

Exercise – 7.1

1. Calculate the mean for the following distribution:

x:	5	6	7	8	9
f:	4	8	14	11	3

Sol:

x	f	fx
5	4	20
6	8	48
7	14	98
8	11	88
9	3	27
	$N = 40$	$\Sigma fx = 281$

$$\begin{aligned}\text{Mean} &= \frac{\Sigma fx}{N} \\ &= \frac{281}{40} = 7.025\end{aligned}$$

2. Find the mean of the following data:

x:	19	21	23	25	27	29	31
f.	13	15	16	18	16	15	13

Sol:

x	f	fx
18	13	247
21	15	315
23	16	368
25	18	450
27	16	432
29	15	435
31	13	403
	$N = 106$	$\Sigma fx = 2620$

$$\text{Mean } (\bar{x}) = \frac{\Sigma fx}{N} = \frac{2620}{106} = 25.$$

3. If the mean of the following data is 20.6. Find the value of p.

x:	10	15	p	25	35
f:	3	10	25	7	5

Sol:

x	F	fx
10	3	30
5	10	150
P	25	25P
25	7	175
35	5	175
	$N = 90$	$\Sigma fx = 530 + 25P$

Given

$$\Rightarrow \text{Mean} = 20.6$$

$$\Rightarrow \frac{\Sigma Px}{N} = 20.6$$

$$\Rightarrow \frac{530 + 25P}{50} = 20.6$$

$$\Rightarrow 25P = 20.6(50) - 530$$

$$\Rightarrow P = \frac{500}{25}$$

$$\Rightarrow P = 20.$$

4. If the mean of the following data is 15, find p.

x:	5	10	15	20	25
f:	6	p	6	10	5

Sol:

x	F	fx
5	6	30
10	P	10P
15	6	90
20	10	200
25	5	125
	$N = P+27$	$\Sigma fx = 10P + 445$

Given

$$\Rightarrow \text{Mean} = 15$$

$$\Rightarrow \frac{\sum Px}{N} = 15$$

$$\Rightarrow \frac{109 + 445}{P + 127} = 15$$

$$\Rightarrow 10P + 445 = 15P + 405$$

$$\Rightarrow 15P - 10P = 445 - 405$$

$$\Rightarrow 5P = 40$$

$$\Rightarrow P = \frac{40}{5}$$

$$\Rightarrow P = 8$$

5. Find the value of p for the following distribution whose mean is 16.6

x:	8	12	15	p	20	25	30
f.	12	16	20	24	16	8	4

Sol:

x	f	fx
8	12	96
12	16	192
15	20	300
P	24	24P
20	16	220
25	8	200
30	4	420
	$N = 100$	$\sum fx = 24P + 1228$

Given

$$\Rightarrow \text{Mean} = 16.6$$

$$\Rightarrow \frac{\sum fx}{N} = 16.6$$

$$\Rightarrow \frac{24P + 1228}{100} = 16.6$$

$$\Rightarrow 24P + 1228 = 1660$$

$$\Rightarrow 24P = 1660 - 1228$$

$$\Rightarrow P = \frac{432}{24}$$

$$\Rightarrow P = 18$$

6. Find the missing value of p for the following distribution whose mean is 12.58

x:	5	8	10	12	p	20	25
f:	2	5	8	22	7	4	2

Sol:

x	f	fx
5	2	10
8	5	40
10	8	80
12	22	264
P	7	70
20	24	480
25	2	50
	$N = 50$	$\Sigma fx = 524P + 7P$

Given

$$\Rightarrow \text{Mean} = 12 = -8$$

$$\Rightarrow 5 \frac{3}{N} = 12.58$$

$$\Rightarrow \frac{528 + 7P}{50} = 12.58$$

$$\Rightarrow 524 + 7P = 629$$

$$\Rightarrow 7P = 629 - 524$$

$$\Rightarrow 7P = 105$$

$$\Rightarrow P = \frac{105}{7}$$

$$\Rightarrow P = 15$$

7. Find the missing frequency (p) for the following distribution whose mean is 7.68.

x:	3	5	7	9	11	13
f:	6	8	15	p	8	4

Sol:

x	f	fx
3	6	18
5	8	40
7	15	105
9	P	9P

11	8	18
13	4	52
	$N = P + 41$	$\sum fx = 9P = 303$

Given

$$\Rightarrow \text{Mean} = 7.68$$

$$\Rightarrow \frac{\sum fx}{N} = 68$$

$$\Rightarrow \frac{7P + 303}{P + 41} = 7.68$$

$$\Rightarrow 9P + 303 = P(7.68) + 314.88$$

$$\Rightarrow 9P - 7.68P = 314.88 - 303$$

$$\Rightarrow 1.32P = 11.88$$

$$\Rightarrow P = \frac{11.88}{1.32}$$

$$\Rightarrow P = 9.$$

8. Find the value of p, if the mean of the following distribution is 20.

x:	15	17	19	$20 + p$	23
f:	2	3	4	$5p$	6

Sol:

x	f	fx
15	2	30
17	3	51
19	4	76
$20+P$	$5P$	$100P+5P^2$
23	6	138
	$N = 5P + 15$	$\sum fx = 295 + 100P + 5P^2$

$$\Rightarrow \text{Given Mean} = 20$$

$$\Rightarrow \frac{\sum fx}{N} = 20$$

$$\Rightarrow \frac{295 + 100P + 5P^2}{5 + 15} = 20$$

$$\Rightarrow 295 + 100P + 5P^2 = 100P + 300$$

$$\Rightarrow 5P^2 - 5 = 0$$

$$\Rightarrow 5(P^2 - 1) = 0$$

$$\Rightarrow P^2 - 1 = 0 \Rightarrow (P+1)(P-1) = 0$$

$$\Rightarrow p^2 = 1$$

$$\Rightarrow p = \pm 1$$

$$\text{If } P+1=0$$

$$P = -1 \quad (\text{Reject})$$

$$\text{Or } P-1=0$$

$$P = 1$$

9. The following table gives the number of boys of a particular age in a class of 40 students. Calculate the mean age of the students

Age (in years):	15	16	17	18	19	20
No. of students:	3	8	10	10	5	4

Sol:

x	f	fx
15	3	45
16	8	128
17	10	170
18	10	180
19	5	95
20	4	80
	$\Sigma f = N = 40$	$\Sigma fx = 498$

$$\text{Mean age} = \frac{\Sigma fx}{N}$$

$$= \frac{498}{40}$$

$$= 12.45 \text{ years}$$

$$\therefore \text{Mean age} = 12.45 \text{ years}$$

10. Candidates of four schools appear in a mathematics test. The data were as follows:

Schools	No. of Candidates	Average Score
I	60	75
II	48	80
III	NA	55
IV	40	50

If the average score of the candidates of all the four schools is 66, find the number of candidates that appeared from school III.

Sol:

Let the number of candidates from school III = P

Schools	No of candidates N_i	Average scores (x_i)
I	60	75
II	48	80
III	P	55
IV	40	50

Given

Average score of all schools = 66.

$$\begin{aligned} \Rightarrow \frac{N_1\bar{x}_1 + N_2\bar{x}_2 + N_3\bar{x}_3 + N_4\bar{x}_4}{N_1 + N_2 + N_3 + N_4} &= 66 \\ \Rightarrow \frac{60 \times 75 + 48 \times 80 + P \times 55 + 40 \times 50}{60 + 48 + P + 40} &= 66 \\ \Rightarrow \frac{4500 + 3340 + 55P + 2000}{148 + P} &= 66 \\ \Rightarrow 10340 + 55P &= 66P + 9768 \\ \Rightarrow 10340 - 9768 &= 66P - 55P \\ \Rightarrow P &= \frac{572}{11} \\ \Rightarrow P &= 52. \end{aligned}$$

11. Five coins were simultaneously tossed 1000 times and at each toss the number of heads were observed. The number of tosses during which 0, 1, 2, 3, 4 and 5 heads were obtained are shown in the table below. Find the mean number of heads per toss.

No. of heads per toss	No. of tosses
0	38
1	144
2	342
3	287
4	164

5	25
Total	1000

Sol:

No. of heads per toss	No. of tosses
0	38
1	144
2	342
3	287
4	164
5	25

No. of heads per toss	No. of tosses	fx
0	28	0
1	144	144
2	342	684
3	287	861
4	164	656
5	25	125

$$\text{Mean number of heads per toss} = \frac{\sum fx}{N}$$

$$= \frac{2470}{1000}$$

$$= 2.47$$

$$\text{Mean} = 2.47$$

12. Find the missing frequencies in the following frequency distribution if it is known that the mean of the distribution is 50.

X: 10 30 50 70 90

f: 17 f_1 32 f_2 19 Total 120.

Sol:

x	f	fx
10	17	170
30	f_1	$30 f_1$
50	32	1600
70	f_2	$70 f_2$
90	19	1710
	$N = 120$	$\sum fx = 30 f_1 + 70 f_2 + 3480.$

Given mean

$$\frac{\Sigma fx}{N} = 50$$

$$\frac{30 f_1 + 70 f_2 + 3480}{120} = 50$$

$$30 f_1 + 70 f_2 + 3480 = 6000 \quad \dots(i)$$

Also,

$$\Sigma f = 120$$

$$17 + f_1 + 32 + f_2 + 19 = 120$$

$$f_1 + f_2 = 52$$

$$f_1 = 52 - f_2$$

Substituting value of f_1 in (i)

$$30(52 - f_2) + 70 f_2 + 3480 = 6000 \Rightarrow 40 f_2 = 960$$

$$\Rightarrow f_2 = 24$$

$$\text{Hence } f_1 = 52 - 24 = 28 \quad \therefore f_1 = 28; f_2 = 24$$

13. The arithmetic mean of the following data is 14. Find the value of k

x_i :	5	10	15	20	25
f_i :	7	k	8	4	5.

Sol:

x	f	fx
10	17	170
30	f_1	$30 f_1$
50	32	1600
70	f_2	$70 f_2$
90	19	1710
	$N = 120$	$\Sigma fx = 30 f_1 + 70 f_2 + 3480.$

Given mean = 50

$$\frac{\Sigma fx}{N} = 50$$

$$\frac{30 f_1 + 70 f_2 + 3480}{120} = 50$$

$$30 f_1 + 70 f_2 + 3480 = 6000 \quad \dots(i)$$

Also,

$$\Sigma f = 120$$

$$17 + f_1 + 32 + f_2 + 19 = 120$$

$$f_1 + f_2 = 52$$

$$f_1 = 52 - f_2$$

Substituting value of f_1 in (i)

$$30(52 - f_2) + 70f_2 + 3480 = 6000 \Rightarrow 40f_2 = 960$$

$$\Rightarrow f_2 = 24$$

$$\text{Hence } f_1 = 52 - 24 = 28 \quad \therefore f_1 = 28; f_2 = 24$$

14. The arithmetic mean of the following data is 25, find the value of k.

$$x_i: \quad 5 \quad 15 \quad 25 \quad 35 \quad 45$$

$$f_i: \quad 3 \quad k \quad 3 \quad 6 \quad 2$$

Sol:

x	f	fx
5	3	15
15	K	15k
25	3	75
35	6	210
45	2	90
	$N = k + 14$	$\Sigma fx = 15k + 390$

$$\Rightarrow \text{Given mean} = 25$$

$$\Rightarrow \frac{\Sigma fx}{N} = 25$$

$$\Rightarrow \frac{15k + 390}{k + 14} = 25$$

$$\Rightarrow 15k + 390 = 25k + 350$$

$$\Rightarrow 25k - 15k = 40$$

$$\Rightarrow 10k = 40$$

$$\Rightarrow k = \frac{40}{10}$$

$$\Rightarrow k = 4.$$

15. If the mean of the following data is 18.75. Find the value of p.

x_i :	10	15	p	25	30
f_i :	5	10	7	8	2

Sol:

x	f	fx
10	5	50
15	10	150
P	7	7P
25	8	200
30	2	60
	$N = 32$	$\sum fx = 1P + 460.$

$\Rightarrow$ Given mean = 18.75

$$\Rightarrow \frac{\sum fx}{N} = 18.75$$

$$\Rightarrow \frac{7P + 460}{32} = 18.75$$

$$\Rightarrow 7P + 460 = 600$$

$$\Rightarrow 7P = -460 + 600$$

$$\Rightarrow 7P = 140$$

$$\Rightarrow P = \frac{140}{7}$$

$$\Rightarrow P = 20$$

Exercise – 7.2

1. The number of telephone calls received at an exchange per interval for 250 successive one-minute intervals are given in the following frequency table:

No. of calls(x):	0	1	2	3	4	5	6
No. of intervals (f):	15	24	29	46	54	43	39

Compute the mean number of calls per interval.

Sol:

Let be assumed mean (A) = 3

No. of calls (x_i)	No. of intervals (f_i)	$u_i = x_i - A = x_i - 3$	$f_i u_i$
0	15	-3	-45
1	24	-2	-48

2	29	-1	-39
3	46	0	0
4	54		54
5	43	2	43(2) = 86
6	39	3	47
	$N = 250$		$\Sigma f_i u_i = 135$

$$\text{Mean number of cells} = A + \frac{\Sigma f_i u_i}{N}$$

$$\begin{aligned}
 &= 3 + \frac{135}{250} \\
 &= \frac{750 + 135}{250} \\
 &= \frac{885}{250} \\
 &= 3.54
 \end{aligned}$$

2. Five coins were simultaneously tossed 1000 times, and at each toss the number of heads was observed. The number of tosses during which 0,1,2,3,4 and 5 heads were obtained are shown in the table below. Find the mean number of heads per toss

No. of heads per toss (x):	0	1	2	3	4	5
No. of tosses (f):	38	144	342	287	164	25

Sol:

Let the assumed mean (A) = 2.

No. of heads per toss (x_i)	No. of intervals (f_i)	$u_i = A - x_i$ $= 2 - x_i$	$f_i u_i$
0	38	-2	-76
1	144	-1	-144
2	342	0	0
3	287	1	287
4	164	2	328
5	25	3	75
	$N = 1000$		$\Sigma f_i u_i = 470$

$$\text{Mean number of per toss} = A + \frac{\Sigma f_i u_i}{N}$$

$$\begin{aligned}
 &= 2 + \frac{470}{1000} \\
 &= 2 + 0.47 \\
 &= 2.47
 \end{aligned}$$

3. The following table gives the number of branches and number of plants in the garden of a school.

No. of branches (x):	2	3	4	5	6
No. of plants (f):	49	43	57	38	13

Calculate the average number of branches per plant.

Sol:

Let the assumed mean $(A) = 4$.

No. of branches (x_i)	No. of plants (f_i)	$u_i = x_i - A$ $= v_i - 4$	$f_i u_i$
2	49	-2	-98
3	43	-1	-43
4	57	0	0
5	$28 + 10 = 38$	1	28
6	13	2	85
	$N = 200$		$\Sigma f_i u_i = -77$

$$\text{Average number of branches per plant} = A + \frac{\Sigma f_i u_i}{N}$$

$$= 4 + \frac{-77}{200}$$

$$= 4 - \frac{77}{200}$$

$$= \frac{800 - 77}{200}$$

$$= 3.615$$

$$= 3.62 (\text{Approx}).$$

4. The following table gives the number of children of 150 families in a village

No. of children (x):	0	1	2	3	4	5
No. of families (f):	10	21	55	42	15	7

Find the average number of children per family.

Sol:

Let the assumed mean $(A) = 2$

No. of children (x_i)	No of families (f_i)	$u_i = x_i - A$ $= x_i - 2$	$f_i u_i$
0	10	-2	-20
1	21	-1	-21
3	42	1	42
4	15	2	30
5	7	5	21

	$N = 20$		$\Sigma f_i u_i = 52$
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$$\therefore \text{Average number of children for family} = A + \frac{\Sigma f_i u_i}{N}$$

$$\begin{aligned}
 &= 2 + \frac{52}{150} \\
 &= \frac{300 + 52}{150} \\
 &= \frac{352}{150} \\
 &= 2.35(\text{approx})
 \end{aligned}$$

5. The marks obtained out of 50, by 102 students in a Physics test are given in the frequency table below:

Marks(x):	15	20	22	24	25	30	33	38	45
Frequency (f):	5	8	11	20	23	18	13	3	1

Find the average number of marks.

Sol:

Let the assumed mean (A) = 25

Marks (x_i)	Frequency (f_i)	$u_i = x_i - A = x_i - 25$	$f_i u_i$
15	5	-10	-50
20	8	-5	-40
22	8	-3	-33
24	20	-1	-20
25	23	0	0
30	18	5	90
33	13	8	104
38	3	12	39
45	3	20	20
	$N = 122$		$\Sigma f_i u_i = 110$

$$\text{Average number of marks} = A + \frac{\Sigma f_i u_i}{N}$$

$$\begin{aligned}
 &= 25 + \frac{110}{102} \\
 &= \frac{2550 + 110}{102} \\
 &= \frac{2660}{102} \\
 &= 26.08(\text{Approx})
 \end{aligned}$$

6. The number of students absent in a class were recorded every day for 120 days and the information is given in the following frequency table:

No. of students absent (x):	0	1	2	3	4	5	6	7
No. of days(f):	1	4	10	50	34	15	4	2

Find the mean number of students absent per day.

Sol:

Let the assumed mean (A) = 3

No. of students absent x_i	No. of days f_i	$u_i = x_i - A$ $= x_i - 3$	$f_i u_i$
3	1	-3	-3
1	4	-2	-8
2	10	-1	-10
3	50	0	0
4	34	1	34
5	15	2	30
6	4	3	12
7	2	4	8
	$N = 120$		$\Sigma f_i u_i = 63$

Mean number of students absent per day = $A + \frac{\Sigma f_i u_i}{N}$

$$\begin{aligned}
 &= 3 + \frac{63}{120} \\
 &= \frac{360 + 63}{120} \\
 &= \frac{423}{120} \\
 &= 2.525 \\
 &= 3.53 (\text{Approx})
 \end{aligned}$$

7. In the first proof reading of a book containing 300 pages the following distribution of misprints was obtained:

No. of misprints per page (x):	0	1	2	3	4	5
No. of pages (f):	154	95	36	9	5	1

Find the average number of misprints per page.

Sol:

Let the assumed mean (A) = 2

No. of misprints per page (x_i)	No. of days (f_i)	$u_i = x_i - A$ $= x_i - 2$	$f_i u_i$
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0	154	-2	-308
1	95	-1	-95
2	36	0	0
3	9	1	9
4	5	2	1
5	1	3	3
	$N = 300$		$\Sigma f_i u_i = -381$

$$\text{Average number of mis prints per day} = A + \frac{f_i u_i}{N}$$

$$= 2 + \frac{381}{300}$$

$$= 2 - \frac{381}{300}$$

$$= \frac{600 - 381}{300}$$

$$= \frac{219}{300}$$

$$= 0.73$$

8. The following distribution gives the number of accidents met by 160 workers in a factory during a month.

No. of accidents (x): 0 1 2 3 4

No. of workers (f): 70 52 34 3 1

Find the average number of accidents per worker.

Sol:

Let the assumed mean (A) = 2

No. of Accidents	No. of workers (f_i)	$u_i = x_i - A$ $= x_i - 2$	$f_i u_i$
0	70	-2	-140
1	52	-1	-52
2	34	0	0
3	3	1	3
4	1	2	2
	$N = 100$		$\Sigma f_i u_i = -100$

Average no of accidents per day workers

$$\begin{aligned} &= A = \frac{f_i u_i}{N} \\ &= x + \frac{-187}{160} \\ &= \frac{320 - 187}{160} \\ &= \frac{133}{160} \\ &= 0.83 \end{aligned}$$



9. Find the mean from the following frequency distribution of marks at a test in statistics:

Marks(x):	5	10	15	20	25	30	35	40	45	50
No. of students (f):	15	50	80	76	72	45	39	9	8	6

Sol:

Let the assumed mean (A) = 25.

Marks (x_i)	No. of students (f_i)	$u_i = x_i - A$ $= x_i - 25$	$f_i u_i$
5	15	-20	-300
10	50	-15	-750
15	80	-10	-800
20	76	-5	-380
25	72	0	0
30	45	5	225
35	39	10	390
40	9	15	135
45	8	20	160
50	6	25	150
	$N = 400$		$\Sigma f_i u_i = -1170$

$$\begin{aligned}
 \text{Mean} &= \frac{\Sigma f_i u_i}{N} \\
 &= 25 + \frac{-1170}{400} \\
 &= \frac{10000 - 1170}{400} \\
 &= 22.075.
 \end{aligned}$$

Exercise – 7.3

1. The following table gives the distribution of total household expenditure (in rupees) of manual workers in a city.

Expenditure (in rupees) (x)	Frequency (f_i)	Expenditure (in rupees) (x_1)	Frequency (f_i)
100 – 150	24	300 – 350	30
150 – 200	40	350 – 400	22
200 – 250	33	400 – 450	16
250 – 300	28	450 – 500	7

Find the average expenditure (in rupees) per household.

Sol:

Let the assumed mean (A) = 275.

Class interval	Mid value (x_i)	$d_i = x_i - 275$	$u_i = \frac{x_i - 275}{50}$	Frequency f_i	$f_i u_i$
100-150	125	-150	-3	24	-12
150-200	175	-100	-2	40	-80
200-250	225	-50	-1	33	-33
250-300	275	0	0	28	0
300-350	325	50	1	30	30
350-400	375	100	2	22	44
400-450	425	150	3	16	48
450-500	475	200	4	7	28
				$N = 200$	$\Sigma f_i u_i = -35$

We have

$$A = 275, h = 50$$

$$\text{Mean} = A + h \times \frac{\Sigma f_i u_i}{N}$$

$$= 275 + 50 \times \frac{-35}{200}$$

$$= 275 - 8.75$$

$$= 266.25$$

2. A survey was conducted by a group of students as a part of their environment awareness program, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants: 0-2 2-4 4-6 6-8 8-10 10-12 12-14

Number of houses: 1 2 1 5 6 2 3

Which method did you use for finding the mean, and why?

Sol:

Let us find class marks (x_i) for each interval by using the relation

$$\text{Class mark } (x_i) = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Now we may compute x_i and $f_i x_i$ as following

Number of plants	Number of house (f_i)	x_i	$f_i x_i$
0-2	1	1	$1 \times 1 = 1$
2-4	2	3	$2 \times 3 = 6$
4-6	1	5	$1 \times 5 = 5$
6-8	5	7	$5 \times 7 = 35$
8-10	6	9	$6 \times 9 = 54$
10-12	2	11	$2 \times 11 = 22$
12-14	3	13	$3 \times 13 = 39$
Total	20		162

From the table we may observe that

$$\Sigma f_i = 20$$

$$\Sigma f_i x_i = 162$$

$$\text{Mean } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{162}{20} = 8.1$$

So mean number of plants per house is 8.1

We have used for the direct method values x_i and f_i are very small

3. Consider the following distribution of daily wages of 50 workers of a factory

Daily wages (in Rs). 100 - 120 120 - 140 140 - 160 160 - 180 180 - 200

Number of workers: 12 14 8 6 10

Find the mean daily wages of the workers of the factory by using an appropriate method.

Sol:

Let the assume mean $(A) = 150$

Class interval	Mid value x_i	$d_i = x_i - 150$	$u_i = \frac{x_i - 150}{20}$	Frequency f_i	$f_i u_i$
100-120	110	-40	-2	12	-24
120-140	130	-20	-1	14	-14
140-160	150	0	0	8	0
160-180	170	20	1	6	6
180-200	190	40	2	10	20
			$N = 50$	$\Sigma f_i u_i = -12$	

We have

$$N = 50, h = 20$$

$$\text{Mean} = A + h \times \frac{\Sigma f_i u_i}{N}$$

$$= 150 + 20 \times \frac{-12}{50}$$

$$= 150 - \frac{24}{5}$$

$$= 150 - 4.8$$

$$= 145.2$$

4. Thirty women were examined in a hospital by a doctor and the number of heart beats per minute recorded and summarized as follows. Find the mean heart beats per minute for these women, choosing a suitable method.

Number of heart 65 - 68 68 - 71 71 - 74 74 - 77 77 - 80 80 - 83 83 - 86

beats per minute:

Number of women: 2 4 3 8 7 4 2

Sol:

We may find class marks of each interval (x_i) by using the relation

$$x_i = \frac{\text{Upper class limit} + \text{lower class limit}}{2}$$

Class size of this data = 3

Now taking 75.5 as assumed mean (a) we

May calculate, $d_i, u_i, f_i u_i$ as following.

Number of heart beats per minute	Number of women (x_i)	x_i	$d_i = x_i - 75.5$	$u_i = \frac{x_i - 75.5}{h}$	$f_i u_i$
65–68	2	66.5	-9	-3	-6
68–71	9	69.5	-6	-2	-8
71–74	3	72.5	-3	-1	-3
74–77	8	75.5	0	0	0
75–80	7	78.5	3	1	7
80–83	4	81.5	2 × 3 × 6	2	8
83–86	2	84.5	9	3	6
	30				4

Now we may observe from table that $\Sigma f_i = 30; \Sigma f_i u_i = 4$

$$\text{Mean } (\bar{x}) = 9r \left[\frac{\Sigma f_i u_i}{\Sigma f_i} \right] \times h = 75.5 + \left(\frac{4}{30} \right) \times 3$$

$$= 75.5 + 0.4 = 75.9$$

So mean hear beats per minute for those women are 75.9 beats per minute.

Find the mean of each of the following frequency distributions: (5 – 14)

5. Class interval: 0 - 6 6 - 12 12 - 18 18 - 24 24-30
 Frequency: 6 8 10 9 7

Sol:

Let a assume mean be 15

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \frac{x_i - 15}{6}$	f_i	$f_i u_i$
0–6	3	-12	-2	6	-12
6–12	9	-6	-1	2	-8
12–18	15	0	0	10	0
18–24	21	6	1	9	9
24–30	27	18	2	7	14
				$N = 40$	3

$$A = 15, h = 6$$

$$\text{Mean} = A + h \frac{\sum f_i x_i}{N}$$

$$= 15 + 6 \times \frac{3}{40}$$

$$= 15 + 0.45$$

$$= 15 + 0.45$$

$$= 15.45$$

6. Class interval: 50 - 70 70 - 90 90 - 110 110 - 130 130 - 150 150 - 170
 Frequency: 18 12 13 27 8 22

Sol:

Let the assumed mean be 100

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \frac{x_i - 15}{6}$	f_i	$f_i u_i$
50 - 70	60	-40	-2	18	-36
70 - 90	80	-20	-1	12	-12
90 - 110	100	0	0	10	0
110 - 130	120	20	1	27	27
130 - 150	140	65	3	22	66
					61

$$A = 100, n = 20$$

$$\text{Mean} = A + h \frac{\sum f_i u_i}{n}$$

$$= 100 + 20 \times \frac{61}{100}$$

$$= 100 + 12.2$$

$$= 112.2$$

7. Class interval: 0-8 8- 16 16- 24 24-32 32-40
 Frequency: 6 7 10 8 9

Sol:

Let the assumed mean (A) = 20

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \frac{x_i - 15}{6}$	f_i	$f_i u_i$
0 - 8	4	-16	-2	6	-12
8 - 16	12	-8	-1	7	-7
16 - 24	20	0	0	10	0
24 - 32	28	8	1	8	8
32 - 40	36	16	2	9	18
				$N = 40$	$\sum f_i u_i = 7$

We have

$$A = 20, N = 40$$

$$\text{Mean } A + h \times \frac{\sum f_i u_i}{N}$$

$$= 20 + 8 \times \frac{7}{40}$$

$$= 20 + 1.4$$

$$= 21.4$$

8. Class interval: 0-6 6- 12 12- 18 18-24 24-30
Frequency: 7 5 10 12 6

Sol:

Let the assume mean (A) = 15

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \frac{x_i - 15}{6}$	Frequency f_i	$f_i u_i$
0-6	3	-12	-2	7	-14
6-12	9	-6	-1	5	-5
12-18	15	0	0	10	0
18-24	21	6	1	12	12
24-30	27	12	2	6	12
				$N = 40$	$\sum f_i u_i = 5$

We have $A = 15$

$$A = 15, h = 6$$

$$\text{Mean, } A + h \times \frac{\sum f_i u_i}{N}$$

$$= 15 + 6 \times \frac{5}{40}$$

$$= 15 + 0.75$$

$$= 15.75$$

9. Class interval: 0- 10 10- 20 20-30 30-40 40-50
Frequency: 9 12 15 10 14

Sol:

Let the assumed mean (A) = 25

Class interval	Mid-value x_i	$d_i = x_i - 25$	$u_i = \frac{x_i - 25}{10}$	Frequency f_i	$f_i u_i$
0-10	5	-20	-2	9	-18
10-20	15	-10	-1	12	-12

20–30	25	0	0	15	0
30–40	35	10	1	10	10
40–50	45	20	2	14	28
				$N = 60$	$\Sigma f_i u_i = 8$

We have $A = 25, h = 10$

$$\text{Mean} = A + h \frac{\Sigma f_i u_i}{N}$$

$$= 25 + 10 \times \frac{8}{60}$$

$$= 25 + \frac{8}{6}$$

$$= 25 + \frac{4}{3}$$

$$= 26.333$$

10. Class interval: 0-8 8- 16 16-24 24-32 32 -40
Frequency: 5 9 10 8 8

Sol:

Let the assumed mean (A) = 20

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \frac{x_i - 15}{6}$	Frequency f_i	$f_i u_i$
0–8	4	-16	-2	5	-10
8–16	12	-8	-1	9	-9
16–24	20	0	0	10	0
24–32	28	8	1	8	8
32–40	36	16	2	8	16
				$N = 40$	$\Sigma f_i u_i = 5$

We have

$$A = 20, h = 8$$

$$\text{Mean} = A + h \times \frac{\Sigma f_i u_i}{N}$$

$$= 20 + 8 \times \frac{5}{40}$$

$$= 20 + 1$$

$$= 21$$

11. Class interval: 0-8 8- 16 16- 24 24-32 32-40
Frequency: 5 6 4 3 2

Sol:

Let the assumed $(A) = 20$.

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \frac{x_i - 15}{6}$	Frequency f_i	$f_i u_i$
0–8	4	-16	5	-2	-10
8–16	12	-8	6	-1	-6
16–24	20	0	4	0	0
24–32	28	8	3	1	3
32–40	36	16	2	8	4
				$N = 20$	$\Sigma f_i u_i = -9$

We have

$$A = 20, h = 18$$

$$\text{Mean} = A + h \times \frac{\Sigma f_i u_i}{N}$$

$$= 20 + 18 \times \frac{-9}{20}$$

$$= 20 - 8.1$$

$$= 11.9$$

12. Class interval: 10-30 30-50 50-70 70-90 90-110 110-130

Frequency: 5 8 12 20 3 2

Sol:

Let the assume mean $(A) = 60$

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \frac{x_i - 15}{6}$	Frequency f_i	$f_i u_i$
10–30	20	-40	-2	5	-10
30–50	40	-20	-1	8	-8
50–70	60	0	0	12	0
70–90	80	20	1	20	20
90–110	100	40	2	3	6
110–130	120	60	3	2	6
				$N = 50$	$\Sigma f_i u_i = 14$

We have

$$A = 60, h = 25$$

$$\text{Mean} = A + h \times \frac{\Sigma f_i u_i}{N}$$

$$= 60 + 25 \times \frac{14}{50}$$

$$= 60 + 7$$

$$= 67$$

13. Class interval: 25-35 35-45 45-55 55 - 65 65 – 75

Frequency: 6 10 8 12 4

Sol:

Let the assume mean (A) = 50

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \frac{x_i - 15}{6}$	Frequency f_i	$f_i u_i$
25 – 35	30	-20	-2	6	-12
35 – 45	40	-10	-1	10	-10
45 – 55	50	0	0	8	0
55 – 65	60	10	0	12	12
65 – 75	70	20	0	4	8
				$N = 40$	$\Sigma f_i u_i = -2$

We have

$$A = 50, h = 10$$

$$\text{Mean} = A + h \frac{\Sigma f_i u_i}{N}$$

$$= 50 + 10 \left(\frac{-2}{40} \right)$$

$$= 50 - 0.5$$

$$= 49.5$$

14. Classes: 25 -29 30-34 35-39 40-44 45-49 50-54 55-59

Frequency: 14 22 16 6 5 3 4

Sol:

Let the assume mean (A) = 42

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \frac{x_i - 15}{6}$	Frequency f_i	$f_i u_i$
25 – 29	27	-15	-3	14	-42
30 – 34	32	-10	-2	22	-44
35 – 39	37	-5	-1	16	-16
40 – 44	42	0	0	6	0
45 – 49	47	5	1	5	5
50 – 54	52	10	2	3	6
55 – 59	57	15	3	4	12
				$N = 70$	$\Sigma f_i u_i = -79$

We have

$$A = 42, h = 5$$

$$\text{Mean} = A + h \times \frac{\Sigma f_i u_i}{N}$$

$$\begin{aligned}
 &= 42 + 5x \frac{-79}{70} \\
 &= 42 - \frac{5 \times 79}{70} \\
 &= 42 - \frac{79}{14} \\
 &= \frac{588 - 79}{14} \\
 &= 36.357
 \end{aligned}$$

15. For the following distribution, calculate mean using all suitable methods:

Size of item:	1-4	4-9	9-16	16-27
Frequency:	6	12	26	20

Sol:

By direct method

Class interval	Mid-value	Frequency f_i	$f_i u_i$
1-4	2.5	6	15
4-9	6.5	12	18
9-16	12.5	26	325
16-27	21.5	20	430
		$N = 64$	$\Sigma f_i u_i = 848$

$$\text{Mean} = \frac{\Sigma f_i x_i}{N} + A$$

$$= \frac{848}{64}$$

$$= 13.25$$

By assuming mean method

Let the assumed mean (A) = 65

Class interval	Mid-value (x_i)	$l_5 = x_i - A$ $= x_i - 65$	Frequency (f_i)	$f_i u_i$
1-4	2.5	-4	6	-24
4-9	6.5	0	12	0
9-16	12.5	6	26	156
16-27	21.5	15	20	300
			$N = 64$	$\Sigma f_i u_i = 432$

$$\text{Mean} = A + \frac{\Sigma f_i u_i}{N}$$

$$= 6.5 + \frac{432}{64}$$

$$= 6 \cdot 5 + \frac{432}{64}$$

$$= 13 \cdot 25$$

16. The weekly observations on cost of living index in a certain city for the year 2004 - 2005 are given below. Compute the weekly cost of living index.

Cost of living Index	Number of Students	Cost of living Index	Number of Students
1400 – 1500	5	1700 – 1800	9
1500 – 1600	10	1800 – 1900	6
1600 – 1700	20	1900 – 2000	2

Sol:

Let the assume mean (A) = 1650

Class interval	Mid-value x_i	$d_i = x_i - A$ $= x_i - 1650$	$u_i = \frac{x_i - 15}{6}$	Frequency f_i	$f_i u_i$
1400-1500	1450	-200	-2	5	-10
1500-1600	1550	-100	-1	0	-10
1600-1700	1650	0	0	20	0
1700-1800	1750	100	1	9	9
1800-1900	1950	300	3	2	6
				$N = 52$	$\Sigma f_i u_i = 7$

We have

$$A = 16, h = 100$$

$$\text{Mean} = A + h \times \frac{\Sigma f_i u_i}{N}$$

$$= 1650 + 100 \times \frac{175}{13}$$

$$= \frac{21450 + 175}{13}$$

$$= \frac{21625}{13}$$

$$= 1663 \cdot 46$$

17. The following table shows the marks scored by 140 students in an examination of a certain paper:

Marks:	0- 10	10-20	20-30	30-40	40-50
Number of students:	20	24	40	36	20

Calculate the average marks by using all the three methods: direct method, assumed mean deviation and shortcut method.

Sol:

Direct method

Class interval	Mid-value	Frequency f_i	$f_i u_i$
0-10	5	20	100
10-20	15	20	350
20-30	25	40	1000
30-40	35	30	1260
40-50	45	20	900
		$N = 140$	8620

$$\text{Mean} = \frac{\sum f_i x_i}{N}$$

$$= \frac{3650}{140}$$

$$= 25.857$$

Assume mean method : Let the assumed mean = 25

$$\text{Mean} = A + \frac{\sum f_i u_i}{N}$$

Class interval	Mid-value	$u_i = x_i - A$	f	$f_i u_i$
0-10	5	-20	20	-400
10-20	15	-10	24	-240
20-30	25=A	0	40	0
30-40	35	10	36	360
40-50	45	20	20	400
			$N = 145$	120

$$\text{Mean} = A + \frac{\sum f_i u_i}{N}$$

$$= 25 + \frac{120}{145}$$

$$= 25 + 0.867$$

$$= 25.857$$

Step deviation method

Let the assumed mean (A) = 25

Class interval	Mid-value x_i	$d_i = x_i - A$ $= x_i - 25$	$u_i = \frac{x_i - 25}{10}$	Frequency f_i	$f_i u_i$
0-10	5	-20	-2	20	-40
10-20	15	-10	-1	24	-24
20-30	25	0	0	40	0
30-40	35	10	1	36	36
40-50	45	20	2	20	40

				$N = 140$	$\Sigma f_i u_i = 12$
--	--	--	--	-----------	-----------------------

$$\text{Mean} = A + \frac{\Sigma f_i u_i}{N} \times h$$

$$= 25 + \frac{120}{140} \times 10 = 25 + 0.857$$

$$= 25.857$$

18. The mean of the following frequency distribution is 62.8 and the sum of all the frequencies is 50. Compute the missing frequency f_1 and f_2 .

Class:	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 - 120
Frequency:	5	f_1	10	f_2	7	8

Sol:

Class interval	Mid-value	Frequency f_i	$f_i u_i$
0-20	10	5	50
20-40	30	f_1	$30 f_1$
40-60	50	10	500
60-80	70	f_2	$70 f_2$
80-100	90	7	630
100-120	110	8	880
		$N = 50$	$\Sigma f_i u_i = 30 f_1 + 70 f_2 + 3060$

Given

Sum of frequency = 50

$$\Rightarrow 5 + f_1 + 10 + f_2 + 7 + 8 = 50$$

$$\Rightarrow f_1 + f_2 = 50 - 5 - 10 - 7 - 8$$

$$\Rightarrow f_1 + f_2 = 20$$

$$\Rightarrow 3f_1 + 3f_2 = 60 \quad \text{.....(1) [multiply it by '3']}$$

And mean = 62.8

$$\Rightarrow \frac{\Sigma f_i x_i}{N} = 62.8$$

$$\Rightarrow \frac{30 f_1 + 70 f_2 + 2060}{50} = 62.8$$

$$\Rightarrow 30 f_1 + 70 f_2 = 3140 - 2060$$

$$\Rightarrow 30 f_1 + 70 f_2 = 1080$$

$$\Rightarrow 3 f_1 + 7 f_2 = 108 \quad \text{.....(2) (Divide it by 10)}$$

Subtract equation (1) from equation (2)

$$\Rightarrow 3 f_1 + 7 f_2 - 3 f_1 = 108 - 60$$

$$\Rightarrow 4f_2 = 48$$

$$\Rightarrow f_2 = 12$$

Put value of f_2 in equation (1)

$$\Rightarrow 3f_1 + 3 \times 12 = 60$$

$$\Rightarrow 3f_1 = 60 - 36 = 24$$

$$\Rightarrow f_1 = \frac{24}{3} = 8$$

$$f_1 = 8 \text{ and } f_2 = 12$$

- 19.** The following distribution shows the daily pocket allowance given to the children of a multistorey building. The average pocket allowance is Rs 18.00. Find out the missing frequency.

Class interval: 11-13 13-15 15-17 17-19 19-21 21-23 23-25

Frequency: 7 6 9 13 - 5 4

Sol:

Given mean = 18, let missing frequency be v

Class interval	Mid-value	Frequency f_i	$f_i u_i$
11-13	12	7	84
13-15	14	6	88
15-17	16	9	144
17-19	18	13	234
19-21	20	x	$20x$
21-23	22	5	110
23-25	14	4	56
		$N = 44 + v$	$752 + 20x$

$$\text{Mean} = \frac{\sum f_i x_i}{N}$$

$$18 = \frac{752 + 20x}{44 + x}$$

$$792 + 18x = 752 + 20x$$

$$2x = 40$$

$$x = 20$$

- 20.** If the mean of the following distribution is 27, find the value of p .

Class: 0 - 10 10 - 20 20 - 30 30 - 40 40-50

Frequency: 8 p 12 13 10

Sol:

Class interval	Mid-value	Frequency	$f_i u_i$
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	(x_i)	f_i	
0-10	5	8	40
10-20	15	P	152
20-30	25	12	300
30-40	35	13	455
40-50	45	16	450
		$N = 43 + P$	$\Sigma f_i x_i = 1245 + 15P$

Given

Mean = 27

$$\Rightarrow \frac{\Sigma f_i x_i}{N} = 27$$

$$\Rightarrow \frac{1245 + 15P}{43 + P} = 27$$

$$\Rightarrow 1245 + 15P = 1245 - 161 + 27P$$

$$\Rightarrow 27P - 15P = 1245 - 1161$$

$$\Rightarrow 12P = 84$$

$$\Rightarrow P = \frac{84}{12} = 7$$

21. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes: 50 - 52 53 - 55 56 - 58 59 - 61 62 - 64

Number of boxes: 15 110 135 115 25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Sol:

Number of mangoes	Number of boxes (f_i)
50-52	15
53-55	110
56-58	135
59-61	115
62-64	25

We may observe that class intervals are not continuous

There is a gap between two class intervals. So we have to add $\frac{1}{2}$ from lower class limit of

each interval and class mark (x_i) may be obtained by using the relation

$$x_i = \frac{\text{Upper class limit} + \text{lower class limit}}{2}$$

Class size (h) of this data = 3

Now, taking 57 as assumed mean (a) we may calculate

$d_i, u_i, f_i u_i$ as follows.

Class interval	f_i	x_i	$d_i = 4 - 57$	$u_i = \frac{x_i - 57}{h}$	$f_i u_i$
49.5–52.5	15	51	-6	-2	-30
52.5–56.5	110	54	-3	-1	-110
55.5–58.5	135	57	0	0	0
58.5–61.5	115	60	3	1	115
61.5–64.5	25	63	6	2	50
Total	400				-25

Now, we have

$$\Sigma f_i = 400$$

$$\Sigma f_i u_i = 25$$

$$\text{Mean} = 4 + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) \times h$$

$$= 57 + \left(\frac{45}{400} \right) \times 3$$

$$= 57 + \frac{3}{16}$$

$$= 57 + 0.1875$$

$$= 57.1875$$

$$= 57.19$$

Clearly mean number of mangoes kept in packing box is 57.19

22. The table below shows the daily expenditure on food of 25 households in a locality

Daily expenditure (in Rs): 100 - 150 150 - 200 200 - 250 250 - 300 300 - 350

Number of households: 4 5 12 2 2

Find the mean daily expenditure on food by a suitable method.

Sol:

We may calculate class mark (x_i) for each interval by using the relation

$$x_i = \frac{\text{Upper class limit} + \text{lower class limit}}{2}$$

Class size = 50

Now, taking 225 as assumed mean can we may calculated $d_i, u_i, f_i u_i$ as follows

Daily expenditure (in Rs)	f_i	x_i	$d_i = x_i - 225$	$u_i = \frac{x_i - 225}{h}$	$f_i u_i$
100-150	4	125	-100	-2	-8
150-200	5	175	-50	-1	-5
200-250	12	225	0	0	0

250-300	2	275	50	1	2
300-350	2	325	100	2	4
					-7

Now we may observe that

$$\Sigma f_i = 25$$

$$\Sigma f_i x_i = -7$$

$$\text{Mean } (\bar{x}) = a + \left(\frac{\Sigma f_i u_i}{\Sigma f} \right) \times h$$

$$= 225 + \left(\frac{-7}{25} \right) \times 50$$

$$= 225 - 14 = 211$$

So, mean daily expenditure on food is RS 211

23. To find out the concentration of SO₂ in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

Concentration of SO ₂ (in ppm)	Frequency
0.00-0.04	4
0.04-0.08	9
0.08-0.12	9
0.12-0.16	2
0.16-0.20	4
0.20-0.24	2

Find the mean concentration of SO₂ in the air.

Sol:

We may find a class marks for each interval by using the relation

$$x = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size of this data = 0.04

Now, taking 0.14 assumed mean can we use may calculated d,u,fu as following

Concentration SO ₂ (in ppm)	Frequency	Class interval (x_i)	$u_i = x_i - 0.14$	v_i	$f_i u_i$
0.00–0.04	4	0.02	-0.12	-3	-112
0.04–0.08	9	0.06	-0.08	-2	-8
0.08–0.12	1	0.10	-0.04	-1	-9
0.12–0.12	2	0.14	0	0	0
0.16–0.20	4	0.18	0.04	1	7
0.20–0.24	2	0.22	0.08	2	4
Total	30				-31

From the table we may observe that

$$\Sigma f_i = 30$$

$$\Sigma f_i u_i = -31$$

$$\text{Mean } \bar{x} = 9 + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) \times h$$

$$= 0.14 + \left(\frac{+31}{30} \right) (0.04)$$

$$= 0.14 - 0.04133$$

$$= 0.099 \text{PPm}$$

So, mean concentration of SO_2 in the air is 0.099PPm

24. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days: 0 - 6 6 - 10 10 - 14 14 - 20 20 - 28 28 - 38 38 - 40

Number of students: 11 10 7 4 4 3 1

Sol:

We may find class mark of each interval by using the relation

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Now, taking 16 as assumed mean (a) we may

Calculate d and $f_i d_i$ as follows

Number of days	Number of students f_i	x_i	$a = x_i + f_i$	$f_i d_i$
0-6	11	3	-13	-143
6-10	10	8	-8	-280
10-14	7	12	-4	-28
14-20	7	16	0	0
20-28	8	24	8	32
28-36	3	33	17	51
30-40	1	39	23	23
Total	70			-145

Now we may observe that

$$\Sigma f_i = 40$$

$$\Sigma f_i d_i = -145$$

$$\text{Mean } (\bar{x}) = a + \left(\frac{\Sigma f_i d_i}{\Sigma f_i} \right)$$

$$= 16 + \left(\frac{-145}{40} \right) = 16 - 3.625$$

$$= 12.375$$

So, mean number of days is 12, 38 days for which student was absent

25. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %): 45 - 55 55 - 65 65 - 75 75 - 85 85 - 95

Number of cities: 3 10 11 8 3

Sol:

We may find class marks by using the relation

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size (h) for this data = 10

Now taking 70 as assumed mean (a) wrong

Calculate d_i, u_i and $f_i u_i$ as follows

Library rate (in r_i)	Number of cities (f_i)	x_i	$d_i = x_i - 30$	$u_i = \frac{d_i}{10}$	$f_i u_i$
45-55	3	10	-20	-2	-6
55-65	10	60	-10	-1	-10
65-75	11	70	0	0	0
75-85	8	80	10	1	8
85-95	3	90	20	2	6
Total	35				-2

Now we may observe that

$$\Sigma f_i = 35$$

$$\Sigma f_i u_i = -2$$

$$\text{Mean } (\bar{x}) = a + \left(\frac{\Sigma f_i u_i}{\Sigma u_i} \right) \times h$$

$$= 70 + \left(\frac{-2}{35} \right) 10$$

$$= 70 - \frac{4}{7}$$

$$= 70 - 0.57 = 69.43$$

So, mean literacy rate is 69.437.

Exercise – 7.4

1. Following are the lives in hours of 15 pieces of the components of aircraft engine. Find the median:

715, 724, 725, 710, 729, 745, 694, 699, 696, 712, 734, 728, 716, 705, 719.

Sol:

Lives in hours of is pieces are

= 715, 724, 725, 710, 729, 745, 694, 699, 696, 712, 734, 728, 719, 705, 705, 719.

Arrange the above data in a sending order

694, 696, 699, 705, 710, 712, 715, 716, 719, 721, 725, 728, 729, 734, 745

$N = 15(\text{odd})$

$$\text{Median} = \left(\frac{N+1}{2} \right)^{\text{th}} \text{ term}$$

$$= \left(\frac{15+1}{2} \right)^{\text{th}} \text{ term}$$

$$= 8^{\text{th}} \text{ term}$$

$$= 716$$

2. The following is the distribution of height of students of a certain class in a certain city:

Height (in cm): 160 - 162 163 - 165 166 - 168 169 - 171 172 - 174

No. of students: 15 118 142 127 18

Find the median height.

Sol:

Class interval (inclusive)	Class interval (inclusive)	Class interval Frequency	Cumulative frequency
160-162	159.2 – 162.5	15	15
163-164	162.5 – 165.5	118	133 (F)
166-168	165.5 – 168.5	142(f)	275
169-171	168.5 – 168.5	127	402
172 – 174	171.5 – 174.5	18	420
		$N = 420$	

We have

$$N = 420$$

$$\frac{N}{2} = \frac{420}{2} = 210$$

The cumulative frequency just greater than $\frac{N}{2}$ is 275 then 165.5 – 168.5 is the median class such, that

$$l = 165.5, f = 142, F = 133 \text{ and } h = 168.5 - 105.5 = 3$$

$$\text{Mean} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$= 165.5 + \frac{10 \times 2}{142} = 10$$

$$= 165.7 + \frac{17 \times 4}{142}$$

$$= 65.5 + 1.63$$

$$= 168.13$$

3. Following is the distribution of I.Q. of 100 students. Find the median I.Q.

I.Q.: 55-64 65-74 75-84 85-94 95-104 105-114 115-124 125-134 135-144

No of Students: 1 2 9 22 33 22 8 2 1

Sol:

Class interval (inclusive)	Class interval (exclusive)	Frequency	Cumulative frequency
55-64	54.5 – 64.5	1	1
65-74	64.5 – 74.5	2	3
75-84	74.5 – 84.5	9	12
85-94	84.5 – 94.5	22	34(f)
95-104	94.5 – 104.5	33(f)	37
105-114	104.5 – 114.5	22	89
115-124	114.5 – 124.5	8	97
125-134	124.5 – 134.5	2	99
135-144	134.5 – 134.5	1	100
		$N = 100$	

We have

$$N = 100$$

$$\frac{N}{2} = \frac{100}{2} = 50$$

The cumulative frequency just greater than $\frac{N}{2}$ is 67 then the median class is

$$94.5 - 104.5 - 94.5 = 10$$

$$\text{Mean} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$= 94.5 + \frac{50 - 34}{33} \times 10$$

$$= 94.5 + \frac{16 \times 10}{33} = 94.5 + 4.88 = 99.35$$

4. Calculate the median from the following data:

Rent (in Rs.):	15-25	25-35	35-45	45-55	55-65	65-75	75-85	85-95
No. of Houses:	8	10	15	25	40	20	15	7

Sol:

Class interval	Frequency	Cumulative frequency
15-25	8	8
25-35	10	18
35-45	15	33(f)
45-55	25	58(f)
55-65	40(f)	28
65-75	20	38
75-85	15	183
85-95	9	140
	$N = 110$	

We have $N = 140$

$$\frac{N}{2} = \frac{140}{2} = 70$$

The cumulative frequency just greater than Σ is 98 then media class is 55-65 such that $l = 55, f = 40, F = 58, h = 65 - 55 = 10$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$= 55 + \frac{70 - 58}{40} \times 10$$

$$= 55 + \frac{12 \times 10}{40}$$

$$= 55 + 3$$

$$= 58$$

$$\therefore \text{Median} = 58$$

5. Calculate the median from the following data:

Marks below:	10	20	30	40	50	60	70	80
No. of students:	15	35	60	84	96	127	198	250

Sol:

Marks below	No of students	Class interval	Frequency	Cumulative frequency
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10	15	0-10	15	15
20	35	10-20	20	35
30	60	20-30	25	60
40	84	30-40	24	84
50	96	40-50	12	96(f)
60	127	50-60	37(f)	127
70	198	60-70	71	198
80	250	70-8	52	250
			$N = 250$	

We have $N = 250$

$$\frac{N}{2} = \frac{250}{2} = 125$$

The cumulative frequency just greater than $\frac{N}{2}$ is 127 then median class is 50-60 such that

$$l = 50, f = 31, F = 96, h = 60 - 50 = 10$$

$$\text{Median} = L + \frac{\frac{N}{2} - F}{f} \times h$$

$$= 50 + \frac{125 - 96}{31} \times 10$$

$$= 50 + \frac{29 \times 10}{31}$$

$$= \frac{155 + 290}{31}$$

$$= \frac{445}{31}$$

$$= 59.35$$

6. An incomplete distribution is given as follows:

Variable:	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
Frequency:	10	20	?	40	?	25	15

You are given that the median value is 35 and the sum of all the frequencies is 170. Using the median formula, fill up the missing frequencies.

Sol:

Class interval	Frequency	Cumulative frequency
0-10	10	10
10-20	20	30
20-30	f_1	$30 + f_1 (F)$

30-40	40(F)	$70 + f_1$
40-50	f_2	$70 + f_1 + f_2$
50-60	25	$95 + f_1 + f_2$
60-70	15	$40 + f_1 + f_2$
	$N = 170$	

Given median = 35

The median class = 30 – 40

$$l = 30, h = 40 - 30 = 10, f = 40, F = 30 + f_1$$

$$\text{Median } l + \frac{\frac{N}{2} - F}{f} \times h$$

$$35 = 30 + \frac{85 - (30 + f_1)}{40} \times 10$$

$$\Rightarrow 5 = \frac{55 - f_1}{4}$$

$$\Rightarrow F_1 = 55 - 20 = 25$$

Given

Sum of frequencies = 170

$$\Rightarrow 10 + 20 + f_1 + 40 + f_2 + 25 + 15 = 170$$

$$\Rightarrow 10 + 20 + 35 + 40 + f_2 + 25 + 15 = 170$$

$$\Rightarrow f_2 = 170 - 145$$

$$\Rightarrow f_2 = 25$$

$$\therefore f_1 = 35 \text{ and } f_2 = 25$$

7. Calculate the missing frequency from the following distribution, it being given that the median of the distribution is 24.

Age in years: 0 - 10	10 - 20	20 - 30	30 - 40	40-50
No. of persons: 5	25	?	18	7

Sol:

Class interval	Frequency	Cumulative frequency
0-10	5	5
10-20	25	30(F)
20-30	$x(f)$	$30 + x$
30-40	18	$48 + x$
40-50	7	$55 + x$
	$N = 170$	

Given

Median = 24

Then median class = 20 – 30

$$l = 20, h = 30 - 20, F = 30$$

$$\text{Median} = l + \frac{\frac{N}{2} - f}{f} h$$

$$\Rightarrow 24 \cdot 20 + \frac{\frac{55+x}{2} - 30}{x} \times 30$$

$$\Rightarrow 4x = 20 \left(\frac{55+x}{2} - 30 \right) \times 10$$

$$\Rightarrow 4x = 275 + 5x - 300$$

$$\Rightarrow 4x - 5x = -25$$

$$\Rightarrow -x = -25$$

$$\Rightarrow x = 25$$

$\therefore$ Missing frequency = 25

8. Find the missing frequencies and the median for the following distribution if the mean is 1.46.

No. of accidents:	0	1	2	3	4	5	Total
Frequency (No. of days):	46	?	?	25	10	5	200

Sol:

No. of accidents (x)	No. of days (f)	fx
0	46	0
1	x	x
2	y	$2y$
3	25	75
4	10	40
5	5	25
	$N = 200$	$\Sigma f_i x_i = x + 2y + 140$

Given, $N = 200$

$$\Rightarrow 46 + x + y + 25 + 10 + 5 + 5 = 200$$

$$\Rightarrow x + y = 200 - 46 - 25 - 10 - 0$$

$$\Rightarrow x + y = 114 \quad \dots(i)$$

And mean = 1.46

$$\Rightarrow \frac{\Sigma fx}{N} = 1.46$$

$$\Rightarrow \frac{x + 2y + 140}{200} = 1.46$$

$$\Rightarrow x + 2y + 140 = 292$$

$$\Rightarrow x + 2y = 292 + 40$$

$$\Rightarrow x + 2y = 152 \quad \dots\dots\dots(2)$$

Subtract equation (1) from equation (2)

$$\Rightarrow x + 2y - x - y = 152 - 114$$

$$\Rightarrow y = 38$$

Put the value of y in (1), we have $x = 114 - 38 = 76$

No. of accidents	No. of days	Cumulative frequency
0	46	46
1	76	122
2	38	160
3	25	185
4	10	195
5	5	200
	$N = 200$	

We have

$$N = 200$$

$$\frac{N}{2} = \frac{200}{2} = 100$$

The cumulative frequency just more than $\frac{N}{2}$ is 122 then the median is 1.

9. An incomplete distribution is given below:

Variable: 10-20 20-30 30-40 40-50 50-60 60-70 70-80

Frequency: 12 30 - 65 - 25 18

You are given that the median value is 46 and the total number of items is 230.

(i) Using the median formula fill up missing frequencies.

(ii) Calculate the AM of the completed distribution.

Sol:

(i)

Class interval	Frequency	Cumulative frequency
10-20	12	12

20-30	30	42
30-40	x	$42 + x(F)$
40-50	$65(f)$	$1107 + x$
50-60	y	$107 + x + y$
60-70	25	$132x + x + y$
70-80	18	$150 + x + y$
	$N = 200$	

Given median = 46

Then, median as = 40 – 50

$$\therefore l = 40, h = 50 - 40 = 10, f = 65, F = 42 + x$$

$$\therefore \text{median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\Rightarrow 46 = 40 + \frac{115 - (42 + x)}{65} \times 10$$

$$\Rightarrow \frac{6 \times 65}{10} = 73 - x$$

$$\Rightarrow 39 = 73 - x$$

$$\Rightarrow x = 73 - 39$$

$$\Rightarrow x = 34$$

Given $N = 230$

$$= 12 + 30 + 34 + 65 + y + 25 + 18 = 230$$

$$\Rightarrow 184 + y = 230$$

$$\Rightarrow y = 230 - 184 = 46$$

(ii)

Class interval	Mid value	Frequency	fx
10-20	15	12	180
20-30	25	30	750
30-40	35	34	1190
40-50	45	65	2925
50-60	55	46	2530
60-70	65	25	1625
70-80	75	18	1650
		$N = 270$	$\Sigma fx = 10550$

$$\text{Mean} = \frac{\Sigma fx}{N}$$

$$= \frac{10550}{270}$$

$$\therefore 4 = 87$$

10. The following table gives the frequency distribution of married women by age at marriage:

Age (in years)	Frequency	Age (in years)	Frequency
15-19	53	40-44	9
20-24	140	45-49	5
25-29	98	50-54	3
30-34	32	55-59	3
35-39	12	60 and above	2

Calculate the median and interpret the results

Sol:

Class interval (exclusive)	Class interval (inclusive)	Frequency	Cumulative frequency
15-19	14.5 – 19.5	53	53(F)
20-24	19.5 – 24.5	140(f)	193
25-29	24.5 – 29.5	98	291
30-34	29.5 – 34.5	32	393
35-39	34.5 – 39.5	12	335
40-44	39.5 – 44.5	9	344
45-49	44.5 – 49.5	5	349
50-54	49.5 – 54.5	3	352
54-59	54.5 – 59.5	3	355
60 and above	59.5 and above	2	357
		$N = 357$	

$$N = 357$$

$$\frac{N}{2} = \frac{357}{2} = 178.5$$

The cumulative frequency just greater than $\frac{N}{2}$ is 193, then the median class is 19.5-24.5

such that

$$l = 19.5, f = 140, F = 53, h = 24.5 - 19.5 = 5$$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\text{Median} = 19.5 + \frac{178.5 - 53}{140} \times 5 = 23.98$$

Nearly half the a women were married between ages 15 and 25.

11. If the median of the following frequency distribution is 28.5 find the missing frequencies:

Class interval:	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency:	5	f_1	20	15	f_2	5	60

Sol:

Class interval	Frequency	Cumulative frequency
0-10	5	5
10-20	f_1	$5 + f_1 (F)$
20-30	$20(F)$	$25 + F_1$
30-40	15	$40 + f_1$
40-50	f_2	$40 + f_1 + f_2$
50-60	5	$45 + f_1 + f_2$
	$N = 60$	

Given

$$\text{Median} = 28.5$$

Then, median class = 20 + 30

$$l = 20, f = 20, F = 5 + f_1, h = 30 - 20 = 10$$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\Rightarrow 28.5 - 20 = \frac{30 - (5 + f_1)}{20} \times 10$$

$$\Rightarrow 28.5 - 20 = \frac{30 - 5 - f_1}{20} \times 10$$

$$\Rightarrow 8.5 = \frac{25 - f_1}{2}$$

$$\Rightarrow f_1 = 25 - 17$$

$$\Rightarrow f_1 = 8$$

Given sum of frequency = 60

$$\Rightarrow 5 + f_1 + 20 + 15 + f_2 + 5 = 60$$

$$\Rightarrow 5 + 8 + 20 + 15 + f_2 + 5 = 60$$

$$\Rightarrow f_2 = 7$$

$$f_1 = 8; f_2 = 7$$

12. The median of the following data is 525. Find the missing frequency, if it is given that there are 100 observations in the data:

Class interval	Frequency	Class interval	Frequency
0-100	2	500-600	20
100-200	5	600-700	f_2
200-300	f_1	700-800	9
300-400	12	800-900	7

400-500	17	900-1000	4
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Sol:

Class interval	Frequency	Cumulative frequency
0-100	2	2
100-200	5	7
200-300	f_1	$7 + f_1$
300-400	12	$19 + f_1$
400-500	17	$36 + f_1 (F)$
500-600	20(f)	$56 + f_1$
600-700	f_2	$56 + f_1 + f_2$
700-800	9	$65 + f_1 + f_2$
800-900	7	$75 + f_1 + f_2$
900-1000	4	$76 + f_1 + f_2$
	$N = 100$	

Given media = 525

Then media class = 500 – 600

 $l = 500, f = 20, F = 36 + f_1, h = 600 - 500 = 100$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\Rightarrow 525 = 500 + \frac{50 - (36 + f_1)}{20} \times 100$$

$$\Rightarrow 525 - 500 = \frac{50 - 36 - f_1}{20} \times 100$$

$$\Rightarrow 25 = (14 - f_1)5$$

$$\Rightarrow 5f_1 = 45 \Rightarrow f_1 = 9$$

Given sum of frequencies = 100

$$\Rightarrow 2 + 5 + f_1 + 12 + 17 + 20 + f_2 + 9 + 7 + 4 = 100$$

$$\Rightarrow 2 + 5 + 9 + 12 + 17 + 20 + f_2 + 9 + 17 + 4 = 100$$

$$\Rightarrow 86 + f_2 = 100 \Rightarrow f_2 = 15$$

$$\therefore f_1 = 9; f_2 = 15$$

13. If the median of the following data is 32.5, find the missing frequencies.

Class interval:	0- 10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequency:	f_1	5	9	12	f_2	3	2	40

Sol:

Class interval	Frequency	Cumulative frequency
0-10	f_1	f_1
10-20	5	$5 + f_1$
20-30	9	$14 + f_1$
30-40	$12(f)$	$26 + f_1$
40-50	f_2	$26 + f_1 + f_2$
50-60	3	$29 + f_1 + f_2$
60-70	2	$31 + f_1 + f_2$
	$N = 40$	

Given

$$\text{Median} = 32.5$$

The median class = 30 – 40

$$l = 30 \therefore 40 - 30 = 10, f = 12, F = 14 + f_1$$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\Rightarrow 32.5 = 30 + \frac{20 - (14 + f_1)}{12} \times 10$$

$$\Rightarrow 2.5 = \frac{6 - f_1}{6} \times 5 \Rightarrow 15 = (6 - f_1)5$$

$$\Rightarrow 3 = 6 - f_1 \Rightarrow \frac{15}{5} = 6 - f_1$$

$$\Rightarrow f_1 = 3$$

Given sum of frequencies = 40

$$\Rightarrow 3 + 5 + 9 + 12 + f_2 + 3 + 2 = 40$$

$$\Rightarrow 34 + f_2 = 40$$

$$\Rightarrow f_2 = 6$$

$$\therefore f_1 = 3; f_2 = 6$$

- 14.** A survey regarding the height (in cm) of 51 girls of X of a school was conducted and the following data was obtained:

(i) Marks	No. of students	(ii) Marks	No. of students
Less than 10	0	More than 150	0
Less than 30	10	More than 140	12
Less than 50	25	More than 130	27
Less than 70	43	More than 120	60
Less than 90	65	More than 110	105

Less than 110	87	More than 100	124
Less than 130	96	More than 90	141
Less than 150	100	More than 80	150

Sol:

(i)

Marks	No. of students	Class interval	Frequency	Cumulative frequency
Less than 10	0	0-10	0	0
Less than 30	10	10-30	10	10
Less than 50	25	30-50	15	25
Less than 70	43	50-70	18	43(F)
Less than 90	65	70-90	22(f)	65
Less than 110	87	90-110	22	87
Less than 130	96	110-130	9	96
Less than 150	100	130-150	8	100
			$N = 100$	

We have $N = 100$

$$\frac{N}{2} = \frac{100}{2} = 50$$

The cumulative frequencies just greater than $\frac{N}{2}$ is 65 then median class is 70-90 such

that $l = 70$, $f = 22$, $F = 43$, $h = 90 - 70 = 20$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$= 70 + \frac{50 - 43}{22} \times 20$$

$$= 70 + \frac{7 \times 20}{22}$$

$$= 70 + \frac{50 - 43}{22} \times 20$$

$$= 70 + \frac{7 \times 20}{22}$$

$$= 70 + 6.36$$

$$= 76.36$$

(ii)

Marks	No. of students	Class interval	Frequency	Cumulative frequency
Less than 80	150	80-90	9	9
Less than 90	141	90-100	17	26
Less than 100	124	100-110	19	45(F)
Less than 110	105	110-120	45(f)	90
Less than 120	60	120-130	33	123

Less than 130	27	130-140	45	138
Less than 140	12	150-160	0	150
Less than 150	0	150-160	0	150
			$N = 150$	

We have $N = 150$

$$\frac{N}{2} = \frac{150}{2} = 75$$

The cumulative frequencies just greater than $\frac{N}{2}$ is 90 then median class is 110-120 such that $l = 110, f = 45, F = 45, h = 120 - 110 = 10$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$= 110 + \frac{75 - 45}{45} \times 10$$

$$= 110 + \frac{30 \times 10}{45}$$

$$= 110 + 6 + 67$$

$$= 116.67.$$

15. A survey regarding the height (in cm) of 51 girls of class X of a school was conducted and the following data was obtained:

Height in cm	Number of Girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

Find the median height.

Sol:

To calculate the median height, we need to find the class intervals and their corresponding frequencies

The given distribution being of the less than type 140, 145, 150, ..., 165 give the upper limits of the corresponding class intervals. So, the classes should be below 140, 145, 150, ..., 160, 165 observe that from the given distribution, we find that there are 4 girls with height less than 140 is 4. Now there are 4 girls with heights less than 140. Therefore, the number of girls with height in the interval 140, 145 is $11 - 4 = 7$, similarly. The frequencies of 145-150 is $29 - 11 = 18$, for 150-155 it is $40 - 29 = 11$, and so on so our frequency distribution becomes.

Class interval	Frequency	Cumulative frequency
below 140	4	4
140-145	7	11
145-150	18	29
150-155	11	40
155-160	6	46
160-165	5	51

Now $n = 51$, So, $\frac{n}{2} = \frac{51}{2} = 25.5$. This observation lies in the class 145–150.

Then,

The lower limit = 145

CFC The cumulative frequency of the class

Preceding 145–150 = 11

F (The frequency of the median as $145 + 800 = 18$,

$h(\text{class limit}) = 5$

$$\text{Median} = 145 + \left(\frac{25.5 - 11}{18} \right) \times 5$$

$$= 145 + \frac{725}{18}$$

$$= 149.03$$

So, the median height of the girls is 149.03cm . This means what the height of be about 50% of the girls in less than this height, and 50% are taller than this height,

- 16.** A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are only given to persons having age 18 years onwards but less than 60 years.

Age in years	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

Sol:

Here class width is not same. There is no need to adjust the frequencies according to class intervals. Now given frequencies table is of less than type represented with upper class

limits. As policies were given only to persons having age 18 years onwards but less than 60 years we can definite class intervals with their respective cumulative frequencies as below

Age (in years)	No of policy planers	Cumulative frequency (cr)
18-20	2	2
20-25	6-2=4	5
25-30	24-6=18	24
30-35	45-24=21	45
35-40	78-45=33	78
40-45	89-78=11	89
45-50	92-89=3	92
50-55	98-92=6	98
55-60	100-98=2	100

Total (n)

Now from the table we may observe that n=100 cumulative frequencies (F) just greater

than $\frac{n}{2}$ (i.e., $\frac{100}{2} = 50$) is 78 belonging to interval 35-40

So median class = 35-40

Lower limit (l) of median class = 35

Class size (h) = 5

Frequencies (f) of median class = 33

Cumulative frequency (F) of class preceding median class = 45

$$\text{Median} = \frac{\left(\frac{n}{2} - cf\right)}{f} \times h$$

$$= 35 + \left(\frac{50 - 45}{33}\right) \times 5$$

$$= 35 + \frac{25}{33}$$

$$= 35.76$$

So, median age is 35.76 years

17. The lengths of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table:

Length (in mm):	118-126	127-135	136-144	145-153	154-162	163-171	172-180
No. of leaves:	3	5	9	12	5	4	2

Find the mean length of life.

Sol:

The given data is not having continuous class intervals is 1. So, we have to add and subtract

$$\frac{1}{2} = 0.5 \text{ to upper class limits and lower class limits}$$

Now continuous class intervals with respective cumulative frequencies can be presented as below

Length (in mm)	Number of leaves f_i	Cumulative frequencies
117.5–126.5	3	3
126.5–135.5	5	8 = 3 + 5
135.5–144.5	9	17 = 8 + 9
144.5–153.5	12	29 = 17 + 12
153.5–162.5	5	37 = 29 + 5
162.5–171.5	4	34 + 4 = 38
171.5–180.5	2	38 + 2 = 40

From the table we may observe that cumulative frequencies just greater than

$\frac{n}{2}$ (i.e., $\frac{40}{2} = 20$) is 29 belonging class interval 144.5–153.5

Median class = 144.5–153.5

Lower limit (L) of median class = 144.5

Class size (h) = 9

Frequencies (f) of median class = 12

Cumulative frequencies (f) of class preceding median class = 17

$$\text{Median} = l + \frac{\left(\frac{n}{2} - cf\right)}{f} \times h$$

$$= 144.5 + \left(\frac{20 - 17}{12}\right) \times 9$$

$$= 144.5 + \frac{9}{4}$$

$$= 146.75$$

So, median length is 146.75 mm

- 18.** The following table gives the distribution of the life time of 400 neon lamps:

Life time: (in hours)	Number of lamps
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

Find the median life.

Sol:

We can find cumulative frequencies with their respective class intervals as below

Life time	Number of lams (f_i)	Cumulative frequencies
1500-2000	14	14
2000-2500	56	14+56=70
2500-3000	60	70+50=130
3000-3500	86	130+86=216
3500-4000	74	216+74=290
4000-4500	62	290+62=352
4500-5000	48	352+48=400
Total	420	

Now we may observe that cumulative frequencies just greater $430 \times \frac{n}{2}$ (i.e., $\frac{400}{2} = 200$) is

216 belonging to class interval 3000–3500

Median class 3000–3500

Lower limit (1) of median class = 3000

Frequencies (f) of median class = 86

Cumulative frequencies (cf) of class preceding

Median class = 130

Class size = 500

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c.f}{f} \right) \times h = 3000 + \left(\frac{200 - 130}{86} \right) \times 500$$

$$= 3000 + \frac{70 \times 500}{86} = 3406.98 \text{ hours}$$

So, median life time is 3406.98 hours

19. The distribution below gives the weight of 30 students in a class. Find the median weight of students:

Weight (in kg): 40-45 45-50 50-55 55-60 60-65 65-70 70-75

No. of students: 2 3 8 6 6 3 2

Sol:

We may find cumulative frequencies with their respective class internals as below

Weight in (kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Number of students (f)	2	3	8	6	6	3	2
cf	2	5	13	19	25	28	30

Cumulative frequencies just great class interval er than $\frac{n}{2}$ (i.e., $\frac{30}{2} = 15$) is 19, belonging to class interval 55-60

Median class = 55 – 60

Lower limit (l) of median class = 55

Frequency of median class = 6

Cumulative frequencies $y(f)$ of median class = 13

Class $h = 5$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$= 55 + \left(\frac{15 - 13}{6} \right) \times 5$$

$$= 55 + \frac{10}{6}$$

$$= 56.666$$

So, median weight is 56.67 kg

Exercise – 7.5

1. Find the mode of the following data:

(i) 3, 5, 7, 4, 5, 3, 5, 6, 8, 9, 5, 3, 5, 3, 6, 9, 7, 4

(ii) 3, 3, 7, 4, 5, 3, 5, 6, 8, 9, 5, 3, 5, 3, 6, 9, 7, 4

(iii) 15, 8, 26, 25, 24, 15, 18, 20, 24, 15, 19, 15

Sol:

(i)

Value (x)	3	4	5	6	7	8	9
Frequency (f)	4	2	5	2	2	1	2

Mode = 5 because it occurs maximum number of times

(ii)

Value (x)	3	4	5	6	7	8	9
Frequency (f)	5	2	4	2	2	1	2

Mode = 3 because it occurs maximum number of times

(iii)

Value (x)	3	4	5	6	7	8	9
Frequency (f)	1	4	1	1	2	1	1

Mode = 15 because it occurs maximum number of times

2. The shirt sizes worn by a group of 200 persons, who bought the shirt from a store, are as follows:

Shirt size:	37	38	39	40	41	42	43	44
Number of persons:	15	25	39	41	36	17	15	12

Find the modal shirt size worn by the group.

Sol:

Shirt size	37	38	39	40	41	42	43	44
Frequency (f)	15	25	39	41	36	17	15	12

Model shirt size = 40 because it occurs maximum number of times

3. Find the mode of the following distribution.

(i) Class-interval: 0-10 10-20 20-30 30-40 40-50 50-60 60-70 70-80

Frequency: 5 8 7 12 28 20 10 10

(ii) Class-interval: 10-15 15-20 20-25 25-30 30-35 35-40

Frequency: 30 45 75 35 25 15

(iii) Class-interval: 25-30 30-35 35-40 40-45 45-50 50-60

Frequency: 25 34 50 42 38 14

Sol:

(i)

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of persons	5	8	7	12	18	20	10	10

Here the maximum frequency is 28 then the corresponding class 40 – 50 is the model class

$L = 40$, $h = 50 - 40 = 10$, $f = 28$, $f_1 = 12$, $f_2 = 20$

$$\text{Mode} = L + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 40 + \frac{28 - 12}{2 \times 28 - 12} \times 10$$

$$= 40 + \frac{16 \times 10}{24}$$

$$= 40 + 160 = 46.67$$

(ii)

Class interval	10-15	15-20	20-25	25-30	30-35	35-40
No. of persons	30	45	75	35	25	15

Here the maximum frequency is 75 then the corresponding class 20 – 25 is the model class

$L = 25$, $h = 25 - 20 = 5$, $f = 75$, $f_1 = 45$, $f_2 = 35$

$$\text{Mode} = L + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 20 + \frac{75 - 45}{2 \times 75 - 45 - 35} \times 5$$

$$= 20 + \frac{30 \times 5}{70}$$

$$= 20 + 2.14$$

$$= 22.14$$

(iii)

Class interval	25-30	30-35	35-40	40-45	45-50	50-55
No. of persons	25	34	50	42	38	14

Here the maximum frequency is 50 then the corresponding class 35 – 40 is the model class

$$L = 35, h = 40 - 35 = 5, f = 50, f_1 = 34, f_2 = 42$$

$$\text{Mode} = L + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 35 + \frac{50 - 34}{2(50) - 34 - 42} \times 5$$

$$= 35 + \frac{16 \times 5}{24}$$

$$= 35 + 3.33$$

$$= 38.33$$

4. Compare the modal ages of two groups of students appearing for an entrance test:

Age (in years): 16-18 18-20 20-22 22-24 24-26

Group A: 50 78 46 28 23

Group B: 54 89 40 25 17

Sol:

Age in years	16 – 18	18 – 20	20 – 22	22 – 24	24 – 26
Group A	50	78	46	28	23
Group B	54	89	40	25	17

For Group A

Here the maximum frequency is 78, then the corresponding class 18 – 20 is model class

$$L = 18, h = 20 - 18 = 2, f = 78, f_1 = 50, f_2 = 46$$

$$\text{Mode} = L + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 18 + \frac{78 - 50}{156 - 50 - 46} \times 2$$

$$= 18 + \frac{56}{60} = 18 + 0.93$$

$$= 18.93 \text{ years}$$

For group B

Here the maximum frequency is 89, then the corresponding class 18 – 20 is model class

$$L = 18, h = 20 - 18 = 2, f = 89, f_1 = 54, f_2 = 40$$

$$\text{Mode} = L + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 18 + \frac{89 - 54}{156 - 54 - 40} \times 2$$

$$= 18 + \frac{70}{84}$$

$$= 18 + 0.83$$

$$= 18.83$$

Hence the mode of age for the group A is higher than group B

5. The marks in science of 80 students of class X are given below: Find the mode of the marks obtained by the students in science.

Marks: 0-10 10-20 20-30 30-40 40-50 50-60 60-70 70-80 80-90 90-100

Frequency: 3 5 16 12 13 20 5 4 1 1

Sol:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	5	16	12	13	20	5	4	1	1

Here the maximum frequency is 20, then the corresponding class 50 – 60 is modal class

$$L = 50, h = 60 - 50 = 10, f = 20, f_1 = 13, f_2 = 5$$

$$\text{Mode} = L + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 50 + \frac{20 - 13}{40 - 13 - 5} \times 10$$

$$= 50 + \frac{7 \times 10}{22}$$

$$= 50 + 3.18$$

$$= 53.18$$

6. The following is the distribution of height of students of a certain class in a certain city:

Height (in cm): 160-162 163-165 166-168 169-171 172-174

No. of students: 15 118 142 127 18

Find the average height of maximum number of students.

Sol:

Height(exclusive)	160-162	163-165	166-168	169-171	172-174
Height(inclusive)	159.5-162.5	162.5-165.5	165.5-168.5	168.5-171.5	171.5-174.5
No. of students	15	118	142	127	18

Here the maximum frequency is 142, then the corresponding class 165.5 – 168.5 is modal class

$$L = 165.5, h = 168.5 - 165.5 = 3, f = 142, f_1 = 118, f_2 = 127$$

$$\text{Mode} = L + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 165.5 + \frac{142 - 118}{2 \times 142 - 118 - 127} \times 3$$

$$= 165.5 + \frac{24 \times 3}{39}$$

$$= 165.5 + 1.85$$

$$= 167.35 \text{ cm}$$

7. The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years): 5-15 15-25 25-35 35-45 45-55 55-65

No. of students: 6 11 21 23 14 5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Sol:

We may observe compute class marks (x_i) as per the relation

$$x_i = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Now taking 30 as assumed mean (a) we may calculate and $f_i d_i$ as follows

Age (in yrs)	No. of patients (f_i)	Class Mark x_i	$d_i = x_i - 30$	$f_i d_i$
5-15	6	10	-20	-120
15-25	11	20	-10	-110
25-35	21	30	0	0
35-45	23	40	10	230
45-55	14	50	20	280
55-65	5	60	30	150
Total	80			430

From the table we may observe that $\sum f_i = 80$

$$\sum f_i d_i = 430$$

$$\text{Mean} = a + \frac{\sum f_i d_i}{\sum f_i}$$

$$= 30 + \left(\frac{430}{80}\right)$$

$$= 30 + 5.375$$

$$= 35.38$$

Clearly mean of this data is 35.38. It represents that on an average the age of patient admitted to hospital was 35.58 years. As we may observe that maximum class frequency 23 belonging to class interval 35 – 45

So, modal class = 35 – 45

Lower limit (L) of modal class = 35

Frequency (f_1) of modal class = 23

Class size (h) = 10

Frequency (f_0) of class preceding the modal = 21

Frequency (f_2) of class succeeding the modal = 14

$$\text{Now mode} = L + \left(\frac{f - f_0}{2f - f_0 - f_2}\right) h$$

$$= 35 + \left[\frac{23 - 21}{2(23) - 21 - 14}\right] \times 10$$

$$= 35 + \frac{20}{11}$$

$$= 35.81$$

$$= 36.8$$

8. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

Lifetimes (in hours):	0-20	20-40	40-60	60-80	80-100	100-120
No. of components:	10	35	52	61	38	29

Determine the modal lifetimes of the components.

Sol:

From data as given above we may observe that maximum class frequency 61 belonging to class interval 60 – 80.

So, modal class 60 – 80

$L = 60$, $h = 20$, $f_0 = 52$, $f_1 = 61$, $f_2 = 38$

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) h$$

$$= 60 + \left(\frac{61 - 52}{2(61) - 52 - 37} \right) 20$$

$$= 60 + \frac{9 \times 20}{32} = 60 + \frac{90}{16} = 60 + 5.625$$

$$= 65.625$$

9. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure:

Expenditure (in Rs.)	Frequency	Expenditure (in Rs.)	Frequency
1000-1500	24	3000-3500	30
1500-2000	40	3500-4000	22
2000-2500	33	4000-4500	16
2500-3000	28	4500-5000	7

Sol:

We may observe that the given data the maximum class frequency is 40 belonging to 1500 – 2000 interval. So modal class = 1500 – 2000

L.L (L) = 1500, f. of M.C (f_1) = 40

Frequency of class preceeding modal class $f_0 = 24$

Frequency of class succeeding modal class $f_2 = 33$

Class size (h) = 50

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) h$$

$$= 1500 + \left[\frac{40 - 24}{2(40) - 24 - 33} \right] \times 500$$

$$= 1500 + \left[\frac{16}{80 - 67} \times 500 \right]$$

$$= 1500 + \frac{8000}{23}$$

$$= 1500 + 347.826$$

$$1847.826 = 1847.83$$

So modal class monthly expenditure was Rs. 1847.83

Now we may find class mark as

$$\text{Class mark} = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Class size (h) of given data = 500

Now taking 2750 as assumed mean(a) we may calculate d, 4 and $f_i 4$ as follows.

Expenditure In Rs.	No. of families f_i	x_i	$d_i = x_i - 2750$	$4i$	$f_i 4i$
1000-1500	24	1250	-1500	-3	-72
1500-2000	40	1750	-1000	-2	-80
2000-2500	33	2250	-500	-1	-33
2500-3000	28	2750	0	0	0
3000-3500	30	3250	500	1	30
3500-4000	22	3750	1000	2	44
4000-4500	16	4250	1500	3	48
4500-5000	7	4750	2000	4	28
Total	200				-35

Now from table we may observe that

$$\sum f_i 4i = 200$$

$$\sum f_i 4i = -35$$

$$(\bar{x}) \text{ mean} = a + \left(\frac{\sum f_i 4i}{\sum f_i} \right) \times h$$

$$(\bar{x}) = 2750 + \left(\frac{-35}{200} \right) \times 500$$

$$= 2750 - 87.5$$

$$= 2662.5$$

So mean, monthly expenditure was Rs. 2662.50 ps.

We may observe them the given data the maximum class frequency is 10 belonging to class interval 30 – 35

So modal class 30 – 35

Class size (h) = 5

Lower limit (L) of modal class = 30

Frequency (f_1) of modal class 10

Frequency (f_0) of class preceeding modal class = 9

Frequency (f_2) of class succeeding modal class = 3

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) h$$

$$= 30 + \left(\frac{10 - 9}{20 - 9 - 3} \right) 5$$

$$= 30 + \frac{5}{8}$$

$$= 30.625$$

Mode = 30.6

It represents that most of states 1 UT have a teacher – student ratio as 30.6

Now we may find class marks by using the relation

$$\text{Class mark} = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Now taking 325 as assumed mean (a) we may calculate d_i and $f_i d_i$ as following.

10. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret, the two measures:

Number of students per teacher	Number of states / U.T.	Number of students per teacher	Number of states / U.T.
15-20	3	35-40	3
20-25	8	40-45	0
25-30	9	45-50	0
30-35	10	50-55	2

Sol:

No. of Students / teacher	No. of states / U.T. (f_i)	x_i	$d_i = x_i - 32.5$	$f_i d_i$	$f_i^2 d_i^2$
15 – 20	3	17.5	-15	-45	675
20 – 25	8	22.5	-10	-80	800
25 – 30	9	27.5	-5	-45	225
30 – 35	10	32.5	0	0	0
35 – 40	3	37.5	5	15	75
40 – 45	0	42.5	10	0	0
45 – 50	0	47.5	15	0	0
50 – 55	2	52.5	20	40	800
Total	35			-75	1500

$$\text{Now mean } (\bar{x}) = a + \left(\frac{\sum f_i d_i}{\sum f_i} \right) h$$

$$= 32.5 + \left(\frac{-75}{35} \times 5 \right)$$

$$= 32.5 - \frac{75}{7}$$

$$= 32.5 - 10.71$$

$$= 21.79$$

So mean of data is 21.79

It represents that on an average teacher.

Student ratio was 21.79

11. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Runs scored	No. of batsman	Runs scored	No. of batsman
3000-4000	4	7000-8000	6
4000-5000	18	8000-9000	3
5000-6000	9	9000-10000	1
6000-7000	7	10000-11000	1

Find the mode of the data.

Sol:

From the given data we may observe that maximum class frequently is 18 belonging to class interval 4000 – 5000

So modal class 4000 – 5000

Lower limit (1) of model class = 4000

Frequently (f_1) of class preceding modal class = 4

Frequently (f_2) of class succeeding modal class = 9

Frequently of modal case (f_i) = 18

Class size = 1000

$$\text{Now mode} = 1 + \left(\frac{1 - f_n}{2 - f_0 - f_2} \right) \times h$$

$$= 4000 + \left(\frac{18 - 4}{2(18) - 4 - 9} \right) \times 1000$$

$$4000 + \frac{14000}{23} = 4608.695$$

So, mode of given data is ~~4608~~ 4608.7 Runs

12. A student noted the number of cars passing through a spot on a road for loo periods each of 3 minutes and summarized it in the table given below. Find the mode of the data:

Sol:

From the given data we may observe that maximum class internal frequency is 200 belonging to modal class 40 – 50

$$l = 40, f_1 = 20, f_0 = 12, f_2 = 11, h = 10$$

$$\text{Mode} = 1 + \left(\frac{f - f_0}{2f - f_0 - f_2} \right) h$$

$$= 40 + \left[\frac{20 - 12}{40 - 12 - 11} \right] \times 10$$

$$= 40 + \frac{180}{17} = 40 + 4 \cdot 7 = 44 \cdot 7$$

13. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.
Monthly consumption - 65-85 85-105 105-125 125-145 145-165 165-185 185-205
(in units)

No. of consumers: 4 5 13 20 14 8 4

Sol:

Class interval	Mid value x	Frequency f	fx	Cumulative frequency
65-75	75	4	300	4
85-105	95	5	475	9
105-125	115	13	1495	22
125-145	135	20	2700	42
145-165	155	17	2170	56
165-185	175	8	1400	64
185-205	195	4	78	68
Total		$N = 68$	$\Sigma fx = 9320$	

$$\text{Mean} = \frac{\Sigma fx}{N} = \frac{9320}{68} = 137 \cdot 08$$

We have $N = 68$

$$\frac{N}{2} = \frac{68}{2} = 34$$

The cumulative frequency just $> \frac{N}{2}$ is 42 then the median mass 125–145 such that

$$l = 125, f = 20, F = 22, h = 20$$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h = 125 + \frac{34 - 22}{20} \times 20 = 137$$

Here the maximum frequently is 20, then the corresponding class 125-145 is the modal class $l = 125, h = 20, f = 20, f_1 = 13, f_2 = 14$

$$\begin{aligned} \text{Mode} &= l + \frac{f - f_1}{2f - f_1 - f_2} \times h = 125 + \frac{20 - 13}{40 - 13 - 14} \times 20 \\ &= 125 + \frac{7 \times 20}{13} = 135 \cdot 77 \end{aligned}$$

14. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

Number of letters: 1-4 4-7 7-10 10-13 13-16 16-19

Number surnames: 6 30 40 16 4 4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames. Also, find the modal size of the surnames.

Sol:

Class interval	Mid value x	Frequency f	fx	Cumulative frequency
1-4	2.5	6	15	6
4-7	5.5	30	165	36
7-10	8.5	40	340	76
10-13	11.5	16	185	92
13-16	14.5	4	58	96
16-19	17.5	4	70	100
		$N = 100$	$\Sigma fx = 832$	

$$\text{Mean} = \frac{\Sigma fx}{N} = \frac{832}{100} = 8.32$$

$$N = 100 \Rightarrow N/2 = 50$$

The cumulative frequency $> \frac{N}{2}$ is 76, median class 7–10

$$l = 7, h = 3, f = 40, F = 36.$$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h = 7 + \frac{50 - 36}{40} \times 3$$

$$= 7 + \frac{14 \times 3}{40} = 8.05$$

Here the maximum frequency is 40, then the corresponding class 7-10 is the modal class

$$l = 7, h = 10 - 7 = 3, f = 40, f_1 = 30, f_2 = 36$$

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h = 7 + \frac{40 - 30}{2 \times 40 - 30 + 36} \times 3$$

$$= 7 + \frac{10 \times 3}{34} = 7.88$$

15. Find the mean, median and mode of the following data:

Classes: 0-20 20-40 40-60 60-80 80-100 100-120 120-140

Frequency: 6 8 10 12 6 5 3

Sol:

Class interval	Mid value x	Frequency f	fx	Cumulative frequency
0-20	10	6	60	6
20-40	30	8	240	17

40-60	50	10	500	24
60-80	70	12	840	36
80-100	90	6	540	42
100-120	110	5	550	47
120-140	130	3	390	50
		$N = 60$	$\Sigma fx = 3120$	

$$\text{Mean} = \frac{\Sigma fx}{N} = \frac{3120}{60} = 52$$

We have $N = 60$

$$\text{Then, } \frac{1}{2} = \frac{50}{2} = 25$$

$c, > \frac{N}{2}$ is 36 then median class 60-80 such that

$$l = 60, h = 20, f = 12, F = 24$$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h = 60 + \frac{25 - 24}{12} \times 20 = 60 + 1.67$$

Modal class $l = 60, h = 20, f = 12, f_1 = 10, f_2 = 6$

$$\text{Mode} = l + \left[\frac{f - f_1}{2f - f_1 - f_2} \right] h = 60 + \left[\frac{12 - 10}{24 - 10 - 6} \right] 20$$

$$= 60 + \frac{40}{8} = 65$$

Mode = 65

- 16.** Find the mean, median and mode of the following data:

Classes: 0-50 50-100 100-150 150-200 200-250 250-300 300-350

Frequency: 2 3 5 6 5 3 1

Sol:

Class interval	Mid value x	Frequency f	fx	Cumulative frequency
0-50	25	2	50	2
50-100	75	3	225	5
100-150	125	5	625	10
150-200	175	6	1050	16
200-250	225	5	1125	21
250-300	275	3	825	24
300-350	325	1	325	25
		$N = 25$	$\Sigma fx = 4225$	

$$\text{Mean} = \frac{\Sigma fx}{N} = \frac{4225}{25} = 169$$

We have $N = 25$ then $\frac{N}{2} = 12.5$

c. $f > \frac{N}{2}$ 16, median class 150–200 such that

$$l = 150, h = 200 - 150 = 50, f = 6, F = 10$$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h = 150 + \frac{12.5 - 10}{6} \times 50$$

$$= 150 + 20.83 = 170.83$$

Here the maximum frequency is 6 then the corresponding class 150-200 is the modal class

$$l = 150, h = 200 - 150 = 50, f = 6, f_1 = 5, f_2 = 5$$

$$\text{Mode} = t + \frac{F - t_1}{2f - f_1 - f_2} \times h = 150 + \frac{6 - 5}{12 - 5 - 5} \times 50$$

$$= 150 + \frac{50}{2} = 175.$$

17. The following table gives the daily income of 50 workers of a factory:

Daily income (in Rs) 100 - 120 120 - 140 140 - 160 160 - 180 180 - 200

Number of workers: 12 14 8 6 10

Find the mean, mode and median of the above data.

Sol:

Class interval	Mid value x	Frequency f	fx	Cumulative frequency
100-200	110	12	1320	12
120-140	130	14	1820	26
140-160	150	8	1200	34
160-180	170	6	1000	40
180-200	190	10	1900	50
		$N = 50$	$\Sigma fx = 7260$	

$$\text{Mean} = \frac{\Sigma fx}{N} = \frac{7260}{50}$$

$$= 145.2$$

We have

$$N = 50$$

$$\text{Then } \frac{N}{2} = \frac{50}{2} = 25$$

The cumulative frequency is $> \frac{N}{2}$ is 26 corresponding class median class 120-140 such that

$$l = 120, h = 140 - 120 = 20, f = 14, F = 12$$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$= 120 + \frac{25 - 12}{14} \times 20$$

$$= 120 + 18.57$$

$$= 138.57$$

Here the maximum frequency is 14, then the corresponding class 120-140 is the modal class

$$l = 120, h = 140 - 120 = 20, f = 14, f_1 = 12, f_2 = 8$$

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 120 + \frac{14 - 12}{2 \times 14 - 12 - 8} \times 20$$

$$= 120 + \frac{2 \times 20}{5}$$

$$\Rightarrow 120 + 8$$

$$= 128$$

$$\text{Mode} = 128$$

Exercise – 7.6

1. Draw a graph by less than method for the following data:

No. of rooms: 1 2 3 4 5 6 7 8 9 10

No. of houses: 4 9 22 28 24 12 8 6 5 2

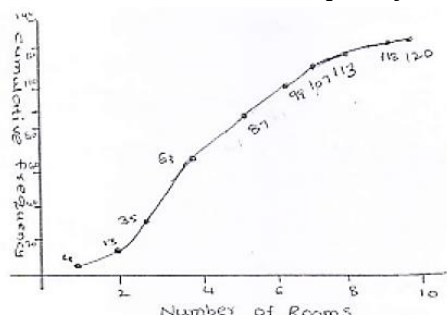
Sol:

We first prepare the cumulative frequency distribution table by less than method as given below

No. of Rooms	No. of houses	Cumulative frequency
Less than or equal to 1	4	4
Less than or equal to 2	9	13
Less than or equal to 3	22	35
Less than or equal to 4	28	63
Less than or equal to 5	24	87
Less than or equal to 6	12	99
Less than or equal to 7	8	107
Less than or equal to 8	6	113
Less than or equal to 9	5	118
Less than or equal to 10	2	120

Now, we mark the upper class limits along x-axis and cumulative frequency along y-axis. Thus we plot the point (1, 4), (2, 35), (3, 35), (4, 63), (5, 87), (6, 99), (7, 107), (8, 113), (9, 118), (10, 120).

Cumulative frequency



2. The marks scored by 750 students in an examination are given in the form of a frequency distribution table:

Marks	No. of students	Marks	No. of students
600 – 640	16	760 – 800	172
640 – 680	45	800 – 840	59
680 – 720	156	840 – 880	18
720 – 760	284		

Prepare a cumulative frequency table by less than method and draw an ogive.

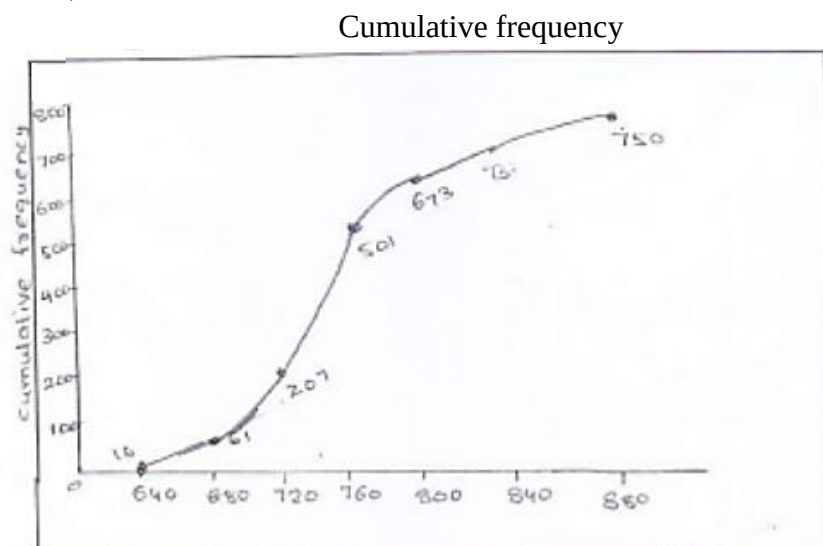
Sol:

We first prepare the cumulative frequency table by less than method as given below

Marks	No. of students	Marks less than	Cumulative frequency
600 – 640	16	640	16
640 – 680	45	680	61
680 – 720	156	720	217
720 – 760	284	760	501
760 – 800	172	800	693
800 – 840	59	840	732
840 – 880	18	880	750

Now, we mark the upper class limits along x-axis and cumulative frequency along y-axis on a suitable gear.

Thus, we plot the points (640, 16), (680, 61), (720, 217), (760, 501), (800, 673), (840, 732) and (880, 750)



3. Draw an ogive to represent the following frequency distribution:

Class-interval: 0 - 4 5 - 9 10 - 14 15 - 19 20-24

No. of students: 2 6 10 5 3

Sol:

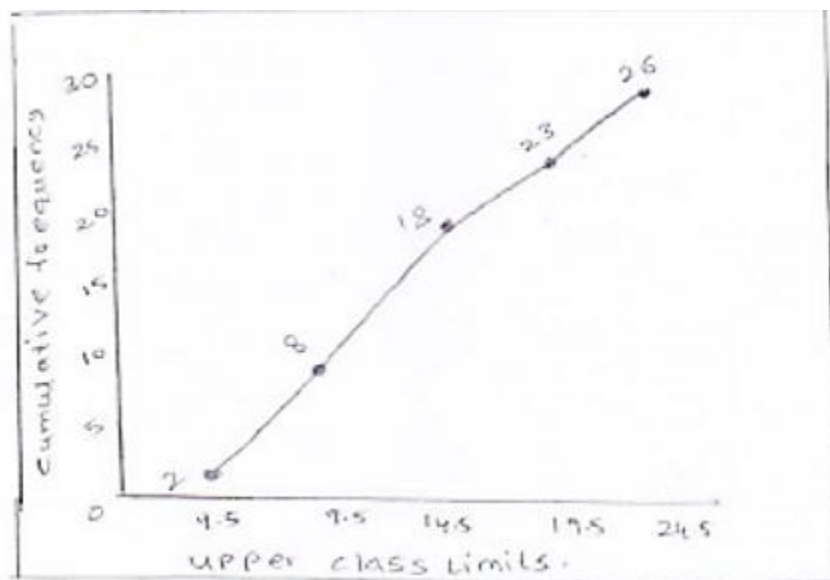
The given frequency of distribution is not continuous so we first make it continuous and prepare the cumulative frequency distribution as under

Class Interval	No. of Students	Less than	Cumulative frequency
0.5 – 4.5	2	4.5	2
4.5 – 9.5	6	9.5	8
9.5 – 14.5	10	14.5	18
14.5 – 19.5	5	19.5	23
19.5 – 24.5	3	24.5	26

Now, we mark the upper class limits along x-axis and cumulative frequency along y-axis.

Thus we plot the points (4, 5, 2), (9, 5, 8), (14, 5, 18), (19, 5, 23) and (24, 5, 26)

Cumulative frequency



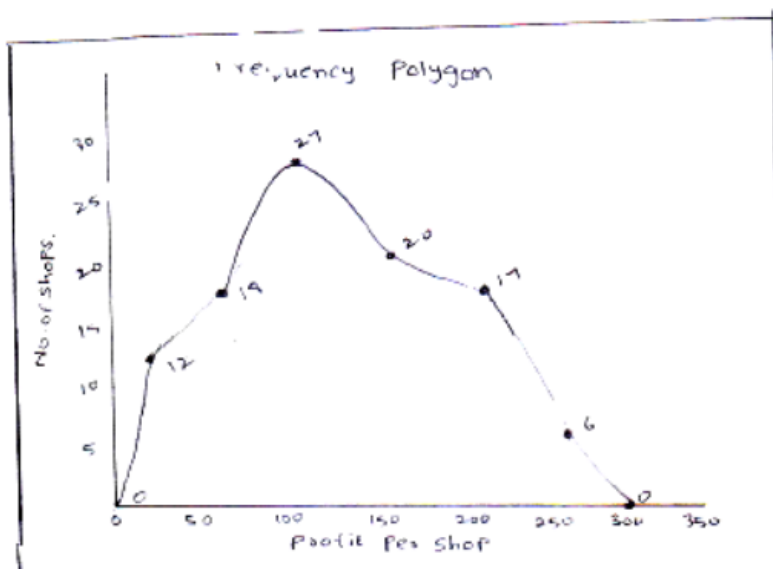
4. The monthly profits (in Rs.) of 100 shops are distributed as follows:
 Profits per shop: 0 - 50 50 - 100 100 - 150 150 - 200 200 - 250 250 - 300
 No. of shops: 12 18 27 20 17 6
 Draw the frequency polygon for it.

Sol:

We have,

Profit per shop	Mid value	No. of shops
Less than 0	0	0
0 - 50	25	12
50 - 100	75	18
100 - 150	125	27
150 - 200	175	20
200 - 250	225	17
250 - 300	275	6

Above 300	300	0
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5. The following table gives the height of trees:

Height	No. of trees
Less than 7	26
Less than 14	57
Less than 21	92
Less than 28	134
Less than 35	216
Less than 42	287
Less than 49	341
Less than 56	360

Draw 'less than' ogive and 'more than' ogive.

Sol:

Less than method,

It is given that,

Height	No of trees
Less than 7	26
Less than 14	57
Less than 21	92
Less than 28	134
Less than 35	216
Less than 42	287
Less than 49	341
Less than 56	360

Now, we mark the upper class limits along x-axis and cumulative frequency along y-axis.

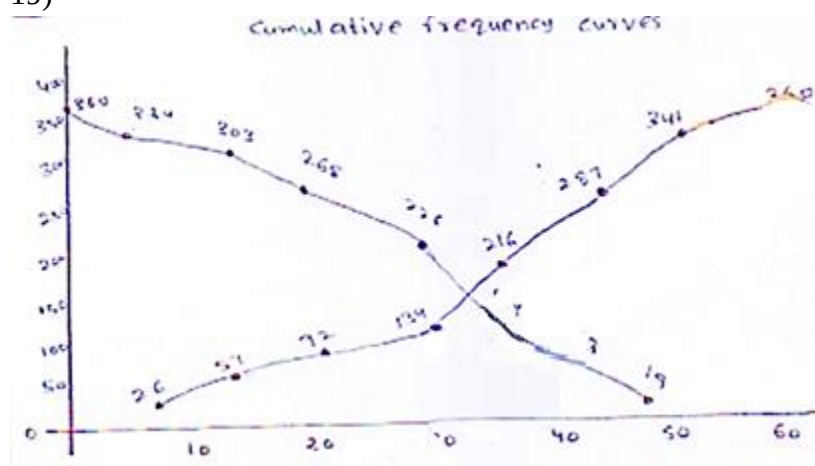
Thus we plot the points (7, 26) (14, 57) (21, 92) (28, 134) (35, 216) (42, 287) (49, 341) (56, 360)

More than method: we prepare the cf table by more than method as given below:

Height	Frequency	Height more than	Cumulative frequency
0 – 7	26	0	360
7 – 14	31	7	334
14 – 21	42	21	268
21 – 28	82	28	226
28 – 35	71	35	144
35 – 42	54	42	73
49 – 56	19	49	19

Now, we mark on x –axis lower class limits, y-axis cumulative frequency

Thus, we plot graph at (0, 360) (7, 334) (14, 303) (21, 268) (28, 226) (35, 144) (42, 73) (49, 19)



6. The annual profits earned by 30 shops of a shopping complex in a locality give rise to the following distribution:

Profit (in lakhs in Rs)	Number of shops (frequency)
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3

Draw both ogives for the above data and hence obtain the median.

Sol:

More than method

Profit (in lakhs in Rs)	No. of shops (frequency)
≥ 5	30

≥ 10	28
≥ 15	16
≥ 20	14
≥ 25	10
≥ 30	7
≥ 35	3

Now, we mark on x- axis lower class limits, y- axis cumulative frequency

Thus, we plot the points (5, 30) (10, 28) (15, 16) (20, 14) (25, 10) (30, 7) and (35, 3)

Less than method

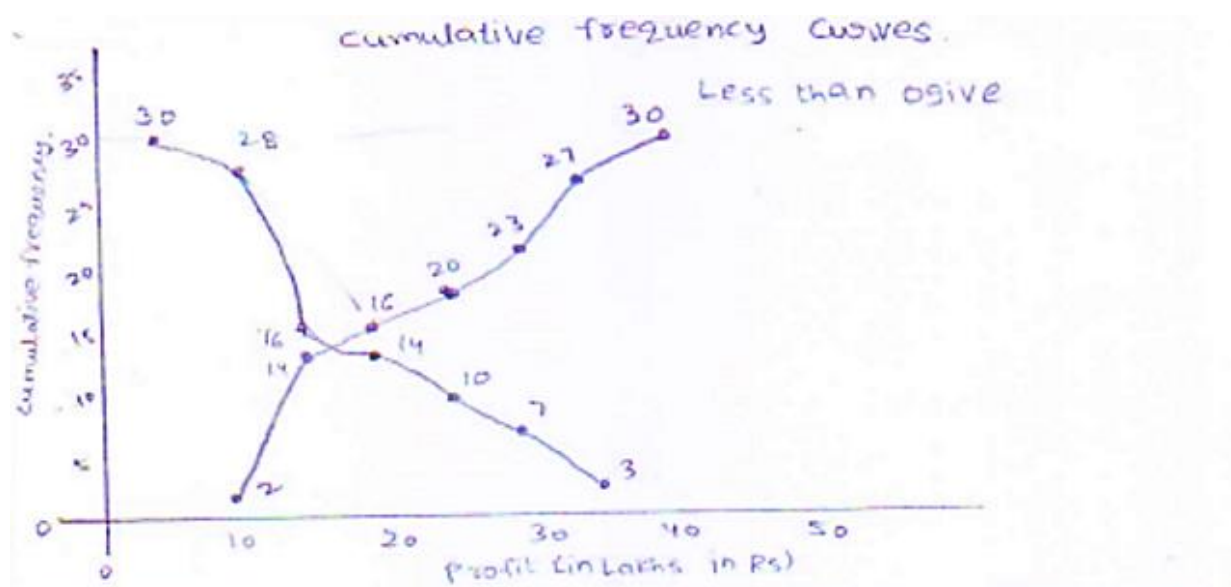
Profit (in lakhs in Rs)	No. of shops (frequency)	Profit less than	Cumulative frequency
0 – 10	2	10	2
10 – 15	12	15	14
15 – 20	2	20	16
20 – 25	4	25	20
25 – 30	3	30	23
30 – 35	4	35	27
35 – 40	3	40	30

Now, we mark the upper class limits along x-axis and cumulative frequency along y-axis.

Thus we plot the points. (10, 2) (15, 14) (20, 16) (25, 20) (30, 23) (35, 27) (40, 30)

We find that the two types of curves intersect at point P from point L it is drawn on x-axis.

The value of a profit corresponding to M is 17.5 lakh, Hence median is 17.5 Lakh



7. The following distribution gives the daily income of 50 workers of a factory:

Daily income (in Rs): 100 - 120 120 - 140 140 - 160 160 - 180 180 - 200

Number of workers: 12 14 8 6 10

Convert the above distribution to a less than type cumulative frequency distribution and draw its ogive.

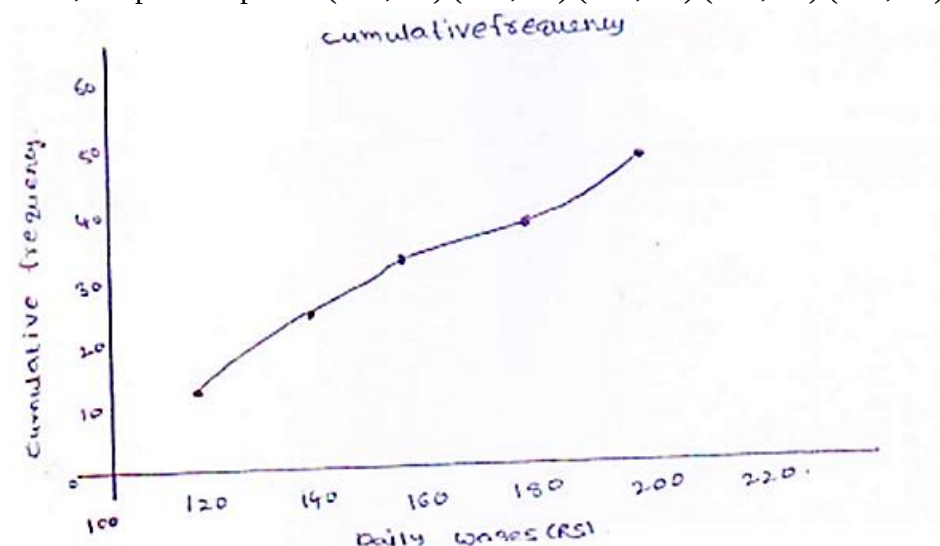
Sol:

We first prepare the cumulative frequency table by less than method as given below.

Daily income (in Rs.)	Cumulative frequency
< 120	12
< 140	26
< 160	34
< 180	40
< 200	50

Now, we mark on x – axis upper class limit, y – axis cumulative frequencies.

Thus, we plot the points (120, 12) (140, 26) (160, 34) (180, 40) (200, 50)



8. The following table gives production yield per hectare of wheat of 100 farms of a village:
Production yield 50 - 55 55 - 60 60 - 65 65 - 70 70 - 75 75 - 80 in kg per hectare:

Number of farms: 2 8 12 24 38 16

Draw 'less than' ogive and 'more than' ogive.

Sol:

Less than method:

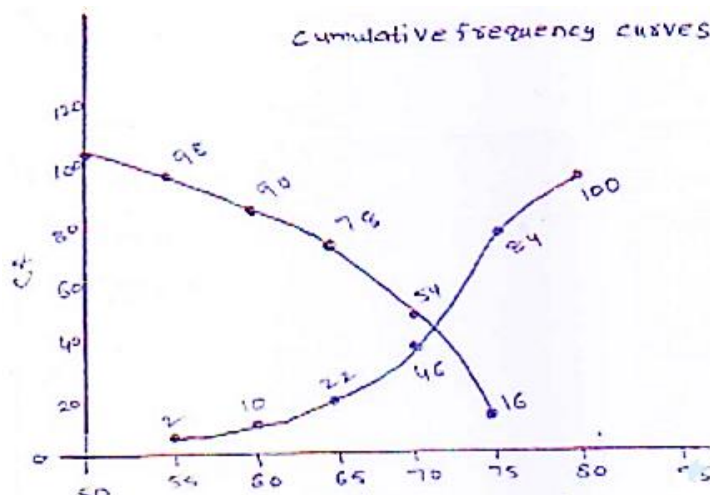
Cumulative frequency table by less than method.

Production yield (integer)	Number of farms	Production yield more than	Cumulative frequency
50 - 55	2	50	100
55 - 60	8	55	98
60 - 65	12	60	90

65 – 70	24	65	78
70 – 75	38	70	54
75 – 80	16	75	16

Now, we mark on x – axis upper class limit, y – axis cumulative frequencies.

We plot the points (50, 100) (55, 98) (60, 90) (65, 78) (70, 54) (75, 16)



9. During the medical check-up of 35 students of a class, their weights were recorded as follows:

Weight (in kg)	No. of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Draw a less than type ogive for the given data. Hence, obtain the median weight from the graph and verify the result by using the formula

Sol:

Less than method

It is given that

On x- axis upper class limits. Y- axis cf.

We plot the points (38, 0) (40, 3) (42, 5) (44, 9) (46, 14) (48, 28) (50, 32) (52, 35)

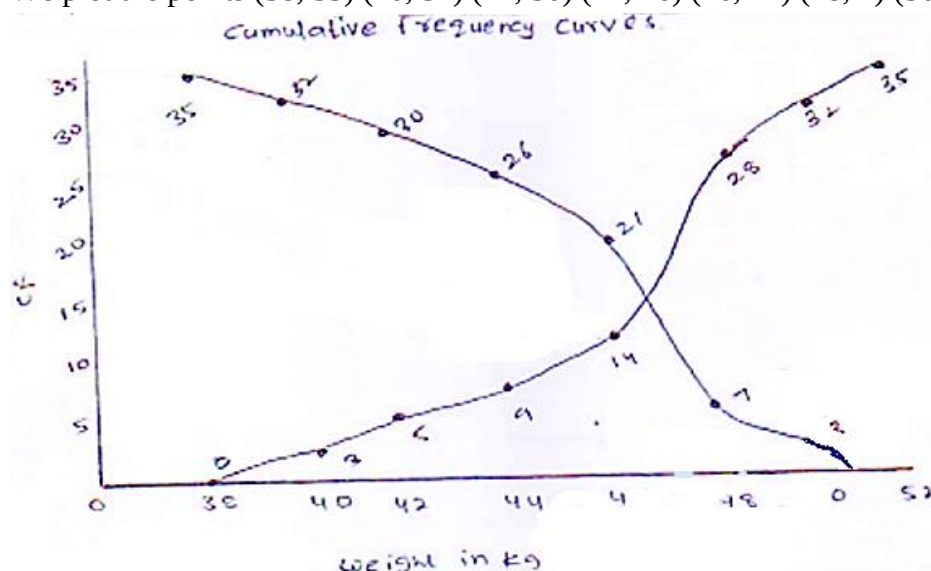
More than method: Cf table

Weight (in kg)	No. of students	Weight more than	Cumulative frequency
38 – 40	3	38	34
40 – 42	2	40	32
42 – 44	4	42	30

44 – 46	5	44	26
46 – 48	14	46	21
48 – 50	4	48	7
50 – 52	3	50	3

x- axis lower class limits on y-axis – cf

We plot the points (38, 35) (40, 32) (42, 30) (44, 26) (46, 21) (48, 7) (50, 3)



We find the two types of curves intersect at a point P. From point P, from P perpendicular PM is draw on x-axis.

The verification,

We have

Weight (in kg)	No. of students	Cumulative frequency
36 – 38	0	0
38 – 40	3	3
40 – 42	2	5
42 – 44	4	9
44 – 46	5	28
46 – 48	14	32
48 – 50	4	32
50 – 0	3	35

Now, $N = 35$

$$\Rightarrow \frac{N}{2} = \frac{35}{2} = 17.5$$

The cumulative frequency just greater than $\frac{N}{2}$ is 28 and the corresponding class is 46 – 48

Thus 46 – 48 is the median class such that

$L = 46$, $f = 14$, $c_1 = 14$ and $h = 2$

$$\therefore \text{Median} = L + \frac{\frac{N}{2} - c_1}{f} \times h$$

$$= 46 + \frac{17.5-14}{14} \times 2$$

$$= 46 + \frac{7}{14}$$

$$= 46.5$$

$\therefore$ Median = 46.5 kg

$\therefore$ Hence verify.

