

**Exercise 2.1**

1. Find the zeroes of each of the following quadratic polynomials and verify the relationship between the zeroes and their co efficient:

(i)  $f(x) = x^2 - 2x - 8$

(ii)  $g(s) = 4s^2 - 4s + 1$

(iii)  $h(t) = t^2 - 15$

(iv)  $p(x) = x^2 + 2\sqrt{2}x + 6$

(v)  $q(x) = \sqrt{3}x^2 + 10x + 7\sqrt{3}$

(vi)  $f(x) = x^2 - (\sqrt{3} + 1)x + \sqrt{3}$

(vii)  $g(x) = a(x^2 + 1) - x(a^2 + 1)$

(viii)  $6x^2 - 3 - 7x$

**Sol:**

(i)  $f(x) = x^2 - 2x - 8$

$$f(x) = x^2 - 2x - 8 = x^2 - 4x + 2x - 8$$

$$= x(x - 4) + 2(x - 4)$$

$$= (x + 2)(x - 4)$$

Zeroes of the polynomials are -2 and 4

$$\text{Sum of the zeroes} = \frac{-\text{co efficient of } x}{\text{co efficient of } x}$$

$$-2 + 4 = \frac{-(-2)}{1}$$

$$2 = 2$$

$$\text{Product of the zeroes} = \frac{\text{constant term}}{\text{co efficient of } x^2}$$

$$= 24 = \frac{-8}{1}$$

$$-8 = -8$$

∴ Hence the relationship verified

(ii)  $9(5) = 45 - 45 + 1 = 45^2 - 25 - 25 + 1 = 25(25 - 1) - 1(25 - 1)$

$$= (25 - 1)(25 - 1)$$

Zeroes of the polynomials are  $\frac{1}{2}$  and  $\frac{1}{2}$

$$\text{Sum of zeroes} = \frac{-\text{co efficient of } s}{\text{co efficient of } s^2}$$

$$\frac{1}{2} + \frac{1}{2} = \frac{-(-4)}{4}$$

$$1 = 1$$

$$\text{Product of the zeroes} = \frac{\text{constant term}}{\text{co efficient of } s^2}$$

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \Rightarrow \frac{1}{4} = \frac{1}{4}$$

∴ Hence the relationship verified.

(iii)  $h(t) = t^2 - 15 = (t^2) - (\sqrt{15})^2 = (t + \sqrt{15})(t - \sqrt{15})$

zeroes of the polynomials are  $-\sqrt{15}$  and  $\sqrt{15}$

$$\text{sum of zeroes} = 0$$

$$-\sqrt{15} + \sqrt{15} = 0$$

$$0 = 0$$

$$\text{Product of zeroes} = \frac{-15}{1}$$

$$-\sqrt{15} \times \sqrt{15} = -15$$

$$-15 = -15$$

$\therefore$  Hence the relationship verified.

$$\begin{aligned} \text{(iv)} \quad p(x) &= x^2 + 2\sqrt{2}x - 6 = x^2 + 3\sqrt{2}x + \sqrt{2} \times 3\sqrt{2} \\ &= x(x + 3\sqrt{2}) - \sqrt{2}(2 + 3\sqrt{2}) = (x - \sqrt{2})(x + 3\sqrt{2}) \end{aligned}$$

Zeroes of the polynomial are  $3\sqrt{2}$  and  $-3\sqrt{2}$

$$\text{Sum of the zeroes} = \frac{-3\sqrt{2}}{1}$$

$$\sqrt{2} - 3\sqrt{2} = -2\sqrt{2}$$

$$-2\sqrt{2} = -2\sqrt{2}$$

$$\text{Product of zeroes} \Rightarrow \sqrt{2} \times -3\sqrt{2} = -\frac{6}{1}$$

$$-6 = -6$$

Hence the relationship verified

$$\begin{aligned} \text{(v)} \quad 2(x) &= \sqrt{3}x^2 + 10x + 7\sqrt{3} = \sqrt{3}x^2 + 7x + 3x + 7\sqrt{3} \\ &= \sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3}) \\ &= (\sqrt{3}x + 7)(x + \sqrt{3}) \end{aligned}$$

Zeroes of the polynomials are  $-\sqrt{3}, \frac{-7}{\sqrt{3}}$

$$\text{Sum of zeroes} = \frac{-10}{\sqrt{3}}$$

$$\Rightarrow -\sqrt{3} - \frac{7}{\sqrt{3}} = \frac{-10}{\sqrt{3}} \Rightarrow \frac{-10}{\sqrt{3}} = \frac{-10}{\sqrt{3}}$$

$$\text{Product of zeroes} = \frac{7\sqrt{3}}{3} \Rightarrow \frac{\sqrt{3}x-7}{\sqrt{30}} = 7$$

$$\Rightarrow 7 = 7$$

Hence, relationship verified.

$$\begin{aligned} \text{(vi)} \quad f(x) &= x^2 - (\sqrt{3} + 1)x + \sqrt{3} = x^2 - \sqrt{3}x - x + \sqrt{3} \\ &= x(x - \sqrt{3}) - 1(x - \sqrt{3}) \\ &= (x - 1)(x - \sqrt{3}) \end{aligned}$$

Zeroes of the polynomials are 1 and  $\sqrt{3}$

$$\text{Sum of zeroes} = \frac{-\{\text{coefficient of } x\}}{\text{coefficient of } x^2} = \frac{-[-\sqrt{3}-1]}{1}$$

$$1 + \sqrt{3} = \sqrt{3} + 1$$

$$\text{Product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{\sqrt{3}}{1}$$

$$1 \times \sqrt{3} = \sqrt{3} = \sqrt{3} = \sqrt{3}$$

$\therefore$  Hence, relationship verified

$$\begin{aligned} \text{(vii)} \quad g(x) &= a[(x^2 + 1) - x(a^2 + 1)]^2 = ax^2 + a - a^2x - x \\ &= ax^2 - [(a^2 + 1) - x] + 0 = ax^2 - a^2x - x + a \end{aligned}$$

$$= ax(x - a) - 1(x - a) = (x - a)(ax - 1)$$

Zeros of the polynomials =  $\frac{1}{a}$  and  $a$

$$\text{Sum of the zeroes} = \frac{-[-a^2-1]}{a}$$

$$\Rightarrow \frac{1}{a} + a = \frac{a^2+1}{a} \Rightarrow \frac{a^2+1}{a} = \frac{a^2+1}{a}$$

$$\text{Product of zeroes} = \frac{a}{a}$$

$$\Rightarrow \frac{1}{a} \times a = \frac{a}{a} \Rightarrow \frac{a^2+1}{a} = \frac{a^2+1}{a}$$

$$\text{Product of zeroes} = \frac{a}{a} \Rightarrow 1 = 1$$

Hence relationship verified

$$(viii) \quad 6x^2 - 3 - 7x = 6x^2 - 7x - 3 = (3x + 11)(2x - 3)$$

Zeros of polynomials are  $+\frac{3}{2}$  and  $-\frac{1}{3}$

$$\text{Sum of zeroes} = \frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{co efficient of } x)}{\text{co efficient of } x^2}$$

$$\text{Product of zeroes} = \frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{constant term}}{\text{co efficient of } x^2}$$

$\therefore$  Hence, relationship verified.

2. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $f(x) = ax^2 + bx + c$ , then evaluate:

$$(i) \quad \alpha - \beta$$

$$(v) \quad \alpha^4 + \beta^4$$

$$(viii) \quad a \left[ \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} \right] +$$

$$(ii) \quad \frac{1}{\alpha} - \frac{1}{\beta}$$

$$(vi) \quad \frac{1}{a\alpha+b} + \frac{1}{a\beta+b}$$

$$b \left[ \frac{\alpha}{a} + \frac{\beta}{a} \right]$$

$$(iii) \quad \frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$$

$$(vii) \quad \frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b}$$

$$(iv) \quad \alpha^2\beta + \alpha\beta^2$$

**Sol:**

$$f(x) = ax^2 + bx + c$$

$$\alpha + \beta = \frac{-b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

since  $\alpha + \beta$  are the roots (or) zeroes of the given polynomials

$$(i) \quad \alpha - \beta$$

The two zeroes of the polynomials are

$$\frac{-b+\sqrt{b^2-4ac}}{2a} - \left( \frac{-b-\sqrt{b^2-4ac}}{2a} \right) = -b + \frac{\sqrt{b^2-4ac} + b + \sqrt{b^2-4ac}}{2a} = \frac{2\sqrt{b^2-4ac}}{2a} = \frac{\sqrt{b^2-4ac}}{a}$$

$$(ii) \quad \frac{1}{\alpha} - \frac{1}{\beta} = \frac{\beta-\alpha}{\alpha\beta} = \frac{-(\alpha-\beta)}{\alpha\beta} \dots (i)$$

$$\text{From (i) we know that } \alpha - \beta = \frac{\sqrt{b^2-4ac}}{a} \text{ [from (i)] } \alpha\beta = \frac{c}{a}$$

$$\text{Putting the values in the (a)} = - \left( \frac{\sqrt{b^2-4ac} \times a}{a \times c} \right) = \frac{-\sqrt{b^2-4ac}}{c}$$

$$(iii) \quad \frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$$

$$\Rightarrow \left[ \frac{\alpha + \beta}{\alpha\beta} \right] - 2\alpha\beta$$

$$\Rightarrow \frac{-b}{a} \times \frac{a}{c} - 2\frac{c}{a} = -2\frac{c}{a} - \frac{b}{c} = \frac{-ab-2c^2}{ac} - \left[ \frac{b}{c} + \frac{2c}{a} \right]$$

$$(iv) \quad \alpha^2 \beta + \alpha \beta^2$$

$$\alpha \beta (\alpha + \beta)$$

$$= \frac{c}{a} \left( \frac{-b}{a} \right)$$

$$= \frac{-bc}{a^2}$$

$$(v) \quad \alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2 + \beta^2$$

$$= ((\alpha + \beta)^2 - 2\alpha\beta)^2 - 2(\alpha\beta)^2$$

$$= \left[ \left( -\frac{b}{a} \right)^2 - 2\frac{c}{a} \right]^2 - \left[ 2 \left( \frac{c}{a} \right)^2 \right]$$

$$= \left[ \frac{b^2 - 2ac}{a^2} \right]^2 - \frac{2c^2}{a^2}$$

$$= \frac{(b^2 - 2ac)^2 - 2a^2c^2}{a^4}$$

$$(vi) \quad \frac{1}{a\alpha + b} + \frac{1}{a\beta + b}$$

$$\Rightarrow \frac{a\beta + b + a\alpha + b}{(3\alpha + b)(a\beta + b)}$$

$$= \frac{a(\alpha + \beta) + 2b}{a^2\alpha\beta + ab\alpha + ab\beta + b^2}$$

$$= \frac{a(\alpha + \beta) + b}{a^2\alpha\beta + \alpha\beta(\alpha^2\beta) + b^2}$$

$$= \frac{a \times \frac{a+2b}{a}}{a \times \frac{c}{a} + \frac{abc(-b) + b^2}{a}} = \frac{b}{ac - b^2 + b^2} = \frac{b}{ac}$$

$$(vii) \quad \frac{\beta}{a\alpha + b} + \frac{\alpha}{a\beta + b}$$

$$= \frac{\beta(a\beta + b) + \alpha(a\alpha + b)}{(a\alpha + b)(a\beta + b)}$$

$$= \frac{a\beta^2 + b\beta + a\alpha^2 + b\alpha}{a^2\alpha\beta + ab\alpha + ab\beta + b^2}$$

$$= \frac{a\alpha^2 + a\beta^2 + b\beta^2 + b\alpha}{a \times \frac{c}{a} + ab(\alpha + \beta) + b^2}$$

$$= \frac{a[(\alpha^2 + \beta^2) + b(\alpha + \beta)]}{ac + ab + b \left( \frac{-b}{a} \right) + b^2}$$

$$= \frac{a[(\alpha + \beta)^2 - 2\alpha\beta] + bx - \frac{b}{a}}{ac}$$

$$= \frac{a \left[ \frac{b^2 - 2c}{a} - \frac{b^2}{a} \right]}{ac} = \frac{a \times \left[ \frac{b^2 - 2c}{a} \right] - b^2}{ac} = \frac{-2}{a}$$

$$(viii) \quad a \left[ \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} \right] + b \left[ \frac{\alpha}{a} + \frac{\beta}{a} \right]$$

$$= a \left[ \frac{\alpha^3 + \beta^3}{\alpha\beta} \right] + b \left( \frac{\alpha^2 + \beta^2}{\alpha\beta} \right)$$

$$\begin{aligned}
 &= \frac{\alpha[(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)]}{\alpha\beta} + b(\alpha + \beta)^2 - 2\alpha\beta \\
 &= \frac{\alpha\left[\left(\frac{-b^3}{a^3}\right) + \frac{3b}{a} \cdot \frac{c}{a} + b\left(\frac{b^2}{a^2} - \frac{2c}{a}\right)\right]}{\frac{c}{a}} \\
 &= \frac{a^2}{c} \left[ \frac{-b^3}{a^3} + \frac{3bc}{a^2} + \frac{b^3}{a^2} - \frac{2bc}{a} \right] \\
 &= \frac{-a^2b^3}{ca^3} + \frac{3a^2bc}{ca^2} + \frac{b^3a^2}{a^2c} - \frac{2bca^2}{ac} \\
 &= \frac{-b^3}{ac} + 3b + \frac{b^3}{ac} - 2b \\
 &= b
 \end{aligned}$$

3. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $f(x) = 6x^2 + x - 2$ , find the value of  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

**Sol:**

$$f(x) = 6x^2 - x - 2$$

Since  $\alpha$  and  $\beta$  are the zeroes of the given polynomial

$$\therefore \text{Sum of zeroes } [\alpha + \beta] = \frac{-1}{6}$$

$$\text{Product of zeroes } (\alpha\beta) = \frac{-1}{3}$$

$$= \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{\left(\frac{-1}{6}\right)^2 - 2 \times \left(\frac{-1}{3}\right)}{\frac{-1}{3}} = \frac{\frac{1}{36} - \frac{2}{3}}{\frac{-1}{3}} = \frac{\frac{1-24}{36}}{\frac{-1}{3}}$$

$$= \frac{\frac{-23}{36}}{\frac{-1}{3}} = \frac{-23}{12}$$

4. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $f(x) = x^2 - x - 4$ , find the value of  $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$

**Sol:**

Since  $\alpha + \beta$  are the zeroes of the polynomial:  $x^2 - x - 4$

$$\text{Sum of the roots } (\alpha + \beta) = 1$$

$$\text{Product of the roots } (\alpha\beta) = -4$$

$$\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{\alpha + \beta}{\alpha\beta} - \alpha\beta$$

$$= \frac{1}{-4} + 4 = \frac{-1}{4} + 4 = \frac{-1+16}{4} = \frac{15}{4}$$

5. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $p(x) = 4x^2 - 5x - 1$ , find the value of  $\alpha^2\beta + \alpha\beta^2$ .

**Sol:**

Since  $\alpha$  and  $\beta$  are the roots of the polynomial:  $4x^2 - 5x - 1$

$$\therefore \text{Sum of the roots } \alpha + \beta = \frac{5}{4}$$

$$\text{Product of the roots } \alpha\beta = \frac{-1}{4}$$

$$\text{Hence } \alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta) = \frac{5}{4} \left( \frac{-1}{4} \right) = \frac{-5}{16}$$

6. If  $\alpha$  and  $3$  are the zeros of the quadratic polynomial  $f(x) = x^2 + x - 2$ , find the value of  $\frac{1}{\alpha} - \frac{1}{\beta}$ .

**Sol:**

Since  $\alpha$  and  $\beta$  are the roots of the polynomial  $x^2 + x - 2$

$$\therefore \text{Sum of roots } \alpha + \beta = 1$$

$$\text{Product of roots } \alpha\beta = -2 \Rightarrow -\frac{1}{\beta}$$

$$\begin{aligned} &= \frac{\beta - \alpha}{\alpha\beta} \cdot \frac{(\alpha - \beta)}{\alpha\beta} \\ &= \frac{\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}}{\alpha\beta} \\ &= \frac{\sqrt{1+8}}{+2} = \frac{3}{2} \end{aligned}$$

7. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $f(x) = x^2 - 5x + 4$ , find the value of  $\frac{1}{\alpha} - \frac{1}{\beta} - 2\alpha\beta$

**Sol:**

Since  $\alpha$  and  $\beta$  are the roots of the quadratic polynomial

$$f(x) = x^2 - 5x + 4$$

$$\text{Sum of roots} = \alpha + \beta = 5$$

$$\text{Product of roots} = \alpha\beta = 4$$

$$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\beta + \alpha}{\alpha\beta} - 2\alpha\beta = \frac{5}{4} - 2 \times 4 = \frac{5}{4} - 8 = \frac{-27}{4}$$

8. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $f(t) = t^2 - 4t + 3$ , find the value of  $\alpha^4\beta^3 + \alpha^3\beta^4$

**Sol:**

Since  $\alpha$  and  $\beta$  are the zeroes of the polynomial  $f(t) = t^2 - 4t + 3$

$$\text{Since } \alpha + \beta = 4$$

$$\text{Product of zeroes } \alpha\beta = 3$$

$$\text{Hence } \alpha^4\beta^3 + \alpha^3\beta^4 = \alpha^3\beta^3(\alpha + \beta) = [3]^3[4] = 108$$

9. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $p(y) = 5y^2 - 7y + 1$ , find the value of  $\frac{1}{\alpha} + \frac{1}{\beta}$

**Sol:**

Since  $\alpha$  and  $\beta$  are the zeroes of the polynomials

$$p(y) = 5y^2 - 7y + 1$$

$$\text{Sum of the zeroes } \alpha + \beta = \frac{7}{5}$$

$$\text{Product of zeroes} = \alpha\beta = \frac{1}{5}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{7 \times 5}{5 \times 1} = 7$$

10. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $p(s) = 3s^2 - 6s + 4$ , find the value of

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2 \left[ \frac{1}{\alpha} + \frac{1}{\beta} \right] + 3\alpha\beta$$

**Sol:**

Since  $\alpha$  and  $\beta$  are the zeroes of the polynomials

$$\text{Sum of the zeroes } \alpha + \beta = \frac{6}{3}$$

$$\text{Product of the zeroes } \alpha\beta = \frac{4}{3}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2 \left[ \frac{1}{\alpha} + \frac{1}{\beta} \right] + 3\alpha\beta$$

$$\Rightarrow \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2 \left[ \frac{\alpha + \beta}{\alpha\beta} \right] + 3\alpha\beta$$

$$\Rightarrow \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} + 2 \left[ \frac{\alpha + \beta}{\alpha\beta} \right] + 3\alpha\beta$$

$$= \frac{[2]^2 - 2 \times \frac{4}{3} + 2 \left[ \frac{2 \times 3}{4} \right] + 3 \left[ \frac{4}{3} \right]}{\frac{4}{3}}$$

$$= \frac{4 - \frac{8}{3}}{\frac{4}{3}} + 7 \Rightarrow \frac{4}{3} \times \frac{3}{4} (1 + 7) \Rightarrow 8$$

11. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $f(x) = x^2 - px + q$ , prove that

$$\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$$

**Sol:**

Since  $\alpha$  and  $\beta$  are the roots of the polynomials

$$f(x) = x^2 - px + q$$

$$\text{sum of zeroes} = p = \alpha + \beta$$

$$\text{Product of zeroes} = q = \alpha\beta$$

$$\text{LHS} = \frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta^2} = \frac{(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$$

$$= \frac{[(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$$

$$= \frac{[(p)^2 - 2q]^2 - 2q^2}{q}$$

$$\begin{aligned}
 &= \frac{p^4 + 4q^2 - 2p^2 \cdot 2q - 2q^2}{q^2} \\
 &= \frac{p^4 + 2q^2 - 4p^2q}{q^2} = \frac{p^4}{q^2} + 2 - \frac{4p^2}{q} \\
 &= \frac{p^4}{q^2} - \frac{4p^2}{q^2} = \frac{p^4}{q^2} + 2 - \frac{4p^2}{q} \\
 &= \frac{p^4}{q^2} - \frac{4p^2}{q} + 2
 \end{aligned}$$

12. If the squared difference of the zeros of the quadratic polynomial  $f(x) = x^2 + px + 45$  is equal to 144, find the value of  $p$ .

**Sol:**

Let the two zeroes of the polynomial be  $\alpha$  and  $\beta$

$$f(x) = x^2 + px + 45$$

$$\text{sum of the zeroes} = -p$$

$$\text{Product of zeroes} = 45$$

$$\Rightarrow (\alpha - \beta)^2 - 4\alpha\beta = 144$$

$$\Rightarrow p^2 - 4 \times 45 = 144$$

$$\Rightarrow p^2 = 144 + 180$$

$$\Rightarrow p^2 = 324$$

$$p = \pm 18$$

13. If the sum of the zeros of the quadratic polynomial  $f(t) = kt^2 + 2t + 3k$  is equal to their product, find the value of  $k$ .

**Sol:**

Let the two zeroes of the  $f(t) = kt^2 + 2t + 3k$  be  $\alpha$  and  $\beta$

$$\text{Sum of the zeroes } (\alpha + \beta)$$

$$\text{Product of the zeroes } \alpha\beta$$

$$\frac{-2}{k} = \frac{3k}{k}$$

$$-2k = 3k^2$$

$$2k + 3k^2 = 0$$

$$k(3k + 2) = 0$$

$$k = 0$$

$$k = \frac{-2}{3}$$

14. If one zero of the quadratic polynomial  $f(x) = 4x^2 - 8kx - 9$  is negative of the other, find the value of  $k$ .

**Sol:**

Let the two zeroes of one polynomial

$$f(x) = 4x^2 - 8kx - 9 \text{ be } \alpha, -\alpha$$

$$\alpha \times \alpha = \frac{-9}{4}$$

$$t\alpha^2 = \frac{+9}{4}$$

$$\alpha = \frac{+3}{2}$$

$$\text{Sum of zeroes} = \frac{8k}{4} = 0$$

$$\text{Hence } 8k = 0$$

$$\text{Or } k = 0$$

15. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $f(x) = x^2 - 1$ , find a quadratic polynomial whose zeroes are  $\frac{2\alpha}{\beta}$  and  $\frac{2\beta}{\alpha}$

**Sol:**

$$f(x) = x^2 - 1$$

$$\text{sum of zeroes } \alpha + \beta = 0$$

$$\text{Product of zeroes } \alpha\beta = -1$$

$$\text{Sum of zeroes} = \frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} = \frac{2\alpha^2 + 2\beta^2}{\alpha\beta}$$

$$= \frac{2((\alpha+\beta)^2 - 2\alpha\beta)}{\alpha\beta}$$

$$= \frac{2[(0)^2 - 2 \times -1]}{-1}$$

$$= \frac{2(2)1}{-1}$$

$$= -4$$

$$\text{Product of zeroes} = \frac{2\alpha \times 2\beta}{\alpha\beta} = \frac{4\alpha\beta}{\alpha\beta}$$

$$\text{Hence the quadratic equation is } x^2 - (\text{sum of zeroes})x + \text{product of zeroes} \\ = k(x^2 + 4x + 14)$$

16. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $f(x) = x^2 - 3x - 2$ , find a quadratic polynomial whose zeroes are  $\frac{1}{2\alpha+\beta} + \frac{1}{2\beta+\alpha}$ .

**Sol:**

$$f(x) = x^2 - 3x - 2$$

$$\text{Sum of zeroes } [\alpha + \beta] = 3$$

$$\text{Product of zeroes } [\alpha\beta] = -2$$

$$\text{Sum of zeroes} = \frac{1}{2\alpha+\beta} + \frac{1}{2\beta+\alpha}$$

$$= \frac{2\beta+\alpha+2\alpha+\beta}{(2\alpha+\beta)(2\beta+\alpha)}$$

$$= \frac{3\alpha+3\beta}{2(\alpha^2+\beta^2)+5\alpha\beta}$$

$$= \frac{3 \times 3}{2[2(\alpha+\beta)^2 - 2\alpha\beta + 5 \times (-2)]}$$

$$= \frac{9}{2[9]-10} = \frac{9}{16}$$

$$\text{Product of zeroes} = \frac{1}{\alpha+\beta} \times \frac{1}{2\beta+\alpha} = \frac{1}{4\alpha\beta+\alpha\beta+2\alpha^2+2\beta^2}$$

$$= \frac{1}{5 \times -2 + 2[(\alpha+\beta)^2 - 2\alpha\beta]}$$

$$= \frac{1}{-10 + 2[9+4]}$$

$$= \frac{1}{10+26}$$

$$= \frac{1}{16}$$

$$\text{Quadratic equation} = x^2 - [\text{sum of zeroes}]x + \text{product of zeroes}$$

$$= x^2 - \frac{9x}{16} + \frac{1}{16}$$

$$= k \left[ x^2 - \frac{9x}{16} + \frac{1}{16} \right]$$

17. If  $\alpha$  and  $\beta$  are the zeros of a quadratic polynomial such that  $\alpha + \beta = 24$  and  $\alpha - \beta = 8$ , find a quadratic polynomial having  $\alpha$  and  $\beta$  as its zeros.

**Sol:**

$$\alpha + \beta = 24$$

$$\alpha - \beta = 8$$

.....

$$2\alpha = 32$$

$$\alpha = 16$$

$$\beta = 8$$

$$\alpha\beta = 16 \times 8 = 128$$

Quadratic equation

$$\Rightarrow x^2 - (\text{sum of zeroes})x + \text{product of zeroes}$$

$$\Rightarrow k[x^2 - 24x + 128]$$

18. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $f(x) = x^2 - p(x+1) - c$ , show that  $(\alpha+1)(\beta+1) = 1 - c$ .

**Sol:**

$$f(x) = x^2 - p(x+1) - c = x^2 - px - p - c$$

$$\text{Sum of zeroes} = \alpha + \beta = p$$

$$\text{Product of zeroes} = -p - c = \alpha\beta$$

$$(\alpha+1)(\beta+1) = \alpha\beta + \alpha + \beta + 1 = -p - c + p + 1$$

$$= 1 - c = \text{R.H.S}$$

$\therefore$  Hence proved

19. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $f(x) = x^2 - 2x + 3$ , find a polynomial whose roots are (i)  $\alpha + 2, \beta + 2$  (ii)  $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$

**Sol:**

$$f(x) = x^2 - 2x + 3$$

$$\text{Sum of zeroes} = 2 = (\alpha + \beta)$$

$$\text{Product of zeroes} = 3 = (\alpha \beta)$$

$$(i) \text{ sum of zeroes} = (\alpha + 2) + (\beta + 2) = \alpha + \beta + 4 = 2 + 4 = 6$$

$$\text{Product of zeroes} = (\alpha + 2)(\beta + 2)$$

$$= \alpha \beta + 2\alpha + 2\beta + 4 = 3 + 2(2) + 4 = 11$$

$$\text{Quadratic equation} = x^2 - 6x + 11 = k[x^2 - 6x + 11]$$

$$(ii) \text{ sum of zeroes} = \frac{\alpha-1}{\alpha+1} + \frac{\beta-1}{\beta+1}$$

$$= \frac{(\alpha-1)(\beta+1) + (\beta-1)(\alpha+1)}{(\alpha+1)(\beta+1)}$$

$$= \frac{\alpha\beta + \alpha - \beta - 1 + \alpha\beta + \beta - \alpha - 1}{3+2+1}$$

$$= \frac{3-1+3-1}{3+2+1} = 4 = \frac{2}{3}$$

$$\text{Product of zeroes} = \frac{\alpha-1}{\beta\alpha+1} \times \frac{\beta-1}{\alpha+1} = \frac{\alpha-1-\alpha-\alpha\beta+1}{\alpha\beta+\alpha+\beta+1}$$

$$= \frac{3-(\alpha+\beta)+1}{3+2+1} = \frac{2}{6} = \frac{1}{3}$$

$$\text{Quadratic equation on } x^2 - \frac{2}{3} \times \frac{+1}{3} = 1 \left[ \frac{x^2-2x}{3} + \frac{1}{3} \right]$$

20. If  $\alpha$  and  $\beta$  are the zeroes of the polynomial  $f(x) = x^2 + px + q$ , form a polynomial whose zeroes are  $(\alpha + \beta)^2$  and  $(\alpha - \beta)^2$ .

**Sol:**

$$f(x) = x^2 + p + q$$

$$\text{Sum of zeroes} = p = \alpha + \beta$$

$$\text{Product of zeroes} = q = \alpha \beta$$

$$\text{Sum of the new polynomial} = (\alpha + \beta)^2 + (\alpha - \beta)^2$$

$$= (-p)^2 + \alpha^2 + \beta^2 - 2\alpha\beta$$

$$= p^2 + (\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta$$

$$= p^2 + p^2 - 4q$$

$$= 2p^2 - 4q$$

$$\text{Product of zeroes} = (\alpha + \beta)^2 \times (\alpha - \beta)^2 = [-p]^2 \times (p^2 - 4q) = (p^2 - 4q)p^2$$

$$\text{Quadratic equation} = x^2 - [2p^2 - 4q] + p^2[-4q + p]$$

$$f(x) = k\{x^2 - 2(p^2 - 28)x + p^2(q^2 - 4q)\}$$

**Exercise 2.2**

1. Verify that the numbers given alongside of the cubic polynomials below are their zeros. Also, verify the relationship between the zeros and coefficients in each case:

(i)  $f(x) = 2x^3 + x^2 - 5x + 2$ ;  $\frac{1}{2}, 1, -2$

(ii)  $g(x) = x^3 - 4x^2 + 5x - 2$ ;  $2, 1, 1$

**Sol:**

(i)  $f(x) = 2x^3 + x^2 - 5x + 2$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$

$$= \frac{2}{8} + \frac{1}{4} - \frac{5}{2} + 2 = \frac{-4}{2} + 2 = 0$$

$$f(1) = 2(1)^3 + (1)^2 - 5(1) + 2 = 2 + 1 - 5 + 2 = 0$$

$$f(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$

$$= -16 + 4 + 10 + 2$$

$$= -16 + 16 = 0$$

$$= \alpha + \beta + \gamma = \frac{-b}{a}$$

$$\frac{1}{2} + 1 - 2 = \frac{-1}{2}$$

$$\frac{1}{2} - 1 = \frac{-1}{2}$$

$$\frac{1}{2} = \frac{-1}{2}$$

$$\alpha\beta + \beta\gamma + r\alpha = \frac{c}{a}$$

$$\frac{1}{2} \times 1 + 1 \times -2 + -2 \times \frac{1}{2} = \frac{-5}{2}$$

$$\frac{1}{2} - 2 - 1 = \frac{-5}{2}$$

$$\frac{-5}{2} = \frac{-5}{2}$$

(ii)  $g(x) = x^3 - 4x^2 + 5x - 2$

$$g(2) = (2)^3 - 4(2)^2 + 5(2) - 2 = 8 - 16 + 10 - 2 = 18 - 18 = 0$$

$$g(1) = [1]^3 - 4[1]^2 + 5[1] - 2 = 1 - 4 + 5 - 2 = 6 - 6 = 0$$

$$\alpha + \beta + \gamma = \frac{-b}{a} \quad (2) + 1 + 1 = -(-4) = 4 = 4$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$2 \times 1 + 1 \times 1 + 1 \times 2 = 5$$

$$2 + 1 + 2 = 5$$

$$5 = 5$$

$$\alpha\beta\gamma = -(-2)$$

$$2 \times 1 \times 1 = 2$$

$$2 = 2$$

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and product of its zeros as 3,  $-1$  and  $-3$  respectively.

**Sol:**

Any cubic polynomial is of the form  $ax^3 + bx^2 + cx + d = x^3 -$

*sum of zeroes* ( $x^2$ ) [*product of zeroes*] + *sum of the products of its zeroes*  $\times$  - *product of zeroes*

$$= x^3 - 2x^2 + (3 - x) + 3$$

$$= k [x^3 - 3x^2 - x - 3]$$

$k$  is any non-zero real numbers

3. If the zeros of the polynomial  $f(x) = 2x^3 - 15x^2 + 37x - 30$  are in A.P., find them.

**Sol:**

Let  $\alpha = a - d$ ,  $\beta = a$  and  $\gamma = a + d$  be the zeroes of polynomial.

$$f(x) = 2x^3 - 15x^2 + 37x - 30$$

$$\alpha + \beta + \gamma = -\left(\frac{-15}{2}\right) = \frac{15}{2}$$

$$\alpha\beta\gamma = -\left(\frac{-30}{2}\right) = 15$$

$$a - d + a + a + d = \frac{15}{2} \text{ and } a(a - d)(a + a) = 15$$

$$3a = \frac{15}{2}, a = \frac{5}{2}$$

$$a(a^2 - d^2) = 15$$

$$a^2 - d^2 = \frac{15 \times 2}{5} \Rightarrow \left(\frac{5}{2}\right)^2 - d^2 = 6 \Rightarrow \frac{25-6}{4} = d^2$$

$$d^2 = \frac{1}{4} \Rightarrow d = \frac{1}{2}$$

$$\therefore \alpha = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$$

$$\beta = \frac{5}{2} = \frac{5}{2}$$

$$\gamma = \frac{5}{2} + \frac{1}{2} = 3$$

4. Find the condition that the zeros of the polynomial  $f(x) = x^3 + 3px^2 + 3qx + r$  may be in A.P.

**Sol:**

$$f(x) = x^3 + 3px^2 + 3qx + q$$

Let  $a - d, a, a + d$  be the zeroes of the polynomial

$$\text{The sum of zeroes} = \frac{-b}{a}$$

$$a + a - d + a + d = \frac{b}{a}$$

$$3a = -3p$$

$$a = -p$$

Since  $a$  is the zero of the polynomial  $f(x)$  therefore  $f(a) = 0 \Rightarrow [a]^2 + 3pa^2 + 3qa + r = 0$

$$\therefore f(a) = 0 \Rightarrow [a]^2 + 3pa^2 + 3qa + r = 0$$

$$\Rightarrow p^3 + 3p(-p)^2 + 3q(-p) + r = 0$$

$$\Rightarrow -p^3 + 3p^2 - pq + r = 0$$

$$\Rightarrow 2p^3 - pq + r = 0$$

5. If the zeroes of the polynomial  $f(x) = ax^3 + 3bx^2 + 3cx + d$  are in A.P., prove that  $2b^3 - 3abc + a^2d = 0$

**Sol:**

Let  $a - d, a, a + d$  be the zeroes of the polynomial  $f(x)$

$$\text{The sum of zeroes} \Rightarrow a - d + a + a + d = \frac{-3b}{a}$$

$$\Rightarrow +3a = -\frac{3b}{a} \Rightarrow a = \frac{-3b}{a \times 3} a = \frac{-b}{a}$$

$$f(a) = 0 \Rightarrow a(a)^2 + 3b(a)^2 + 3c(a) + d = 0$$

$$= a \left( \frac{-b}{a} \right)^3 + \frac{3b^2}{a^2} - \frac{3bc}{a} + d = 0$$

$$\Rightarrow \frac{2b^3}{a^2} - \frac{3bc}{a} + d = 0$$

$$\Rightarrow \frac{2b^3 - 3abc + a^2d}{a^2} = 0$$

$$\Rightarrow 2b^3 - 3abc + a^2d = 0$$

6. If the zeroes of the polynomial  $f(x) = x^3 - 12x^2 + 39x + k$  are in A.P., find the value of  $k$ .

**Sol:**

$$f(x) = x^3 - 12x^2 + 39x - k$$

Let  $a - d, a, a + d$  be the zeroes of the polynomial  $f(x)$

$$\text{The sum of the zeroes} = 12$$

$$3a = 12$$

$$a = 4$$

$$f(a), -a(x)^3 - l^2(4)^2 + 39(4) + k = 0$$

$$64 - 192 + 156 + k = 0$$

$$= -28 = k$$

$$k = -28$$

## Exercise 2.3

1. Apply division algorithm to find the quotient  $q(x)$  and remainder  $r(x)$  on dividing  $f(x)$  by  $g(x)$  in each of the following:

(i)  $f(x) = x^3 - 6x^2 + 11x - 6$ ,  $g(x) = x^2 + x + 1$

(ii)  $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 105$ ,  $g(x) = 2x^2 + 7x + 1$

(iii)  $f(x) = 4x^3 + 8x^2 + 8x + 7$ ,  $g(x) = 2x^2 - x + 1$

(iv)  $f(x) = 15x^3 - 20x^2 + 13x - 12$ ,  $g(x) = x^2 - 2x + 2$

**Sol:**

(i)  $f(x) = x^3 - 6x^2 + 11x - 6$

$g(x) = x^2 + x + 1$

$$\begin{array}{r|l}
 & x - 7 \\
 x^2 + x + 1 & x^3 - 6x^2 + 11x - 6 \\
 & \underline{x^3 + x^2 + x} \\
 & -7x^2 - 7x - 7 \\
 & \underline{-7x^2 - 7x - 7} \\
 & 17x - 1
 \end{array}$$

(ii)  $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 105$ ,  $g(x) = 2x^2 + 7x + 1$

$$\begin{array}{r|l}
 & 5x^2 - 9x - 2 \\
 2x^2 + 7x + 1 & 10x^4 + 17x^3 - 62x^2 + 30x - 3 \\
 & \underline{10x^4 + 35x^3 + 5x^2} \\
 & -18x^3 - 67x^2 + 30x \\
 & \underline{-18x^3 + 63x^2 + 9x} \\
 & -4x^2 + 39x - 3 \\
 & \underline{\pm 4x^2 \pm 14x \pm 2} \\
 & 53x - 1
 \end{array}$$

(iii)  $f(x) = 4x^3 + 8x^2 + 8x + 7$ ,  $g(x) = 2x^2 - x + 1$

$$\begin{array}{r|l}
 & 2x - 5 \\
 2x^2 - x + 1 & 4x^3 + 8x^2 + 8x + 7 \\
 & \underline{4x^3 + 2x^2 + 2x} \\
 & 10x^2 + 6x + 7 \\
 & \underline{10x^2 + 5x + 5} \\
 & 11x - 2
 \end{array}$$

(iv)  $f(x) = 15x^3 - 20x^2 + 13x - 12$ ,  $g(x) = x^2 - 2x + 2$

$$\begin{array}{r|l}
 & 15x + 10 \\
 x^2 - 2x + 2 & 15x^3 - 20x^2 + 13x - 12 \\
 & \underline{15x^3 + 30x^2 + 30x} \\
 & 10x^2 - 17x - 12 \\
 & \underline{10x^2 + 20x + 20} \\
 & 3x - 32
 \end{array}$$

2. Check whether the first polynomial is a factor of the second polynomial by applying the division algorithm:

(i)  $g(t) = t^2 - 3; f(t) = 2t^4 + 3t^3 - 2t^2 - 9t$

(ii)  $g(x) = x^2 - 3x + 1, f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

(iii)  $g(x) = 2x^2 - x + 3, f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

**Sol:**

(i)  $g(t) = t^2 - 3; f(t) = 2t^4 + 3t^3 - 2t^2 - 9t$

|           |                           |
|-----------|---------------------------|
| $t^2 - 3$ | $2t^2 + 3t + 4$           |
|           | $2t^4 + 3t^3 - 2t^2 - 9t$ |
|           | $2t^2 - 6t^2$             |
|           | <hr/>                     |
|           | $3t^3 + 4t - 9t$          |
|           | $3t^3 + 4t - 9t$          |
|           | <hr/>                     |
|           | $4t^2 - 12$               |
|           | $4t^2 \mp 12$             |

(ii)  $g(x) = x^2 - 3x + 1, f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

|                |                             |
|----------------|-----------------------------|
| $x^2 - 3x + 1$ | $x^2 - 1$                   |
|                | $x^5 - 4x^3 + x^2 + 3x + 1$ |
|                | $x^5 - 3x^3 + x^2$          |
|                | <hr/>                       |
|                | $-x^3 + 3x + 1$             |
|                | $-x^3 + 3x - 1$             |
|                | <hr/>                       |
|                | $2$                         |

(iii)  $g(x) = 2x^2 - x + 3, f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

|                |                                     |
|----------------|-------------------------------------|
| $2x^2 - x + 3$ | $3x^3 + x^2 - 2x - 5$               |
|                | $6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$ |
|                | $6x^5 - 3x^4 + 9x^3$                |
|                | <hr/>                               |
|                | $2x^4 - 5x^3 - 5x^2$                |
|                | $2x^4 \mp x^3 \pm 3x^2$             |
|                | <hr/>                               |
|                | $-4x^3 - 8x^2 - x$                  |
|                | $\mp 4x^3 \pm 2x^2 - 6x$            |
|                | <hr/>                               |
|                | $-10x^2 - 5x - 15$                  |
|                | $\mp 10x \pm 15x \mp 15$            |
|                | <hr/>                               |
|                | $0$                                 |

3. Obtain all zeros of the polynomial  $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$ , if two of its zeros are  $-2$  and  $-1$ .

**Sol:**

$$f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$$

If the two zeroes of the polynomial are  $-2$  and  $-1$ , then its factors are  $(x + 2)$  and  $(x + 1)$

$$(x + 2)(x + 1) = x^2 + x + 2x = x^2 + 3x + 2$$

|                |                                |
|----------------|--------------------------------|
| $x^2 + 3x + 2$ | $2x^2 - 5x - 3$                |
|                | $2x^4 + x^3 - 14x^2 - 19x - 6$ |
|                | $2x^4 + 6x^3 + 4x^2$           |
|                | $-5x^3 - 18x^2 - 19x$          |
|                | $-5x^3 \mp 15x^2 \mp 10x$      |
|                | $-3x^2 - 9x - 6$               |
|                | $-3x^2 - 9x - 6$               |

$$\therefore 2x^4 + x^3 - 14x^2 - 19x - 6$$

$$= (2x^2 - 5x - 3)[x^2 + 3x + 2] = [2x + 1][x - 3][x + 2][x + 1]$$

$$\therefore \text{zero all } x = \frac{-1}{2}, 3, -2, -1$$

4. Obtain all zeros of  $f(x) = x^3 + 13x^2 + 32x + 20$ , if one of its zeros is  $-2$ .

**Sol:**

$$f(x) = x^3 + 13x^2 + 32x + 20$$

|         |                          |
|---------|--------------------------|
| $x + 2$ | $x^2 + 11x + 10$         |
|         | $x^3 + 13x^2 + 32x + 20$ |
|         | $x^3 \pm 2x^2$           |
|         | $11x^2 + 32x + 20$       |
|         | $11x^2 \pm 22x$          |
|         | $10x + 20$               |
|         | $10x + 20$               |
|         | $0$                      |

$$(x^2 + 11x + 10) = x^2 + 10x + x + 20(x + 10) + 1(x + 10) = (x + 1)(x + 10)$$

$\therefore$  The zeroes of the polynomial are  $-1, -10, -2$ .

5. Obtain all zeros of the polynomial  $f(x) = x^4 - 3x^2 = x^2 + 9x - 6$  if two of its zeros are  $-\sqrt{3}$ , and  $\sqrt{3}$ .

**Sol:**

$$f(x) = (x^2 - 3x + 2) = (x + \sqrt{3})(x - \sqrt{3}) = x^2 - 3$$

|           |                             |
|-----------|-----------------------------|
| $x^2 - 3$ | $x^2 - 3x + 2$              |
|           | $x^4 - 3x^2 = x^2 + 9x - 6$ |
|           | $x^4 - 3x^2$                |
|           | $-3x^2 + 2x^2 + 9x$         |
|           | $-3x^2 \pm 9x$              |
|           | $2x^2 - 6$                  |
|           | $2x^2 - 6$                  |

$$(x^2 - 3)(x^2 - 3x + 2) = (x + \sqrt{3})(x - \sqrt{3})(x^2 - 2x - x + 2)$$

$$= (x + \sqrt{3})(x - \sqrt{3})(x - 2)(x - 2)$$

Zeroes are  $-\sqrt{3}, \sqrt{3}, 1, 2$

6. Find all zeros of the polynomial  $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$ , if its two zeroes are  $-\sqrt{\frac{3}{2}}$  and  $\sqrt{\frac{3}{2}}$

**Sol:**

If the zeroes of the polynomial are  $-\sqrt{\frac{3}{2}}$  and  $\sqrt{\frac{3}{2}}$

Its factors are  $\left(x + \frac{\sqrt{3}}{2}\right)\left(x - \sqrt{\frac{3}{2}}\right) = \frac{x^2 - 3}{2}$

$$\begin{aligned} x &= -1, 2, \sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}} \\ &= [2x^2 - 2x - 4]\left(x^2 - \frac{3}{2}\right) \\ &= (2x^2 - 4x + 2x - 4)\left(x + \sqrt{\frac{3}{2}}\right) \\ &= [2[x(x + 2) + 2(x - 2)]] \\ &= \left[x + \frac{\sqrt{3}}{2}\right]\left[x - \sqrt{\frac{3}{2}}\right] \\ &= (x + 2)(x - 2)\left[x + \sqrt{\frac{3}{2}}\right]\left[x - \sqrt{\frac{3}{2}}\right] \\ x &= -1, 2, \sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}} \end{aligned}$$

7. What must be added to the polynomial  $f(x) = x^4 + 2x^3 - 2x^2 + x - 1$  so that the resulting polynomial is exactly divisible by  $x^2 + 2x - 3$ ?

**Sol:**

|                |                                                                                                                                                        |
|----------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|
| $x^2 + 2x - 3$ | $\begin{array}{r} x^2 - 1 \\ \hline x^4 + 2x^3 - 2x^2 + x - 1 \\ x^4 + 2x^3 - 3x^2 \\ \hline x^2 + x - 1 \\ x^2 + 2x - 3 \\ \hline -x + 2 \end{array}$ |
|----------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|

we must add  $x - 2$  in order to get the resulting polynomial exactly divisible by  $x^2 + 2x - 3$

8. What must be subtracted from the polynomial  $x^4 + 2x^3 - 13x^2 - 12x + 21$ , so that the resulting polynomial is exactly divisible by  $x^2 - 4x + 3$ ?

**Sol:**

|                |                                 |
|----------------|---------------------------------|
|                | $x^2 + 6x + 8$                  |
| $x^2 - 4x + 3$ | $x^4 + 2x^3 - 13x^2 - 12x + 21$ |
|                | $x^4 - 4x^3 + 3x^2$             |
|                | $6x^3 - 16x^2 - 12x$            |
|                | $6x^3 - 24x^2 - 18x$            |
|                | $8x^2 - 30x + 21$               |
|                | $8x^2 - 32x + 21$               |
|                | $2x - 2$                        |

We must subtract  $[2x - 2] + 10m$  the given polynomial so as to get the resulting polynomial exactly divisible by  $x^2 - x + 3$

9. Find all the zeroes of the polynomial  $x^4 + x^3 - 34x^2 - 4x + 120$ , if two of its zeroes are 2 and -2.

**Sol:**

$$\Rightarrow f(x) = x^4 + x^3 - 34x^2 - 4x + 120$$

$$\Rightarrow x = -2 \text{ is a solution}$$

$$x = -2 \text{ is a factor}$$

$$x = -2 \text{ is a solution}$$

$$x = +2 \text{ is a factor}$$

here,

$$(x - 2)(x + 2) \text{ is a factor of } f(x)$$

$$x^2 - 4 \text{ is a factor}$$

|           |                                |
|-----------|--------------------------------|
|           | $x^2 + x - 30$                 |
| $x^2 - 4$ | $x^4 + x^3 - 34x^2 - 4x + 120$ |
|           | $-x^4 \quad - 4x^2$            |
|           | $x^3 - 30x^2 - 4x + 120$       |
|           | $x^3 \quad - 4x$               |
|           | $-30x^2 \quad + 120$           |
|           | $-30x^2 \quad + 120$           |
|           | 0                              |

$$\text{Hence, } x^4 + x^3 - 34x^2 - 4x + 120 = (x^2 - 4)(x^2 + x - 30)$$

$$x^4 + x^3 - 34x^2 - 4x + 120 = (x^2 - 4)(x^2 + 6x - 5x - 30)$$

$$x^4 + x^3 - 34x^2 - 4x + 120 = (x^2 - 4)[(x(x + 6) - 5(x + 6))]$$

$$x^4 + x^3 - 34x^2 - 4x + 120 = (x^2 - 4)(x + 6)(x - 5)$$

Other zeroes are

$$x + 6 = 0 \quad \Rightarrow x - 5 = 0$$

$$x = -6 \quad x = 5$$

Set of zeroes for  $f(x)$   $[2, -2, -6, 5]$

10. Find all zeros of the polynomial  $2x^4 + 7x^3 - 19x^2 - 14x + 30$ , if two of its zeros are  $\sqrt{2}$  and  $-\sqrt{2}$ .

**Sol:**

$$f(x) = 2x^4 + 7x^3 - 19x^2 - 14x + 30$$

$x = \sqrt{2}$  is a solution

$x - \sqrt{2}$  is a solution

$x + \sqrt{2}$  is a solution

$x + \sqrt{2}$  is a factor

Here,  $(x + \sqrt{2})(x - \sqrt{2})$  is a factor of  $f(x)$

$x^2 - 2$  is a factor of  $f(x)$

|                                  |                      |
|----------------------------------|----------------------|
| $2x^2 + 7x - 15$                 |                      |
| $2x^4 + 7x^3 - 19x^2 - 14x + 30$ |                      |
| $2x^4$                           | $-4x^2$              |
|                                  | $7x^3 - 15x^2 - 14x$ |
|                                  | $7x^3 - \quad -14x$  |
|                                  | $-15x^2 + 30$        |
|                                  | $-15x^2 + 30$        |
|                                  | $0$                  |

$$\text{Hence, } 2x^4 + 7x^3 - 19x^2 - 14x + 30 = (x^2 - 2)(2x^2 + 7x - 15)$$

$$= (x^2 - 2)(2x^2 + 10x - 3x - 15)$$

$$= (x^2 - 2)(2x(x + 5) - 3(x + 5))$$

$$= (x^2 - 2)(x + 5)(x - 3)$$

Other zeroes are:

$$x + 5 = 0 \quad 2x - 3 = 0$$

$$x = -5 \quad 2x = 3$$

$$x = \frac{3}{2}$$

Hence the set of zeroes for  $f(x)$   $\left\{-5, \frac{3}{2}, \sqrt{2}, -\sqrt{2}\right\}$

11. Find all the zeros of the polynomial  $2x^3 + x^2 - 6x - 3$ , if two of its zeros are  $-\sqrt{3}$  and  $\sqrt{3}$ .

**Sol:**

$$f(x) = 2x^3 + x^2 - 6x - 3$$

$x = -\sqrt{3}$  is a solution

$x + \sqrt{3}$  is a factor

$x = \sqrt{3}$  is a solution

$x - \sqrt{3}$  is a factor

Here,  $(x + \sqrt{3})(x - \sqrt{3})$  is a factor of  $f(x)$

$x^2 - 3$  is a factor of  $f(x)$

|           |                       |
|-----------|-----------------------|
| $x^2 - 3$ | $2x + 1$              |
|           | $2x^3 + x^2 - 6x - 3$ |
|           | $2x^3 \qquad - 6x$    |
|           | $x^2 \qquad - 3$      |
|           | $x^2 \qquad - 3$      |
|           | $0$                   |

Hence,  $2x^3 + x^2 - 6x - 3 = (x^2 - 3)(2x + 1)$

Other zeroes of  $f(x)$  is  $2x + 1 = 0$

$$x = -\frac{1}{2}$$

Set of zeroes  $\left\{\sqrt{3}, -\sqrt{3}, -\frac{1}{2}\right\}$

12. Find all the zeros of the polynomial  $x^3 + 3x^2 - 2x - 6$ , if two of its zeros are  $-\sqrt{2}$  and  $\sqrt{2}$ .

**Sol:**

Since  $-\sqrt{2}$  and  $\sqrt{2}$  are zeroes of polynomial  $f(x) = x^3 + 3x^2 - 2x - 6$

$(x + \sqrt{2})(x - \sqrt{2}) = x^2 - 2$  is a factor of  $f(x)$

Now we divide  $f(x) = x^3 + 3x^2 - 2x - 6$  by

$g(x) = x^2 - 2$  to find the other zeroes of  $f(x)$

|           |                       |
|-----------|-----------------------|
| $x^2 - 2$ | $x + 3$               |
|           | $x^3 + 3x^2 - 2x - 6$ |
|           | $x^3 \qquad - 2x$     |
|           | $3x^2 \qquad - 6$     |
|           | $3x^2 \qquad - 6$     |
|           | $0$                   |

By division algorithm, we have

$$\Rightarrow x^3 + 3x^2 - 2x - 6 = (x^2 - 2)(x + 3)$$

$$\Rightarrow x^3 + 3x^2 - 2x - 6 = (x + \sqrt{2})(x - \sqrt{2})(x + 3)$$

Here the zeroes of the given polynomials are  $-\sqrt{2}, \sqrt{2}$  and  $-3$