2} + b}{\sqrt{a^2 - b^2}}\right] + c$$

स्थिति II : जब $a^2 < b^2$, तब

$$\int \frac{dx}{a + b\sin x} = \frac{1}{\sqrt{b^2 - a^2}} \log\left|\frac{(a\tan \frac{x}{2} + b) - (\sqrt{b^2 - a^2})}{(a\tan \frac{x}{2} + b) + \sqrt{b^2 - a^2}}\right| + c$$

स्थिति III : जब $a^2 = b^2$

$b = a$ या $b = -a$

(a) जब $b = a$, तब

$$\int \frac{dx}{a + b\sin x} = \frac{-1}{a}\cot\left(\frac{\pi}{4} + \frac{x}{2}\right) + c = \frac{1}{a}[\tan x - \sec x] + c$$

(b) जब $b = -a$, तब $\int \frac{dx}{a + b\sin x} = \frac{1}{a}\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + c$.

$\int \frac{dx}{a + b\cos x + c\sin x}$, $\int \frac{dx}{a\sin x + b\cos x}$ रूप का समाकलन

(1) $\int \frac{dx}{a + b\cos x + c\sin x}$ **रूप का समाकलन :** इस प्रकार के समाकलन को ज्ञात करने के लिए, सर्वप्रथम $b = r\cos\alpha$ और $c = r\sin\alpha$ रखने पर

अतः $r^2 = b^2 + c^2$ तथा $\alpha = \tan^{-1}\frac{c}{b}$.

$$\therefore I = \int \frac{dx}{a + r(\cos\alpha\cos x + \sin\alpha\sin x)} = \int \frac{dx}{a + r\cos(x - \alpha)}$$

$x - \alpha = t \Rightarrow dx = dt$, रखने पर $I = \int \frac{dt}{a + r\cos t}$

इस प्रकार के समाकलन को पहले दर्शाया जा चुका है।

(2) $\int \frac{dx}{a\sin x + b\cos x}$ **रूप का समाकलन :** इस प्रकार के समाकलन को ज्ञात करने के लिए, सर्वप्रथम $a = r\cos\theta$, $b = r\sin\theta$ रखने पर $r = \sqrt{a^2 + b^2}$, $\alpha = \tan^{-1}\frac{b}{a}$

$$\therefore \int \frac{dx}{a\sin x + b\cos x} = \frac{1}{r}\int \frac{dx}{\sin(x + \alpha)} = \frac{1}{r}\int \operatorname{cosec}(x + \alpha)dx$$

$$= \frac{1}{r}\log\left|\tan\left(\frac{x}{2} + \frac{\alpha}{2}\right)\right| = \frac{1}{\sqrt{a^2 + b^2}}\log\left|\tan\left(\frac{x}{2} + \frac{1}{2}\tan^{-1}\frac{b}{a}\right)\right| + c$$

$\cos x = \frac{1 - \tan^2 x/2}{1 + \tan^2 x/2}$ और $\sin x = \frac{2\tan(x/2)}{1 + \tan^2(x/2)}$ का उपयोग उपरोक्त समाकलन को ज्ञात करने के लिए करते हैं।

$\int \frac{dx}{a + b\cos^2 x}$, $\int \frac{dx}{a + b\sin^2 x}$, $\int \frac{dx}{a\sin^2 x + b\cos^2 x}$, $\int \frac{dx}{(a\sin x + b\cos x)^2}$, $\int \frac{dx}{a + b\sin^2 x + c\cos^2 x}$ रूप का समाकलन

इस प्रकार के समाकलन को ज्ञात करने के लिए निम्नांकित बिन्दुओं का अनुसरण करते हैं :

(1) अंश व हर दोनों को $\cos^2 x$ से भाग देते हैं।

(2) हर में $\sec^2 x$ की जगह $(1 + \tan^2 x)$ रखते हैं।

(3) अब $\tan x = t \Rightarrow \sec^2 x dx = dt$ रखते हैं।

(4) अब प्राप्त हुए समाकलन को पूर्व में दी गयी विधि द्वारा हल करते हैं।

$\int \frac{a\sin x + b\cos x}{c\sin x + d\cos x}$ और $\int \frac{a\sin x + b\cos x + q}{c\sin x + d\cos x + r}$ रूप का समाकलन

(1) $\int \frac{a\sin x + b\cos x}{c\sin x + d\cos x}dx$ **रूप का समाकलन :** इस प्रकार के समाकलन को ज्ञात करने के लिए अंश को निम्न प्रकार से व्यक्त करते हैं।

अंश = M (हर का अवकलन) + N (हर)

अर्थात्

$$a\sin x + b\cos x = M\frac{d}{dx}(c\sin x + d\cos x) + N(c\sin x + d\cos x)$$

जहाँ M और N अचर हैं जिनका मान दोनों पक्षों में $\sin x$ तथा $\cos x$ के गुणांकों की तुलना कर ज्ञात करते हैं।

$$I = \int \frac{a\sin x + b\cos x}{c\sin x + d\cos x}dx$$

$$= \int \frac{M(c\cos x - d\sin x) + N(c\sin x + d\cos x)}{c\sin x + d\cos x}dx$$

$$= M\int \frac{c\cos x - d\sin x}{c\sin x + d\cos x}dx + N\int 1dx$$

$$= M\log|\, c\sin x + d\cos x\,| + Nx + c.$$

(2) $\int \frac{a\sin x + b\cos x + q}{c\sin x + d\cos x + r}dx$ **रूप का समाकलन :** इस प्रकार के समाकलन को ज्ञात करने के लिए अंश को निम्न प्रकार से व्यक्त करते हैं।

अंश $= M(\text{हर}) + N(\text{हर का अवकलन}) + P$

$$(c\sin x + b\cos x + q) = M(c\sin x + d\cos x + r) + N(c\cos x - d\sin x) + P.$$

जहाँ M, N, P अचर हैं, जिनका मान दोनों पक्षों में $\sin x, \cos x$ तथा अचर पद के गुणांकों की तुलना कर ज्ञात करते हैं।

$$\therefore \int \frac{a\sin x + b\cos x + q}{c\sin x + d\cos x + r}dx$$

$$= \int M\,dx + N\int \frac{\text{हर का अवकलन}}{\text{हर}}dx + \int \frac{dx}{c\sin x - d\cos x + r}$$

$$= Mx + N\log|\text{हर}| + P\int \frac{dx}{c\sin x + d\cos x + r}.$$

आंशिक भिन्नों द्वारा परिमेय बीजीय फलनों का समाकलन (Integration of rational functions by using partial fractions)

यदि दिया गया फलन दो बहुपदों का भागफल है, तो इसके समाकलन के लिए इसे आंशिक भिन्नों में विभक्त करते हैं (यदि सम्भव है)। पूर्व अध्याय में आंशिक भिन्नों को हल करने की विधि हम पढ़ चुके हैं।

त्रिकोणमितीय फलनों का समाकलन
(Integration of trigonometric functions)

(i) $\int \sin^m x\cos^n x\,dx$ **रूप का समाकलन :** (i) इस प्रकार के समाकलन को ज्ञात करने के लिए $I = \int \sin^m x\cos^n x\,dx$, जहाँ m और n परिमेय संख्यायें है

(a) यदि n विषम हो, तब $\sin x = t$, प्रतिस्थापित करते हैं।

(b) यदि m विषम हो, तब $\cos x = t$, प्रतिस्थापित करते हैं।

(c) यदि $m+n$ ऋणात्मक सम पूर्णांक हो, तब $\tan x = t$, प्रतिस्थापित करते हैं।

(d) यदि $\frac{1}{2}(n-1)$ एक पूर्णांक हो, तब $\cot x = t$ प्रतिस्थापित करते हैं।

(e) यदि m और n परिमेय संख्याएँ हो तथा $\left(\frac{m+n-2}{2}\right)$ एक ऋणात्मक पूर्णांक हो, तब $\cos x = t$ या $\tan x = t$ प्रतिस्थापित करते हैं।

(ii) $\int R(\sin x, \cos x)dx$ **रूप का समाकलन** :

$\int R(\sin x, \cos x)dx$, के रूप का समाकलन जहाँ R, $\sin x$ तथा $\cos x$ का परिमेय फलन है। इस प्रकार के समाकलन को $\tan\frac{x}{2} = t$ (जहाँ $-\pi < x < \pi$) रखकर, परिमेय फलन के समाकलन में परिवर्तित कर देते हैं।

कभी–कभी $\cot\frac{x}{2} = t$ (जहाँ $0 < x < 2\pi$) रखकर, परिमेय फलन के समाकलन के रूप में आसानी से परिवर्तित कर सकते हैं।

(a) यदि $\int R(\sin x)\cos x\,dx$ के रूप का समाकलन हो, तब $\sin x = t$ प्रतिस्थापित करते हैं।

(b) यदि $\int R(\cos x)\sin x\,dx$ के रूप का समाकलन हो, तब $\cos x = t$ प्रतिस्थापित करते हैं।

(c) यदि समाकलन केवल $\tan x$ पर आश्रित हों, तब $\tan x = t$, $dx = \frac{dt}{1+t^2}$ प्रतिस्थापित करते हैं।

(d) यदि $R(-\sin x, \cos x) = -R(\sin x, \cos x)$, तब $\cos x = t$ प्रतिस्थापित करते हैं।

(e) यदि $R(\sin x, -\cos x) = -R(\sin x, \cos x)$, तब $\sin x = t$ प्रतिस्थापित करते हैं।

(f) यदि $R(-\sin x, -\cos x) = -R(\sin x, \cos x)$, तब $\tan x = t$ प्रतिस्थापित करते हैं।

(2) **विशेष प्रकार के समानयन सूत्र**

(i) $\int \sin^n x\,dx = \frac{-\cos x\,.\sin^{n-1}x}{n} + \frac{n-1}{n}\int \sin^{n-2}x\,dx$

(ii) $\int \cos^n x\,dx = \frac{\sin x\cos^{n-1}x}{n} + \frac{n-1}{n}\int \cos^{n-2}x\,dx$

(iii) $\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx$

(iv) $\int \cot^n x\,dx = \frac{-1}{n-1}\cot^{n-1}x - \int \cot^{n-2}x\,dx$

(v) $\int \sec^n x\,dx = \frac{1}{(n-1)}\left[\sec^{n-2}x.\tan x + (n-2)\int \sec^{n-2}x\,dx\right]$

(vi) $\int \text{cosec}^n x dx = \frac{1}{(n-1)}[-\text{cosec}^{n-2}x\cot x + (n-2)\int \text{cosec}^{n-2}x dx]$

(vii) $\int \sin^p x\cos^q x\,dx = -\frac{\sin^{q+1}x.\cos^{p-1}x}{p+q} + \frac{p-1}{p+q}\int \sin^{p-2}x.\cos^q x\,dx$

(viii) $\int \sin^p x\cos^q x\,dx = \frac{\sin^{p+1}x.\cos^{q-1}x}{p+q} + \frac{p-1}{p+q}\int \sin^p x.\cos^{q-2}x\,dx$

(ix) $\int \frac{dx}{(x^2+k)^n} = \frac{x}{k(2n-2)(x^2+k)^{n-1}} + \frac{(2n-3)}{k(2n-2)}\int \frac{dx}{(x^2+k)^{n-1}}$

अतिपरवलयिक फलनों का समाकलन
(Integration of hyperbolic functions)

(1) $\int \sinh x\,dx = \cos h x + c$ (2) $\int \cosh x\,dx = \sinh x + c$

(3) $\int \sec h^2 x\,dx = \tan h\,x + c$

(4) $\int \text{cosech}^2 x\,dx = -\coth x + c$

(5) $\int \sec h x\tan h x\,dx = -\sec h x + c$

(6) $\int \text{cosec}\,h x\cot h\,dx = -\text{cosec}\,h x + c$

Tips & Tricks

✍ यदि अन्तराल $[a, b]$ में फलन $f(x)$ के दो प्रतिअवकलज $F_1(x)$ तथा $F_2(x)$ हों, तब उनका अन्तर एक अचर होता है।

✍ सिग्नम फलन का प्रति अवकलज उस अन्तराल में होता है, जिसमें बिन्दु $x = 0$ शामिल नहीं है।

✍ प्रत्येक समफलन का प्रतिअवकलज विषम फलन होता है तथा प्रत्येक विषम फलन का प्रतिअवकलज सम फलन होता है।

✍ यदि $I_n = \int x^n . e^{ax} dx$, तो $I_n = \frac{x^n e^{ax}}{a} - \frac{n}{a} I_{n-1}$

✍ यदि $I_n = \int (\log x) dx$, तो $I_n = x \log x - x$

✍ यदि $I_n = \int \frac{1}{\log x} dx$, तो

$$I_n = \log(\log x) + \log x + \frac{(\log x)^2}{2.(2!)} + \frac{(\log x)^3}{3(3!)} + \text{..... ....}$$

✍ यदि $I_n = \int (\log x)^n dx$; तो $I_n = x(\log x)^n - n.I_{n-1}$

✍ उत्तरोत्तर खण्डशः समाकलन का उपयोग तब कर सकते हैं, जबकि एक फलन x^n (n धनात्मक पूर्णांक है) है, जो उत्तरोत्तर अवकलित होगा एवं दूसरा फलन जो कि $\sin ax, \cos ax, e^{ax}, e^{-ax}, (x+a)^m$ में से कोई एक है, उत्तरोत्तर समाकलित होगा।

✍ श्रृंखला नियम : $\int u.v dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \text{..... ..}$

$$+ (-1)^{n-1} u^{n-1} v_n + (-1)^n \int u^n . v_n dx$$

जहाँ $u = \frac{d^n u}{dx^n}$ तथा v, v के n वें समाकलन को व्यक्त करता है।

✍ $\int x\, e^{ax} \sin(bx + c) dx = \frac{x\, e^{ax}}{a^2 + b^2}[a \sin(bx + c) - b \cos(bx + c)]$

$$- \frac{e^{ax}}{(a^2 + b^2)^2}[(a^2 - b^2)\sin(bx + c) - 2ab \cos(bx + c)] + k$$

✍ $\int x . e^{ax} \cos(bx + c) dx = \frac{x . e^{ax}}{a^2 + b^2}[a \cos(bx + c) + b \sin(bx + c)]$

$$- \frac{e^{ax}}{(a^2 + b^2)^2}[(a^2 - b^2)\cos(bx + c) + 2ab \sin(bx + c)] + k$$

✍ $\int a^x . \sin(bx + c) dx = \frac{a^x}{(\log a)^2 + b^2}$

$$[(\log a)\sin(bx + c) - b \cos(bx + c)] + k$$

✍ $\int a^x . \cos(bx + c) dx = \frac{a^x}{(\log a)^2 + b^2}$

$$[(\log a)\cos(bx + c) + b \sin(bx + c)] + k$$

✍ $\int \frac{a \cos x + b \sin x}{c \cos x + d \sin x} dx$

$$= \frac{ac + bd}{c^2 + d^2} x + \frac{ad - bc}{c^2 + d^2} \log| c \cos x + d \sin x| + k .$$

✍ $\int \sin mx \cos nx\, dx$, $\int \sin mx . \sin nx\, dx$, $\int \cos mx . \cos nx\, dx$ तथा $\int \cos mx . \sin nx\, dx$ रूप के समाकलन को ज्ञात करने के लिए निम्न त्रिकोणमितीय गुणधर्मों का उपयोग करते हैं।

✍ $\sin mx . \cos nx = \frac{1}{2}[\sin(m - n)x + \sin(m + n)x]$

$$\Rightarrow \cos mx . \sin nx = \frac{1}{2}[\sin(m + n)x - \sin(m - n)x]$$

✍ $\sin mx . \sin nx = \frac{1}{2}[\cos(m - n)x - \cos(m + n)x]$

$$\Rightarrow \cos mx . \cos nx = \frac{1}{2}[\cos(m - n)x + \cos(m + n)x]$$

✍ $I_{(n,m)} = \int \frac{\sin^n x}{\cos^m x} dx$ के लिए समानयन सूत्र

$$I_{(n,m)} = \frac{1}{m-1} . \frac{\sin^{n-1} x}{\cos^{m-1} x} - \frac{(n-1)}{(m-1)} . I_{(n-2, m-2)} \text{ है।}$$

Ordinary Thinking

Objective Questions

मूलभूत समाकलन

1. $\int \sqrt{1 - \sin 2x}\, dx = \text{.....}$, $x \in (0, \pi/4)$ **[MP PET 1987]**

(a) $-\sin x + \cos x$ (b) $\sin x - \cos x$

(c) $\tan x + \sec x$ (d) $\sin x + \cos x$

2. $\int \frac{1 + \cos^2 x}{\sin^2 x} dx =$ **[MP PET 1993; BIT Ranchi 1982]**

(a) $-\cot x - 2x + c$ (b) $-2\cot x - 2x + c$

(c) $-2\cot x - x + c$ (d) $-2\cot x + x + c$

3. $\int \sin^{-1}(\cos x) dx =$

(a) $\frac{\pi x}{2}$ (b) $\frac{\pi x^2}{2}$

(c) $\frac{\pi x - x^2}{2}$ (d) $\frac{\pi x + x^2}{2}$

4. $\int \frac{dx}{\tan x + \cot x} =$ **[MP PET 1991]**

(a) $\frac{\cos 2x}{4} + c$ (b) $\frac{\sin 2x}{4} + c$

(c) $-\frac{\sin 2x}{4} + c$ (d) $-\frac{\cos 2x}{4} + c$

5. $\int (e^{a \log x} + e^{x \log a}) dx =$

(a) $x^{a+1} + \frac{a^x}{\log a} + c$ (b) $\frac{x^{a+1}}{a+1} + a^x \log a + c$

(c) $\frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + c$ (d) इनमें से कोई नहीं

6. यदि $f'(x) = x^2 + 5$ और $f(0) = -1$, तब $f(x) =$

(a) $x^3 + 5x - 1$ (b) $x^3 + 5x + 1$

(c) $\frac{1}{3}x^3 + 5x - 1$ (d) $\frac{1}{3}x^3 + 5x + 1$

7. $\int \tan^{-1} \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}\, dx =$

(a) $2x^2 + c$ (b) $x^2 + c$

(c) $\frac{x^2}{2}+c$ (d) $2x+c$

8. $\int \frac{dx}{\sqrt{x}+\sqrt{x-2}} =$ **[MP PET 1990]**

(a) $\frac{1}{3}[x^{3/2}-(x-2)^{3/2}]+c$ (b) $\frac{2}{3}[x^{3/2}-(x-2)^{3/2}]+c$

(c) $\frac{1}{3}[(x-2)^{3/2}-x^{3/2}]+c$ (d) $\frac{2}{3}[(x-2)^{3/2}-x^{3/2}]+c$

9. $\int \frac{\sin x}{\sin(x-\alpha)}dx =$

[RPET 1999; Kerala (Engg.) 2002; AIEEE 2004]

(a) $x\cos\alpha-\sin\alpha\log\sin(x-\alpha)+c$

(b) $x\cos\alpha+\sin\alpha\log\sin(x-\alpha)+c$

(c) $x\sin\alpha-\sin\alpha\log\sin(x-\alpha)+c$

(d) इनमें से कोई नहीं

10. $\int \frac{\cos x-1}{\cos x+1}dx =$ **[MP PET 1989, 92]**

(a) $2\tan\frac{x}{2}-x+c$ (b) $\frac{1}{2}\tan\frac{x}{2}-x+c$

(c) $x-\frac{1}{2}\tan\frac{x}{2}+c$ (d) $x-2\tan\frac{x}{2}+c$

11. $\int \frac{dx}{1-\sin x} =$ **[MP PET 1991]**

(a) $x+\cos x+c$ (b) $1+\sin x+c$

(c) $\sec x-\tan x+c$ (d) $\sec x+\tan x+c$

12. यदि $\int(\sin 2x-\cos 2x)\,dx=\frac{1}{\sqrt{2}}\sin(2x-a)+b$, तब

[Roorkee 1978; MP PET 2001]

(a) $a=\frac{\pi}{4}, b=0$

(b) $a=-\frac{\pi}{4}, b=0$

(c) $a=\frac{5\pi}{4}$, $b=$ कोई स्थिरांक

(d) $a=-\frac{5\pi}{4}$, $b=$ कोई स्थिरांक

13. $\int\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.....\right)dx =$

(a) $-e^x+c$ (b) e^x+c

(c) $e^{-x}+c$ (d) $-e^{-x}+c$

14. $\int \frac{\cot x\tan x}{\sec^2 x-1}dx =$

(a) $\cot x-x+c$ (b) $-\cot x+x+c$

(c) $\cot x+x+c$ (d) $-\cot x-x+c$

15. $\int(\sec x+\tan x)^2\,dx =$ **[MP PET 1987, 92]**

(a) $2(\sec x+\tan x)-x+c$ (b) $1/3(\sec x+\tan x)^3+c$

(c) $\sec x(\sec x+\tan x)+c$ (d) $2(\sec x+\tan x)+c$

16. $\int x^{51}(\tan^{-1}x+\cot^{-1}x)\,dx =$ **[MP PET 1991]**

(a) $\frac{x^{52}}{52}(\tan^{-1}x+\cot^{-1}x)+c$ (b)

(c) $\frac{\pi x^{52}}{104}+\frac{\pi}{2}+c$ (d) $\frac{x^{52}}{52}+\frac{\pi}{2}+c$

17. $\int 5\sin x dx =$ **[MP PET 1988]**

(a) $5\cos x+c$ (b) $-5\cos x+c$

(c) $5\sin x+c$ (d) $-5\sin x+c$

18. $\int \frac{\tan x}{\sec x+\tan x}dx =$

(a) $\sec x+\tan x-x+c$ (b) $\sec x-\tan x+x+c$

(c) $\sec x+\tan x+x+c$ (d) $-\sec x-\tan x+x+c$

19. $\int \frac{dx}{\sin^2 x\cos^2 x} =$ **[Roorkee 1976; RPET 1991]**

(a) $\tan x+\cot x+c$ (b) $\cot x-\tan x+c$

(c) $\tan x-\cot x+c$ (d) इनमें से कोई नहीं

20. $\int\left(x+\frac{1}{x}\right)^3 dx =$

(a) $\frac{1}{4}\left(x+\frac{1}{x}\right)^4+c$

(b) $\frac{x^4}{4}+\frac{3x^2}{2}+3\log x-\frac{1}{2x^2}+c$

(c) $\frac{x^4}{4}+\frac{3x^2}{2}+3\log x+\frac{1}{x^2}+c$

(d) इनमें से कोई नहीं

21. $\int\sqrt{1+\sin\frac{x}{2}}\,dx =$

[IIT 1980; MP PET 1989; Pb. CET 2003]

(a) $\frac{1}{4}\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)+c$ (b) $4\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)+c$

(c) $4\left(\sin\frac{x}{4}-\cos\frac{x}{4}\right)+c$ (d) $4\left(\sin\frac{x}{4}+\cos\frac{x}{4}\right)+c$

22. $\int(\sin^{-1}x+\cos^{-1}x)\,dx =$ **[MP PET 1990]**

(a) $\frac{1}{2}\pi x+c$ (b) $x(\sin^{-1}x-\cos^{-1}x)+c$

(c) $x(\cos^{-1}x+\sin^{-1}x)+c$ (d) $\frac{\pi}{2}+x+c$

23. $\int\frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}dx =$ **[MP PET 1990]**

(a) $\sin x+c$ (b) $\cos x+c$

(c) $x+c$ (d) x^2+c

24. $\int \frac{x-1}{(x+1)^2}\,dx =$

(a) $\log(x+1)+\frac{2}{x+1}+c$ (b) $\log(x+1)-\frac{2}{x+1}+c$

(c) $\frac{2}{x+1}-\log(x+1)+c$ (d) इनमें से कोई नहीं

25. $\int \frac{\text{cosec}\,\theta-\cot\theta}{\text{cosec}\,\theta+\cot\theta}\,d\theta =$

(a) $2\text{cosec}\,\theta-2\cot\theta-\theta+c$

(b) $2\,\text{cosec}\,\theta-2\cot\theta+\theta+c$

(c) $2\,\text{cosec}\,\theta+2\cot\theta-\theta+c$

(d) इनमें से कोई नहीं

26. $\int (1+2x+3x^2+4x^3+.....)\,dx =$

(a) $(1+x)^{-1}+c$ (b) $(1-x)^{-1}+c$

(c) $(1-x)^{-1}-1+c$ (d) इनमें से कोई नहीं

27. यदि $\int (\cos x-\sin x)\,dx=\sqrt{2}\sin(x+\alpha)+c$, तब $\alpha =$

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $-\frac{\pi}{4}$

28. $\int \frac{3x^3-2\sqrt{x}}{x}\,dx =$ **[Roorkee 1976]**

(a) $x^3-\sqrt{x}+c$ (b) $x^3+\sqrt{x}+c$

(c) $x^3-2\sqrt{x}+c$ (d) $x^3-4\sqrt{x}+c$

29. यदि $\int \frac{dx}{1+\sin x}=\tan\left(\frac{x}{2}+a\right)+b$, तब **[Roorkee 1979]**

(a) $a=\frac{\pi}{4}, b=3$

(b) $a=-\frac{\pi}{4}, b=3$

(c) $a=\frac{\pi}{4}, b=$ कोई स्थिरांक

(d) $a=-\frac{\pi}{4}, b=$ कोई स्थिरांक

30. $\int \frac{dx}{\sin x+\cos x} =$ **[BIT Ranchi 1990; RPET 1997; Karnataka CET 1999; Orissa JEE 2004]**

(a) $\log\tan\left(\frac{\pi}{8}+\frac{x}{2}\right)+c$ (b) $\log\tan\left(\frac{\pi}{8}-\frac{x}{2}\right)+c$

(c) $\frac{1}{\sqrt{2}}\log\tan\left(\frac{\pi}{8}+\frac{x}{2}\right)+c$ (d) इनमें से कोई नहीं

31. $\int \frac{1}{x\sqrt{x^2-1}}\,dx =$ **[MP PET 1988]**

(a) $\cos^{-1}x+c$ (b) $\sec^{-1}x+c$

(c) $\cot^{-1}x+c$ (d) $\tan^{-1}x+c$

32. $\int \frac{\cos 2x+2\sin^2 x}{\cos^2 x}\,dx =$

(a) $2\sec x+c$ (b) $2\tan x+c$

(c) $\tan x+c$ (d) इनमें से कोई नहीं

33. $\int \frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}\,dx =$

(a) $\tan x+\cot x+c$ (b) $\tan x-\cot x+c$

(c) $\text{cosec}\,x-\cot x+c$ (d) $\sec x-\text{cosec}\,x+c$

34. $\int \left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2 dx =$

(a) $x+\cos x+c$ (b) $2\cos^2\frac{x}{2}+c$

(c) $\frac{1}{3}\left(\cos\frac{x}{2}-\frac{x}{2}\right)^3+c$ (d) $x-\cos x+c$

35. $\int \frac{1}{x^2}(2x+1)^3 dx =$

(a) $4x^2+12x+6\log x-\frac{1}{x}+c$

(b) $4x^2+12x-6\log x-\frac{2}{x}+c$

(c) $2x^2+8x+3\log x-\frac{2}{x}+c$

(d) $8x^2+6x+6\log x+\frac{2}{x}+c$

36. $\int \frac{5(x^6+1)}{x^2+1}dx =$

(a) $5(x^7+x)\tan^{-1}x+c$

(b) $x^5-\frac{5}{3}x^3+5x+c$

(c) $3x^4-5x^2+15x+c$

(d) $5\tan^{-1}(x^2+1)+\log(x^2+1)+c$

37. $\int \frac{ax^{-2}+bx^{-1}+c}{x^{-3}}\,dx =$

(a) $2ax^2+3bx^3+4cx^4+k$

(b) $6ax^2+4bx^3+3cx^4+k$

(c) $a+b+cx^2+k$

(d) $\frac{1}{2}ax^2+\frac{1}{3}bx^3+\frac{1}{4}cx^4+k$

38. $\int \frac{dx}{\sin x+\sqrt{3}\cos x} =$

(a) $\log\tan\left(\frac{x}{2}+\frac{\pi}{2}\right)+c$ (b) $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)+c$

(c) $\log\cot\left(\frac{x}{2}+\frac{\pi}{6}\right)+c$ (d) $\frac{1}{2}\log\cot\left(\frac{x}{2}+\frac{\pi}{6}\right)+c$

39. $\int \frac{x^2+x-6}{(x-2)(x-1)}dx =$

(a) $x+2\log(x-1)+c$ (b) $2x+2\log(x-1)+c$

(c) $x+4\log(1-x)+c$ (d) $x+4\log(x-1)+c$

40. $\int \frac{dx}{4\cos^3 2x-3\cos 2x}=$

(a) $\frac{1}{3}\log[\sec 6x+\tan 6x]+c$ (b) $\frac{1}{6}\log[\sec 6x+\tan 6x]+c$

(c) $\log[\sec 6x+\tan 6x]+c$ (d) इनमें से कोई नहीं

41. $\int \frac{\sin 3x}{\sin x}dx=$

(a) $x+\sin 2x+c$ (b) $3x+\sin 2x+c$

(c) $3x+\sin^2 x+c$ (d) इनमें से कोई नहीं

42. यदि $\int \frac{f(x)\,dx}{\log\sin x}=\log\log\sin x$, तब $f(x)=$

(a) $\sin x$ (b) $\cos x$

(c) $\log\sin x$ (d) $\cot x$

43. $\int \frac{\sin x+\text{cosec } x}{\tan x}dx=$

(a) $\sin x-\text{cosec } x+c$ (b) $\text{cosec } x-\sin x+c$

(c) $\log\tan x+c$ (d) $\log\cot x+c$

44. $\int \frac{1}{\sqrt{1+\sin x}}dx=$ **[RPET 1996]**

(a) $2\sqrt{2}\log\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)+c$ (b) $\frac{1}{\sqrt{2}}\log\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)+c$

(c) $\sqrt{2}\log\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)+c$ (d) $\frac{1}{2\sqrt{2}}\log\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)+c$

45. $\int (\tan x-\cot x)^2\,dx=$

(a) $\tan x+\cot x+c$ (b) $\sec x\tan x+c$

(c) $\text{cosec}\,x\cot x+c$ (d) इनमें से कोई नहीं

46. $\int \{1+2\tan x(\tan x+\sec x)\}^{1/2}dx=$ **[Roorkee 1987]**

(a) $\log(\sec x+\tan x)+c$

(b) $\log(\sec x+\tan x)^{1/2}+c$

(c) $\log\sec x(\sec x+\tan x)+c$

(d) इनमें से कोई नहीं

47. $\int \frac{2x}{(2x+1)^2}dx=$ **[DSSE 1985]**

(a) $\frac{1}{2}\log(2x+1)+\frac{1}{2(2x+1)}+c$

(b) $\frac{1}{2}\log(2x+1)-\frac{1}{2(2x+1)}+c$

(c) $2\log(2x+1)+\frac{1}{2(2x+1)}+c$

(d) $2\log(2x+1)-\frac{1}{2(2x+1)}+c$

48. $\int \frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx=$ **[MP PET 1996]**

(a) $\tan x+\cot x+c$ (b) $\tan x+\text{cosec } x+c$

(c) $-\tan x+\cot x+c$ (d) $\tan x+\sec x+c$

49. $\int (3\,\text{cosec}^2 x+2\sin 3x)\,dx=$ **[AI CBSE 1981]**

(a) $3\cot x+\frac{2}{3}\cos 3x+c$

(b) $-\left(3\cot x+\frac{2}{3}\cos 3x\right)+c$

(c) $3\cot x-\frac{2}{3}\cos 3x+c$

(d) इनमें से कोई नहीं

50. यदि $f'(x)=\frac{1}{x}+x$ और $f(1)=\frac{5}{2}$, तब $f(x)=$

(a) $\log x+\frac{x^2}{2}+2$ (b) $\log x+\frac{x^2}{2}+1$

(c) $\log x-\frac{x^2}{2}+2$ (d) $\log x-\frac{x^2}{2}+1$

51. $\int \frac{dx}{\sqrt{1+x}+\sqrt{x}}=$

(a) $\frac{2}{3}(1+x)^{2/3}-\frac{2}{3}x^{2/3}+c$

(b) $\frac{3}{2}(1+x)^{2/3}+\frac{3}{2}x^{2/3}+c$

(c) $\frac{3}{2}(1+x)^{3/2}+\frac{3}{2}x^{3/2}+c$

(d) $\frac{2}{3}(1+x)^{3/2}-\frac{2}{3}x^{3/2}+c$

52. $\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx=$ **[MP PET 1994]**

(a) $2[\sin x+x\cos\alpha]+c$ (b) $2[\sin x+\sin\alpha]+c$

(c) $2[-\sin x+x\cos\alpha]+c$ (d) $-2[\sin x+\sin\alpha]+c$

53. $\int \frac{1-\tan x}{1+\tan x}dx=$ **[MP PET 1994]**

(a) $\log\sec\left(\frac{\pi}{4}-x\right)+c$ (b) $\log\cos\left(\frac{\pi}{4}+x\right)+c$

(c) $\log\sin\left(\frac{\pi}{4}+x\right)+c$ (d) इनमें से कोई नहीं

54. $\int a^x\,da=$ **[MP PET 1994, 96]**

(a) $\frac{a^x}{\log_e a}+c$ (b) $a^x\log_e a+c$

(c) $\frac{a^{x+1}}{x+1}+c$ (d) $xa^{x-1}+c$

55. $\int \cot x\,dx$ का मान है **[RPET 1995]**

(a) $\log\cos x+c$ (b) $\log\tan x+c$

(c) $\log\sin x+c$ (d) $\log\sec x+c$

56. $\int \frac{1}{x^4}dx$ का मान है **[RPET 1995]**

(a) $\frac{1}{-3x^3}+c$ (b) $\frac{1}{3x^3}+c$

(c) $\frac{1}{-4x^3}+c$ (d) $-\frac{1}{3x^2}+c$

57. $\int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} dx =$ **[MNR 1985]**

(a) $e \cdot 3^{-3x} + c$ (b) $e^3 \log x + c$

(c) $\frac{x^3}{3} + c$ (d) इनमें से कोई नहीं

58. $\int \frac{x^4 + x^2 + 1}{x^2 - x + 1} dx =$

(a) $\frac{1}{3}x^3 + \frac{1}{2}x^2 + x + c$ (b) $\frac{1}{3}x^3 - \frac{1}{2}x^2 + x + c$

(c) $\frac{1}{3}x^3 + \frac{1}{2}x^2 - x + c$ (d) इनमें से कोई नहीं

59. $\int \sec x \, dx =$ **[MP PET 1988, 95; RPET 1996]**

(a) $\log \tan\left(\frac{\pi}{8} + \frac{x}{2}\right) + c$ (b) $-\log(\sec x - \tan x) + c$

(c) $\log(\sec x - \tan x) + c$ (d) इनमें से कोई नहीं

60. $\int \sqrt{1 + \sin x} \, dx =$ **[MP PET 1995]**

(a) $\frac{1}{2}\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) + c$ (b) $\frac{1}{2}\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) + c$

(c) $2\sqrt{1 + \sin x} + c$ (d) $-2\sqrt{1 - \sin x} + c$

61. $\int \text{cosec}^2 x \, dx$ का मान है **[MP PET 1999]**

(a) $\cot x + c$ (b) $-\cot x + c$

(c) $\tan^2 x + c$ (d) $-\cot^2 x + c$

62. $\int \left(2\sin x + \frac{1}{x}\right) dx$ का मान है **[MP PET 1999]**

(a) $-2\cos x + \log x + c$ (b) $2\cos x + \log x + c$

(c) $-2\sin x - \frac{1}{x^2} + c$ (d) $-2\cos x + \frac{1}{x^2} + c$

63. $\int \sqrt{1 + \cos x} \, dx =$ **[RPET 1996]**

(a) $2\sqrt{2}\sin\frac{x}{2} + c$ (b) $-2\sqrt{2}\sin\frac{x}{2} + c$

(c) $-2\sqrt{2}\cos\frac{x}{2} + c$ (d) $2\sqrt{2}\cos\frac{x}{2} + c$

64. $\int 2\sin x \cos x \, dx =$ **[SCRA 1996]**

(a) $\cos 2x + c$ (b) $\sin 2x + c$

(c) $\cos^2 x + c$ (d) $\sin^2 x + c$

65. $\int \tan^2 x \, dx =$ **[SCRA 1996]**

(a) $\tan x + x + c$ (b) $\tan x - x + c$

(c) $\sec x + x + c$ (d) $\sec x - x + c$

66. $\int e^{\log(\sin x)} dx =$ **[MP PET 1995]**

(a) $\sin x + c$ (b) $-\cos x + c$

(c) $e^{\log(\cos x)} + c$ (d) इनमें से कोई नहीं

67. $\int e^{x\log a} \cdot e^x \, dx =$ **[Kerala (Engg.) 2005]**

(a) $(ae)^x + c$ (b) $\frac{(ae)^x}{\log(ae)} + c$

(c) $\frac{e^x}{1 + \log a} + c$ (d) इनमें से कोई नहीं

68. $\int \frac{1}{\sqrt{1 + \cos x}} dx =$

(a) $\sqrt{2}\log\left(\sec\frac{x}{2} + \tan\frac{x}{2}\right) + K$

(b) $\frac{1}{\sqrt{2}}\log\left(\sec\frac{x}{2} + \tan\frac{x}{2}\right) + K$

(c) $\log\left(\sec\frac{x}{2} + \tan\frac{x}{2}\right) + K$

(d) इनमें से कोई नहीं

69. $\int \frac{\cos 2x - 1}{\cos 2x + 1} dx =$ **[MP PET 2000]**

(a) $\tan x - x + c$ (b) $x + \tan x + c$

(c) $x - \tan x + c$ (d) $-x - \cot x + c$

70. $\int \frac{ax^3 + bx^2 + c}{x^4} dx =$ **[RPET 2001]**

(a) $a\log x + \frac{b}{x^2} + \frac{c}{3x^3} + c$ (b) $a\log x + \frac{b}{x} - \frac{c}{3x^3} + c$

(c) $a\log x - \frac{b}{x} - \frac{c}{3x^3} + c$ (d) इनमें से कोई नहीं

71. $\int \frac{1}{(x-5)^2} dx$ का मान है

[Karnataka CET 2001; Pb. CET 2002]

(a) $\frac{1}{x-5} + c$ (b) $-\frac{1}{x-5} + c$

(c) $\frac{2}{(x-5)^3} + c$ (d) $-2(x-5)^3 + c$

72. यदि $\int f(x)dx = f(x)$, तब $\int [f(x)]^2 \, dx =$ **[DCE 2002]**

(a) $\frac{1}{2}[f(x)]^2$ (b) $[f(x)]^3$

(c) $\frac{[f(x)]^3}{3}$ (d) $[f(x)]^2$

73. यदि $\int \sqrt{2}\sqrt{1 + \sin x} \, dx = -4\cos(ax + b) + c$, तब (a, b) का मान है **[UPSEAT 2002]**

(a) $\frac{1}{2}, \frac{\pi}{4}$ (b) $1, \frac{\pi}{2}$

(c) 1, 1 (d) इनमें से कोई नहीं

74. $\int 13^x dx =$ **[Kerala (Engg.) 2002]**

(a) $\frac{13^x}{\log 13} + c$ (b) $13^{x+1} + c$

(c) $14x + c$ (d) $14^{x+1} + c$

75. $\int a^x \, dx =$ **[RPET 2003]**

(a) $\frac{a^x}{\log a}+c$ (b) $a^x \log a + c$

(c) $\log a + c$ (d) $a^x + c$

76. $\int \sec x \tan x \, dx =$ **[RPET 2003]**

(a) $\sec x + \tan x + c$ (b) $\sec x + c$

(c) $\tan x + c$ (d) $-\sec x + c$

77. $\int \left(\sin^4 x - \cos^4 x\right) dx =$ **[RPET 2003]**

(a) $-\frac{\cos 2x}{2}+c$ (b) $-\frac{\sin 2x}{2}+c$

(c) $\frac{\sin 2x}{2}+c$ (d) $\frac{\cos 2x}{2}+c$

78. $\int \frac{(x+1)^2\, dx}{x(x^2+1)} =$ **[MP PET 2003]**

(a) $\log_e x + c$ (b) $\log_e x + 2\tan^{-1} x + c$

(c) $\log_e \frac{1}{x^2+1}+c$ (d) $\log_e\{x(x^2+1)\}+c$

79. $\int \frac{dx}{\sqrt{1-x}} =$ **[Pb. CET 2001]**

(a) $2\sqrt{1-x}+c$ (b) $-2\sqrt{1-x}+c$

(c) $-\sin^{-1}\sqrt{x}+c$ (d) $\sin^{-1}\sqrt{x}+c$

80. $\int \frac{dx}{1-x^2} =$ **[MP PET 1987, 92, 2000]**

(a) $\tan^{-1} x + c$ (b) $\sin^{-1} x + c$

(c) $\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|+c$ (d) $\frac{1}{2}\ln\left|\frac{1-x}{1+x}\right|+c$

81. $\int \frac{dx}{4x^2+9} =$ **[MP PET 1991; Roorkee 1977; MNR 1974]**

(a) $\frac{1}{2}\tan^{-1}\left(\frac{2x}{3}\right)+c$ (b) $\frac{3}{2}\tan^{-1}\left(\frac{2x}{3}\right)+c$

(c) $\frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right)+c$ (d) $\frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right)+c$

82. $\int \sqrt{1+x^2}\, dx =$ **[MP PET 1987, 89]**

(a) $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log(x+\sqrt{1+x^2})+c$

(b) $\frac{2}{3}(1+x^2)^{3/2}+c$

(c) $\frac{2}{3}x(1+x^2)^{3/2}+c$

(d) इनमें से कोई नहीं

83. $\int \frac{dx}{\sqrt{x^2-a^2}} =$ **[SCRA 1996]**

(a) $\sin^{-1}\left(\frac{x}{a}\right)+c$ (b) $\log_e|\, x+\sqrt{x^2-a^2}\,|+c$

(c) $\log_e|\, x-\sqrt{x^2-a^2}\,|+c$ (d) $\frac{x\sqrt{x^2-a^2}}{2+c}$

84. $\int \frac{x^2}{x^2+4}\, dx =$ **[RPET 2001]**

(a) $x-2\tan^{-1}(x/2)+c$ (b) $x+2\tan^{-1}(x/2)+c$

(c) $x-4\tan^{-1}(x/2)+c$ (d) $x+4\tan^{-1}(x/2)+c$

85. $\int \sqrt{x^2+a^2}\, dx =$ **[RPET 2001]**

(a) $\frac{x}{2}\sqrt{x^2+a^2}-\frac{a^2}{2}\log\{x+\sqrt{x^2+a^2}\}+c$

(b) $\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\{x+\sqrt{x^2+a^2}\}+c$

(c) $\frac{x}{2}\sqrt{x^2+a^2}-\frac{a^2}{2}\log\{x-\sqrt{x^2+a^2}\}+c$

(d) $\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\{x-\sqrt{x^2+a^2}\}+c$

86. $\int \frac{dx}{a^2-x^2} =$ **[EAMCET 2002]**

(a) $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$ (b) $\frac{1}{2a}\sin^{-1}\left(\frac{a-x}{a+x}\right)$

(c) $\frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)$ (d) $\frac{1}{2a}\log\left(\frac{a-x}{a+x}\right)$

87. $\int \frac{dx}{\sqrt{x^2+a^2}} =$ **[MP PET 2004]**

(a) $\frac{1}{2}x\sqrt{x^2+a^2}+\frac{1}{2}a^2\log(x+\sqrt{x^2+a^2})+c$

(b) $\frac{1}{2}\log(x^2+a^2)+c$

(c) $\log(x+\sqrt{x^2+a^2})+c$

(d) $\log(x-\sqrt{x^2+a^2})+c$

प्रतिस्थापन द्वारा समाकलन

1. $\int \frac{dx}{1+e^x} =$ **[MP PET 1991; Roorkee 1977]**

(a) $\log(1+e^x)$ (b) $-\log(1+e^{-x})$

(c) $-\log(1-e^{-x})$ (d) $\log(e^{-x}+e^{-2x})$

2. $\int \frac{dx}{e^x+e^{-x}} =$ **[Bihar CEE 1976; MNR 1974]**

(a) $\tan^{-1}(e^{-x})$ (b) $\tan^{-1}(e^x)$

(c) $\log(e^x-e^{-x})$ (d) $\log(e^x+e^{-x})$

3. $\int \frac{e^{\sqrt{x}}\cos e^{\sqrt{x}}}{\sqrt{x}}dx =$

(a) $2\sin e^{\sqrt{x}}$ (b) $\sin e^{\sqrt{x}}$

(c) $2\cos e^{\sqrt{x}}$ (d) $-2\sin e^{\sqrt{x}}$

4. $\int \frac{dx}{x + x\log x} =$ **[MP PET 1993; Roorkee 1977]**

(a) $\log(1+\log x)$ (b) $\log\log(1+\log x)$

(c) $\log x + \log(\log x)$ (d) इनमें से कोई नहीं

5. $\int \frac{1+\log x}{x} dx$ का मान ज्ञात करने के लिये उचित प्रतिस्थापन है **[MP PET 1988]**

(a) $\log x = t$ (b) $1 + \log x = t$

(c) $\frac{1}{x} = t$ (d) इनमें से कोई नहीं

6. $\int \frac{\sec x\, dx}{\sqrt{\cos 2x}} =$

(a) $\sin^{-1}(\tan x)$ (b) $\tan x$

(c) $\cos^{-1}(\tan x)$ (d) $\frac{\sin x}{\sqrt{\cos x}}$

7. $\int \frac{dx}{x\sqrt{2ax - x^2}}$ का मान ज्ञात करने के लिये उचित प्रतिस्थापन है

(a) $x = a\cos t$ (b) $x = 2a\cos t$

(c) $x = 2at$ (d) $x = 2a\sin^2 t$

8. $\int \frac{x\, dx}{1 - x\cot x} =$

(a) $\log(\cos x - x\sin x) + c$ (b) $\log(x\sin x - \cos x) + c$

(c) $\log(\sin x - x\cos x) + c$ (d) इनमें से कोई नहीं

9. $\int \frac{\sin 2x}{1 + \sin^2 x} dx =$ **[Roorkee 1976]**

(a) $\log\sin 2x + c$ (b) $\log(1 + \sin^2 x) + c$

(c) $\frac{1}{2}\log(1 + \sin^2 x) + c$ (d) $\tan^{-1}(\sin x) + c$

10. $\int \frac{x^3}{\sqrt{x^2 + 2}} dx =$

(a) $\frac{1}{3}(x^2+2)^{3/2} + 2(x^2+2)^{1/2} + c$

(b) $\frac{1}{3}(x^2+2)^{3/2} - 2(x^2+2)^{1/2} + c$

(c) $\frac{1}{3}(x^2+2)^{3/2} + (x^2+2)^{1/2} + c$

(d) $\frac{1}{3}(x^2+2)^{3/2} - (x^2+2)^{1/2} + c$

11. $\int \frac{x^{e-1} + e^{x-1}}{x^e + e^x} dx =$

(a) $\log(x^e + e^x) + c$ (b) $e\log(x^e + e^x) + c$

(c) $\frac{1}{e}\log(x^e + e^x) + c$ (d) इनमें से कोई नहीं

12. $\int \frac{\sin x\, dx}{a^2 + b^2\cos^2 x} =$

(a) $\log(a^2 + b^2\cos^2 x) + c$ (b) $\frac{1}{ab}\tan^{-1}\left(\frac{a\cos x}{b}\right) + c$

(c) $\frac{1}{ab}\cot^{-1}\left(\frac{b\cos x}{a}\right) + c$ (d) $\frac{1}{ab}\cot^{-1}\left(\frac{a\cos x}{b}\right) + c$

13. $\int \sec x\log(\sec x + \tan x)\, dx =$

(a) $[\log(\sec x + \tan x)]^2 + c$

(b) $\frac{1}{2}[\log(\sec x + \tan x)]^2 + c$

(c) $\sec^2 x + \tan x\sec x + c$

(d) इनमें से कोई नहीं

14. $\int \frac{x-2}{x^2 - 4x + 3} dx =$ **[MP PET 1987]**

(a) $\log\sqrt{x^2 - 4x + 3} + c$ (b) $x\log(x-3) - 2\log(x-2) + c$

(c) $\log[(x-3)(x-1)]$ (d) इनमें से कोई नहीं

15. $\int \frac{3x^2}{x^6 + 1} dx =$ **[MNR 1981; MP PET 1988; RPET 1995]**

(a) $\log(x^6 + 1) + c$ (b) $\tan^{-1}(x^3) + c$

(c) $3\tan^{-1}(x^3) + c$ (d) $3\tan^{-1}\left(\frac{x^3}{3}\right) + c$

16. $\int \frac{\cot x}{\log\sin x} dx =$ **[MNR 1974]**

(a) $\log(\log\sin x) + c$ (b) $\log(\log\operatorname{cosec} x) + c$

(c) $2\log(\log\sin x) + c$ (d) इनमें से कोई नहीं

17. $\int \frac{(1+\log x)^2}{x} dx =$ **[Roorkee 1977]**

(a) $(1+\log x)^3 + c$ (b) $3(1+\log x)^3 + c$

(c) $\frac{1}{3}(1+\log x)^3 + c$ (d) इनमें से कोई नहीं

18. $\int \sec^p x\tan x\, dx =$

(a) $\frac{\sec^{p+1} x}{p+1} + c$ (b) $\frac{\sec^p x}{p} + c$

(c) $\frac{\tan^{p+1} x}{p+1} + c$ (d) $\frac{\tan^p x}{p} + c$

19. $\int \frac{dx}{e^x - 1} =$ **[MP PET 1989]**

(a) $\ln(1 - e^{-x}) + c$ (b) $-\ln(1 - e^{-x}) + c$

(c) $\ln(e^x - 1) + c$ (d) इनमें से कोई नहीं

20. $\int x^2\sec x^3\, dx =$ **[MNR 1986; Roorkee 1975]**

(a) $\log(\sec x^3 + \tan x^3)$ (b) $3(\sec x^3 + \tan x^3)$

(c) $\frac{1}{3}\log(\sec x^3 + \tan x^3)$ (d) इनमें से कोई नहीं

21. $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx =$ **[RPET 1995]**

(a) $\cot^{-1}(\tan^2 x) + c$ (b) $\tan^{-1}(\tan^2 x) + c$

(c) $\cot^{-1}(\cot^2 x)+c$ (d) $\tan^{-1}(\cot^2 x)+c$

22. $\int \frac{x-2}{x(2\log x - x)}dx =$

(a) $\log(2\log x - x)+c$ (b) $\log\left(\frac{1}{2\log x - x}\right)+c$

(c) $\log(x - 2\log x)+c$ (d) $\log\left(\frac{1}{x - 2\log x}\right)+c$

23. $\int x\sqrt{1+x^2}\,dx =$ **[MP PET 1989]**

(a) $\frac{1+2x^2}{\sqrt{1+x^2}}+c$ (b) $\sqrt{1+x^2}+c$

(c) $3(1+x^2)^{3/2}+c$ (d) $\frac{1}{3}(1+x^2)^{3/2}+c$

24. $\int \frac{e^x(x+1)}{\cos^2(xe^x)}dx =$

[Roorkee 1979; MP PET 1995; Pb. CET 2001]

(a) $\tan(xe^x)+c$ (b) $\sec(xe^x)\tan(xe^x)+c$

(c) $-\tan(xe^x)+c$ (d) इनमें से कोई नहीं

25. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx =$

[MP PET 1987; IIT 1990; SCRA 1996; RPET 2001]

(a) $2\cos\sqrt{x}+c$ (b) $2\sin\sqrt{x}+c$

(c) $\sin\sqrt{x}+c$ (d) $\frac{1}{2}\cos\sqrt{x}+c$

26. $\int \frac{x+1}{\sqrt{1+x^2}}dx =$ **[MP PET 1991]**

(a) $\sqrt{1+x^2}+\tan^{-1}x+c$

(b) $\sqrt{1+x^2}-\log\{x+\sqrt{1+x^2}\}+c$

(c) $\sqrt{1+x^2}+\log\{x+\sqrt{1+x^2}\}+c$

(d) $\sqrt{1+x^2}+\log(\sec x+\tan x)+c$

27. $\int \frac{\sin x\cos x}{a\cos^2 x + b\sin^2 x}dx =$ **[AI CBSE 1988, 89]**

(a) $\frac{1}{2(b-a)}\log(a\cos^2 x + b\sin^2 x)+c$

(b) $\frac{1}{b-a}\log(a\cos^2 x + b\sin^2 x)+c$

(c) $\frac{1}{2}\log(a\cos^2 x + b\sin^2 x)+c$

(d) इनमें से कोई नहीं

28. $\int \frac{e^{\tan^{-1}x}}{1+x^2}dx =$ **[MP PET 1987]**

(a) $\log(1+x^2)+c$ (b) $\log e^{\tan^{-1}x}+c$

(c) $e^{\tan^{-1}x}+c$ (d) $\tan^{-1}e^{\tan^{-1}x}+c$

29. $\int \frac{1}{x(\log x)^2}dx =$

(a) $\frac{1}{\log x}+c$ (b) $-\frac{1}{\log x}+c$

(c) $\log\log x+c$ (d) $-\log\log x+c$

30. $\int \frac{1}{\sqrt{x}}\tan^4\sqrt{x}\sec^2\sqrt{x}\,dx =$

(a) $2\tan^5\sqrt{x}+c$ (b) $\frac{1}{5}\tan^5\sqrt{x}+c$

(c) $\frac{2}{5}\tan^5\sqrt{x}+c$ (d) इनमें से कोई नहीं

31. $\int \frac{a^x}{\sqrt{1-a^{2x}}}dx =$ **[MNR 1983, 87]**

(a) $\frac{1}{\log a}\sin^{-1}a^x+c$ (b) $\sin^{-1}a^x+c$

(c) $\frac{1}{\log a}\cos^{-1}a^x+c$ (d) $\cos^{-1}a^x+c$

32. $\int \frac{\sqrt{\tan x}}{\sin x\cos x}dx =$

[Bihar CEE 1974; MP PET 2002; Kerala (Engg.) 2002]

(a) $2\sqrt{\sec x}+c$ (b) $2\sqrt{\tan x}+c$

(c) $\frac{2}{\sqrt{\tan x}}+c$ (d) $\frac{2}{\sqrt{\sec x}}+c$

33. $\int \frac{\sin 2x}{a^2+b^2\sin^2 x}dx =$ **[Roorkee 1977]**

(a) $\frac{1}{b^2}\log(a^2+b^2\sin^2 x)+c$

(b) $\frac{1}{b}\log(a^2+b^2\sin^2 x)+c$

(c) $\log(a^2+b^2\sin^2 x)+c$

(d) $b^2\log(a^2+b^2\sin^2 x)+c$

34. $\int \frac{1}{x\sqrt{1+\log x}}dx =$ **[Roorkee 1977]**

(a) $\frac{2}{3}(1+\log x)^{3/2}+c$ (b) $(1+\log x)^{3/2}+c$

(c) $2\sqrt{1+\log x}+c$ (d) $\sqrt{1+\log x}+c$

35. $\int \frac{\sec^2 x}{1+\tan x}dx =$ **[MP PET 1987]**

(a) $\log(\cos x+\sin x)+c$ (b) $\log(\sec^2 x)+c$

(c) $\log(1+\tan x)+c$ (d) $-\frac{1}{(1+\tan x)^2}+c$

36. $\int \frac{e^{2x}-1}{e^{2x}+1}dx =$ **[MP PET 1987]**

(a) $\frac{e^{2x}-1}{e^{2x}+1}+c$ (b) $\log(e^{2x}+1)-x+c$

(c) $\log(e^{2x}+1)+c$ (d) इनमें से कोई नहीं

37. $\int \frac{\text{cosec}\,x}{\log\tan\frac{x}{2}}dx =$

(a) $\log\left(\log\tan\frac{x}{2}\right)+c$ (b) $2\log\left(\log\tan\frac{x}{2}\right)+c$

(c) $\frac{1}{2}\log\left(\log\tan\frac{x}{2}\right)+c$ (d) इनमें से कोई नहीं

38. $\int \frac{1}{\cos^2 x(1-\tan x)^2}dx =$

(a) $\frac{1}{\tan x-1}+c$ (b) $\frac{1}{1-\tan x}+c$

(c) $-\frac{1}{3}\frac{1}{(1-\tan x)^3}+c$ (d) इनमें से कोई नहीं

39. $\int \frac{10x^9+10^x\log_e 10}{10^x+x^{10}}dx =$ **[MNR 1979]**

(a) $-\frac{1}{2}\frac{1}{(10^x+x^{10})^2}+c$ (b) $\log(10^x+x^{10})+c$

(c) $\frac{1}{2}\frac{1}{(10^x+x^{10})^2}+c$ (d) इनमें से कोई नहीं

40. $\int \frac{1}{(e^x+e^{-x})^2}dx =$

(a) $-\frac{1}{2(e^{2x}+1)}+c$ (b) $\frac{1}{2(e^{2x}+1)}+c$

(c) $-\frac{1}{e^{2x}+1}$ (d) इनमें से कोई नहीं

41. $\int \frac{\cos 2x}{(\cos x+\sin x)^2}dx =$

(a) $\log\sqrt{\cos x+\sin x}+c$ (b) $\log(\cos x-\sin x)+c$

(c) $\log(\cos x+\sin x)+c$ (d) $-\frac{1}{\cos x+\sin x}+c$

42. $\int \frac{\tan(\log x)}{x}dx =$

(a) $\log\cos(\log x)+c$ (b) $\log\sin(\log x)+c$

(c) $\log\sec(\log x)+c$ (d) $\log\operatorname{cosec}(\log x)+c$

43. $\int \cos^3 x\, e^{\log(\sin x)}\, dx =$

(a) $-\frac{\sin^4 x}{4}+c$ (b) $-\frac{\cos^4 x}{4}+c$

(c) $\frac{e^{\sin x}}{4}+c$ (d) इनमें से कोई नहीं

44. $\int \tan(3x-5)\sec(3x-5)\,dx =$ **[MP PET 1988]**

(a) $\sec(3x-5)+c$ (b) $\frac{1}{3}\sec(3x-5)+c$

(c) $\tan(3x-5)+c$ (d) इनमें से कोई नहीं

45. $\int \frac{x}{1+x^4}dx =$ **[IIT 1978; UPSEAT 2002]**

(a) $\frac{1}{2}\cot^{-1}x^2+c$ (b) $\frac{1}{2}\tan^{-1}x^2+c$

(c) $\cot^{-1}x^2+c$ (d) $\tan^{-1}x^2+c$

46. $\int \frac{e^{-x}}{1+e^x}dx =$

(a) $\log(1+e^x)-x-e^{-x}+c$ (b) $\log(1+e^x)+x-e^{-x}+c$

(c) $\log(1+e^x)-x+e^{-x}+c$ (d) $\log(1+e^x)+x+e^{-x}+c$

47. $\int \frac{1}{\sqrt{1-e^{2x}}}dx =$ **[MP PET 1993, 2002; RPET 1999]**

(a) $x-\log[1+\sqrt{1-e^{2x}}]+c$ (b) $x+\log[1+\sqrt{1-e^{2x}}]+c$

(c) $\log[1+\sqrt{1-e^{2x}}]-x+c$ (d) इनमें से कोई नहीं

48. $\int \frac{3x^2}{\sqrt{9-16x^6}}dx =$

(a) $\frac{1}{4}\sin^{-1}\left(\frac{4x^3}{3}\right)+c$ (b) $\frac{1}{3}\sin^{-1}\left(\frac{4x^3}{3}\right)+c$

(c) $\frac{1}{4}\sin^{-1}x^3+c$ (d) $\frac{1}{3}\sin^{-1}x^3+c$

49. $\int \cos x\sqrt{4-\sin^2 x}\,dx =$

(a) $\frac{1}{2}\sin x\sqrt{4-\sin^2 x}-2\sin^{-1}\left(\frac{1}{2}\sin x\right)+c$

(b) $\frac{1}{2}\sin x\sqrt{4-\sin^2 x}+2\sin^{-1}\left(\frac{1}{2}\sin x\right)+c$

(c) $\frac{1}{2}\sin x\sqrt{4-\sin^2 x}+\sin^{-1}\left(\frac{1}{2}\sin x\right)+c$

(d) इनमें से कोई नहीं

50. $\int x^2(3)^{x^3+1}dx =$

(a) $(3)^{x^3}+c$ (b) $\frac{(3)^{x^3}}{\log 3}+c$

(c) $\log 3(3)^{x^3}+c$ (d) इनमें से कोई नहीं

51. $\int \sec^{2/3}x\operatorname{cosec}^{4/3}x\,dx =$

(a) $-3(\tan x)^{1/3}+c$ (b) $-3(\tan x)^{-1/3}+c$

(c) $3(\tan x)^{-1/3}+c$ (d) $(\tan x)^{-1/3}+c$

52. $\int \cos^5 x\,dx =$

(a) $\sin x-\frac{2}{3}\sin^3 x+\frac{1}{5}\sin^5 x+c$

(b) $\sin x+\frac{2}{3}\sin^3 x+\frac{1}{5}\sin^5 x+c$

(c) $\sin x-\frac{2}{3}\sin^3 x-\frac{1}{5}\sin^5 x+c$

(d) इनमें से कोई नहीं

53. $\int \sec x\tan^3 x\,dx =$

(a) $\frac{1}{3}\sec^3 x-\sec x+c$ (b) $\sec^3 x-\sec x+c$

(c) $\frac{1}{3}\sec^3 x+\sec x+c$ (d) इनमें से कोई नहीं

54. $\int \frac{d\theta}{\sin\theta\cos^3\theta} =$

(a) $\log\tan\theta+\tan^2\theta+c$ (b) $\log\tan\theta-\frac{1}{2}\tan^2\theta+c$

(c) $\log\tan\theta+\frac{1}{2}\tan^2\theta+c$ (d) इनमें से कोई नहीं

55. $\int\frac{1}{\cos^{-1}x.\sqrt{1-x^2}}dx=$

(a) $\log(\cos^{-1}x)+c$ (b) $-\log(\cos^{-1}x)+c$

(c) $-\frac{1}{2(\cos^{-1}x)^2}+c$ (d) इनमें से कोई नहीं

56. $\int x^3e^{3x^2+5}dx$ का मान ज्ञात करने के लिये सरलतम तरीका है

(a) $x^2=t$ के प्रतिस्थापन द्वारा

(b) $(3x^2+5)=t$ के प्रतिस्थापन द्वारा

(c) खण्डशः समाकलन द्वारा

(d) इनमें से कोई नहीं

57. $\int\frac{\sec^2x}{(1+\tan x)(2+\tan x)}dx$ का मान ज्ञात करने के लिये उचित प्रतिस्थापन है

(a) $1+\tan x=t$ (b) $2+\tan x=t$

(c) $\tan x=t$ (d) इनमें से कोई नहीं

58. $\int\frac{\text{cosec}^2x}{1+\cot x}dx=$ **[MNR 1973]**

(a) $\log(1+\cot x)+c$ (b) $-\log(1+\cot x)+c$

(c) $\frac{1}{2(1+\cot x)^2}+c$ (d) इनमें से कोई नहीं

59. $\int\frac{1}{\sqrt{x}}\sin\sqrt{x}\,dx=$ **[MP PET 1989]**

(a) $-\frac{1}{2}\cos\sqrt{x}+c$ (b) $-2\cos\sqrt{x}+c$

(c) $\frac{1}{2}\cos\sqrt{x}+c$ (d) $2\cos\sqrt{x}+c$

60. $\int e^x\tan^2(e^x)dx=$

(a) $\tan(e^x)-x+c$ (b) $e^x(\tan e^x-1)+c$

(c) $\sec(e^x)+c$ (d) $\tan(e^x)-e^x+c$

61. $\int\frac{dx}{e^{-2x}(e^{2x}+1)^2}=$

(a) $\frac{-1}{2(e^{2x}+1)}+c$ (b) $\frac{1}{2(e^{2x}+1)}+c$

(c) $\frac{1}{e^{2x}+1}+c$ (d) $\frac{-1}{e^{2x}+1}+c$

62. $\int\tan^4x\,dx=$

(a) $\tan^3x-\tan x+x+c$ (b) $\frac{1}{3}\tan^3x-\tan x+x+c$

(c) $\frac{1}{3}\tan^3x+\tan x+x+c$ (d) $\frac{1}{3}\tan^3x+\tan x+2x+c$

63. $\int\frac{dx}{x\sqrt{1-(\log x)^2}}=$

(a) $\cos^{-1}(\log x)+c$ (b) $x\log(1-x^2)+c$

(c) $\sin^{-1}(\log x)+c$ (d) $\frac{1}{2}\cos^{-1}(\log x)+c$

64. $\int\frac{f'(x)}{[f(x)]^2}dx=$

(a) $-[f(x)]^{-1}+c$ (b) $\log[f(x)]+c$

(c) $e^{f(x)}+c$ (d) इनमें से कोई नहीं

65. निम्न में से किस फलन के लिये $x^2=t$ प्रतिस्थापन करने योग्य है

(a) $\int x^6\tan^{-1}x^3\,dx$ (b) $\int\tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$

(c) $\int x^3\cos x^2\,dx$ (d) इनमें से कोई नहीं

66. $\int\tan x\sec^2x\sqrt{1-\tan^2x}\,dx=$

(a) $-\frac{1}{3}(1-\tan^2x)^{3/2}+c$ (b) $\frac{1}{3}(1-\tan^2x)^{3/2}+c$

(c) $-\frac{2}{3}(1-\tan^2x)^{2/3}+c$ (d) इनमें से कोई नहीं

67. $\int\frac{\sin 2x}{\sin 5x\sin 3x}dx=$

(a) $\log\sin 3x-\log\sin 5x+c$

(b) $\frac{1}{3}\log\sin 3x+\frac{1}{5}\log\sin 5x+c$

(c) $\frac{1}{3}\log\sin 3x-\frac{1}{5}\log\sin 5x+c$

(d) $3\log\sin 3x-5\log\sin 5x+c$

68. $\int\frac{e^x\,dx}{\sqrt{1-e^{2x}}}=$

(a) $\cos^{-1}(e^x)+c$ (b) $-\cos^{-1}(e^x)+c$

(c) $\cos^{-1}(e^{2x})+c$ (d) $\sqrt{1-e^{2x}}+c$

69. $\int\frac{1}{\log a}(a^x\cos a^x)dx=$

(a) $\sin a^x+c$ (b) $a^x\sin a^x+c$

(c) $\frac{1}{(\log a)^2}\sin a^x+c$ (d) $\log\sin a^x+c$

70. $\int\frac{\sin x\,dx}{(a+b\cos x)^2}=$

(a) $\frac{1}{b}(a+b\cos x)+c$ (b) $\frac{1}{b(a+b\cos x)}+c$

(c) $\frac{1}{b}\log(a+b\cos x)+c$ (d) इनमें से कोई नहीं

71. $\int\frac{1}{x^3}[\log x^x]^2\,dx=$

(a) $\frac{x^3}{3}(\log x)+x+c$ (b) $\frac{1}{3}(\log x)^3+c$

(c) $3\log(\log x)+c$ (d) इनमें से कोई नहीं

72. $\int\frac{1}{x}\sec^2(\log x)dx=$

(a) $\tan(\log x)+c$ (b) $\log(\sec x)+c$

(c) $\log(\tan x)+c$ (d) $\sec(\log x).\tan(\log x)+c$

73. $\int \frac{dx}{x \log x \log(\log x)} =$

(a) $2\log(\log x)+c$ (b) $\log[\log(\log x)]+c$

(c) $\log(x \log x)+c$ (d) इनमें से कोई नहीं

74. $\int \frac{\sec^2 x\, dx}{\sqrt{\tan^2 x+4}} =$

(a) $\log\left[\tan x+\sqrt{\tan^2 x+4}\right]+c$

(b) $\frac{1}{2}\log\left[\tan x+\sqrt{\tan^2 x+4}\right]+c$

(c) $\log\left[\frac{1}{2}\tan x+\frac{1}{2}\sqrt{\tan^2 x+4}\right]+c$

(d) इनमें से कोई नहीं

75. $\int \frac{2x\tan^{-1}x^2}{1+x^4}dx =$ **[Roorkee 1982]**

(a) $[\tan^{-1}x^2]^2+c$ (b) $\frac{1}{2}[\tan^{-1}x^2]^2+c$

(c) $2[\tan^{-1}x^2]^2+c$ (d) इनमें से कोई नहीं

76. $\int \frac{a^{\sqrt{x}}}{\sqrt{x}}dx =$ **[Roorkee 1990; MP PET 2001]**

(a) $2a^{\sqrt{x}}\log_e a+c$ (b) $2a^{\sqrt{x}}\log_a e+c$

(c) $2a^{\sqrt{x}}\log_{10} a+c$ (d) $2a^{\sqrt{x}}\log_a 10+c$

77. $\int \frac{x^3}{\sqrt{1-x^8}}dx =$

(a) $\frac{1}{2}\sin^{-1}(x^4)+c$ (b) $\frac{1}{3}\sin^{-1}(x^4)+c$

(c) $\frac{1}{4}\sin^{-1}(x^4)+c$ (d) इनमें से कोई नहीं

78. $\int 2x\cos^3 x^2 \sin x^2 dx =$

(a) $-\frac{1}{4}\cos^4 x^2+c$ (b) $\frac{1}{4}\cos^4 x^2+c$

(c) $\cos^4 x^2+c$ (d) इनमें से कोई नहीं

79. $\int \sec^4 x\tan x\, dx =$ **[AI CBSE 1980, 81; SCRA 1996]**

(a) $\frac{1}{4}\sec^4 x+c$ (b) $4\sec^4 x+c$

(c) $\frac{\sec^3 x}{3}+c$ (d) $3\sec^3 x+c$

80. $\int e^{-x}\text{cosec}^2(2e^{-x}+5)\,dx =$ **[AISSE 1988]**

(a) $\frac{1}{2}\cot(2e^{-x}+5)+c$ (b) $-\frac{1}{2}\cot(2e^{-x}+5)+c$

(c) $2\cot(2e^{-x}+5)+c$ (d) $-2\cot(2e^{-x}+5)+c$

81. $\int \sin^3 x\,.\cos x\, dx =$ **[SCRA 1996]**

(a) $\frac{\sin^4 x\cos^2 x}{8}+c$ (b) $\frac{\sin^4 x}{4}+c$

(c) $\frac{\sin^2 x}{2}+c$ (d) $4\sin^4 x+c$

82. $\int a^{3x+3}dx =$ **[Roorkee 1977]**

(a) $\frac{a^{3x+3}}{\log a}+c$ (b) $\frac{a^{3x+3}}{3\log a}+c$

(c) $a^{3x+3}\log a+c$ (d) $3a^{3x+3}\log a+c$

83. $\int \frac{\cos 2x+x+1}{x^2+\sin 2x+2x}dx =$ **[AI CBSE 1980]**

(a) $\log(x^2+\sin 2x+2x)+c$

(b) $-\log(x^2+\sin 2x+2x)+c$

(c) $\frac{1}{2}\log(x^2+\sin 2x+2x)+c$

(d) इनमें से कोई नहीं

84. $\int \frac{1+\tan x}{x+\log\sec x}dx =$ **[AI CBSE 1986]**

(a) $\log(x+\log\sec x)+c$ (b) $-\log(x+\log\sec x)+c$

(c) $\log(x-\log\sec x)+c$ (d) इनमें से कोई नहीं

85. $\int \frac{(x+1)(x+\log x)^2}{x}dx =$ **[AI CBSE 1986]**

(a) $\frac{1}{3}(x+\log x)+c$ (b) $\frac{1}{3}(x+\log x)^2+c$

(c) $\frac{1}{3}(x+\log x)^3+c$ (d) इनमें से कोई नहीं

86. $\int \frac{1+x^2}{\sqrt{1-x^2}}dx =$ **[IIT 1977]**

(a) $\frac{3}{2}\sin^{-1}x-\frac{1}{2}x\sqrt{1-x^2}+c$

(b) $\frac{3}{2}\sin^{-1}x+\frac{1}{2}x\sqrt{1-x^2}+c$

(c) $\frac{3}{2}\cos^{-1}x-\frac{1}{2}x\sqrt{1-x^2}+c$

(d) $\frac{3}{2}\cos^{-1}x+\frac{1}{2}x\sqrt{1-x^2}+c$

87. $\int \frac{\cos x-\sin x}{1+\sin 2x}dx =$ **[AISSE 1985]**

(a) $-\frac{1}{\cos x+\sin x}+c$ (b) $\frac{1}{\cos x+\sin x}+c$

(c) $\frac{1}{\cos x-\sin x}+c$ (d) इनमें से कोई नहीं

88. $\int x^3\sqrt{3+5x^4}\,dx =$ **[DSSE 1982]**

(a) $(3+5x^4)^{3/2}+c$ (b) $\frac{1}{5}(3+5x^4)^{3/2}+c$

(c) $\frac{1}{30}(3+5x^4)^{3/2}+c$ (d) इनमें से कोई नहीं

89. $\int \sqrt{\frac{x}{a^3-x^3}}\,dx =$

(a) $\sin^{-1}\left(\frac{x}{a}\right)^{3/2}+c$ (b) $\frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2}+c$

(c) $\frac{3}{2}\sin^{-1}\left(\frac{x}{a}\right)^{3/2}+c$ (d) $\frac{3}{2}\sin^{-1}\left(\frac{x}{a}\right)^{2/3}+c$

90. $\int \frac{1}{x\cos^2(1+\log x)}dx =$

(a) $\tan(1+\log x)+c$ (b) $\cot(1+\log x)+c$

(c) $-\tan(1+\log x)+c$ (d) $-\cot(1+\log x)+c$

91. $\int \frac{1}{x^2\sqrt{1+x^2}}dx =$

(a) $-\frac{\sqrt{1+x^2}}{x}+c$ (b) $\frac{\sqrt{1+x^2}}{x}+c$

(c) $-\frac{\sqrt{1-x^2}}{x}+c$ (d) $-\frac{\sqrt{x^2-1}}{x}+c$

92. $\int \frac{1}{(x^2-1)\sqrt{x^2+1}}dx =$

(a) $\frac{1}{2\sqrt{2}}\log\left\{\frac{\sqrt{1+x^2}+x\sqrt{2}}{\sqrt{1+x^2}-x\sqrt{2}}\right\}+c$

(b) $\frac{1}{2\sqrt{2}}\log\left\{\frac{\sqrt{1+x^2}-\sqrt{2}}{\sqrt{1+x^2}+\sqrt{2}}\right\}+c$

(c) $\frac{1}{2\sqrt{2}}\log\left\{\frac{\sqrt{1+x^2}-x\sqrt{2}}{\sqrt{1+x^2}+x\sqrt{2}}\right\}+c$

(d) इनमें से कोई नहीं

93. $\int \frac{\log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}dx =$

(a) $\frac{1}{2}[\log(x+\sqrt{1+x^2})]^2+c$ (b) $\log(x+\sqrt{1+x^2})^2+c$

(c) $\log(x+\sqrt{1+x^2})+c$ (d) इनमें से कोई नहीं

94. $\int e^x\sin(e^x)\,dx =$ **[MP PET 1995]**

(a) $-\cos e^x+c$ (b) $\cos e^x+c$

(c) $-\text{cosec}\, e^x+c$ (d) इनमें से कोई नहीं

95. $\int \frac{x^5\,dx}{\sqrt{(1+x^3)}} =$ **[IIT 1975]**

(a) $\frac{2}{3}\sqrt{(1+x^3)}(x^3+2)$ (b) $\frac{2}{9}\sqrt{(1+x^3)}(x^3-4)$

(c) $\frac{2}{9}\sqrt{(1+x^3)}(x^3+4)$ (d) $\frac{2}{9}\sqrt{(1+x^3)}(x^3-2)$

96. $\int \frac{(x^4-x)^{1/4}}{x^5}dx =$

(a) $\frac{4}{15}\left(1-\frac{1}{x^3}\right)^{5/4}+c$ (b) $\frac{4}{5}\left(1-\frac{1}{x^3}\right)^{5/4}+c$

(c) $\frac{4}{15}\left(1+\frac{1}{x^3}\right)^{5/4}+c$ (d) इनमें से कोई नहीं

97. $\int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}}dx =$

(a) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4}+c$ (b) $\frac{4}{3}\left(\frac{x+2}{x-1}\right)^{1/4}+c$

(c) $\frac{1}{3}\left(\frac{x-1}{x+2}\right)^{1/4}+c$ (d) $\frac{1}{3}\left(\frac{x+2}{x-1}\right)^{1/4}+c$

98. $\int \frac{1}{1+\sin^2 x}dx =$

(a) $\frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2}\tan x)+k$ (b) $\sqrt{2}\tan^{-1}(\sqrt{2}\tan x)+k$

(c) $-\frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2}\tan x)+k$ (d) $-\sqrt{2}\tan^{-1}(\sqrt{2}\tan x)+k$

99. $\int \frac{\sin x}{\cos^2 x}dx =$

(a) $\sin x+k$ (b) $\tan x+k$

(c) $\sec x+k$ (d) $\tan x+\sec x+k$

100. $\int e^x\sec^2(e^x)\,dx =$

(a) $\tan(e^x)+k$ (b) $\tan(e^x).e+k$

(c) $e^x\tan x+k$ (d) $\frac{\tan(e^x)}{e^x}+k$

101. $\int \frac{dx}{x\sqrt{x^4-1}} =$

(a) $\frac{1}{2}\sec^{-1}x^2+k$ (b) $\log x\sqrt{x^4-1}+k$

(c) $x\log\sqrt{x^4-1}+k$ (d) $\log\sqrt{x^4-1}+k$

102. $\int \frac{t}{e^{3t^2}}dt =$ **[MP PET 1997]**

(a) $\frac{1}{6}e^{3t^2}+c$ (b) $-\frac{1}{6}e^{3t^2}+c$

(c) $\frac{1}{6}e^{-3t^2}+c$ (d) $-\frac{1}{6}e^{-3t^2}+c$

103. यदि $\int \frac{1}{(1+x)\sqrt{x}}dx = f(x)+A$, जहाँ A कोई स्वेच्छ नियतांक है, तब फलन $f(x)$ है **[MP PET 1998]**

(a) $2\tan^{-1}x$ (b) $2\tan^{-1}\sqrt{x}$

(c) $2\cot^{-1}\sqrt{x}$ (d) $\log_e(1+x)$

104. $\int x\cos x^2\,dx =$ **[MP PET 1999; Pb. CET 2000]**

(a) $-\frac{1}{2}\sin^2 x+c$ (b) $\frac{1}{2}\sin^2 x+c$

(c) $-\frac{1}{2}\sin x^2+c$ (d) $\frac{1}{2}\sin x^2+c$

105. $\int \frac{x^2\tan^{-1}x^3}{1+x^6}dx =$

[MP PET 1999; UPSEAT 1999]

(a) $\tan^{-1}(x^3)+c$ (b) $\frac{1}{6}(\tan^{-1}x^3)^2+c$

(c) $-\frac{1}{2}(\tan^{-1}x^3)^2+c$ (d) $\frac{1}{2}(\tan^{-1}x^2)^3+c$

106. $\int \frac{x^2+1}{x(x^2-1)}\,dx$ = **[MP PET 1999]**

(a) $\log\frac{x^2-1}{x}+c$ (b) $-\log\frac{x^2-1}{x}+c$

(c) $\log\frac{x}{x^2+1}+c$ (d) $-\log\frac{x}{x^2+1}+c$

107. $\int \frac{e^{2x}+1}{e^{2x}-1}\,dx$ = **[RPET 1996]**

(a) $\log(e^x-e^{-x})+c$ (b) $\log(e^x+e^{-x})+c$

(c) $\log(e^{-x}-e^x)+c$ (d) $\log(1-e^{-x})+c$

108. $\int \frac{\cos x-\sin x}{\sqrt{\sin 2x}}\,dx$ = **[RPET 1996]**

(a) $\cosh^{-1}(\sin x+\cos x)+c$

(b) $\sinh^{-1}(\sin x+\cos x)+c$

(c) $-\cosh^{-1}(\sin x+\cos x)+c$

(d) $-\sinh^{-1}(\sin x+\cos x)+c$

109. $\int \left(1+\frac{1}{x^2}\right)e^{\left(x-\frac{1}{x}\right)}\,dx$ =

[Kurukshetra CEE 1998]

(a) $e^{x-\frac{1}{x}}+c$ (b) $e^{x+\frac{1}{x}}+c$

(c) $e^{x^2-\frac{1}{x}}+c$ (d) $e^{x^2+\frac{1}{x^2}}+c$

110. $\int (x+3)(x^2+6x+10)^9\,dx$ = **[SCRA 1996]**

(a) $\frac{1}{20}(x^2+6x+10)^{10}+c$

(b) $\frac{1}{20}(x+3)^2(x^2+6x+10)^{10}+c$

(c) $\frac{1}{16}(x^2+6x+10)^8+c$

(d) $\frac{1}{38}(x+3)^{19}+\frac{1}{2}(x+3)+c$

111. $\int \frac{x}{x^2+1}dx$ = **[SCRA 1996]**

(a) $\log_e(x^2+1)$ (b) $x\tan^{-1}x$

(c) $\frac{\log_e(x^2+1)}{2}$ (d) $\frac{1}{2}x\tan^{-1}x$

112. $\int \sin^3 x\,dx$ = **[SCRA 1996]**

(a) $\sin^2 x+1$ (b) $\sin x^2+x^2+1$

(c) $\frac{\cos^3 x}{3}-\cos x$ (d) $\frac{1}{4}\sin^4 x-\frac{3}{4}\sin^2 x$

113. $\int \frac{1}{x}\log x\,dx$ = **[SCRA 1996]**

(a) $\frac{1}{2}\log x+c$ (b) $\frac{1}{2}(\log x)^2+c$

(c) $\frac{1}{2}\log(x)^2+c$ (d) $\log x+c$

114. $\int \sin^2 x\cos x\,dx$ = **[SCRA 1996]**

(a) $\frac{\cos^2 x}{2}+c$ (b) $\frac{\sin^2 x}{3}+c$

(c) $\frac{\sin^3 x}{3}+c$ (d) $-\frac{\cos^2 x}{2}+c$

115. $\int e^{x^2}x\,dx$ = **[SCRA 1996]**

(a) e^{x^2} (b) $\frac{1}{2}e^{x^2}$

(c) $2e^{x^2}$ (d) $\frac{e^{x^2}-x^2}{2}$

116. $\int \frac{x^3}{\sqrt{1+x^4}}\,dx$ = **[SCRA 1996]**

(a) $(1+x^4)^{\frac{1}{2}}+c$ (b) $-(1+x^4)^{\frac{1}{2}}+c$

(c) $\frac{1}{2}(1+x^4)^{\frac{1}{2}}+c$ (d) $-\frac{1}{2}(1+x^4)^{\frac{1}{2}}+c$

117. $I=\int\frac{dx}{(1+e^x)(1+e^{-x})}$ का क्या मान है **[DCE 1999]**

(a) $\frac{-1}{1+e^x}$ (b) $\frac{e^x}{1+e^x}$

(c) $\frac{1}{1+e^x}$ (d) इनमें से कोई नहीं

118. $\int\frac{e^{\sqrt{x}}}{\sqrt{x}}dx=$ **[DCE 1999]**

(a) $e^{\sqrt{x}}$ (b) $\frac{e^{\sqrt{x}}}{2}$

(c) $2e^{\sqrt{x}}$ (d) $\sqrt{x}.e^{\sqrt{x}}$

119. $\int\frac{\sin^3 2x}{\cos^5 2x}dx=$ **[Karnataka CET 1999]**

(a) $\tan^4 x+C$ (b) $\tan 4x+C$

(c) $\tan^4 2x+x+C$ (d) $\frac{1}{8}\tan^4 2x+C$

120. $\int x^x(1+\log x)\,dx$ = **[RPET 2000]**

(a) x^x (b) x^{2x}

(c) $x^x\log x$ (d) $\frac{1}{2}(1+\log x)^2$

121. $\int\frac{dx}{(a^2+x^2)^{3/2}}$ = **[RPET 2000]**

(a) $\frac{x}{\left(a^2+x^2\right)^{1/2}}$ (b) $\frac{x}{a^2\left(a^2+x^2\right)^{1/2}}$

(c) $\frac{1}{a^2\left(a^2+x^2\right)^{1/2}}$ (d) इनमें से कोई नहीं

122. $\int \frac{e^{m\tan^{-1}x}}{1+x^2}dx$ = **[RPET 2001]**

(a) $e^{\tan^{-1}x}$ (b) $\frac{1}{m}e^{\tan^{-1}x}$

(c) $\frac{1}{m}e^{m\tan^{-1}x}$ (d) इनमें से कोई नहीं

123. $\int\frac{1+\tan^2 x}{1-\tan^2 x}dx$ = **[RPET 2001]**

(a) $\log\left(\frac{1-\tan x}{1+\tan x}\right)+c$ (b) $\log\left(\frac{1+\tan x}{1-\tan x}\right)+c$

(c) $\frac{1}{2}\log\left(\frac{1-\tan x}{1+\tan x}\right)+c$ (d) $\frac{1}{2}\log\left(\frac{1+\tan x}{1-\tan x}\right)+c$

124. $\int\frac{2\,dx}{\sqrt{1-4x^2}}$ का मान है

[Karnataka CET 2001; Pb. CET 2001]

(a) $\tan^{-1}(2x)+c$ (b) $\cot^{-1}(2x)+c$

(c) $\cos^{-1}(2x)+c$ (d) $\sin^{-1}(2x)+c$

125. $\int e^{3\log x}(x^4+1)^{-1}\,dx$ = **[MP PET 2001]**

(a) $\log(x^4+1)+c$ (b) $\frac{1}{4}\log(x^4+1)+c$

(c) $-\log(x^4+1)+c$ (d) इनमें से कोई नहीं

126. $\int\frac{dx}{2\sqrt{x}(1+x)}=$ **[RPET 2002]**

(a) $\frac{1}{2}\tan^{-1}(\sqrt{x})+c$ (b) $\tan^{-1}(\sqrt{x})+c$

(c) $2\tan^{-1}(\sqrt{x})+c$ (d) इनमें से कोई नहीं

127. $\int\text{cosec}^4 x\,dx$ = **[RPET 2002]**

(a) $\cot x+\frac{\cot^3 x}{3}+c$ (b) $\tan x+\frac{\tan^3 x}{3}+c$

(c) $-\cot x-\frac{\cot^3 x}{3}+c$ (d) $-\tan x-\frac{\tan^3 x}{3}+c$

128. $\int xe^{x^2}dx=$ **[RPET 2003]**

(a) $-\frac{e^{x^2}}{2}+c$ (b) $\frac{e^{x^2}}{2}+c$

(c) $\frac{e^x}{2}+c$ (d) $-\frac{e^x}{2}+c$

129. यदि $\int f(x)\,dx=g(x)$, तब $\int f^{-1}(x)\,dx$ =

[MP PET 2003]

(a) $g^{-1}(x)$ (b) $xf^{-1}(x)-g(f^{-1}(x))$

(c) $xf^{-1}(x)-g^{-1}(x)$ (d) $f^{-1}(x)$

130. $\int\frac{e^x}{e^x+1}dx$ का मान है **[Pb. CET 2000]**

(a) e^x+c (b) $(e^x+1)+c$

(c) $\log(e^x+1)+c$ (d) इनमें से कोई नहीं

131. $\int\frac{\sin x-\cos x}{\sin x+\cos x}dx$ का मान है **[Pb. CET 2000]**

(a) $\frac{1}{\sin x+\cos x}+c$ (b) $\frac{1}{\sin x-\cos x}+c$

(c) $\log(\sin x+\cos x)+c$ (d) $\log\left(\frac{1}{\sin x+\cos x}\right)+c$

132. $\int\frac{(\tan^{-1}x)^3}{1+x^2}dx=$ **[UPSEAT 2004]**

(a) $(\tan^{-1}x)^4+c$ (b) $\frac{(\tan^{-1}x)^4}{4}+c$

(c) $2\tan^{-1}x+c$ (d) $2(\tan^{-1}x)^2+c$

133. $\int\sqrt{\frac{1-x}{1+x}}\,dx=$ **[IIT 1971]**

(a) $\sin^{-1}x-\frac{1}{2}\sqrt{1-x^2}+c$ (b) $\sin^{-1}x+\frac{1}{2}\sqrt{1-x^2}+c$

(c) $\sin^{-1}x-\sqrt{1-x^2}+c$ (d) $\sin^{-1}x+\sqrt{1-x^2}+c$

134. $\int\frac{\sqrt{x}}{1+x}dx=$

(a) $\sqrt{x}-\tan^{-1}\sqrt{x}+c$ (b) $2(\sqrt{x}-\tan^{-1}\sqrt{x})+c$

(c) $2(\sqrt{x}+\tan^{-1}x)+c$ (d) $\sqrt{1+x}+c$

135. $\int\frac{\sin x}{\sin x-\cos x}dx=$ **[Roorkee 1988]**

(a) $\frac{1}{2}\log(\sin x-\cos x)+x+c$

(b) $\frac{1}{2}[\log(\sin x-\cos x)+x]+c$

(c) $\frac{1}{2}\log(\cos x-\sin x)+x+c$

(d) $\frac{1}{2}[\log(\cos x-\sin x)+x]+c$

136. $\int\sqrt{\frac{1+x}{1-x}}\,dx=$ **[RPET 2002]**

(a) $-\sin^{-1}x-\sqrt{1-x^2}+c$ (b) $\sin^{-1}x+\sqrt{1-x^2}+c$

(c) $\sin^{-1}x-\sqrt{1-x^2}+c$ (d) $-\sin^{-1}x-\sqrt{x^2-1}+c$

137. $\int\frac{x}{\sqrt{4-x^4}}dx=$ **[Roorkee 1976]**

(a) $\cos^{-1}\frac{x^2}{2}$ (b) $\frac{1}{2}\cos^{-1}\frac{x^2}{2}$

(c) $\sin^{-1}\frac{x^2}{2}$ (d) $\frac{1}{2}\sin^{-1}\frac{x^2}{2}$

138. $\int\frac{\sin x\,dx}{3+4\cos^2 x}=$ **[Karnataka CET 2000]**

(a) $\log(3+4\cos^2 x)+c$ (b) $\frac{-1}{2\sqrt{3}}\tan^{-1}\left(\frac{\cos x}{\sqrt{3}}\right)+c$

(c) $\frac{-1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+c$ (d) $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+c$

139. $\int\frac{dx}{\sqrt{x}\,(x+9)}dx$ का मान है **[Pb. CET 2002]**

(a) $\tan^{-1}\sqrt{x}$ (b) $\tan^{-1}\left(\frac{\sqrt{x}}{3}\right)$

(c) $\frac{2}{3}\tan^{-1}\sqrt{x}$ (d) $\frac{2}{3}\tan^{-1}\left(\frac{\sqrt{x}}{3}\right)$

140. $\int\left\{\frac{(\log x-1)}{1+(\log x)^2}\right\}^2 dx =$ **[AIEEE 2005]**

(a) $\frac{xe^x}{1+x^2}+c$ (b) $\frac{x}{(\log x)^2+1}+c$

(c) $\frac{\log x}{(\log x)^2+1}+c$ (d) $\frac{x}{x^2+1}+c$

141. $\int\frac{\sin 2x dx}{1+\cos^2 x}=$ **[Karnataka CET 2005]**

(a) $\frac{1}{2}\log(1+\cos^2 x)+c$ (b) $2\log(1+\cos^2 x)+c$

(c) $\frac{1}{2}\log(1+\cos 2x)+c$ (d) $-\log(1+\cos^2 x)+c$

142. यदि $\int\frac{\cos 4x+1}{\cos x-\tan x}dx = k\cos 4x+c$ तब **[DCE 2005]**

(a) $k=-1/2$ (b) $k=-1/8$

(c) $k=-1/4$ (d) इनमें से कोई नहीं

143. यदि $\int\frac{1}{x+x^5}dx = f(x)+c$, तब $\int\frac{x^4}{x+x^5}dx$ का मान है **[DCE 2005]**

(a) $\log x - f(x)+c$ (b) $f(x)+\log x+c$

(c) $f(x)-\log x+c$ (d) इनमें से कोई नहीं

144. माना $f(x)=\int\frac{x^2dx}{(1+x^2)\left(1+\sqrt{1+x^2}\right)}$ और $f(0)=0$, तब $f(1)$ का मान है **[AMU 2005]**

(a) $\log(1+\sqrt{2})$ (b) $\log(1+\sqrt{2})-\frac{\pi}{4}$

(c) $\log(1+\sqrt{2})+\frac{\pi}{2}$ (d) इनमें से कोई नहीं

145. $\int\sqrt{e^x-1}dx =$ **[Kerala (Engg.) 2005]**

(a) $2\left[\sqrt{e^x-1}-\tan^{-1}\sqrt{e^x-1}\right]+c$

(b) $\sqrt{e^x-1}-\tan^{-1}\sqrt{e^x-1}+c$

(c) $\sqrt{e^x-1}+\tan^{-1}\sqrt{e^x-1}+c$

(d) $2\left[\sqrt{e^x-1}+\tan^{-1}\sqrt{e^x-1}\right]+c$

(e) $2\left[\sqrt{e^x-1}-\tan^{-1}\sqrt{e^x+1}\right]+c$

146. $\int\frac{dx}{\sin(x-a)\sin(x-b)} =$ **[Kerala (Engg.) 2005]**

(a) $\frac{1}{\sin(a-b)}\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right|+c$

(b) $\frac{-1}{\sin(a-b)}\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right|+c$

(c) $\log\sin(x-a)\sin(x-b)+c$

(d) $\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right|+c$

(e) $\frac{1}{\sin(x-a)}\log\sin(x-a)\sin(x-b)+c$

147. $\int\frac{(\sin\theta+\cos\theta)}{\sqrt{\sin 2\theta}}d\theta =$ **[Kerala (Engg.) 2005]**

(a) $\log\left|\cos\theta-\sin\theta+\sqrt{\sin 2\theta}\right|$

(b) $\log\left|\sin\theta-\cos\theta)+\sqrt{\sin 2\theta}\right|$

(c) $\sin^{-1}(\sin\theta-\cos\theta)+c$

(d) $\sin^{-1}(\sin\theta+\cos\theta)+c$

(e) $\sin^{-1}(\cos\theta-\sin\theta)+c$

148. $\int\cos^{-3/7}x\sin^{-11/7}x\,dx =$ **[Kerala (Engg.) 2005]**

(a) $\log|\sin^{4/7}x|+c$ (b) $\frac{4}{7}\tan^{4/7}x+c$

(c) $\frac{-7}{4}\tan^{-4/7}x+c$ (d) $\log|\cos^{3/7}x|+c$

(e) $\frac{7}{4}\tan^{-4/7}x+c$

खण्डशः समाकलन, $\int e^x(F(x)+F'(x))\,dx$ तथा $\int e^{kx}(kF(x)+F'(x))\,dx$ के रूप का समाकलन

1. $\int x\sec^2 x\,dx =$

[RPET 1996, 2003; MP PET 1987, 97; Pb. CET 2002]

(a) $\tan x+\log\cos x+c$ (b) $\frac{x^2}{2}\sec^2 x+\log\cos x+c$

(c) $x\tan x+\log\sec x+c$ (d) $x\tan x+\log\cos x+c$

2. $\int\sin(\log x)dx =$

(a) $\frac{1}{2}x[\cos(\log x)-\sin(\log x)]$

(b) $\cos(\log x)-x$

(c) $\frac{1}{2}x[\sin(\log x)-\cos(\log x)]$

(d) $-\cos\log x$

3. यदि $\int x\sin x dx = -x\cos x+A$, तब $A=$

[MP PET 1992, 2000; RPET 1997]

(a) $\sin x$ + अचर (b) $\cos x$ + अचर

(c) अचर (d) इनमें से कोई नहीं

4. $\int x \log x dx =$ **[MP PET 1987]**

(a) $\frac{x^2}{2}\log x - \frac{x^2}{2} + c$ (b) $\frac{x^2}{2}\log x - \frac{x^2}{4} + c$

(c) $\frac{x^2}{2}\log x + \frac{x^2}{2} + c$ (d) इनमें से कोई नहीं

5. $\int x \cos x \, dx =$ **[MP PET 1988]**

(a) $x \sin x + \cos x$ (b) $x \sin x - \cos x$

(c) $x \cos x + \sin x$ (d) $x \cos x - \sin x$

6. $\int x \cos^2 x dx =$ **[IIT 1972]**

(a) $\frac{x^4}{4} - \frac{1}{4}x \sin 2x - \frac{1}{8}\cos 2x + c$

(b) $\frac{x^2}{4} + \frac{1}{4}x \sin 2x + \frac{1}{8}\cos 2x + c$

(c) $\frac{x^4}{4} - \frac{1}{4}x \sin 2x + \frac{1}{8}\cos 2x + c$

(d) $\frac{x^4}{4} + \frac{1}{4}x \sin 2x - \frac{1}{8}\cos 2x + c$

7. $\int \tan^{-1} x \, dx =$ **[Roorkee 1977]**

(a) $x \tan^{-1} x + \frac{1}{2}\log(1 + x^2)$ (b) $x \tan^{-1} x - \frac{1}{2}\log(1 + x^2)$

(c) $(x-1)\tan^{-1} x$ (d) $x \tan^{-1} x - \log(1 + x^2)$

8. $\int x \tan^{-1} x dx =$ **[Roorkee 1979]**

(a) $\frac{1}{2}(x^2 + 1)\tan^{-1} x - \frac{1}{2}x + c$

(b) $\frac{1}{2}(x^2 - 1)\tan^{-1} x - \frac{1}{2}x + c$

(c) $\frac{1}{2}(x^2 + 1)\tan^{-1} x + \frac{1}{2}x + c$

(d) $\frac{1}{2}(x^2 + 1)\tan^{-1} x - x + c$

9. $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx =$

(a) $x \log(\log x) + \frac{x}{\log x} + c$ (b) $x \log(\log x) - \frac{x}{\log x} + c$

(c) $x \log(\log x) + \frac{\log x}{x} + c$ (d) $x \log(\log x) - \frac{\log x}{x} + c$

10. $\int [\sin(\log x) + \cos(\log x)] \, dx =$ **[MP PET 1991]**

(a) $x \cos(\log x) + c$ (b) $\sin(\log x) + c$

(c) $\cos(\log x) + c$ (d) $x \sin(\log x) + c$

11. $\int \frac{\log x \, dx}{x^3} =$ **[Roorkee 1986]**

(a) $\frac{1}{4x^2}(2\log x - 1) + c$ (b) $-\frac{1}{4x^2}(2\log x + 1) + c$

(c) $\frac{1}{4x^2}(2\log x + 1) + c$ (d) $\frac{1}{4x^2}(1 - 2\log x) + c$

12. $\int e^{-2x} \sin 3x \, dx =$

(a) $\frac{1}{13}e^{-2x}[\sin 3x + \cos 3x] + c$

(b) $-\frac{1}{13}e^{-2x}[\sin 3x + \cos 3x] + c$

(c) $\frac{1}{13}e^{-2x}[2\sin 3x + 3\cos 3x] + c$

(d) $-\frac{1}{13}e^{-2x}[2\sin 3x + 3\cos 3x] + c$

13. $\int x \sin x \sec^3 x \, dx =$

(a) $\frac{1}{2}[\sec^2 x - \tan x] + c$ (b) $\frac{1}{2}[x \sec^2 x - \tan x] + c$

(c) $\frac{1}{2}[x \sec^2 x + \tan x] + c$ (d) $\frac{1}{2}[\sec^2 x + \tan x] + c$

14. $\int x \sin^2 x \, dx =$ **[BIT Ranchi 1977; IIT 1972]**

(a) $\frac{x^2}{4} + \frac{x}{4}\sin 2x + \frac{1}{8}\cos 2x + c$

(b) $\frac{x^2}{4} - \frac{x}{4}\sin 2x + \frac{1}{8}\cos 2x + c$

(c) $\frac{x^2}{4} + \frac{x}{4}\sin 2x - \frac{1}{8}\cos 2x + c$

(d) $\frac{x^2}{4} - \frac{x}{4}\sin 2x - \frac{1}{8}\cos 2x + c$

15. $\int e^{2x + \log x} dx =$

(a) $\frac{1}{4}(2x - 1)e^{2x} + c$ (b) $\frac{1}{4}(2x + 1)e^{2x} + c$

(c) $\frac{1}{2}(2x + 1)e^{2x} + c$ (d) $\frac{1}{2}(2x - 1)e^{2x} + c$

16. $\int \log(x + 1) dx =$ **[Roorkee 1974]**

(a) $(x+1)\log(x+1) - x + c$ (b) $(x+1)\log(x+1) + x + c$

(c) $(x-1)\log(x+1) - x + c$ (d) $(x-1)\log(x+1) + x + c$

17. यदि $\int \ln(x^2 + x) dx = x \ln(x^2 + x) + A$, तब $A =$

[MP PET 1992]

(a) $2x + \ln(x+1) +$ अचर (b) $2x - \ln(x+1) +$ अचर

(c) अचर (d) इनमें से कोई नहीं

18. $\int x^2 \sin 2x \, dx =$ **[IIT 1974]**

(a) $\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + c$

(b) $-\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + c$

(c) $\frac{1}{2}x^2\cos 2x - \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$

(d) इनमें से कोई नहीं

19. $\int \log x dx =$ **[MNR 1979; BIT Ranchi 1992; SCRA 1996]**

(a) $x + x\log x + c$ (b) $x\log x - x + c$

(c) $x^2\log x + c$ (d) $\frac{1}{x}\log x + x + c$

20. $\int \log_{10} x\, dx =$ **[Roorkee 1973]**

(a) $x\log_{10} x + c$ (b) $x(\log_{10} x + \log_{10} e) + c$

(c) $\log_{10} x + c$ (d) $x(\log_{10} x - \log_{10} e) + c$

21. $\int \frac{1}{\log_x e} dx =$ **[MP PET 1994]**

(a) $\log\log_x e + c$ (b) $\frac{1}{(\log_x e)^2} + c$

(c) $x\log\left(\frac{x}{e}\right) + c$ (d) इनमें से कोई नहीं

22. यदि $\int e^x \sin x\, dx = \frac{1}{2}e^x . a + c$, तब $a =$ **[MP PET 1989]**

(a) $\sin x - \cos x$ (b) $\cos x - \sin x$

(c) $-\cos x - \sin x$ (d) $\cos x + \sin x$

23. $\int x^n \log x\, dx =$

(a) $\frac{x^{n+1}}{n+1}\left\{\log x + \frac{1}{n+1}\right\} + c$ (b) $\frac{x^{n+1}}{n+1}\left\{\log x + \frac{2}{n+1}\right\} + c$

(c) $\frac{x^{n+1}}{n+1}\left\{2\log x - \frac{1}{n+1}\right\} + c$ (d) $\frac{x^{n+1}}{n+1}\left\{\log x - \frac{1}{n+1}\right\} + c$

24. $\int \log x(\log x + 2)\, dx =$

(a) $x(\log x)^2 + c$ (b) $x(1 + \log x)^2 + c$

(c) $x[1 + (\log x)^2] + c$ (d) इनमें से कोई नहीं

25. $\int \left[\frac{1}{\log x} - \frac{1}{(\log x)^2}\right] dx =$

(a) $\frac{1}{\log x} + c$ (b) $\frac{x}{\log x} + c$

(c) $\frac{x}{(\log x)^2}$ (d) इनमें से कोई नहीं

26. $\int \frac{\log x}{(1 + \log x)^2} dx =$

(a) $\frac{1}{1 + \log x} + c$ (b) $\frac{x}{(1 + \log x)^2} + c$

(c) $\frac{x}{1 + \log x} + c$ (d) $\frac{1}{(1 + \log x)^2} + c$

27. $\int \left(\frac{2 + \sin 2x}{1 + \cos 2x}\right) e^x dx =$ **[AISSE 1982]**

(a) $e^x \cot x + c$ (b) $-e^x \cot x + c$

(c) $-e^x \tan x + c$ (d) $e^x \tan x + c$

28. $\int e^x \sin x\, dx =$ **[IIT 1978; AI CBSE 1980; MP PET 1999]**

(a) $\frac{1}{2}e^x(\sin x + \cos x) + c$ (b) $\frac{1}{2}e^x(\sin x - \cos x) + c$

(c) $e^x(\sin x + \cos x) + c$ (d) $e^x(\sin x - \cos x) + c$

29. $\int (1 - x^2)\log x\, dx =$ **[DSSE 1982]**

(a) $\left(x - \frac{x^3}{3}\right)\log x - \left(x - \frac{x^3}{9}\right) + c$

(b) $\left(x - \frac{x^3}{3}\right)\log x + \left(x - \frac{x^3}{9}\right) + c$

(c) $\left(x + \frac{x^3}{3}\right)\log x + \left(x + \frac{x^3}{9}\right) + c$

(d) इनमें से कोई नहीं

30. $\int e^{2x}(-\sin x + 2\cos x)\, dx =$ **[DSSE 1987]**

(a) $e^{2x}\sin x + c$ (b) $-e^{2x}\sin x + c$

(c) $-e^{2x}\cos x + c$ (d) $e^{2x}\cos x + c$

31. $\int [f(x)g''(x) - f''(x)g(x)]\, dx =$ **[MP PET 2001]**

(a) $\frac{f(x)}{g'(x)}$ (b) $f'(x)g(x) - f(x)g'(x)$

(c) $f(x)g'(x) - f'(x)g(x)$ (d) $f(x)g'(x) + f'(x)g(x)$

32. $\int x\sin kx\, dx =$

(a) $\frac{\sin kx}{k} + c$ (b) $\frac{\cos kx}{k} + c$

(c) $\frac{\sin x}{k} + c$ (d) $-\frac{x\cos kx}{k} + \frac{\sin kx}{k^2} + c$

33. $\int xe^x\, dx =$ **[SCRA 1996]**

(a) $(x + 1)e^x + c$ (b) $(x - 1)e^x + c$

(c) $xe^x + 1 + c$ (d) $xe^x - 1 + c$

34. $\int x^3 e^{x^2} dx =$ **[MNR 1980]**

(a) $\frac{1}{2}(x^2 + 1)e^{x^2} + c$ (b) $(x^2 + 1)e^{x^2} + c$

(c) $\frac{1}{2}(x^2 - 1)e^{x^2} + c$ (d) $(x^2 - 1)e^{x^2} + c$

35. $\int \frac{\log x}{(x+1)^2} dx$ का मान है **[UPSEAT 1999]**

(a) $\frac{-\log x}{x+1} + \log x - \log(x+1)$

(b) $\frac{\log x}{(x+1)} + \log x - \log(x+1)$

(c) $\frac{\log x}{x+1} - \log x - \log(x+1)$

(d) $\frac{-\log x}{x+1} - \log x - \log(x+1)$

36. यदि $\int xe^{2x}\,dx$ का मान $e^{2x} f(x) + C$ है, जहाँ C समाकलन स्थिरांक है, तब $f(x)$ = **[UPSEAT 2001]**

(a) $(3x-1)/4$ (b) $(2x+1)/2$

(c) $(2x-1)/4$ (d) $(x-4)/6$

37. यदि $\frac{d}{dx} f(x) = x\cos x + \sin x$ और $f(0) = 2$, तब $f(x) =$ **[MP PET 1989]**

(a) $x\sin x$ (b) $x\cos x + \sin x + 2$

(c) $x\sin x + 2$ (d) $x\cos x + 2$

38. $\int \cos^{-1}\left(\frac{1}{x}\right) dx$ = **[RPET 2002]**

(a) $x\sec^{-1} x + \cosh^{-1} x + C$ (b) $x\sec^{-1} x - \cosh^{-1} x + C$

(c) $x\sec^{-1} x - \sin^{-1} x + C$ (d) इनमें से कोई नहीं

39. $\int x^3 \log x\,dx =$ **[Karnataka CET 2002]**

(a) $\frac{x^4 \log x}{4} + c$ (b) $\frac{1}{16}[4x^4 \log x - x^4] + c$

(c) $\frac{1}{8}[x^4 \log x - 4x^2] + c$ (d) $\frac{1}{16}[4x^4 \log x + x^4] + c$

40. $\int \cos(\log_e x)dx$ = **[MP PET 2003]**

(a) $\frac{1}{2}x\{\cos(\log_e x) + \sin(\log_e x)\}$

(b) $x\{\cos(\log_e x) + \sin(\log_e x)\}$

(c) $\frac{1}{2}x\{\cos(\log_e x) - \sin(\log_e x)\}$

(d) $x\{\cos(\log_e x) - \sin(\log_e x)\}$

41. $\int e^x(1+\tan x)\sec x\,dx =$ **[Karnataka CET 2005]**

(a) $e^x \cot x$ (b) $e^x \tan x$

(c) $e^x \sec x$ (d) $e^x \cos x$

42. $\int \frac{xe^x}{(1+x)^2} dx =$ **[MP PET 1997; UPSEAT 2001; RPET 2002]**

(a) $\frac{e^{-x}}{1+x} + c$ (b) $-\frac{e^{-x}}{1+x} + c$

(c) $\frac{e^x}{1+x} + c$ (d) $-\frac{e^x}{1+x} + c$

43. $\int e^x[\tan x - \log(\cos x)]\,dx =$ **[MP PET 1991]**

(a) $e^x \log(\sec x) + c$ (b) $e^x \log(\operatorname{cosec} x) + c$

(c) $e^x \log(\cos x) + c$ (d) $e^x \log(\sin x) + c$

44. $\int e^x \sin x(\sin x + 2\cos x)\,dx =$ **[MP PET 1988]**

(a) $e^x \sin^2 x + c$ (b) $e^x \sin x + c$

(c) $e^x \sin 2x + c$ (d) इनमें से कोई नहीं

45. $\int e^{2x} \frac{1+\sin 2x}{1+\cos 2x} dx =$

(a) $e^{2x}\tan x + c$ (b) $e^{2x}\cot x + c$

(c) $\frac{e^{2x}\tan x}{2} + c$ (d) $\frac{e^{2x}\cot x}{2} + c$

46. $\int \frac{e^x(x-1)}{x^2} dx =$

(a) $\frac{1}{x}e^x + c$ (b) $xe^{-x} + c$

(c) $\frac{1}{x^2}e^x + c$ (d) $\left(x - \frac{1}{x}\right)e^x + c$

47. $\int e^x\left[\frac{1+x\log x}{x}\right] dx =$

(a) $e^x + \log x + c$ (b) $\frac{e^x}{\log x} + c$

(c) $e^x - \log x + c$ (d) $e^x \log x + c$

48. $\int e^x\left[\sin^{-1}\frac{x}{a} + \frac{1}{\sqrt{a^2 - x^2}}\right] dx =$

(a) $\frac{1}{a}e^x \sin^{-1}\frac{x}{a} + c$ (b) $ae^x \sin^{-1}\frac{x}{a} + c$

(c) $e^x \sin^{-1}\frac{x}{a} + c$ (d) $\frac{e^x}{\sqrt{a^2 - x^2}} + c$

49. $\int e^x \frac{(x^2+1)}{(x+1)^2} dx =$

(a) $\left(\frac{x-1}{x+1}\right)e^x + c$ (b) $e^x\left(\frac{x+1}{x-1}\right) + c$

(c) $e^x(x+1)(x-1) + c$ (d) इनमें से कोई नहीं

50. $\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right) dx =$ **[AISSE 1983; MP PET 1994, 96]**

(a) $-\frac{e^x}{x^2} + c$ (b) $\frac{e^x}{x^2} + c$

(c) $\frac{e^x}{x} + c$ (d) $-\frac{e^x}{x} + c$

51. $\int e^x(1 + \tan x + \tan^2 x)\,dx =$ **[Karnataka CET 1999]**

(a) $e^x \sin x + c$ (b) $e^x \cos x + c$

(c) $e^x \tan x + c$ (d) $e^x \sec x + c$

52. $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right) dx$ = **[RPET 2000]**

(a) $-e^x \tan(x/2)$ (b) $-e^x \cot(x/2)$

(c) $-\frac{1}{2}e^x \tan\left(\frac{x}{2}\right)$ (d) $\frac{1}{2}e^x \cot\left(\frac{x}{2}\right)$

53. $\int \frac{(x+3)e^x}{(x+4)^2} dx =$ **[Karnataka CET 2000]**

(a) $\frac{1}{(x+4)^2}+c$ (b) $\frac{e^x}{(x+4)^2}+c$

(c) $\frac{e^x}{x+4}+c$ (d) $\frac{e^x}{x+3}+c$

54. $\int\left(\frac{x+2}{x+4}\right)^2 e^x\,dx =$ **[AMU 2000]**

(a) $e^x\left(\frac{x}{x+4}\right)+c$ (b) $e^x\left(\frac{x+2}{x+4}\right)+c$

(c) $e^x\left(\frac{x-2}{x+4}\right)+c$ (d) $\left(\frac{2xe^x}{x+4}\right)+c$

55. $\int e^x\left[f(x)+f'(x)\right]dx =$ **[DCE 2002]**

(a) $e^x f(x)$ (b) e^x

(c) $e^x f'(x)$ (d) इनमें से कोई नहीं

56. $\int e^x(1-\cot x+\cot^2 x)\,dx =$ **[MP PET 2002]**

(a) $e^x\cot x+c$ (b) $e^x\text{cosec } x+c$

(c) $-e^x\cot x+c$ (d) $-e^x\text{cosec } x+c$

57. $\int \sin^{-1}x\,dx =$ **[MP PET 2004]**

(a) $\frac{1}{\sqrt{1-x^2}}+c$ (b) $x\sin^{-1}x-\sqrt{1-x^2}+c$

(c) $\cos^{-1}x+c$ (d) $x\sin^{-1}x+\sqrt{1-x^2}+c$

58. $\int\frac{x-\sin x}{1-\cos x}dx =$ **[AISSE 1989]**

(a) $x\cot\frac{x}{2}+c$ (b) $-x\cot\frac{x}{2}+c$

(c) $\cot\frac{x}{2}+c$ (d) इनमें से कोई नहीं

59. $\int\frac{x+\sin x}{1+\cos x}dx =$ **[Roorkee 1980; UPSEAT 1999]**

(a) $-x\tan\frac{x}{2}+c$ (b) $x\tan\frac{x}{2}+c$

(c) $x\tan x+c$ (d) $\frac{1}{2}x\tan x+c$

60. यदि $\int\frac{e^x(1+\sin x)dx}{1+\cos x}=e^x f(x)+c$, तब $f(x)=$ **[RPET 1997; Karnataka CET 2003, 05; Orissa JEE 2004]**

(a) $\sin\frac{x}{2}$ (b) $\cos\frac{x}{2}$

(c) $\tan\frac{x}{2}$ (d) $\log\frac{x}{2}$

61. $\int\sqrt{x}e^{\sqrt{x}}\,dx =$ **[Karnataka CET 2004]**

(a) $2\sqrt{x}-e^{\sqrt{x}}-4\sqrt{x}\,e^{\sqrt{x}}+c$

(b) $(2x-4\sqrt{x}+4)e^{\sqrt{x}}+c$

(c) $(2x+4\sqrt{x}+4)e^{\sqrt{x}}+c$

(d) $(1-4\sqrt{x})e^{\sqrt{x}}+c$

62. $\int 32x^3(\log x)^2\,dx =$ **[MP PET 2004]**

(a) $x^4\{8(\log x)^2-4(\log x)+1\}+c$

(b) $x^3\{(\log x)^2+2\log x\}+c$

(c) $x^4\{8(\log x)^2-4\log x\}+c$

(d) $8x^4(\log x)^2+c$

63. $\int\sin^{-1}(3x-4x^3)dx =$ **[AISSE 1986; DSSE 1984]**

(a) $x\sin^{-1}x+\sqrt{1-x^2}+c$ (b) $x\sin^{-1}x-\sqrt{1-x^2}+c$

(c) $2[x\sin^{-1}x+\sqrt{1-x^2}]+c$ (d) $3[x\sin^{-1}x+\sqrt{1-x^2}]+c$

64. $\int\cos\sqrt{x}\,dx =$ **[BIT Ranchi 1990; IIT 1977; RPET 1999]**

(a) $2[\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x}]+c$

(b) $2[\sqrt{x}\sin\sqrt{x}-\cos\sqrt{x}]+c$

(c) $2[\cos\sqrt{x}-\sqrt{x}\sin\sqrt{x}]+c$

(d) $-2[\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x}]+c$

65. $\int\tan^{-1}\frac{2x}{1-x^2}dx =$ **[MP PET 1991]**

(a) $x\tan^{-1}x+c$

(b) $x\tan^{-1}x-\log(1+x^2)+c$

(c) $2x\tan^{-1}x+\log(1+x^2)+c$

(d) $2x\tan^{-1}x-\log(1+x^2)+c$

66. $\int\frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx =$ **[MNR 1978; EAMCET 1982; IIT 1984]**

(a) $x-\sqrt{1-x^2}\sin^{-1}x+c$ (b) $x+\sqrt{1-x^2}\sin^{-1}x+c$

(c) $\sqrt{1-x^2}\sin^{-1}x-x+c$ (d) इनमें से कोई नहीं

67. $\int\frac{\sin^{-1}x}{(1-x^2)^{3/2}}dx =$ **[AISSE 1983, 87]**

(a) $\frac{x}{\sqrt{1-x^2}}\sin^{-1}x+\frac{1}{2}\log(1-x^2)+c$

(b) $\frac{x}{\sqrt{1-x^2}}\sin^{-1}x-\frac{1}{2}\log(1-x^2)+c$

(c) $\frac{1}{\sqrt{1-x^2}}\sin^{-1}x-\frac{1}{2}\log(1-x^2)+c$

(d) $\frac{1}{\sqrt{1-x^2}}\sin^{-1}x+\frac{1}{2}\log(1-x^2)+c$

68. $\int\frac{x\tan^{-1}x}{(1+x^2)^{3/2}}dx =$

(a) $\frac{x+\tan^{-1}x}{\sqrt{1+x^2}}+c$ (b) $\frac{x-\tan^{-1}x}{\sqrt{1+x^2}}+c$

(c) $\frac{\tan^{-1}x-x}{\sqrt{1+x^2}}+c$ (d) इनमें से कोई नहीं

69. $\int x^5 . e^{x^2} dx =$

(a) $\frac{1}{2}x^4 e^{x^2} - x^2 e^{x^2} + e^{x^2} + c$

(b) $\frac{1}{2}x^4 e^{x^2} + x^2 e^{x^2} + e^{x^2} + c$

(c) $\frac{1}{2}x^4 e^{x^2} - x^2 e^{x^2} - e^{x^2} + c$

(d) इनमें से कोई नहीं

70. $\int e^{\tan^{-1} x}\left(\frac{1+x+x^2}{1+x^2}\right)dx$ का मान है

(a) $xe^{\tan^{-1} x} + c$ (b) $x^2 e^{\tan^{-1} x} + c$

(c) $\frac{1}{x}e^{\tan^{-1} x} + c$ (d) इनमें से कोई नहीं

71. $\int e^{\sqrt{x}}\, dx$ का मान है **[MP PET 1998]**

(a) $e^{\sqrt{x}} + A$ (b) $\frac{1}{2}e^{\sqrt{x}} + A$

(c) $2(\sqrt{x}-1)e^{\sqrt{x}} + A$ (d) $2(\sqrt{x}+1)e^{\sqrt{x}} + A$

(जहाँ A कोई स्वेच्छ अचर है)

72. $I_1 = \int \sin^{-1} x\, dx$ और $I_2 = \int \sin^{-1}\sqrt{1-x^2}\, dx$, तब **[Kerala (Engg.) 2005]**

(a) $I_1 = I_2$ (b) $I_2 = \pi/2 I_1$

(c) $I_1 + I_2 = \pi/2x$ (d) $I_1 + I_2 = \pi/2$

(e) $I_1 - I_2 = \pi/2x$

73. यदि $f(x)$ का प्रतिअवकलज e^x और $g(x)$ का प्रतिअवकलज $\cos x$ है तब $\int f(x)\cos x\, dx + \int g(x)e^x dx =$ **[Kerala (Engg.) 2005]**

(a) $f(x)g(x)+c$ (b) $f(x)+g(x)+c$

(c) $e^x \cos x + c$ (d) $f(x) - g(x)+c$

(e) $e^x \cos x + f(x)g(x) + c$

आंशिक भिन्नों द्वारा परिमेय फलनों के समाकलन, समाकलन के विभिन्न रूपों का मान ज्ञात करना

1. $\int \frac{dx}{(x-x^2)} =$ **[Roorkee 1982]**

(a) $\log x - \log(1-x) + c$ (b) $\log(1-x^2) + c$

(c) $-\log x + \log(1-x) + c$ (d) $\log(x-x^2) + c$

2. $\int \frac{dx}{1+x+x^2+x^3} =$ **[MP PET 1991]**

(a) $\log\sqrt{1+x} - \frac{1}{2}\log\sqrt{1+x^2} + \frac{1}{2}\tan^{-1} x + c$

(b) $\log\sqrt{1+x} - \log\sqrt{1+x^2} + \tan^{-1} x + c$

(c) $\log\sqrt{1+x^2} - \log\sqrt{1+x} + \frac{1}{2}\tan^{-1} x + c$

(d) $\log\sqrt{1+x} + \tan^{-1} x + \log\sqrt{1+x^2} + c$

3. $\int \frac{x-1}{(x-3)(x-2)}dx =$ **[Roorkee 1978]**

(a) $\log(x-3) - \log(x-2) + c$

(b) $\log(x-3)^2 - \log(x-2) + c$

(c) $\log(x-3) + \log(x-2) + c$

(d) $\log(x-3)^2 + \log(x-2) + c$

4. $\int \frac{1}{\cos x(1+\cos x)}dx =$

(a) $\log(\sec x + \tan x) + 2\tan\frac{x}{2} + c$

(b) $\log(\sec x + \tan x) - 2\tan\frac{x}{2} + c$

(c) $\log(\sec x + \tan x) + \tan\frac{x}{2} + c$

(d) $\log(\sec x + \tan x) - \tan\frac{x}{2} + c$

5. $\int \frac{dx}{(x+1)(x+2)} =$ **[MP PET 1987]**

(a) $\log\frac{x+2}{x+1} + c$ (b) $\log(x+1) + \log(x+2) + c$

(c) $\log\frac{x+1}{x+2} + c$ (d) इनमें से कोई नहीं

6. $\int \frac{x}{(x-2)(x-1)}dx$ का सही मान है **[MP PET 1993]**

(a) $\log_e\frac{(x-2)^2}{(x-1)} + p$ (b) $\log_e\frac{(x-1)}{(x-2)} + p$

(c) $\frac{x-1}{x-2} + p$ (d) $2\log_e\left(\frac{x-2}{x-1}\right) + p$

(जहाँ p एक स्वेच्छ अचर है)

7. $\int \frac{1}{(x-1)(x^2+1)}dx =$ **[Roorkee 1984]**

(a) $\frac{1}{2}\log(x-1) - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1} x + c$

(b) $\frac{1}{2}\log(x-1) + \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1} x + c$

(c) $\frac{1}{2}\log(x-1) - \frac{1}{2}\log(x^2+1) - \frac{1}{2}\tan^{-1} x + c$

(d) इनमें से कोई नहीं

8. $\int \frac{x^2+x-1}{x^2+x-6}dx =$ **[AISSE 1988]**

(a) $x + \log(x+3) + \log(x-2) + c$

(b) $x - \log(x+3) + \log(x-2) + c$

(c) $x - \log(x+3) - \log(x-2) + c$

(d) इनमें से कोई नहीं

9. $\int \frac{x^2}{(x^2+2)(x^2+3)}dx =$ **[AISSE 1990]**

(a) $-\sqrt{2}\tan^{-1}x+\sqrt{3}\tan^{-1}x+c$

(b) $-\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}}+\sqrt{3}\tan^{-1}\frac{x}{\sqrt{3}}+c$

(c) $\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}}+\sqrt{3}\tan^{-1}\frac{x}{\sqrt{3}}+c$

(d) इनमें से कोई नहीं

10. $\int \frac{dx}{(x^2+1)(x^2+4)}=$ **[MP PET 1995]**

(a) $\frac{1}{3}\tan^{-1}x-\frac{1}{3}\tan^{-1}\frac{x}{2}+c$

(b) $\frac{1}{3}\tan^{-1}x+\frac{1}{3}\tan^{-1}\frac{x}{2}+c$

(c) $\frac{1}{3}\tan^{-1}x-\frac{1}{6}\tan^{-1}\frac{x}{2}+c$

(d) $\tan^{-1}x-2\tan^{-1}\frac{x}{2}+c$

11. $\int \frac{1}{x-x^3}dx=$ **[MP PET 1996]**

(a) $\frac{1}{2}\log\frac{(1-x^2)}{x^2}+c$ (b) $\log\frac{(1-x)}{x(1+x)}+c$

(c) $\log x(1-x^2)+c$ (d) $\frac{1}{2}\log\frac{x^2}{(1-x^2)}+c$

12. यदि $\int \sin 5x\cos 3x\,dx=-\frac{\cos 8x}{16}+A$, तब $A=$ **[MP PET 1992]**

(a) $\frac{\sin 2x}{16}+$ अचर (b) $-\frac{\cos 2x}{4}+$ अचर

(c) अचर (d) इनमें से कोई नहीं

13. $\int \sin^3 x\cos^2 x\,dx=$

(a) $\frac{\cos^5 x}{5}-\frac{\cos^3 x}{3}+c$ (b) $\frac{\cos^5 x}{5}+\frac{\cos^3 x}{3}+c$

(c) $\frac{\sin^5 x}{5}-\frac{\sin^3 x}{3}+c$ (d) $\frac{\sin^5 x}{5}+\frac{\sin^3 x}{3}+c$

14. $\int \sin 2x\cos 3x\,dx=$ **[Roorkee 1976]**

(a) $\frac{1}{2}\left(\cos x+\frac{1}{5}\cos 5x\right)+c$ (b) $\frac{1}{2}\left(\cos x-\frac{1}{5}\cos 5x\right)+c$

(c) $\cos x+\frac{1}{5}\cos 5x+c$ (d) $\cos x-\frac{1}{5}\cos 5x+c$

15. $\int \frac{\cos x}{(1+\sin x)(2+\sin x)}dx=$ **[Roorkee 1979]**

(a) $\log[(1+\sin x)(2+\sin x)]+c$

(b) $\log\frac{2+\sin x}{1+\sin x}+c$

(c) $\log\frac{1+\sin x}{2+\sin x}+c$

(d) इनमें से कोई नहीं

16. $\int \frac{e^x}{(1+e^x)(2+e^x)}dx=$

(a) $\log[(1+e^x)(2+e^x)]+c$ (b) $\log\left[\frac{1+e^x}{2+e^x}\right]+c$

(c) $\log[(1+e^x)\sqrt{2+e^x}]+c$ (d) इनमें से कोई नहीं

17. $\int \frac{dx}{e^x+1-2e^{-x}}=$

(a) $\log(e^x-1)-\log(e^x+2)+c$

(b) $\frac{1}{2}\log(e^x-1)-\frac{1}{3}\log(e^x+2)+c$

(c) $\frac{1}{3}\log(e^x-1)-\frac{1}{3}\log(e^x+2)+c$

(d) $\frac{1}{3}\log(e^x-1)+\frac{1}{3}\log(e^x+2)+c$

18. $\int \frac{x}{x^4-1}dx=$

(a) $\frac{1}{4}\log\left[\frac{x^2-1}{x^2+1}\right]+c$ (b) $\frac{1}{4}\log\left[\frac{x^2+1}{x^2-1}\right]+c$

(c) $\frac{1}{2}\log\left[\frac{x^2-1}{x^2+1}\right]+c$ (d) $\frac{1}{2}\log\left[\frac{x^2+1}{x^2-1}\right]+c$

19. $\int \sin^5 x\cos^4 x\,dx=$

(a) $-\frac{1}{5}\cos^5 x+\frac{2}{7}\cos^7 x-\frac{1}{9}\cos^9 x+c$

(b) $\frac{1}{5}\cos^5 x+\frac{2}{7}\cos^7 x-\frac{1}{9}\cos^9 x+c$

(c) $\frac{1}{5}\cos^5 x+\frac{2}{7}\cos^7 x+\frac{1}{9}\cos^9 x+c$

(d) इनमें से कोई नहीं

20. $\int \sqrt{x^2-8x+7}\,dx=$

(a) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log[x-4+\sqrt{x^2-8x+7}]+c$

(b) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log[x-4+\sqrt{x^2-8x+7}]+c$

(c) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log[x-4+\sqrt{x^2-8x+7}]+c$

(d) इनमें से कोई नहीं

21. $\int \frac{dx}{\sqrt{2x-x^2}}=$ **[MP PET 1991; Karnataka CET 2002]**

(a) $\cos^{-1}(x-1)+c$ (b) $\sin^{-1}(x-1)+c$

(c) $\cos^{-1}(1+x)+c$ (d) $\sin^{-1}(1-x)+c$

22. $\int \frac{x\,dx}{(x^2-a^2)(x^2-b^2)}=$ **[Roorkee 1976]**

(a) $\frac{1}{a^2-b^2}\log\left(\frac{x^2-a^2}{x^2-b^2}\right)+c$

(b) $\frac{1}{a^2-b^2}\log\left(\frac{x^2-b^2}{x^2-a^2}\right)+c$

(c) $\frac{1}{2(a^2-b^2)}\log\left(\frac{x^2-a^2}{x^2-b^2}\right)+c$

(d) $\frac{1}{2(a^2-b^2)}\log\left(\frac{x^2-b^2}{x^2-a^2}\right)+c$

23. $\int \frac{dx}{5+4\cos x} =$ **[Roorkee 1983; RPET 1997]**

(a) $\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan x\right)+c$ (b) $\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\tan x\right)+c$

(c) $\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)+c$ (d) $\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)+c$

24. $\int \frac{1}{(x^2+a^2)(x^2+b^2)}dx =$

(a) $\frac{1}{(a^2-b^2)}\left[\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)-\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]+c$

(b) $\frac{1}{(b^2-a^2)}\left[\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)-\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]+c$

(c) $\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)-\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$

(d) $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)-\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)+c$

25. $\int \frac{1}{1+\cos^2 x}dx =$

(a) $\frac{1}{\sqrt{2}}\tan^{-1}(\tan x)+c$ (b) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{2}\tan x\right)+c$

(c) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\tan x\right)+c$ (d) इनमें से कोई नहीं

26. $\int \frac{dx}{1+3\sin^2 x} =$ **[Roorkee 1989; DCE 2001]**

(a) $\frac{1}{3}\tan^{-1}(3\tan^2 x)+c$ (b) $\frac{1}{2}\tan^{-1}(2\tan x)+c$

(c) $\tan^{-1}(\tan x)+c$ (d) इनमें से कोई नहीं

27. $\int \frac{dx}{2x^2+x+1} =$ **[RPET 1997]**

(a) $\frac{1}{\sqrt{7}}\tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)+c$ (b) $\frac{1}{2\sqrt{7}}\tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)+c$

(c) $\frac{1}{2}\tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)+c$ (d) इनमें से कोई नहीं

28. $\int \frac{dx}{7+5\cos x} =$ **[EAMCET 2002]**

(a) $\frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{1}{\sqrt{6}}\tan\frac{x}{2}\right)+c$ (b) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)+c$

(c) $\frac{1}{4}\tan^{-1}\left(\tan\frac{x}{2}\right)+c$ (d) $\frac{1}{7}\tan^{-1}\left(\tan\frac{x}{2}\right)+c$

29. $\int \frac{dx}{x^2+4x+13} =$ **[Kerala (Engg.) 2002]**

(a) $\log(x^2+4x+13)+c$ (b) $\frac{1}{3}\tan^{-1}\left(\frac{x+2}{3}\right)+c$

(c) $\log(2x+4)+c$ (d) $\frac{2x+4}{(x^2+4x+13)^2}+c$

30. $\int \frac{dx}{\cos x-\sin x} =$ **[AIEEE 2004]**

(a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+c$

(b) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right|+c$

(c) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|+c$

(d) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+c$

31. $\int \frac{dx}{x^2+2x+2} =$ **[Karnataka CET 2004]**

(a) $\sin^{-1}(x+1)+c$ (b) $\sinh^{-1}(x+1)+c$

(c) $\tanh^{-1}(x+1)+c$ (d) $\tan^{-1}(x+1)+c$

32. $\int \frac{3\sin x+2\cos x}{3\cos x+2\sin x}dx =$

(a) $\frac{12}{13}x-\frac{5}{13}\log(3\cos x+2\sin x)$

(b) $\frac{12}{13}x+\frac{5}{13}\log(3\cos x+2\sin x)$

(c) $\frac{13}{12}x+\frac{5}{13}\log(3\cos x+2\sin x)$

(d) इनमें से कोई नहीं

33. $\int \frac{dx}{x[(\log x)^2+4\log x-1]} =$

(a) $\frac{1}{2\sqrt{5}}\log\left[\frac{\log x+2-\sqrt{5}}{\log x+2+\sqrt{5}}\right]+c$

(b) $\frac{1}{\sqrt{5}}\log\left[\frac{\log x+2-\sqrt{5}}{\log x+2+\sqrt{5}}\right]+c$

(c) $\frac{1}{2\sqrt{5}}\log\left[\frac{\log x+2+\sqrt{5}}{\log x+2-\sqrt{5}}\right]+c$

(d) $\frac{1}{\sqrt{5}}\log\left[\frac{\log x+2+\sqrt{5}}{\log x+2-\sqrt{5}}\right]+c$

34. $\int \frac{dx}{x(x^n+1)} =$ **[Roorkee 1979]**

(a) $n\log\frac{x^n}{x^n+1}+c$ (b) $n\log\frac{x^n+1}{x^n}+c$

(c) $\frac{1}{n}\log\frac{x^n}{x^n+1}+c$ (d) $\frac{1}{n}\log\frac{x^n+1}{x^n}+c$

35. $\int \frac{dx}{x(x^7+1)} =$ **[Karnataka CET 2004]**

(a) $\log\left(\frac{x^7}{x^7+1}\right)+c$ (b) $\frac{1}{7}\log\left(\frac{x^7}{x^7+1}\right)+c$

(c) $\log\left(\frac{x^7+1}{x^7}\right)+c$ (d) $\frac{1}{7}\log\left(\frac{x^7+1}{x^7}\right)+c$

36. $\int \frac{dx}{x(x^5+1)} =$ **[UPSEAT 2004]**

(a) $\frac{1}{5}\log x^5(x^5+1)+c$ (b) $\frac{1}{5}\log x^5\left(\frac{1+x^5}{x^5}\right)+c$

(c) $\frac{1}{5}\log x^5\left(\frac{x^5}{x^5+1}\right)+c$ (d) इनमें से कोई नहीं

37. $\int \frac{x^2+1}{x^4+1}dx =$ **[AISSE 1990]**

(a) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{2x}\right)+c$ (b) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+c$

(c) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{2\sqrt{x}}\right)+c$ (d) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+c$

38. $\int \frac{x^2-1}{x^4+x^2+1}dx =$ **[AISSE 1990]**

(a) $\frac{1}{2}\log\left(\frac{x^2+x+1}{x^2-x+1}\right)+c$ (b) $\frac{1}{2}\log\left(\frac{x^2-x-1}{x^2+x+1}\right)+c$

(c) $\log\left(\frac{x^2-x+1}{x^2+x+1}\right)+c$ (d) $\frac{1}{2}\log\left(\frac{x^2-x+1}{x^2+x+1}\right)+c$

39. $\int \frac{x\,dx}{x^2+4x+5} =$ **[RPET 2002]**

(a) $\frac{1}{2}\log(x^2+4x+5)+2\tan^{-1}(x)+c$

(b) $\frac{1}{2}\log(x^2+4x+5)-\tan^{-1}(x+2)+c$

(c) $\frac{1}{2}\log(x^2+4x+5)+\tan^{-1}(x+2)+c$

(d) $\frac{1}{2}\log(x^2+4x+5)-2\tan^{-1}(x+2)+c$

40. यदि $\int \frac{2x^2+3.dx}{(x^2-1)(x^2-4)} = \log\left(\frac{x-2}{x+2}\right)^a\left(\frac{x+1}{x-1}\right)^b+c$, तब a व b के मान क्रमशः हैं **[AMU 2005]**

(a) $\frac{11}{12},\frac{5}{6}$ (b) $\frac{11}{12},\frac{-5}{6}$

(c) $-\frac{11}{12},\frac{5}{6}$ (d) इनमें से कोई नहीं

Critical Thinking

Objective Questions

1. $\int \frac{dx}{\cos(x-a)\cos(x-b)} =$

(a) $\text{cosec}\ (a-b)\log\frac{\sin(x-a)}{\sin(x-b)}+c$

(b) $\text{cosec}\,(a-b)\log\frac{\cos(x-a)}{\cos(x-b)}+c$

(c) $\text{cosec}\,(a-b)\log\frac{\sin(x-b)}{\sin(x-a)}+c$

(d) $\text{cosec}\,(a-b)\log\frac{\cos(x-b)}{\cos(x-a)}+c$

2. $\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}} =$ **[AISSE 1989]**

(a) $\frac{2}{3(b-a)}[(x+a)^{3/2}-(x+b)^{3/2}]+c$

(b) $\frac{2}{3(a-b)}[(x+a)^{3/2}-(x+b)^{3/2}]+c$

(c) $\frac{2}{3(a-b)}[(x+a)^{3/2}+(x+b)^{3/2}]+c$

(d) इनमें से कोई नहीं

3. $\int \frac{3\cos x+3\sin x}{4\sin x+5\cos x}dx =$ **[EAMCET 1991]**

(a) $\frac{27}{41}x-\frac{3}{41}\log(4\sin x+5\cos x)$

(b) $\frac{27}{41}x+\frac{3}{41}\log(4\sin x+5\cos x)$

(c) $\frac{27}{41}x-\frac{3}{41}\log(4\sin x-5\cos x)$

(d) इनमें से कोई नहीं

4. यदि $\int(\sin 2x+\cos 2x)\,dx = \frac{1}{\sqrt{2}}\sin(2x-c)+a$, तब a व c के मान हैं **[Roorkee 1978]**

(a) $c=\pi/4$ और $a=k$ (एक स्वेच्छ अचर)

(b) $c=-\pi/4$ और $a=\pi/2$

(c) $c=\pi/2$ और a एक स्वेच्छ अचर है

(d) इनमें से कोई नहीं

5. $\int \frac{x^3-x-2}{(1-x^2)}dx =$ **[AI CBSE 1985]**

(a) $\log\left(\frac{x+1}{x-1}\right)-\frac{x^2}{2}+c$ (b) $\log\left(\frac{x-1}{x+1}\right)+\frac{x^2}{2}+c$

(c) $\log\left(\frac{x+1}{x-1}\right)+\frac{x^2}{2}+c$ (d) $\log\left(\frac{x-1}{x+1}\right)-\frac{x^2}{2}+c$

6. $\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx =$ **[IIT 1986]**

(a) $\sin 2x+c$ (b) $-\frac{1}{2}\sin 2x+c$

(c) $\frac{1}{2}\sin 2x+c$ (d) $-\sin 2x+c$

7. $\int \frac{x^2dx}{(a+bx)^2} =$ **[IIT 1979]**

(a) $\frac{1}{b^2}\left[x+\frac{2a}{b}\log(a+bx)-\frac{a^2}{b}\frac{1}{a+bx}\right]$

(b) $\frac{1}{b^2}\left[x-\frac{2a}{b}\log(a+bx)+\frac{a^2}{b}\frac{1}{a+bx}\right]$

(c) $\frac{1}{b^2}\left[x+\frac{2a}{b}\log(a+bx)+\frac{a^2}{b}\frac{1}{a+bx}\right]$

(d) $\frac{1}{b^2}\left[x+\frac{a}{b}-\frac{2a}{b}\log(a+bx)-\frac{a^2}{b}\frac{1}{a+bx}\right]$

8. $\int \frac{dx}{(1+x^2)\sqrt{p^2+q^2(\tan^{-1}x)^2}} =$

(a) $\frac{1}{q}\log[q\tan^{-1}x+\sqrt{p^2+q^2(\tan^{-1}x)^2}]+c$

(b) $\log[q\tan^{-1}x+\sqrt{p^2+q^2(\tan^{-1}x)^2}]+c$

(c) $\frac{2}{3q}(p^2+q^2\tan^{-1}x)^{3/2}+c$

(d) इनमें से कोई नहीं

9. $\int \frac{x^5}{\sqrt{1+x^3}}dx =$ **[IIT 1985]**

(a) $\frac{2}{9}(1+x^3)^{3/2}+c$

(b) $\frac{2}{9}(1+x^3)^{3/2}+\frac{2}{3}(1+x^3)^{1/2}+c$

(c) $\frac{2}{9}(1+x^3)^{3/2}-\frac{2}{3}(1+x^3)^{1/2}+c$

(d) इनमें से कोई नहीं

10. $\int \frac{dx}{\sin x-\cos x+\sqrt{2}} =$ **[MP PET 2002]**

(a) $-\frac{1}{\sqrt{2}}\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)+c$ (b) $\frac{1}{\sqrt{2}}\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)+c$

(c) $\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)+c$ (d) $-\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)+c$

11. $\int \frac{a\,dx}{b+ce^x} =$ **[MP PET 1988; BIT Ranchi 1979]**

(a) $\frac{a}{b}\log\left(\frac{e^x}{b+ce^x}\right)+c$ (b) $\frac{a}{b}\log\left(\frac{b+ce^x}{e^x}\right)+c$

(c) $\frac{b}{a}\log\left(\frac{e^x}{b+ce^x}\right)+c$ (d) $\frac{b}{a}\log\left(\frac{b+ce^x}{e^x}\right)+c$

12. $\int \sin\sqrt{x}\,dx =$ **[Roorkee 1977]**

(a) $2[\sin\sqrt{x}-\cos\sqrt{x}]+c$ (b) $2[\sin\sqrt{x}-\sqrt{x}\cos\sqrt{x}]+c$

(c) $2[\sin\sqrt{x}+\cos\sqrt{x}]+c$ (d) $2[\sin\sqrt{x}+\sqrt{x}\cos\sqrt{x}]+c$

13. $\int \frac{x^2}{(9-x^2)^{3/2}}dx =$

(a) $\frac{x}{\sqrt{9-x^2}}-\sin^{-1}\frac{x}{3}+c$ (b) $\frac{x}{\sqrt{9-x^2}}+\sin^{-1}\frac{x}{3}+c$

(c) $\sin^{-1}\frac{x}{3}-\frac{x}{\sqrt{9-x^2}}+c$ (d) इनमें से कोई नहीं

14. $\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx =$

(a) $\frac{1}{2}[\sin^{-1}x^2+\sqrt{1-x^4}]+c$

(b) $\frac{1}{2}[\sin^{-1}x^2+\sqrt{1-x^2}]+c$

(c) $\sin^{-1}x^2+\sqrt{1-x^4}+c$

(d) $\sin^{-1}x^2+\sqrt{1-x^2}+c$

15. यदि $\int f(x)\sin x\cos x\,dx=\frac{1}{2(b^2-a^2)}\log(f(x))+c$, तब $f(x)=$

(a) $\frac{1}{a^2\sin^2x+b^2\cos^2x}$ (b) $\frac{1}{a^2\sin^2x-b^2\cos^2x}$

(c) $\frac{1}{a^2\cos^2x+b^2\sin^2x}$ (d) $\frac{1}{a^2\cos^2x-b^2\sin^2x}$

16. $\int \frac{dx}{4\sin^2x+5\cos^2x} =$ **[AISSE 1986]**

(a) $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right)+c$ (b) $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{\tan x}{\sqrt{5}}\right)+c$

(c) $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right)+c$ (d) इनमें से कोई नहीं

17. $\int \frac{x^2+1}{x^4-x^2+1}dx =$ **[MP PET 1991]**

(a) $\tan^{-1}\left(\frac{1+x^2}{x}\right)+c$ (b) $\cot^{-1}\left(\frac{1+x^2}{x}\right)+c$

(c) $\tan^{-1}\left(\frac{x^2-1}{x}\right)+c$ (d) $\cot^{-1}\left(\frac{x^2-1}{x}\right)+c$

18. $\int (\log x)^2\,dx =$ **[IIT 1971, 77]**

(a) $x(\log x)^2-2x\log x-2x+c$

(b) $x(\log x)^2-2x\log x-x+c$

(c) $x(\log x)^2-2x\log x+2x+c$

(d) $x(\log x)^2-2x\log x+x+c$

19. $\int \frac{\sqrt{(x^2-a^2)}}{x}dx$ का मान होगा **[UPSEAT 1999]**

(a) $\sqrt{(x^2-a^2)}-a\tan^{-1}\left[\frac{\sqrt{(x^2-a^2)}}{a}\right]$

(b) $\sqrt{(x^2-a^2)}+a\tan^{-1}\left[\frac{\sqrt{(x^2-a^2)}}{a}\right]$

(c) $\sqrt{(x^2-a^2)}+a^2\tan^{-1}[\sqrt{x^2-a^2}]$

(d) $\tan^{-1}x/a+c$

20. $\int \tan^3 2x\sec 2x\,dx =$ **[IIT 1977]**

(a) $\frac{1}{6}\sec^3 2x-\frac{1}{2}\sec 2x+c$ (b) $\frac{1}{6}\sec^3 2x+\frac{1}{2}\sec 2x+c$

(c) $\frac{1}{9}\sec^2 2x-\frac{1}{3}\sec 2x+c$ (d) इनमें से कोई नहीं

21. $\int x\sin^{-1}x\,dx =$ **[MP PET 1991]**

(a) $\left(\frac{x^2}{2}-\frac{1}{4}\right)\sin^{-1}x+\frac{x}{4}\sqrt{1-x^2}+c$

(b) $\left(\frac{x^2}{2}+\frac{1}{4}\right)\sin^{-1}x+\frac{x}{4}\sqrt{1-x^2}+c$

(c) $\left(\frac{x^2}{2}-\frac{1}{4}\right)\sin^{-1}x-\frac{x}{4}\sqrt{1-x^2}+c$

(d) $\left(\frac{x^2}{2}+\frac{1}{4}\right)\sin^{-1}x-\frac{x}{4}\sqrt{1-x^2}+c$

22. $\int\sqrt{\frac{a-x}{x}}\,dx=$

(a) $a\left[\sin^{-1}\sqrt{\frac{x}{a}}+\sqrt{\frac{x}{a}}\sqrt{\frac{a-x}{a}}\right]+c$

(b) $\sin^{-1}\frac{x}{a}+\frac{x}{a}\sqrt{a^2-x^2}+c$

(c) $a\left[\sin^{-1}\frac{x}{a}-\frac{x}{a}\sqrt{a^2-x^2}\right]+c$

(d) $\sin^{-1}\frac{x}{a}-\frac{x}{a}\sqrt{a^2-x^2}+c$

23. यदि $x\in\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$, तब $\int\frac{\sin x-\cos x}{\sqrt{1-\sin 2x}}e^{\sin x}\cos x\,dx=$

(a) $e^{\sin x}+c$ (b) $e^{\sin x-\cos x}+c$

(c) $e^{\sin x+\cos x}+c$ (d) $e^{\cos x-\sin x}+c$

24. यदि $\int\frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx=Ax+B\log(9e^{2x}-4)+C$, तब A, B और C हैं **[IIT 1990]**

(a) $A=\frac{3}{2},\ B=\frac{36}{35},\ C=\frac{3}{2}\log 3+$ अचर

(b) $A=\frac{3}{2},\ B=\frac{35}{36},\ C=\frac{3}{2}\log 3+$ अचर

(c) $A=-\frac{3}{2},\ B=-\frac{35}{36},\ C=-\frac{3}{2}\log 3+$ अचर

(d) इनमें से कोई नहीं

25. $\int\sec^3 x\,dx$ का मान होगा **[UPSEAT 1999]**

(a) $\frac{1}{2}\left[\sec x\tan x+\log(\sec x+\tan x)\right]$

(b) $\frac{1}{3}\left[\sec x\tan x+\log(\sec x+\tan x)\right]$

(c) $\frac{1}{4}\left[\sec x\tan x+\log(\sec x+\tan x)\right]$

(d) $\frac{1}{8}\left[\sec x\tan x+\log(\sec x+\tan x)\right]$

26. $\int\frac{x-1}{(x+1)^3}e^x\,dx=$ **[IIT 1983; MP PET 1990]**

(a) $\frac{-e^x}{(x+1)^2}+c$ (b) $\frac{e^x}{(x+1)^2}+c$

(c) $\frac{e^x}{(x+1)^3}+c$ (d) $\frac{-e^x}{(x+1)^3}+c$

27. यदि $I=\int e^x\sin 2x\,dx$, तब K के किस मान के लिये $KI=e^x(\sin 2x-2\cos 2x)+$ अचर होगा **[MP PET 1992]**

(a) 1 (b) 3

(c) 5 (d) 7

28. $\int\frac{dx}{3-2x-x^2}$ का मान होगा **[UPSEAT 1999]**

(a) $\frac{1}{4}\log\left(\frac{3+x}{1-x}\right)$ (b) $\frac{1}{3}\log\left(\frac{3+x}{1-x}\right)$

(c) $\frac{1}{2}\log\left(\frac{3+x}{1-x}\right)$ (d) $\log\left(\frac{1-x}{3+x}\right)$

29. $\int x\sqrt{2x+3}\,dx=$ **[AISSE 1985]**

(a) $\frac{x}{3}(2x+3)^{3/2}-\frac{1}{15}(2x+3)^{5/2}+c$

(b) $\frac{x}{3}(2x+3)^{3/2}+\frac{1}{15}(2x+3)^{5/2}+c$

(c) $\frac{x}{2}(2x+3)^{3/2}+\frac{1}{6}(2x+3)^{5/2}+c$

(d) इनमें से कोई नहीं

30. $\int\cos 2\theta\log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)d\theta=$ **[IIT 1994]**

(a) $(\cos\theta-\sin\theta)^2\log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)$

(b) $(\cos\theta+\sin\theta)^2\log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)$

(c) $\frac{(\cos\theta-\sin\theta)^2}{2}\log\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)$

(d) $\frac{1}{2}\sin 2\theta\log\tan\left(\frac{\pi}{4}+\theta\right)-\frac{1}{2}\log\sec 2\theta$

31. $\int\frac{x^2}{(x\sin x+\cos x)^2}dx=$ **[MNR 1989; RPET 2000]**

(a) $\frac{\sin x+\cos x}{x\sin x+\cos x}$ (b) $\frac{x\sin x-\cos x}{x\sin x+\cos x}$

(c) $\frac{\sin x-x\cos x}{x\sin x+\cos x}$ (d) इनमें से कोई नहीं

32. यदि $u=\int e^{ax}\cos bx\,dx$ और $v=\int e^{ax}\sin bx\,dx$, तब $(a^2+b^2)(u^2+v^2)=$

(a) $2e^{ax}$ (b) $(a^2+b^2)e^{2ax}$

(c) e^{2ax} (d) $(a^2-b^2)e^{2ax}$

33. यदि $I_n=\int(\log x)^n\,dx$, तब $I_n+nI_{n-1}=$ **[Karnataka CET 2003]**

(a) $x(\log x)^n$ (b) $(x\log x)^n$

(c) $(\log x)^{n-1}$ (d) $n(\log x)^n$

34. $\int e^{x/2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx=$ **[Roorkee 1982]**

(a) $e^{x/2}\cos\frac{x}{2}+c$ (b) $\sqrt{2}e^{x/2}\cos\frac{x}{2}+c$

(c) $e^{x/2}\sin\frac{x}{2}+c$ (d) $\sqrt{2}e^{x/2}\sin\frac{x}{2}+c$

35. यदि $\int \frac{2x+3}{x^2-5x+6}dx = 9\ln(x-3)-7\ln(x-2)+A$, तब $A=$ **[MP PET 1992]**

(a) $5\ln(x-2)+$ अचर (b) $-4\ln(x-3)+$ अचर

(c) अचर (d) इनमें से कोई नहीं

36. $\int \frac{dx}{2+\cos x} =$

(a) $2\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)+c$ (b) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)+c$

(c) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)+c$ (d) इनमें से कोई नहीं

37. $\int \frac{x}{x^4+x^2+1}dx =$ **[MP PET 2004]**

(a) $\frac{1}{3}\tan^{-1}\left(\frac{2x^2+1}{3}\right)$ (b) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$

(c) $\frac{1}{\sqrt{3}}\tan^{-1}(2x^2+1)$ (d) इनमें से कोई नहीं

38. $\int \frac{dx}{(\sin x+\sin 2x)} =$ **[IIT 1984]**

(a) $\frac{1}{6}\log(1-\cos x)+\frac{1}{2}\log(1+\cos x)-\frac{2}{3}\log(1+2\cos x)$

(b) $6\log(1-\cos x)+2\log(1+\cos x)-\frac{2}{3}\log(1+2\cos x)$

(c) $6\log(1-\cos x)+\frac{1}{2}\log(1+\cos x)+\frac{2}{3}\log(1+2\cos x)$

(d) इनमें से कोई नहीं

39. यदि $\int \frac{2x+3}{(x-1)(x^2+1)}dx = \log_e\left\{(x-1)^{\frac{5}{2}}(x^2+1)^a\right\}-\frac{1}{2}\tan^{-1}x+A$, जहाँ A कोई स्वेच्छ अचर है, तब 'a' का मान है **[MP PET 1998]**

(a) 5/4 (b) – 5/3

(c) – 5/6 (d) – 5/4

40. यदि $\int \frac{(2x^2+1)\,dx}{(x^2-4)(x^2-1)} = \log\left[\left(\frac{x+1}{x-1}\right)^a\left(\frac{x-2}{x+2}\right)^b\right]+C$, तब a व b के मान क्रमशः हैं **[Roorkee 2000]**

(a) $\frac{1}{2}, \frac{3}{4}$ (b) $-1, \frac{3}{2}$

(c) $1, \frac{3}{2}$ (d) $-\frac{1}{2}, \frac{3}{4}$

Answers

मूलभूत समाकलन

1	d	2	c	3	c	4	d	5	c
6	c	7	c	8	a	9	b	10	d
11	d	12	d	13	b	14	d	15	a
16	a	17	b	18	b	19	c	20	b
21	c	22	a,c	23	c	24	a	25	a
26	b	27	c	28	d	29	d	30	c
31	b	32	c	33	d	34	a	35	a
36	b	37	d	38	b	39	d	40	b
41	a	42	d	43	a	44	c	45	d
46	c	47	a	48	a	49	b	50	a
51	d	52	a	53	c	54	c	55	c
56	a	57	c	58	a	59	b	60	d
61	b	62	a	63	a	64	d	65	b
66	b	67	b	68	a	69	c	70	c
71	b	72	a	73	a	74	a	75	a
76	b	77	b	78	b	79	b	80	c
81	c	82	a	83	b	84	a	85	b
86	c	87	c						

प्रतिस्थापन द्वारा समाकलन

1	b	2	b	3	a	4	a	5	b
6	a	7	d	8	c	9	b	10	b
11	c	12	c	13	b	14	a	15	b
16	a	17	c	18	b	19	a	20	c
21	b	22	b	23	d	24	a	25	b
26	c	27	a	28	c	29	b	30	c
31	a	32	b	33	a	34	c	35	c
36	b	37	a	38	b	39	b	40	a
41	c	42	c	43	b	44	b	45	b
46	a	47	a	48	a	49	b	50	b
51	b	52	a	53	a	54	c	55	b
56	b	57	c	58	b	59	b	60	d
61	a	62	b	63	c	64	a	65	c
66	a	67	c	68	b	69	c	70	b
71	b	72	a	73	b	74	a	75	b
76	b	77	c	78	a	79	a	80	a
81	b	82	b	83	c	84	a	85	c
86	a	87	a	88	c	89	b	90	a
91	a	92	c	93	a	94	a	95	d
96	a	97	a	98	a	99	c	100	a
101	a	102	d	103	b	104	d	105	b
106	a	107	a	108	a	109	a	110	a
111	c	112	c	113	b	114	c	115	b
116	c	117	a	118	c	119	d	120	a
121	b	122	c	123	d	124	d	125	b
126	b	127	c	128	b	129	b	130	c
131	d	132	b	133	d	134	b	135	b
136	c	137	d	138	c	139	d	140	b
141	d	142	b	143	a	144	b	145	a
146	a	147	c	148	c				

खण्डशः समाकलन, $\int e^x(F(x) + F'(x))\,dx$ तथा $\int e^{kx}(kF(x) + F'(x))\,dx$ के रूप का समाकलन

1	d	2	c	3	a	4	b	5	a
6	b	7	b	8	a	9	b	10	d
11	b	12	d	13	b	14	d	15	a
16	a	17	d	18	b	19	b	20	d
21	c	22	a	23	d	24	a	25	b
26	c	27	d	28	b	29	a	30	d
31	c	32	d	33	b	34	c	35	a
36	c	37	c	38	b	39	b	40	a
41	c	42	c	43	a	44	a	45	c
46	a	47	d	48	c	49	a	50	c
51	c	52	b	53	c	54	a	55	a
56	c	57	d	58	b	59	b	60	c
61	b	62	a	63	d	64	a	65	d
66	a	67	a	68	b	69	a	70	a
71	c	72	c	73	c				

आंशिक भिन्नों द्वारा परिमेय फलनों के समाकलन, समाकलन के विभिन्न रूपों का मान ज्ञात करना

1	a	2	a	3	b	4	d	5	c
6	a	7	a	8	b	9	b	10	c
11	d	12	b	13	a	14	b	15	c
16	b	17	c	18	a	19	a	20	c
21	b	22	c	23	c	24	a	25	c
26	b	27	d	28	a	29	b	30	a
31	d	32	a	33	a	34	c	35	b
36	d	37	d	38	d	39	d	40	a

Critical Thinking Questions

1	b	2	b	3	a	4	a	5	d
6	b	7	d	8	a	9	c	10	d
11	a	12	b	13	a	14	a	15	a
16	c	17	c	18	c	19	a	20	a
21	a	22	a	23	a	24	d	25	a
26	b	27	c	28	a	29	a	30	d
31	c	32	c	33	a	34	d	35	c
36	b	37	b	38	a	39	d	40	a

AS Answers and Solutions

मूलभूत समाकलन

1. (d) $\int \sqrt{1-\sin 2x}\ dx$

$= \int (\cos x - \sin x)dx = \sin x + \cos x + c.$

2. (c) $\int \frac{1+\cos^2 x}{\sin^2 x}dx = \int (\text{cosec}^2 x + \cot^2 x)dx$

$= \int (2\text{cosec}^2 x - 1)dx = -2\cot x - x + c.$

3. (c) $\int \sin^{-1}(\cos x)dx = \int \left\{\frac{\pi}{2} - \cos^{-1}(\cos x)\right\}dx$

$= \frac{\pi}{2}x - \frac{x^2}{2} = \frac{\pi x - x^2}{2}.$

4. (d) $\int \frac{dx}{\tan x + \cot x} = \int \frac{dx}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$

$= \frac{1}{2}\int 2\sin x \cos x\, dx = \frac{-\cos 2x}{4} + c.$

5. (c) $\int (e^{a\log x} + e^{x\log a})dx = \int (e^{\log_e x^a} + e^{\log_e a^x})dx$

$= \int (x^a + a^x)dx = \frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + c.$

6. (c) दिया गया है, $f'(x) = x^2 + 5$ और $f(0) = -1$

$\Rightarrow f(x) = \frac{x^3}{3} + 5x + c$.यदि $x = 0$, तब $f(0) = c \Rightarrow c = -1$.

अतः $f(x) = \frac{x^3}{3} + 5x - 1.$

7. (c) $\int \tan^{-1}\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}\,dx = \int \tan^{-1}\tan x\,dx$

$= \int x\,dx = \frac{x^2}{2} + c.$

8. (a) $\int \frac{dx}{\sqrt{x}+\sqrt{x-2}} = \frac{1}{2}\int \frac{x-(x-2)}{\sqrt{x}+\sqrt{x-2}}dx$

$= \frac{1}{2}\int (\sqrt{x} - \sqrt{x-2})dx = \frac{1}{2}\left[\frac{x^{3/2}}{3/2} - \frac{(x-2)^{3/2}}{3/2}\right] + c$

$= \frac{1}{3}\left\{x^{3/2} - (x-2)^{3/2}\right\} + c.$

9. (b) $\int \frac{\sin x}{\sin(x-\alpha)}dx = \int \frac{\sin(x-\alpha+\alpha)}{\sin(x-\alpha)}dx$

$= \int \frac{\{(\sin(x-\alpha)\cos\alpha + \cos(x-\alpha)\sin\alpha\}}{\sin(x-\alpha)}dx$

$= \int \cos\alpha\,dx + \int \sin\alpha . \cot(x-\alpha)dx$

$= x\cos\alpha + \sin\alpha . \log\sin(x-\alpha) + c.$

10. (d) $\int \frac{\cos x - 1}{\cos x + 1}dx = -\int \tan^2\frac{x}{2}dx$

$= -\int\left(\sec^2\frac{x}{2} - 1\right)dx = \int\left(1 - \sec^2\frac{x}{2}\right)dx = x - 2\tan\frac{x}{2} + c.$

11. (d) $\int \frac{dx}{1-\sin x} = \int \frac{(1+\sin x)}{1-\sin^2 x}dx$

$= \int \sec^2 x\,dx + \int \tan x . \sec x\,dx = \tan x + \sec x + c.$

12. (d) $\int (\sin 2x - \cos 2x)dx = \frac{1}{\sqrt{2}}\sin(2x-a) + b$

$\Rightarrow -\frac{1}{2}(\sin 2x + \cos 2x) = \frac{1}{\sqrt{2}}\sin(2x-a) + b$

$\Rightarrow -\left[\frac{1}{\sqrt{2}}\sin 2x + \frac{1}{\sqrt{2}}\cos 2x\right] = \sin(2x-a) + b\sqrt{2}$

$\Rightarrow \sin\left(2x + \frac{5\pi}{4}\right) = \sin(2x-a) + b\sqrt{2}$

$\Rightarrow b$ कोई अचर है तथा $a = \frac{-5\pi}{4}$.

13. (b) $\int \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\right)dx = \int e^x dx = e^x + c.$

14. (d) $\int \frac{\cot x \tan x}{\sec^2 x - 1}dx = \int \frac{1}{\tan^2 x}dx = \int \cot^2 x\,dx$

$= \int (\text{cosec}^2 x - 1)dx = -\cot x - x + c.$

15. (a) $\int (\sec x + \tan x)^2 dx$

$= \int (\sec^2 x + \tan^2 x + 2\sec x \tan x)dx$

$= \int (2\sec^2 x - 1 + 2\sec x \tan x)dx$

$= 2\tan x + 2\sec x - x + c = 2(\sec x + \tan x) - x + c.$

16. (a) $\int x^{51}(\tan^{-1} x + \cot^{-1} x)dx = \int x^{51}.\frac{\pi}{2}dx,$

$\left\{\because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\right\}$

$= \frac{\pi x^{52}}{104} + c = \frac{x^{52}}{52}(\tan^{-1} x + \cot^{-1} x) + c.$

17. (b) $\int 5\sin x\ dx = -5\cos x + c.$

18. (b) $\int \frac{\tan x}{(\sec x + \tan x)}dx = \int \frac{\tan x(\sec x - \tan x)}{(\sec x + \tan x)(\sec x - \tan x)}dx$

अंश व हर को $(\sec x - \tan x)$ से गुणा करने पर,

$= \int \frac{\tan x(\sec x - \tan x)}{(\sec^2 x - \tan^2 x)}dx = \int (\sec x \tan x - \tan^2 x)dx$

$= \int \sec x \tan x\,dx - \int (\sec^2 x - 1)dx$

$= \int \sec x \tan x\, dx - \int \sec^2 x\, dx + \int 1\, dx$

$= \sec x - \tan x + x + c$.

ट्रिक : जाँच द्वारा,

$\frac{d}{dx}\{\sec x + \tan x\} = \sec x \tan x + \sec^2 x$

$= \sec x(\sec x + \tan x) = \frac{\sec x}{\sec x - \tan x}$

$\Rightarrow \frac{d}{dx}\{\sec x - \tan x + x + c\} = \sec x \tan x - \sec^2 x + 1$

$= -\sec x(\sec x - \tan x) + 1 = \frac{-\sec x}{\sec x + \tan x} + 1$

$= \frac{\tan x}{\sec x + \tan x}$.

19. (c) $\int \frac{dx}{\sin^2 x \cos^2 x} = \int \frac{(\cos^2 x + \sin^2 x)dx}{\cos^2 x \sin^2 x}$

$= \int \text{cosec}^2 x\, dx + \int \sec^2 x\, dx = -\cot x + \tan x + c$.

20. (b) $\int \left(x + \frac{1}{x}\right)^3 dx = \int \left(x^3 + \frac{1}{x^3} + 3x + \frac{3}{x}\right)dx$

$= \frac{x^4}{4} - \frac{1}{2x^2} + \frac{3x^2}{2} + 3\log x + c$

$= \frac{x^4}{4} + \frac{3x^2}{2} + 3\log x - \frac{1}{2x^2} + c$.

21. (c) $\int \sqrt{1 + \sin\frac{x}{2}}\, dx = \int \sqrt{\left(\sin^2\frac{x}{4} + \cos^2\frac{x}{4} + 2\sin\frac{x}{4}\cos\frac{x}{4}\right)}\, dx$

$= \int \left(\sin\frac{x}{4} + \cos\frac{x}{4}\right)dx = 4\left(\sin\frac{x}{4} - \cos\frac{x}{4}\right) + c$.

22. (a, c) $\int (\sin^{-1} x + \cos^{-1} x)\, dx = \int \left(\frac{\pi}{2}\right)dx = \frac{\pi x}{2} + c$

$\left(\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}\right)$

साथ ही, $\int (\sin^{-1} x + \cos^{-1} x)dx = x(\cos^{-1} x + \sin^{-1} x) + c$.

23. (c) $\int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}}dx = \int \frac{\sin x + \cos x}{\sqrt{(\sin x + \cos x)^2}}dx = \int dx = x + c$.

24. (a) $\int \frac{x-1}{(x+1)^2}dx = \int \frac{x+1-2}{(x+1)^2}dx$

$= \int \frac{1}{x+1}dx - \int \frac{2}{(x+1)^2}dx = \log(x+1) + \frac{2}{(x+1)} + c$.

25. (a) $\int \frac{\text{cosec}\,\theta - \cot\theta}{\text{cosec}\,\theta + \cot\theta}\, d\theta = \int (\text{cosec}\,\theta - \cot\theta)^2 d\theta$

$= \int \text{cosec}^2\theta\, d\theta + \int \cot^2\theta\, d\theta - 2\int \text{cosec}\,\theta \cot\theta\, d\theta$

$= \int (2\text{cosec}^2\theta - 1)\, d\theta - 2\int \text{cosec}\,\theta \cot\theta\, d\theta$

$= 2\text{cosec}\,\theta - 2\cot\theta - \theta + c$.

26. (b) $\int (1 + 2x + 3x^2 + 4x^3 + ...)\, dx$

$= \int (1-x)^{-2} dx = (1-x)^{-1} + c$.

27. (c) दिया गया है, $\int (\cos x - \sin x)dx = \sqrt{2}\sin(x + \alpha) + c$

$\Rightarrow \sin x + \cos x + c = \sqrt{2}\sin(x + \alpha) + c$

$\Rightarrow \sqrt{2}\left(\frac{\sin x}{\sqrt{2}} + \frac{\cos x}{\sqrt{2}}\right) + c = \sqrt{2}\sin(x + \alpha) + c$

$\Rightarrow \sqrt{2}\sin\left(x + \frac{\pi}{4}\right) + c = \sqrt{2}\sin(x + \alpha) + c$

$\Rightarrow \sin\left(x + \frac{\pi}{4}\right) = \sin(x + \alpha) \Rightarrow \alpha = \frac{\pi}{4}$.

28. (d) $\int \frac{3x^3 - 2\sqrt{x}}{x}dx = \int 3x^2 dx - 2\int x^{-1/2}dx = x^3 - 4\sqrt{x} + c$.

29. (d) $\int \frac{dx}{1 + \sin x} = \tan x - \sec x + c = -\frac{1 - \sin x}{\cos x}$

$= -\frac{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}} + c = -\frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}} + c$

$= \frac{\tan\frac{x}{2} - 1}{1 + \tan\frac{x}{2}} + c = \tan\left(\frac{x}{2} - \frac{\pi}{4}\right) + c$

$\Rightarrow a = -\frac{\pi}{4}$, $b =$ स्वेच्छ अचर.

30. (c) $\int \frac{dx}{\sin x + \cos x} = \frac{1}{\sqrt{2}}\int \frac{dx}{\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}}$

$= \frac{1}{\sqrt{2}}\int \text{cosec}\left(x + \frac{\pi}{4}\right)dx = \frac{1}{\sqrt{2}}\log\tan\left(\frac{\pi}{8} + \frac{x}{2}\right) + c$.

31. (b) $\int \frac{1}{x\sqrt{x^2 - 1}}dx = \sec^{-1} x + c$ (सूत्र).

32. (c) $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}dx = \int \frac{2(\cos^2 x + \sin^2 x) - 1}{\cos^2 x}dx$

$= \int \sec^2 x\, dx = \tan x + c$.

33. (d) $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}dx = \int \left(\frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}\right)dx$

$= \int (\sec x \tan x + \text{cosec}\, x \cot x)\, dx = \sec x - \text{cosec}\, x + c$.

34. (a) $\int \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2 dx$

$= \int \left\{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}\right\}dx$

$= \int (1 - \sin x)dx = x + \cos x + c$.

35. (a) $\int \frac{1}{x^2}(2x+1)^3\,dx = \int \frac{(8x^3+1+12x^2+6x)}{x^2}dx$

$= \int \left(8x+12+\frac{6}{x}+\frac{1}{x^2}\right)dx = 4x^2+12x+6\log x-\frac{1}{x}+c$.

36. (b) $\int \frac{5(x^6+1)}{x^2+1}dx = \int \frac{5(x^2+1)(x^4-x^2+1)}{(x^2+1)}dx$

$= \int 5(x^4-x^2+1)dx = x^5-\frac{5}{3}x^3+5x+c.$

37. (d) $\int \frac{ax^{-2}+bx^{-1}+c}{x^{-3}}dx = \int (ax+bx^2+cx^3)dx$

$= \frac{1}{4}cx^4+\frac{1}{3}bx^3+\frac{1}{2}ax^2+k.$

38. (b) $\int \frac{dx}{\sin x+\sqrt{3}\cos x} = \frac{1}{2}\int \frac{dx}{\frac{\sin x}{2}+\frac{\sqrt{3}}{2}\cos x}$

$= \frac{1}{2}\int \frac{dx}{\sin\left(x+\frac{\pi}{3}\right)} = \frac{1}{2}\int \operatorname{cosec}\left(x+\frac{\pi}{3}\right)$

$= \frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)+c.$

39. (d) $\int \frac{x^2+x-6}{(x-2)(x-1)}dx = \int \frac{(x+3)(x-2)}{(x-2)(x-1)}dx = \int \frac{x+3}{x-1}dx$

$= \int \frac{x-1}{x-1}dx + \int \frac{4}{x-1}dx = x+4\log(x-1)+c$.

40. (b) $\int \frac{dx}{4\cos^3 2x-3\cos 2x} = \int \frac{dx}{\cos 6x} = \int \sec 6x dx$

$= \frac{1}{6}\log(\sec 6x+\tan 6x)+c.$

41. (a) $\int \frac{\sin 3x}{\sin x}dx = \int \frac{3\sin x-4\sin^3 x}{\sin x}dx$

$= \int 3\,dx - 4\int \sin^2 x\,dx = 3x-2\int (1-\cos 2x)dx + c$

$= 3x-2x+\sin 2x+c = x+\sin 2x+c.$

42. (d) $\int \frac{f(x)dx}{\log\sin x} = \log\log\sin x$

दोनों पक्षों का अवकलन करने पर,

$\frac{f(x)}{\log\sin x} = \frac{\cot x}{\log\sin x} \Rightarrow f(x) = \cot x.$

43. (a) $\int \frac{\sin x+\operatorname{cosec} x}{\tan x}dx = \int (\cos x+\operatorname{cosec} x\cot x)dx$

$= \sin x - \operatorname{cosec} x + c$.

44. (c) $\int \frac{1}{\sqrt{1+\sin x}}dx = \int \frac{1}{\sqrt{2}\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}dx$

$= \frac{1}{\sqrt{2}}\int \operatorname{cosec}\left(\frac{x}{2}+\frac{\pi}{4}\right)dx = \sqrt{2}\log\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)+c.$

45. (d) $\int (\tan x-\cot x)^2 dx = \int (\tan^2 x+\cot^2 x-2)dx$

$= \int \sec^2 x\,dx + \int \operatorname{cosec}^2 x\,dx - \int 4\,dx$

$= \tan x-\cot x-4x+c.$

46. (c) $\int (1+2\tan^2 x+2\tan x\sec x)^{1/2}dx$

$= \int (\sec^2 x+\tan^2 x+2\tan x\sec x)^{1/2}dx$

$= \int (\sec x+\tan x)dx = \log(\sec x+\tan x)+\log\sec x+c$

$= \log\sec x(\sec x+\tan x)+c$.

47. (a) $\int \frac{2x}{(2x+1)^2}dx = \int \frac{2x+1-1}{(2x+1)^2}dx$

$= \int \frac{1}{(2x+1)}dx - \int (2x+1)^{-2}dx$

$= \frac{1}{2}\log(2x+1)+\frac{1}{2(2x+1)}+c$.

48. (a) $\int \frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx = \int \sec^2 x\,dx - \int \operatorname{cosec}^2 x\,dx$

$= \tan x+\cot x+c.$

49. (b) $\int (3\operatorname{cosec}^2 x+2\sin 3x)dx$

$= -3\cot x-\frac{2\cos 3x}{3}+c = -[3\cot x+\frac{2}{3}\cos 3x]+c.$

50. (a) $f(x) = \int f'(x)dx = \int \left(\frac{1}{x}+x\right)dx = \log x+\frac{x^2}{2}+c$

$x=1$ रखने पर, $\frac{5}{2} = 0+\frac{1}{2}+c \Rightarrow c = 2$

अतः $f(x) = \log x+\frac{x^2}{2}+2.$

51. (d) $\int \frac{dx}{\sqrt{1+x}+\sqrt{x}} = \int \left[\frac{(x+1)-x}{\sqrt{1+x}+\sqrt{x}}\right]dx$

$\int (\sqrt{x+1}-\sqrt{x})dx = \frac{(x+1)^{3/2}}{3/2}-\frac{x^{3/2}}{3/2}+c$

$= \frac{2}{3}[(x+1)^{3/2}-x^{3/2}]+c = \frac{2}{3}(x+1)^{3/2}-\frac{2}{3}x^{3/2}+c.$

52. (a) $\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx = \int \frac{2(\cos^2 x-\cos^2\alpha)}{\cos x-\cos\alpha}dx$

$= 2\int (\cos x+\cos\alpha)dx = 2(\sin x+x\cos\alpha).$

53. (c) $\int \frac{1-\tan x}{1+\tan x}dx = \int \tan\left(\frac{\pi}{4}-x\right)dx$

$= \int \frac{\sin\left(\frac{\pi}{4}-x\right)}{\cos\left(\frac{\pi}{4}-x\right)}dx = \log\cos\left(\frac{\pi}{4}-x\right)+c$

$= \log\sin\left(\frac{\pi}{4}+x\right)+c$.

54. (c) $\int a^x da = \frac{a^{x+1}}{x+1}+c.$

55. (c) यह स्पष्ट है।

56. (a) $\int x^{-4}dx = \frac{x^{-3}}{-3}+c = -\frac{1}{3x^3}+c.$

57. (c) $\int \frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}dx=\int \frac{x^5-x^4}{x^3-x^2}dx$

$=\int \frac{x^4(x-1)}{x^2(x-1)}dx=\int x^2dx=\frac{x^3}{3}+c$.

58. (a) $\int \frac{x^4+x^2+1}{x^2-x+1}dx=\int (x^2+x+1)dx=\frac{x^3}{3}+\frac{x^2}{2}+x+c.$

59. (b) $\int \sec x\,dx=\log(\sec x+\tan x)+c$

$=\log\left(\frac{1}{\sec x-\tan x}\right)+c=-\log(\sec x-\tan x)+c$.

60. (d) $\int \sqrt{1+\sin x}\,dx=\int \sqrt{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}dx$

$=\int \sin\frac{x}{2}dx+\int \cos\frac{x}{2}dx=-2\cos\frac{x}{2}+2\sin\frac{x}{2}+c$

$=-2\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)+c=-2\sqrt{(1-\sin x)}+c.$

61. (b) $\int \text{cosec}^2x\,dx=-\cot x+c$.

62. (a) $\int \left(2\sin x+\frac{1}{x}\right)dx=-2\cos x+\log x+c$.

63. (a) $I=\int \sqrt{1+\cos x}\,dx=\int \sqrt{2\cos^2(x/2)}dx$

$I=\sqrt{2}\int \cos(x/2)dx=2\sqrt{2}\sin(x/2)+c$.

64. (d) $I=\int 2\sin x.\cos x\,dx=\int \sin 2x\,dx$

$=-\frac{\cos 2x}{2}+c=-\frac{(1-2\sin^2 x)}{2}+c$

$=-\frac{1}{2}+\sin^2 x+c=\sin^2 x+c$.

65. (b) $\int \tan^2 x\,dx=\int (\sec^2 x-1)dx=\tan x-x+c$.

66. (b) $\int e^{\log(\sin x)}dx=\int \sin x\,dx=-\cos x+c.$

67. (b) $\int e^{x\log a}e^x dx=\int e^{\log a^x}.e^x dx=\int a^x e^x dx$

$=\int (ae)^x dx=\frac{(ae)^x}{\log(ae)}+C.$

68. (a) $\int \frac{1}{\sqrt{1+\cos x}}dx=\int \frac{dx}{\sqrt{2\cos^2(x/2)}}=\frac{1}{\sqrt{2}}\int \sec\frac{x}{2}dx$

$=\frac{1}{\sqrt{2}}\left\{\log\left(\sec\frac{x}{2}+\tan\frac{x}{2}\right)\right\}.\frac{1}{1/2}=\sqrt{2}\log\left(\sec\frac{x}{2}+\tan\frac{x}{2}\right)+K$.

69. (c) $\int \frac{\cos 2x-1}{\cos 2x+1}dx$

$\Rightarrow I=-\int \frac{(1-\cos 2x)}{(1+\cos 2x)}dx=-\int \frac{2\sin^2 x}{2\cos^2 x}dx$

$\Rightarrow I=-\int \tan^2 x\,dx=-\int (\sec^2 x-1)dx$

$\Rightarrow I=-\int \sec^2 x\,dx+\int 1\,dx=-\tan x+x+c$

$\Rightarrow I=x-\tan x+c$

70. (c) $I=\int \frac{ax^3+bx^2+c}{x^4}dx=\int\left[\frac{a}{x}+\frac{b}{x^2}+\frac{c}{x^4}\right]dx$

$=a\log x-\frac{b}{x}-\frac{c}{3x^3}+c$.

71. (b) $I=\int \frac{1}{(x-5)^2}dx=\frac{(x-5)^{-2+1}}{-2+1}+c=\frac{(x-5)^{-1}}{-1}+c$

$=-\frac{1}{(x-5)}+c$.

72. (a) **ट्रिक :** माना $f(x)=e^x$, तब $\int f(x)dx=f(x)$

$\therefore \int (e^x)^2dx=\frac{e^{2x}}{2}=\frac{1}{2}[f(x)]^2.$

73. (a) $I=\int \sqrt{2}\sqrt{1+\sin x}\,dx=\sqrt{2}\int\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)dx$

$=2\int \sin\left(\frac{\pi}{4}+\frac{x}{2}\right)dx=-4\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)+c$

तुलना करने पर, $a=\frac{1}{2},b=\frac{\pi}{4}.$

74. (a) यह स्पष्ट है।

75. (a) यह स्पष्ट है।

76. (b) यह स्पष्ट है।

77. (b) $\int(\sin^4 x-\cos^4 x)dx=\int(\sin^2 x-\cos^2 x)(\sin^2 x+\cos^2 x)dx$

$=\int(\sin^2 x-\cos^2 x)dx=-\int(\cos^2 x-\sin^2 x)dx$

$=-\int \cos 2x\,dx=\frac{-\sin 2x}{2}+c$.

78. (b) $\int \frac{(x+1)^2}{x(x^2+1)}dx=\int \frac{x^2+1+2x}{x(x^2+1)}dx$

$=\int \frac{x^2+1}{x(x^2+1)}dx+2\int \frac{x}{x(x^2+1)}dx$

$=\int \frac{dx}{x}+2\int \frac{dx}{x^2+1}=\log_e x+2\tan^{-1}x+c$.

79. (b) यहाँ $\int \frac{dx}{\sqrt{1-x}}$ या $I=\int (1-x)^{-1/2}dx$

$I=\frac{(1-x)^{\frac{-1}{2}+1}}{(-1)\left(-\frac{1}{2}+1\right)}+c \Rightarrow I=-2\sqrt{1-x}+c$.

80. (c) $\int \frac{dx}{1-x^2}=\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)+c=\frac{1}{2}\text{ In}\left|\frac{1+x}{1-x}\right|+c.$

81. (c) $\int \frac{dx}{4x^2+9}=\frac{1}{4}\int \frac{dx}{x^2+(3/2)^2}$

$=\frac{1}{4}.\frac{2}{3}.\tan^{-1}\left(\frac{2x}{3}\right)+c=\frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right)+c.$

82. (a) $\int \sqrt{1+x^2}dx=\frac{x}{2}\sqrt{x^2+1}+\frac{1}{2}\log(x+\sqrt{x^2+1})+c$.

83. (b) $I=\int \frac{dx}{\sqrt{x^2-a^2}}$

$x=a\sec\theta$ रखने पर $\Rightarrow dx=a\sec\theta.\tan\theta\,d\theta$

$\therefore\ I = \int \frac{a\sec\theta.\tan\theta\, d\theta}{a\tan\theta} = \int \sec\theta\, d\theta$

$= \log(\sec\theta + \tan\theta) + = \log\left(\frac{x}{a} + \frac{\sqrt{x^2 - a^2}}{a}\right) + c$

$= \log(x + \sqrt{x^2 - a^2}) + c$.

84. (a) $I = \int \frac{x^2}{x^2+4} dx = \int \frac{x^2+4-4}{(x^2+4)} dx$

$\Rightarrow I = \int \left[1 - \frac{4}{x^2+4}\right] dx = \int dx - \int \frac{4}{x^2+4} dx$

$\Rightarrow I = x - 4\int \frac{dx}{x^2 + (2)^2} = x - \frac{4}{2}\tan^{-1}(x/2) + c$

$\Rightarrow\ I = x - 2\tan^{-1}\frac{x}{2} + c$.

85. (b) $I = \int \sqrt{x^2+a^2}\, dx = \int \sqrt{x^2+a^2}\,.1\, dx$

$= \sqrt{x^2+a^2}\int 1dx - \int\left[\frac{d}{dx}\left(\sqrt{x^2+a^2}\right)\int 1\, dx\right] dx$

$= x\sqrt{x^2+a^2} - \int\left[\frac{2x}{2\sqrt{x^2+a^2}} x\right] dx$

$= x\sqrt{x^2+a^2} - \int\left[\frac{x^2+a^2-a^2}{\sqrt{x^2+a^2}}\right] dx$

$= x\sqrt{x^2+a^2} - \int\left[\sqrt{x^2+a^2} - \frac{a^2}{\sqrt{x^2+a^2}}\right] dx$

$= x\sqrt{x^2+a^2} - \int\sqrt{x^2+a^2}\, dx + a^2\int \frac{dx}{\sqrt{x^2+a^2}}$

$= \sqrt{x^2+a^2} - I + a^2\log\left[x + \sqrt{x^2+a^2}\right] + C$

$\Rightarrow\ 2I = x\sqrt{x^2+a^2} + a^2\log\left[x + \sqrt{x^2+a^2}\right] + C$

$\Rightarrow\ I = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left[x + \sqrt{x^2+a^2}\right] + C$

86. (c) यह सूत्र है।

87. (c) यह आधारभूत तथ्य है।

प्रतिस्थापन द्वारा समाकलन

1. (b) $\int \frac{dx}{1+e^x} = \int \frac{e^{-x}}{1+e^{-x}} dx$

$1 + e^{-x} = t$ रखने पर, $e^{-x}dx = -dt$

$\therefore\ -\int \frac{dt}{t} = -\log t = -\log(1+e^{-x}) + c.$

2. (b) $\int \frac{dx}{e^x + e^{-x}} = \int \frac{e^x}{e^{2x}+1} dx = \int \frac{dt}{t^2+1} = \tan^{-1}(t)$

$= \tan^{-1}(e^x) + c$, { $e^x = t$ रखने पर, $e^x dx = dt$}.

3. (a) $e^{\sqrt{x}} = t$ रखने पर, $\frac{e^{\sqrt{x}}}{\sqrt{x}} = 2dt$, (स्वयं हल करें).

4. (a) $\int \frac{dx}{x + x\ \log x} = \int \frac{dx}{x(1+\log x)}$

$1 + \log x = t$ रखने पर, $\frac{1}{x}dx = dt$

$\therefore\ \int \frac{dt}{t} = \log(t) = \log(1+\log x) + c.$

5. (b) माना $1 + \log x = t \Rightarrow \frac{1}{x}dx = dt$

अतः, $\int \frac{1+\log x}{x} dx = \int t\, dt = \frac{t^2}{2} = \frac{(1+\log x)^2}{2} + c.$

6. (a) $\int \frac{\sec x\, dx}{\sqrt{\cos 2x}} = \int \frac{\sec x}{\sqrt{\cos^2 x - \sin^2 x}} dx$

$= \int \frac{\sec^2 x\, dx}{\sqrt{1-\tan^2 x}}$ {अंश व हर को $\sec x$ से गुणा करने पर}

अब $\tan x = t$ रखने पर, $\sec^2 x\, dx = dt$,

अतः समाकलन का मान $= \sin^{-1} t = \sin^{-1}(\tan x) + c$

ट्रिक : चूँकि $\frac{d}{dx}\{\sin^{-1}(\tan x)\} = \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}$

$= \frac{\sec^2 x.\cos x}{\sqrt{\cos^2 x - \sin^2 x}} = \frac{\sec x}{\sqrt{\cos 2x}}$.

7. (d) $\int \frac{dx}{x\sqrt{2ax - x^2}}$ का मान ज्ञात करने के लिये उचित प्रतिस्थापन $2a\sin^2 t = x$ है।

8. (c) $\int \frac{x\, dx}{1 - x\cot x} = \int \frac{x\, dx}{1 - x\frac{\cos x}{\sin x}} = \int \frac{x\sin x}{\sin x - x\cos x} dx$

{ $\sin x - x\cos x = t$ रखने पर,

$[\cos x - (-x\sin x + \cos x)]dx = dt \Rightarrow x\sin x\, dx = dt$}

$= \int \frac{dt}{t} = \log t = \log(\sin x - x\cos x) + c.$

9. (b) $(1 + \sin^2 x) = t$ रखने पर, $\sin 2x\, dx = dt$

अतः $\int \frac{\sin 2x}{1+\sin^2 x} dx = \int \frac{1}{t} dt = \log(1+\sin^2 x) + c.$

10. (b) $\int \frac{x^3}{\sqrt{x^2+2}} dx = \int \frac{x^2.x}{\sqrt{x^2+2}} dx$

$x^2 + 2 = t^2$ रखने पर, $x\, dx = t\, dt$ और $x^2 = t^2 - 2$,

$\therefore\ \int \frac{(t^2-2)}{t} t\, dt = \int (t^2-2) dt$

$= \frac{t^3}{3} - 2t + c = \frac{(x^2+2)^{3/2}}{3} - 2(x^2+2)^{1/2} + c.$

11. (c) $x^e + e^x = t$ रखने पर, $e(x^{e-1} + e^{x-1})dx = dt$

अतः $\int \frac{x^{e-1} + e^{x-1}}{x^e + e^x} dx = \frac{1}{e}\int \frac{dt}{t} = \frac{1}{e}\log t = \frac{1}{e}\log(x^e + e^x) + c$

12. (c) $b\cos x = t$ रखकर हल करें।

13. (b) माना $\log(\sec x + \tan x) = t \Rightarrow \sec x\, dx = dt$

$\therefore \int \sec x \log(\sec x + \tan x) dx = \int t\, dt$

$$= \frac{t^2}{2} + c = \frac{[\log(\sec x + \tan x)]^2}{2} + c.$$

14. (a) $x^2 - 4x + 3 = t$ रखने पर, $(2x-4)dx = dt$

$$\Rightarrow (x-2)dx = \frac{1}{2}dt,$$

$$\therefore \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\log t + c = \frac{1}{2}\log(x^2 - 4x + 3) + c.$$

15. (b) माना $x^3 = t \Rightarrow 3x^2 dx = dt,$

$$\therefore \int \frac{3x^2}{x^6+1}dx = \int \frac{1}{t^2+1}dt = \tan^{-1}(t) + c = \tan^{-1}(x^3) + c.$$

16. (a) $\log \sin x = t$ रखकर हल करें।

17. (c) $(1 + \log x) = t$ रखने पर, $\frac{1}{x}dx = dt$

$$\therefore \int \frac{(1+\log x)^2}{x}dx = \int t^2 dt = \frac{t^2}{3} + c = \frac{(1+\log x)^3}{3} + c.$$

18. (b) $\sec x = t$ रखने पर, $\sec x \tan x\, dx = dt,$

$$\therefore \int \sec^p x \tan x\, dx = \int t^{p-1}dt = \frac{t^p}{p} + c = \frac{\sec^p x}{p} + c.$$

19. (a) $\int \frac{dx}{e^x - 1} = \int \frac{e^{-x}}{1 - e^{-x}}dx$

$1 - e^{-x} = t$ रखने पर, $e^{-x}dx = dt,$

$$\therefore \int \frac{dt}{t} = \log t + c = \log(1 - e^{-x}) + c.$$

20. (c) $x^3 = t$ रखने पर, $3x^2 dx = dt$

अब आगे स्वयं हल करें।

21. (b) $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x}dx$

$$= \int \frac{2\sin x \cos x}{\sin^4 x + \cos^4 x}dx = \int \frac{2\tan x \sec^2 x}{1 + \tan^4 x}dx$$

$\tan^2 x = t$ रखने पर, $2 \tan x \sec^2 x\, dx = dt,$

$$\therefore \int \frac{dt}{1+t^2} = \tan^{-1} t + c = \tan^{-1}(\tan^2 x) + c.$$

ट्रिक : जाँच द्वारा,

$$\frac{d}{dx}\{\cot^{-1}(\tan^2 x)\} = -\frac{1(2\tan x . \sec^2 x)}{1 + \tan^4 x} = -\frac{\sin 2x}{\cos^4 x + \sin^4 x}$$

$$\Rightarrow \frac{d}{dx}\left\{\tan^{-1}(\tan^2 x)\right\} = \frac{\sin 2x}{\sin^4 x + \cos^4 x}.$$

22. (b) $\int \frac{x-2}{x(2\log x - x)}dx = -\int \frac{\left(\frac{2}{x} - 1\right)}{(2\log x - x)}dx$

$(2\log x - x) = t$ रखने पर, $\left(\frac{2}{x} - 1\right)dx = dt,$

$$\therefore -\int \frac{1}{t}dt = -\log t = -\log(2\log x - x)$$

$$= \log\left(\frac{1}{2\log x - x}\right) + c.$$

23. (d) $1 + x^2 = t$ रखने पर, $x\, dx = \frac{dt}{2}$

$$\therefore \frac{1}{2}\int t^{1/2}dt = \frac{1}{2} \times \frac{t^{3/2}}{3/2} = \frac{1}{3}(1 + x^2)^{3/2} + c.$$

24. (a) $\int \frac{e^x(x+1)}{\cos^2(xe^x)} = \int e^x(x+1)\sec^2(xe^x)dx$

$xe^x = t$ रखने पर, $(x+1)e^x dx = dt$

$$\therefore \int \sec^2 t\, dt = \tan t + c = \tan(xe^x) + c.$$

25. (b) $\sqrt{x} = t$ रखने पर, $\frac{1}{2\sqrt{x}}dx = dt.$

अब आगे स्वयं हल करें।

26. (c) $\int \frac{x+1}{\sqrt{x^2+1}}dx = \int \frac{x}{\sqrt{x^2+1}}dx + \int \frac{1}{\sqrt{x^2+1}}dx$

$x^2 + 1 = t$ रखने पर, $2x\, dx = dt$

$$\therefore \frac{1}{2}\int \frac{dt}{t^{1/2}} + \int \frac{1}{\sqrt{x^2+1}}dx = \frac{1}{2}.2.t^{1/2} + \log(x + \sqrt{x^2+1}) + c$$

$$= (x^2+1)^{1/2} + \log(x + \sqrt{x^2+1}) + c.$$

27. (a) $a\cos^2 x + b\sin^2 x = t$ रखने पर $\Rightarrow 2(b-a)\sin x \cos x = dt,$

तब $\int \frac{\sin x \cos x\, dx}{a\cos^2 x + b\sin^2 x} = \frac{1}{2(b-a)}\int \frac{1}{t}dt$

$$= \frac{1}{2(b-a)}\log(a\cos^2 x + b\sin^2 x) + c.$$

28. (c) $t = \tan^{-1} x$ रखने पर, $dt = \frac{1}{1+x^2}dx$

अतः $\int \frac{e^{\tan^{-1} x}}{1+x^2}dx = \int e^t dt = e^t + c = e^{\tan^{-1} x} + c.$

29. (b) $\log x = t$ रखने पर, $\frac{1}{x}dx = dt,$ तब

$$\int \frac{1}{x(\log x)^2}dx = \int \frac{1}{t^2}dt = -\frac{1}{t} + c = -\frac{1}{\log x} + c.$$

30. (c) $\int \frac{1}{\sqrt{x}}\tan^4 \sqrt{x} . \sec^2 \sqrt{x}\, dx$

$\tan \sqrt{x} = t$ रखने पर, $\frac{\sec^2 \sqrt{x}}{2\sqrt{x}}dx = dt$

$$\therefore 2\int t^4 dt = \frac{2}{5}(\tan \sqrt{x})^5 + c = \frac{2}{5}\tan^5 \sqrt{x} + c.$$

31. (a) $a^x = t$ रखने पर, $a^x \log_e a\, dx = dt$ तब

$$\int \frac{a^x}{\sqrt{1 - a^{2x}}}dx = \frac{1}{\log_e a}\int \frac{dt}{\sqrt{1-t^2}}$$

$$= \frac{1}{\log_e a}\sin^{-1}(t) + c = \frac{\sin^{-1}(a^x)}{\log_e a} + c.$$

32. (b) $\int \frac{\sqrt{\tan x}}{\sin x \cos x}dx = \int \frac{\tan x}{\sqrt{\tan x}\sin x \cos x}dx$

$$= \int \frac{\sin x \sec x}{\sqrt{\tan x}\sin x \cos x}dx = \int \frac{\sec^2 x}{\sqrt{\tan x}}dx$$

$t = \tan x$ रखने पर, $dt = \sec^2 x\, dx$

$\therefore \int \frac{1}{\sqrt{t}}dt = 2t^{1/2} + c = 2\sqrt{\tan x} + c$.

33. (a) $a^2 + b^2 \sin^2 x = t$ रखने पर, $b^2 \sin 2x\, dx = dt$ तब

$$\int \frac{\sin 2x}{a^2 + b^2 \sin^2 x}dx = \frac{1}{b^2}\int \frac{dt}{t} = \frac{1}{b^2}\log t + c$$

$$= \frac{1}{b^2}\log(a^2 + b^2 \sin^2 x) + c.$$

34. (c) $t = 1 + \log x$ रखने पर, $dt = \frac{1}{x}dx$

अतः $\int \frac{dx}{x\sqrt{1+\log x}} = \int \frac{dt}{t^{1/2}} = 2t^{1/2} + c = 2(1+\log x)^{1/2} + c$.

35. (c) $t = 1 + \tan x$ रखने पर, $dt = \sec^2 x\, dx$

$$\therefore \int \frac{\sec^2 x}{1+\tan x}dx = \int \frac{1}{t}dt = \log t + c = \log(1+\tan x) + c.$$

36. (b) $\int \frac{e^{2x}-1}{e^{2x}+1}dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$

$e^x + e^{-x} = t$ रखने पर, $(e^x - e^{-x})dx = dt$

$$\therefore \int \frac{dt}{t} = \log t = \log(e^x + e^{-x}) = \log(e^{2x}+1) - x + c.$$

37. (a) $\log \tan \frac{x}{2} = t$ रखने पर, $\frac{1}{\tan \frac{x}{2}} \cdot \frac{1}{2}\sec^2 \frac{x}{2}dx = dt$

$\Rightarrow \text{cosec } x\, dx = dt,$

$$\therefore \int \frac{\text{cosec } x}{\log \tan \frac{x}{2}}dx = \int \frac{1}{t}dt = \log t + c = \log\left(\log \tan \frac{x}{2}\right) + c$$

38. (b) $\int \frac{1}{\cos^2 x(1-\tan x)^2}dx = \int \frac{\sec^2 x\, dx}{(\tan x - 1)^2}$

$\tan x - 1 = t$ रखने पर, $\sec^2 x\, dx = dt$

$$\therefore \int \frac{1}{t^2}dt = \frac{-1}{\tan x - 1} + c = \frac{1}{1-\tan x} + c.$$

39. (b) $x^{10} + 10^x = t$ रखने पर, $(10x^9 + 10^x \log_e 10)dx = dt$

तब $\int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}}dx = \int \frac{1}{t}dt = \log t + c$

$$= \log(x^{10} + 10^x) + c.$$

40. (a) $\int \frac{1}{(e^x + e^{-x})^2}dx = \int \frac{e^{2x}}{(e^{2x}+1)^2}dx$

$e^{2x} + 1 = t$ रखने पर, $2e^{2x}dx = dt$

$$\therefore \frac{1}{2}\int \frac{1}{t^2}dt = -\frac{1}{2}\cdot\frac{1}{t} + c = -\frac{1}{2(e^{2x}+1)} + c.$$

41. (c) $\int \frac{\cos 2x}{(\cos x + \sin x)^2}dx = \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)^2}dx$

$$= \int \frac{\cos x - \sin x}{\cos x + \sin x}dx$$

$t = \sin x + \cos x$ रखने पर, $dt = (\cos x - \sin x)dx$

$$\therefore \int \frac{1}{t}\, dt = \log t + c = \log(\sin x + \cos x) + c.$$

42. (c) $\log x = t$ रखने पर, $\frac{1}{x}dx = dt$

$$\therefore \int \frac{\tan(\log x)}{x}dx = \int \tan t\, dt$$

$$= \log \sec t + c = \log \sec(\log x) + c.$$

43. (b) $\int \cos^3 x\, e^{\log \sin x}dx = \int \cos^3 x \sin x\, dx$

$= -\int t^3 dt = -\frac{t^4}{4} + c = -\frac{\cos^4 x}{4} + c$, {$t = \cos x$ रखने पर}.

44. (b) $t = 3x - 5$ रखने पर, $dt = 3dx$

$$\therefore \int \tan(3x-5)\sec(3x-5)dx = \frac{1}{3}\int \tan t.\sec t\, dt$$

$$= \frac{\sec t}{3} + c = \frac{\sec(3x-5)}{3} + c.$$

45. (b) $t = x^2$ रखने पर, $dt = 2x\, dx$

$$\therefore \int \frac{x}{1+x^4}dx = \frac{1}{2}\int \frac{1}{1+t^2}dt = \frac{1}{2}\tan^{-1} t + c = \frac{1}{2}\tan^{-1} x^2 + c.$$

46. (a) $\int \frac{e^{-x}}{1+e^x}dx = \int \frac{e^{-x}e^{-x}}{e^{-x}+1}dx$

$e^{-x} + 1 = t$ रखने पर, $-e^{-x}dx = dt$

$$\therefore -\int \frac{(t-1)}{t}dt = \int \left(\frac{1}{t} - 1\right)dt$$

$$= \log t - t + c = \log(e^{-x}+1) - (e^{-x}+1) + c$$

$$= \log(e^x + 1) - x - e^{-x} - 1 + c$$

$$= \log(e^x + 1) - x - e^{-x} + c, \quad (\because 1 = \text{अचर}).$$

47. (a) $\int \frac{1}{\sqrt{1-e^{2x}}}dx = \int \frac{e^{-x}}{\sqrt{e^{-2x}-1}}dx$

$e^{-x} = t$ रखने पर, $-e^{-x}dx = dt$

$$\therefore -\int \frac{1}{\sqrt{t^2-1}}dt = -\log\left[t + \sqrt{t^2-1}\right] + c$$

$$= -\log\left[e^{-x} + \sqrt{e^{-2x}-1}\right] = -\log\left[\frac{1}{e^x} + \frac{\sqrt{1-e^{2x}}}{e^x}\right]$$

$$= -\log\left[1 + \sqrt{1-e^{2x}}\right] + \log e^x + c$$

$$= x - \log\left[1 + \sqrt{1-e^{2x}}\right] + c.$$

48. (a) $\int \frac{3x^2}{\sqrt{9-16x^6}}dx = \int \frac{3x^2}{\sqrt{(3)^2 - (4x^3)^2}}dx$

$4x^3 = t$ रखने पर, $12x^2dx = dt$

$$\therefore \frac{1}{4}\int \frac{dt}{\sqrt{(3)^2 - t^2}} = \frac{1}{4}\cdot\frac{1}{1}\sin^{-1}\left(\frac{t}{3}\right) + c = \frac{1}{4}\sin^{-1}\left(\frac{4x^3}{3}\right) + c.$$

49. (b) $\sin x = t$ रखने पर, $\cos x\, dx = dt$

$$\therefore \int \cos x\sqrt{4-\sin^2 x}\, dx = \int \sqrt{4-t^2}dt = \int \sqrt{(2)^2 - t^2}dt$$

$$= \frac{t}{2}\sqrt{4-t^2} + \frac{4}{2}\sin^{-1}\frac{t}{2} + c$$

$$= \frac{1}{2}\sin x\sqrt{4-\sin^2 x} + 2\sin^{-1}\left(\frac{1}{2}\sin x\right) + c.$$

50. (b) $\int x^2(3)^{x^3+1}dx = \int 3x^2.(3)^{x^3}dx$

$x^3 = t$ रखने पर, $3x^2\,dx = dt$

$$= \int 3^t dt = \frac{3^t}{\log 3} + c = \frac{(3)^{x^3}}{\log 3} + c\,.$$

51. (b) $\int \sec^{2/3} x\,\text{cosec}^{4/3} x\,dx = \int \frac{dx}{\sin^{4/3} x\cos^{2/3} x}$

अंश व हर को $\cos^2 x$, से गुणा करने पर,

$\{\tan x = t$ रखने पर, $\sec^2 x\,dx = dt\}$

$$\therefore \int \frac{\sec^2 x\,dx}{\tan^{4/3} x} = \int \frac{dt}{t^{4/3}} = \frac{t^{-1/3}}{(-1/3)} + c = -3(\tan x)^{-1/3} + c$$

52. (a) $\int \cos^5 x\,dx = \int \cos^4 x\cos x\,dx = \int (1-\sin^2 x)^2\cos x dx$

$\sin x = t$ रखने पर, $\cos x\,dx = dt$

$$\therefore \int (1-t^2)^2 dt = \int (1+t^4-2t^2)\,dt = \frac{t^5}{5} - \frac{2t^3}{3} + t + c$$

$$= \frac{\sin^5 x}{5} - \frac{2\sin^3 x}{3} + \sin x + c\,.$$

53. (a) $\int \sec x\tan^3 x\,dx = \int \sec x(\sec^2 x - 1)\tan x\,dx$

$$= \int \sec x\tan x\sec^2 x\,dx - \int \sec x\tan x\,dx$$

$$= \frac{\sec^3 x}{3} - \sec x + c\,,$$ (प्रथम भाग में $\sec x = t$ रखने पर).

54. (c) $\int \frac{d\theta}{\sin\theta\cos^3\theta} = \int \frac{\sec^2\theta\,d\theta}{\sin\theta\cos\theta} = \int \frac{\sec^2\theta(1+\tan^2\theta)}{\tan\theta}d\theta$

$t = \tan\theta$ रखने पर, $dt = \sec^2\theta\,d\theta$

$$\therefore \int \frac{1+t^2}{t}dt = \int \left(\frac{1}{t} + t\right)dt$$

$$= \log t + \frac{t^2}{2} + c = \log\tan\theta + \frac{\tan^2\theta}{2} + c.$$

55. (b) $\cos^{-1} x = t$ रखने पर, $-\frac{1}{\sqrt{1-x^2}}dx = dt$

तब $\int \frac{1}{\cos^{-1}\sqrt{1-x^2}}dx = -\int \frac{1}{t}dt = -\log t + c = \log\frac{1}{t} + c$

$$= -\log(\cos^{-1} x) + c.$$

56. (b) $\int x^3 e^{3x^2+5}dx$

$(3x^2+5) = t$ सरलतम प्रतिस्थापन है

$t = 3x^2 + 5$ रखने पर, $dx = \frac{dt}{6x}$

तब $\int x^3 e^{3x^2+5}dx = \frac{1}{6}\int \left(\frac{t-5}{3}\right)e^t dt$

$$= \frac{1}{18}\int [te^t - 5e^t]dt = \frac{1}{18}\int te^t dt - \frac{5}{18}\int e^t dt$$

$$= \frac{1}{18}\left[te^t - \int e^t dt\right] - \frac{5}{18}\int e^t dt + c = \frac{1}{18}(te^t) - \frac{1}{3}e^t + c$$

$$= \frac{1}{18}(3x^2+5)e^{3x^2+5} - \frac{1}{3}e^{3x^2+5} + c.$$

57. (c) सबसे उपयुक्त प्रतिस्थापन $\tan x = t$ है।

58. (b) $1 + \cot x = t$ रखने पर, $\text{cosec}^2 x\,dx = -dt$

तब $\int \frac{\text{cosec}^2 x}{1+\cot x}dx = -\int \frac{1}{t}dt = -\log t + c = -\log(1+\cot x) + c$

59. (b) $\sqrt{x} = t$ रखने पर $\Rightarrow \frac{1}{\sqrt{x}}dx = 2dt,$

$$\therefore 2\int \sin t\,dt = -2\cos t + c = -2\cos\sqrt{x} + c.$$

60. (d) $e^x = t$ रखने पर, $e^x dx = dt$

तब $\int e^x\tan^2(e^x)dx = \int \tan^2 t\,dt = \int (\sec^2 t - 1)dt$

$$= \tan t - t + c = \tan(e^x) - e^x + c.$$

61. (a) $\int \frac{dx}{e^{-2x}(e^{2x}+1)^2} = \int \frac{e^{2x}dx}{(e^{2x}+1)^2}$

$t = e^{2x} + 1$ रखने पर, $\frac{dt}{2} = e^{2x}dx$

$$\therefore \frac{1}{2}\int \frac{1}{t^2}dt = -\frac{1}{2t} + c = \frac{-1}{2(e^{2x}+1)} + c.$$

62. (b) $\int \tan^4 x\,dx = \int \tan^2 x(\sec^2 x - 1)dx$

$$= \int \tan^2 x\sec^2 x\,dx - \int \tan^2 x\,dx = \frac{\tan^3 x}{3} - \tan x + x + c\,.$$

63. (c) $\log x = t$ रखने पर, $\frac{1}{x}dx = dt$

$$\therefore \int \frac{dt}{\sqrt{1-t^2}} = \sin^{-1} t = \sin^{-1}(\log x) + c.$$

64. (a) $f(x) = t$ रखने पर, $f'(x)dx = dt$

तब $\int \frac{f'(x)}{|f(x)|^2}dx = \int \frac{1}{t^2}dt = -\frac{1}{t} + c = -\frac{1}{f(x)} + c.$

65. (c) $\int x^3\cos x^2\,dx$ के लिये $x^2 = t$ उपयुक्त प्रतिस्थापन है

$$= \frac{1}{2}\int t\cos t\,dt = \frac{1}{2}(t\sin t - \int \sin t\,dt + c)$$

$$= \frac{1}{2}(t\sin t + \cos t + c) = \frac{1}{2}(x^2\sin x^2 + \cos x^2 + c).$$

66. (a) $\int \tan x\,.\sec^2 x\sqrt{1-\tan^2 x}\,dx$

माना $\tan x = t \Rightarrow \sec^2 x\,dx = dt\,, \therefore \int t\sqrt{1-t^2}\,dt$

पुनः $1 - t^2 = u$ रखने पर, $-2tdt = du$

$$\therefore -\frac{1}{2}\int \sqrt{u}\,du = -\frac{1}{3}u^{3/2} + c = -\frac{1}{3}(1-\tan^2 x)^{3/2} + c.$$

67. (c) $\int \frac{\sin 2x}{\sin 5x\sin 3x}dx = \int \frac{\sin(5x-3x)}{\sin 5x\sin 3x}dx$

$$= \int \frac{\sin 5x \cos 3x - \cos 5x \sin 3x}{\sin 5x \sin 3x} dx$$

$$= \frac{1}{3}\log \sin 3x - \frac{1}{5}\log \sin 5x + c.$$

68. (b) $e^x = t$ रखने पर $\Rightarrow e^x dx = dt$, तब

$$\int \frac{e^x dx}{\sqrt{1-e^{2x}}} = \int \frac{dt}{\sqrt{1-t^2}} = -\cos^{-1} t + c = -\cos^{-1}(e^x) + c.$$

69. (c) $\int \frac{1}{\log a}(a^x \cos a^x) dx$

अब $a^x = t$ रखने पर $\Rightarrow a^x dx = \frac{dt}{\log a}$,

$$\therefore \int \frac{1}{(\log a)^2} \cos t \, dt = \frac{1}{(\log a)^2} \sin t + c = \frac{1}{(\log a)^2} \sin a^x + c.$$

70. (b) $a + b\cos x = t$ रखने पर $\Rightarrow dx = -\frac{dt}{b \sin x}$

तब $\int \frac{\sin x}{(a + b\cos x)^2} dx = -\frac{1}{b}\int \frac{1}{t^2} dt = \frac{1}{b}\frac{1}{t} + c$

$$= \frac{1}{b(a + b\cos x)} + c.$$

71. (b) $\int \frac{1}{x^3}[\log x^x]^2 \, dx = \int \frac{1}{x^3}[x \log x]^2 dx$

$= \int \frac{1}{x}(\log x)^2 dx = \frac{1}{3}(\log x)^3 + c$, {$\log x = t$ रखने पर}.

72. (a) $t = \log x$ रखने पर $\Rightarrow x\, dt = dx$

तब $\int \frac{1}{x}\sec^2(\log x) dx = \int \sec^2 t \, dt = \tan t + c$

$$= \tan(\log x) + c.$$

73. (b) $\int \frac{dx}{x \log x . \log(\log x)}$

अब $\log x = t$ रखने पर, $\int \frac{dt}{t . \log(t)}$

पुनः $\log t = z$ रखने पर,

$$\int \frac{dz}{z} = \log z = \log[\log(\log x)] + c.$$

74. (a) $t = \tan x$ रखने पर $\Rightarrow dt = \sec^2 x \, dx$

तब $\int \frac{\sec^2 x \, dx}{\sqrt{\tan^2 x + 4}} = \int \frac{1}{\sqrt{t^2 + 2^2}} dt$

$$= \log[\tan x + \sqrt{\tan^2 x + 4}] + c.$$

75. (b) $t = \tan^{-1} x^2$ रखने पर $\Rightarrow dt = \frac{1}{1+x^4} 2x \, dx$, तब

$$\int \frac{2x \tan^{-1} x^2}{1+x^4} dx = \int t \, dt = \frac{t^2}{2} + c = \frac{1}{2}(\tan^{-1} x^2)^2 + c.$$

76. (b) $\sqrt{x} = t$ रखने पर $\Rightarrow \frac{1}{2}\frac{1}{\sqrt{x}} dx = dt$, तब

$$\int \frac{a^{\sqrt{x}}}{\sqrt{x}} dx = 2\int a^t dt = \frac{2a^t}{\log_e a} + c = 2a^{\sqrt{x}} \log_a e + c.$$

77. (c) $\int \frac{x^3}{\sqrt{1-x^8}} dx = \int \frac{x^3}{\sqrt{1-(x^4)^2}}$

$x^4 = t$ रखने पर $\Rightarrow 4x^3 dx = dt$,

$$\therefore \frac{1}{4}\int \frac{dt}{\sqrt{1-x^2}} = \frac{1}{4}[\sin^{-1}(t)] + c = \frac{1}{4}\sin^{-1}(x^4) + c.$$

78. (a) $t = \cos x^2$ रखने पर $\Rightarrow dt = -2x \sin x^2 dx$, तब

$$\int 2x \cos^3 x^2 \sin x^2 dx = -\int t^3 dt = -\frac{t^4}{4} + c$$

$$= -\frac{1}{4}\cos^4 x^2 + c.$$

79. (a) $\int \sec^4 x \tan x \, dx = \int \sec^3 x \sec x \tan x \, dx$

$t = \sec x$ रखने पर $\Rightarrow dt = \sec x \tan x \, dx$, तब

$$\int t^3 dt = \frac{t^4}{4} + c = \frac{1}{4}\sec^4 x + c.$$

80. (a) $2e^{-x} + 5$ रखने पर $= t \Rightarrow -2e^{-x} dx = dt$, तब

$$\int e^{-x}\text{cosec}^2(2e^{-x} + 5) dx = -\frac{1}{2}\int \text{cosec}^2 t \, dt$$

$$= \frac{1}{2}\cot t = \frac{1}{2}\cot(2e^{-x} + 5) + c.$$

81. (b) $\int \sin^3 x . \cos x \, dx$

$\sin x = t$ रखने पर, $\cos x \, dx = dt$

$$\therefore \int t^3 dt = \frac{t^4}{4} = \frac{\sin^4 x}{4} + c.$$

82. (b) $t = 3x + 3$ रखने पर $\Rightarrow dt = 3\, dx$, तब

$$\int a^{3x+3} dx = \frac{1}{3}\int a^t dt = \frac{1}{3}\frac{a^t}{\log_e a} + c = \frac{a^{3x+3}}{3\log_e a} + c.$$

83. (c) माना $x^2 + \sin 2x + 2x = t$

$$\therefore \frac{1}{2}\int \frac{1}{t} dt = \frac{1}{2}\log t + c = \frac{1}{2}\log(x^2 + \sin 2x + 2x) + c.$$

84. (a) $t = x + \log \sec x$ रखने पर $\Rightarrow dt = (1 + \tan x) dx$, तब

$$\int \frac{1 + \tan x}{x + \log \sec x} dx = \int \frac{1}{t} dt = \log t + c$$

$$= \log(x + \log \sec x) + c.$$

85. (c) $t = x + \log x$ रखने पर $\Rightarrow dt = \left(1 + \frac{1}{x}\right) dx$, तब

$$\int \frac{(x+1)(x + \log x)^2}{x} dx = \int t^2 dt = \frac{t^3}{3} + c$$

$$= \frac{1}{3}(x + \log x)^3 + c.$$

86. (a) $x = \sin\theta$ रखने पर $\Rightarrow dx = \cos\theta \, d\theta$,

$$\therefore \int (1 + \sin^2\theta) d\theta = \theta + \frac{1}{2}\int (1 - \cos 2\theta) d\theta$$

$$= \frac{3\theta}{2} - \frac{1}{2}\sin\theta\sqrt{1 - \sin^2\theta} + c$$

$$= \frac{3}{2}\sin^{-1} x - \frac{1}{2}x\sqrt{1 - x^2} + c.$$

87. (a) $\int \frac{\cos x - \sin x}{1 + \sin 2x} dx = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx$

$\sin x + \cos x = t$ रखने पर $\Rightarrow (\cos x - \sin x)dx = dt$

अभीष्ट समाकलन $= -\frac{1}{\sin x + \cos x} + c$.

88. (c) माना $3 + 5x^4 = t \Rightarrow 20x^3 dx = dt$, तब

$$\int x^3\sqrt{3+5x^4}\,dx = \frac{1}{20}\int t^{1/2}dt$$

$$= \frac{2}{3}\times\frac{1}{20}.t^{3/2} + c = \frac{1}{30}(3+5x^4)^{3/2} + c.$$

89. (b) $x = a(\sin\theta)^{2/3}$ रखने पर $\Rightarrow dx = \frac{2}{3}a(\sin\theta)^{-1/3}\cos\theta\, d\theta$

$$\therefore \int \sqrt{\frac{x}{a^3 - x^3}}\,dx = \int \frac{a^{1/2}(\sin\theta)^{1/3}\frac{2}{3}a(\sin\theta)^{-1/3}\cos\theta}{\sqrt{a^3 - a^3\sin^2\theta}}d\theta$$

$$= \frac{2}{3}a^{3/2}\int \frac{\cos\theta\, d\theta}{a^{3/2}\sqrt{1-\sin^2\theta}} = \frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2} + c.$$

90. (a) $1 + \log x = t$ रखने पर $\Rightarrow \frac{1}{x}dx = dt$, तब

$$\int \frac{1}{x\cos^2(1+\log x)}dx = \int \frac{dt}{\cos^2 t} = \int \sec^2 t\, dt$$

$$= \tan t + c = \tan(1+\log x) + c.$$

91. (a) $x = \tan\theta$ रखने पर $\Rightarrow dx = \sec^2\theta\, d\theta$, तब

$$\int \frac{1}{x^2\sqrt{1+x^2}}dx = \int \frac{\sec^2\theta\, d\theta}{\tan^2\theta\sec\theta} = \int \text{cosec}\,\theta\cot\theta\, d\theta$$

$$= -\text{cosec}\,\theta + c = \frac{-\sqrt{x^2+1}}{x} + c.$$

92. (c) $x = \tan\theta$ रखने पर $\Rightarrow dx = \sec^2\theta\, d\theta$, तब

$$\int \frac{dx}{(x^2-1)\sqrt{x^2+1}} = \int \frac{\sec^2\theta\, d\theta}{(\tan^2\theta - 1)\sec\theta} = \int \frac{\cos\theta\, d\theta}{(2\sin^2\theta - 1)}$$

पुनः $t = \sin\theta$ रखने पर $\Rightarrow dt = \cos\theta\, d\theta$,

$$\therefore \int \frac{dt}{(2t^2-1)} = \frac{1}{2}\int \frac{dt}{t^2 - \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{1}{2\sqrt{2}}\log\left(\frac{t - \frac{1}{\sqrt{2}}}{t + \frac{1}{\sqrt{2}}}\right) + c$$

$$= \frac{1}{2\sqrt{2}}\log\left(\frac{\sqrt{1+x^2} - x\sqrt{2}}{\sqrt{1+x^2} + x\sqrt{2}}\right) + c.$$

93. (a) $\log(x + \sqrt{1+x^2}) = t$ रखने पर $\Rightarrow \frac{1}{\sqrt{1+x^2}}dx = dt$, तब

$$\int \frac{\log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}dx = \int t\, dt$$

$$= \int \frac{t^2}{2}dt = \frac{1}{2}\left[\log(x+\sqrt{1+x^2})\right]^2 + c.$$

94. (a) $\int e^x \sin(e^x)dx$

अब $e^x = t$ रखने पर, $e^x dx = dt$

$$\therefore \int \sin t\, dt = -\cos t + c = -\cos(e^x) + c.$$

95. (d) यहाँ $x^5 = x^3x^2$ और x^3 का अवकल गुणांक $3x^2$ है। भिन्न घातांकों को हटाने के लिये

$1 + x^3 = t^2 \Rightarrow 3x^2dx = 2t.dt$; साथ ही $x^3 = t^2 - 1$

अतः $I = \int \frac{(t^2-1)}{t}\left(\frac{2}{3}t\,dt\right) = \frac{2}{3}\int (t^2-1)dt$

$$= \frac{2}{3}\left(\frac{t^3}{3} - t\right) = \frac{2}{9}t(t^2-3) = \frac{2}{9}\sqrt{(1+x^3)}(x^3-2)$$

$(\because t^2 = 1 + x^3)$.

96. (a) $\int \frac{(x^4-x)^{1/4}}{x^5}dx = \int \frac{1}{x^4}\left(1 - \frac{1}{x^3}\right)^{1/4}dx$

$$= \frac{1}{3}\int t^{1/4}dt = \frac{4}{15}t^{5/4} + c = \frac{4}{15}\left(1 - \frac{1}{x^3}\right)^{5/4} + c$$

$\left\{1 - \frac{1}{x^3} = t \text{ रखने पर}, \frac{1}{x^4}dx = \frac{1}{3}dt\right\}$.

97. (a) $\int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}}dx = \int \frac{1}{\left(\frac{x-1}{x+2}\right)^{3/4}(x+2)^2}dx$

$$= \frac{1}{3}\int \frac{1}{t^{3/4}}dt, \qquad \left\{\because \frac{x-1}{x+2} = t \Rightarrow \frac{3}{(x+2)^2}dx = dt\right\}$$

$$= \frac{1}{3}\left(\frac{t^{1/4}}{1/4}\right) + c = \frac{4}{3}t^{1/4} + c = \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c.$$

98. (a) $I = \int \frac{1}{1+\sin^2 x}dx = \int \frac{dx}{2\sin^2 x + \cos^2 x}$

$$= \int \frac{\sec^2 x\, dx}{2\tan^2 x + 1} = \frac{1}{2}\int \frac{\sec^2 x\, dx}{\tan^2 x + \frac{1}{2}}$$

$\tan x = t$ रखने पर $\Rightarrow \sec^2 x\, dx = dt$, तब

$$I = \frac{1}{2}\int \frac{dt}{t^2 + \frac{1}{2}} = \frac{1}{2}.\frac{1}{1/\sqrt{2}}\tan^{-1}\frac{t}{1/\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2}\tan x) + k.$$

99. (c) दिया है, $I = \int \frac{\sin x}{\cos^2 x}dx$

$\cos x = t$ रखने पर, $\Rightarrow \sin x\, dx = -dt$

$$\therefore I = -\int \frac{dt}{t^2} = \frac{1}{t} + k = \sec x + k.$$

100. (a) $I = \int e^x \sec^2(e^x)dx$

$e^x = t$ रखने पर $\Rightarrow e^x dx = dt$

$$\therefore I = \int \sec^2 t\, dt = \tan t + k = \tan(e^x) + k.$$

101. (a) $I = \int \frac{dx}{x\sqrt{x^4-1}}$

$x^2 = t$ रखने पर $\Rightarrow 2x\, dx = dt \Rightarrow dx = \frac{dt}{2x} = \frac{dt}{2\sqrt{t}}$

$$\therefore I = \int \frac{dt}{2t\sqrt{t^2-1}} = \frac{1}{2}\sec^{-1}t + k = \frac{1}{2}\sec^{-1}x^2 + k$$

102. (d) $I = \int t.e^{-3t^2} dt$

$-3t^2 = z$ रखने पर $\Rightarrow -6t\,dt = dz \Rightarrow t\,dt = \frac{-1}{6}dz$

$\therefore\ I = -\frac{1}{6}\int e^z dt = \frac{-e^z}{6} + c = -\frac{e^{-3t^2}}{6} + c.$

103. (b) $I = \int \frac{dx}{\sqrt{x}(1+(\sqrt{x})^2)}$

$\sqrt{x} = t$ रखने पर $\Rightarrow \frac{1}{2\sqrt{x}}dx = dt$

$\therefore\ I = \int \frac{2\,dt}{1+t^2} = 2\tan^{-1} t + A$

अतः $I = 2\tan^{-1}\sqrt{x} + A$; $\therefore\ f(x) = 2\tan^{-1}\sqrt{x}$.

104. (d) $x^2 = t$ रखने पर $\Rightarrow dt = 2x\,dx$

$\therefore$ दिया गया समाकलन

$= \frac{1}{2}\int \cos t\,dt = \frac{1}{2}\sin t = \frac{1}{2}\sin x^2 + c$.

105. (b) $x^3 = t$ रखने पर $\Rightarrow dt = 3x^2\,dx$

अब $\int \frac{x^2 \tan^{-1} x^3\,dx}{1+x^6} = \frac{1}{3}\int \frac{\tan^{-1} t}{1+t^2}dt$

$z = \tan^{-1} t$ रखने पर $\Rightarrow dz = \frac{dt}{1+t^2}$

$\therefore \frac{1}{3}\int z\,dz = \frac{1}{3}\frac{z^2}{2} = \frac{z^2}{6} = \frac{1}{6}(\tan^{-1} x^3)^2 + c$.

106. (a) $I = \int \frac{x^2+1}{x(x^2-1)}dx = \int \frac{1+\left(\frac{1}{x^2}\right)}{x-\left(\frac{1}{x}\right)}dx$

$x - \frac{1}{x} = t$ रखने पर $\Rightarrow (1+\frac{1}{x^2})dx = dt$

$\therefore\ I = \int \frac{dt}{t} = \log t + c = \log\frac{x^2-1}{x} + c$.

107. (a) $I = \int \frac{e^{2x}+1}{e^{2x}-1} = \int \frac{e^x + e^{-x}}{e^x - e^{-x}}dx$

$e^x - e^{-x} = t$ रखने पर $\Rightarrow (e^x + e^{-x})dx = dt$

$\therefore\ I = \int \frac{dt}{t}dt = \log t + c = \log(e^x - e^{-x}) + c$.

108. (a) $I = \int \frac{\cos x - \sin x}{\sqrt{\sin 2x}}dx = \int \frac{\cos x - \sin x}{\sqrt{(\sin x + \cos x)^2 - 1}}dx$

$\sin x + \cos x = t$ रखने पर $\Rightarrow (\cos x - \sin x)\,dx = dt$

$\therefore\ I = \int \frac{dt}{\sqrt{t^2-1}} = \cosh^{-1} t + c = \cosh^{-1}(\sin x + \cos x) + c$.

109. (a) $I = \int \left(1+\frac{1}{x^2}\right)e^{x-\frac{1}{x}}dx$

$x - \frac{1}{x} = t$ रखने पर $\Rightarrow \left(1+\frac{1}{x^2}\right)dx = dt$

$\therefore\ I = \int e^t dt = e^t + c = e^{x-\frac{1}{x}} + c$.

110. (a) $\int (x+3)(x^2+6x+10)^9\,dx$

$= \frac{1}{2}\int (2x+6)(x^2+6x+10)^9\,dx$

$= \frac{1}{2}\frac{(x^2+6x+10)^{10}}{10} + c = \frac{1}{20}(x^2+6x+10)^{10} + c$.

111. (c) $f(x) = \frac{x}{1+x^2}$, $\therefore\ I = \int f(x) = \int \frac{x}{1+x^2}dx$

$1+x^2 = t$ रखने पर

$\Rightarrow 2x\,dx = dt \Rightarrow x\,dx = dt/2$

$\therefore\ I = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\log t + c$; $I = \frac{1}{2}\log(1+x^2) + c$.

112. (c) $\int \sin^3 x\,dx = \int \sin^2 x.\sin x\,dx$

$= \int \sin x(1-\cos^2 x)dx$

$= \int \sin x\,dx - \int \cos^2 x.\sin x\,dx = -\cos x + \frac{\cos^3 x}{3}$.

113. (b) $I = \int \frac{1}{x}\log x\,dx$

$\log x = t$ रखने पर $\Rightarrow \frac{1}{x}dx = dt$

$\therefore\ I = \int t\,dt = \frac{t^2}{2} + c = \frac{(\log x)^2}{2} + c$.

114. (c) $I = \int \sin^2 x.\cos x\,dx$

$\sin x = t$ रखने पर $\Rightarrow \cos x\,dx = dt$

$\therefore\ I = \int t^2 dt = \frac{t^3}{3} + c = \frac{\sin^3 x}{3} + c$.

115. (b) $\int e^{x^2}.x\,dx = \frac{1}{2}\int (2x)e^{x^2}dx$

$x^2 = t$ रखने पर, $dt = 2x\,dx$

$= \frac{1}{2}\int e^t dt = \frac{1}{2}e^t = \frac{1}{2}e^{x^2}$.

116. (c) $\int \frac{x^3}{\sqrt{1+x^4}}dx = \frac{1}{4}\int \frac{4x^3}{\sqrt{1+x^4}}dx$

$1+x^4 = t$ रखने पर

$\Rightarrow\ 4x^3 dx = dt$

$= \frac{1}{4}\int \frac{dt}{t^{1/2}} = \frac{1}{4}\frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \frac{1}{2}\sqrt{t} = \frac{1}{2}\sqrt{1+x^4} + c.$

117. (a) $I = \int \frac{dx}{(1+e^x)\left(1+\frac{1}{e^x}\right)} = \int \frac{e^x dx}{(1+e^x)^2}$

माना $1+e^x = t$, $\therefore e^x\,dx = dt$

$\therefore I = \int \frac{dt}{t^2} = \int t^{-2}dt = \frac{t^{-1}}{-1} = \frac{(1+e^x)^{-1}}{-1} = \frac{-1}{1+e^x}$.

118. (c) $I = \int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$

$\sqrt{x} = t$ रखने पर, $\therefore \frac{1}{2\sqrt{x}}dx = dt$

अतः $I = 2\int e^t dt = 2e^t + C = 2e^{\sqrt{x}} + C$.

119. (d) $I = \int \frac{\sin^3 2x}{\cos^5 2x} dx$

$\Rightarrow I = \int \frac{\sin^3 2x}{\cos^3 2x} . \frac{1}{\cos^2 2x} dx = \int \tan^3 2x . \sec^2 2x \, dx$.

$\tan 2x = t$ रखने पर $\Rightarrow 2\sec^2 2x \, dx = dt$

$\therefore I = \int t^3 \frac{dt}{2} = \frac{1}{2} . \frac{t^4}{4} + C = \frac{1}{8}(\tan^4 2x) + C$.

120. (a) $I = \int x^x (1 + \log x) dx$

$x^x = t$ रखने पर $\Rightarrow x^x (1 + \log x) dx = dt$

$\therefore I = \int dt \Rightarrow I = t + C \Rightarrow I = x^x + C$.

121. (b) $I = \int \frac{dx}{(a^2 + x^2)^{3/2}}$

$x = a \tan\theta$ रखने पर $\Rightarrow dx = a \sec^2\theta \, d\theta$

$\therefore I = \int \frac{a \sec^2\theta}{(a^2 + a^2 \tan^2\theta)^{3/2}} d\theta = \int \frac{a \sec^2\theta}{a^3 (\sec^2\theta)^{3/2}} d\theta$

$\Rightarrow I = \frac{1}{a^2} \int \frac{d\theta}{\sec\theta} = \frac{1}{a^2} \int \cos\theta \, d\theta = \frac{1}{a^2} \sin\theta + c$

$\Rightarrow I = \frac{x}{a^2 (x^2 + a^2)^{1/2}} + c$

122. (c) $I = \int \frac{e^{m \tan^{-1} x}}{1 + x^2} dx$

$m \tan^{-1} x = t$ रखने पर

$\Rightarrow \frac{m}{1 + x^2} dx = dt \Rightarrow \frac{dx}{1 + x^2} = \frac{dt}{m}$

अतः $I = \frac{1}{m} \int e^t . dt = \frac{1}{m} e^t + c = \frac{1}{m} e^{m \tan^{-1} x} + c$.

123. (d) $I = \int \frac{1 + \tan^2 x}{1 - \tan^2 x} dx = \int \frac{\sec^2 x}{1 - \tan^2 x} dx$

$\tan x = t$ रखने पर $\Rightarrow \sec^2 x . dx = dt \Rightarrow I = \int \frac{dt}{1 - t^2}$

$= \frac{1}{2 \times 1} \log\left[\frac{1 + t}{1 - t}\right] + c = \frac{1}{2} \log\left|\frac{1 + \tan x}{1 - \tan x}\right| + c$.

124. (d) $I = \int \frac{2dx}{\sqrt{1 - 4x^2}}$

$2x = \sin\theta$ रखने पर $\Rightarrow 2dx = \cos\theta \, d\theta$

$\Rightarrow I = \int \frac{\cos\theta \, d\theta}{\sqrt{1 - \sin^2\theta}} = \int \frac{\cos\theta}{\cos\theta} d\theta = \int d\theta + c = \theta + c$.

$\therefore I = \sin^{-1}(2x) + c$.

125. (b) $I = \int e^{3 \log x} (x^4 + 1)^{-1} dx = \int e^{\log x^3} (x^4 + 1)^{-1} dx$

$= \frac{1}{4} \int \frac{4x^3}{(x^4 + 1)} dx = \frac{1}{4} \log(x^4 + 1) + c$.

126. (b) $I = \int \frac{dx}{2\sqrt{x}(1 + x)}$

$\sqrt{x} = t$ रखने पर $\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$\therefore I = \int \frac{dt}{1 + t^2} \tan^{-1} t + c = \tan^{-1}(\sqrt{x}) + c$.

127. (c) $I = \int \text{cosec}^4 x \, dx = \int \text{cosec}^2 x . \text{cosec}^2 x dx$

$= \int \text{cosec}^2 x (1 + \cot^2 x) dx$

$= \int \text{cosec}^2 x \, dx + \int \cot^2 x . \text{cosec}^2 x \, dx$

$= -\cot x - \frac{\cot^3 x}{3} + c$.

128. (b) $I = \int x e^{x^2} dx = \frac{1}{2} \int e^t dt = \frac{e^{x^2}}{2} + c$.

{$x^2 = t$ रखने पर $\Rightarrow 2xdx = dt$}

129. (b) $\int f(x) dx = g(x)$

$\int f^{-1}(x) . 1 dx = f^{-1}(x) \int dx - \int \left\{\frac{d}{dx} f^{-1}(x) \int dx\right\} dx$

$= x f^{-1}(x) - \int x \frac{d}{dx} f^{-1}(x) dx$

$= x f^{-1}(x) - \int x d\{f^{-1}(x)\}$

माना $f^{-1}(x) = t \Rightarrow x = f(t)$ और $d\{f^{-1}(x)\} = dt$

$= x f^{-1}(x) - \int f(t) dt = x f^{-1}(x) - g(t) = x f^{-1}(x) - g\{f^{-1}(x)\}$.

ट्रिक : $f(x) = x^2$ रखने पर, विकल्प (b) सही हैं

130. (c) $e^x + 1 = t$ रखने पर $\Rightarrow e^x dx = dt$

$\therefore \int \frac{e^x}{e^x + 1} dx = \int \frac{dt}{t} = \log t + c = \log(e^x + 1) + c$.

131. (d) माना $\sin x + \cos x = t \Rightarrow (\cos x - \sin x) dx = dt$

$\Rightarrow -(\sin x - \cos x) dx = dt$

$\therefore \int \frac{\sin x - \cos x}{\sin x + \cos x} dx = -\int \frac{dt}{t} = -\log t + c = \log\left(\frac{1}{t}\right) + c$

अतः $\int \frac{\sin x - \cos x}{\sin x + \cos x} dx = \log\left(\frac{1}{\sin x + \cos x}\right) + c$.

132. (b) $\tan^{-1} x = t$ रखने पर $\Rightarrow \frac{1}{1 + x^2} dx = dt$

$\therefore \int \frac{(\tan^{-1} x)^3}{1 + x^2} dx = \int t^3 dt = \frac{t^4}{4} + c = \frac{(\tan^{-1} x)^4}{4} + c$.

133. (d) $\int \sqrt{\frac{1 - x}{1 + x}} dx = \int \frac{1 - x}{\sqrt{1 - x^2}} dx = \int \frac{1}{\sqrt{1 - x^2}} dx - \int \frac{x \, dx}{\sqrt{1 - x^2}}$

हल करने पर अभीष्ट परिणाम प्राप्त होता है।

134. (b) $\int \frac{\sqrt{x}}{1 + x} dx = \int \frac{\sqrt{x}\sqrt{x}}{\sqrt{x}(1 + x)} dx$

$= \int \frac{x + 1}{\sqrt{x}(x + 1)} dx - \int \frac{1}{\sqrt{x}(x + 1)} dx$

$= \int \frac{1}{\sqrt{x}} dx - \int \frac{1}{\sqrt{x}(x + 1)} dx$

$= 2x^{1/2} - 2\tan^{-1}\sqrt{x} + c = 2(\sqrt{x} - \tan^{-1}\sqrt{x}) + c$.

135. (b) $\int \frac{\sin x \, dx}{\sin x - \cos x} = \frac{1}{2} \int \frac{2 \sin x}{\sin x - \cos x} dx$

$$= \frac{1}{2}\int \frac{(\sin x - \cos x + \sin x + \cos x)}{\sin x - \cos x}dx$$

$$= \frac{1}{2}\int \left(1 + \frac{\sin x + \cos x}{\sin x - \cos x}\right)dx$$

$$= \frac{1}{2}[x + \log(\sin x - \cos x)] + c\ .$$

136. (c) $I = \int \sqrt{\frac{1+x}{1-x}}dx = \int \frac{1+x}{\sqrt{1-x^2}}dx$

$$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x}{\sqrt{1-x^2}}dx = \sin^{-1}x - \sqrt{1-x^2} + c\ .$$

137. (d) $\int \frac{x}{\sqrt{4-x^4}}dx = \int \frac{x}{\sqrt{2^2-(x^2)^2}}dx$

$x^2 = t$ रखने पर $\Rightarrow 2x\,dx = dt,$

अतः अभीष्ट समाकलन $= \frac{1}{2}\sin^{-1}\frac{x^2}{2}$.

138. (c) $I = \int \frac{\sin x}{3 + 4\cos^2 x}dx$

$\cos x = t$ रखने पर $\Rightarrow -\sin x\ dx = dt$

$$\therefore I = \int \frac{-dt}{3+4t^2} = \frac{-1}{4}\int \frac{dt}{t^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow I = -\frac{1}{4.\frac{\sqrt{3}}{2}}.\tan^{-1}\frac{t}{\left(\frac{\sqrt{3}}{2}\right)} + c = \frac{-1}{2\sqrt{3}}\tan^{-1}\frac{2t}{\sqrt{3}} + c$$

$$\Rightarrow I = \frac{-1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right) + c\ .$$

139. (d) हमें ज्ञात है कि $I = \int \frac{dx}{\sqrt{x}(x+9)}$

$\sqrt{x} = t$ रखकर दोनों तरफ वर्ग करने पर, $x = t^2$

$\Rightarrow dx = 2tdt$

$$\therefore I = 2\int \frac{dt}{t^2 + 3^2} = \frac{2}{3}\tan^{-1}\left(\frac{t}{3}\right) \Rightarrow I = \frac{2}{3}\tan^{-1}\left(\frac{\sqrt{x}}{3}\right).$$

140. (b) $\int \left\{\frac{\log x - 1}{1 + (\log x)^2}\right\}^2 dx\ .$

$\log x = t$ रखने पर, $\Rightarrow dx = e^t dt$

$$\therefore \text{अभीष्ट समाकलन} = \int e^t\left[\frac{1}{1+t^2} - \frac{2t}{(1+t^2)^2}\right]dt$$

$$[\because \int e^x[f(x) + f'(x)]\,dx = e^x f(x) + c\ ]$$

$$= \frac{e^t}{1+t^2} + C = \frac{x}{1+(\log x)^2} + C\ .$$

141. (d) $I = \int \frac{\sin 2x}{1 + \cos^2 x}dx = \int \frac{2\sin x\cos x}{1+\cos^2 x}dx$

$1 + \cos^2 x = t$ रखने पर $\Rightarrow -2\sin x\cos x\ dx = dt$

$\Rightarrow \sin 2x = -dt\ .$

$$\therefore I = -\int\left(\frac{dt}{t}\right) = -\log t + c$$

$$= -\log(1 + \cos^2 x) + c\ .$$

142. (b) $\int \frac{1+\cos 4x}{\cot x - \tan x}dx = \int \frac{2\cos^2 2x}{\cos^2 x - \sin^2 x}.\sin x\cos x\ dx$

$$= \int \cos 2x \sin 2x\ dx = -\frac{1}{8}\cos 4x + c\ ,$$

$$\therefore k = -\frac{1}{8}\ .$$

143. (a) $\frac{x^4 dx}{x + x^5} = \int \frac{(x^4+1)dx}{x+x^5} - \int \frac{dx}{x+x^5}$

$$= \int \frac{(x^4+1)dx}{x(1+x^4)} - \int \frac{dx}{x(x^4+1)} = \int \frac{dx}{x} - \int \frac{dx}{x+x^5}$$

$= \log x - f(x) - c_2 + c_1 = \log x - f(x) + c\ ,$

जहाँ $c_1 - c_2 = c =$ एक नया अचर.

144. (b) $f(x) = \int \frac{x^2 dx}{(1+x^2)\left(1+\sqrt{1+x^2}\right)}$

माना $x = \tan\theta, dx = \sec^2\theta\, d\theta = (1+x^2).d\theta$

$$f(x) = \int \frac{x^2 dx}{(1+x^2)\left(1+\sqrt{1+x^2}\right)} = \int \frac{\tan^2\theta\sec^2\theta d\theta}{\sec^2\theta(1+\sec\theta)}$$

$$= \int \frac{\tan^2\theta\ d\theta}{1+\sec\theta}\theta = \int \frac{\sin^2\theta\, d\theta}{\cos\theta(1+\cos\theta)} = \int \frac{1-\cos^2\theta\, d\theta}{\cos\theta(1+\cos\theta)}$$

$$= \int \frac{(1-\cos\theta)d.\theta}{\cos\theta} = \int \sec\theta d\theta - \int d\theta$$

$= \log(x + \sqrt{1+x^2}) - \tan^{-1}x + c$

$f(0) = \log(0 + \sqrt{1+0} - \tan^{-1}(0) + c$

$0 = \log 1 - 0 + c \Rightarrow c = 0$

$f(1) = \log(1 + \sqrt{1+1^2}) - \tan^{-1}(1) = \log(1+\sqrt{2}) - \frac{\pi}{4}.$

145. (a) $A = \int \sqrt{e^x - 1}\,dx$

माना $e^x - 1 = t^2 \Rightarrow e^x dx = 2t\,dt$

$\Rightarrow dx = \frac{2t}{t^2+1}dt$

$$\therefore A = \int t\frac{2t}{t^2+1}dt = \int \frac{2t^2}{t^2+1}dt$$

$$= \int \frac{2(t^2+1)-2}{t^2+1}dt = \int 2dt - \int \frac{2dt}{t^2+1}$$

$= 2t - 2\tan^{-1}t + c = 2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x - 1} + c\ .$

146. (a) माना $I = \int \frac{dx}{\sin(x-a)\sin(x-b)}$

$$= \frac{1}{\sin(a-b)} \int \frac{\sin\{(x-b)-(x-a)\}}{\sin(x-a)\sin(x-b)}\, dx$$

$$= \frac{1}{\sin(a-b)} \int \frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}\, dx$$

$$= \frac{1}{\sin(a-b)} \left[\int \cot(x-a)dx - \int \cot(x-b)dx\right]$$

$$= \frac{1}{\sin(a-b)} \left[\log\sin(x-a) - \log\sin(x-b)\right] + c$$

$$\Rightarrow I = \frac{1}{\sin(a-b)} \log\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + c\ .$$

147. (c) माना $I = \int \frac{\sin\theta + \cos\theta}{\sqrt{2\sin\theta\cos\theta}} d\theta$

$\therefore I = \int \frac{\sin\theta + \cos\theta}{\sqrt{1-(1-2\sin\theta\cos\theta)}} d\theta$

$\Rightarrow I = \int \frac{(\sin\theta + \cos\theta)}{\sqrt{1-(\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta)}} d\theta$

$\Rightarrow I = \int \frac{\sin\theta + \cos\theta}{\sqrt{1-(\sin\theta - \cos\theta)^2}} d\theta$

माना $(\sin\theta - \cos\theta) = t \Rightarrow (\cos\theta + \sin\theta)d\theta = dt$

$\therefore I = \int \frac{dt}{\sqrt{1-t^2}} = \sin^{-1}(t) + c = \sin^{-1}(\sin\theta - \cos\theta) + c$.

148. (c) $m + n = -\frac{3}{7} + \left(\frac{-11}{7}\right) = -2$, (ऋणात्मक पूर्णांक)

अब, $I = \int \cos^{-3/7} x\,(\sin^{(-2+3/7)} x)\,dx$

$= \int \cos^{-3/7} x \sin^{-2} x \sin^{3/7} x dx$

$I = \int \frac{\text{cosec}^2 x}{\left(\frac{\cos^{3/7} x}{\sin^{3/7} x}\right)} dx = \int \frac{\text{cosec}^2 x}{\cot^{3/7} x} dx$

$\cot x = t$ रखने पर $\Rightarrow -\text{cosec}^2 x\, dx = dt$

$\therefore I = -\int \frac{dt}{t^{3/7}} = -\frac{t^{-\frac{3}{7}+1}}{-\frac{3}{7}+1} + c = -\frac{7}{4}t^{4/7} + c$

अतः $I = -\frac{7}{4}\cot^{4/7} x + c = -\frac{7}{4}\tan^{-4/7} x + c$.

खण्डशः समाकलन($\int e^x(F(x) + F'(x))\,dx$ तथा $\int e^{kx}(kF(x) + F'(x))dx$ के रूप का समाकलन

1. (d) $\int x\sec^2 x\, dx = x\tan x - \int \tan x\, dx$

$= x\tan x + \log(\cos x) + c.$

2. (c) माना $I = \int \sin(\log x)dx$

$\log x = t$ रखने पर $\Rightarrow x = e^t \Rightarrow dx = e^t dt$, तब

$I = \int \sin t . e^t\, dt = \sin t . e^t - \int e^t . \cos t\, dt$

$= \sin t . e^t - [\cos t . e^t + \int e^t . \sin t\, dt]$

$\Rightarrow 2I = \sin t . e^t - \cos t . e^t$

$\Rightarrow I = \int \sin(\log x)dx = \frac{1}{2}x[\sin(\log x) - \cos(\log x)]$.

3. (a) $\because \int x\sin x\, dx = -x\cos x + A$

$\Rightarrow -x\cos x + \sin x +$ अचर $= -x\cos x + A$

तुलना करने पर, $A = \sin x +$ अचर.

4. (b) $\int x\log x\, dx = \frac{x^2}{2}\log x - \int \frac{1}{x}.\frac{x^2}{2}dx + c = \frac{x^2\log x}{2} - \frac{x^2}{4} + c$.

5. (a) $\int x\cos x\, dx = x\sin x - \int \sin x\, dx + c = x\sin x + \cos x + c$.

6. (b) $\int x\cos^2 x\, dx = \frac{1}{2}\int x(1+\cos 2x)dx$

$= \frac{x^2}{4} + \frac{1}{2}\left[\frac{x\sin 2x}{2} - \int \frac{\sin 2x}{2}dx\right] + c$

$= \frac{x^2}{4} + \frac{x\sin 2x}{4} + \frac{\cos 2x}{8} + c.$

7. (b) $\int \tan^{-1} x\, dx = x\tan^{-1} x - \int \frac{x}{1+x^2}dx + c$

$= x\tan^{-1} x - \frac{1}{2}\log(1+x^2) + c.$

नोट : विद्यार्थी इस प्रश्न को सूत्र मानकर याद रखें।

8. (a) $\int x.\tan^{-1} x\, dx = \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int \frac{x^2+1-1}{1+x^2}dx$

$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}x + \frac{1}{2}\tan^{-1} x + c$

$= \frac{1}{2}\tan^{-1} x.(x^2+1) - \frac{1}{2}x + c$.

9. (b) $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right]dx = \int \log(\log x)dx + \int \frac{1}{(\log x)^2}dx$

$= x\log(\log x) - \int \frac{x}{x\log x}dx + \int \frac{1}{(\log x)^2}dx$

$= x\log(\log x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2}dx + \int \frac{1}{(\log x)^2}dx$

$= x\log(\log x) - \frac{x}{\log x} + c.$

10. (d) $\int \sin(\log x)dx + \int \cos(\log x)dx$

$= x\sin(\log x) - \int \frac{x\cos(\log x)}{x}dx + \int \cos(\log x)dx + c$

$= x\sin(\log x) + c.$

11. (b) $\int \frac{\log x}{x^3}dx = \int x^{-3}\log x\, dx$

$= -\frac{\log x}{2x^2} + \int \frac{1}{x}.\frac{1}{2x^2} + c = -\frac{\log x}{2x^2} + \frac{1}{2}.\frac{x^{-2}}{-2} + c$

$= -\frac{\log x}{2x^2} - \frac{1}{4x^2} + c = -\frac{1}{4x^2}(2\log x + 1) + c$.

12. (d) माना $I = \int e^{-2x}\sin 3x\, dx$

$= -\frac{e^{-2x}\cos 3x}{3} - \int \frac{2e^{-2x}\cos 3x}{3}dx$

$= -\frac{e^{-2x}\cos 3x}{3} - \frac{2}{3}\left[\frac{e^{-2x}\sin 3x}{3} + \int \frac{2e^{-2x}\sin 3x}{3}dx\right]$

$$\Rightarrow I = -\frac{e^{-2x}\cos 3x}{3} - \frac{2e^{-2x}\sin 3x}{9} - \frac{4}{9}I$$

$$\Rightarrow \frac{13}{9}I = -e^{-2x}\left[\frac{3\cos 3x + 2\sin 3x}{9}\right]$$

अतः, $I = -\frac{1}{13}e^{-2x}[3\cos 3x + 2\sin 3x]$.

13. (b) $\int x\sin x\sec^3 x\,dx = \int x\sin x\frac{1}{\cos^3 x}dx$

$$= \int x\tan x.\sec^2 x\,dx$$

$\tan x = t$ रखने पर, $\sec^2 x\,dx = dt$ और $x = \tan^{-1} t$,

$$\therefore \int \tan^{-1} t.t\,dt = \frac{x\tan^2 x}{2} - \frac{1}{2}t + \frac{1}{2}\tan^{-1} t$$

$$= \frac{x(\sec^2 x - 1)}{2} - \frac{1}{2}\tan x + \frac{1}{2}x = \frac{1}{2}[x\sec^2 x - \tan x] + c.$$

14. (d) $\int x\sin^2 x\,dx = \int x.\frac{(1-\cos 2x)}{2}dx$

$$= \frac{1}{2}\left[\int x\,dx - \int x.\cos 2x\,dx\right] = \frac{x^2}{4} - \frac{x}{4}\sin 2x - \frac{1}{8}\cos 2x + c.$$

15. (a) $\int e^{2x+\log x}dx = \int xe^{2x}dx$

$$= \frac{xe^{2x}}{2} - \int \frac{1}{2}e^{2x}dx + c = \frac{e^{2x}}{4}(2x-1) + c.$$

16. (a) $\int \log(x+1)dx = x\log(x+1) - \int \frac{x}{x+1}dx + c$

$$= x\log(x+1) - x + \log(x+1) + c = (x+1)\log(x+1) - x + c.$$

17. (d) $\int \log(x^2+x)dx = \int \log x\,dx + \int \log(x+1)dx$

$$= x\log x - x + x\log(x+1) - x + \log(x+1)$$
$$= x\{(\log x + \log(x+1)\} - 2x + \log(x+1)$$
$$= x\log(x^2+x) - 2x + \log(x+1)$$

दिये गये समाकलन से तुलना करने पर,

$A = -2x + \log(x+1)$.

18. (b) माना $I = \int x^2\sin 2x\,dx = \frac{-x^2\cos 2x}{2} + \int \frac{2x\cos 2x}{2}dx + c$

$$= -\frac{x^2\cos 2x}{2} + \frac{x\sin 2x}{2} + \frac{\cos 2x}{4} + c.$$

19. (b) $\int \log x\,dx = x\log x - \int x.\frac{1}{x}dx + c = x\log x - x + c$

नोट : विद्यार्थी इस प्रश्न को सूत्र समझकर याद रखें।

20. (d) $\int \log_{10} x\,dx = \int \frac{\log x}{\log 10}dx$

$$= \frac{1}{\log 10}[x\log x - x] = x[\log_{10} x - \log_{10} e] + c.$$

21. (c) $\int \frac{1}{\log_x e}dx = \int \log_e x\,dx = x\log x - x + c$

$$= x(\log_e x - \log_e e) + c = x\log_e\left(\frac{x}{e}\right) + c.$$

22. (a) दिया गया है, $\int e^x\sin x\,dx = \frac{1}{2}e^x a + c$ .....(i)

माना $I = \int e^x\sin x\,dx = -e^x\cos x + \int e^x\cos x\,dx + c$

$$= -e^x\cos x + e^x\sin x - \int e^x\sin x\,dx + c$$

$\Rightarrow 2I = e^x(-\cos x + \sin x) + c$.

अतः (i) से, $\frac{1}{2}e^x a = \frac{1}{2}e^x(\sin x - \cos x) \Rightarrow a = \sin x - \cos x$.

23. (d) $\int x^n\log x\,dx = \log x.\frac{x^{n+1}}{n+1} - \int \frac{x^{n+1}}{n+1}.\frac{1}{x}dx$

$$= \frac{x^{n+1}}{n+1}\log x - \frac{x^{n+1}}{(n+1)^2} + c = \frac{x^{n+1}}{n+1}\left[\log x - \frac{1}{n+1}\right] + c.$$

24. (a) $I = \int \log x(\log x + 2)dx$

$\log x = t$ रखने पर $\Rightarrow e^t = x \Rightarrow e^t dt = dx$

तब $I = \int t(t+2)e^t dt = t^2.e^t + c = x(\log x)^2 + c.$

25. (b) $\int\left[\frac{1}{\log x} - \frac{1}{(\log x)^2}\right]dx = \int \frac{1}{\log x}dx - \int \frac{1}{(\log x)^2}dx$

$$= \frac{x}{\log x} + \int \frac{1}{(\log x)^2}.\frac{1}{x}x\,dx - \int \frac{1}{(\log x)^2}dx + c = \frac{x}{\log x} + c.$$

26. (c) $\int \frac{\log x}{(1+\log x)^2}dx$

$1 + \log x = t$ रखने पर $\Rightarrow \frac{1}{x}dx = dt$

$\Rightarrow dx = x\,dt = e^{t-1}dt$,

$$\therefore \int \frac{(t-1)e^{t-1}}{t^2}dt = \int e^{t-1}\left(\frac{1}{t} - \frac{1}{t^2}\right)dt = \frac{e^{t-1}}{t} = \frac{x}{1+\log x} + c.$$

27. (d) $\int\left(\frac{2+\sin 2x}{1+\cos 2x}\right)e^x dx = \int\left(\frac{2e^x}{1+\cos 2x}\right)dx + \int \frac{e^x\sin 2x}{1+\cos 2x}dx$

$$= \int e^x\sec^2 x\,dx + \int e^x\tan x\,dx = e^x\tan x + c.$$

28. (b) माना $I = \int e^x\sin x\,dx = e^x\sin x - \int e^x\cos x\,dx + c$

$$= e^x\sin x - e^x\cos x - \int e^x\sin x\,dx + c$$

$\Rightarrow 2I = e^x(\sin x - \cos x) + c \Rightarrow I = \frac{1}{2}e^x(\sin x - \cos x) + c$.

29. (a) $\int (1-x^2)\log x\,dx = \int \log x\,dx - \int x^2\log x\,dx$

$$= x(\log x - 1) - \frac{x^3\log x}{3} + \frac{x^3}{9} + c$$
$$= \left(x - \frac{x^3}{3}\right)\log x - \left(x - \frac{x^3}{9}\right) + c.$$

30. (d) $\int e^{2x}(-\sin x + 2\cos x)dx$

$= -\int e^{2x} \sin x\, dx + 2\int e^{2x} \cos x\, dx$

$= e^{2x} \cos x - 2\int e^{2x} \cos x\, dx + 2\int e^{2x} \cos x\, dx + c$

$= e^{2x} \cos x + c.$

वैकल्पिक : $\int e^{2x}(2\cos x - \sin x)dx = e^{2x}\cos x + c$

$\left\{\because \int e^{kx}\{k f(x) + f'(x)\}dx = e^{kx} f(x) + c\right\}$.

31. (c) $\int [f(x)g''(x) - f''(x)g(x)]dx$

$= \int f(x)g''(x)dx - \int f''(x)g(x)dx$

$= \left(f(x)g'(x) - \int f'(x)g'(x)dx\right) - \left(g(x)f'(x) - \int g'(x)f'(x)dx\right)$

$= f(x)g'(x) - f'(x)g(x).$

32. (d) $I = \int x \sin kx\, dx = \frac{-x\cos kx}{k} + \frac{\sin kx}{k^2} + c$.

33. (b) $\int xe^x dx = x.e^x - \int 1.e^x dx$

$= xe^x - e^x + c = e^x(x-1) + c$.

34. (c) $x^2 = t$ रखने पर $\Rightarrow 2x\, dx = dt$, तब

$\int x^3 e^{x^2} dx = \frac{1}{2}\int te^t dt$

$= \frac{1}{2}\left[te^t - e^t\right] + c = \frac{1}{2}e^{x^2}(x^2 - 1) + c.$

35. (a) $\int \frac{\log x}{(x+1)^2}dx = \int \log x\,(x+1)^{-2}dx$

$= \log x.\left\{-(x+1)^{-1}\right\} - \int \frac{1}{x}.\{-(x+1)^{-1}\}dx$

$= \frac{-\log x}{(x+1)} + \int \frac{1}{x(x+1)}dx = \frac{-\log x}{(x+1)} + \int\left[\frac{1}{x} - \frac{1}{x+1}\right]dx$

$= \frac{-\log x}{x+1} + \log x - \log(x+1)$.

36. (c) $\int xe^{2x}dx = \frac{xe^{2x}}{2} - \int 1.\frac{e^{2x}}{2}dx = \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + c$

$= e^{2x}\left(\frac{2x-1}{4}\right) + c \Rightarrow f(x) = \frac{(2x-1)}{4}$.

37. (c) $\frac{d}{dx}f(x) = x\cos x + \sin x$

$\Rightarrow f(x) = \int (x\cos x + \sin x)\, dx = x\sin x + c$

$\because f(0) = 2 \Rightarrow c = 2$; $\therefore f(x) = x\sin x + 2$.

38. (b) $I = \int \cos^{-1}\left(\frac{1}{x}\right)dx = \int \sec^{-1} x.1\, dx$

$= \sec^{-1} x\int dx - \int\left[\frac{d}{dx}\sec^{-1} x\int dx\right]dx$

$= x\sec^{-1} x - \int \frac{1}{x\sqrt{x^2-1}}x.\, dx$

$= x\sec^{-1} x - \int \frac{1}{\sqrt{x^2-1}}dx$

$= x\sec^{-1} x - \cosh^{-1} x + c$.

39. (b) $I = \int x^3 \log x\, dx = \frac{x^4}{4}\log x - \int \frac{x^4}{4}\frac{1}{x}dx + c$

$= \frac{x^4}{4}\log x - \int \frac{x^3}{4}dx = \frac{x^4}{4}\log x - \frac{x^4}{16} + c$

$= \frac{1}{16}[4x^4 \log x - x^4] + c$

40. (a) माना $I = \int \cos(\log_e x)dx = \int \cos(\log_e x).1\, dx$

$I = \cos(\log_e x).x - \int \frac{-\sin(\log_e x)}{x}.x\, dx$

$= x\cos(\log_e x) + \int \sin(\log_e x)\, dx$

$= x\cos(\log_e x) + \int \sin(\log_e x)\, 1\, dx$

$= x\cos(\log_e x) + \sin(\log_e x).x - \int \frac{\cos(\log_e x)}{x}x\, dx$

$= x\cos(\log_e x) + x\sin(\log_e x) - I$

$\Rightarrow 2I = x[\cos(\log_e x) + \sin(\log_e x)]$

$\Rightarrow I = \frac{x}{2}[\cos(\log_e x) + \sin(\log_e x)]$.

41. (c) $\int e^x(1+\tan x)\sec x\, dx = \int e^x \sec x\, dx + \int e^x \tan x \sec x\, dx$

$= e^x \sec x - \int e^x \sec x \tan x\, dx + \int e^x \sec x \tan x\, dx$

$= e^x \sec x + c.$

वैकल्पिक : $\int e^x(\sec x + \sec x \tan x)dx = e^x \sec x + c$

स्पष्टतः यह $\int e^x\{f(x) + f'(x)\}dx$ प्रकार का है।

42. (c) $\int \frac{xe^x}{(1+x)^2}dx = \int \frac{(x+1-1)}{(1+x)^2}e^x dx$

$= \int e^x\left(\frac{1}{1+x} - \frac{1}{(1+x)^2}\right)dx = \frac{e^x}{1+x} + c$.

43. (a) $\int e^x[\tan x - \log(\cos x)]dx = \int e^x[\tan x + \log(\sec x)]dx$

$= e^x \log(\sec x) + c \quad \left\{\text{चूँकि} \int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c\right\}$.

44. (a) $\int e^x \sin x(\sin x + 2\cos x)dx$

$= \int e^x \sin^2 x\, dx + \int e^x 2\sin x \cos x dx$

$= \int e^x \sin^2 x\, dx + \int e^x \sin 2x\, dx$

$= e^x \sin^2 x - \int e^x \sin 2x\, dx + \int e^x \sin 2x\, dx + c$

$= e^x \sin^2 x + c.$

वैकल्पिक : $\int e^x(\sin^2 x + \sin 2x)dx = e^x \sin^2 x + c.$

45. (c) $\int e^{2x} \frac{1+\sin 2x}{1+\cos 2x} dx = \int e^{2x}\left[\frac{1}{1+\cos 2x} + \frac{\sin 2x}{1+\cos 2x}\right]dx$

$= \int e^{2x}\left[\frac{\sec^2 x}{2} + \tan x\right]dx$

$= \frac{1}{2}\int e^{2x} \sec^2 x\, dx + \int e^{2x} \tan x\, dx$

$= \frac{e^{2x}\tan x}{2} - \int \frac{e^{2x}\sec^2 x}{2} dx + \int \frac{e^{2x}\sec^2 x}{2} dx + c$

$= \frac{e^{2x}\tan x}{2} + c.$

46. (a) $\int \frac{e^x(x-1)}{x^2} dx = \int e^x\left[\frac{1}{x} - \frac{1}{x^2}\right]dx = \frac{e^x}{x} + c.$

47. (d) $\int e^x\left[\frac{1 + x\log x}{x}\right]dx = \int e^x\left(\log x + \frac{1}{x}\right)dx = e^x \log x + c.$

48. (c) $\int e^x\left[\sin^{-1}\frac{x}{a} + \frac{1}{\sqrt{a^2 - x^2}}\right]dx$

$= \int e^x \sin^{-1}\frac{x}{a} dx + \int \frac{e^x}{\sqrt{a^2 - x^2}} dx$

$= e^x \sin^{-1}\frac{x}{a} - \int \frac{e^x}{\sqrt{a^2 - x^2}} dx + \int \frac{e^x}{\sqrt{a^2 - x^2}} dx + c$

$= e^x \sin^{-1}\frac{x}{a} + c.$

49. (a) माना $I = \int \frac{e^x(x^2+1)}{(x+1)^2} dx = \int \frac{e^x(x^2 - 1 + 2)}{(x+1)^2} dx$

$= \int e^x\left[\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right]dx = \int e^x[f(x) + f'(x)]dx,$

जहाँ $f(x) = \frac{x-1}{x+1}$ और $f'(x)$

अतः $I = \frac{2}{(x+1)^2} = e^x\left(\frac{x-1}{x+1}\right) + c.$

50. (c) $\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx = e^x \frac{1}{x} + c$

$\left(\because \int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c\right).$

51. (c) $I = \int e^x(1 + \tan x + \tan^2 x)dx$

$\Rightarrow \int e^x(1 + \tan x + \tan^2 x)dx = \int e^x(\tan x + \sec^2 x)dx.$

$\Rightarrow I = e^x \tan x + c, \quad (\because \int e^x[f(x) + f'(x)] + x = e^x f(x) + c).$

52. (b) $I = \int e^x\left(\frac{1 - \sin x}{1 - \cos x}\right)dx = \int e^x\left(\frac{1 - \sin x}{2\sin^2(x/2)}\right)dx$

$\Rightarrow I = \int e^x\left(\frac{1}{2}\operatorname{cosec}^2\frac{x}{2} - \cot\frac{x}{2}\right)dx,$

$\left(\because \int e^x(f(x) + f'(x)) = e^x f(x) + c\right)$

$\therefore I = e^x\left(-\cot\frac{x}{2}\right) + c = -e^x \cot\frac{x}{2} + c.$

53. (c) $I = \int \frac{(x+3)e^x}{(x+4)^2} dx = \int \frac{(x+4-1)e^x dx}{(x+4)^2}$

$\Rightarrow I = \int e^x\left(\frac{1}{x+4} - \frac{1}{(x+4)^2}\right)dx$

$\therefore I = \frac{e^x}{x+4} + c.$

54. (a) $I = \int\left(\frac{x+2}{x+4}\right)^2 e^x dx = \int e^x\left[\frac{x^2 + 4x + 4}{(x+4)^2}\right]dx$

$\Rightarrow I = \int e^x\left[\frac{x(x+4)}{(x+4)^2} + \frac{4}{(x+4)^2}\right]dx$

$= e^x\left[\frac{x}{x+4} + \frac{4}{(x+4)^2}\right]dx = e^x\left(\frac{x}{x+4}\right) + c.$

55. (a) यह आधारभूत गुणधर्म है।

56. (c) $I = \int e^x(1 - \cot x + \cot^2 x)dx = \int e^x(-\cot x + \operatorname{cosec}^2 x)dx$

$= e^x(-\cot x) + c = -e^x \cot x + c.$

57. (d) $I = \int \sin^{-1} x.1\, dx\, dx$

$I = \sin^{-1} x.x - \int \frac{1}{\sqrt{1 - x^2}}.x\, dx$

द्वितीय समाकलन में $1 - x^2 = t^2 \Rightarrow -2xdx = 2tdt$ रखकर हल करने पर, $I = x\sin^{-1} x. + \sqrt{1 - x^2} + c.$

58. (b) $\int \frac{x - \sin x}{1 - \cos x} dx = \int \frac{x}{1 - \cos x} dx - \int \frac{\sin x}{1 - \cos x} dx$

$= \frac{1}{2}\int x\operatorname{cosec}^2\left(\frac{x}{2}\right)dx - \int \frac{2\sin(x/2)\cos(x/2)}{2\sin^2(x/2)} dx$

$= \frac{1}{2}\int x\operatorname{cosec}^2\left(\frac{x}{2}\right)dx - \int \cot\left(\frac{x}{2}\right)dx = -x\cot\left(\frac{x}{2}\right) + c.$

59. (b) $\int \frac{x + \sin x}{1 + \cos x} dx = \frac{1}{2}\int x\sec^2\frac{x}{2} dx + \int \tan\frac{x}{2} dx$

$= \frac{1}{2}\frac{x\tan\frac{x}{2}}{\frac{1}{2}} - \int \tan\frac{x}{2} dx + \int \tan\frac{x}{2} dx = x\tan\frac{x}{2} + c.$

ट्रिक : जाँच द्वारा, $\frac{d}{dx}\left\{x\tan\frac{x}{2} + c\right\}$

$$= \frac{x}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2} = \frac{1}{2}\left[\frac{x}{\cos^2\frac{x}{2}} + \frac{2\sin\frac{x}{2}}{\cos\frac{x}{2}}\right] = \frac{x+\sin x}{1+\cos x}.$$

60. (c) $I = \int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx = \int e^x\left[\frac{1+2\sin(x/2)\cos(x/2)}{2\cos^2(x/2)}\right]dx$

$$I = \int e^x\left[\frac{1}{2}\sec^2(x/2) + \tan(x/2)\right]dx = e^x.\tan(x/2) + c$$

$$\{\because \int e^x[f(x) + f'(x)]dx = e^x.f(x) + c\}$$

61. (b) $I = \int \sqrt{x}.e^{\sqrt{x}}dx$. माना $x = t^2 \Rightarrow dx = 2t\,dt$

$$\therefore I = 2\int t^2.e^t dt \Rightarrow I = 2(t^2.e^t - 2te^t + 2e^t] + c$$

$$\Rightarrow I = 2[x.e^{\sqrt{x}} - 2\sqrt{x}\,e^{\sqrt{x}} + 2e^{\sqrt{x}}] + c$$

अर्थात्, $I = e^{\sqrt{x}}[2x - 4\sqrt{x} + 4] + c$.

62. (a) माना $I = \int 32x^3(\log x)^2dx = 32\int x^3(\log x)^2dx$

$$= 32\left[(\log x)^2\int x^3dx - \int\left(\frac{d}{dx}(\log x)^2\int x^3dx\right)dx\right]$$

$$= 32\left[(\log x)^2.\frac{x^4}{4} - \int 2\log x.\frac{1}{x}.\frac{x^4}{4}dx\right]$$

$$= 32\left[(\log x)^2\frac{x^4}{4} - \frac{1}{2}\int x^3\log x\,dx\right]$$

$$= 32\left[\frac{(\log x)^2x^4}{4} - \frac{1}{2}\left(\frac{\log x.x^4}{4} - \int\frac{1}{x}.\frac{x^4}{4}dx\right)\right]$$

$$= 32\left[\frac{(\log x)^2x^4}{4} - \frac{1}{2}\left(\frac{x^4\log x}{4} - \frac{1}{4}.\frac{x^4}{4}\right)\right] + c$$

$$= 8\left[(\log x)^2x^4 - \frac{1}{2}\left(x^4\log x - \frac{x^4}{4}\right)\right] + c$$

$$= 8x^4\left[(\log x)^2 - \frac{\log x}{2} + \frac{1}{8}\right] + c$$

$$= x^4[8(\log x)^2 - 4\log x + 1] + c.$$

63. (d) $x = \sin\theta$ रखने पर $\Rightarrow dx = \cos\theta\,d\theta$,

$$\int \sin^{-1}(3x - 4x^3)dx = \int \sin^{-1}(\sin 3\theta)\cos\theta\,d\theta$$

$$= \int 3\theta\cos\theta\,d\theta = 3\left\{\theta\sin\theta - \int\sin\theta\,d\theta\right\}$$

$$= 3\{\theta\sin\theta + \cos\theta\} + c = 3\left\{x\sin^{-1}x + \sqrt{1-x^2}\right\} + c.$$

64. (a) $\sqrt{x} = t$ रखने पर $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}}dx = dt \Rightarrow dx = 2t\,dt$

$$\therefore \int 2t.\cos t\,dt = 2\left[t.\sin t - \int\sin t\,dt\right]$$

$$= 2t\sin t + 2\cos t = 2[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}] + c.$$

65. (d) $x = \tan\theta$ रखने पर $\Rightarrow dx = \sec^2\theta\,d\theta$, तब

$$\int \tan^{-1}\frac{2x}{1-x^2}dx = \int \tan^{-1}\frac{2\tan\theta}{1-\tan^2\theta}\sec^2\theta\,d\theta$$

$$= \int \tan^{-1}(\tan 2\theta)\sec^2\theta\,d\theta = \int 2\theta\sec^2\theta\,d\theta$$

$$= 2\left[\theta\tan\theta - \int\tan\theta\,d\theta\right] = 2x\tan^{-1}x - \log(x^2+1) + c.$$

66. (a) $\sin^{-1}x = t$ रखने पर $\Rightarrow \frac{1}{\sqrt{1-x^2}}dx = dt$

$$\therefore \int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx = \int t\sin t\,dt = -t\cos t + \sin t + c$$

$$= -\sin^{-1}x\cos(\sin^{-1}x) + \sin(\sin^{-1}x) + c$$

$$= x - \sin^{-1}x\sqrt{1-x^2} + c.$$

67. (a) $t = \sin^{-1}x$ रखने पर $\Rightarrow \sin t = x \Rightarrow \cos t\,dt = dx$, तब

$$\int \frac{\sin^{-1}x}{(1-x^2)^{3/2}}dx = \int t\sec^2t\,dt = t\tan t + \log\cos t + c$$

$$= \sin^{-1}x\tan(\sin^{-1}x) + \log\cos(\sin^{-1}x) + c$$

$$= \frac{x}{\sqrt{1-x^2}}\sin^{-1}x + \frac{1}{2}\log(1-x^2) + c.$$

68. (b) $x = \tan\theta$ रखने पर $\Rightarrow dx = \sec^2\theta\,d\theta$, तब

$$\int \frac{x\tan^{-1}x}{(1+x^2)^{3/2}}dx = \int \frac{\theta\tan\theta\sec^2\theta\,d\theta}{(1+\tan^2\theta)^{3/2}}$$

$$= \int \theta\sin\theta\,d\theta = -\theta\cos\theta + \sin\theta + c$$

$$= \frac{x}{\sqrt{x^2+1}} - \tan^{-1}x\frac{1}{\sqrt{x^2+1}} = \frac{x - \tan^{-1}x}{\sqrt{1+x^2}} + c.$$

69. (a) $x^2 = t$ रखने पर $\Rightarrow 2x\,dx = dt$, तब

$$\int x^5e^{x^2}dx = \frac{1}{2}\int t^2e^t dt = \frac{1}{2}\left[e^tt^2 - 2\int te^t dt\right] + c$$

$$= \frac{t^2e^t}{2} - \left[te^t - e^t\right] + c = \frac{1}{2}x^4e^{x^2} - x^2e^{x^2} + e^{x^2} + c.$$

70. (a) $\tan^{-1}x = t$ रखने पर $\Rightarrow \frac{dx}{1+x^2} = dt$

$$\therefore \int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx = \int e^t(\tan t + \sec^2 t)dt$$

$$= e^t\tan t + c = x\,e^{\tan^{-1}x} + c$$

$$\left[\int e^x\{f(x) + f'(x)\}dx = e^xf(x) + C \text{ का प्रयोग करने पर }\right].$$

71. (c) $I = \int e^{\sqrt{x}}.dx$

$\sqrt{x} = t$ रखने पर, $\frac{1}{2\sqrt{x}}dx = dt \Rightarrow dx = 2t\,dt$

$$\therefore I = \int e^t.2t\,dt = 2[t.e^t - e^t] + A = 2[\sqrt{x}.e^{\sqrt{x}} - e^{\sqrt{x}}] + A$$

$$\Rightarrow I = 2(\sqrt{x} - 1).e^{\sqrt{x}} + A.$$

72. (c) $I_1 = \int \sin^{-1} x dx$

माना $\sin^{-1} x = \theta \Rightarrow x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

$I_1 = \int \theta \cos\theta\, d\theta = \theta \sin\theta - \int \sin\theta d\theta = \theta \sin\theta + \cos\theta$

$= x \sin^{-1} x + \sqrt{1-x^2}$

$I_2 = \int \sin^{-1}\sqrt{1-x^2}\, dx = \int \cos^{-1} x dx$

माना $\cos\phi = x$, अतः $-\sin\phi\, d\phi = dx$

$I_2 = -\int \phi \sin\phi d\phi = \phi\cos\phi + \int -\cos\phi d\phi$

$= \phi\cos\phi - \sin\phi = x\cos^{-1} x - \sqrt{1-x^2}$

$I_1 + I_2 = x(\cos^{-1} x + \sin^{-1} x) = \frac{\pi}{2} x$.

73. (c) $\int f(x)\cos x dx + \int g(x) e^x dx$

$= \int e^x \cos x dx + \int (-\sin x) e^x dx$

$= \frac{e^x}{2}(\cos x + \sin x) - \frac{e^x}{2}(\sin x - \cos x) + c$

$= \frac{e^x}{2}(2\cos x) + c = e^x \cos x + c$.

आंशिक भिन्नों द्वारा परिमेय फलनों के समाकलन, समाकलन के विभिन्न रूपों का मान ज्ञात करना

1. (a) $\int \frac{dx}{(x - x^2)} = \int \left(\frac{1}{x} + \frac{1}{1-x}\right) dx = \log x - \log(1-x) + c$.

2. (a) $\int \frac{dx}{1 + x + x^2 + x^3} = \int \frac{dx}{(1+x)(1+x^2)}$

$= \frac{1}{2}\int \frac{1}{1+x^2} dx + \frac{1}{2}\int \frac{1}{1+x} dx - \frac{1}{2}\int \frac{x}{1+x^2} dx$

$= \frac{1}{2}\tan^{-1} x + \log\sqrt{1+x} - \frac{1}{2}\log\sqrt{1+x^2} + c$.

3. (b) $\int \frac{x-1}{(x-3)(x-2)} dx$

$= \int \frac{x-3}{(x-3)(x-2)} dx + \int \frac{2}{(x-3)(x-2)} dx$

$= \log\left[\frac{(x-2)(x-3)^2}{(x-2)^2}\right] + c = \log\left[\frac{(x-3)^2}{(x-2)}\right] + c.$

ट्रिक: जाँच द्वारा $\frac{d}{dx}\{\log(x-3) - \log(x-2)\}$

$= \frac{1}{x-3} - \frac{1}{x-2} = \frac{1}{(x-3)(x-2)}$

$\Rightarrow \frac{d}{dx}\{2\log(x-3) - \log(x-2)\}$

$= \frac{2}{x-3} - \frac{1}{x-2} = \frac{x-1}{(x-3)(x-2)}$.

4. (d) $\int \frac{1}{\cos x(1+\cos x)} dx = \int \frac{dx}{\cos x} - \int \frac{dx}{1+\cos x}$

$= \int \sec x\, dx - \frac{1}{2}\int \sec^2 \frac{x}{2} dx$

$= \log(\sec x + \tan x) - \tan\frac{x}{2} + c.$

5. (c) $\int \frac{dx}{(x+1)(x+2)} = \int \left(\frac{1}{x+1} - \frac{1}{x+2}\right) dx$

$= \log(x+1) - \log(x+2) + c = \log\frac{x+1}{x+2} + c.$

6. (a) $\int \frac{x}{(x-2)(x-1)} dx = -\int \frac{1}{x-1} dx + \int \frac{2}{x-2} dx$

$= -\log_e(x-1) + 2\log_e(x-2) + c = \log_e \frac{(x-2)^2}{(x-1)} + p.$

7. (a) हमें ज्ञात है कि, $\frac{1}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+1)}$

$\Rightarrow 1 = A(x^2+1) + (Bx+C)(x-1)$

यदि $x = 1$, तब $A = \frac{1}{2}$ .....(i)

$A - C = 1 \Rightarrow C = -\frac{1}{2}$ .....(ii)

$A + B = 0 \Rightarrow B = -\frac{1}{2}$ .....(iii)

इन मानों को रखने पर,

$\therefore \frac{1}{(x-1)(x^2+1)} = \frac{1}{2}\cdot\frac{1}{(x-1)} - \frac{x+1}{2(x^2+1)}$

$\therefore \int \frac{1}{(x-1)(x^2+1)} dx = \frac{1}{2}\int \frac{dx}{(x-1)} - \frac{1}{2}\int \frac{x+1}{x^2+1} dx$

$= \frac{1}{2}\log(x-1) - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1} x + c.$

8. (b) $\int \frac{x^2+x-1}{x^2+x-6} dx = \int \left[1 + \frac{5}{x^2+x-6}\right] dx$

$= \int \left[1 + \frac{5}{(x+3)(x-2)}\right] dx = \int dx + \int \frac{dx}{x-2} - \int \frac{dx}{x+3}$

$= x + \log(x-2) - \log(x+3) + c$.

9. (b) $\int \frac{x^2}{(x^2+2)(x^2+3)} dx = \int \left[\frac{3}{x^2+3} - \frac{2}{x^2+2}\right] dx$

$= \frac{3}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - \frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + c$

$= \sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + c.$

10. (c) $\int \frac{dx}{(x^2+1)(x^2+4)} = \frac{1}{3}\left[\int \frac{dx}{x^2+1} - \int \frac{dx}{x^2+4}\right]$

$= \frac{1}{3}\left[\tan^{-1} x - \frac{1}{2}\tan^{-1}\frac{x}{2}\right] + c = \frac{1}{3}\tan^{-1} x - \frac{1}{6}\tan^{-1}\frac{x}{2} + c$.

11. (d) $\int \frac{1}{x - x^3} dx = \int \frac{1}{x(1+x)(1-x)} dx$

$= \frac{1}{2}\int \left(\frac{2}{x} - \frac{1}{1+x} + \frac{1}{1-x}\right) dx$

$$= \frac{1}{2}[2\log x - \log(1+x) - \log(1-x)] = \frac{1}{2}\log\frac{x^2}{(1-x^2)} + c\,.$$

12. (b) $\int \sin 5x \cos 3x\, dx = \frac{1}{2}\int (\sin 8x + \sin 2x)\, dx$

$$= \frac{-\cos 8x}{16} - \frac{\cos 2x}{4} + c$$

दिये गये मानों से तुलना करने पर $A = \frac{-\cos 2x}{4} + c$.

13. (a) $\int \sin^3 x \cos^2 x\, dx = \int (1-\cos^2 x)\cos^2 x \,.\sin x\, dx$

$\cos x = t$ रखने पर, $-\sin x\, dx = dt$

$$\therefore -\int (t^2 - t^4)dt = \frac{t^5}{5} - \frac{t^3}{3} + c = \frac{(\cos x)^5}{5} - \frac{(\cos x)^3}{3} + c$$

14. (b) $\int \sin 2x \cos 3x\, dx = \frac{1}{2}\int 2(\sin 2x \cos 3x)\, dx$

$$= \frac{1}{2}\int (\sin 5x - \sin x)\, dx = \frac{1}{2}\left[-\frac{\cos 5x}{5} + \cos x\right] + c$$

$$= \frac{1}{2}\left[\cos x - \frac{\cos 5x}{5}\right] + c.$$

15. (c) $\sin x = t$ रखने पर, $\cos x\, dx = dt$, तब

$$\int \frac{\cos x}{(1+\sin x)(2+\sin x)}dx = \int \frac{dt}{(t+1)(t+2)}$$

$$= \int \frac{1}{t+1}dt - \int \frac{1}{t+2}dt = \log\left(\frac{t+1}{t+2}\right) + c = \log\left(\frac{\sin x + 1}{\sin x + 2}\right) + c\,.$$

16. (b) $\int \frac{e^x}{(1+e^x)(2+e^x)}dx = \int \left\{\frac{e^x}{1+e^x} - \frac{e^x}{2+e^x}\right\}dx$

$1 + e^x = t$ और $2 + e^x = t$, रखने पर तब अभीष्ट समाकलन

$$= \log(1+e^x) - \log(2+e^x) = \log\left(\frac{1+e^x}{2+e^x}\right) + c.$$

17. (c) $\int \frac{e^x dx}{e^{2x} + e^x - 2} = \int \frac{dt}{t^2 + t - 2} \quad \{\because e^x = t \Rightarrow e^x dx = dt\}$

$$= \int \frac{dt}{(t+2)(t-1)} = \int \frac{1}{3}\left[\frac{1}{t-1} - \frac{1}{t+2}\right]dt$$

$$= \frac{1}{3}\log(e^x - 1) - \frac{1}{3}\log(e^x + 2) + c.$$

18. (a) $\int \frac{x}{(x^4-1)}dx = \frac{1}{2}\int \left[\frac{x}{x^2-1} - \frac{x}{x^2+1}\right]dx$

$$= \frac{1}{4}\log(x^2-1) - \frac{1}{4}\log(x^2+1) = \frac{1}{4}\log\left(\frac{x^2-1}{x^2+1}\right) + c\,.$$

वैकल्पिक : $t = x$ रखने पर, $dt = 2x\, dx$, तब

$$\int \frac{x}{x^4-1}dx = \frac{1}{2}\int \frac{dt}{t^2-1} = \frac{1}{2}.\frac{1}{2}\log\frac{t-1}{t+1} + c$$

$$= \frac{1}{4}\log\left[\frac{x^2-1}{x^2+1}\right] + c.$$

19. (a) $\cos x = t$ रखने पर, $-\sin x\, dx = dt$, तब

$$\int (1-\cos^2 x)^2 . \cos^4 x \sin x\, dx = -\int (1-t^2)^2 . t^4 dt$$

$$= -\frac{t^5}{5} + \frac{2}{7}t^7 - \frac{1}{9}t^9 + c = -\frac{\cos^5 x}{5} + \frac{2}{7}\cos^7 x - \frac{1}{9}\cos^9 x + c\,.$$

वैकल्पिक : समानयन सूत्र द्वारा

20. (c) $\int \sqrt{x^2 - 8x + 7}\, dx = \int \sqrt{(x-4)^2 - (3)^2}\, dx$

$\int \sqrt{x^2 - a^2}\, dx$. सूत्र प्रयोग करने पर

21. (b) $\int \frac{dx}{\sqrt{2x - x^2}} = \int \frac{dx}{\sqrt{1-(x-1)^2}} = \sin^{-1}(x-1) + c.$

22. (c) $\int \frac{x}{(x^2-a^2)(x^2-b^2)}dx$

$$= \frac{1}{a^2-b^2}\left[\int \frac{x}{x^2-a^2}dx - \int \frac{x\,dx}{x^2-b^2}\right].$$

23. (c) $\int \frac{dx}{5 + 4\cos x} = \int \frac{dx}{5 + 4\left[\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right]} = \int \frac{\sec^2\frac{x}{2}}{9 + \tan^2\frac{x}{2}}dx$

$\tan\frac{x}{2} = t$, रखने पर, $2\int \frac{dt}{3^2 + t^2} = \frac{2}{3}\tan^{-1}\left[\frac{1}{3}\tan\frac{x}{2}\right] + c$

वैकल्पिक : सूत्र का प्रयोग करने पर अर्थात्,

$\int \frac{1}{a + b\cos x}dx$, $\{a > b\}$

$$= \frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right] + c$$

$$\therefore \int \frac{dx}{5 + 4\cos x} = \frac{2}{3}\tan^{-1}\left\{\frac{1}{3}\tan\frac{x}{2}\right\} + c.$$

24. (a) $\int \frac{1}{(x^2+b^2)(x^2+a^2)}dx$

$$= \frac{1}{a^2-b^2}\int \left[\frac{1}{x^2+b^2} - \frac{1}{x^2+a^2}\right]dx$$

$$= \frac{1}{(a^2-b^2)}\left[\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right) - \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right] + c\,.$$

नोट : विद्यार्थियों को यह प्रश्न सूत्र की तरह याद रखना चाहिये।

25. (c) $\int \frac{dx}{1+\cos^2 x} = \int \frac{\sec^2 x\, dx}{\sec^2 x + 1} = \int \frac{\sec^2 x}{\tan^2 x + 2}dx$

$$= \int \frac{dt}{t^2+2} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right) + c \quad \{\tan x = t \text{ रखने पर}\}$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\tan x\right) + c\,.$$

ट्रिक : जाँच द्वारा,

$$\frac{d}{dx}\left\{\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\tan x\right)\right\}=\frac{1}{\sqrt{2}}\left(\frac{1}{1+\frac{\tan^2 x}{2}}\right)\frac{1}{\sqrt{2}}\sec^2 x$$

$$=\frac{1}{2}\cdot\frac{2\sec^2 x}{(2+\tan^2 x)}=\frac{\sec^2 x}{1+\sec^2 x}=\frac{1}{1+\cos^2 x}.$$

26. (b) $\int\frac{dx}{1+3\sin^2 x}=\int\frac{dx}{\sin^2 x+\cos^2 x+3\sin^2 x}$

$$=\int\frac{dx}{4\sin^2 x+\cos^2 x}=\int\frac{\sec^2 x\,dx}{4\tan^2 x+1}=\frac{1}{4}\int\frac{\sec^2 x\,dx}{\tan^2 x+\frac{1}{4}}$$

$t=\tan x$ रखने पर $\Rightarrow dt=\sec^2 x\,dx$,

$$\therefore\ \frac{1}{4}\int\frac{dt}{t^2+\left(\frac{1}{2}\right)^2}=\frac{1}{4}2\tan^{-1}(2t)+c$$

$$=\frac{1}{2}\tan^{-1}(2t)+c=\frac{1}{2}\tan^{-1}(2\tan x)+c.$$

27. (d) $I=\int\frac{dx}{2x^2+x+1}=\int\frac{dx}{2\left(x^2+\frac{x}{2}+\frac{1}{2}\right)}$

$$=\frac{1}{2}\int\frac{dx}{x^2+\frac{x}{2}+\frac{1}{16}-\frac{1}{16}+\frac{1}{2}}=\frac{1}{2}\int\frac{dx}{\left(x+\frac{1}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2}$$

$$=\frac{1}{2}\frac{1}{\frac{\sqrt{7}}{4}}\tan^{-1}\frac{[x+(1/4)]}{\sqrt{7}/4}=\frac{2}{\sqrt{7}}\tan^{-1}\frac{(4x+1)}{\sqrt{7}}+C.$$

28. (a) $I=\frac{dx}{7+5\cos x}=\int\frac{dx}{7+5\left(\frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}\right)}$

$$=\int\frac{\sec^2(x/2)dx}{7+7\tan^2(x/2)+5-5\tan^2(x/2)}$$

$$=\int\frac{\sec^2(x/2)dx}{12+2\tan^2(x/2)}=\int\frac{\frac{1}{2}\sec^2(x/2).dx}{6+\tan^2(x/2)}$$

$\tan\frac{x}{2}=t$ रखने पर $\Rightarrow\frac{1}{2}\sec^2\frac{x}{2}dx=dt$

$$I=\int\frac{dt}{t^2+(\sqrt{6})^2}=\frac{1}{\sqrt{6}}\tan^{-1}\frac{t}{\sqrt{6}}+c$$

$$=\frac{1}{\sqrt{6}}\tan^{-1}\left|\frac{\tan(x/2)}{\sqrt{6}}\right|+c.$$

29. (b) $\int\frac{dx}{x^2+4x+13}=\int\frac{dx}{(x+2)^2+9}=\frac{1}{3}\tan^{-1}\frac{(x+2)}{3}+c.$

30. (a) हमें ज्ञात है कि $I=\int\frac{dx}{\cos x-\sin x}=\frac{1}{\sqrt{2}}\int\frac{dx}{\cos\left(\frac{\pi}{4}+x\right)}$

$$I=\frac{1}{\sqrt{2}}\int\sec\left(x+\frac{\pi}{4}\right)dx=\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}+\frac{\pi}{8}\right)\right|+c$$

$$I=\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+c.$$

31. (d) $\int\frac{dx}{x^2+2x+2}=\int\frac{dx}{(x+1)^2+1}=\tan^{-1}(x+1)+c.$

32. (a) अंश को निम्न प्रकार से लिख सकते हैं

अंश $=l$(हर)$+m$ (हर का अवकल गुणांक D^r).

माना $3\sin x+2\cos x=l(3\cos x+2\sin x)+m(-3\sin x+2\cos x)$

दोनों पक्षों में $\sin x$ व $\cos x$ के गुणांको की तुलना करने पर $3=2l-3m$ तथा $2=3l+2m$, हल करने पर

$$l=\frac{12}{13},\ \ m=-\frac{5}{13},$$

$$\therefore I=l\int dx+m\int\frac{-3\sin x+2\cos x}{3\cos x+2\sin x}dx$$

$$=lx+m\log(3\cos x+2\sin x)=\frac{12}{13}x-\frac{5}{13}\log(3\cos x+2\sin x).$$

33. (a) $\log x=t$ रखने पर $\Rightarrow\frac{1}{x}dx=dt$, तब

$$\int\frac{dx}{x[(\log x)^2+4\log x-1]}=\int\frac{dt}{t^2+4t-1}$$

$$=\int\frac{dt}{(t+2)^2-(\sqrt{5})^2}=\frac{1}{2\sqrt{5}}\log\left[\frac{t+2-\sqrt{5}}{t+2+\sqrt{5}}\right]$$

$$=\frac{1}{2\sqrt{5}}\log\left[\frac{\log x+2-\sqrt{5}}{\log x+2+\sqrt{5}}\right]+c.$$

34. (c) $x^n=t$ रखने पर $\Rightarrow nx^{n-1}dx=dt$

$$\Rightarrow\frac{nx^n}{x}dx=dt\Rightarrow\frac{1}{x}dx=\frac{dt}{nt},$$

$$\therefore\ \int\frac{dt}{nt(t+1)}=\frac{1}{n}\left[\int\frac{dt}{t(t+1)}\right]$$

$$=\frac{1}{n}\left[\int\frac{1}{t}dt-\int\frac{1}{t+1}dt\right]=\frac{1}{n}\log\frac{x^n}{x^n+1}+c.$$

35. (b) दिया है $\int\frac{dx}{x(x^7+1)}=\int\frac{dx}{x^8\left(1+\frac{1}{x^7}\right)}$

$1+\frac{1}{x^7}=t$ रखने पर $\Rightarrow\frac{-7}{x^8}dx=dt$

$$\therefore\ I=\frac{-1}{7}\int\frac{dt}{t}=\frac{-1}{7}\log t+c$$

$$\Rightarrow I=-\frac{1}{7}\log\left(\frac{x^7+1}{x^7}\right)+c\ \Rightarrow\ I=\frac{1}{7}\log\left(\frac{x^7}{x^7+1}\right)+c.$$

36. (d) हमें ज्ञात है कि $I=\int\frac{dx}{x(x^5+1)}=\int\frac{dx}{x^6\left(1+\frac{1}{x^5}\right)}$

$1+\frac{1}{x^5}=t$ रखने पर $\Rightarrow \frac{-5}{x^6}dx=dt$

$\Rightarrow I=-\frac{1}{5}\int\frac{dt}{t}=-\frac{1}{5}\log t+c$

$I=-\frac{1}{5}\log\left(1+\frac{1}{x^5}\right)+c=-\frac{1}{5}\log\left(\frac{x^5+1}{x^5}\right)+c$

$\therefore\ I=\frac{1}{5}\log\left(\frac{x^5}{x^5+1}\right)+c$.

37. (d) $\int\frac{x^2+1}{x^4+1}dx=\int\frac{\left(1+\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2}\right)}dx=\int\frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+2}$

$x-\frac{1}{x}=t$ रखने पर $\Rightarrow\left(1+\frac{1}{x^2}\right)dx=dt$, तब अभीष्ट समाकल $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+c$ है।

38. (d) दिये गये समाकल को निम्न प्रकार से लिखा जा सकता है

$\int\frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)^2-1}dx$

$x+\frac{1}{x}=t$ रखने पर $\Rightarrow\left(1-\frac{1}{x^2}\right)dx=dt$,

$\therefore\ \int\frac{dt}{t^2-1}=\frac{1}{2}\log\left|\frac{t-1}{t+1}\right|+c$

$=\frac{1}{2}\log\left(\frac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1}\right)+c=\frac{1}{2}\log\left(\frac{x^2-x+1}{x^2+x+1}\right)+c.$

39. (d) $I=\int\frac{x\ dx}{x^2+4x+5}=\int\frac{x+2-2}{(x+2)^2+1}dx$

$=\frac{1}{2}\int\frac{2(x+2)\ dx}{(x+2)^2+1}-2\int\frac{dx}{1+(x+2)^2}$

$=\frac{1}{2}\int\frac{dt}{t}-2\int\frac{dx}{1+(x+2)^2}$

प्रथम व्यंजक में $[1+(x+2)^2=t$ रखने पर $\Rightarrow 2(x+2)dx=dt]$

$=\frac{1}{2}\log t-2\tan^{-1}(x+2)+c$

$=\frac{1}{2}\log(x^2+4x+5)-2\tan^{-1}(x+2)+c$.

40. (a) $I=\int\frac{2x^2+3}{(x^2-1)(x^2-4)}$

$\frac{2x^2+3}{(x^2-1)(x^2-4)}=\frac{A}{x^2-1}+\frac{B}{x^2-4}$

$2x^2+3=A(x^2-4)+B(x^2-1)$

$2x^2+3=x^2(A+B)-4A-B$

दोनों पक्षों में x^2 व अचर पदों के गुणांकों की तुलना करने पर

$A+B=2$ .....(i)

$4A+B=-3$ .....(ii)

दोनों समीकरणों को हल करने पर $A=-\frac{5}{3}$, $B=\frac{11}{3}$

$\int\frac{2x^2+3.dx}{(x^2-1)(x^2-4)}=\int\frac{-\frac{5}{3}dx}{(x^2-1)}+\int\frac{\frac{11}{3}dx}{(x^2-4)}$

$=-\frac{5}{3}\int\frac{dx}{(x+1)(x-1)}+\frac{11}{3}\int\frac{dx}{(x+2)(x-2)}$

$=-\frac{5}{3}\cdot\frac{1}{2}\int\frac{dx}{x-1}+\frac{5}{6}\int\frac{dx}{x+1}+\frac{11}{3}\cdot\frac{1}{4}\int\frac{dx}{x-2}-\frac{11}{12}\int\frac{dx}{x+2}+c$

$=-\frac{5}{6}\log(x-1)+\frac{5}{6}\log(x+1)+\frac{11}{12}\log(x-2)-\frac{11}{12}\log(x+2)+c$

$=\frac{5}{6}\log\left(\frac{x+1}{x-1}\right)+\frac{11}{12}\log\left(\frac{x-2}{x+2}\right)+c$

$=\log\left(\frac{x+1}{x-1}\right)^{5/6}+\log\left(\frac{x-2}{x+2}\right)^{11/12}+c$

$\Rightarrow a=\frac{11}{12}$ और $b=\frac{5}{6}$.

Critical Thinking Questions

1. (b) $\int\frac{dx}{\cos(x-a)\cos(x-b)}$

$=\frac{1}{\sin(a-b)}\int\frac{\sin\{(x-b)-(x-a)\}}{\cos(x-a).\cos(x-b)}dx$

$=\frac{1}{\sin(a-b)}\int\left\{\frac{\sin(x-b)}{\cos(x-b)}-\frac{\sin(x-a)}{\cos(x-a)}\right\}dx$

$=\text{cosec}\,(a-b)\log\frac{\cos(x-a)}{\cos(x-b)}+c$.

2. (b) $\int\frac{dx}{\sqrt{x+a}+\sqrt{x+b}}=\int\frac{\sqrt{x+a}-\sqrt{x+b}}{(x+a)-(x+b)}dx$

$=\frac{1}{(a-b)}\int(x+a)^{1/2}dx-\frac{1}{(a-b)}\int(x+b)^{1/2}dx$

$=\frac{2}{3(a-b)}[(x+a)^{3/2}-(x+b)^{3/2}]+c.$

3. (a) स्वयं करें।

4. (a) $\int(\sin 2x+\cos 2x)dx=-\frac{\cos 2x}{2}+\frac{\sin 2x}{2}+k$

$=\frac{1}{\sqrt{2}}\left(\sin 2x\cos\frac{\pi}{4}-\cos 2x\sin\frac{\pi}{4}\right)+k$

$=\frac{1}{\sqrt{2}}\sin\left(2x-\frac{\pi}{4}\right)+k$

$\Rightarrow c=\frac{\pi}{4}$ तथा $a=k$, एक स्वेच्छ अचर

5. (d) $\int\frac{x^3-x-2}{(1-x^2)}dx=\int\frac{-x(1-x^2)}{(1-x^2)}dx-\int\frac{2}{1-x^2}dx$

$=-\int x\,dx-2\int\frac{1}{1-x^2}dx=\frac{-x^2}{2}+\log\left(\frac{x-1}{x+1}\right)+c.$

6. (b) $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$

$= \int \frac{(\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x)}{(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x} dx$

$= \int (\sin^4 x - \cos^4 x) dx$

$= \int (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) dx$

$= \int (\sin^2 x - \cos^2 x) dx = \int -\cos 2x \, dx = -\frac{\sin 2x}{2} + c.$

7. (d) $a + bx = t$ रखने पर $\Rightarrow x = \frac{t-a}{b}$ और $dx = \frac{dt}{b}$

$\therefore I = \int \left(\frac{t-a}{b}\right)^2 \times \frac{1}{t^2} \frac{dt}{b}$

$= \frac{1}{b^2} \int \left(1 - \frac{2a}{t} + a^2 . t^{-2}\right) dt = \frac{1}{b^2}\left[t - 2a \log t - \frac{a^2}{t}\right]$

$= \frac{1}{b^2}\left[x + \frac{a}{b} - \frac{2a}{b}\log(a+bx) - \frac{a^2}{b}\frac{1}{(a+bx)}\right].$

8. (a) $q \tan^{-1} x = t$ रखने पर

$\Rightarrow \frac{q}{1+x^2} dx = dt \Rightarrow \frac{1}{1+x^2} dx = \frac{dt}{q}$

$\Rightarrow \int \frac{dx}{(1+x^2)\sqrt{p^2 + q^2(\tan^{-1} x)^2}} = \frac{1}{q} \int \frac{dt}{\sqrt{p^2 + t^2}}$

$= \frac{1}{q} \log\left[q \tan^{-1} x + \sqrt{p^2 + q^2 (\tan^{-1} x)^2}\right] + c.$

9. (c) $1 + x^3 = t^2$ रखने पर $\Rightarrow 3x^2 dx = 2t\,dt$ और $x^3 = t^2 - 1$

$\therefore \int \frac{x^5}{\sqrt{1+x^3}} dx = \int \frac{x^2 . x^3}{\sqrt{1+x^3}} dx$

$= \frac{2}{3} \int \frac{(t^2 - 1).t\,dt}{t} = \frac{2}{3} \int (t^2 - 1) dt = \frac{2}{3}\left[\frac{t^3}{3} - t\right] + c$

$= \frac{2}{3}\left[\frac{(1+x^3)^{3/2}}{3} - (1+x^3)^{1/2}\right] + c.$

10. (d) $I = \int \frac{dx}{\sin x - \cos x + \sqrt{2}}$

$= \int \frac{dx}{\sqrt{2}(\sin x . \sin\frac{\pi}{4} - \cos x \cos\frac{\pi}{4} + 1)}$

$= \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos(x + \frac{\pi}{4})} = \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos 2\left(\frac{x}{2} + \frac{\pi}{8}\right)}$

$= \frac{1}{\sqrt{2}} \int \frac{dx}{2\sin^2\left(\frac{x}{2} + \frac{\pi}{8}\right)} = \frac{1}{2\sqrt{2}} \int \text{cosec}^2\left(\frac{x}{2} + \frac{\pi}{8}\right) dx$

$= \frac{1}{2\sqrt{2}} \frac{-\cot\left(\frac{x}{2} + \frac{\pi}{8}\right)}{1/2} + c = \frac{-1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c.$

11. (a) $\int \frac{a\,dx}{b + c\,e^x} = \int \frac{ae^x}{be^x + c\,e^{2x}} dx$

$e^x = t$, रखने पर $\Rightarrow e^x dx = dt$

$a \int \frac{dt}{t(ct+b)} = a \int -\frac{1}{b}\left\{\frac{c}{ct+b} - \frac{1}{t}\right\} dt$ {आंशिक भिन्न द्वारा}

$= \frac{a}{b} \log\left(\frac{e^x}{b + ce^x}\right) + c.$

12. (b) $\sqrt{x} = t$ रखने पर $\Rightarrow \frac{1}{2\sqrt{x}} dx = dt \Rightarrow dx = 2t\,dt$, तब

$\int \sin\sqrt{x}\, dx = 2\int t \sin t\, dt = 2(-t\cos t + \sin t) + c$

$= 2(\sin\sqrt{x} - \sqrt{x}\cos\sqrt{x}) + c.$

13. (a) $x = 3\sin\theta$ रखने पर $\Rightarrow dx = 3\cos\theta\, d\theta$,

$\therefore \int \frac{x^2}{(9-x^2)^{3/2}} dx = \int \frac{9\sin^2\theta}{(9 - 9\sin^2\theta)^{3/2} . 3\cos\theta} d\theta$

$= \int \frac{27\sin^2\theta\cos\theta}{27\cos^3\theta} d\theta = \int \tan^2\theta\, d\theta = \int (\sec^2\theta - 1) d\theta$

$= \tan\theta - \theta + c = \tan\left\{\sin^{-1}\left(\frac{x}{3}\right)\right\} - \sin^{-1}\left(\frac{x}{3}\right) + c$

$= \tan\tan^{-1}\left(\frac{\left(\frac{x}{3}\right)}{\sqrt{1 - (x^2/9)}}\right) - \sin^{-1}\left(\frac{x}{3}\right) + c$

$= \frac{x}{\sqrt{9 - x^2}} - \sin^{-1}\left(\frac{x}{3}\right) + c.$

14. (a) $\int x\sqrt{\frac{1-x^2}{1+x^2}} dx = \int \frac{x.(1-x^2)}{\sqrt{1-x^4}} dx$

{अंश व हर को $(1-x^2)^{1/2}$ से गुणा करने पर}

$= \int \frac{x}{\sqrt{1-x^4}} dx - \int \frac{x^3}{\sqrt{1-x^4}} dx$

$= \frac{1}{2}[\sin^{-1}(x^2) + \sqrt{1-x^4}] + c.$

($x^2 = t$ तथा $\sqrt{1-x^4} = \sqrt{t}$ रखने पर)

15. (a) चूँकि $\int f(x)\sin x \cos x\, dx = \frac{1}{2(b^2 - a^2)} \log(f(x)) + c$

$\therefore \quad f(x)\sin x\cos x = \frac{1}{2(b^2 - a^2)} . \frac{1}{f(x)} f'(x)$

दोनों पक्षों का x के सापेक्ष अवकलन करने पर,

$\Rightarrow 2(b^2 - a^2)\sin x \cos x = \frac{f'(x)}{f(x)^2}$

$\Rightarrow \int (2b^2 \sin x\cos x - 2a^2 \sin x \cos x) dx = \int \frac{f'(x)}{\{f(x)\}^2} dx$

$\Rightarrow \pm(-b^2\cos^2 x - a^2 \sin^2 x) = -\frac{1}{f(x)}$

$\Rightarrow f(x) = \pm \frac{1}{(a^2 \sin^2 x + b^2 \cos^2 x)}.$

16. (c) $\int \frac{dx}{4\sin^2 x + 5\cos^2 x} = \int \frac{\sec^2 x\,dx}{4\tan^2 x + 5} = \frac{1}{4}\int \frac{\sec^2 x\,dx}{\tan^2 x + \frac{5}{4}}$

$\tan x = t$ रखने पर $\Rightarrow \sec^2 x\,dx = dt$,

$\therefore \frac{1}{4}\int \frac{dt}{t^2 + \left(\frac{\sqrt{5}}{2}\right)^2} = \frac{2}{4\sqrt{5}}\tan^{-1}\left(\frac{2t}{\sqrt{5}}\right) + c$

$= \frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right) + c.$

17. (c) $\int \frac{x^2+1}{x^4 - x^2 + 1}dx = \int \frac{\left(1+\frac{1}{x^2}\right)}{x^2 + \frac{1}{x^2} - 1}dx$

$= \int \frac{1+\frac{1}{x^2}}{\left(x - \frac{1}{x}\right)^2 + 1}dx = \int \frac{dt}{t^2+1} = \tan^{-1} t + c$

$\left\{x - \frac{1}{x} = t \text{ रखने पर } \Rightarrow \left(1 + \frac{1}{x^2}\right)dx = dt\right\}$

$= \tan^{-1}\left(x - \frac{1}{x}\right) + c = \tan^{-1}\left(\frac{x^2-1}{x}\right) + c.$

18. (c) $\int (\log x)^2 dx$; $\log x = t$ रखने पर $\Rightarrow e^t = x \Rightarrow dx = e^t dt$, तब

$\int t^2 . e^t dt = t^2 e^t - 2te^t + 2e^t + c$

$= x(\log x)^2 - 2x\log x + 2x + c$.

19. (a) माना $\sqrt{(x^2 - a^2)} = t \Rightarrow x^2 - a^2 = t^2$

$\Rightarrow x^2 = a^2 + t^2 \Rightarrow xdx = tdt$

$\therefore \int \frac{\sqrt{(x^2-a^2)}}{x}dx = \int \frac{\sqrt{(x^2-a^2)}\,x}{x^2}dx$

$\Rightarrow I = \int \frac{t}{a^2+t^2}tdt = \int \frac{t^2}{a^2+t^2}dt$

$\Rightarrow I = \int\left(1 - \frac{a^2}{a^2+t^2}\right)dt = t - a^2\frac{1}{a}\tan^{-1}\left(\frac{t}{a}\right)$

$\Rightarrow I = \sqrt{(x^2-a^2)} - a\tan^{-1}\left[\frac{\left\{\sqrt{(x^2-a^2)}\right\}}{a}\right] + c$

20. (a) $\int \tan^3 2x \sec 2x\,dx = \int (\sec^2 2x - 1)\sec 2x \tan 2x\,dx$

$= \int (\sec^3 2x \tan 2x - \sec 2x \tan 2x)dx$

$= \int \sec^3 2x \tan 2x\,dx - \int \sec 2x \tan 2x\,dx$ .....(i)

$\int \sec^3 2x \tan 2x\,dx$ लेने पर

$\sec 2x = t$ रखने पर $\Rightarrow \sec 2x \tan 2x dx = \frac{dt}{2}$

$\therefore \frac{1}{2}\int t^2 dt = \frac{t^3}{6} = \frac{\sec^3 2x}{6}$

(i) से, $\int \sec^3 2x \tan 2x\,dx - \int \sec 2x \tan 2x\,dx$

$= \frac{\sec^3 2x}{6} - \frac{\sec 2x}{2} + c.$

ट्रिक : माना $\sec 2x = t$, तब $\sec 2x \tan 2x\,dx = \frac{1}{2}dt$

$\therefore \frac{1}{2}\int (t^2 - 1)dt = \frac{1}{6}t^3 - \frac{1}{2}t + c = \frac{1}{6}\sec^3 2x - \frac{1}{2}\sec 2x + c$.

21. (a) $\int x\sin^{-1} x dx = \frac{x^2}{2}\sin^{-1} x - \int \frac{1}{\sqrt{1-x^2}} . \frac{x^2}{2}dx + c$

$= \frac{x^2}{2}\sin^{-1} x - \frac{1}{2}\int -\frac{(1-x^2)+1}{\sqrt{1-x^2}}dx + c$

$= \frac{x^2}{2}\sin^{-1} x + \frac{1}{2}\int \sqrt{1-x^2}\,dx - \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx + c$

$= \frac{x^2}{2}\sin^{-1} x + \frac{x}{4}\sqrt{1-x^2} + \frac{1}{4}\sin^{-1} x - \frac{1}{2}\sin^{-1} x + c$

$= \frac{x^2}{2}\sin^{-1} x + \frac{x}{4}\sqrt{1-x^2} - \frac{1}{4}\sin^{-1} x$

$= \left(\frac{x^2}{2} - \frac{1}{4}\right)\sin^{-1} x + \frac{x}{4}\sqrt{1-x^2} + c$.

22. (a) $I = \int \sqrt{\frac{a-x}{x}}dx$.

$x = a\sin^2\theta$ रखने पर $\Rightarrow dx = 2a\sin\theta\cos\theta\,d\theta$, तब

$I = \int \sqrt{\frac{\cos^2\theta}{\sin^2\theta}} . 2a\sin\theta\cos\theta\,d\theta$

$= a\int 2\cos^2\theta\,d\theta = a\int (1 + \cos 2\theta)d\theta$

$= a\left[\sin^{-1}\sqrt{\frac{x}{a}} + \sqrt{\frac{x}{a}}.\sqrt{\frac{a-x}{a}}\right] + c$.

23. (a) $\int \frac{\sin x - \cos x}{\sqrt{1 - \sin 2x}} e^{\sin x}\cos x\,dx = \int \frac{\sin x - \cos x}{\sin x - \cos x} e^{\sin x}\cos x\,dx$

$= \int e^{\sin x}\cos x\,dx = e^{\sin x} + c$.

24. (d) $I = \int \frac{4e^x + 6e^{-x}}{9e^{2x} - 4e^{-x}}dx = \frac{4}{9}\int \frac{9e^{2x}dx}{9e^{2x} - 4} + 6\int \frac{dx}{9e^{2x} - 4}$

$\therefore \int \frac{dx}{9e^{2x} - 4} = \frac{1}{8}\log(9e^{2x} - 4) - \frac{1}{4}\log 3 - \frac{1}{4}x + \text{अचर}$

$\therefore I = \frac{35}{36}\log(9e^{2x} - 4) - \frac{3}{2}x - \frac{3}{2}\log 3 + \text{ अचर}$

दिये गये समाकल से तुलना करने पर

$A = -\frac{3}{2}$, $B = \frac{35}{36}$, $C = -\frac{3}{2}\log 3 + \text{ अचर}$

25. (a) माना $I = \int \sec^3 x dx = \int \sec x \sec^2 x dx$

$\Rightarrow I = \sec x \tan x - \int \sec x \tan^2 x\, dx$

$\Rightarrow I = \sec x \tan x - \int \sec x (\sec^2 x - 1) dx$

$\Rightarrow I = \sec x \tan x - \int \sec^3 x\, dx + \int \sec x\, dx$

$\Rightarrow I = \sec x \tan x - I + \log(\sec x + \tan x)$

$\Rightarrow 2I = \sec x \tan x + \log(\sec x + \tan x)$

$\Rightarrow I = \frac{1}{2}[\sec x \tan x + \log(\sec x + \tan x)]$.

26. (b) $\int \frac{x-1}{(x+1)^3} e^x dx = \int e^x \left(\frac{(x+1)}{(x+1)^3} - \frac{2}{(x+1)^3}\right) dx$

$= \int e^x \left(\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}\right) dx = \frac{e^x}{(x+1)^2} + c$.

$\left\{\because \frac{d}{dx}\left(\frac{1}{(x+1)^2}\right) = -\frac{2}{(x+1)^3}\right\}$

27. (c) $I = \int e^x \sin 2x\, dx = \sin 2x . e^x - 2\int \cos 2x . e^x dx$

$= \sin 2x . e^x - 2\cos 2x . e^x - 4\int e^x \sin 2x\, dx$

$\Rightarrow 5I = e^x(\sin 2x - 2\cos 2x) +$ अचर

दिये गये मान से तुलना करने पर $K = 5$.

28. (a) $\int \frac{dx}{3 - 2x - x^2} = \int \frac{dx}{4 - (x^2 + 2x + 1)}$

$= \int \frac{dx}{4 - (x+1)^2} = \int \frac{dt}{(2)^2 - t^2}$

जहाँ $x + 1 = t, \quad \therefore dx = dt$

$\therefore I = \frac{1}{2.2}\log\left(\frac{2+t}{2-t}\right) = \frac{1}{4}\log\left(\frac{2+x+1}{2-x-1}\right) = \frac{1}{4}\log\left(\frac{3+x}{1-x}\right)$.

29. (a) $\int x(2x+3)^{1/2} dx$

$= x\frac{(2x+3)^{3/2}}{3/2}\frac{1}{2} - \int \frac{(2x+3)^{3/2}}{3/2}\frac{1}{2} dx + c$

$= \frac{1}{3}x(2x+3)^{3/2} - \frac{1}{3}\int (2x+3)^{3/2} dx + c$

$= \frac{1}{3}x(2x+3)^{3/2} - \frac{1}{15}(2x+3)^{5/2} + c$.

30. (d) हम जानते है कि

$\log\left(\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\right) = \log\left(\frac{1+\tan\theta}{1-\tan\theta}\right) = \log\tan\left(\frac{\pi}{4} + \theta\right)$

$\int \sec\theta\, d\theta = \log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$

$\therefore \int \sec 2\theta\, d\theta = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \theta\right)$

$\therefore 2\sec 2\theta = \frac{d}{d\theta}\log\tan\left(\frac{\pi}{4} + \theta\right)$ .....(i)

दिये गये व्यंजक का खण्डशः समाकलन करने पर,

$I = \frac{1}{2}\sin 2\theta \log\tan\left(\frac{\pi}{4} + \theta\right) - \frac{1}{2}\int \sin 2\theta . 2\sec 2\theta\, d\theta$,{(i) से}

$I = \frac{1}{2}\sin 2\theta \log\tan\left(\frac{\pi}{4} + \theta\right) - \int \tan 2\theta\, d\theta$

$I = \frac{1}{2}\sin 2\theta \log\tan\left(\frac{\pi}{4} + \theta\right) - \frac{1}{2}\log\sec 2\theta$.

31. (c) $x\sin x + \cos x$ का अवकलन $x\cos x$ है, तब

$I = \int \frac{x^2 dx}{(x\sin x + \cos x)^2} = \int \frac{x\cos x}{(x\sin x + \cos x)^2} . \frac{x}{\cos x} dx$

खण्डशः समाकलन द्वारा $\left[\int \frac{1}{t^2} dt = -\frac{1}{t}\right]$

$\therefore I = \frac{-1}{(x\sin x + \cos x)} . \frac{x}{\cos x}$

$+ \int \frac{1}{(x\sin x + \cos x)} . \frac{\cos x . 1 - x(-\sin x)}{\cos^2 x} dx$

$= -\frac{1}{x\sin x + \cos x} . \frac{x}{\cos x} + \int \sec^2 x\, dx$

$= -\frac{1}{x\sin x + \cos x} . \frac{x}{\cos x} + \frac{\sin x}{\cos x}$

$= \frac{-x + x\sin^2 x + \sin x\cos x}{(x\sin x + \cos x)\cos x}$

$= \frac{\sin x\cos x - x(1 - \sin^2 x)}{(x\sin x + \cos x)\cos x} = \frac{\sin x - x\cos x}{x\sin x + \cos x}$.

32. (c) $u = \int e^{ax}\cos bx\, dx = e^{ax}\frac{\sin bx}{b} - \frac{a}{b}\int e^{ax} . \sin bx\, dx$

$= \frac{e^{ax}\sin bx}{b} - \frac{a}{b}v \Rightarrow bu + av = e^{ax}\sin bx$ .....(i)

इसी प्रकार $bv - au = -e^{ax}\cos bx$ .....(ii)

(i) व (ii) को वर्ग करके जोड़ने पर,

$(a^2 + b^2)(u^2 + v^2) = e^{2ax}$.

33. (a) $I_n = \int (\log x)^n dx$ .....(i)

$\therefore I_{n-1} = \int (\log x)^{n-1} dx$ .....(ii)

अब $I_n = \int (\log x)^n . dx = (\log x)^n x - n\int (\log x)^{n-1}\frac{1}{x} x\, dx$

$= x(\log x)^n - n\int (\log x)^{n-1}\,dx$

$I_n = x(\log x)^n - nI_{n-1}\ ;\ \therefore I_n + n\,I_{n-1} = x(\log x)^n$.

34. (d) माना $I = \int e^{x/2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$

$= 2\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)e^{x/2} - \int \cos\left(\frac{x}{2}+\frac{\pi}{4}\right)\frac{1}{2}2e^{x/2}dx + c$

$= 2\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)e^{x/2} - 2e^{x/2}\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)$

$-\int \sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\frac{1}{2}2e^{x/2}$

$\therefore\ 2I = 2e^{x/2}\left\{\sin\left(\frac{x}{2}+\frac{\pi}{4}\right) - \cos\left(\frac{x}{2}+\frac{\pi}{4}\right)\right\}$

$\Rightarrow I = e^{x/2}\left\{\sin\left(\frac{x}{2}+\frac{\pi}{4}\right) - \cos\left(\frac{x}{2}+\frac{\pi}{4}\right)\right\}$

$= \sqrt{2}e^{x/2}\left(\sin\frac{x}{2}\right) = \sqrt{2}e^{x/2}\sin\frac{x}{2} + c.$

ट्रिक : जाँच द्वारा,

$\frac{d}{dx}\left\{\sqrt{2}e^{x/2}\sin\frac{x}{2}+c\right\} = \sqrt{2}\left[\frac{1}{2}e^{x/2}\cos\frac{x}{2} + \frac{1}{2}e^{x/2}\sin\frac{x}{2}\right]$

$= e^{x/2}\left[\frac{1}{\sqrt{2}}\cos\frac{x}{2} + \frac{1}{\sqrt{2}}\sin\frac{x}{2}\right] = e^{x/2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right).$

35. (c) $\int \frac{2x+3}{x^2-5x+6}dx = \int \frac{2x-5}{x^2-5x+6}dx + \int \frac{8}{x^2-5x+6}dx$

$= \log(x^2-5x+6) + 8\int \frac{1}{(x-2)(x-3)}dx + c\ .$

$= \log[(x-2)(x-3)] + 8\int\left[\frac{1}{x-3} - \frac{1}{x-2}\right]dx + c\ .$

$= \log(x-2) + \log(x-3) + 8\log(x-3) - 8\log(x-2) + c$

$= 9\log(x-3) - 7\log(x-2) + c$ .....(i)

दिया गया है

$\int \frac{2x+3}{x^2-5x+6}\,dx = 9\log(x-3) - 7\log(x-2) + A$

इसकी (i) से तुलना करने पर $A =$ अचर

36. (b) $\int \frac{dx}{2+\cos x} = \int \frac{dx}{2\sin^2\left(\frac{x}{2}\right) + 2\cos^2\left(\frac{x}{2}\right) + \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)}$

$= \int \frac{dx}{\sin^2\left(\frac{x}{2}\right) + 3\cos^2\left(\frac{x}{2}\right)} = \int \frac{\sec^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+3}dx$

$\tan\left(\frac{x}{2}\right) = t$ रखने पर $\Rightarrow \sec^2\left(\frac{x}{2}\right)dx = 2dt,$

$\therefore\ 2\int \frac{dt}{t^2+3} = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + c = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right) + c\cdot$

37. (b) $I = \int \frac{x}{x^4+x^2+1}dx = \int \frac{xdx}{(x^2+x+1)(x^2-x+1)}$

$I = \frac{1}{2}\int \frac{dx}{x^2-x+1} - \frac{1}{2}\int \frac{dx}{x^2+x+1}$

$I = \frac{1}{2}\int \frac{dx}{\left(x-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} - \frac{1}{2}\int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$I = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)$

$I = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right).$

38. (a) $I = \int \frac{dx}{\sin x(1+2\cos x)} = \int \frac{\sin x\,dx}{\sin^2 x(1+2\cos x)}$

$= \int \frac{\sin x\,dx}{(1-\cos x)(1+\cos x)(1+2\cos x)}$

$\cos x$ का अवकल गुणांक $-\sin x$ है जो कि अंश में दिया गया है और इस कारण हम प्रतिस्थापन $\cos x = t$

$\Rightarrow -\sin x\,dx = dt$ का प्रयोग करते हैं।

$\therefore\ I = -\int \frac{dt}{(1-t)(1+t)(1+2t)}$

समाकल को आंशिक भिन्न में रखने पर,

$\therefore\ I = -\int\left[\frac{1}{6(1-t)} - \frac{1}{2(1+t)} + \frac{4}{3(1+2t)}\right]dt$

39. (d) $I = \int \frac{2x+3}{(x-1)(x^2+1)}dx$

$= \int \frac{5\,dx}{2(x-1)} + \int \frac{-\left(\frac{5}{2}x+\frac{1}{2}\right)}{x^2+1}dx$, (आंशिक भिन्न द्वारा)

$I = \frac{5}{2}\log(x-1) - \frac{5}{2}\int \frac{x\,dx}{1+x^2} - \frac{1}{2}\int \frac{dx}{1+x^2}$

$I = \frac{5}{2}\log(x-1) - \frac{5}{4}\log(1+x^2) - \frac{1}{2}\tan^{-1}x + A$

$I = \log(x-1)^{5/2}(1+x^2)^{-5/4} - \frac{1}{2}\tan^{-1}x + A$

तुलना करने पर, $a = -\frac{5}{4}$

40. (a) $I = \int \frac{(2x^2+1)}{(x^2-4)(x^2-1)}dx$

$$\frac{2x^2+1}{(x^2-4)(x^2-1)} = \frac{3}{(x^2-4)} - \frac{1}{x^2-1}$$

$$\therefore I = \int \left[\frac{3}{(x^2-4)} - \frac{1}{x^2-1}\right]dx$$

$$= \frac{3}{2\times 2}\log\left|\frac{x-2}{x+2}\right| - \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + c$$

$$= \frac{3}{4}\log\left|\frac{x-2}{x+2}\right| + \log\left|\frac{x+1}{x-1}\right|^{1/2} + c$$

$$= \log\left|\frac{x-2}{x+2}\right|^{3/4} + \log\left|\frac{x+1}{x-1}\right|^{1/2} + c$$

$$= \log\left[\left(\frac{x+1}{x-1}\right)^{1/2}\left(\frac{x-2}{x+2}\right)^{3/4}\right] + c$$

अनिश्चित समाकलन

SET Self Evaluation Test - 23

1. निम्न वाक्यों का अवलोकन कीजिये :

प्रथक्कथन (A) : $\frac{1}{x^2+a^2}$ का समाकलन $x = a\tan\theta$ प्रतिस्थापन द्वारा ज्ञात किया जा सकता है

कारण (R) : क्योंकि सभी समाकलों को प्रतिस्थापन विधि द्वारा ही समाकलित कर सकते हैं।

इन वाक्यों में से **[SCRA 1996]**

(a) दोनों A व R सत्य है और R, A का सही कारण है

(b) दोनों A व R सत्य है पर R, A का सही कारण नहीं है

(c) A सत्य है पर R असत्य है

(d) A असत्य है पर R सत्य है

2. $\int\sqrt{1+\frac{\sin x}{4}}\,dx =$ **[Karnataka CET 2003]**

(a) $8\left(\sin\frac{x}{8}-\cos\frac{x}{8}\right)+c$ (b) $\left(\sin\frac{x}{8}+\cos\frac{x}{8}\right)+c$

(c) $\frac{1}{8}\left(\sin\frac{x}{8}-\cos\frac{x}{8}\right)+c$ (d) $8\left(\cos\frac{x}{8}-\sin\frac{x}{8}\right)+c$

3. $\int\frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}}\,dx =$ **[EAMCET 2003]**

(a) $\frac{1}{2}\sqrt{1+x}+c$ (b) $\frac{2}{3}(1+x)^{3/2}+c$

(c) $\sqrt{1+x}+c$ (d) $2(1+x)^{3/2}+c$

4. $\int(1+x-x^{-1})e^{x+x^{-1}}\,dx =$ **[EAMCET 2003]**

(a) $(x+1)e^{x+x^{-1}}+c$ (b) $(x-1)e^{x+x^{-1}}+c$

(c) $-xe^{x+x^{-1}}+c$ (d) $xe^{x+x^{-1}}+c$

5. $\int e^{\cos^2 x}\sin 2x\,dx =$ **[AI CBSE 1980]**

(a) $e^{\cos^2 x}+c$ (b) $-e^{\cos^2 x}+c$

(c) $\frac{1}{2}e^{\cos^2 x}+c$ (d) इनमें सें कोई नहीं

6. $\int\frac{dx}{(1+x^2)\sqrt{1-x^2}} =$ **[MNR 1985]**

(a) $\frac{1}{\sqrt{2}}\tan^{-1}\left[\frac{\sqrt{1-x^2}}{x\sqrt{2}}\right]+c$ (b) $\frac{1}{\sqrt{2}}\tan^{-1}\left[\frac{x\sqrt{2}}{\sqrt{1-x^2}}\right]+c$

(c) $\sqrt{2}\tan^{-1}\left[\frac{\sqrt{1-x^2}}{x\sqrt{2}}\right]+c$ (d) $-\sqrt{2}\tan^{-1}\left[\frac{\sqrt{1-x^2}}{x\sqrt{2}}\right]+c$

7. $\int\sqrt{\frac{a+x}{a-x}}\,dx =$

(a) $a\cos^{-1}x/a+\sqrt{a^2-x^2}+c$

(b) $a\cos^{-1}x/a-\sqrt{a^2-x^2}+c$

(c) $-a\cos^{-1}x/a+\sqrt{a^2-x^2}+c$

(d) $-a\cos^{-1}x/a-\sqrt{a^2-x^2}+c$

8. $\int\sqrt{\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)}\,dx =$ **[IIT 1985]**

(a) $\cos^{-1}\sqrt{x}+\sqrt{1-x}\,.(\sqrt{x}-2)+c$

(b) $\cos^{-1}\sqrt{x}-\sqrt{1-x}\,.(\sqrt{x}-2)+c$

(c) $\cos^{-1}\sqrt{x}+\sqrt{1-x}\,.(\sqrt{x-2})+c$

(d) इनमें सें कोई नहीं

9. $\int\frac{1}{x^2(x^4+1)^{3/4}}dx =$

[IIT 1984; RPET 2000; UPSEAT 2001]

(a) $\frac{(x^4+1)^{1/4}}{x}+c$ (b) $-\frac{(x^4+1)^{1/4}}{x}+c$

(c) $\frac{3}{4}\frac{(x^4+1)^{3/4}}{x}+c$ (d) $\frac{4}{3}\frac{(x^4+1)^{3/4}}{x}+c$

10. $\int\sqrt{2+\sin 3x}\,.\cos 3x\,dx =$ **[IIT 1976]**

(a) $\frac{2}{9}(2+\sin 3x)^{1/2}+c$ (b) $\frac{2}{3}(2+\sin 3x)^{2/3}+c$

(c) $\frac{2}{3}(2+\sin 3x)^{3/2}+c$ (d) $\frac{2}{9}(2+\sin 3x)^{3/2}+c$

11. $\int\frac{dx}{(2\sin x+\cos x)^2} =$ **[MP PET 1994]**

(a) $\frac{1}{2}\left(\frac{1}{2\tan x+1}\right)+c$

(b) $\frac{1}{2}\log(2\tan x+1)+c$

(c) $\frac{1}{2+\cot x}+c$

(d) $-\frac{1}{2}\left(\frac{1}{2\tan x-1}\right)+c$

12. $\int \frac{\sqrt{x^2+1}[\log(x^2+1)-2\log x]}{x^4}dx =$

(a) $\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{1/2}\left[\log\left(1+\frac{1}{x^2}\right)+\frac{2}{3}\right]+c$

(b) $-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right]+c$

(c) $\frac{2}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right)+\frac{2}{3}\right]+c$

(d) इनमें सें कोई नहीं

13. $\int \sqrt{\frac{\cos x-\cos^3 x}{1-\cos^3 x}}\,dx =$

(a) $\frac{2}{3}\sin^{-1}(\cos^{3/2}x)+c$ (b) $\frac{3}{2}\sin^{-1}(\cos^{3/2}x)+c$

(c) $\frac{2}{3}\cos^{-1}(\cos^{3/2}x)+c$ (d) इनमें सें कोई नहीं

14. यदि $l'(x)$ का अर्थ $\log\log\log.....\log x$ है, $\log$ r बार पुनरावृत्त होता है, तब $\int \frac{1}{xl(x)l^2(x)l^3(x).....l^r(x)}dx =$

(a) $l^{r+1}(x)+c$ (b) $\frac{l^{r+1}(x)}{r+1}+c$

(c) $l^r(x)+c$ (d) इनमें सें कोई नहीं

15. यदि c कोई स्वेच्छ अचर है, तो $\int 2^{2^{2^x}}2^{2^x}2^x dx =$ **[J & K 2005]**

(a) $\frac{2^{2^{2^x}}}{(\ln 2)^3}+c$ (b) $\frac{2^{2^{2^x}}}{(\ln 2)^3}+c$

(c) $2^{2^{2^x}}(\ln 2)^3+c$ (d) इनमें सें कोई नहीं

16. यदि $\int \frac{1}{(\sin x+4)(\sin x-1)}dx = A\frac{1}{\tan\frac{x}{2}-1}+B\tan^{-1}(f(x))+C$,

तब

(a) $A=\frac{1}{5}, B=\frac{-2}{5\sqrt{15}}, f(x)=\frac{4\tan x+3}{\sqrt{15}}$

(b) $A=-\frac{1}{5}, B=\frac{1}{\sqrt{15}}, f(x)=\frac{4\tan\left(\frac{x}{2}\right)+1}{\sqrt{15}}$

(c) $A=\frac{2}{5}, B=\frac{-2}{5}, f(x)=\frac{4\tan x+1}{5}$

(d) $A=\frac{2}{5}, B=\frac{-2}{5\sqrt{15}}, f(x)=\frac{4\tan\frac{x}{2}+1}{\sqrt{15}}$

17. $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} =$ **[AMU 2001]**

(a) $\sqrt{\tan x}+\frac{\tan^{5/2}x}{5}+c$ (b) $\sqrt{\tan x}+\frac{2}{5}\tan^{5/2}x+c$

(c) $2\sqrt{\tan x}+\frac{2}{5}\tan^{5/2}x+c$ (d) इनमें सें कोई नहीं

18. $\int \frac{dx}{1-\cos x-\sin x} =$ **[EAMCET 2002]**

(a) $\log|1+\cot x/2|+c$ (b) $\log|1-\tan x/2|+c$

(c) $\log|1-\cot x/2|+c$ (d) $\log|1+\tan x/2|+c$

19. $\int \frac{3^x}{\sqrt{9^x-1}}dx =$ **[EAMCET 2002]**

(a) $\frac{1}{\log 3}\log|3^x+\sqrt{9^x-1}|+c$

(b) $\frac{1}{\log 3}\log|9^x+\sqrt{9^x-1}|+c$

(c) $\frac{1}{\log 9}\log|3^x+\sqrt{9^x-1}|+c$

(d) $\frac{1}{\log 9}\log|3^x-\sqrt{9^x-1}|+c$

20. $\int \frac{e^x dx}{\sqrt{a+be^x}} =$

(a) $\frac{2}{b}\sqrt{a+be^x}+c$ (b) $2b\sqrt{a+be^x}+c$

(c) $\frac{1}{2b}\sqrt{a+be^x}$ (d) $\frac{a}{b}\sqrt{a+be^x}+c$

21. $\int u\frac{d^2v}{dx^2}dx-\int v\frac{d^2u}{dx^2}dx =$ **[MP PET 1990]**

(a) $u\frac{dv}{dx}-v\frac{du}{dx}+c$ (b) $2\frac{du}{dx}\frac{dv}{dx}+c$

(c) $uv+c$ (d) c

22. यदि $f(x)=g(x)$, तब $\int f'(x).g(x)\,dx$ का मान है **[BIT Ranchi 1991]**

(a) $\{f(x)\}^2+c$ (b) $\{g(x)\}^2+c$

(c) $\frac{1}{2}\{f(x)\}^2+c$ (d) $\frac{1}{2}\{g(x)\}^2+c$

23. $\int e^{2x}\left(\frac{\sin 4x-2}{1-\cos 4x}\right)dx =$ **[Mathematics Olympiad 1986]**

(a) $\frac{1}{2}e^{2x}\cot 2x+c$ (b) $-\frac{1}{2}e^{2x}\cot 2x+c$

(c) $-2e^{2x}\cot 2x+c$ (d) $2e^{2x}\cot 2x+c$

24. $\int e^{2x}(2\sin 3x + 3\cos 3x)\,dx$ = [MP PET 2003]

(a) $e^{2x}\sin 3x$ (b) $e^{2x}\cos 3x$

(c) e^{2x} (d) $e^{2x}(2\sin 3x)$

25. $\int \frac{1}{x^2}\log(x^2+a^2)dx$ = [MNR 1980]

(a) $\frac{1}{x}\log(x^2+a^2)+\frac{2}{a}\tan^{-1}\frac{x}{a}+c$

(b) $-\frac{1}{x}\log(x^2+a^2)+\frac{2}{a}\tan^{-1}\frac{x}{a}+c$

(c) $-\frac{1}{x}\log(x^2+a^2)-\frac{2}{a}\tan^{-1}\frac{x}{a}+c$

(d) इनमें सें कोई नहीं

26. $\int \frac{x^2-1}{x^4+x^2+1}dx$ = [MP PET 2004]

(a) $\log(x^4+x^2+1)+c$ (b) $\frac{1}{2}\log\frac{x^2-x+1}{x^2+x+1}+c$

(c) $\frac{1}{2}\log\frac{x^2+x+1}{x^2-x+1}+c$ (d) $\log\frac{x^2-x+1}{x+x+1}+c$

27. $\int \frac{x^4}{(x-1)(x^2+1)}dx$ = [Roorkee 1986]

(a) $\frac{x(x+2)}{2}+\frac{\log(x-1)}{2}-\frac{\log(x^2+1)}{4}-\frac{\tan^{-1}x}{2}+c$

(b) $\frac{x(x+2)}{2}+\frac{\log(x-1)}{2}+\frac{\log(x^2+1)}{4}-\frac{\tan^{-1}x}{2}+c$

(c) $\frac{x(x+2)}{2}+\frac{\log(x-1)}{2}+\frac{\log(x^2+1)}{4}+\frac{\tan^{-1}x}{2}+c$

(d) इनमें सें कोई नहीं

28. $\int \frac{x^3-1}{x^3+x}dx$ = [Roorkee 1988; MP PET 2001]

(a) $x-\log x+\frac{1}{2}\log(x^2+1)+\tan^{-1}x+c$

(b) $x-\log x+\log\sqrt{x^2+1}-\tan^{-1}x+c$

(c) $x+\log x+\log\sqrt{x^2+1}+\tan^{-1}x+c$

(d) इनमें सें कोई नहीं

29. $x>1$ के लिये, $\int \frac{1}{x(x^4-1)}dx$ = [RPET 1997]

(a) $\log\frac{x^4-1}{x^4}+K$

(b) $\frac{1}{4}\log\frac{x^4-1}{x^4}+K$

(c) $\log\frac{x^4-1}{x}+K$

(d) $\frac{1}{4}\log\frac{x^4-1}{x}+K$

30. यदि $\int \frac{2x^2+3}{(x^2-1)(x^2+4)}dx = a\log\left(\frac{x-1}{x+1}\right)+b\tan^{-1}\frac{x}{2}+c$, तब a व b के मान हैं [RPET 2000]

(a) (1, –1)

(b) (–1, 1)

(c) $\left(\frac{1}{2},-\frac{1}{2}\right)$

(d) $\left(\frac{1}{2},\frac{1}{2}\right)$

1. (c) A सत्य है किन्तु R असत्य है

2. (a) $\int \sqrt{1+\sin\left(\frac{x}{4}\right)}\,dx$

$$= \int \sqrt{\left(\sin^2\frac{x}{8}+\cos^2\frac{x}{8}\right)+\left(2\sin\frac{x}{8}\cos\frac{x}{8}\right)}\,dx$$

$$= \int \sqrt{\left(\sin\frac{x}{8}+\cos\frac{x}{8}\right)^2}\,dx = \int\left(\sin\frac{x}{8}+\cos\frac{x}{8}\right)dx$$

$$= \frac{-\cos\frac{x}{8}}{\left(\frac{1}{8}\right)}+\frac{\sin\frac{x}{8}}{\left(\frac{1}{8}\right)}+c = 8\left(\sin\frac{x}{8}-\cos\frac{x}{8}\right)+c\,.$$

3. (b) $\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}}\,dx = \int \frac{\sqrt{1+x}[\sqrt{1+x}+\sqrt{x}]}{(\sqrt{x}+\sqrt{1+x})}\,dx$

$$= \int \sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{3/2}+c\,.$$

4. (d) $\int (1+x-x^{-1})e^{x+x^{-1}}\,dx$

$$= \int x\,e^{x+x^{-1}}\left(1-\frac{1}{x^2}\right)+e^{x+x^{-1}}]\,dx$$

$$\left(\because \int [x\,f'(x)+f(x)]dx = x\,f(x)+c\right)$$

$$\therefore \int (1+x-x^{-1})e^{x+x^{-1}}\,dx = xe^{x+x^{-1}}+c\,.$$

5. (b) $t=\cos^2 x$ रखने पर $\Rightarrow dt=-\sin 2x\,dx$, तब

$$\int e^{\cos^2 x}\sin 2x\,dx = -\int e^t dt = -e^t+c = -e^{\cos^2 x}+c.$$

6. (b) $x=\sin\theta$ रखने पर $\Rightarrow dx=\cos\theta\,d\theta$, तब

$$\int \frac{dx}{(1+x^2)\sqrt{1-x^2}} = \int \frac{1}{1+\sin^2\theta}\,d\theta = \int \frac{\sec^2\theta}{1+2\tan^2\theta}\,d\theta$$

पुनः $t=\tan\theta$ रखने पर $\Rightarrow dt=\sec^2\theta\,d\theta$,

$$\therefore \int \frac{1}{1+2t^2}\,dt = \frac{1}{2}\int \frac{1}{t^2+\left(\frac{1}{\sqrt{2}}\right)^2}\,dt$$

$$= \frac{1}{2}\left(\frac{1}{(1/\sqrt{2})}\right)\tan^{-1}\left(\frac{t}{(1/\sqrt{2})}\right)+c$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2}\tan\theta)+c = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x\sqrt{2}}{\sqrt{1-x^2}}\right)+c.$$

वैकल्पिक : पहले $x=\frac{1}{t}$ रखने पर तथा तब $t^2-1=z^2$.

7. (d) $\int \sqrt{\frac{a+x}{a-x}}\,dx$; $x=a\cos\theta$ रखने पर

$\Rightarrow dx=-a\sin\theta\,d\theta$,

$$\therefore -a\int \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}(\sin\theta)\,d\theta$$

$$= -2a\int \sqrt{\frac{2\cos^2(\theta/2)}{2\sin^2(\theta/2)}}.\sin\frac{\theta}{2}\cos\frac{\theta}{2}\,d\theta$$

$$= -a\int (1+\cos\theta)\,d\theta = -a\left[\cos^{-1}\frac{x}{a}+\sqrt{\frac{a^2-x^2}{a}}\right]+c$$

$$= -a\cos^{-1}\frac{x}{a}-\sqrt{a^2-x^2}+c\,.$$

8. (a) $x=\cos^2\theta$ रखने पर $\Rightarrow dx=-2\cos\theta\sin\theta\,d\theta$, तब

$$\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx = -4\int \sin^2\frac{\theta}{2}\cos\theta\,d\theta$$

$$= -2\int (1-\cos\theta)\cos\theta\,d\theta = \theta+\frac{1}{2}\sin 2\theta-2\sin\theta$$

$$= \theta+\sin\theta\cos\theta-2\sin\theta$$

$$= \cos^{-1}\sqrt{x}+(\sqrt{1-x})(\sqrt{x}-2)+c$$

9. (b) $\int \frac{1}{x^2(x^4+1)^{3/4}}\,dx = \int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$

$1+\frac{1}{x^4}=t$ रखने पर $\Rightarrow \frac{-4}{x^5}dx=dt$,

$$\therefore -\frac{1}{4}\int \frac{dt}{t^{3/4}} = -\frac{1}{4}\frac{4}{1}t^{1/4}+c = -t^{1/4}+c$$

$$= -\left(1+\frac{1}{x^4}\right)^{1/4}+c = -\frac{(x^4+1)^{1/4}}{x}+c.$$

10. (d) $2+\sin 3x=t$ रखने पर $\Rightarrow 3\cos 3x\,dx=dt$, तब

$$\int \sqrt{2+\sin 3x}\cos 3x\,dx = \frac{1}{3}\int t^{1/2}dt = \frac{2}{9}t^{3/2}+c$$

$$= \frac{2}{9}(2+\sin 3x)^{3/2}+c\,.$$

11. (c) $\int \frac{dx}{(2\sin x+\cos x)^2} = \int \frac{dx}{\sin^2 x(2+\cot x)^2} = \int \frac{\operatorname{cosec}^2 x\,dx}{(2+\cot x)^2}$

$(2+\cot x)=t$ रखने पर $\Rightarrow -\operatorname{cosec}^2 x\,dx=dt$

$$= \int \frac{-dt}{t^2} = \frac{1}{t}+c = \frac{1}{2+\cot x}+c.$$

12. (b) $\int \frac{\sqrt{x^2+1}\left[\log(x^2+1)-2\log x\right]}{x^4}\,dx$

$$= \int \sqrt{1+\frac{1}{x^2}}\log\left(1+\frac{1}{x^2}\right).\frac{1}{x^3}\,dx$$

$$\left\{1+\frac{1}{x^2}=t \text{ रखने पर}, -\frac{2}{x^3}dx=dt\right\}$$

$$= -\frac{1}{2}\int \sqrt{t}\log t\,dt = -\frac{1}{2}\left\{\log t\left(\frac{t^{3/2}}{3/2}\right)-\frac{2}{3}\int \frac{1}{t}t^{3/2}dt\right\}$$

$$= -\frac{1}{2}\left\{\frac{2}{3}\log t.t^{3/2}-\frac{2}{3}.\left(\frac{2}{3}t^{3/2}\right)\right\}+c$$

$$= -\frac{1}{3}t^{3/2}\log t + \frac{2}{9}t^{3/2} + c$$

$$= -\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right) - \frac{2}{3}\right] + c\,.$$

13. (c) $\int \sqrt{\frac{\cos x - \cos^3 x}{1-\cos^3 x}}\,dx = \int \sqrt{\frac{\cos x}{1-\cos^3 x}}\sin x\,dx$

$$= -\int \sqrt{\frac{t}{1-t^3}}\,dt = -\int \frac{\sqrt{t}}{\sqrt{1-(t^{3/2})^2}}\,dt = -\frac{2}{3}\int \frac{\frac{3}{2}\sqrt{t}}{\sqrt{1-(t^{3/2})^2}}\,dt$$

$$= \frac{2}{3}\cos^{-1}(t^{3/2}) + c = \frac{2}{3}\cos^{-1}(\cos^{3/2} x) + c\,.$$

14. (a) $l^{r+1}(x) = t$ रखने पर $\Rightarrow \frac{1}{xl(x)l^2(x)....l^r(x)}dx = dt,$

$$\therefore \int \frac{1}{xl(x)l^2(x)....l^r(x)}dx = \int 1\,.\,dt = t + c = l^{r+1}(x) + c.$$

15. (b) $2^{2^{2^x}} = t$ रखने पर $\Rightarrow 2^{2^{2^x}}\,2^{2^x}\,2^x(\log 2)^3\,dx = dt,$

$$\int 2^{2^{2^x}}.2^{2^x}.2^x\,dx = \frac{1}{(\log 2)^3}\int 1\,.\,dt = \frac{t}{(\log 2)^3} + c$$

$$= \frac{2^{2^{2^x}}}{(\log 2)^3} + c\,.$$

16. (d) $\int \frac{1}{(\sin x+4)(\sin x-1)}dx = \frac{1}{5}\int \frac{(\sin x+4)-(\sin x-1)}{(\sin x+4)(\sin x-1)}dx$

$$= \frac{1}{5}\int \frac{1}{\sin x - 1}dx - \frac{1}{5}\int \frac{1}{\sin x + 4}dx$$

$$= \frac{1}{5}\int \frac{2\,dt}{2t - 1 - t^2} - \frac{1}{5}\int \frac{2\,dt}{2t + 4(1+t^2)}$$

$\left(\tan\frac{x}{2} = t \text{ रखने पर}\right)$

$$= -\frac{2}{5}\int \frac{dt}{t^2 - 2t + 1} - \frac{1}{10}\int \frac{dt}{t^2 + \frac{1}{2}t + 1}$$

$$= -\frac{2}{5}\int \frac{1}{(t-1)^2}dt - \frac{1}{10}\int \frac{dt}{\left(t+\frac{1}{4}\right)^2 + \left(\frac{\sqrt{15}}{4}\right)^2}$$

$$= \frac{2}{5}\frac{1}{(t-1)} - \frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4t+1}{\sqrt{15}}\right) + c$$

$$= \frac{2}{5}.\frac{1}{\tan\frac{x}{2} - 1} - \frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4\tan\frac{x}{2}+1}{\sqrt{15}}\right) + c$$

$$\therefore\ A = \frac{2}{5},\quad B = \frac{-2}{5\sqrt{15}},\quad f(x) = \frac{4\tan\frac{x}{2}+1}{\sqrt{15}}.$$

17. (a) $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = \int \frac{dx}{\cos^3 x\sqrt{4\sin x\cos x}}$

$$= \frac{1}{2}\int \frac{dx}{\cos^{7/2} x\ \sin^{1/2} x}$$

$$= \frac{1}{2}\int \frac{\sec^4 x}{\sqrt{\tan x}}dx = \frac{1}{2}\int \frac{(1+\tan^2 x)\sec^2 x}{\sqrt{\tan x}}dx$$

$$= \frac{1}{2}\int \frac{1+t^2}{\sqrt{t}}dt \quad (\tan x = t \text{ रखने पर},\ \therefore \sec^2 x\,dx = dt)$$

$$= \frac{1}{2}\int t^{-1/2}dt + \frac{1}{2}\int t^{3/2}dt = t^{1/2} + \frac{t^{5/2}}{5} + c$$

$$= \sqrt{\tan x} + \frac{1}{5}\tan^{5/2} x + c\,.$$

18. (c) $I = \int \frac{dx}{1 - \cos x - \sin x}$

$$= \int \frac{dx}{1 - \frac{[(1-\tan^2(x/2)]}{[(1+\tan^2(x/2)]} - \frac{2\tan(x/2)}{1+\tan^2(x/2)}}$$

$$= \int \frac{\sec^2(x/2).dx}{1+\tan^2(x/2) - 1 + \tan^2(x/2) - 2\tan(x/2)}$$

$$= \int \frac{\sec^2(x/2)dx}{2\tan^2(x/2) - 2\tan(x/2)} = \int \frac{\frac{1}{2}.\sec^2\left(\frac{x}{2}\right)dx}{\tan^2\left(\frac{x}{2}\right) - \tan\left(\frac{x}{2}\right)}$$

$\tan(x/2) = t$ रखने पर $\Rightarrow \frac{1}{2}\sec^2(x/2)dx = dt$

$$I = \int \frac{dt}{t^2 - t} = \int \frac{dt}{t(t-1)} = \int\left[\frac{1}{t-1} - \frac{1}{t}\right]dt$$

$$= \int \frac{dt}{t-1} - \int \frac{dt}{t} = \log(t-1) - \log t + c = \log\left|\frac{t-1}{t}\right| + c$$

$$= \log\left|\frac{\tan(x/2) - 1}{\tan(x/2)}\right| + c = \log\left|1 - \cot\frac{x}{2}\right| + c\,.$$

19. (a) $I = \int \frac{3^x}{\sqrt{9^x - 1}}dx = \int \frac{3^x dx}{\sqrt{(3^x)^2 - 1}}$

$3^x = t$ रखने पर $\Rightarrow 3^x \log 3\,dx = dt \ \Rightarrow 3^x\,dx = dt/\log 3$

$$\Rightarrow I = \frac{1}{\log 3}\int \frac{dt}{\sqrt{t^2 - 1}} = \frac{1}{\log 3}\log\left[t + \sqrt{t^2 - 1}\right] + c$$

$$= \frac{1}{\log_e 3}\log\left[3^x + \sqrt{9^x - 1}\right] + c\,.$$

20. (a) $a + be^x = t$ रखने पर $\Rightarrow be^x dx = dt,$ तब

$$\int \frac{e^x dx}{\sqrt{a+be^x}} = \frac{1}{b}\int \frac{1}{\sqrt{t}}dt = \frac{2}{b}\sqrt{t} + c = \frac{2\sqrt{a+be^x}}{b} + c.$$

21. (a) $\int u\frac{d^2v}{dx^2}dx - \int v\frac{d^2u}{dx^2}dx$

$$= u\frac{dv}{dx} - \int \frac{du}{dx}.\frac{dv}{dx}dx - v\frac{du}{dx} + \int \frac{dv}{dx}.\frac{du}{dx}dx + c$$

$$= u\frac{dv}{dx} - v\frac{du}{dx} + c.$$

22. (c, d) दिया गया है $f(x)=g(x)$ अब $\int f'(x)g(x)dx$

$= g(x)\int f'(x)dx - \int\left[g'(x)\int f'(x)dx\right]dx + c$

$= g(x)f(x) - \int g'(x)f(x)dx + c$

$\Rightarrow \int f'(x)g(x)dx = g(x)f(x) - \int f'(x)g(x)dx + c$

$\because f(x)=g(x) \Rightarrow f'(x)=g'(x)$ .....(i)

(i)से, $2\int f'(x)g(x)dx = g(x)f(x)+c$

$\therefore \int f'(x)g(x)dx = \frac{1}{2}\{g(x)\}^2 + c$

साथ ही $\int f'(x).g(x)dx = \int f'(x).f(x)dx$

$= \frac{1}{2}\{f(x)\}^2 + c$.

वैकल्पिक : $\int f'(x).g(x)dx = \int f'(x).f(x)dx$

$= \int t\,dt$, $\{f(x)=t \Rightarrow f'(x)dx = dt\}$

$= \frac{t^2}{2}+c = \frac{\{f(x)\}^2}{2}+c = \frac{\{g(x)\}^2}{2}+c$.

23. (a) $\int e^{2x}\left(\frac{\sin 4x-2}{1-\cos 4x}\right)dx = \int \frac{e^{2x}\sin 4x}{1-\cos 4x}dx - 2\int \frac{e^{2x}}{1-\cos 4x}dx$

$= \int e^{2x}\cot 2x\,dx - \int e^{2x}\operatorname{cosec}^2 2x\,dx$

$= \frac{e^{2x}\cot 2x}{2} + \int 2\frac{e^{2x}}{2}\operatorname{cosec}^2 2x\,dx - \int e^{2x}\operatorname{cosec}^2 x\,dx$

$= \frac{1}{2}(e^{2x}\cot 2x)+c.$

24. (a) $\int e^{2x}(2\sin 3x + 3\cos 3x)\,dx$

$= e^{2x}\sin 3x + c$ $\left\{\because \int e^{mx}[mf(x)+f'(x)]dx = e^{mx}f(x)+c\right\}$.

25. (b) $\int \frac{1}{x^2}\log(x^2+a^2)dx = \int x^{-2}\log(x^2+a^2)dx$

$= \frac{-\log(x^2+a^2)}{x} + \int \frac{2x}{(x^2+a^2)}.\frac{1}{x}+c$

$= \frac{-\log(x^2+a^2)}{x} + \frac{2}{a}\tan^{-1}\frac{x}{a}+c.$

26. (b) $I = \int \frac{x^2-1}{x^4+x^2+1}dx = \int \frac{x^2\left(1-\frac{1}{x^2}\right)}{x^2\left[\left(x+\frac{1}{x}\right)^2-1\right]}dx$

$\left(x+\frac{1}{x}\right)=t$ रखने पर $\Rightarrow \left(1-\frac{1}{x^2}\right)dx = dt$

$I = \int \frac{dt}{t^2-1} = \frac{1}{2}\log\left|\frac{t-1}{t+1}\right|+c$

$\therefore I = \frac{1}{2}\log\left|\frac{x^2-x+1}{x^2+x+1}\right|+c \Rightarrow a=\frac{1}{2}, b=\frac{1}{2}$.

27. (a) $\int \frac{x^4}{(x-1)(x^2+1)}dx = \int \frac{x^4-1}{(x-1)(x^2+1)} + \int \frac{1}{(x-1)(x^2+1)}dx$

$= \int \frac{(x+1)(x-1)(x^2+1)}{(x-1)(x^2+1)}dx + \int \frac{dx}{(x-1)(x^2+1)}$

$= \int (x+1)dx + \int \frac{dx}{(x-1)(x^2+1)}$

$= \frac{x^2}{2} + x + \left[\frac{1}{2}\log(x-1) - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x\right]+c$.

28. (b) $\int \frac{x^3-1}{x^3+x}dx = \int \frac{x^3}{x(x^2+1)}dx - \int \frac{1}{x(x^2+1)}dx$

$= \int \frac{x^2}{x^2+1}dx - \int\left(\frac{1}{x}-\frac{x}{x^2+1}\right)dx$

$= \int\left(1-\frac{1}{x^2+1}\right)dx - \int \frac{1}{x}dx + \int \frac{x}{x^2+1}dx$

$= x - \tan^{-1}x - \log x + \log\sqrt{x^2+1} + c.$

29. (b) $\int \frac{1}{x(x^4-1)}dx = \frac{1}{4}\int\left[\frac{4x^3}{(x^4-1)} - \frac{4}{x}\right]dx$

$= \frac{1}{4}[\log(x^4-1) - 4\log x] + c = \frac{1}{4}\log\frac{x^4-1}{x^4}+c$.

30. (d) $I = \int \frac{2x^2+3}{(x^2-1)(x^2+4)}dx$

$\because \frac{2x^2+3}{(x^2-1)(x^2+4)} = \frac{1}{(x^2-1)} + \frac{1}{(x^2+4)}$

$\therefore I = \int \frac{dx}{(x^2-1)} + \int \frac{dx}{x^2+4}$

$I = \frac{1}{2\times 1}\log\left|\frac{x-1}{x+1}\right| + \frac{1}{2}\tan^{-1}\frac{x}{2}+c$

$\Rightarrow I = \frac{1}{2}\log\left(\frac{x-1}{x+1}\right) + \frac{1}{2}\tan^{-1}\frac{x}{2}+c$

$\Rightarrow a=\frac{1}{2},\ b=\frac{1}{2}$.

* * *