
Exercise 1.1

1. Prove that the product of two consecutive positive integers is divisible by 2.

Sol:

Let, $(n - 1)$ and n be two consecutive positive integers

$\therefore$ Their product $= n(n - 1)$

$$= n^2 - n$$

We know that any positive integer is of the form $2q$ or $2q + 1$, for some integer q

When $n = 2q$, we have

$$n^2 - n = (2q)^2 - 2q$$

$$= 4q^2 - 2q$$

$$2q(2q - 1)$$

Then $n^2 - n$ is divisible by 2.

When $n = 2q + 1$, we have

$$n^2 - n = (2q + 1)^2 - (2q + 1)$$

$$= 4q^2 + 4q + 1 - 2q - 1$$

$$= 4q^2 + 2q$$

$$= 2q(2q + 1)$$

Then $n^2 - n$ is divisible by 2.

Hence the product of two consecutive positive integers is divisible by 2.

2. If a and b are two odd positive integers such that $a > b$, then prove that one of the two numbers $\frac{a+b}{2}$ and $\frac{a-b}{2}$ is odd and the other is even.

Sol:

Let $a = 2q + 3$ and $b = 2q + 1$ be two positive odd integers such that $a > b$

$$\text{Now, } \frac{a+b}{2} = \frac{2q+3+2q+1}{2} = \frac{4q+4}{2} = 2q + 2 = \text{an even number}$$

$$\text{and } \frac{a-b}{2} = \frac{(2q+3)-(2q+1)}{2} = \frac{2q+3-2q-1}{2} = \frac{2}{2} = 1 = \text{an odd number}$$

Hence one of the two numbers $\frac{a+b}{2}$ and $\frac{a-b}{2}$ is odd and the other is even for any two positive odd integer

3. Show that the square of an odd positive integer is of the form $8q + 1$, for some integer q .

Sol:

By Euclid's division algorithm

$$a = bq + r, \text{ where } 0 \leq r < b$$

$$\text{Put } b = 4$$

$$a = 4q + r, \text{ where } 0 \leq r < 4$$

If $r = 0$, then $a = 4q$ even

If $r = 1$, then $a = 4q + 1$ odd

If $r = 2$, then $a = 4q + 2$ even

If $r = 3$, then $a = 4q + 3$ odd

Now, $(4q + 1)^2 = (4q)^2 + 2(4q)(1) + (1)^2$

$$= 16q^2 + 8q + 1$$

$$= 8(2q^2 + q) + 1$$

$$= 8m + 1 \text{ where } m \text{ is some integer}$$

Hence the square of an odd integer is of the form $8q + 1$, for some integer q

4. Show that any positive odd integer is of the form $6q + 1$ or, $6q + 3$ or, $6q + 5$, where q is some integer.

Sol:

Let a be any odd positive integer we need to prove that a is of the form $6q + 1$, or $6q + 3$, $6q + 5$, where q is some integer

Since a is an integer consider $b = 6$ another integer applying Euclid's division lemma we get

$$a = 6q + r \text{ for some integer } q \geq 0, \text{ and } r = 0, 1, 2, 3, 4, 5 \text{ since } 0 \leq r < 6.$$

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

However since a is odd so a cannot take the values $6q$, $6q + 2$ and $6q + 4$ (since all these are divisible by 2)

Also, $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$, where k_1 is a positive integer

$6q + 3 = (6q + 2) + 1 = 2(3q + 1) + 1 = 2k_2 + 1$, where k_2 is an integer

$6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k_3 + 1$, where k_3 is an integer

Clearly, $6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where k is an integer

Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are odd numbers.

Therefore, any odd integer can be expressed is of the form

$6q + 1$, or $6q + 3$, $6q + 5$ where q is some integer

Concept insight: In order to solve such problems Euclid's division lemma is applied to two integers a and b the integer b must be taken in accordance with what is to be proved, for example here the integer b was taken 6 because a must be of the form $6q + 1$, $6q + 3$, $6q + 5$

Basic definition of even (divisible by 2) and odd numbers (not divisible by 2) and the fact that addition and multiplication of integers is always an integer are applicable here.

5. Prove that the square of any positive integer is of the form $3m$ or, $3m + 1$ but not of the form $3m + 2$.

Sol:

By Euclid's division algorithm

$$a = bq + r, \text{ where } 0 \leq r < b$$

Put $b = 3$

$$a = 3q + r, \text{ where } 0 \leq r < 3$$

If $r = 0$, then $a = 3q$

If $r = 1$, then $a = 3q + 1$

If $r = 2$, then $a = 3q + 2$

Now, $(3q)^2 = 9q^2$

$$= 3 \times 3q^2$$

$= 3m$, where m is some integer

$$(3q + 1)^2 = (3q)^2 + 2(3q)(1) + (1)^2$$

$$= 9q^2 + 6q + 1$$

$$= 3(3q^2 + 2q) + 1$$

$= 3m + 1$, where m is some integer

$$(3q + 2)^2 = (3q)^2 + 2(3q)(2) + (2)^2$$

$$= 9q^2 + 12q + 4$$

$$= 9q^2 + 12q + 4$$

$$= 3(3q^2 + 4q + 1) + 1$$

$= 3m + 1$, where m is some integer

Hence the square of any positive integer is of the form $3m$, or $3m + 1$

But not of the form $3m + 2$

6. Prove that the square of any positive integer is of the form $4q$ or $4q + 1$ for some integer q .

Sol:

By Euclid's division Algorithm

$$a = bm + r, \text{ where } 0 \leq r \leq b$$

Put $b = 4$

$$a = 4m + r, \text{ where } 0 \leq r \leq 4$$

If $r = 0$, then $a = 4m$

If $r = 1$, then $a = 4m + 1$

If $r = 2$, then $a = 4m + 2$

If $r = 3$, then $a = 4m + 3$

Now, $(4m)^2 = 16m^2$

$$= 4 \times 4m^2$$

$= 4q$ where q is some integer

$$(4m + 1)^2 = (4m)^2 + 2(4m)(1) + (1)^2$$

$$= 16m^2 + 8m + 1$$

$$= 4(4m^2 + 2m) + 1$$

$= 4q + 1$ where q is some integer

$$(4m + 2)^2 = (4m)^2 + 2(4m)(2) + (2)^2$$

$$= 16m^2 + 24m + 4$$

$$= 16m^2 + 24m + 8 + 1$$

$$= 4(4m^2 + 6m + 2) + 1$$

$= 4q + 1$, where q is some integer

Hence, the square of any positive integer is of the form $4q$ or $4q + 1$ for some integer m

7. Prove that the square of any positive integer is of the form $5q$, $5q + 1$, $5q + 4$ for some integer q .

Sol:

By Euclid's division algorithm

$a = bm + r$, where $0 \leq r \leq b$

Put $b = 5$

$a = 5m + r$, where $0 \leq r \leq 4$

If $r = 0$, then $a = 5m$

If $r = 1$, then $a = 5m + 1$

If $r = 2$, then $a = 5m + 2$

If $r = 3$, then $a = 5m + 3$

If $r = 4$, then $a = 5m + 4$

Now, $(5m)^2 = 25m^2$

$= 5(5m^2)$

$= 5q$ where q is some integer

$(5m + 1)^2 = (5m)^2 + 2(5m)(1) + (1)^2$

$= 25m^2 + 10m + 1$

$= 5(5m^2 + 2m) + 1$

$= 5q + 1$ where q is some integer

$(5m + 1)^2 = (5m)^2 + 2(5m)(1)(1)^2$

$= 25m^2 + 10m + 1$

$= 5(5m^2 + 2m) + 1$

$= 5q + 1$ where q is some integer

$(5m + 2)^2 = (5m)^2 + 2(5m)(2) + (2)^2$

$= 25m^2 + 20m + 4$

$= 5(5m^2 + 4m) + 4$

$= 5q + 4$, where q is some integer

$(5m + 3)^2 = (5m)^2 + 2(5m)(3) + (3)^2$

$= 25m^2 + 30m + 9$

$= 25m^2 + 30m + 5 + 4$

$= 5(5m^2 + 6m + 1) + 4$

$= 5q + 1$, where q is some integer

$(5m + 4)^2 = (5m)^2 + 2(5m)(4) + (4)^2$

$= 25m^2 + 40m + 16$

$= 25m^2 + 40m + 15 + 1$

$= 5(5m^2) + 2(5m)(4) + (4)^2$

$= 5q + 1$, where q is some integer

Hence, the square of any positive integer is of the form $5q$ or $5q + 1$, $5q + 4$ for some integer q .

8. Prove that if a positive integer is of the form $6q + 5$, then it is of the form $3q + 2$ for some integer q , but not conversely.

Sol:

Let, $n = 6q + 5$, when q is a positive integer

We know that any positive integer is of the form $3k$, or $3k + 1$, or $3k + 2$

$\therefore q = 3k$ or $3k + 1$, or $3k + 2$

If $q = 3k$, then

$$n = 6q + 5$$

$$= 6(3k) + 5$$

$$= 18k + 5$$

$$= 18k + 3 + 2$$

$$= 3(6k + 1) + 2$$

$$= 3m + 2, \text{ where } m \text{ is some integer}$$

If $q = 3k + 1$, then

$$n = 6q + 5$$

$$= 6(3k + 1) + 5$$

$$= 18k + 6 + 5$$

$$= 18k + 11$$

$$= 3(6k + 3) + 2$$

$$= 3m + 2, \text{ where } m \text{ is some integer}$$

If $q = 3k + 2$, then

$$n = 6q + 5$$

$$= 6(3k + 2) + 5$$

$$= 18k + 12 + 5$$

$$= 18k + 17$$

$$= 3(6k + 5) + 2$$

$$= 3m + 2, \text{ where } m \text{ is some integer}$$

Hence, if a positive integer is of the form $6q + 5$, then it is of the form $3q + 2$ for some integer q .

Conversely

Let $n = 3q + 2$

We know that a positive integer can be of the form $6k + 1$, $6k + 2$, $6k + 3$, $6k + 4$ or $6k + 5$

So, now if $q = 6k + 1$ then

$$n = 3(6k + 1) + 2$$

$$= 18k + 5$$

$$= 6(3k) + 5$$

$= 6m + 5$, where m is some integer

So, now if $q = 6k + 2$ then

$$n = 3(6k + 2) + 2$$

$$= 18k + 8$$

$$= 6(3k + 1) + 2$$

$= 6m + 2$, where m is some integer

Now, this is not of the form $6m + 5$

Hence, if n is of the form $3q + 2$, then it necessarily won't be of the form $6q + 5$ always.

9. Prove that the square of any positive integer of the form $5q + 1$ is of the same form.

Sol:

Let $n = 5q + 1$ where q is a positive integer

$$\therefore n^2 = (5q + 1)^2$$

$$= 25q^2 + 10q + 1$$

$$= 5(5q^2 + 2q) + 1$$

$$= 5m + 1, \text{ where } m \text{ is some integer}$$

Hence, the square of any positive integer of the form $5q + 1$ is of the same form.

10. Prove that the product of three consecutive positive integer is divisible by 6.

Sol:

Let, n be any positive integer. Since any positive integer is of the form $6q$ or $6q + 1$ or $6q + 2$ or, $6q + 3$ or $6q + 4$ or $6q + 5$.

If $n = 6q$, then

$$n(n + 1)(n + 2) = (6q + 1)(6q + 2)(6q + 3)$$

$$= 6[(6q + 1)(3q + 1)(2q + 1)]$$

$$= 6m, \text{ which is divisible by 6}$$

If $n = 6q + 1$, then

$$n(n + 1)(n + 2) = (6q + 2)(6q + 3)(6q + 4)$$

$$= 6[(6q + 1)(3q + 1)(2q + 1)]$$

$$= 6m, \text{ which is divisible by 6}$$

If $n = 6q + 2$, then

$$n(n + 1)(n + 2) = (6q + 2)(6q + 3)(6q + 4)$$

$$= 6[(3q + 1)(2q + 1)(6q + 4)]$$

$$= 6m, \text{ which is divisible by 6.}$$

If $n = 6q + 3$, then

$$n(n + 1)(n + 2) = (6q + 3)(6q + 4)(6q + 5)$$

$$= 6[(6q + 1)(3q + 2)(2q + 5)]$$

$$= 6m, \text{ which is divisible by 6.}$$

If $n = 6q + 4$, then

$$n(n + 1)(n + 2) = (6q + 4)(6q + 5)(6q + 6)$$

$$= 6[(6q + 4)(3q + 5)(2q + 1)]$$

= 6m, which is divisible by 6.

If $n = 6q + 5$, then

$$n(n + 1)(n + 2) = (6q + 5)(6q + 6)(6q + 7)$$

$$= 6[(6q + 5)(q + 1)(6q + 7)]$$

= 6m, which is divisible by 6.

Hence, the product of three consecutive positive integer is divisible by 6.

11. For any positive integer n , prove that $n^3 - n$ is divisible by 6.

Sol:

$$\text{We have } n^3 - n = n(n^2 - 1) = (n - 1)(n)(n + 1)$$

Let, n be any positive integer. Since any positive integer is of the form $6q$ or $6q + 1$ or, $6q + 2$ or, $6q + 3$ or, $6q + 4$ or, $6q + 5$.

If $n = 6q$, then

$$(n - 1)(n)(n + 1) = (6q - 1)(6q)(6q + 1)$$

$$= 6[(6q - 1)(q)(6q + 1)]$$

= 6m, which is divisible by 6

If $n = 6q + 1$, then

$$(n - 1)(n + 1) = (6q)(6q + 1)(6q + 2)$$

$$= 6[(q)(6q + 1)(6q + 2)]$$

= 6m, which is divisible by 6

If $n = 6q + 2$, then

$$(n - 1)(n)(n + 1) = (6q + 1)(6q + 2)(6q + 3)$$

$$= 6[(6q + 1)(3q + 1)(2q + 1)]$$

= 6m, which is divisible by 6

If $n = 6q + 3$, then

$$(n - 1)(n)(n + 1) = (6q + 3)(6q + 4)(6q + 5)$$

$$= 6[(3q + 1)(2q + 1)(6q + 4)]$$

= 6m, which is divisible by 6

If $n = 6q + 4$, then

$$(n - 1)(n)(n + 1) = (6q + 3)(6q + 4)(6q + 5)$$

$$= 6[(2q + 1)(3q + 2)(6q + 5)]$$

= 6m, which is divisible by 6

If $n = 6q + 5$, then

$$(n - 1)(n)(n + 1) = (6q + 4)(6q + 5)(6q + 6)$$

$$= 6[(6q + 4)(6q + 5)(q + 1)]$$

= 6m, which is divisible by 6

Hence, for any positive integer n , $n^3 - n$ is divisible by 6.

Exercise 1.2

1. Define HCF of two positive integers and find the HCF of the following pairs of numbers:

(i) 32 and 54 (ii) 18 and 24 (iii) 70 and 30 (iv) 56 and 88
(v) 475 and 495 (vi) 75 and 243 (vii) 240 and 6552 (viii) 155 and 1385
(ix) 100 and 190 (x) 105 and 120

Sol:

By applying Euclid's division lemma

(i) $5y = 32 \times 1 + 22$

Since remainder $\neq 0$, apply division lemma on division of 32 and remainder 22.

$$32 = 22 \times 1 + 10$$

Since remainder $\neq 0$, apply division lemma on division of 22 and remainder 10.

$$22 = 10 \times 2 + 2$$

Since remainder $\neq 0$, apply division lemma on division of 10 and remainder 2.

$$10 = 2 \times 5 \text{ [remainder 0]}$$

Hence, HCF of 32 and 54 is 2

(ii) By applying division lemma

$$24 = 18 \times 1 + 6$$

Since remainder $= 6$, apply division lemma on divisor of 18 and remainder 6.

$$18 = 6 \times 3 + 0$$

$\therefore$ Hence, HCF of 18 and 24 is 6

(iii) By applying Euclid's division lemma

$$70 = 30 \times 2 + 10$$

Since remainder $\neq 0$, apply division lemma on divisor of 30 and remainder 10.

$$30 = 10 \times 3 + 0$$

$\therefore$ Hence HCF of 70 and 30 is 10.

(iv) By applying Euclid's division lemma

$$88 = 56 \times 1 + 32$$

Since remainder $\neq 0$, apply division lemma on divisor of 56 and remainder 32.

$$56 = 32 \times 1 + 24$$

Since remainder $\neq 0$, apply division lemma on divisor of 32 and remainder 24.

$$32 = 24 \times 1 + 8$$

Since remainder $\neq 0$, apply division lemma on divisor of 24 and remainder 8.

$$24 = 8 \times 3 + 0$$

$\therefore$ HCF of 56 and 88 is 8.

(v) By applying Euclid's division lemma

$$495 = 475 \times 1 + 20$$

Since remainder $\neq 0$, apply division lemma on divisor of 475 and remainder 20.

$$475 = 20 \times 23 + 15$$

Since remainder $\neq 0$, apply division lemma on divisor of 20 and remainder 15.

$$20 = 15 \times 1 + 5$$

Since remainder $\neq 0$, apply division lemma on divisor of 15 and remainder 5.

$$15 = 5 \times 3 + 0$$

$\therefore$ HCF of 475 and 495 is = 5.

- (vi) By applying Euclid's division lemma

$$243 = 75 \times 3 + 18$$

Since remainder $\neq 0$, apply division lemma on divisor of 75 and remainder 18.

$$75 = 18 \times 4 + 3$$

Since remainder $\neq 0$, apply division lemma on divisor of 18 and remainder 3.

$$18 = 3 \times 6 + 0$$

$\therefore$ HCF of 243 and 75 is = 3.

- (vii) By applying Euclid's division lemma

$$6552 = 240 \times 27 + 72$$

Since remainder $\neq 0$, apply division lemma on divisor of 240 and remainder 72.

$$210 = 72 \times 3 + 24$$

Since remainder $\neq 0$, apply division lemma on divisor of 72 and remainder 24.

$$72 = 24 \times 3 + 0$$

$\therefore$ HCF of 6552 and 240 is = 24.

- (viii) By applying Euclid's division lemma

$$1385 = 155 \times 8 + 145$$

Since remainder $\neq 0$, applying division lemma on divisor 155 and remainder 145

$$155 = 145 \times 1 + 10$$

Since remainder $\neq 0$, applying division lemma on divisor 10 and remainder 5

$$10 = 5 \times 2 + 0$$

$\therefore$ Hence HCF of 1385 and 155 = 5.

- (ix) By applying Euclid's division lemma

$$190 = 100 \times 1 + 90$$

Since remainder $\neq 0$, applying division lemma on divisor 100 and remainder 90.

$$90 = 10 \times 9 + 0$$

$\therefore$ HCF of 100 and 190 = 10

- (x) By applying Euclid's division lemma

$$120 = 105 \times 1 + 15$$

Since remainder $\neq 0$, applying division lemma on divisor 105 and remainder 15.

$$105 = 15 \times 7 + 0$$

$\therefore$ HCF of 105 and 120 = 15

2. Use Euclid's division algorithm to find the HCF of

- (i) 135 and 225 (ii) 196 and 38220

Sol:

- (i) 135 and 225

Step 1: Since $225 > 135$. Apply Euclid's division lemma to $a = 225$ and $b = 135$ to find q and r such that $225 = 135q + r$, $0 \leq r < 135$

On dividing 225 by 135 we get quotient as 1 and remainder as '90'

i.e., $225 = 135 \times 1 + 90$

Step 2: Remainder 5 which is 90 7, we apply Euclid's division lemma to $a = 135$ and $b = 90$ to find whole numbers q and r such that $135 = 90 \times q + r$ $0 \leq r < 90$ on dividing 135 by 90 we get quotient as 1 and remainder as 45

i.e., $135 = 90 \times 1 + 45$

Step3: Again remainder $r = 45$ to so we apply division lemma to $a = 90$ and $b = 45$ to find q and r such that $90 = 45 \times q + r$. $0 \leq r < 45$. On dividing 90 by 45 we get quotient as 2 and remainder as 0 i.e., $90 = 2 \times 45 + 0$

Step 4: Since the remainder = 0, the divisor at this stage will be HCF of (135, 225) Since the divisor at this stage is 45. Therefore the HCF of 135 and 225 is 45.

(ii) 867 and 255:

Step 1: Since $867 > 255$, apply Euclid's division

Lemma a to $a = 867 = 255q + r$, $0 < r < 255$

On dividing 867 by 255 we get quotient as 3 and the remainder as low

Step 2: Since the remainder 102 to, we apply the division lemma to $a = 255$ and $b = 102$ to find $255 = 102q + 51 = 102r - 151$

Step 3: Again remainder 0 is non-zero, so we apply the division lemma to $a = 102$ and $b = 51$ to find whole numbers q and r such that $102 = q \times r$ when $0 \leq r < 51$

On dividing 102 by 51 quotient = 2 and remainder is '0'

i.e., $102 = 51 \times 2 + 0$

Since the remainder is zero, the divisional this stage is the HCF.

Since the divisor at this stage is 51, $\therefore$ HCF of 867 and 255 is '51'.

3. Find the HCF of the following pairs of integers and express it as a linear combination of them.

(i) 963 and 657 (ii) 592 and 252 (iii) 506 and 1155 (iv) 1288 and 575

Sol:

(i) 963 and 657

By applying Euclid's division lemma $963 = 657 \times 1 + 306$... (i)

Since remainder $\neq 0$, apply division lemma on divisor 657 and remainder 306

$657 = 306 \times 2 + 45$ (ii)

Since remainder $\neq 0$, apply division lemma on divisor 306 and remainder 45

$306 = 45 \times 6 + 36$ (iii)

Since remainder $\neq 0$, apply division lemma on divisor 45 and remainder 36

$45 = 36 \times 1 + 9$ (iv)

Since remainder $\neq 0$, apply division lemma on divisor 36 and remainder 9

$36 = 9 \times 4 + 0$

$$\therefore \text{HCF} = 9$$

$$\text{Now } 9 = 45 - 36 \times 1 \quad [\text{from (iv)}]$$

$$= 45 - [306 - 45 \times 6] \times 1 \quad [\text{from (iii)}]$$

$$= 45 - 306 \times 1 + 45 \times 6$$

$$= 45 \times 7 - 306 \times 1$$

$$= 657 \times 7 - 306 \times 14 - 306 \times 1 \quad [\text{from (ii)}]$$

$$= 657 \times 7 - 306 \times 15$$

$$= 657 \times 7 - [963 - 657 \times 1] \times 15 \quad [\text{from (i)}]$$

$$= 657 \times 22 - 963 \times 15$$

(ii) 595 and 252

By applying Euclid's division lemma

$$595 = 252 \times 2 + 91 \quad \dots (i)$$

Since remainder $\neq 0$, apply division lemma on divisor 252 and remainder 91

$$252 = 91 \times 2 + 70 \quad \dots (ii)$$

Since remainder $\neq 0$, apply division lemma on divisor 91 and remainder 70

$$91 = 70 \times 1 + 21 \quad \dots (iii)$$

Since remainder $\neq 0$, apply division lemma on divisor 70 and remainder 20

$$70 = 21 \times 3 + 7 \quad \dots (iv)$$

Since remainder $\neq 0$, apply division lemma on divisor 21 and remainder 7

$$21 = 7 \times 3 + 0$$

$$\text{H.C.F} = 7$$

$$\text{Now, } 7 = 70 - 21 \times 3 \quad [\text{from (iv)}]$$

$$= 70 - [90 - 70 \times 1] \times 3 \quad [\text{from (iii)}]$$

$$= 70 - 91 \times 3 + 70 \times 3$$

$$= 70 \times 4 - 91 \times 3$$

$$= [252 - 91 \times 2] \times 4 - 91 \times 3 \quad [\text{from (ii)}]$$

$$= 252 \times 4 - 91 \times 8 - 91 \times 3$$

$$= 252 \times 4 - 91 \times 11$$

$$= 252 \times 4 - [595 - 252 \times 2] \times 11 \quad [\text{from (i)}]$$

$$= 252 \times 4 - 595 \times 11 + 252 \times 22$$

$$= 252 \times 6 - 595 \times 11$$

(iii) 506 and 1155

By applying Euclid's division lemma

$$1155 = 506 \times 2 + 143 \quad \dots (i)$$

Since remainder $\neq 0$, apply division lemma on division 506 and remainder 143

$$506 = 143 \times 3 + 77 \quad \dots (ii)$$

Since remainder $\neq 0$, apply division lemma on division 143 and remainder 77

$$143 = 77 \times 1 + 66 \quad \dots (iii)$$

Since remainder $\neq 0$, apply division lemma on division 77 and remainder 66

$$77 = 66 \times 1 + 11 \quad \dots (iv)$$

Since remainder $\neq 0$, apply division lemma on divisor 36 and remainder 9

$$66 = 11 \times 6 + 0$$

$$\therefore \text{HCF} = 11$$

$$\text{Now, } 11 = 77 - 6 \times 11 \quad [\text{from (iv)}]$$

$$= 77 - [143 - 77 \times 1] \times 1 \quad [\text{from (iii)}]$$

$$= 77 - 143 \times 1 + 77 \times 1$$

$$= 77 \times 2 - 143 \times 1$$

$$= [506 - 143 \times 3] \times 2 - 143 \times 1 \quad [\text{from (ii)}]$$

$$= 506 \times 2 - 143 \times 6 - 143 \times 1$$

$$= 506 \times 2 - 143 \times 7$$

$$= 506 \times 2 - [1155 - 506 \times 27 \times 7] \quad [\text{from (i)}]$$

$$= 506 \times 2 - 1155 \times 7 + 506 \times 14$$

$$= 506 \times 16 - 115 \times 7$$

(iv) 1288 and 575

By applying Euclid's division lemma

$$1288 = 575 \times 2 + 138 \quad \dots(i)$$

Since remainder $\neq 0$, apply division lemma on division 575 and remainder 138

$$575 = 138 \times 4 + 23 \quad \dots(ii)$$

Since remainder $\neq 0$, apply division lemma on division 138 and remainder 23 ... (iii)

$$\therefore \text{HCF} = 23$$

$$\text{Now, } 23 = 575 - 138 \times 4 \quad [\text{from (ii)}]$$

$$= 575 - [1288 - 575 \times 2] \times 4 \quad [\text{from (i)}]$$

$$= 575 - 1288 \times 4 + 575 \times 8$$

$$= 575 \times 9 - 1288 \times 4$$

4. Express the HCF of 468 and 222 as $468x + 222y$ where x, y are integers in two different ways.

Sol:

Given integers are 468 and 222 where $468 > 222$.

By applying Euclid's division lemma, we get $468 = 222 \times 2 + 24 \dots(i)$

Since remainder $\neq 0$, apply division lemma on division 222 and remainder 24

$$222 = 24 \times 9 + 6 \dots(ii)$$

Since remainder $\neq 0$, apply division lemma on division 24 and remainder 6

$$24 = 6 \times 4 + 0 \dots(iii)$$

We observe that the remainder = 0, so the last divisor 6 is the HCF of the 468 and 222

From (ii) we have

$$6 = 222 - 24 \times 9$$

$$\Rightarrow 6 = 222 - [468 - 222 \times 2] \times 9 \quad [\text{Substituting } 24 = 468 - 222 \times 2 \text{ from (i)}]$$

$$\Rightarrow 6 = 222 - 468 \times 9 + 222 \times 18$$

$$\Rightarrow 6 = 222 \times 19 - 468 \times 9$$

$$\Rightarrow 6 = 222y + 468x, \text{ where } x = -9 \text{ and } y = 19$$

5. If the HCF of 408 and 1032 is expressible in the form $1032m - 408 \times 5$, find m .

Sol:

General integers are 408 and 1032 where $408 < 1032$

By applying Euclid's division lemma, we get

$$1032 = 408 \times 2 + 216$$

Since remainder $\neq 0$, apply division lemma on division 408 and remainder 216

$$408 = 216 \times 1 + 192$$

Since remainder $\neq 0$, apply division lemma on division 216 and remainder 192

$$216 = 192 \times 1 + 24$$

Since remainder $\neq 0$, apply division lemma on division 192 and remainder 24

$$192 = 24 \times 8 + 32$$

We observe that 32m under in 0. So the last divisor 24 is the H.C.F of 408 and 1032

$$\therefore 216 = 1032m - 408 \times 5$$

$$\Rightarrow 1032m = 24 + 408 \times 5$$

$$\Rightarrow 1032m = 24 + 2040$$

$$\Rightarrow 1032m = 2064$$

$$\Rightarrow m = \frac{2064}{1032} = 2$$

6. If the HCF of 657 and 963 is expressible in the form $657x + 963y - 15$, find x .

Sol:

657 and 963

By applying Euclid's division lemma

$$963 = 657 \times 1 + 306$$

Since remainder $\neq 0$, apply division lemma on division 657 and remainder 306

$$657 = 306 \times 2 + 45$$

Since remainder $\neq 0$, apply division lemma on division 306 and remainder 45

$$306 = 45 \times 6 + 36$$

Since remainder $\neq 0$, apply division lemma on division 45 and remainder 36

$$45 = 36 \times 1 + 9$$

Since remainder $\neq 0$, apply division lemma on division 36 and remainder 9

$$36 = 9 \times 4 + 0$$

$$\therefore \text{HCF} = 9$$

$$\text{Given HCF} = 657x + 963y - 15$$

$$\Rightarrow 9 = 657x - 1445y$$

$$\Rightarrow 9 + 1445y = 657x$$

$$\Rightarrow 657x = 1445y$$

$$\Rightarrow x = \frac{1445y}{657}$$

$$\Rightarrow x = 22$$

7. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Sol:

Members in arms = 616

Members in Band = 32

$\therefore$ Maximum numbers of columns

= HCF of 616 and 32

By applying Euclid's division lemma

$$616 = 32 \times 19 + 8$$

$$32 = 8 \times 4 + 0$$

$$\therefore \text{HCF} = 8$$

Hence the maximum remainder number of columns in which they can each is 8

8. Find the largest number which divides 615 and 963 leaving remainder 6 in each case.

Sol:

The required number when the divides 615 and 963

Leaves remainder 616 is means $615 - 6 = 609$ and $963 - 957$ are completely divisible by the number

$\therefore$ the required number

= HCF of 609 and 957

By applying Euclid's division lemma

$$957 = 609 \times 1 + 348$$

$$609 = 348 \times 1 + 261$$

$$348 = 261 \times 1 + 87$$

$$261 = 87 \times 3 + 0$$

$$\text{HCF} = 87$$

Hence the required number is '87'

9. Find the greatest number which divides 285 and 1249 leaving remainders 9 and 7 respectively.

Sol:

The require number when divides 285 and 1249, leaves remainder 9 and 7, this means $285 - 9 = 276$ and $1249 - 7 = 1242$ are completely divisible by the number

$\therefore$ The required number = HCF of 276 and 1242

By applying Euclid's division lemma

$$1242 = 276 \times 4 + 138$$

$$276 = 138 \times 2 + 0$$

$$\therefore \text{HCF} = 138$$

Hence remainder is = 0

Hence required number is 138

10. Find the largest number which exactly divides 280 and 1245 leaving remainders 4 and 3, respectively.

Sol:

The required number when divides 280 and 1245 leaves the remainder 4 and 3, this means $280 - 4 = 276$ and $1245 - 3 = 1242$ are completely divisible by the number

$\therefore$ The required number = HCF of 276 and 1242

By applying Euclid's division lemma

$$1242 = 276 \times 4 + 138$$

$$276 = 138 \times 2 + 0$$

$$\therefore \text{HCF} = 138$$

Hence the required numbers is 138

11. What is the largest number that divides 626, 3127 and 15628 and leaves remainders of 1, 2 and 3 respectively.

Sol:

The required number when divides 626, 3127 and 15628, leaves remainder 1, 2 and 3. This means $626 - 1 = 625$, $3127 - 2 = 3125$ and

$15628 - 3 = 15625$ are completely divisible by the number

$\therefore$ The required number = HCF of 625, 3125 and 15625

First consider 625 and 3125

By applying Euclid's division lemma

$$3125 = 625 \times 5 + 0$$

$$\text{HCF of } 625 \text{ and } 3125 = 625$$

Now consider 625 and 15625

By applying Euclid's division lemma

$$15625 = 625 \times 25 + 0$$

$$\therefore \text{HCF of } 625, 3125 \text{ and } 15625 = 625$$

Hence required number is 625

12. Find the greatest number that will divide 445, 572 and 699 leaving remainders 4, 5 and 6 respectively.

Sol:

The required number when divides 445, 572 and 699 leaves remainders 4, 5 and 6

This means $445 - 4 = 441$, $572 - 5 = 567$ and

$699 - 6 = 693$ are completely divisible by the number

$\therefore$ The required number = HCF of 441, 567 and 693

First consider 441 and 567

By applying Euclid's division lemma

$$567 = 441 \times 1 + 126$$

$$441 = 126 \times 3 + 63$$

$$126 = 63 \times 2 + 0$$

$$\therefore \text{HCF of 441 and 567} = 63$$

Now consider 63 and 693

By applying Euclid's division lemma

$$693 = 63 \times 11 + 0$$

$$\therefore \text{HCF of 441, 567 and 693} = 63$$

Hence required number is 63

13. Find the greatest number which divides 2011 and 2623 leaving remainders 9 and 5 respectively.

Sol:

The required number when divides 2011 and 2623

Leaves remainders 9 and 5 the means

$2011 - 9 = 2002$ and $2623 - 5 = 2618$ are completely divisible by the number

$$\therefore \text{The required number} = \text{HCF of 2002 and 2618}$$

By applying Euclid's division lemma

$$2618 = 2002 \times 1 + 616$$

$$2002 = 616 \times 3 + 154$$

$$616 = 754 \times 4 + 0$$

$$\therefore \text{HCF of 2002 and 2618} = 154$$

Hence required number is 154

14. The length, breadth and height of a room are 8 m 25 cm, 6 m 75 cm and 4 m 50 cm, respectively. Determine the longest rod which can measure the three dimensions of the room exactly.

Sol:

$$\text{Length of room} = 8\text{m } 25\text{cm} = 825 \text{ cm}$$

$$\text{Breadth of room} = 6\text{m } 75\text{cm} = 675 \text{ cm}$$

$$\text{Height of room} = 4\text{m } 50\text{cm} = 450 \text{ cm}$$

$$\therefore \text{The required longest rod}$$

$$= \text{HCF of 825, 675 and 450}$$

First consider 675 and 450

By applying Euclid's division lemma

$$675 = 450 \times 1 + 225$$

$$450 = 225 \times 2 + 0$$

$$\therefore \text{HCF of 675 and 450} = 225$$

Now consider 625 and 825

By applying Euclid's division lemma

$$825 = 225 \times 3 + 150$$

$$225 = 150 \times 1 + 75$$

$$150 = 75 \times 2 + 0$$

$$\text{HCF of 825, 675 and 450} = 75$$

15. 105 goats, 140 donkeys and 175 cows have to be taken across a river. There is only one boat which will have to make many trips in order to do so. The lazy boatman has his own conditions for transporting them. He insists that he will take the same number of animals in every trip and they have to be of the same kind. He will naturally like to take the largest possible number each time. Can you tell how many animals went in each trip?

Sol:

$$\text{Number of goats} = 205$$

$$\text{Number of donkey} = 140$$

$$\text{Number of cows} = 175$$

$\therefore$ The largest number of animals in one trip = HCF of 105, 140 and 175

First consider 105 and 140

By applying Euclid's division lemma

$$140 = 105 \times 1 + 35$$

$$105 = 35 \times 3 + 0$$

$\therefore$ HCF of 105 and 140 = 35

Now consider 35 and 175

By applying Euclid's division lemma

$$175 = 35 \times 5 + 0$$

$$\text{HCF of 105, 140 and 175} = 35$$

16. 15 pastries and 12 biscuit packets have been donated for a school fete. These are to be packed in several smaller identical boxes with the same number of pastries and biscuit packets in each. How many biscuit packets and how many pastries will each box contain?

Sol:

$$\text{Number of pastries} = 15$$

$$\text{Number of biscuit packets} = 12$$

$\therefore$ The required no of boxes to contain equal number = HCF of 15 and 12

By applying Euclid's division lemma

$$15 = 12 \times 1 + 3$$

$$12 = 2 \times 3 + 0$$

$\therefore$ No. of boxes required = 3

Hence each box will contain $\frac{15}{3} = 5$ pastries and $\frac{12}{3} = 4$ biscuit packets

17. A mason has to fit a bathroom with square marble tiles of the largest possible size. The size of the bathroom is 10 ft. by 8 ft. What would be the size in inches of the tile required that has to be cut and how many such tiles are required?

Sol:

Size of bathroom = 10ft by 8ft

= (10 × 12) inch by (8 × 12) inch

= 120 inch by 96 inch

The largest size of tile required = HCF of 120 and 96

By applying Euclid's division lemma

$$120 = 96 \times 1 + 24$$

$$96 = 24 \times 4 + 0$$

$$\therefore \text{HCF} = 24$$

$$\therefore \text{Largest size of tile required} = 24 \text{ inches}$$

$$\therefore \text{No. of tiles required} = \frac{\text{Area of bathroom}}{\text{area of 2 tile}}$$

$$= \frac{120 \times 96}{24 \times 24}$$

$$= 5 \times 4$$

$$= 20 \text{ tiles}$$

18. Two brands of chocolates are available in packs of 24 and 15 respectively. If I need to buy an equal number of chocolates of both kinds, what is the least number of boxes of each kind I would need to buy?

Sol:

Number of chocolates of 1st brand in one pack = 24

Number of chocolates of 2nd brand in one pack = 15

$\therefore$ The least number of chocolates I need to purchase

= LCM of 24 and 15

$$= 2 \times 2 \times 2 \times 3 \times 5$$

$$= 120$$

$$\therefore \text{The number of packet of 1st brand} = \frac{120}{24} = 5$$

$$\text{And the number of packet of 2nd brand} = \frac{120}{15} = 8$$

$\therefore$ Largest size of tile required = 24 inches

$$\therefore \text{No of tiles required} = \frac{\text{area of bath room}}{\text{area of 1 tile}} = \frac{120 \times 96}{24 \times 24} = 5 \times 4 = 20 \text{ tiles}$$

No of chocolates of 1st brand in one pack = 24

No of chocolate of 2nd brand in one pack = 15

$\therefore$ The least number of chocolates I need to purchase

= LCM of 24 and 15

$$= 2 \times 2 \times 2 \times 3 \times 5$$

$$= 120$$

$\therefore$ The number of packet of 1st brand = $\frac{120}{24} = 5$

All the number of packet of 2nd brand = $\frac{120}{15} = 8$

19. 144 cartons of Coke Cans and 90 cartons of Pepsi Cans are to be stacked in a Canteen. If each stack is of the same height and is to contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

Sol:

Number of cartons of coke cans = 144

Number of cartons of pepsi cans = 90

$\therefore$ The greatest number of cartons in one stock = HCF of 144 and 90

By applying Euclid's division lemma

$$144 = 90 \times 1 + 54$$

$$90 = 54 \times 1 + 36$$

$$54 = 36 \times 1 + 18$$

$$36 = 18 \times 2 + 0$$

$$\therefore \text{HCF} = 18$$

Hence the greatest number cartons in one stock = 18

20. During a sale, colour pencils were being sold in packs of 24 each and crayons in packs of 32 each. If you want full packs of both and the same number of pencils and crayons, how many of each would you need to buy?

Sol:

Number of color pencils in one pack = 24

No of crayons in pack = 32

$\therefore$ The least number of both colors to be purchased

= LCM of 24 and 32

$$= 2 \times 2 \times 2 \times 2 \times 3$$

$$= 96$$

$$\therefore \text{Number of packs of pencils to be bought} = \frac{96}{24} = 4$$

$$\text{And number of packs of crayon to be bought} = \frac{96}{32} = 3$$

21. A merchant has 120 liters of oil of one kind, 180 liters of another kind and 240 liters of third kind. He wants to sell the oil by filling the three kinds of oil in tins of equal capacity. What should be the greatest capacity of such a tin?

Sol:

Quantity of oil A = 120 liters

Quantity of oil B = 180 liters

Quantity of oil C = 240 liters

We want to fill oils A, B and C in tins of the same capacity

$\therefore$ The greatest capacity of the tin that can hold oil. A, B and C = HCF of 120, 180 and 240

By fundamental theorem of arithmetic

$$120 = 2^3 \times 3 \times 5$$

$$180 = 2^2 \times 3^2 \times 5$$

$$240 = 2^4 \times 3 \times 5$$

$$\text{HCF} = 2^2 \times 3 \times 5 = 4 \times 3 \times 5 = 60 \text{ litres}$$

The greatest capacity of tin = 60 liters
