

Exercise 8.1

1. Which of the following are quadratic equations?

- (i) $x^2 + 6x - 4 = 0$
- (ii) $\sqrt{3x^2} - 2x + \frac{1}{2} = 0$
- (iii) $x^2 + \frac{1}{x^2} = 5$
- (iv) $x - \frac{3}{x} = x^2$
- (v) $2x^2 - \sqrt{3x} + 9 = 0$
- (vi) $x^2 - 2x - \sqrt{x} - 5 = 0$
- (vii) $3x^2 - 5x + 9 = x^2 - 7x + 3$
- (viii) $x + \frac{1}{x} = 1$
- (ix) $x^2 - 3x = 0$
- (x) $\left(x + \frac{1}{x}\right)^2 = 3\left(1 + \frac{1}{x}\right) + 4$
- (xi) $(2x + 1)(3x + 2) = 6(x - 1)(x - 2)$
- (xii) $x + \frac{1}{x} = x^2, x \neq 0$
- (xiii) $16x^2 - 3 = (2x + 5)(5x - 3)$
- (xiv) $(x + 2)^3 = x^3 - 4$
- (xv) $x(x + 1) + 8 = (x + 2)(x - 2)$

Sol:

- (i) $x^2 + 6x - 4 = 0$
- (ii) $\sqrt{3x^2} - 2x + \frac{1}{2} = 0$
- (iii) $3x^2 - 5x + 9 = x^2 - 7x + 3$
- (iv) $x + \frac{1}{x} = 1$
- (v) $(2x + 1)(3x + 2) = 6(x - 1)(x - 2)$
- (vi) $16x^2 - 3 = (2x + 5)(5x - 3)$
- (vii) $(x + 2)^3 = x^3 - 4$

These are all quadratic equations

2. In each of the following, determine whether the given values are solutions of the given equation or not:

- (i) $x^2 - 3x + 2 = 0, x = 2, x = -1$
- (ii) $ax^2 - 3abx + 2b^2 = 0, x = \frac{a}{b}$ and $x = \frac{b}{a}$
- (iii) $x^2 - \sqrt{2}x - 4 = 0, x = -\sqrt{2}$ and $x = -8\sqrt{2}$
- (iv) $2x^2 - x + 9 = x^2 + 4x + 3, x = 2$ and $x = 3$

(v) $x + \frac{1}{2} = \frac{13}{6} = x = \frac{5}{6}, x = \frac{4}{3}$

(vi) $x^2 - 3\sqrt{3}x + 6 = 0, x = \sqrt{3}, x = -8\sqrt{3}$

(vii) $x^2 + x + 1 = 0, x = 0, x = 1$

Sol:

(i) $x^2 - 3x + 2 = 0, x = 2, x = -1$

Here LHS = $x^2 - 3x + 2$

and RHS = 0

Now, substitute x = 2 in LHS

We get $(2)^2 - 3(2) + 2 = 4 - 6 + 2$

$= 6 - 6$

$= 0$

\Rightarrow RHS

Since, LHS = RHS

x = 2 is a solution for the given equation.

Similarly,

Now substitute x = -1 in LHS

We get $(-1)^2 - 3(-1) + 2$

$\Rightarrow 1 + 3 + 2 = 6 \neq$ RHS

Since LHS \neq RHS

x = -1 is not a solution for the given equation

(ii) $x^2 + x + 1 = 0, x = 0, x = 1$

Here LHS = $x^2 + x + 1$ and RHS = 0

Now substitute x = 0 and x = 1 in LHS

$\Rightarrow 0^2 + 0 + 1$ and $(1)^2 + (1) + 1$

$\Rightarrow 1$ and $1 + 1 + 1 = 3$

\neq RHS \neq RHS

\therefore x = 0, x = 1 are not solutions of the given equation

(iii) $x^2 - 3\sqrt{3}x + 6 = 0, x = \sqrt{3}, x = -8\sqrt{3}$

Here LHS = $x^2 - 3\sqrt{3}x + 6$ and RHS = 0

Substitute $x = \sqrt{3}$ and $x = -2\sqrt{3}$ in LHS

$\Rightarrow (\sqrt{3})^2 - 3\sqrt{3}(\sqrt{3}) + 6$ and $(-2\sqrt{3})^2 - 3\sqrt{3}(-2\sqrt{3}) + 6$

$\Rightarrow 3 - 9 + 6$ and $18 + 18 + 6$

$\Rightarrow 0$ and 36

\Rightarrow RHS \neq RHS

$\therefore x = \sqrt{3}$ is a solution and $x = -2\sqrt{3}$ is not a solution for the given equation

(iv) $x + \frac{1}{2} = \frac{13}{6} = x = \frac{5}{6}, x = \frac{4}{3}$

Here LHS = $x + \frac{1}{x}$ and RHS = $\frac{13}{6}$

Substitute $x = \frac{5}{6}$ and $x = \frac{4}{3}$ in the LHS

$$\Rightarrow \frac{5}{6} + \frac{1}{\left(\frac{5}{6}\right)} \text{ and } \frac{4}{3} + \frac{1}{\left(\frac{4}{9}\right)}$$

$$\Rightarrow \frac{5}{6} + \frac{6}{5} \text{ and } \frac{4}{3} + \frac{3}{4}$$

$$\Rightarrow \frac{85+36}{30} \text{ and } \frac{16+9}{18}$$

$$\Rightarrow \frac{61}{30} \text{ and } \frac{85}{18}$$

\neq RHS \neq RHS

$\therefore x = \frac{5}{6}$ and $x = \frac{4}{3}$ are not solutions of the given equation

$$(v) \quad 2x^2 - x + 9 = x^2 + 4x + 3, x = 2 \text{ and } x = 3$$

$$\Rightarrow 2x^2 - x^2 - x - 4x + 9 - 3 = 0$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

Here, LHS = $x^2 - 5x + 6$ and RHS = 0

Substitute $x = 2$ and $x = 3$ in LHS

$$\Rightarrow (2)^2 - 5(2) + 6 \text{ and } (3)^2 - 5(3) + 6$$

$$\Rightarrow 4 - 10 + 6 \text{ and } 9 - 15 + 6$$

$$\Rightarrow 10 - 10 \text{ and } 15 - 15$$

$$\Rightarrow 0 \text{ and } \Rightarrow 0$$

$=$ RHS $=$ RHS

$x = 3$ and $x = 2$ are solutions of the given equation.

$$(vi) \quad x^2 - \sqrt{2}x - 4 = 0, x = -\sqrt{2} \text{ and } x = -8\sqrt{2}$$

Here, LHS = $x^2 - \sqrt{2}x - 4$ and RHS = 0

Substitute $x = -\sqrt{2}$ and $x = -2\sqrt{2}$ in LHS

$$\Rightarrow (-\sqrt{2})^2 - \sqrt{2}(-\sqrt{2}) - 4 \text{ and } (-2\sqrt{2})^2 - \sqrt{2}(-2\sqrt{2}) - 4$$

$$\Rightarrow 2 + 2 - 4 \text{ and } 8 + 4 - 4$$

$$\Rightarrow 4 - 4 \text{ and } 8 - 4$$

$$\Rightarrow 0 \text{ and } 8$$

$=$ RHS \neq RHS

$\therefore x = -\sqrt{2}$ is a solution and $x = -2\sqrt{2}$ is not a solution in the given equation.

(vii) $ax^2 - 3abx + 2b^2 = 0, x = \frac{a}{b}$ and $x = \frac{b}{a}$

Here, LHS = $ax^2 - 3abx + 2b^2$ and RHS = 0

Substitute $x = \frac{a}{b}$ and $x = \frac{b}{a}$ in LHS

$$\Rightarrow a^2 \left(\frac{a}{b} \right)^2 - 3ab \left(\frac{a}{b} \right) + 2b^2 \text{ and } a^2 \left(\frac{b}{a} \right)^2 - 3ab \left(\frac{b}{a} \right) + 2b^2$$

$$\Rightarrow a^2 \left(\frac{a^2}{b^2} \right) - 3a \times a + 2b^2 \text{ and } a^2 \times \frac{b^2}{a^2} - 3b \times b + 2b^2$$

$$\Rightarrow \frac{a^2}{b^2} - 3a^2 + 2b^2 \text{ and } b^2 - 3b^2 + 2b^2$$

$$\Rightarrow \frac{a^4}{b^2} - 3a^2 + 2b^2 \text{ and } 3b^2 - 3b^2 = 0$$

$\Rightarrow \neq$ RHS = RHS

$\therefore x = \frac{b}{a}$ is a solution and $x = \frac{a}{b}$ is not a solution for the given equation.

3. In each of the following, find the value of k for which the given value is a solution of the given equation:

(i) $7x^2 + kx - 3 = 0, x = \frac{2}{3}$

(ii) $x^2 - x(a + b) + k = 0, x = a$

(iii) $kx^2 + \sqrt{2}x - 4 = 0, x = \sqrt{2}$

(iv) $x^2 + 3ax + k = 0, x = -a$

Sol:

(i) Given that $x = \frac{2}{3}$ is a root of the given equation

$$\Rightarrow x = \frac{2}{3} \text{ satisfies the equation}$$

$$i.e. 7 \left(\frac{2}{3} \right)^2 + k \left(\frac{2}{3} \right) - 3 = 0$$

$$\Rightarrow 7 \times \frac{4}{9} + 2 \frac{k}{3} - 3 = 0$$

$$\Rightarrow 2 \frac{k}{3} = 3 - \frac{28}{9}$$

$$\Rightarrow 2 \frac{k}{3} = \frac{27 - 28}{9}$$

$$\Rightarrow 2 \frac{k}{\beta} = -\frac{1}{\gamma} \Rightarrow \boxed{k = \frac{-1}{6}}$$

(ii) Given that $x = a$ is a root of the given equation

$$x^2 - x(a+b) + k = 0$$

$\Rightarrow x = a$ Satisfies the equation

$$i \cdot e(a)^2 - a(a+b) + k = 0$$

$$\Rightarrow a^2 - a^2 - ab + k = 0 \Rightarrow -ab + k = 0$$

$$\Rightarrow \boxed{k = ab}$$

(iii) Given that $x = \sqrt{2}$ is a root at the given equation

$$kx^2 + \sqrt{2}x - 4 = 0$$

$\Rightarrow x = \sqrt{2}$ Satisfies the equation

$$i \cdot e k(\sqrt{2})^2 + \sqrt{2}(\sqrt{2}) - 4 = 0$$

$$\Rightarrow 2k + 2 - 4 = 0$$

$$\Rightarrow 2k - 2 = 0 \Rightarrow 2k = 2$$

$$\Rightarrow \boxed{k = 1}$$

(iv) Given that $x = -0$ is a root of the given equation $x^2 + 3ax + k = 0$

$\Rightarrow x = -a$ Satisfies the equation

$$i \cdot e(-a)^2 + 3a(-a) + k = 0$$

$$\Rightarrow a^2 - 3a^2 + k = 0 \Rightarrow -2a^2 + k = 0$$

$$\Rightarrow \boxed{k = 2a^2}$$

4. If $x = \frac{2}{3}$ and $x = -3$ are the roots of the equation $ax^2 + 7x + b = 0$, find the values of a and b.

Sol:

$$a = 3, b = -6$$

5. Determine if, 3 is a root of the equation given below:

$$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 16}$$

Sol:

Given to check whether 3 is a root of the equation

$$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 16}$$

Here LHS = $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9}$ and RHS = $\sqrt{4x^2 - 14x + 16}$

Substitute $x = 3$ in LHS

$$\Rightarrow \sqrt{3^2 - 4(3) + 3} + \sqrt{(3)^2} = 9$$

$$\Rightarrow \sqrt{9 - 18 + 3} + \sqrt{9 - 9}$$

$$\Rightarrow \sqrt{0} + \sqrt{0} \Rightarrow 0 \quad \therefore \text{LHS} = 0$$

Similarly, substitute $x = 3$ in RHS.

$$\Rightarrow \sqrt{4(3)^2 - 14(3) + 16}$$

$$\Rightarrow \sqrt{4 \times 9 - 42 + 16} \Rightarrow \sqrt{36 + 42 + 16}$$

$$\Rightarrow \sqrt{52 - 42} \Rightarrow \sqrt{10}$$

$$\therefore \text{RHS} = \sqrt{10}$$

Now, we can observe that

$\text{LHS} \neq \text{RHS}$

$\therefore x = 3$ is not a solution or root for the equation

$$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 16}$$

Exercise 8.2

1. The product of two consecutive positive integers is 306. Form the quadratic equation to find the integers, if x denotes the smaller integer.

Sol:

Given that the smallest integer of 2 consecutive integer is denoted by x

\Rightarrow The two integer will be x and $(x+1)$

Product of two integers $\Rightarrow x(x+1)$

Given that the product is 306

$$\therefore x(x+1) = 306$$

$$\Rightarrow x^2 + x = 306 \Rightarrow x^2 + x - 306 = 0$$

\therefore The required quadratic equation is $x^2 + x - 306 = 0$

2. John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 128. Form the quadratic equation to find how many marbles they had to start with, if John had x marbles.

Sol:

Given that John and Jivani together have 45 marbles and John has x marbles

\Rightarrow Jivani had $(45 - x)$ marbles

No. of marbles John had after loosing 5 marbles $= x - 5$

No. of marbles Jivani had after loosing 5 marbles $= (45 - x) - 5$

$$= 45 - 5 - x$$

$$= 40 - x$$

Given that product of the no of marbles they now have = 128

$$\Rightarrow (x-5)(40-x) = 128$$

$$\Rightarrow 40x - x^2 - 40 \times 5 + 5x = 128$$

$$\Rightarrow 45x - x^2 - 200 = 128 \Rightarrow x^2 - 45x + 128 + 200 = 0$$

$$\Rightarrow x^2 - 45x + 328 = 0$$

∴ The required quadratic equation is $x^2 - 45x + 328 = 0$

3. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of articles produced in a day. On a particular day, the total cost of production was Rs. 750. If x denotes the number of toys produced that day, form the quadratic equation to find x .

Sol:

Given that x denotes the no of toys produced in a day

⇒ The cost of production of each by = 55 – no. of toys produced in a day

$$= (55 - x)$$

Total cost of production is nothing but product of no. of toys produced in a day and cost of production of each toy

$$\Rightarrow x(55 - x)$$

But total cost of production = Rs 750

$$\Rightarrow x(55 - x) = 750$$

$$\Rightarrow 55x - x^2 = 750$$

$$\Rightarrow x^2 - 55x + 750 = 0$$

∴ The required quadratic from of the given data is $x^2 - 55x + 750 = 0$

4. The height of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, form the quadratic equation to find the base of the triangle.

Sol:

Given that in a right triangle is 7cm less than its base

Let base of the triangle be denoted by x

⇒ Height of the triangle = $(x - 7)$ cm

We have hypotenuse of the triangle = 13cm

We know that, in a right triangle

$$(\text{base})^2 + (\text{Height})^2 = (\text{Hypotenuse})^2$$

$$\Rightarrow (x)^2 + (x - 7)^2 = (13)^2$$

$$\begin{aligned}
 &\Rightarrow x^2 + x^2 - 14x + 49 = 169 \\
 &\Rightarrow 2x^2 - 14x + 49 - 169 = 0 \\
 &\Rightarrow 2x^2 - 14x - 120 = 0 \\
 &\Rightarrow 2(x^2 - 7x - 60) = 0 \\
 &\Rightarrow x^2 - 7x - 60 = 0 \\
 \therefore & \text{ The required quadratic equation is } x^2 - 7x - 60 = 0
 \end{aligned}$$

5. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore. If the average speed of the express train is 11 km/hr more than that of the passenger train, form the quadratic equation to find the average speed of express train.

Sol:

Let the average speed of express train be denoted by x km/hr

Given that average speed of express train is 11 km/hr more than that of the passenger train

$$\Rightarrow \text{Average speed of passenger train} = (x - 11) \text{ km/hr}$$

Total distance travelled by the train = 132 km

We know that,

$$\text{Time taken to travel} = \frac{\text{Distance travelled}}{\text{Average speed}}$$

$$\Rightarrow \text{Time taken by express train} = \frac{\text{Distance travelled}}{\text{Average speed of express train}}$$

$$= \frac{132}{x} \text{ hr}$$

$$\Rightarrow \text{Time taken by express train} = \frac{132}{(x - 11)} \text{ hr}$$

Given that time taken by express train is 1 hour less than that of passenger train.

$$\Rightarrow \text{Time taken by passenger train} - \text{Time taken by express train} = 1 \text{ hour}$$

$$\Rightarrow \frac{132}{x-11} - \frac{132}{x} = 1$$

$$\Rightarrow 132 \left(\frac{1}{x-11} - \frac{1}{x} \right) = 1$$

$$\Rightarrow 132 \left(\frac{x - (x-11)}{x(x-11)} \right) = 1$$

$$\Rightarrow 132(x - 2 + 11) = x(x - 11)$$

$$\Rightarrow 132(11) = x^2 - 11x$$

$$\Rightarrow x^2 - 11x = 1452$$

$$\Rightarrow x^2 - 11x - 1452 = 0$$

The required quadratic is $x^2 - 11x - 1452 = 0$

6. A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Form the quadratic equation to find the speed of the train.

Sol:

Let Speed of train be $x \text{ km/hr}$

Distance travelled by train = 360 km

We know that

$$\text{Time of travel} = \frac{\text{Distance travelled}}{\text{Speed of the train}} = \frac{360}{x} \text{ hr}$$

If speed had been 5 km/hr more $\Rightarrow (x+5) \text{ km/hr}$

$$\text{Time of travel} = \frac{\text{Distance travelled}}{\text{Speed of the train}} = \frac{360}{x+5} \text{ hr}$$

Given that,

Time of travel when speed is increased is 1 hour less than of the actual time of travel

$$\Rightarrow \frac{360}{x} - \frac{360}{x+5} = 1$$

$$\Rightarrow 360 \left(\frac{1}{x} - \frac{1}{x+5} \right) = 1$$

$$\Rightarrow 360 \left(\frac{x+5-x}{x(x+5)} \right) = 1$$

$$\Rightarrow 360(5) = x(x+5)$$

$$\Rightarrow x^2 + 5x = 1800$$

$$\Rightarrow x^2 + 5x + 1800 = 0$$

\therefore The required quadratic equation to find the speed of the train is $x^2 + 5x - 1800 = 0$

Exercise 8.3

Solve the following quadratic equations by factorization:

1. $(x-4)(x+2) = 0$

Sol:

We have

$$(x-4)(x+2) = 0$$

$$\Rightarrow \text{either } (x-4)=0 \text{ or } (x+2)=0$$

$$\Rightarrow x=4 \text{ or } x=-2$$

Thus, $x=4$ and $x=-2$ are two roots of the equation $(x-4)(x+2)=0$

2. $(2x+3)(3x-7)=0$

Sol:

We have,

$$(2x+3)(3x-7)=0$$

$$\Rightarrow (2x+3)=0 \text{ or } (3x-7)=0$$

$$\Rightarrow 2x=-3 \text{ or } 3x=7$$

$$\Rightarrow x=\frac{-3}{2} \text{ or } x=\frac{7}{3}$$

Thus, $x=\frac{-3}{2}$ and $x=\frac{7}{3}$ are two roots of the equation $(2x+3)(3x-7)=0$

3. $4x^2 + 5x = 0$

Sol:

We have $4x^2 + 5x = 0$

$$\Rightarrow x(4x+5)=0$$

$$\Rightarrow \text{either } x=0 \text{ or } 4x+5=0$$

$$\Rightarrow x=0 \text{ or } 4x=-5$$

$$\Rightarrow x=0 \text{ or } x=\frac{-5}{4}$$

Thus, $x=0$ and $x=\frac{-5}{4}$ are two roots of equation $4x^2 + 5x = 0$

4. $9x^2 - 3x - 2 = 0$

Sol:

We have $9x^2 - 3x - 2 = 0$

$$\Rightarrow 9x^2 - 6x + 3x - 2 = 0$$

$$\Rightarrow 3x(3x-2) + 1(3x-2) = 0$$

$$\Rightarrow (3x-2)(3x+1) = 0$$

$$\Rightarrow \text{either } 3x-2=0 \text{ or } 3x+1=0$$

$$\Rightarrow 3x=2 \text{ or } 3x=-1$$

$$\Rightarrow x=\frac{2}{3} \text{ or } x=-\frac{1}{3}$$

Thus, $x = \frac{2}{3}$ and $x = -\frac{1}{3}$ are two roots of the equation $9x^2 - 3x - 2 = 0$

5. $6x^2 - x - 2 = 0$

Sol:

We have $6x^2 - x - 2 = 0$

$$\Rightarrow 6x^2 + 3x - 4x - 2 = 0$$

$$\Rightarrow 3x(2x+1) - 2(2x+1) = 0$$

$$\Rightarrow (2x+1)(3x-2) = 0$$

$$\Rightarrow \text{either } 2x+1 = 0 \text{ or } 3x-2 = 0$$

$$\Rightarrow 2x = -1 \text{ or } 3x = 2$$

$$\Rightarrow x = -\frac{1}{2} \text{ or } x = \frac{2}{3}$$

Thus, $x = -\frac{1}{2}$ and $x = \frac{2}{3}$ are two roots of the equation $6x^2 - x - 2 = 0$

6. $6x^2 + 11x + 3 = 0$

Sol:

We have

$$6x^2 + 11x + 3 = 0$$

$$\Rightarrow 6x^2 + 9x + 2x + 3 = 0$$

$$\Rightarrow 3x(2x+3) + 1(2x+3) = 0$$

$$\Rightarrow (2x+3)(3x+1) = 0$$

$$\Rightarrow 2x+3 = 0 \text{ or } x = -\frac{1}{3}$$

Thus, $x = -\frac{3}{2}$ and $x = -\frac{1}{3}$ are the two roots of the given equation.

7. $5x^2 - 3x - 2 = 0$

Sol:

We have,

$$5x^2 - 3x - 2 = 0$$

$$\Rightarrow 5x^2 - 5x + 2(x-1) = 0$$

$$\Rightarrow 5x(x-1) + 2(x-1) = 0$$

$$\Rightarrow (x-1)(5x+2) = 0$$

$$\Rightarrow (x-1) = 0 \text{ or } 5x+2 = 0$$

$$\Rightarrow x=1 \text{ or } x=-\frac{2}{5}$$

$\therefore x=1$ and $x=-\frac{2}{5}$ are the two roots of the given equation.

8. $48x^2 - 13x - 1 = 0$

Sol:

We have

$$\begin{aligned} 48x^2 - 13x - 1 &= 0 \\ \Rightarrow 48x^2 - 16x + 3x - 1 &= 0 \\ \Rightarrow 16x(3x - 1) + 1(3x - 1) &= 0 \\ \Rightarrow (3x - 1)(16x + 1) &= 0 \\ \Rightarrow 3x - 1 &= 0 \text{ or } 16x + 1 = 0 \\ \Rightarrow x = \frac{1}{3} \text{ or } x = -\frac{1}{16} & \end{aligned}$$

$\therefore x = -\frac{1}{16}$ and $x = \frac{1}{3}$ are the two roots of the given equation.

9. $3x^2 = -11x - 10$

Sol:

We have

$$\begin{aligned} 3x^2 &= -11x - 10 \\ \Rightarrow 3x^2 + 11x + 10 &= 0 \\ \Rightarrow 3x^2 + 6x + 5x + 10 &= 0 \\ \Rightarrow 3x(x + 2) + 5(x + 2) &= 0 \\ \Rightarrow (x + 2)(3x + 5) &= 0 \\ \Rightarrow (x + 2) = 0 \text{ or } x = -\frac{5}{3} & \therefore x = 2 \text{ and } x = -\frac{5}{3} \text{ are the two roots of the quadratic equation} \\ 3x^2 &= -11x - 10 \end{aligned}$$

10. $25x(x + 1) = -4$

Sol:

We have

$$\begin{aligned} (x + 1) &= -4 \\ \Rightarrow (25x) \times x + (25x) \times 10 - 4 & \\ \Rightarrow 25x^2 + 25x + 4 &= 0 \quad [25 \times 4 = 100 \Rightarrow 25 = 20 + 5 \Rightarrow 100 = 20 \times 5] \\ \Rightarrow 25x^2 + 20x + 5x + 4 &= 0 \end{aligned}$$

$$\Rightarrow 5x(5x+4) + 1(5x+4) = 0$$

$$\Rightarrow (5x+4)(5x+1) = 0$$

$$\Rightarrow 5x+4=0 \text{ or } 5x+1=0$$

$$\Rightarrow x = -\frac{4}{3} \text{ or } x = -\frac{1}{3}$$

$\therefore x = -\frac{4}{3}$ and $x = -\frac{1}{3}$ are the two solutions of the quadratic equation $25x(x+1) = -4$

11. $10x - \frac{1}{x} = 3$

Sol:

We have

$$10x - \frac{1}{x} = 3$$

$$\Rightarrow \frac{10x^2 - 1}{x} = 3$$

$$\Rightarrow 10x^2 - 1 = 3x$$

$$\Rightarrow 10x^2 - 3x - 1 = 0 \quad [10x - 1 = -10 \Rightarrow -10 = -5 \times 2 \text{ and } -3 = -5 + 2]$$

$$\Rightarrow 10x^2 - 5x^2 + 2x - 1 = 0$$

$$\Rightarrow 5x(2x-1) + 1(2x-1) = 0$$

$$\Rightarrow (2x-1)(5x+1) = 0$$

$$\Rightarrow 2x-1=0 \text{ or } 5x+1=0$$

$$\Rightarrow x = \frac{1}{2} \text{ or } x = -\frac{1}{5}$$

$\therefore x = \frac{1}{2}$ and $x = -\frac{1}{5}$ are the two roots of the given equation

12. $\frac{2}{2^2} - \frac{5}{x} + 2 = 0$

Sol:

We have,

$$\frac{2}{2^2} - \frac{5}{x} + 2 = 0$$

$$\Rightarrow \frac{2-5x+2x^2}{x^2} = 0$$

$$\Rightarrow 2x^2 - 5x + 2 = 0$$

$$\begin{aligned}
 & [2 \times 2 = 4 \Rightarrow 4 = -4 \times -1 \Rightarrow -5 = -4 = 1] \\
 & \Rightarrow 2x^2 - 4x - x + 8 = 0 \\
 & \Rightarrow 2x(x-2) - 1(x-2) = 0 \\
 & \Rightarrow (x-2)(2x-1) = 0 \\
 & \Rightarrow x-2=0 \text{ or } 2x-1=0 \\
 & \Rightarrow x=2 \text{ or } x=\frac{1}{2} \\
 & \therefore x=2 \text{ and } x=\frac{1}{2} \text{ are the two roots at the given quadratic equation}
 \end{aligned}$$

13. $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

Sol:

We have,

$$\begin{aligned}
 & 4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0 \\
 & [4\sqrt{3} \times 2\sqrt{3} = -8 \times 3 = -24 \Rightarrow -24 = -8 \times 3 = -3 \times 8 \Rightarrow 5 = -3 + 8] \\
 & \Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0 \\
 & \Rightarrow 4x(\sqrt{3}x+2) - \sqrt{3}(\sqrt{3}x+2) = 0 \\
 & \Rightarrow (4x - \sqrt{3})(\sqrt{3}x + 2) = 0 \\
 & \Rightarrow 4x - \sqrt{3} = 0 \text{ or } \sqrt{3}x = -2 \\
 & \Rightarrow x = \frac{\sqrt{3}}{4} \text{ or } x = -\frac{2}{\sqrt{3}} \\
 & \therefore x = \frac{\sqrt{3}}{4} \text{ and } x = -\frac{2}{\sqrt{3}} \text{ are the two roots of the given quadratic equation}
 \end{aligned}$$

14. $\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

Sol:

We have,

$$\begin{aligned}
 & \sqrt{2}x^2 - 3x - 2\sqrt{2} = 0 \\
 & [\sqrt{2} \times -2\sqrt{2} = -2 \times 2 = -4 \Rightarrow -4 = -4 \times 1 \Rightarrow -3 = -4 + 1] \\
 & \Rightarrow \sqrt{2}x^2 - 4x + x - 2\sqrt{2} = 0 \\
 & \Rightarrow \sqrt{2}x^2 - (2\sqrt{2}x\sqrt{2})x + x - 2\sqrt{2} = 0 \\
 & \Rightarrow \sqrt{2}x - (x - 2\sqrt{2}) + 1(x - 2\sqrt{2}) = 0 \\
 & \Rightarrow (x - 2\sqrt{2})(\sqrt{2}x + 1) = 0
 \end{aligned}$$

$$\Rightarrow x - 2\sqrt{2} = 0 \text{ or } \sqrt{2}x + 0 = 0$$

$$\Rightarrow x = 2\sqrt{2} \text{ or } x = \frac{-1}{\sqrt{2}}$$

$\therefore x = -\frac{1}{\sqrt{2}}$ and $x = 2\sqrt{2}$ are the two roots of the given quadratic equation.

15. $a^2x^2 - 30bx + 2b^2 = 0$

Sol:

We have,

$$a^2x^2 - 30bx + 2b^2 = 0$$

$$\Rightarrow a^2x^2 - abx - 2abx + 2b^2 = 0$$

$$\left[a^2 \times 2b^2 = 2a^2b^2 \Rightarrow 2a^2b^2 = 2ab \times ab = -2ab \times -ab \Rightarrow -3ab = -2ab - ab \right]$$

$$\Rightarrow ax(ax - b) - 2b(ax - b) = 0$$

$$\Rightarrow (ax - 2b)(ax - b) = 0$$

$$\Rightarrow ax - 2b = 0 \text{ or } ax - b = 0$$

$$\Rightarrow ax = 2b \text{ or } ax = b$$

$$\Rightarrow x = \frac{2b}{a} \text{ or } x = \frac{b}{a}$$

$\therefore x = \frac{b}{a}$ and $x = \frac{2b}{a}$ are the two roots of the given quadratic equation

16. $x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$

Sol:

We have,

$$x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$$

$$\Rightarrow x^2 - \sqrt{2}x - 1 \times x + \sqrt{2} = 0$$

$$\left[1 \times \sqrt{2} = \sqrt{2} \Rightarrow \sqrt{2} = -\sqrt{2} \times -1 \right]$$

$$\Rightarrow x^2 - \sqrt{2}x - x + \sqrt{2} = 0$$

$$\Rightarrow x(x - \sqrt{2}) - 1(x - \sqrt{2}) = 0$$

$$\Rightarrow (x - \sqrt{2})(x - 1) = 0$$

$$\Rightarrow x = \sqrt{2} = 0 \text{ or } x - 1 = 0$$

$$\Rightarrow x = \sqrt{2} \text{ or } x = 1$$

$\therefore x = 1$ and $x = \sqrt{2}$ are the roots of the given quadratic equation

17. $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$

Sol:

We have,

$$x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$$

$$\Rightarrow x^2 - \sqrt{3}x - 1 \times x + \sqrt{3} = 0$$

$$[\sqrt{3} \times 1 = \sqrt{3} \Rightarrow \sqrt{3} = -\sqrt{3} \times -1 \Rightarrow (\sqrt{3} + 1) = -\sqrt{3} - 1]$$

$$\Rightarrow x(x - \sqrt{3}) - 1(x - \sqrt{3}) = 0$$

$$\Rightarrow (x - \sqrt{3})(x - 1) = 0$$

$$\Rightarrow x - \sqrt{3} = 0 \text{ or } x - 1 = 0$$

$$\Rightarrow x = \sqrt{3} \text{ or } x = 1$$

$\therefore x = 1$ and $x = \sqrt{3}$ are the two roots of the given quadratic equation

18. $4x^2 + 4bx - (a^2 - b^2) = 0$

Sol:

We have,

$$4x^2 + 4bx - (a^2 - b^2) = 0$$

$$[4x - (a^2 - b^2) = 4x - (a - b)(a + b) = -2(a - b) \times 2(a + b) = 2(b - a) \times 2(a + b) \Rightarrow 4b = 2b + 2b = 8(b - a) +$$

$$\Rightarrow 4x^2 + (2(b - a) + 2(a + b))x - (a - b)(a + b) = 0$$

$$\Rightarrow 2x^2 + (2x + b - a) + (a + b)(2x + (b - a)) = 0$$

$$\Rightarrow (2x + b - a) \text{ or } 2x + a + b = 0$$

$$\Rightarrow 2x = a - b \text{ or } 2x = -a - b$$

$$\Rightarrow x = \frac{a - b}{2} \text{ or } 2x = -(a + b) \Rightarrow x = -\frac{(a + b)}{2}$$

$\therefore x = -\frac{(a + b)}{2}$ and $x = \frac{a - b}{2}$ are the two roots of the given quadratic equation

19. $ax^2 + (4a^2 - 3b)x - 12ab = 0$

Sol:

We have,

$$ax^2 + (4a^2 - 3b)x - 12ab = 0$$

$$\begin{aligned}
 & \left[a \times 12ab = -12a^2b^2 = 4a^2 \times -3b \right] \\
 \Rightarrow & ax^2 + 4a^2x - 3bx + (4a \times (-3b)) = 0 \\
 \Rightarrow & ax(x + 4a) - 3b(x + 4a) = 0 \\
 \Rightarrow & (a + 4a)(ax - 3b) = 0 \\
 \Rightarrow & (x + 4a) = 0 \text{ or } (ax - 3b) = 0 \\
 \Rightarrow & x = -4a \text{ or } x = \frac{3b}{a} \\
 \therefore & x = \frac{3b}{a} \text{ and } x = -4a \text{ are the two roots of the given equations}
 \end{aligned}$$

20. $\left(x - \frac{1}{2}\right)^2 = 4$

Sol:

We have,

$$\begin{aligned}
 & \left(x - \frac{1}{2}\right)^2 = 4 \\
 \Rightarrow & \left(x - \frac{1}{2}\right)^2 - 4 = 0 \\
 \Rightarrow & \left(x - \frac{1}{2}\right)^2 - (2)^2 = 0 \\
 \Rightarrow & \left[\left(x - \frac{1}{2}\right) + 2\right] \left[\left(x - \frac{1}{2}\right) - 2\right] = 0 \quad [\because a^2 - b^2 = (a+b)(a-b)] \\
 \Rightarrow & \left(x - \frac{1}{2} + 2\right) = 0 \text{ or } \left(x - \frac{1}{2} - 2\right) = 0 \\
 \Rightarrow & x = 2 - \frac{1}{2} \text{ or } x = 2 - \frac{1}{2} \\
 \Rightarrow & x = \frac{4-1}{2} \text{ or } x = \frac{4+1}{2} \\
 \Rightarrow & x = \frac{3}{2} \text{ or } x = \frac{5}{2} \\
 \Rightarrow & x = \frac{3}{2} \text{ and } x = \frac{5}{2} \text{ are the two roots of the given equations}
 \end{aligned}$$

21. $x^2 - 4\sqrt{2}x + 6 = 0$

Sol:

We have,

$$x^2 - 4\sqrt{2}x + 6 = 0$$

$$[1 \times 6 = 6 \Rightarrow 6 = -3\sqrt{2}x - \sqrt{2} \text{ and } -4\sqrt{2} = -3\sqrt{2} - \sqrt{2}]$$

$$\Rightarrow x^2 - 3\sqrt{2}x - \sqrt{2}x + (-3\sqrt{2} \times -2) = 0$$

$$\Rightarrow x(x - 3\sqrt{2})\sqrt{2}(x - 3\sqrt{2}) = 0$$

$$\Rightarrow (x - 3\sqrt{2})(x - \sqrt{2}) = 0$$

$$\Rightarrow x = 3\sqrt{2} = 0 \text{ or } x - \sqrt{2} = 0$$

$$\Rightarrow x = 3\sqrt{2} \text{ or } x = \sqrt{2}$$

 $\therefore x = 3\sqrt{2}$ and $x = \sqrt{2}$ are the two roots of the given equation.

22. $\frac{x+3}{x+2} = \frac{3x-7}{2x-3}$

Sol:

We have, $\frac{x+3}{x+2} = \frac{3x-7}{2x-3}$

$$\Rightarrow (x+3)(2x-3) = (x+2)(3x-7)$$

$$\Rightarrow 2x^2 - 3x + 6x - 9 = 3x^2 - x - 14$$

$$\Rightarrow 2x^2 + 3x - 9 = 3x^2 - x - 14$$

$$\Rightarrow x^2 - 3x - x - 14 + 9 = 0$$

$$\Rightarrow x^2 - 4x - 5 = 0$$

$$[1x - 5 = -5 - 4 = -5 + 1]$$

$$\Rightarrow x^2 - 5x + x - 5 = 0$$

$$\Rightarrow x(x - 5) + 1(x - 5) = 0$$

$$\Rightarrow (x - 5)(x + 1) = 0$$

$$\Rightarrow x - 5 = 0 \text{ or } x + 1 = 0$$

$$\Rightarrow x = 5 \text{ or } x = -1$$

 $\therefore x = 5$ and $x = -1$ are the two roots of the given quadratic equation.

23. $\frac{2x}{x-4} + \frac{2x-5}{x-3} = \frac{25}{3}$

Sol:

We have, $\frac{2x}{x-4} + \frac{2x-5}{x-3} = \frac{25}{3}$

$$\Rightarrow \frac{2x(x-3)7(x-4)(2x-5)}{(x-4)(x-3)} = \frac{25}{3}$$

$$\Rightarrow \frac{2x^2 - 6x + 2x^2 - 5x - 5x + 20}{x^2 - 4x - 3x + 12} = \frac{25}{3}$$

$$\Rightarrow \frac{4x^2 - 19x + 20}{x^2 - 7x + 12} = \frac{25}{3}$$

$$\Rightarrow 3(4x^2 - 19x + 20) = 25(x^2 - 7x + 12)$$

$$\Rightarrow 12x^2 - 57x + 60 = 25x^2 - 175x + 300$$

$$\Rightarrow 25x^2 - 12x^2 - 175x + 57x + 300 - 60 = 0$$

$$\Rightarrow 13x^2 - 118x + 240 = 0$$

$$\Rightarrow 13x^2 - 78x - 40x + 240 = 0$$

$$[\because 13 \times 240 = 3120 \Rightarrow 3180 = -78 \times 40 \text{ and } -118 = -78 - 40]$$

$$\Rightarrow 13x(x-6) - 40(x-6) = 0$$

$$\Rightarrow (x-6)(13x-40) = 0$$

$$\Rightarrow x-6=0 \text{ or } 13x-40=0$$

$$\Rightarrow x=6 \text{ or } x=\frac{40}{13}$$

$\therefore x=6$ and $x=\frac{40}{13}$ are the two roots of the given equation.

24. $\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$

Sol:

We have,

$$\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$$

$$\Rightarrow \frac{x(x+3) - (x-2)(1-x)}{x(x-2)} = \frac{17}{4}$$

$$\Rightarrow \frac{x^2 + 3x - (x - x^2 - 2 + 2x)}{x^2 - 2x} = \frac{17}{4}$$

$$\Rightarrow \frac{x^2 + 3x - x + x^2 + 2 - 2x}{x^2 - 2x} = \frac{17}{4}$$

$$\Rightarrow \frac{2x^2 + 2}{x^2 - 2x} = \frac{17}{4}$$

$$\begin{aligned}
 &\Rightarrow 4(2x^2 + 2) = 17(x^2 - 2x) \\
 &\Rightarrow 8x^2 + 8 = 17x^2 - 34x \\
 &\Rightarrow 8x^2 + 8 = 17x^2 - 34x \\
 &\Rightarrow (17 - 8)x^2 - 34x - 8 = 0 \\
 &\Rightarrow 9x^2 - 34x - 8 = 0 \\
 &\left[9 \times -8 = -72 \Rightarrow -72 = -36 \times 2 \text{ and } -34 = -36 + 2 \right] \\
 &\Rightarrow 9x^2 - 36x + 2x - 8 = 0 \\
 &\Rightarrow 9x(x - 4) + 2(x - 4) = 0 \\
 &\Rightarrow (x - 4)(9x + 2) = 0 \\
 &\Rightarrow (x - 4) = 0 \text{ or } 9x + 2 = 0 \\
 &\Rightarrow x = 4 \text{ or } x = -\frac{8}{9} \\
 &\therefore x = 4 \text{ and } x = -\frac{8}{9} \text{ are the two roots of the given equations}
 \end{aligned}$$

25. $\frac{x-3}{x+3} - \frac{x+3}{x-3} = \frac{48}{7}, x \neq 3, x \neq -3$

Sol:

$$-4, \frac{9}{4}$$

26. $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}, x \neq 0$

Sol:

$$\text{We have, } \frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}, x \neq 0$$

$$\Rightarrow \frac{(x+1)+1(x-2)}{(x-2)(x-1)} = \frac{6}{x}$$

$$\Rightarrow \frac{x-1+2x-4}{x^2-2x-x+2} = \frac{6}{x}$$

$$\Rightarrow \frac{3x-5}{x^2-3x+2} = \frac{6}{x}$$

$$\Rightarrow x(3x-5) = 6(x^2-3x+2)$$

$$\Rightarrow 3x^2 - 5x = 6x^2 - 16x + 12$$

$$\Rightarrow 3x^2 - 18x + 5x + 18 = 0$$

$$\Rightarrow 3x^2 - 13x + 18 = 0$$

$$[\because 3 \times 18 = 36 \Rightarrow -9x - 4 \text{ and } -13 = -9 - 4]$$

$$\Rightarrow 3x^2 - 9x - 4x + 12 = 0$$

$$\Rightarrow 3x(x-3) - 4(x-3) = 0$$

$$\Rightarrow (x-3)(3x-4) = 0$$

$$\Rightarrow x-3=0 \text{ or } 3x-4=0$$

$$\Rightarrow x=3 \text{ or } x=\frac{4}{3}$$

$\therefore x=3$ and $x=\frac{4}{3}$ are the two roots of the given equation

27. $\frac{x+1}{x-1} - \frac{x-1}{x+1} = \frac{5}{6}, x \neq 1 \text{ and } x \neq -1$

Sol:

We have

$$\frac{x+1}{x-1} - \frac{x-1}{x+1} = \frac{5}{6}, x \neq 1 \text{ and } x \neq -1$$

$$\Rightarrow \frac{(x+1)(x+1) - (x-1)(x-1)}{(x-1)(x+1)} = \frac{5}{6}$$

$$\Rightarrow \frac{(x+1)^2 - (x-1)^2}{x^2 - 1^2} = \frac{5}{6}$$

$$\Rightarrow \frac{4x}{x^2 - 1} = \frac{5}{6} \quad \left[\because (x+b)^2 - (a-b)^2 = 4ab \text{ and } (a-b)(a+b) = a^2 - b^2 \right]$$

$$\Rightarrow 6(4x) = 5(x^2 - 1)$$

$$\Rightarrow 24x = 5x^2 - 5$$

$$\Rightarrow 5x^2 - 5 - 24x = 0$$

$$\Rightarrow 5x^2 - 24x - 5 = 0 \quad \left[\because 5x - 5 = -25 \Rightarrow -25 = -25 \times 1 - 24 = -25 + 1 \right]$$

$$\Rightarrow 5x^2 - 25x + x - 5 = 0$$

$$\Rightarrow 5x(x-5) + 1(x-5) = 0$$

$$\Rightarrow (x-5)(5x+1) = 0$$

$$\Rightarrow x-5=0 \text{ or } 5x+1=0$$

$$\Rightarrow x=5 \text{ or } x=-\frac{1}{5}$$

$\therefore x=5$ and $x=-\frac{1}{5}$ are the two roots of the given equation.

28.
$$\frac{x-1}{2x+1} - \frac{2x+1}{x-1} = \frac{5}{2}, x \neq -\frac{1}{2}, 1$$

Sol:

We have

$$\frac{x-1}{2x+1} - \frac{2x+1}{x-1} = \frac{5}{2}, x \neq -\frac{1}{2}, 1$$

$$\Rightarrow \frac{(x-1)(x-1) - (2x+1)(2x+1)}{(2x+1)(x-1)} = \frac{5}{2}$$

$$\Rightarrow \frac{(x-1)^2 + (2x+1)^2}{2x^2 - 2x + x - 1} = \frac{5}{2}$$

$$\Rightarrow \frac{x^2 - 2x + 1 + 4x^2 + 4x + 1}{2x^2 - x - 1} = -\frac{5}{2} \quad \left[\because (a+b)^2 = a^2 + b^2 + 2ab, (a-b)^2 = a^2 + b^2 - 2ab \right]$$

$$\Rightarrow \frac{5x^2 + 2x + 2}{2x^2 - x - 1} = \frac{5}{2}$$

$$\Rightarrow 2(5x^2 + 2x + 2) = 5(2x^2 - x - 1)$$

$$\Rightarrow 10x^2 + 4x + 4 = 10x^2 - 5x - 5$$

$$\Rightarrow 4x + 5x + 4 + 5 = 0$$

$$\Rightarrow 9x + 9 = 0$$

$$\Rightarrow 9x = -9$$

$$\Rightarrow \boxed{x = -1}$$

$\therefore x = -1$ is the only root for the given equation

29.
$$3x^2 - 14x - 5 = 0$$

Sol:

We have, $3x^2 - 14x - 5 = 0$

$$\Rightarrow 3x^2 - 15x + x - 5 = 0$$

$$\Rightarrow 3x(x-5) + 1(x-5) = 0 \quad \left[\because 3 \times -5 = -15 \Rightarrow -15 = -15 \times 1 \text{ and } +4 = +5 + 1 \right]$$

$$\Rightarrow (x-5)(3x+1) = 0$$

$$\Rightarrow x-5=0 \text{ or } 3x+1=0$$

$$\Rightarrow x=5 \text{ or } x=-\frac{1}{3}$$

$\therefore x=5$ and $x=-\frac{1}{3}$ are the two roots of the given quadratic equation

30.
$$\frac{m}{n}x^2 + \frac{n}{m} = 1 - 2x$$

Sol:

We have given,

$$\begin{aligned} \frac{m}{n}x^2 + \frac{n}{m} &= 1 - 2x \\ \Rightarrow \frac{m^2x^2 + n^2}{mn} &= 1 - 2x \\ \Rightarrow m^2x^2 + 2mnx + (n^2 - mn) &= 0 \end{aligned}$$

Now we solve the above quadratic equation using factorization method.

Therefore,

$$\begin{aligned} & \left[m^2x^2 + mnx + m\sqrt{mn}x \right] + \left[mnx - m\sqrt{mn}x + (n + \sqrt{mn})(n - \sqrt{mn}) \right] = 0 \\ \Rightarrow & \left[m^2x^2 + mnx + m\sqrt{mn}x \right] + \left[(mx)(n - \sqrt{mn}) + (n + \sqrt{mn})(n - \sqrt{mn}) \right] = 0 \\ \Rightarrow & (mx)(mx + n + \sqrt{mn}) + (n - \sqrt{mn})(mx + n + \sqrt{mn}) = 0 \\ \Rightarrow & (mx + n + \sqrt{mn})(mx + n - \sqrt{mn}) = 0 \end{aligned}$$

Now, one of the products must be equal to zero for the whole product to be zero. Hence we equate both the product to zero. In order to find the value of x . Therefore,

$$mx + n + \sqrt{mn} = 0$$

$$\Rightarrow mx = -n - \sqrt{mn}$$

$$\Rightarrow x = \frac{-n - \sqrt{mn}}{m}$$

Or

$$mx + n - \sqrt{mn} = 0$$

$$\Rightarrow mx = -n + \sqrt{mn}$$

$$\Rightarrow x = \frac{-n + \sqrt{mn}}{m}$$

$$\text{Hence } x = \frac{-n - \sqrt{mn}}{m} \text{ or } n = \frac{-n + \sqrt{mn}}{m}.$$

31.
$$\frac{x-a}{x-b} + \frac{x-b}{x-a} = \frac{a}{b} + \frac{b}{a}$$

Sol:

We have,

$$\frac{x-a}{x-b} + \frac{x-b}{x-a} = \frac{a}{b} + \frac{b}{a}$$

$$\begin{aligned}
& \Rightarrow \frac{(x-a)(x-a)+(x-b)(x-b)}{(x-b)(x-a)} = \frac{a^2+b^2}{ab} \\
& \Rightarrow \frac{(x-a)^2+(x-b)^2}{x^2-ax-bx+ab} = \frac{a^2+b^2}{ab} \\
& \Rightarrow \frac{x^2-2ax+a^2+x^2-2bx+b^2}{x^2-(a+b)x+ab} = \frac{a^2+b^2}{ab} \\
& \Rightarrow (2x^2-2x(a+b)+a^2+b^2)ab = (a^2+b^2)(x^2-(a+b)x+ab) \\
& \Rightarrow 2abx^2-2abx(a+b)+ab(a^2+b^2) = (a^2+b^2)x^2-(a^2+b^2)(a+b)x+ab(a^2+b^2) \\
& \Rightarrow (a^2+b^2-2ab)x^2-(a+b)(a^2+b^2-2ab)x = 0 \\
& \Rightarrow (a-b)^2 x^2 - (a+b)(a-b)^2 x = 0 \\
& \Rightarrow (a-b)^2 (x - (a+b)) = 0 \\
& \Rightarrow x(x - (a+b)) = 0 \\
& \Rightarrow x = 0 \text{ or } x - (a+b) = 0 \Rightarrow x = a+b \\
& \therefore x = 0 \text{ and } x = (a+b) \text{ are the two roots of the equation}
\end{aligned}$$

32. $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} + \frac{1}{(x-3)(x-4)} = \frac{1}{6}$

Sol:

We have,

$$\begin{aligned}
& \frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} + \frac{1}{(x-3)(x-4)} = \frac{1}{6} \\
& \Rightarrow \frac{(x-3)(x-4) + (x-1)(x-4) + (x-1)(x-8)}{(x-1)(x-2)(x-3)(x-4)} = \frac{1}{6} \\
& \Rightarrow \frac{(x-3)(x-4) + (x-1)[(x-4) + (x-8)]}{(x-1)(x-2)(x-3)(x-4)} = \frac{1}{6} \\
& \Rightarrow \frac{(x-3)(x-4) + (x-1)(2x-12)}{(x-1)(x-2)(x-3)(x-4)} = \frac{1}{6} \\
& \Rightarrow \frac{(x-3)(x-4) + (x-1) \times 2(x-6)}{(x-1)(x-2)(x-3)(x-4)} = \frac{1}{6} \\
& \Rightarrow \frac{(x-3)(x-4) + (x-1) \times 2(x-6)}{(x-1)(x-2)(x-3)(x-4)} = \frac{1}{6} \\
& \Rightarrow \frac{(x-3)[x-4+2x-12]}{(x-1)(x-2)(x-3)(x-4)} = \frac{1}{6}
\end{aligned}$$

$$\begin{aligned}
 & \Rightarrow \frac{(x-3)(3x-6)}{(x-1)(x-2)(x-3)(x-4)} = \frac{1}{6} \\
 & \Rightarrow \frac{(x-3)(x-2) \times 3}{(x-1)(x-2)(x-3)(x-4)} = \frac{1}{6} \\
 & \Rightarrow \frac{3}{(x-1)(x-4)} = \frac{1}{6} \\
 & \Rightarrow (x-1)(x-4) = 3 \times 6 \\
 & \Rightarrow x^2 - 4x - x + 4 = 18 \\
 & \Rightarrow x^2 - 5x - 14 = 0 \quad [\because -14 = -7 \times 8 \text{ and } -5 = -7 + 8] \\
 & \Rightarrow x^2 - 7x + 8x - 14 = 0 \\
 & \Rightarrow x(x-7) + 8(x-7) = 0 \\
 & \Rightarrow (x-7)(x+8) = 0 \\
 & \Rightarrow x-7=0 \text{ or } x+2=0 \\
 & \Rightarrow x=7 \text{ or } x=-2 \\
 & \therefore x=7 \text{ and } x=-8 \text{ are the two roots of the given equation.}
 \end{aligned}$$

33. $(x-5)(x-6) = \frac{25}{(24)^2}$

Sol:

We have,

$$\begin{aligned}
 (x-5)(x-6) &= \frac{25}{(24)^2} \\
 \Rightarrow x^2 - 5x - 6x + 30 - \frac{25}{(24)^2} &= 0 \\
 \Rightarrow x^2 - 11x + \frac{30(24)^2 - 25}{(24)^2} &= 0 \\
 \Rightarrow x^2 - 11x + \frac{17280 - 25}{(24)^2} &= 0 \\
 \Rightarrow x^2 - 11x + \frac{17255 - 25}{(24)^2} &= 0 \\
 \Rightarrow x^2 - 11x + \frac{119}{24} \times \frac{145}{24} &= 0 \quad [\because 17255 = 145 \times 119] \\
 \Rightarrow x^2 - \frac{264}{24}x + \frac{119}{24} \times \frac{145}{24} &= 0 \quad [\because 11 \times 24 = 264]
 \end{aligned}$$

$$\begin{aligned}
 & \Rightarrow x^2 - \left(\frac{119+145}{24} \right) x + \frac{119}{24} \times \frac{145}{24} = 0 \\
 & \Rightarrow x^2 - \frac{119}{24} x - \frac{145}{24} x + \frac{119}{24} \times \frac{145}{24} = 0 \\
 & \Rightarrow x \left(x - \frac{119}{24} \right) - \frac{145}{24} \left(x - \frac{119}{24} \right) = 0 \\
 & \Rightarrow \left(x - \frac{119}{24} \right) \left(x - \frac{145}{24} \right) = 0 \\
 & \Rightarrow x - \frac{119}{24} = 0 \text{ and } x - \frac{145}{25} = 0 \\
 & \Rightarrow x = \frac{119}{24} \text{ or } x = \frac{145}{24} \\
 & \Rightarrow x = 4\frac{23}{24} \text{ or } x = 6\frac{1}{24} \\
 & \therefore x = 4\frac{23}{24} \text{ and } x = 6\frac{1}{24} \text{ are the two roots of the given equation.}
 \end{aligned}$$

34. $7x + \frac{3}{x} = 35\frac{3}{5}$

Sol:

$$\begin{aligned}
 & \text{We have, } 7x + \frac{3}{x} = 35\frac{3}{5} \\
 & \Rightarrow \frac{7x^2 + 3}{x} = 35 + \frac{3}{5} \\
 & \Rightarrow 7x^2 + 3 = \left(35 + \frac{3}{5} \right) x \\
 & \Rightarrow 7x^2 - \left(35 + \frac{3}{5} \right) x + 3 = 0 \\
 & \Rightarrow 7x^2 - 35x - \frac{3}{5}x + 3 = 0 \\
 & \Rightarrow 7x^2 - 35x - \frac{1}{5}(3x - 3 \times 5) = 0 \\
 & \Rightarrow 7x(x-5) - \frac{3}{5}(x-5) = 0 \\
 & \Rightarrow (x-5) \left(7x - \frac{3}{5} \right) = 0 \\
 & \Rightarrow (x-5) = 0 \text{ or } 7x - \frac{3}{5} = 0
 \end{aligned}$$

$$\Rightarrow x=5 \text{ or } 7x=\frac{3}{5} \Rightarrow x=\frac{3}{35}$$

$\therefore x=5$ and $x=\frac{3}{35}$ are the two roots of the given equation.

35. $\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$

Sol:

We have,

$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$$

$$\Rightarrow \frac{a(x-b) + b(x-a)}{(x-a)(x-b)} = \frac{2c}{x-c}$$

$$\Rightarrow \frac{ax-ab+bx-ab}{x^2-ax-bx+ab} = \frac{2c}{x-c}$$

$$\Rightarrow (x-c)((a+b)x-2ab) = 2c(x^2-(a+b)x+ab)$$

$$\Rightarrow (a+b)x^2 - 2abx - (a+b)c x + 2abc = 2cx^2 - 2c(a+b)x + 2abc$$

$$\Rightarrow (a+b-8c)x^2 - 2abx - (a+b)xc + 8c(a+b)x = 0$$

$$\Rightarrow (a+b-8c)x^2 + x(-8ab-ac-bc+8ac+8bc) = 0$$

$$\Rightarrow (a+b-8c)x^2 + x(-2ab+ac+bc) = 0$$

$$\Rightarrow x[x(a+b-2c)+(ac+bc-2ab)] = 0$$

$$\Rightarrow x=0 \text{ or } x(a+b-2c)+(ac+bc-2ab)=0$$

$$\Rightarrow x=0 \text{ or } x = -\frac{(ac+bc-8ab)}{a+b-8c}$$

$$\Rightarrow x = \frac{8ab-ac-bc}{a+b-2c}$$

$\therefore x=0$ and $x=\frac{2ab-ac-bc}{a+b-8c}$ are the two roots of the given equation.

$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$$

$$\Rightarrow \frac{a(x-b) + b(x-a)}{(x-a)(x-b)} = \frac{2c}{x-c}$$

$$\Rightarrow \frac{ax-ab+bx-ab}{x^2-ax-bx+ab} = \frac{2c}{x-c}$$

$$\Rightarrow (x-c)((a+b)x-2ab) = 2c(x^2-(a+b)x+ab)$$

$$\Rightarrow (a+b)x^2 - 2abx - (a+b)c x + 2abc = 2cx^2 - 2c(a+b)x + 2abc$$

36. $x^2 + 2ab = (2a+b)x$

Sol:

We have

$$x^2 + 2ab = (2a+b)x$$

$$\Rightarrow x^2 - (2a+b)x + 2ab = 0 \quad [\because 2ab = -8a \times -b \Rightarrow -(8a+b) = -8a-b]$$

$$\Rightarrow x^2 - 2ax - bx + 2ab = 0$$

$$\Rightarrow x - (x-8a) - b(x-2a) = 0$$

$$\Rightarrow (x-8a)(x-b) = 0$$

$$\Rightarrow x-8a = 0 \text{ or } x-b = 0$$

$$\Rightarrow x = 8a \text{ or } x = b$$

$\therefore x = 8a$ and $x = b$ are the two roots of the given equation.

37. $(a+b)^2 x^2 - (4ab)x - (a-b)^2 = 0$

Sol:

We have,

$$(a+b)^2 x^2 - (4ab)x - (a-b)^2 = 0$$

$$\Rightarrow (a+b)^2 x^2 - \left((a+b)^2 - (a-b)^2 \right)x - (a-b)^2 = 0 \quad [\because (a+b)^2 - (a-b)^2 = 4ab]$$

$$\Rightarrow (a+b)^2 x^2 - (a+b)^2 x + (a-b)^2 x - (a-b)^2 = 0$$

$$\Rightarrow (a+b)^2 x(x-1) + (a-b)^2 (x-1) = 0$$

$$\Rightarrow (x-1) \left((a+b)^2 x + (a-b)^2 \right) = 0$$

$$\Rightarrow x-1 = 0 \text{ or } (a+b)^2 x + (a-b)^2 = 0$$

$$\Rightarrow x = 1 \text{ or } x = -\frac{(a-b)^2}{(a+b)^2} = -\left[\frac{a-b}{a+b} \right]^2$$

$\therefore x = 1$ and $x = -\left[\frac{a-b}{a+b} \right]^2$ are the two roots of the given equation

38. $a(x^2 + 1) - x(a^2 + 1) = 0$

Sol:

We have

$$a(x^2 + 1) - x(a^2 + 1) = 0$$

$$\begin{aligned}
 & \Rightarrow ax^2 - a^2x - x + a = 0 \quad \left[\because a \times a = a^2 \Rightarrow a^2 = -a^2 \times -1 - (a^2 + 1) = a^2 - 1 \right] \\
 & \Rightarrow a \times (x - a) - 1(x - a) = 0 \\
 & \Rightarrow (x - a)(ax - 1) = 0 \\
 & \Rightarrow x - a = 0 \text{ or } ax - 1 = 0 \\
 & \Rightarrow x = a \text{ or } x = \frac{1}{a} \\
 & \therefore x = a \text{ and } x = \frac{1}{a} \text{ are the two roots of the given equation}
 \end{aligned}$$

39. $x^2 - x - x(a+1) = 0$

Sol:

We have,

$$\begin{aligned}
 & x^2 - x - x(a+1) = 0 \\
 & \Rightarrow x^2 - (a+1-a)x - a(a+1) = 0 \quad \left[\because -a(a+1) = -(a+1) \times a - 1 = a - (a+1) \right] \\
 & \Rightarrow x^2 - (a+1)x + ax + a(-a-1) = 0 \\
 & \Rightarrow x(x - (a+1)) + a(x - (a+1)) = 0 \\
 & \Rightarrow (x - (a+1))(x + a) = 0 \\
 & \Rightarrow x - (a+1) = 0 \text{ or } x + a = 0 \\
 & \Rightarrow x = a+1 \text{ or } x = -a \\
 & \therefore x = (a+1) \text{ and } x = -a \text{ are the two roots of the given equation.}
 \end{aligned}$$

40. $x^2 + \left(a + \frac{1}{a}\right)x + 1 = 0$

Sol:

We have,

$$\begin{aligned}
 & x^2 + \left(a + \frac{1}{a}\right)x + 1 = 0 \\
 & \Rightarrow x^2 + ax + \frac{1}{a}x + a \times \frac{1}{a} = 0 \quad \left[\because 1 = a \times \frac{1}{a} \left(a + \frac{1}{a} \right) x = ax + \frac{1}{a}x \right] \\
 & \Rightarrow x(x+a) + \frac{1}{a}(x+a) = 0 \\
 & \Rightarrow (x+a)\left(x + \frac{1}{a}\right) = 0
 \end{aligned}$$

$$\Rightarrow x+a=0 \text{ or } x+\frac{1}{a}=0$$

$$\Rightarrow x=-a \text{ or } x=-\frac{1}{a}$$

$\therefore x=a$ and $x=-\frac{1}{a}$ are the two roots of the given equation.

41. $abx^2 + (b^2 - ac)x - bc = 0$

Sol:

We have,

$$abx^2 + (b^2 - ac)x - bc = 0$$

$$[abx - bc = -ab^2c \Rightarrow -ab^2c = b^2 \times -ac \text{ and } b^2 - ac = b^2 + (-ac)]$$

$$\Rightarrow abx^2 + b^2x - acx - bc = 0$$

$$\Rightarrow bx(ax + b) - c(ax + b) = 0$$

$$\Rightarrow (ax + b)(bx - c) = 0$$

$$\Rightarrow ax + b = 0 \text{ or } bx - c = 0$$

$$\Rightarrow x = -\frac{b}{a} \text{ or } x = \frac{c}{b}$$

$\therefore x = -\frac{b}{a}$ and $x = \frac{c}{b}$ are the two roots of the given equation

42. $a^2b^2x^2 + b^2x - a^2x - 1 = 0$

Sol:

We have, $a^2b^2x^2 + b^2x - a^2x - 1 = 0$

$$[-1 \times a^2b^2 = -a^2b^2 \Rightarrow -a^2b^2 = -a^2 \times b^2]$$

$$\Rightarrow a^2b^2x^2 + b^2x - a^2x - 1 = 0$$

$$\Rightarrow b^2 \times (a^2x + 1) - 1(a^2x - 1) = 0$$

$$\Rightarrow (a^2x + 1)(b^2x - 1) = 0$$

$$\Rightarrow a^2x + 1 = 0 \text{ or } b^2x - 1 = 0$$

$$\Rightarrow x = -\frac{1}{a^2} \text{ and } x = \frac{1}{b^2} \text{ are the two roots of the given equation}$$

43.
$$\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}, x \neq 2 \text{ and } x \neq 4$$

Sol:

We have,

$$\begin{aligned}
 & \frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}, x \neq 2 \text{ and } x \neq 4 \\
 & \Rightarrow \frac{(x-1)(x-4) + (x-3)(x-2)}{(x-2)(x-4)} = \frac{10}{3} \\
 & \Rightarrow \frac{x^2 - x - 4x + 4 + x^2 - 3x - 2x + 6}{x^2 - 2x - 4x + 8} = \frac{10}{3} \\
 & \Rightarrow \frac{2x^2 - 10x + 10}{x^2 - 6x + 8} = \frac{10}{3} \\
 & \Rightarrow 2(x^2 - 5x + 5) \times 3 = 5(x^2 - 6x + 8) \\
 & \Rightarrow 3x^2 - 15x + 15 = 5x^2 - 30x + 40 \\
 & \Rightarrow 2x^2 - 30x + 15x + 40 - 15 = 0 \\
 & \Rightarrow 2x^2 - 15x + 25 = 0 \\
 & \Rightarrow 2x^2 - 10x - 5x + 25 = 0 \\
 & \Rightarrow 2x(x-5) - 5(x-5) = 0 \\
 & \Rightarrow (x-5)(2x-5) = 0 \\
 & \Rightarrow (x-5) = 0 \text{ or } 2x-5 = 0 \\
 & \Rightarrow x = 5 \text{ or } x = \frac{5}{2} \\
 & \therefore x = 5 \text{ and } x = \frac{5}{2} \text{ are the two roots of the given equation}
 \end{aligned}$$

44.
$$3x^2 - 2\sqrt{6}x + 2 = 0$$

Sol:

We have $3x^2 - 2\sqrt{6}x + 2 = 0$ Now we solve the above quadratic equation using factorization method.

Therefore

$$\begin{aligned}
 & 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0 \\
 & \Rightarrow \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0 \\
 & \Rightarrow (\sqrt{3} - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0 \\
 & \Rightarrow (\sqrt{3}x - \sqrt{2}) = 0 \text{ or } (\sqrt{3}x - \sqrt{2}) = 0
 \end{aligned}$$

$$\Rightarrow \sqrt{3}x = \sqrt{2} \text{ or } \sqrt{3}x = -\sqrt{2}$$

$$\Rightarrow x = \frac{\sqrt{2}}{\sqrt{3}} \text{ or } x = -\frac{\sqrt{2}}{\sqrt{3}}$$

$$\Rightarrow x = \sqrt{\frac{2}{3}} \text{ or } x = -\sqrt{\frac{2}{3}}$$

$$\text{Hence } x = \sqrt{\frac{2}{3}} \text{ or } x = -\sqrt{\frac{2}{3}}$$

$$45. \quad \frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}, x \neq 1, -5$$

Sol:

We have

$$\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}, x \neq 1, -5$$

$$\Rightarrow \frac{x+5-(x-1)}{(x-1)(x+5)} = \frac{6}{7}$$

$$\Rightarrow \frac{x-5-x+1}{x^2+5x-x-5} = \frac{6}{7}$$

$$\Rightarrow \frac{6}{x^2+4x-5} = \frac{6}{7}$$

$$\Rightarrow x^2+4x-5=7$$

$$\Rightarrow x^2+4x-5-7=0$$

$$\Rightarrow x^2+4x-18=0$$

$$\Rightarrow x^2+6x-2x-12=0$$

$$\Rightarrow x(x+6)-2(x+6)=0$$

$$\Rightarrow x+6=0 \text{ or } x-8=0$$

$$\Rightarrow x=-6 \text{ or } x=8$$

$\therefore x = -6$ and $x = 8$ are the two roots of the given equation.

$$46. \quad \frac{1}{x} - \frac{1}{x-2} = 3, x \neq 0, 2$$

Sol:

We have,

$$\frac{1}{x} - \frac{1}{x-2} = 3, x \neq 0, 2$$

$$\Rightarrow \frac{x-2-x}{x(x-2)} = 3$$

$$\begin{aligned}
&\Rightarrow -2 = 3(x^2 - 2x) \\
&\Rightarrow 3x^2 - 6x + 2 = 0 \\
&\Rightarrow 3x^2 - (3+3)x + 2 = 0 \\
&\Rightarrow 3x^2 - (3+\sqrt{3}+3\sqrt{3})x + (3-1) = 0 \\
&\Rightarrow 3x^2 - \sqrt{3}(\sqrt{3}+1)x - \sqrt{3}(\sqrt{3}-1)x + [(\sqrt{3}+1)(\sqrt{3}-1)] = 0 \\
&\quad [\because a^2 - b^2 = (a+b)(a-b)] \\
&\Rightarrow (\sqrt{3})^2 x^2 - \sqrt{3}(\sqrt{3}-1)x + (\sqrt{3}+1)(\sqrt{3}-1) = 0 \\
&\Rightarrow \sqrt{3}(\sqrt{3}+1) - (\sqrt{3}-1)(\sqrt{3}x - (\sqrt{3}+1)) = 0 \\
&\Rightarrow [\sqrt{3}x(\sqrt{3}+1)][\sqrt{3}x - (\sqrt{3}-1)] = 0 \\
&\Rightarrow \sqrt{3}x - (\sqrt{3}+1) = 0 \text{ or } \sqrt{3}x - (\sqrt{3}-1) = 0
\end{aligned}$$

47. $x - \frac{1}{x} = 3, x \neq 0$

Sol:

We have,

$$\begin{aligned}
&x - \frac{1}{x} = 3, x \neq 0 \\
&\Rightarrow \frac{x^2 - 1}{x} = 3 \\
&\Rightarrow x^2 - 1 = 3x \\
&\Rightarrow x^2 - 3x - 1 = 0 \\
&\Rightarrow x^2 - \left(\frac{3}{2} + \frac{3}{2}\right)x - 1 = 0 \\
&\Rightarrow x^2 - \left(\frac{3+\sqrt{13}}{2} + \frac{3-\sqrt{13}}{2}\right)x + (-1) = 0 \\
&\Rightarrow x^2 - \left(\frac{3+\sqrt{13}}{2}\right)x - \left(\frac{3-\sqrt{13}}{2}\right)x + (-1) = 0 \\
&\Rightarrow x^2 - \left(\frac{3+\sqrt{13}}{2}\right)x - \left(\frac{3-\sqrt{13}}{2}\right)x + \left(\frac{-4}{4}\right) = 0 \\
&\Rightarrow x^2 - \left(\frac{3+\sqrt{13}}{2}\right)x - \left(\frac{3-\sqrt{13}}{2}\right)x + \left(\frac{9-13}{4}\right) = 0
\end{aligned}$$

$$\begin{aligned}
& \Rightarrow x^2 - \left(\frac{3+\sqrt{13}}{2} \right) x - \left(\frac{3-\sqrt{13}}{2} \right) x + \left(\frac{3^2 - (\sqrt{13})^2}{2^2} \right) = 0 \\
& \Rightarrow x^2 - \left(\frac{3+\sqrt{13}}{2} \right) x - \left(\frac{3-\sqrt{13}}{2} \right) x + \frac{(3+\sqrt{13})}{2} + \frac{(3-\sqrt{13})}{2} = 0 \\
& \Rightarrow x \left(x - \left(\frac{3+\sqrt{13}}{2} \right) \right) - \left(\frac{3-\sqrt{13}}{2} \right) \left(x - \left(\frac{3+\sqrt{13}}{2} \right) \right) = 0 \\
& \Rightarrow \left(x - \frac{3+\sqrt{13}}{2} \right) \left(x - \frac{3-\sqrt{13}}{2} \right) = 0 \\
& \Rightarrow x - \left(\frac{3+\sqrt{13}}{2} \right) = 0 \text{ or } x - \left(\frac{3-\sqrt{13}}{2} \right) = 0 \\
& \Rightarrow x = \frac{3+\sqrt{13}}{2} \text{ or } x = \frac{3-\sqrt{13}}{2} \\
& \therefore x = \frac{3+\sqrt{13}}{2} \text{ and } x = \frac{3-\sqrt{13}}{2} \text{ are the two roots of the given equation.}
\end{aligned}$$

48. $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq 4, 7$

Sol:

We have,

$$\begin{aligned}
& \frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq 4, 7 \\
& \Rightarrow \frac{x-7 - (x+4)}{(x+4)(x-7)} = \frac{11}{30} \\
& \Rightarrow \frac{x-7-x-4}{x^2-7x+4x-28} = \frac{11}{30} \\
& \Rightarrow \frac{-11}{x^2-3x-28} = \frac{11}{30} \\
& \Rightarrow (-1) \times 30 = 1 \times (x^2 - 3x - 28) \\
& \Rightarrow -30 = x^2 - 3x - 28 \\
& \Rightarrow x^2 - 3x - 28 + 30 = 0 \\
& \Rightarrow x^2 - 3x + 2 = 0 \quad [\because 2 = -2 \times -1 - 3 = -2 - 1] \\
& \Rightarrow x^2 - 2x - x + 2 = 0
\end{aligned}$$

$$\Rightarrow x(x-8) - 1(x-8) = 0$$

$$\Rightarrow (x-8)(x-1) = 0$$

$$\Rightarrow x-8=0 \text{ or } x-1=0$$

$$\Rightarrow x=8 \text{ or } x=1$$

$\therefore x=2$ and $x=1$ are the two roots of the given equation.

Exercise 8.4

Find the roots of the following quadratic equations (if they exist) by the method of completing the square.

1. $x^2 - 4\sqrt{2}x + 6 = 0$

Sol:

We have,

$$x^2 - 4\sqrt{2}x + 6 = 0$$

$$\Rightarrow x^2 - 2 \times x \times 2\sqrt{2} + (2\sqrt{2})^2 - (2\sqrt{2})^2 + 6 = 0$$

$$\Rightarrow (x - 2\sqrt{2})^2 = (2\sqrt{2})^2 - 6$$

$$\Rightarrow (x - 2\sqrt{2})^2 = (4 \times 2) - 6$$

$$\Rightarrow (x - 2\sqrt{2})^2 = 8 - 6$$

$$\Rightarrow (x - 2\sqrt{2})^2 = 2$$

$$\Rightarrow x - 2\sqrt{2} = \pm\sqrt{2}$$

$$\Rightarrow x - 2\sqrt{2} = \sqrt{2} \text{ or } x - 2\sqrt{2} = -\sqrt{2}$$

$$\Rightarrow x = 3\sqrt{2} \text{ or } x = \sqrt{2}$$

$\therefore x = \sqrt{2}$ and $x = 3\sqrt{2}$ are the roots of the given equation.

2. $2x^2 - 7x + 3 = 0$

Sol:

We have,

$$2x^2 - 7x + 3 = 0$$

$$2\left(x^2 - \frac{7}{2}x + \frac{3}{2}\right) = 0$$

$$\begin{aligned}
 & \Rightarrow x^2 - 2 \times \frac{7}{2} \times \frac{1}{2} \times x + \frac{3}{2} = 0 \\
 & \Rightarrow x^2 - 2 \times \frac{7}{4} \times x + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0 \\
 & \Rightarrow x^2 - 2 \times \frac{7}{4} \times x + \left(\frac{7}{4}\right)^2 - \frac{49}{16} + \frac{3}{2} = 0 \\
 & \Rightarrow \left(x - \frac{7}{4}\right)^2 - \frac{49}{16} + \frac{3}{2} = 0 \\
 & \Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{49}{16} - \frac{3}{2} \\
 & \Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{49 - 86}{16} \\
 & \Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16} \\
 & \Rightarrow x - \frac{7}{4} = \pm \left(\frac{5}{4}\right) \\
 & \Rightarrow x - \frac{7}{4} = \pm \frac{5}{4} \\
 & \Rightarrow x - \frac{7}{4} = \frac{5}{4} \text{ or } x - \frac{7}{4} = -\frac{5}{4} \\
 & \Rightarrow x = \frac{5}{4} + \frac{7}{4} \text{ or } x = \frac{7}{4} - \frac{5}{4} \\
 & \Rightarrow x = \frac{12}{4} \text{ or } x = \frac{2}{4} \\
 & \Rightarrow x = 3 \text{ or } x = \frac{1}{2} \\
 & \therefore x = 3 \text{ and } x = \frac{1}{2} \text{ are the roots of the given quadratic equation.}
 \end{aligned}$$

3. $3x^2 + 11x + 10 = 0$

Sol:

We have,

$$\begin{aligned}
 & 3x^2 + 11x + 10 = 0 \\
 & \Rightarrow x^2 + \frac{11}{3}x + \frac{10}{3} = 0
 \end{aligned}$$

$$\begin{aligned}
 & \Rightarrow x^2 + 2 \times \frac{1}{2} \times \frac{11}{3}x + \frac{10}{3} = 0 \\
 & \Rightarrow x^2 + 2 \times \frac{11}{6} \times 2 + \left(\frac{11}{6}\right)^2 - \left(\frac{11}{6}\right)^2 + \frac{10}{3} = 0 \\
 & \Rightarrow \left(x + \frac{11}{6}\right)^2 = \left(\frac{11}{6}\right)^2 - \frac{10}{3} \\
 & \Rightarrow \left(x + \frac{11}{6}\right)^2 = \frac{121}{36} - \frac{10}{3} \\
 & \Rightarrow \left(x + \frac{11}{6}\right)^2 = \frac{121 - 120}{36} \\
 & \Rightarrow \left(x + \frac{11}{6}\right)^2 = \frac{1}{36} \\
 & \Rightarrow \left(x + \frac{11}{6}\right)^2 = \left(\frac{1}{6}\right)^2 \\
 & \Rightarrow x + \frac{11}{6} = \pm \frac{1}{6} \\
 & \Rightarrow x + \frac{11}{6} = \frac{1}{6} \text{ or } x + \frac{11}{6} = -\frac{1}{6} \\
 & \Rightarrow x = \frac{1}{6} - \frac{11}{6} \text{ or } x = -\frac{1}{6} - \frac{11}{6} \\
 & \Rightarrow x = -\frac{10}{6} \text{ or } x = -\frac{12}{6} = -2 \\
 & \Rightarrow x = -\frac{5}{3} \text{ or } x = -2 \\
 & \therefore x = -\frac{5}{3} \text{ or } x = -2 \text{ are the two roots of the given equation.}
 \end{aligned}$$

4. $2x^2 + x - 4 = 0$

Sol:

We have,

$$\begin{aligned}
 & 2x^2 + x - 4 = 0 \\
 & \Rightarrow 2\left(x^2 + \frac{x}{2} - \frac{4}{2}\right) = 0 \\
 & \Rightarrow x^2 + 2 \times \frac{1}{2} \times \frac{1}{2}x - 2 = 0
 \end{aligned}$$

$$\begin{aligned}
 & \Rightarrow x^2 + 2x \times \frac{1}{4} \times x + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 2 = 0 \\
 & \Rightarrow \left(x + \frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 + 2 \\
 & \Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1}{16} + 2 \\
 & \Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1+2 \times 16}{16} \\
 & \Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1+32}{16} \\
 & \Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16} \\
 & \Rightarrow x + \frac{1}{4} = \pm \sqrt{\frac{33}{16}} \\
 & \Rightarrow x + \frac{1}{4} = \pm \frac{\sqrt{33}}{4} \text{ or } x + \frac{1}{4} = -\frac{\sqrt{33}}{4} \\
 & \Rightarrow x = \frac{\sqrt{33}}{4} - \frac{1}{4} \text{ or } x = -\frac{\sqrt{33}}{4} - \frac{1}{4} \\
 & \Rightarrow x = \frac{\sqrt{33} - 1}{4} \text{ or } x = \frac{-\sqrt{33} - 1}{4} \\
 & \therefore x = \frac{\sqrt{33} - 1}{4} \text{ or } x = -\frac{\sqrt{33} - 1}{4} \text{ are the two roots of the given equation}
 \end{aligned}$$

5. $2x^2 + x + 4 = 0$

Sol:

We have,

$$2x^2 + x + 4 = 0$$

$$\Rightarrow x^2 + \frac{x}{2} + 2 = 0$$

$$\Rightarrow x^2 + 2 \times \frac{1}{2} \times \frac{1}{2} \times x + 2 = 0$$

$$\Rightarrow x^2 + 2 \times \frac{1}{4} \times x + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 2$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1}{16} - 2$$

$$\begin{aligned}
 & \Rightarrow \left(x + \frac{1}{4} \right)^2 = \frac{1-36}{16} \\
 & \Rightarrow \left(x + \frac{1}{4} \right)^2 = -\frac{31}{16} \\
 & \Rightarrow x + \frac{1}{4} = \sqrt[+]{\frac{-31}{16}} \\
 & \Rightarrow x + \frac{1}{4} = \frac{\sqrt{-31}}{4} \text{ or } x + \frac{1}{4} = -\frac{\sqrt{31}}{4} \\
 & \Rightarrow x = \frac{\sqrt{-31}-1}{4} \text{ or } x = \frac{-\sqrt{31}-1}{4}
 \end{aligned}$$

Since, $\sqrt{-31}$ is not a real number

\therefore The roots are not real roots.

6. $4x^2 + 4\sqrt{3}x + 3 = 0$

Sol:

We have,

$$\begin{aligned}
 & 4x^2 + 4\sqrt{3}x + 3 = 0 \\
 & \Rightarrow x^2 + \frac{4\sqrt{3}}{4}x + \frac{3}{4} = 0 \\
 & \Rightarrow x^2 + 2 \times \frac{1}{2} \times \sqrt{3} \times x + \frac{3}{4} = 0 \\
 & \Rightarrow x^2 + 2 \times \frac{\sqrt{3}}{2} \times x + \left(\frac{\sqrt{3}}{2} \right)^2 - \left(\frac{\sqrt{3}}{2} \right)^2 + \frac{3}{4} = 0 \\
 & \Rightarrow \left(x + \frac{\sqrt{3}}{2} \right)^2 - \frac{3}{4} + \frac{3}{4} = 0 \\
 & \Rightarrow \left(x + \frac{\sqrt{3}}{2} \right)^2 = 0 \\
 & \Rightarrow x + \frac{\sqrt{3}}{2} = 0 \text{ and } x - \frac{\sqrt{3}}{2} = 0 \\
 & \Rightarrow x = \frac{-\sqrt{3}}{2} \text{ and } x = \frac{\sqrt{3}}{2} \\
 & \therefore x = \frac{-\sqrt{3}}{2} \text{ and } x = \frac{\sqrt{3}}{2} \text{ are the two roots of the given equation as it is a perfect square.}
 \end{aligned}$$

7. $\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

Sol:

We have,

$$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$$

$$\Rightarrow x^2 - \frac{3x}{\sqrt{2}} - \frac{2\sqrt{2}}{\sqrt{2}} = 0$$

$$\Rightarrow x^2 - \frac{3}{\sqrt{2}}x - 8 = 0$$

$$\Rightarrow x^2 - 2 \times \frac{1}{2} \times \frac{3}{\sqrt{2}}x - 8 = 0$$

$$\Rightarrow x^2 - 2 \times \frac{3}{2\sqrt{2}} \times x + \left(\frac{3}{2\sqrt{2}}\right)^2 - \left(\frac{3}{2\sqrt{2}}\right)^2 - 8 = 0$$

$$\Rightarrow \left(x - \frac{3}{2\sqrt{2}}\right)^2 = \frac{9}{8} + 8$$

$$\Rightarrow \left(x - \frac{3}{2\sqrt{2}}\right)^2 = \frac{9+16}{8}$$

$$\Rightarrow \left(x - \frac{3}{2\sqrt{2}}\right)^2 = \frac{25}{8}$$

8. $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

Sol:

We have

$$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$$

$$\Rightarrow x^2 + \frac{10}{\sqrt{3}}x + \frac{7\sqrt{3}}{\sqrt{3}} = 0$$

$$\Rightarrow x^2 + 2 \times \frac{1}{2} \times \frac{10}{\sqrt{3}}x + 7 = 0$$

$$\Rightarrow x^2 + 2 \times \frac{5}{\sqrt{3}} \times x + \left(\frac{5}{\sqrt{3}}\right)^2 - \left(\frac{5}{\sqrt{3}}\right)^2 + 7 = 0$$

$$\Rightarrow \left(x + \frac{5}{\sqrt{3}}\right)^2 = \frac{25}{3} - 7$$

$$\Rightarrow \left(x + \frac{5}{\sqrt{3}}\right)^2 = \frac{25-21}{3}$$

$$\begin{aligned}
 & \Rightarrow \left(x + \frac{5}{\sqrt{3}} \right)^2 = \frac{4}{3} \\
 & \Rightarrow x + \frac{5}{\sqrt{3}} = \pm \sqrt{\frac{4}{3}} \\
 & \Rightarrow x + \frac{5}{\sqrt{3}} = +\frac{2}{\sqrt{3}} \text{ or } x + \frac{5}{\sqrt{3}} = -\frac{2}{\sqrt{3}} \\
 & \Rightarrow x = \frac{-3}{\sqrt{3}} \text{ or } x = -\frac{7}{\sqrt{3}} \\
 & \Rightarrow x = -\sqrt{3} \text{ or } x = -\frac{7}{\sqrt{3}}
 \end{aligned}$$

$\therefore x = -\sqrt{3}$ and $x = -\frac{7}{\sqrt{3}}$ are the roots of the given equation.

9. $x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$

Sol:

We have,

$$\begin{aligned}
 & x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0 \\
 & \Rightarrow x^2 - 2 \times \frac{1}{2} (\sqrt{2} + 1)x + \sqrt{2} = 0 \\
 & \Rightarrow x^2 - 2 \times \frac{\sqrt{2} + 1}{2} x + \left(\frac{\sqrt{2} - 1}{2} \right)^2 - \left(\frac{\sqrt{2} + 1}{2} \right)^2 + \sqrt{2} = 0 \\
 & \Rightarrow \left(x - \frac{\sqrt{2} + 1}{2} \right)^2 = \left(\frac{\sqrt{2} + 1}{2} \right)^2 - \sqrt{2} \\
 & \Rightarrow \left(x - \frac{\sqrt{2} + 1}{2} \right)^2 = \frac{3 + 2\sqrt{2} - 4\sqrt{2}}{4} \\
 & \Rightarrow \left(x - \frac{\sqrt{2} + 1}{2} \right)^2 = \frac{3 - 2\sqrt{2}}{4} \\
 & \Rightarrow \left(x - \frac{\sqrt{2} + 1}{2} \right)^2 = \frac{2 + 1 - 2\sqrt{2}}{4} \\
 & \Rightarrow \left(x - \frac{\sqrt{2} + 1}{2} \right)^2 = \frac{(\sqrt{2})^2 - 2\sqrt{2} + 1}{2^2}
 \end{aligned}$$

$$\begin{aligned}
 & \Rightarrow \left(x - \frac{\sqrt{2}+1}{2} \right)^2 = \frac{(\sqrt{2}-1)^2}{2^2} \\
 & \Rightarrow \left(x - \frac{\sqrt{2}+1}{2} \right)^2 = \left(\frac{\sqrt{2}-1}{2} \right)^2 \\
 & \Rightarrow x - \frac{\sqrt{2}+1}{2} = \pm \left(\frac{\sqrt{2}-1}{2} \right) \\
 & \Rightarrow x - \frac{\sqrt{2}+1}{2} = \frac{\sqrt{2}-1}{2} \text{ or } x - \frac{\sqrt{2}+1}{2} = -\frac{\sqrt{2}-1}{2} \\
 & \Rightarrow x = \frac{\sqrt{2}-1}{2} + \frac{\sqrt{2}+1}{2} \text{ or } x = \frac{\sqrt{2}-1}{2} - \frac{\sqrt{2}+1}{2} \\
 & \Rightarrow x = \frac{\sqrt{2}-1+\sqrt{2}+1}{2} \text{ or } x = \frac{-\sqrt{2}+1+\sqrt{2}+1}{2} \\
 & \Rightarrow x = \frac{\sqrt{2}}{2} \text{ or } x = \frac{1}{2} \\
 & \Rightarrow x = \sqrt{2} \text{ or } x = 1 \\
 & \therefore x = \sqrt{2} \text{ and } x = 1 \text{ are the roots of the given equation}
 \end{aligned}$$

10. $x^2 - 4ax + 4a^2 - b^2 = 0$

Sol:

We have,

$$\begin{aligned}
 & x^2 - 4ax + 4a^2 - b^2 = 0 \\
 & \Rightarrow x^2 - 2 \times (2a) \times x + (2a)^2 - b^2 = 0 \\
 & \Rightarrow (x - 2a)^2 = b^2 \\
 & \Rightarrow x - 2a = \pm b \\
 & \Rightarrow x - 2a = b \text{ or } x - 2a = -b \\
 & \Rightarrow x = 2a + b \text{ or } x = 2a - b \\
 & \therefore x = 2a + b \text{ and } x = 2a - b \text{ are the two roots of the given quadratic equation.}
 \end{aligned}$$

Exercise 8.5

1. Write the discriminant of the following quadratic equation:

(i) $2x^2 - 5x + 3 = 0$ (ii) $x^2 + 2x + 4 = 0$ (iii) $(x-1)(2x-1) = 0$ (iv) $x^2 - 2x + k = 0, K \in R$
 (v) $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$ (vi) $x^2 - x + 1 = 0$ (vii) $3x^2 + 2x + k = 0$ (viii) $4x^2 - 3kx + 1 = 0$

Sol: (i) $2x^2 - 5x + 3 = 0$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = 2, b = -5$ and $c = 3$

The discriminant $D = b^2 - 4ac$

$$\Rightarrow (-5)^2 - 4 \times 2 \times 3$$

$$\Rightarrow 25 - 24 = 1$$

∴ The discriminant of the following quadratic equation is 1.

(ii) $x^2 + 2x + 4 = 0$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = 1, b = 2$ and $c = 4$

The discriminant is $D = b^2 - 4ac$

$$\Rightarrow (2)^2 - 4 \times 1 \times 4$$

$$\Rightarrow 4 - 16 = -12$$

∴ The discriminant of the following quadratic equation is -12.

(iii) $(x-1)(2x-1) = 0$

The given equation is $(x-1)(2x-1) = 0$

By solving it, we get $2x^2 - 3x + 1 = 0$

∴ This equation is in the form of $ax^2 + bx + c = 0$

here $a = 2, b = -3, c = -1$

The discriminant is $D = b^2 - 4ac$

$$\Rightarrow (-3)^2 - 4 \times 2 \times 1$$

$$\Rightarrow 9 - 8 = 1$$

∴ The discriminant D, for the following quadratic equation is 1

(iv) $x^2 - 2x + k = 0, K \in R$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = 1, b = -2, c = k$ [given $k \in R$]

The discriminant is $D = b^2 - 4ac$

$$\Rightarrow (-2)^2 - 4 \times 1 \times k$$

$$\Rightarrow 4 - 4k$$

\therefore The discriminant D, of the following quadratic equation is $4 - 4k$, where $K \in R$

$$(v) \sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = \sqrt{3}$, $b = 2\sqrt{2}x$ and $c = -2\sqrt{3}$

The discriminant is $D = b^2 - 4ac$

$$\Rightarrow (2\sqrt{2})^2 - 4 \times \sqrt{3} \times -2\sqrt{3}$$

$$\Rightarrow 8 + 24$$

$$\Rightarrow 32$$

\therefore The discriminant D, of the following quadratic equation is 32.

$$(vi) x^2 - x + 1 = 0$$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = 1$, $b = -1$ and $c = 1$

The discriminant is $D = b^2 - 4ac$

$$\Rightarrow (-1)^2 - 4 \times 1 \times 1$$

$$\Rightarrow 1 - 4 = -3$$

\therefore The discriminant D, of the following quadratic equation is -3.

$$(vii) 3x^2 + 2x + k = 0$$

The given equation has $3x^2 + 2x + k = 0$

here $a = 3$, $b = 2$, $c = k$

\Rightarrow given that the quadratic equation has real roots.

i.e., $D = b^2 - 4ac \geq 0$

$$\Rightarrow 4 - 4 \times 3 \times k \geq 0$$

$$\Rightarrow 4 - 12k \geq 0$$

$$\Rightarrow 4 \geq 12k$$

$$\Rightarrow 4 \leq \frac{4}{12}$$

$$\Rightarrow k \leq \frac{1}{3}$$

The 'k' value should not exceed $\frac{1}{3}$ to have the real roots for the given equation.

$$(viii) 4x^2 - 3kx + 1 = 0$$

The given equation has $4x^2 - 3kx + 1 = 0$

here $a = 4, b = -3k, c = 1$

given that quadratic equation has real roots

$$\text{i.e., } D = b^2 - 4ac \geq 0$$

$$\Rightarrow (2)^2 - 4 \times 3 \times k \geq 0$$

$$\Rightarrow 4 - 4 \times 3 \times k \geq 0$$

$$\Rightarrow 4 - 12k \geq 0$$

$$\Rightarrow 4 \geq 12k$$

$$\Rightarrow k \leq \frac{12}{4} \Rightarrow k \leq 3$$

2. In the following, determine whether the given quadratic equation have real roots and if so, find the roots:

$$(i) 16x^2 = 24x + 1 \quad (ii) x^2 + x + 2 = 0 \quad (iii) \sqrt{3}x^2 + 10x - 8\sqrt{3} = 0 \quad (iv) 3x^2 - 2x + 2 = 0$$

$$(v) 2x^2 - 2\sqrt{6}x + 3 = 0 \quad (vi) 3a^2x^2 + 8abx + 4b^2 = 0, a \neq 0 \quad (vii) 3x^2 + 2\sqrt{b}x - b = 0$$

$$(viii) x^2 - 2x + 1 = 0 \quad (ix) 2x^2 + 5\sqrt{3} + 6 = 0 \quad (x) \sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$$

$$(xi) 2x^2 - 2\sqrt{2}x + 1 = 0 \quad (xii) 3x^2 - bx + 2 = 0$$

Sol:

$$(i) 16x^2 = 24x + 1$$

The given equation is in the form of $16x^2 - 24x - 1 = 0$

Hence, the equation is in the form of $ax^2 + bx + c = 0$

$$\text{Here, } a = 16, b = -24, c = -1, D = b^2 - 4ac = (-24)^2 - 4 \times 16 \times -1 = 576 + 64 = 640 > 0$$

As $D > 0$, the given equation has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \Rightarrow \frac{-(-24) + \sqrt{640}}{2 \times 16} = \frac{3 + \sqrt{10}}{4}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} \Rightarrow \frac{-(-24) - \sqrt{640}}{2 \times 16} = \frac{3 - \sqrt{10}}{4}$$

$$\therefore \text{The roots of the equation are } \frac{3 \pm \sqrt{10}}{4}$$

$$(ii) x^2 + x + 2 = 0$$

The given equation is in the form of $ax^2 + bx + c = 0$

$$a = 1, b = 1, c = 2.$$

$$D = b^2 - 4ac = (1)^2 - 4 \times 1 \times 2 = 1 - 8 = -7 < 0$$

As $Q < 0$, the equation has no real roots

$$(iii) \sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$$

The given equation is in the form of $ax^2 + bx + c = 0$

$$\text{here } a = \sqrt{3}, b = 10 \text{ and } c = -8\sqrt{3}$$

$$D = b^2 - 4ac \Rightarrow (10)^2 - 4 \times \sqrt{3} \times -8\sqrt{3} = 100 + 96 = 196 > 0$$

As $Q > 0$, the given equation has real roots, given by

$$\alpha = \frac{-b + \sqrt{196}}{2a} \Rightarrow \frac{10 + \sqrt{196}}{2 \times \sqrt{3}} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}} \quad [\because \text{Multiplying and dividing by } \sqrt{3}]$$

$$\beta = \frac{-b - \sqrt{196}}{2a} \Rightarrow \frac{-10 - \sqrt{196}}{2 \times \sqrt{3}} = -4\sqrt{3}$$

\therefore The roots of the equation are $\frac{2}{\sqrt{3}}$ and $-4\sqrt{3}$

$$(iv) 3x^2 - 2x + 2 = 0$$

The given equation is in the form of $ax^2 + bx + c = 0$

$$\text{here } a = 3, b = -2, c = 2$$

$$\text{The discriminant } Q = b^2 - 4ac$$

$$\Rightarrow (-2)^2 - 4 \times 3 \times 2 = 4 - 24$$

$$\Rightarrow -20 < 0$$

Hence as $Q < 0$,

The given equation has no real roots.

$$(v) 2x^2 - 2\sqrt{6}x + 3 = 0$$

The given equation is in the form of $ax^2 + bx + c = 0$

$$\text{here } a = 2, b = -2\sqrt{6}, c = 3$$

$$\text{The discriminant } Q = b^2 - 4ac$$

$$\Rightarrow (-2\sqrt{6})^2 - 4 \times 2 \times 3 = 24 - 24$$

$$\Rightarrow 0$$

As $Q = 0$, the given equation has real and equal roots, They are

$$\alpha = \frac{-b + \sqrt{D}}{2a} \Rightarrow -\frac{(-2\sqrt{6}) + \sqrt{0}}{2 \times 2} = \frac{-2\sqrt{6}}{4} = \frac{\sqrt{3}}{2}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} \Rightarrow -\frac{(-2\sqrt{6}) - \sqrt{0}}{2 \times 2} = \frac{2\sqrt{6}}{4} = \frac{\sqrt{3}}{2}$$

∴ The roots of the given equation is $\sqrt{\frac{3}{2}}$

(vi) $3a^2x^2 + 8abx + 4b^2 = 0, a \neq 0$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = 3a^2, b = 8ab, c = 4b^2$ [given $a \neq 0$]

$$D = b^2 - 4ac = (8ab)^2 - 4 \times 3a^2 \times 4b^2 = 64a^2b^2 - 48a^2b^2 = 16a^2b^2 > 0$$

As $Q = 0$, the given equation has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \Rightarrow -\frac{(8ab) + \sqrt{16a^2b^2}}{2 \times 3a^2} = \frac{-2b}{a}$$

$$\beta = \frac{-b + \sqrt{D}}{2a} \Rightarrow -\frac{(8ab) - \sqrt{16a^2b^2}}{2 \times 3a^2} = \frac{-2b}{a}$$

∴ The roots of the given equation are $\frac{-2b}{a}, \frac{-2b}{3a}$

(vii) $3x^2 + 2\sqrt{b}x - b = 0$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = 3, b = 2\sqrt{5}, c = -5$

The discriminant $Q = b^2 - 4ac$

$$\Rightarrow (2\sqrt{5})^2 - 4 \times 3 \times -5 = 20 + 4 \times 3 \times 5$$

$$\Rightarrow 20 + 60 - 80 > 0$$

As $Q = 0$, the given equation has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \Rightarrow -\frac{-(2\sqrt{5}) + \sqrt{80}}{2 \times 3} = \frac{\sqrt{5}}{3}$$

$$\beta = \frac{-b + \sqrt{D}}{2a} \Rightarrow -\frac{-(2\sqrt{5}) - \sqrt{80}}{2 \times 3} = -\frac{\sqrt{5}}{3}$$

∴ The roots of the given equation is $\sqrt{\frac{3}{2}}$

$$(viii) x^2 - 2x + 1 = 0$$

The given equation is in the form of $ax^2 + bx + c = 0$

Here $a = 1, b = -2$ and $c = 1$

The discriminant $D = b^2 - 4ac$

$$\Rightarrow (-2)^2 - 4 \times 1 \times 1 = 0$$

As $Q = 0$, the given equation has real and equal roots

$$\Rightarrow \alpha = -\frac{b + \sqrt{D}}{2a}, \beta = -\frac{b - \sqrt{D}}{2a} \text{ i.e., } \alpha \text{ and } \beta = -\frac{b}{2a} [\because 0 = 0]$$

$$\Rightarrow \alpha \text{ and } \beta = -\frac{b}{2a} = -\frac{(-2)}{2 \times 2} = \frac{2}{2} = 1$$

∴ The roots of the given equation α and β is 1.

$$(ix) 2x^2 + 5\sqrt{3} + 6 = 0$$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = 2, b = 5\sqrt{3}, c = 6$

The discriminant $D = b^2 - 4ac$

$$\Rightarrow (5\sqrt{3})^2 - 4 \times 2 \times 6 = 75 - 48$$

$$\Rightarrow 27 > 0$$

As $Q = 0$, the given equation has real roots, given by

$$\boxed{\alpha = \frac{-b + \sqrt{D}}{2a} \Rightarrow \frac{-(5\sqrt{3}) + \sqrt{27}}{2 \times 2} = \frac{\sqrt{3}(-5 + 3)}{4} = \frac{\sqrt{3} \times -2}{4} = \frac{\sqrt{3}}{2}}$$

$$\boxed{\beta = \frac{-b + \sqrt{D}}{2a} \Rightarrow \frac{-(5\sqrt{3}) - \sqrt{27}}{2 \times 2} = \frac{-\sqrt{3}[5 + 3]}{4} = \frac{-8\sqrt{3}}{4} = -2\sqrt{3}}$$

∴ The roots of the given equation are $-\frac{\sqrt{3}}{2}, -2\sqrt{3}$

$$(x) \sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = \sqrt{2}, b = 7, c = 5\sqrt{2}$

The discriminant $Q = b^2 - 4ac$

$$\Rightarrow (7)^2 - 4 \times \sqrt{2} \times 5\sqrt{2} = 49 - 40$$

$$\Rightarrow 9 > 0$$

As $D = 0$, the given equation has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \Rightarrow \frac{-7 + \sqrt{9}}{2 \times \sqrt{2}} = \frac{-7 + \sqrt{3}}{2\sqrt{2}} = \frac{4^2}{2\sqrt{2}} = -\sqrt{2}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} \Rightarrow \frac{-7 - \sqrt{9}}{2 \times \sqrt{2}} = \frac{-7 - \sqrt{3}}{2\sqrt{2}} = \frac{5}{2\sqrt{2}} = -\frac{5}{\sqrt{2}}$$

\therefore The roots of the given equation are $-\sqrt{2}, -\frac{5}{\sqrt{2}}$

$$(xi) \ 2x^2 - 2\sqrt{2}x + 1 = 0$$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = 2, b = -2\sqrt{2}, c = 1$

$$D = b^2 - 4ac \Rightarrow \frac{(-2\sqrt{2})}{2 \times 2} = \frac{2^1 \sqrt{2}}{4^2} = \frac{1}{\sqrt{2}}$$

as $D > 0$, the given equation has Real and equal roots

hence $\alpha \text{ and } \beta = -\frac{b}{2a} = -\frac{(-2\sqrt{2})}{2 \times 2} = \frac{2^1 \sqrt{2}}{4^2} = \frac{1}{\sqrt{2}}$

\therefore The roots $\frac{1}{\sqrt{2}}$ is obtained by multiplying and dividing $\frac{\sqrt{3}}{2}$ by $\sqrt{2}$

\therefore The roots of the given equation is $\frac{1}{\sqrt{2}}$

$$(xii) \ 3x^2 - bx + 2 = 0$$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = 3, b = -5, c = 2$

$$D = b^2 - 4ac \Rightarrow (-5)^2 - 4 \times 3 \times 2 = 25 - 24 = 1 > 0$$

as $D > 0$, the given equation has Real roots, giving by

$$\alpha = -\frac{b + \sqrt{D}}{2a} \Rightarrow \frac{-(-5) + \sqrt{1}}{2 \times 3} = \frac{5 + \sqrt{1}}{6} = \frac{6^1}{6} = 1$$

$$\beta = -\frac{b - \sqrt{D}}{2a} \Rightarrow \frac{-(-5) - \sqrt{1}}{2 \times 3} = \frac{5 - \sqrt{1}}{6} = \frac{4^2}{6} = \frac{2}{3}$$

∴ The roots of the given equation are 1 and $\frac{2}{3}$

3. Solve for x : $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}; x \neq 2, 4$

Sol: Given,

$$\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$$

$$\Rightarrow \frac{(x-1)(x-4) + (x-3)(x-2)}{(x-2)(x-4)} = \frac{10}{3} \quad [\text{Solving improper fraction}]$$

$$\Rightarrow \frac{(x^2 - 5x + 4) + (x^2 - 5x + 6)}{(x-2)(x-4)} = \frac{10}{3}$$

This can also be written as. .

$$3[2x^2 - 10x + 10] = 10[(x-2)(x-4)]$$

$$\Rightarrow 6x^2 - 30x + 30 = 10x^2 - 60x + 80$$

By solving them, by taking all to one side, we get

$$\Rightarrow (10x^2 - 60x + 80) - (6x^2 - 30x + 30) = 0$$

$$\Rightarrow 4x^2 - 30x + 50 = 0, \text{ here } a = 4, b = -30, c = 50$$

Hence we get x by
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = -\frac{(-30) \pm \sqrt{(-30)^2 - 4 \times 4 \times 50}}{2(4)} = 5$$

$$x = -\frac{(-30) - \sqrt{(-30)^2 - 4 \times 4 \times 50}}{2 \times 4} = \frac{5}{2}$$

∴ The value of x are 5 and $\frac{5}{2}$

4. Solve for x : $\frac{1}{x} - \frac{1}{x-7} = 3, x \neq 0, 2$

Sol: Given $\frac{1}{x} - \frac{1}{x-2} = 3$

$$\Rightarrow \frac{x-2}{x(x-2)} = 3$$

$$\Rightarrow \frac{-2}{x(x-2)} = 3$$

This can be written as $-2 = 3(x^2 - 2x)$

The equation hence is $3x^2 - 6x + 2 = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here $a = 3, b = -6$ and $c = 2$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-6) + \sqrt{(-6)^2 - 4 \times 3 \times 2}}{2 \times 3} = \frac{6 + \sqrt{36 - 24}}{6} = \frac{3 + \sqrt{3}}{3}$$

$$\Rightarrow x = \frac{-(-6) - \sqrt{(-6)^2 - 4 \times 3 \times 2}}{2 \times 3} = \frac{6 - \sqrt{36 - 24}}{6} = \frac{3 - \sqrt{3}}{3}$$

\therefore The value of x are $\frac{3 \pm \sqrt{3}}{3}$

5. $x + \frac{1}{x} = 3, x \neq 0$

Sol: Given $x + \frac{1}{x} = 3, x \neq 0$

Hence this equation can be written as $\frac{x^2 + 1}{x} = 3$

$$\Rightarrow x^2 + 1 = 3x = x^2 - 3x + 1 = 0$$

\therefore The equation is in the form of $ax^2 + bx + c = 0$

Here $a = 1, b = -3, c = 1$.

The value of 'x' can be solved by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow x = \frac{-b + \sqrt{(-3)^2 - 4 \times 1 \times 1}}{2 \times 1} = \frac{3 + \sqrt{9 - 4}}{2} = \frac{3 + \sqrt{5}}{2}$$

$$\Rightarrow x = \frac{-b - \sqrt{(-3)^2 - 4 \times 1 \times 1}}{2 \times 1} = \frac{3 - \sqrt{9 - 4}}{2} = \frac{3 - \sqrt{5}}{2}$$

\therefore The value of 'x' are $\frac{3 \pm \sqrt{5}}{2}$

Exercise 8.6

1. Determine the nature of the roots of the following quadratic equations:

(i) $2x^2 - 3x + 5 = 0$ (ii) $2x^2 - 3x + 5 = 0$ (iii) $\frac{3}{5}x^2 - \frac{2}{3}x + 1 = 0$ (iv) $3x^2 - 4\sqrt{3}x + 4 = 0$
 (v) $3x^2 - 2\sqrt{6}x + 2 = 0$ (vi) $(x-2a)(x-2b) = 4ab$ (vii) $2(a^2 + b^2)x^2 + 2(a+b)x + 1 = 0$
 (viii) $2(a^2 + b^2)x^2 + 2(a+b)x + 1 = 0$ (ix) $(b+c)x^2 - (a+b+c)x + a = 0$

Sol:

(i) $2x^2 - 3x + 5 = 0$

The given quadratic equation is $2x^2 - 3x + 5 = 0$

here $a = 2, b = -3, c = 5$

$$D = b^2 + 4ac \Rightarrow (-3)^2 - 4 \times 2 \times 5 = 9 - 40 = -31 < 0$$

As $D < 0$, The discriminant of equation is negative, then the expression has no real roots

(ii) $2x^2 - 3x + 5 = 0$

The given quadratic equation is $2x^2 - 3x + 5 = 0$

here $a = 2, b = -3$ and $c = 5$

$$\therefore D = b^2 - 4ac \Rightarrow (-3)^2 - 4 \times 2 \times 5 = 9 - 40 = -31 < 0$$

As $D < 0$, the discriminant of equation is negative, then the expression has no real roots

(iii) $\frac{3}{5}x^2 - \frac{2}{3}x + 1 = 0$

The given quadratic equation is $\frac{3}{5}x^2 - \frac{2}{3}x + 1 = 0$ can also be written as $9x^2 - 10x + 15 = 0$

here $a = 9, b = -10, c = 15$

$$D = b^2 - 4ac \Rightarrow (-10)^2 - 4 \times 9 \times 15 = 100 - 540 = -440 < 0$$

\therefore as $D < 0$, the equation has no real roots

(iv) $3x^2 - 4\sqrt{3}x + 4 = 0$

The given quadratic equation is $3x^2 - 4\sqrt{3}x + 4 = 0$

here $a = 3, b = -4\sqrt{3}, c = 4$

The discriminant $D = b^2 - 4ac$

$$\Rightarrow (-4\sqrt{3})^2 - 4 \times 3 \times 4 = 48 - 48 = 0$$

as $D > 0$, the equation has real and equal roots

$$(v) 3x^2 - 2\sqrt{6}x + 2 = 0$$

The given quadratic equation is $3x^2 - 2\sqrt{6}x + 2 = 0$

Here. The equation is in the form of $ax^2 + b + c = 0$

Where $a = 3, b = -2\sqrt{6}$ and $c = 2$

$$D = b^2 - 4ac \Rightarrow (-2\sqrt{6})^2 - 4 \times 3 \times 2 = 24 - 24 = 0$$

as $D = 0$, the given quadratic equation has real and equal roots

$$(vi) (x - 2a)(x - 2b) = 4ab$$

The given equation $(x - 2a)(x - 2b) = 4ab$ can also be written as $x^2 - x(2a + 2b)$ and $c = 0$ $[4ab - 4ab = 0]$

$$D = b^2 - 4ac \Rightarrow [-(2a + 2b)]^2 - 4 \times 1 \times 0 = (2a + 2b)^2 > 0$$

\Rightarrow as equal root of any integers is always positive

$\Rightarrow D > 0$, hence the discriminant of the equation is positive

$$(vii) 2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$$

The given equation is $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$

It is in the form of the equation $ax^2 + bx + c = 0$

Here $a = 2(a^2 + b^2), b = 2(a + b)$ and $c = 1$

$$\therefore [D = b^2 - 4ac]$$

$$\Rightarrow (-2abcd)^2 - 9 \times 9a^2b^2 \times 16c^2d^2$$

$$\Rightarrow b + 6a^2b^2c^2d^2 - 576a^2b^2c^2d^2 = 0$$

Hence as $D = 0$, the equation has Real and equal roots

$$(viii) 2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$$

The given equation is $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$

It is in the farm of the equation $ax^2 + bx + c = 0$

Here $a = 2(a^2 + b^2), b = 2(a + b)$ and $c = 1$

$$\therefore [D = b^2 - 4ac] \Rightarrow [2(a + b)]^2 - 4 \times 2(a^2 + b^2) \times 1$$

$$\Rightarrow 4a^2 + ab^2 + 8ab - 8a^2 - 8b^2$$

$$\Rightarrow 8ab + 4(a^2 + b^2) < 0$$

as $D < 0$, The discriminant is negative and the nature of the roots are not real

$$(ix) (b+c)x^2 - (a+b+c)x + a = 0$$

The given equation is $(b+c)x^2 - (a+b+c)x + a = 0$

Here $a = b+c$, $b = -(a+b+c)$ and $c = a$

$$\therefore [D = b^2 - 4ac] \Rightarrow [-(a+b+c)]^2 - 4 \times (b+c) \times a$$

$$\Rightarrow (a+b+c)^2 - 4abc > 0$$

\therefore as $D > 0$, the discriminant is positive and the nature of the roots are real and unequal

2. Find the values of k for which the roots are real and equal in each of the following equation:

$$(i) kx^2 + 4x + 1 = 0 \quad (ii) kx^2 - 2\sqrt{5}x + 4 = 0 \quad (iii) 3x^2 - 5x + 2k = 0 \quad (iv) 4x^2 + kx + 9 = 0$$

$$(v) 2kx^2 - 40x + 25 = 0 \quad (vi) 9x^2 - 24x + k = 0 \quad (vii) 4x^2 - 3kx + 1 = 0$$

$$(viii) x^2 - 2(5+2k)x + 3(7+10k) = 0$$

Sol:

$$(i) kx^2 + 4x + 1 = 0$$

Sol:

The given equation $kx^2 + 4x + 1 = 0$ is in the form of $ax^2 + bx + c = 0$ where

$$a = k, b = 4, c = 1$$

\Rightarrow given that, the equation has real and equal roots

$$\text{i.e., } [D = b^2 - 4ac = 0]$$

$$\Rightarrow (4)^2 - 4 \times k \times 1 = 0$$

$$\Rightarrow 16 = 4k \Rightarrow [k = 4]$$

\therefore The value of $k = 4$

$$(ii) kx^2 - 2\sqrt{5}x + 4 = 0$$

Sol:

The given equation $kx^2 - 2\sqrt{5}x + 4 = 0$ is in the form of $ax^2 + bx + c = 0$ where

$$a = k, b = -2\sqrt{5} \text{ and } c = 4$$

\Rightarrow given that, the equation has real and equal roots

$$\text{i.e., } [D = b^2 - 4ac = 0]$$

$$\Rightarrow (-2\sqrt{5})^2 - 4 \times k \times 4 = 0$$

$$\Rightarrow 20 = 16k \Rightarrow k = \frac{20}{16} = \frac{5}{4} \quad \therefore k = \frac{5}{4}$$

$$\therefore \text{The value of } k = \frac{5}{4}$$

$$(iii) 3x^2 - 5x + 2k = 0$$

Sol:

The given equation is $3x^2 - 5x + 2k = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here, $a = 3, b = -5$ and $c = 2k$

\Rightarrow given that, the equation has real and equal roots

$$\text{i.e., } D = b^2 - 4ac = 0$$

$$\Rightarrow (-5)^2 - 4 \times 3 \times (2k) = 0$$

$$\Rightarrow 25 = 24k$$

$$\Rightarrow k = \frac{25}{24}$$

$$\therefore \text{The value of } k = \frac{25}{24}$$

$$(iv) 4x^2 + kx + 9 = 0$$

Sol:

The given equation is $4x^2 + kx + 9 = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here, $a = 4, b = k$ and $c = 9$

\Rightarrow given that, the equation has real and equal roots

$$\text{i.e., } D = b^2 - 4ac = 0$$

$$\Rightarrow k^2 - 4 \times 4 \times 9 = 0$$

$$\Rightarrow k^2 - 16 \times 9$$

$$\Rightarrow k = \sqrt{16 \times 9} = 4 \times 3 = 12$$

$$\therefore \text{The value of } k = 12$$

$$(v) 2kx^2 - 40x + 25 = 0$$

Sol:

The given equation is $2kx^2 - 40x + 25 = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here, $a = 2k, b = -40$ and $c = 25$

\Rightarrow given that, the equation has real and equal roots

i.e., $D = b^2 - 4ac = 0$

$$\Rightarrow (-10)^2 - 4 \times 2k \times 25 = 0$$

$$\Rightarrow 1600 - 200k = 0$$

$$\Rightarrow k = \frac{1600}{200} = 8 \quad [\therefore k = 8]$$

\therefore The value of $k = 8$

(vi) $9x^2 - 24x + k = 0$

Sol:

The given equation is $9x^2 - 24x + k = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here, $a = 9, b = -24$, and $c = k$

\Rightarrow given that, the nature of the roots of this equation is real and equal

i.e., $D = b^2 - 4ac = 0$

$$\Rightarrow (-24)^2 - 4 \times 4 \times k = 0$$

$$\Rightarrow 576 - 36k = 0$$

$$\Rightarrow k = \frac{576}{36} = 16 \quad [\therefore k = 16]$$

\therefore The value of $k = 16$

(vii) $4x^2 - 3kx + 1 = 0$

Sol:

The given equation is $4x^2 - 3kx + 1 = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here, $a = 4, b = -3k$, and $c = 1$

\Rightarrow given that, the nature of the roots of this equation is real and equal

i.e., $D = b^2 - 4ac = 0$

$$\Rightarrow (-3k)^2 - 4 \times 4 \times 1 = 0$$

$$\Rightarrow k^2 = \frac{16}{9} \Rightarrow k = \sqrt{\frac{16}{9}} = \pm \frac{4}{3}$$

$$\Rightarrow k = \pm \frac{4}{3}$$

$$\therefore \text{The value of } k \text{ is } \pm \frac{4}{3}$$

$$(viii) x^2 - 2(5+2k)x + 3(7+10k) = 0$$

Sol:

$$\text{The given equation is } x^2 - 2(5+2k)x + 3(7+10k) = 0$$

$$\text{Here, } a = 1, b = -2(5+2k) \text{ and } c = 3(7+10k)$$

\Rightarrow given that, the nature of the roots of this equation are real and equal

$$\text{i.e., } D = b^2 - 4ac = 0$$

$$\Rightarrow \{[-2(5+2k)]\} - 4 \times 1 \times 3(7+10k) = 0$$

$$\Rightarrow 4(5+2k)^2 - 12(7+10k) = 0$$

$$\Rightarrow 25 + 4k^2 + 20k - 21 - 30k = 0$$

$$\Rightarrow 4k^2 - 10k + 4 = 0 = 2k^2 - 5k + 2 = 0$$

$$\Rightarrow k2(k-2) - 1(k-2) \Rightarrow k = 2 \text{ or } k = \frac{1}{2}$$

\therefore The value of k is 2 or $\frac{1}{2}$

$$(ix) (3k+1)x^2 + 2(k+1)x + k = 0$$

Sol:

$$\text{The given equation is } (3k+1)x^2 + 2(k+1)x + k = 0$$

This equation is in the form of $ax^2 + bx + c = 0$

$$\text{Here } a = 3k+1, b = 2(k+1) \text{ and } c = k$$

\Rightarrow Given that the nature of the roots of this equation are real and equal

$$\text{i.e., } \boxed{D = b^2 - 4ac = 0}$$

$$\Rightarrow [2(k+1)]^2 - 4 \times (3k+1) \times 0$$

$$\Rightarrow 4[k+1]^2 = 4k[3k+1] = 0$$

$$\Rightarrow (k+1)^2 - k(3k+1) = 0$$

$$\Rightarrow k^2 + 1 + 2k - 3k^2 - k = 0$$

$$\Rightarrow -2k^2 + k + 1 = 0$$

This equation can also be written as $2k^2 - k - 1 = 0$

The value of k can obtain by

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where $a = 2, b = -1, c = 1$ from equation 2

$$k = \frac{-(-1) + \sqrt{(-1)^2 - 4 \times 2 \times (-1)}}{2 \times 2} = \frac{1 + \sqrt{9}}{4} = \frac{4}{4} = 1$$

$$k = \frac{-(-1) + \sqrt{(-1)^2 - 4 \times 2 \times (-1)}}{2 \times 2} = \frac{1 - \sqrt{9}}{4} = \frac{2}{4} = \frac{-1}{2}$$

∴ the value of k are 1 and $\frac{-1}{2}$

$$(x) kx^2 + kx + 1 = -4x^2 - x$$

Sol:

The given equation is $kx^2 + kx + 1 = -4x^2 - x$ bringing all the 'x' components to one side, we get the equation as $x^2(4+k) + x(k+1) + 1 = 0$

This equation is in the form of the general quadratic equation i.e., $ax^2 + bx + c = 0$

Here $a = 4+k, b = k+1$ and $c = 1$

⇒ Given that the nature of the roots of the given equation are real and equal

i.e., $D = b^2 - 4ac = 0$

$$\Rightarrow (k+1)^2 - 4x(4+k) \times 1 = 0$$

$$\Rightarrow k^2 + 1 + 2k - 16 - 4k = 0$$

$$\Rightarrow k^2 - 2k - 10 = 0 \quad \dots \dots \dots (2)$$

The equation (2) is as of the form $ax^2 + bx + c$ here $a = 1, b = -2, c = -16 + 1 = -15$

The value of k is obtained by
$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow k = \frac{-(-2) + \sqrt{(-2)^2 - 4 \times 1 \times -15}}{2 \times 1} = 5$$

$$\Rightarrow k = \frac{-(-2) + \sqrt{(-2)^2 - 4 \times 1 \times -15}}{2 \times 1} = -3$$

∴ The value of k are 5 and -3 respectively for the given quadratic equation.

$$(xi) (k+1)x^2 + 20 = (k+3)x + (k+8) = 0$$

Sol:

The given equation is $(k+1)x^2 + 2(k+3)x + (k+8) = 0$

Here $a = k+1, b = 2(k+3)$ and $c = k+8$

⇒ given that the nature of the roots of this equation are real and equal i.e.,

$$D = b^2 - 4ac = 0$$

$$\begin{aligned}
 &\Rightarrow [2(K+3)]^2 - 4 \times (k+1) \times (k+8) = 0 \\
 &\Rightarrow 4(k+3)^2 - 4(k+1) \times (k+8) = 0 \\
 &\Rightarrow (k+3)^2 - (k+1)(k+8) = 0 \\
 &\Rightarrow k^2 + 9 + 6k - [k^2 + 9k + 8] = 0 \\
 &\Rightarrow k^2 + 9 + 6k - k^2 - 9k - 8 = 0 \\
 &\Rightarrow -3k + 1 = 0 \Rightarrow k = \frac{1}{3} \\
 \therefore \text{The value of } 'k' \text{ for the given equation is } \frac{1}{3}
 \end{aligned}$$

(xii) $x^2 - 2kx + 7k - 12 = 0$

Sol:

The given equation is $x^2 - 2kx + 7k - 12 = 0$

Here $a = 1, b = -2k$ and $c = 7k - 12$

\Rightarrow given that the nature of the roots of this equation are real and equal i.e.,

$$D = b^2 - 4ac = 0$$

$$\Rightarrow (-2k)^2 - 4 \times 1 \times (7k - 12) = 0$$

$$\Rightarrow 4k^2 - 28k + 48 = 0$$

$$\Rightarrow k^2 - 7k + 12 = 0$$

The value of k can be obtained by
$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1, b = -7, c = 12 \Rightarrow k = \frac{-(-7) \pm \sqrt{49 - 48}}{2} = \frac{7 \pm \sqrt{1}}{2} = 4, 3$$

\therefore The value of 'k' for the equation is 4 and 3

(xiii) $(k+1)x^2 - 2(3k+1)x + 8k + 1 = 0$

Sol:

The given equation is $(k+1)x^2 - 2(3k+1)x + 8k + 1 = 0$

It is in the form of the equation $ax^2 + bx + c = 0$

Here, $a = k+1, b = -2(3k+1)$ and $c = 8k+1$

\Rightarrow given that the nature of the roots of the given equation are real and equal i.e.,

$$D = b^2 - 4ac = 0$$

$$\Rightarrow [-2(3k+1)]^2 - 4 \times (k+1)x(8k+1) = 0$$

$$\Rightarrow 4(3k+1)^2 - 4(k+1)(8k+1) = 0$$

$$\Rightarrow (3k+1)^2 - (k+1)(8k+1) = 0$$

$$\Rightarrow 9k^2 + 6k + 1 - [8k^2 + 9k + 1] = 0$$

$$\Rightarrow 9k^2 + 6k + 1 - 8k^2 - 9k - 1 = 0$$

$$\Rightarrow k^2 - 3k = 0$$

$$\Rightarrow k(k-3) = 0$$

$$\therefore k = 0 \text{ or } k = 3$$

\therefore The values of 'k' for the given quadratic equation are 0 and 3

$$(xiv) 5x^2 - 4x + 2 + k(4x^2 - 2x + 1) = 0$$

Sol:

$$\text{The given equation is } 5x^2 - 4x + 2 + k(4x^2 - 2x + 1) = 0$$

$$\text{This can be written as } x^2[5+4k] - x[4+2k] + 2 - k = 0$$

$$\text{This equation is in the form of } ax^2 + bx + c = 0 \quad \dots \dots \dots (1)$$

$$\text{Here } a = 5+4k, b = -(4+2k) \text{ and } c = -2k$$

\Rightarrow given that the nature of the roots of this equation are real and equal i.e.,

$$D = b^2 - 4ac = 0$$

$$\Rightarrow [-(4+2k)]^2 - 4(5+4k)(2-k) = 0$$

$$\Rightarrow (4+2k)^2 - 4(5+4k)(2-k) = 0$$

$$\Rightarrow 16 + 4k^2 + 16k - 4[10 - 5k + 8k - 4k^2] = 0$$

$$\Rightarrow 16 + 4k^2 + 16k - 40 + 20k - 32k + 16k^2 = 0$$

$$\Rightarrow 20k^2 - 4k - 24 = 0$$

$$5k^2 - k - 6 = 0 \quad \dots \dots \dots (2)$$

As equation (2) is of the form (1), k can be obtained

$$\text{By } k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ where } a = 5, b = -1, c = -6$$

$$\Rightarrow k = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) - \sqrt{1 - 4 \times 5 \times -6}}{2 \times 5} = +\frac{6}{5}$$

$$\Rightarrow k = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) - \sqrt{1 - 4 \times 5 \times -6}}{2 \times 5} = -1$$

The values of k for the given equation are $+\frac{6}{5}$ and -1

$$(xv) (4-k)x^2 + (2k+4)x + (8k+1) = 0$$

Sol:

$$\text{The given equation is } (4-k)x^2 + (2k+4)x + (8k+1) = 0$$

This equation is in the form of $ax^2 + bx + c = 0$

Here $a = 4-k$, $b = 2k+4$ and $c = 8k+1$

\Rightarrow given that the nature of the roots of this equation are real and equal i.e.,

$$D = b^2 - 4ac = 0$$

$$\Rightarrow (2k+4)^2 - 4(4-k)(8k+1) = 0$$

$$\Rightarrow 4k^2 + 16 + 16k - 4[-8k^2 + 32k + 4 - k] = 0$$

$$\Rightarrow 4k^2 + 16 + 16k + (8k^2 \times 4) - (31 \times 4)k - 16 = 0$$

$$\Rightarrow 4k^2 + 16 + 16k + 32k^2 - 124k - 16 = 0$$

$$\Rightarrow 36k^2 - 108k = 0$$

$$\Rightarrow 9k^2 - 27k = 0$$

$$\Rightarrow k^2 - 3k = 0$$

$$\Rightarrow k(k-3) = 0$$

Hence $k = 0$ or $k = 3$

\therefore The value of 'k' for the given quadratic equation is 0 and 3

$$(xvi) (2k+1)x^2 + 2(k+3)x + (k+5) = 0$$

Sol:

$$\text{The given equation is } (2k+1)x^2 + 2(k+3)x + (k+5) = 0$$

This equation is in the form of $ax^2 + bx + c = 0$

Here $a = 2k+1$, $b = 2(k+3)$ and $c = k+5$

\Rightarrow given that the nature of the roots for this equation are real and equal i.e.,

$$D = b^2 - 4ac = 0$$

$$\Rightarrow [2(k+3)]^2 - 4[2k+1][k+5] = 0$$

$$\Rightarrow (k+3)^2 - (2k+1)(k+5) = 0$$

$$\Rightarrow k^2 + 9 + 6k - [2k^2 + 11k + 5] = 0$$

$$\Rightarrow -k^2 - 5k + 4 = 0$$

$$\Rightarrow k^2 + 5k - 4 = 0 \quad \dots\dots\dots(2)$$

$$\Rightarrow k^2 + 4k + k - 4 = 0$$

The value of 'k' can be obtained by

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where from (2), $a = 1, b = 5, c = -4$

$$k = \frac{-5 \pm \sqrt{25 - 4 \times 1 \times -4}}{2 \times 1} = \frac{-5 \pm \sqrt{25 + 16}}{2} = \frac{-5 \pm \sqrt{41}}{2}$$

$$k = \frac{-5 - \sqrt{25 - 4 \times 1 \times -4}}{2 \times 1} = \frac{-5 - \sqrt{25 + 16}}{2} = \frac{-5 - \sqrt{41}}{2}$$

\therefore The value of 'k' from the given equation are $\frac{-5 \pm \sqrt{41}}{2}$

$$(vii) 4x^2 - 2(k + 1)x + (k + 4) = 0$$

Sol:

The given equation is $4x^2 - 2(k + 1)x + (k + 4) = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here $a = 4, b = -2(k + 1), c = k + 4$

\Rightarrow Given that the nature of the roots of this equation is real and equal i.e. $0 = b^2 - 4ac = 0$

$$\Rightarrow [-2(k + 1)]^2 - 4x + x(k + 4) = 0$$

$$\Rightarrow 4(k + 1)^2 - 16(k + 4) = 0$$

$$\Rightarrow (k + 1)^2 - 4(k + 4) = 0$$

$$\Rightarrow k^2 + 1 + 2k - 4k - 16 = 0$$

$$\Rightarrow k^2 - 2k - 15 = 0$$

The value of 'k' can be obtained by the formula

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ where } a = 1, b = -2, c = -15$$

$$\Rightarrow k = \frac{-(-2) \pm \sqrt{4 - 4 \times 1 \times -15}}{2 \times 1} = \frac{2 \pm \sqrt{69}}{2 \times 1} = k = -3$$

\therefore The value of 'k' for the given equation are 5 and -3

$$(xviii) x^2 - 2(k + 1)x + (k + 4) = 0$$

Sol:

The given equation is $x^2 - 2(k + 1)x + (k + 4) = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here $a = 1, b = -2(k + 1)$ and $c = k + 4$

\Rightarrow The nature of the roots of this equation is given that it is real and equal

i.e., $0 = b^2 - 4ac = 0$

$$\Rightarrow [-2(k + 1)]^2 - 4 \times 1 \times k + 4 = 0$$

$$\Rightarrow 4(k + 1)^2 - 4(k + 4) = 0$$

$$\Rightarrow 4(k^2 + 1 + 2k) - 4k - 16 = 0$$

(xxi) $2x^2 + kx + 3 = 0$

Sol:

The given equation is $2x^2 + kx + 3 = 0$

It is in the form of the equation $ax^2 + bx + c = 0$

Here $a = 2$, $b = k$, and $c = 3$

\Rightarrow Given that the roots of the equation are real and equal i.e., $D = b^2 - 4ac = 0$

$$\Rightarrow k^2 - 4 \times 2 \times 3 = 0$$

$$\Rightarrow k^2 = 24$$

$$\Rightarrow k = \sqrt{24} = \pm 2\sqrt{6}$$

\therefore The value of k for the given equation is $\pm 2\sqrt{6}$

(xxii) $kx(x - 2) + 6 = 0$

Sol:

The given equation is $kx^2 - 2kx + 6 = 0$

$a = 6$, $b = -2k$, $c = 6$

\Rightarrow Given that the roots are real and equal

i.e., $D = b^2 - 4ac = 0 \Rightarrow 4k^2 - 4k \times 6 = 0$

$$\Rightarrow k^2 - 6k = 0$$

$$\Rightarrow k(k - 6) = 0$$

$$\Rightarrow k = 0 \text{ or } 6$$

\therefore The value of k for the given equation is 0 or 6

(xxiii) $x^2 - 4kx + k = 0$

Sol:

The given equation is $x^2 - 4kx + k = 0$

$a = 1$, $b = -4k$, $c = k$

\Rightarrow Given that the roots are real and equal i.e., $D = b^2 - 4ac = 0$

$$\Rightarrow 16k^2 - 4k = 0$$

$$\Rightarrow 4k^2 - k = 0$$

$$\Rightarrow k(4k + 1) = 0$$

$$k = 0, k = \frac{1}{4}$$

\therefore The value of k for the given equation is 0 or $\frac{1}{4}$

3. In the following determine the set of values of k for which the given quadratic equation has real roots:

(i) $2x^2 + 3x + k = 0$ (ii) $2x^2 + kx + 3 = 0$ (iii) $2x^2 - 5x - k = 0$ (iv) $kx^2 + 6x + 1 = 0$

(v) $x^2 - kx + 9 = 0$

Sol:

$$(i) 2x^2 + 3x + k = 0$$

Sol:

The given equation is $2x^2 + 3x + k = 0$

\Rightarrow given that the quadratic equation has real roots i.e., $D = b^2 - 4ac \geq 0$

Given here $a = 2, b = 3, c = k$

$$\Rightarrow 9 - 4 \times 2 \times k \geq 0$$

$$\Rightarrow 9 - 8k \geq 0$$

$$\Rightarrow 9 \geq 8k \Rightarrow k \leq \frac{9}{8}$$

The value of k does not exceed $\frac{9}{8}$ to have roots

$$(ii) 2x^2 + kx + 3 = 0$$

Sol:

The given equation is $2x^2 + kx + 3 = 0$

\Rightarrow given that the quadratic equation has real roots i.e., $D = b^2 - 4ac \geq 0$

here $a = 2, b = k, c = 3$

$$\Rightarrow k^2 - 4 \times 2 \times 3 \geq 0$$

$$\Rightarrow k^2 - 24 \geq 0$$

$$\Rightarrow k^2 \geq 24$$

$$\Rightarrow k \geq \sqrt{24} \Rightarrow k \geq \pm 2\sqrt{6} \text{ or } k \leq -2\sqrt{6}$$

\therefore The value of k does not exceed $2\sqrt{6}$ and $-2\sqrt{6}$ to have real roots

$$(iii) 2x^2 - 5x - k = 0$$

Sol:

The given equation is $2x^2 - 5x - k = 0$

\Rightarrow given that the equation has real roots i.e., $D = b^2 - 4ac \geq 0$

$$\Rightarrow 25 - 4 \times 2 \times -k \geq 0$$

$$\Rightarrow 25 + 8k \geq 0$$

$$\Rightarrow 8k \geq -25$$

The value of k should not exceed $\frac{25}{8}$ to have real roots.

$$(iv) kx^2 + 6x + 1 = 0$$

Sol:

The given equation is $kx^2 + 6x + 1 = 0$

Here $a = k, b = 6, c = 1$

\Rightarrow given that the equation has real roots

$$\text{i.e., } D = b^2 - 4ac \geq 0$$

$$\Rightarrow 36 - 4 \times k \times 1 \geq 0$$

$$\Rightarrow 36 \geq 4k$$

$$\Rightarrow k \geq \frac{36}{7}$$

$$\Rightarrow k \leq 9$$

The value of k should not exceed the value '9' to have real roots.

$$(v) x^2 - kx + 9 = 0$$

Sol:

The given equation is $x^2 - kx + 9 = 0$

Here $a = 1, b = -k, c = 9$

\Rightarrow given that the equation has real roots

$$\text{i.e., } D = b^2 - 4ac \geq 0$$

$$\Rightarrow (-k)^2 - 4 \times 1 \times 9 \geq 0$$

$$\Rightarrow k^2 - 36 \geq 0$$

$$\Rightarrow k^2 \geq 36$$

$$\Rightarrow [k \geq 6] \text{ or } [k \leq -6]$$

The 'k' value exists between -6 and 6 to have the real roots for the given equation.

$$(vi) 2x^2 + kx + 2 = 0$$

Sol:

The given equation is $2x^2 + kx + 2 = 0$

Here $a = 2, b = k, c = 2$

\Rightarrow Given that the equation has real roots

$$\text{i.e., } D = b^2 - 4ac \geq 0$$

$$\Rightarrow k^2 - 4 \times 2 \times 2 \geq 0$$

$$\Rightarrow k^2 - 16 \geq 0$$

$$\Rightarrow k \geq 16$$

$$\Rightarrow k \geq \sqrt{16}$$

$$\Rightarrow k \geq 4 \text{ or } k \leq -4$$

\therefore The k value lies between -4 and 4 to have the real roots for the given equation.

$$(vii) 3x^2 + 2x + k = 0$$

Sol:

The given equation has $3x^2 + 2x + k = 0$

Here $a = 3, b = 2, c = k$

\Rightarrow Given that the quadratic equation has real roots i.e., $D = b^2 - 4ac \geq 0$

$$\Rightarrow 4 - 4 \times 3 \times k \geq 0$$

$$\Rightarrow 4 - 12k \geq 0$$

$$\Rightarrow k \leq \frac{4}{12}$$

$$\Rightarrow k \leq \frac{1}{3}$$

The 'k' value should not exceed $\frac{1}{3}$ to have the real roots for the given equation

$$(viii) 4x^2 - 3k + 1 = 0$$

Sol:

The given equation has $4x^2 - 3k + 1 = 0$

Here $a = 4, b = -3k, c = 1$

\Rightarrow Given that that quadratic equation has real roots i.e., $D = b^2 - 4ac \geq 0$

$$= 9k^2 - 16 \geq 0$$

$$\Rightarrow 9k^2 \geq 16 = k^2 \geq \frac{16}{9}$$

$$\Rightarrow k \geq \sqrt{\frac{16}{9}} \Rightarrow k \geq \frac{4}{3} \text{ or } k \leq -\frac{4}{3}$$

\therefore The value of k should be in between $-\frac{4}{3}$ and $\frac{4}{3}$ to have real roots for the given equation.

$$(ix) 2x^2 + kx - 4 = 0$$

Sol:

The given equation is $2x^2 + kx - 4 = 0$

Here $a = 2, b = k, c = -4$

\Rightarrow Given that the quadratic equation has real roots i.e., $D = b^2 - 4ac \geq 0$

$$\Rightarrow k^2 + 32 \geq 0$$

$$\Rightarrow k \leq \sqrt{32}$$

$$\Rightarrow k \in \mathbb{R}$$

\therefore The $k \in \mathbb{R}$ for the equation to have the real roots

4. For what value of $k, (4-k)x^2 + (2k+4)x + (8k+1) = 0$, is a perfect square

Sol:

The given equation is $(4-k)x^2 + (2k+4)x + (8k+1) = 0$,

Here $a = 4 - k, b = 2k + 4, c = 8k + 1$

The discriminant $D = b^2 - 4ac$

$$= (2k+4)^2 - 4 \times (4-k)(8k+1)$$

$$\Rightarrow 4k^2 + 16 + 4k - 4[32k + 4 - 8k^2 - k]$$

$$\Rightarrow [4k^2 + 8k^2 + 4k - 31k + 4 - 4]$$

$$\Rightarrow 4[9k^2 - 27k]$$

$$\Rightarrow D = 4[9k^2 - 27k]$$

The given equation is a perfect square

$$D = 0$$

$$\Rightarrow 4[9k^2 - 27k] = 0$$

$$\Rightarrow 9k^2 - 27k = 0$$

$$\Rightarrow 3k^2 - 9k = 0$$

$$\Rightarrow k^2 - 3k = 0$$

$$\Rightarrow k(k - 3) = 0$$

$$\therefore k = 0 \text{ or } k = 3$$

\therefore The value of k is '0' or '3' for the equation to be a perfect square

5. Find the least positive value of k for which the equation $x^2 + kx + 4 = 0$ has real roots

Sol:

The given equation is $x^2 + kx + 4 = 0$

\Rightarrow given that the equation has real roots

$$\text{i.e., } D = b^2 - 4ac \geq 0$$

$$\Rightarrow k^2 - 4 \times 1 \times 4 \geq 0$$

$$\Rightarrow k^2 - 16 \geq 0$$

$$\Rightarrow k \geq 4 \text{ or } k \leq -4$$

\therefore The least positive value of $k = 4$, for the equation to have real roots

6. Find the value of k for which the given quadratic equation has real and distinct roots

$$(i) kx^2 + 2x + 1 = 0 \quad (ii) kx^2 + 6x + 1 = 0 \quad (iii) x^2 - kx + 9 = 0$$

Sol:

$$(i) kx^2 + 2x + 1 = 0$$

The given equation is $kx^2 + 2x + 1 = 0$

Here $a = k, b = 2, c = 1$

\Rightarrow given that the equation has real and distinct roots

$$\text{i.e., } D = b^2 - 4ac \geq 0$$

$$\Rightarrow 4 - 4 \times 1 \times k \geq 0$$

$$\Rightarrow 4 - 4k \geq 0 \Rightarrow 4k \leq 4$$

$$\Rightarrow 4 \leq \frac{4}{k}$$

\therefore The value of k is 1 i.e., $k < 1$ for which the quadratic equation has real and distinct roots

$$(ii) kx^2 + 6x + 1 = 0$$

The given equation is $kx^2 + 6x + 1 = 0$

Here $a = k, b = 6, c = 1$

\Rightarrow given that the equation has real and distinct roots

Hence $D = b^2 - 4ac \geq 0$

$$\Rightarrow 36 - 4 \times 1 \times k \geq 0$$

$$\Rightarrow 36 - 4k \geq 0$$

$$\Rightarrow 4k \leq 36$$

$$\Rightarrow k \leq \frac{36}{4} \Rightarrow k > 9$$

$\therefore k < 9$ for the equation to have real and distinct roots

$$(iii) x^2 - kx + 9 = 0$$

The given equation is $x^2 - kx + 9 = 0$

Here $a = 1, b = -k, c = 9$

\Rightarrow given that the equation is having real and distinct roots

Hence $D = b^2 - 4ac \geq 0$

$$\Rightarrow k^2 - 4 \times 1 \times 9 \geq 0$$

$$\Rightarrow k^2 - 36 \geq 0$$

$$\Rightarrow k^2 \geq 36$$

$$\Rightarrow k \geq 6 \text{ or } k \leq -6$$

\therefore The value of "k" lies in between -6 and 6 to have the real roots for the given equation

7. If the roots of the equation $(b-c)x^2 + (c-a)x + (a-b) = 0$ are equal, then prove that

$$2b = a + c$$

Sol:

$$\text{The given equation is } (b-c)x^2 + (c-a)x + (a-b) = 0$$

This equation has the general form i.e., $ax^2 + bx + c = 0$

Here $a = b-c, b = c-a$ and $c = a-b$

\Rightarrow given that the equation has real and equal roots

Hence $b^2 - 4ac = D = 0$

$$\Rightarrow (c-a)^2 - 4 \times (b-c)(a-b) = 0$$

$$\Rightarrow c^2 + a^2 - 2ac = 4[ab - b^2 - ac + cb] = 0$$

$$\Rightarrow c^2 + a^2 - 2ac - 4ab + 4b^2 + 4ac - 4cb = 0$$

$$\Rightarrow c^2 + a^2 + 2ac - 4ab + 4b^2 - 4cb = 0$$

$$\Rightarrow (a+c)^2 - 4ab + 4b^2 + 4cb = 0$$

$$\Rightarrow (c+a-2b)^2 = 0$$

$$\Rightarrow c+a-2b = 0$$

$$\Rightarrow c+a = 2b$$

Hence, it is proved that $c+a = 2b$

8. If the roots of the equation $(a^2 + b^2)x^2 - 2(x + bd)x + (c^2 + d^2) = 0$ are equal, prove that

$$\frac{a}{b} = \frac{c}{d}.$$

Sol:

$$\text{The given equation is } (a^2 + b^2)x^2 - 2(x + bd)x + (c^2 + d^2) = 0$$

This equation has the general form $ax^2 + bx + c = 0$

$$\text{Here } a = a^2 + b^2, b = -2(x + bd), c = c^2 + d^2 = 0$$

\Rightarrow given here that the nature of the real and equal

$$\text{i.e., } D = b^2 - 4ac \geq 0$$

$$\Rightarrow [-2(x + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2) = 0$$

$$\Rightarrow 4(x + bd)^2 - 4(a^2 + b^2)(c^2 + d^2) = 0$$

$$\Rightarrow (x + bd)^2 - (a^2 + b^2)(c^2 + d^2) = 0$$

$$\Rightarrow a^2c^2 + b^2d^2 + 2abcd - [a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2] = 0$$

$$\Rightarrow a^2c^2 + 2^2d^2 + 2abcd - [a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2] = 0$$

$$\Rightarrow 2abcd - a^2d^2 - b^2c^2 = 0$$

$$\Rightarrow abcd + abcd - a^2d^2 - b^2c^2 = 0$$

$$\Rightarrow ad(bc - ad) + bc(ad - bc) = 0$$

$$\Rightarrow (ad - bc)(bc - ad) = 0$$

Case i:

$$\Rightarrow ab - bc = 0$$

$$\Rightarrow ad = bc$$

$$\Rightarrow \boxed{\frac{a}{b} = \frac{c}{d}}$$

Case ii:

$$\Rightarrow (bc - ad) = 0$$

$$\Rightarrow bc = ad$$

$$\Rightarrow \boxed{\frac{a}{b} = \frac{c}{d}}$$

∴ Hence, it is proved that $\frac{a}{b} = \frac{c}{d}$

9. If the roots the equation $ax^2 + 2bx + c = 0$ and $bx^2 - 2\sqrt{c}ax + b = 0$ are simultaneously real, then prove that $b^2 - ac$

Sol:

Given equations are $ax^2 + 2bx + c = 0$ and $bx^2 - 2\sqrt{c}ax + b = 0$

Then two equations are of the form $ax^2 + bx + c = 0$

\Rightarrow given that the roots of these two equations are real. Hence $D \geq 0$ i.e., $\boxed{b^2 - 4ac \geq 0}$

Let us assume that $ax^2 + 2bx + c = 0$ be equations(1)

and $bx^2 - 2\sqrt{c}ax + b = 0$ be equation(2)

from equation (1) $\Rightarrow b^2 - 4ac \geq 0$

$\Rightarrow 4b^2 - 4ac \geq 0$ (3)

From equation (2) $\Rightarrow b^2 - 4ac \geq 0$

$\Rightarrow (-2\sqrt{c})^2 - 4b^2 \geq 0$ (4)

Given that the roots of (1) and (2) are simultaneously real hence equation (3) equation (4)

$$\Rightarrow 4b^2 - 4ac = 4ac - 4b^2$$

$$\Rightarrow 8ac = 8b^2$$

$$\Rightarrow \boxed{b^2 = ac}$$

∴ Hence, it is proved that $b^2 - ac$

10. If p, q are real and $p \neq q$, then show that the roots of the equation

$$(p-q)x^2 + 5(p+q)x - 2(p-q) = 0$$
 are real and unequal

Sol:

The given equation is $(p-q)x^2 + 5(p+q)x - 2(p-q) = 0$

\Rightarrow given p, q are real and $p \neq q$

The discriminant $\boxed{D = b^2 - 4ac}$

$$\Rightarrow [5(p+q)]^2 - 4 \times (p-q) \times (-2(p-q))$$

$$\Rightarrow 25(p+q)^2 + 8(p-q)^2$$

We know that the square of any integer is always positive i.e., greater than zero

Hence $D = b^2 - 4ac \geq 0$

As given that p, q are real and $p \neq q$

$$\therefore 25(p+q)^2 + 8(p-q)^2 > 0 \text{ i.e., } D > 0$$

\therefore The roots of this equation are real and unequal

11. If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ are equal, prove that either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$

Sol:

The given equation is $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here $a = c^2 - ab, b = -2(a^2 - bc)$ and $c = b^2 - ac$

\Rightarrow given that the roots of this equation are equal

Hence $D = 0$ i.e., $b^2 - 4ac = 0$

$$\Rightarrow [-2(a^2 - bc)]^2 - 4(c^2 - ab)(b^2 - ac) = 0$$

$$\Rightarrow 4(a^2 - bc)^2 - 4(c^2 - ab)(b^2 - ac) = 0$$

$$\Rightarrow 4a(a^3 + b^3 + c^3 - 3abc) = 0$$

$$\Rightarrow 4a = 0 \text{ or } a^3 + b^3 + c^3 - 3abc = 0$$

$$\Rightarrow a = 0 \text{ or } a^3 + b^3 + c^3 = 3abc$$

\therefore hence, it is proved

12. Show that the equation $2(a^2 + b^2)x^2 + 2(a+b)x + 1 = 0$ has no real roots, when $a \neq b$.

Sol:

The given equation is $2(a^2 + b^2)x^2 + 2(a+b)x + 1 = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here $a = 2(a^2 + b^2), b = 2(a+b)$ and $c = 1$

\Rightarrow given that $a \neq b$

The discriminant $D = b^2 - 4ac$

$$\Rightarrow [2(a+b)]^2 - 4 \times 2(a^2 + b^2) \times (1)$$

$$\Rightarrow 4(a+b)^2 - 8(a^2 + b^2)$$

$$\Rightarrow 4[a^2 + b^2 + 2ab] - 8a^2 - 8b^2$$

$$\Rightarrow -4a^2 - 4b^2 + 2ab$$

As given that $a \neq b$, as the discriminant 0 has negative squares, 0 will be less than zero

Hence $0 < 0$, when $a \neq b$

13. Prove that both the roots of the equation $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ are real but they are equal only when $a=b=c$

Sol:

The given equation is $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$

By solving the equation, we get it as

$$3x^2 - 2x(a+b+c) + (ab+bc+ca) = 0$$

This equation is in the form of $ax^2 + bx + c = 0$

Here $a = 3, b = -2(a+b+c)$ and $c = ab+bc+ca$

The discriminant $D = b^2 - 4ac$

$$D = (-2a+b+c)^2 - 4(3)(ab+bc+ca)$$

$$\Rightarrow 4(a+b+c)^2 - 12(ab+bc+ca)$$

$$\Rightarrow 4[(a+b+c)^2 - 3(ab+bc+ca)]$$

$$\Rightarrow 4[a^2 + b^2 + c^2 - ab - bc - ca]$$

$$\Rightarrow 2[2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca]$$

$$\Rightarrow 2[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

Here clearly $0 \geq 0$, if $0 = 0$ then

$$(a-b)^2 + (b-c)^2 + (c-a)^2 = 0$$

$$\Rightarrow a-b=0, b-c=0, c-a=0$$

Hence $a=b=c$

Hence, it is proved.

14. If a, b, c are real numbers such that $ac \neq 0$, then show that at least one of the equations $ax^2 + bx + c = 0$ and $-ax^2 + bx + c = 0$ has real roots

Sol:

The given equations are $ax^2 + bx + c = 0$ (1)

And $-ax^2 + bx + c = 0$ (2)

Given equations are of the form $ax^2 + bx + c = 0$ also given that a, b, c are real numbers and $ac \neq 0$

The discriminant $D = b^2 - 4ac$

For equation (1) $\Rightarrow b^2 - 4ac$ (3)

For equation (2) $\Rightarrow b^2 - 4(-a) \times (c)$

$$\Rightarrow b^2 + 4ac \quad \dots \dots \dots (4)$$

As a, b, c are real and given that $ac \neq 0$ hence $b^2 - 4ac > 0$ and $b^2 + 4ac > 0$

$$\therefore 0 > 0$$

Exercise 8.7

- Find the consecutive numbers whose squares have the sum 85.

Sol:

Let the two consecutive natural numbers be 'x' and 'x + 1'

\Rightarrow Given that the sum of their squares is 85.

Then by hypothesis, we get

$$x^2 + (x + 1)^2 = 85$$

$$\Rightarrow x^2 + x^2 + 2x + 1 = 85$$

$$\Rightarrow 2x^2 + 2x + 1 - 85 = 0$$

$$\Rightarrow 2x^2 + 2x + 84 = 0 \Rightarrow 2[x^2 + x - 42] = 0$$

$$\Rightarrow x^2 + 7x - 6x - 42 = 0 \quad [\text{by the method of factorisation}]$$

$$\Rightarrow x(x + 7) - 6(x + 7) = 0$$

$$\Rightarrow (x - 6)(x + 7) = 0$$

$$\Rightarrow x = 6 \text{ or } x = 7$$

Case i: if $x = 6$ $x + 1 = 6 + 1 = 7$

Case ii: If $x = 7$ $x + 1 = -7 + 1 = -6$

\therefore The consecutive numbers that the sum of their squares be 85 are 6, 7 and -6, -7.

- Divide 29 into two parts so that the sum of the squares of the parts is 425.

Sol:

Let the two parts be 'x' and $29 - x$

\Rightarrow Given that the sum of the squares of the parts is 425.

Then, by hypothesis, we have

$$\Rightarrow x^2 + (29 - x)^2 = 425$$

$$\Rightarrow 2x^2 - 58x + 841 - 425 = 0$$

$$\Rightarrow 2x^2 - 58x + 416 = 0$$

$$\Rightarrow 2[x^2 - 29x + 208] = 0$$

$$\Rightarrow x^2 - 29x + 208 = 0$$

$$\Rightarrow x^2 - 13x - 16x + 208 = 0 \quad [\text{By the method of factorisation}]$$

$$\Rightarrow x(x - 13) - 16(x - 13) = 0$$

$$\Rightarrow (x - 13)(x - 16) = 0$$

$$\Rightarrow x = 13 \text{ or } x = 16$$

Case i: If $x = 13$; $29 - x = 29 - 13 = 16$

Case ii: $x = 16$; $29 - x = 29 - 16 = 13$

\therefore The two parts that the sum of the squares of the parts is 425 are 13, 16.

3. Two squares have sides x cm and $(x + 4)$ cm. The sum of their areas is 656 cm^2 . Find the sides of the squares.

Sol:

The given sides of two squares = x cm and $(x + 4)$ cm

The sum of their areas = 656 cm^2 .

The area of the square = side \times side.

\therefore Area of the square = $x(x + 4) \text{ cm}^2$.

\Rightarrow Given that sum of the areas is 656 cm^2 .

Hence by hypothesis, we have

$$\Rightarrow x(x + 4) + x(x + 4) = 656$$

$$\Rightarrow 2x(x + 4) = 656$$

$$\Rightarrow x^2 + 4x = 328 \quad [\text{dividing both sides by 2}]$$

$$\Rightarrow x^2 + 4x - 328 = 0$$

$$\Rightarrow x^2 + 20x - 16x - 328 = 0 \quad [\because \text{By the method of factorisation}]$$

$$\Rightarrow x(x + 20) - 16(x + 20) = 0$$

$$\Rightarrow (x + 20)(x - 16) = 0 \Rightarrow x = -20 \text{ or } x = 16$$

Case i: If $x = 16$; $x + 4 = 20$

\therefore The sides of the squares are 16 cm and 20 cm.

Note: No negative value is considered as the sides will never be measured negatively.

4. The sum of two numbers is 48 and their product is 432. Find the numbers

Sol:

Given the sum of two numbers is 48

Let the two numbers be x and $48 - x$ also given their product is 432.

Hence $x(48 - x) = 432$

$$\Rightarrow 48x - x^2 = 432$$

$$\Rightarrow 48x - x^2 - 432 = 0$$

$$\Rightarrow x^2 - 48x + 432 = 0$$

$$\Rightarrow x^2 - 36x - 12x + 432 = 0 \quad [\text{By method of factorisation}]$$

$$\Rightarrow x(x - 36) - 12(x - 36) = 0$$

$$\Rightarrow (x - 36)(x - 12) = 0$$

$$\Rightarrow x = 36 \text{ or } x = 12$$

\therefore The two numbers are 12, 36.

5. If an integer is added to its square, the sum is 90. Find the integer with the help of quadratic equation.

Sol:

Let the integer be 'x'

Given that if an integer is added to its square, the sum is 90.

$$\Rightarrow x + x^2 = 90$$

$$\Rightarrow x + x^2 - 90 = 0$$

$$\Rightarrow x^2 + 10x - 9x - 90 = 0$$

$$\Rightarrow x(x + 10) - 9(x + 10) = 0$$

$$\Rightarrow x = -10 \text{ or } x = 9$$

∴ The value of an integer are -10 or 9.

6. Find the whole numbers which when decreased by 20 is equal to 69 times the reciprocal of the members.

Sol:

Let the whole number be x as it is decreased by 20 $\Rightarrow (x - 20) = 69 \cdot \left(\frac{1}{x}\right)$

$$\Rightarrow x \cdot 20 = 69 \cdot \left(\frac{1}{x}\right)$$

$$\Rightarrow x(x - 20) = 69$$

$$\Rightarrow x^2 - 20x - 69 = 0$$

$$\Rightarrow x^2 - 23 + 3x - 69 = 0$$

$$\Rightarrow x(x - 23) + 3(x - 23) = 0$$

$$\Rightarrow (x - 23)(x + 3) = 0$$

$$\Rightarrow x = 23; x = -3$$

As the whole numbers are always positive, $x = -3$ is not considered.

∴ The whole number $x = 23$.

7. Find the two consecutive natural numbers whose product is 20.

Sol:

Let the two consecutive natural numbers be 'x' and 'x + 2'

\Rightarrow Given that the product of the natural numbers is 20

Hence $\Rightarrow x(x + 1) = 20$

$$\Rightarrow x^2 + x = 20$$

$$\Rightarrow x^2 + x - 20 = 0$$

$$\Rightarrow x^2 + 5x - 4x - 20 = 0$$

$$\Rightarrow x(x + 5) - 4(x + 5) = 0$$

$$\Rightarrow x = -5 \text{ or } x = 4$$

Considering positive value of x as $x \in \mathbb{N}$

For $r = 4$, $x + 1 = 4 + 1 = 5$

∴ The two consecutive natural numbers are 4 as 5.

8. The sum of the squares of the two consecutive odd positive integers as 394. Find them.

Sol:

Let the consecutive odd positive integers are $2x - 1$ and $2x + 1$

Given that the sum of the squares is 394.

$$\Rightarrow (2x - 1)^2 + (2x + 1)^2 = 394$$

$$\Rightarrow 4x^2 + 1 - 4x + 4x^2 + 1 + 4x = 394$$

$$\Rightarrow 8x^2 + 2 = 394$$

$$\Rightarrow 4x^2 = 392$$

$$\Rightarrow x^2 = 36$$

$$\Rightarrow x = 6$$

$$\text{As } x = 6, 2x - 1 = 2 \times 6 - 1 = 11$$

$$2x + 1 = 2 \times 6 + 1 = 13$$

\therefore The two consecutive odd positive numbers are 11 and 13.

9. The sum of two numbers is 8 and 15 times the sum of their reciprocals is also 8. Find the numbers.

Sol:

Let the numbers be x and $8 - x$

Given that the sum of these numbers is 8

And 15 times the sum of their reciprocals as 8

$$\Rightarrow 15 \left(\frac{1}{x} + \frac{1}{8-x} \right) = 8$$

$$\Rightarrow 15 \left(\frac{(8-x)+x}{x(8-x)} \right) = 8$$

$$\Rightarrow 15 ((8-x) + x) = 8(x(8-x))$$

$$\Rightarrow 15 [8 - x + x] = 8x(8 - x)$$

$$\Rightarrow 120 = 64x - 8x^2$$

$$\Rightarrow 8x^2 - 64x + 120 = 0$$

$$\Rightarrow 8[x^2 - 8x + 15] = 0$$

$$\Rightarrow x^2 - 5x - 3x + 15 = 0$$

$$\Rightarrow (x - 5)(x - 3) = 0$$

$$\Rightarrow x = 5 \text{ or } x = 3$$

\therefore The two numbers are 5 and 3.

10. The sum of a numbers and its positive square root is $\frac{6}{25}$. Find the numbers.

Sol:

Let the number be x

By the hypothesis, we have

$$\Rightarrow x + \sqrt{x} = \frac{6}{25}$$

\Rightarrow let us assume that $x = y^2$, we get

$$\Rightarrow y^2 + y = \frac{6}{25}$$

$$\Rightarrow 25y^2 + 25y - 6 = 0$$

The value of 'y' can be obtained by $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Where $a = 25$, $b = 25$, $c = -6$

$$\Rightarrow y = \frac{-25 \pm \sqrt{625 - 600}}{50}$$

$$\Rightarrow y = \frac{-25 \pm 35}{50} \Rightarrow y = \frac{1}{5} \text{ or } \frac{-11}{10}$$

$$x = y^2 = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$$

$$\therefore \text{The number } x = \frac{1}{25}.$$

11. The sum of a number and its square is $63/4$. Find the numbers.

Sol:

Let the number be x .

Given that the sum of x and its square $= \frac{63}{4}$

$$\Rightarrow x + x^2 = \frac{63}{4}$$

$$\Rightarrow 4x + 4x^2 - 63 = 0$$

$$\Rightarrow 4x^2 + 4x - 63 = 0$$

$$\Rightarrow 4x^2 + 4x - 63 = 0 \quad \dots \text{(i)}$$

The value of x can be found by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

\Rightarrow here $a = 4$, $b = 4$ and $c = -63$ from (i)

$$x = \frac{-4 \pm \sqrt{16 - 4 \times 4 \times -63}}{2 \times 4}$$

$$= \frac{-4 \pm \sqrt{16 + 16 \times 63}}{2 \times 4}$$

$$x = \frac{-4 \pm \sqrt{16 + 1008}}{8} = \frac{7}{2}; x = \frac{-4 - \sqrt{16 + 1008}}{8} = \frac{-9}{2}$$

$$\therefore \text{The values of } x \text{ i. e., the numbers is } \frac{7}{2}, \frac{-9}{2}.$$

12. There are three consecutive integers such that the square of the first increased by the product of the first increased by the product of the others the two gives 154. What are the integers?

Sol:

Let the three consecutive numbers x , $x+1$ and $x+8$.

According to the hypothesis given

$$x^2 + (x+1)(x+2) = 154.$$

$$\Rightarrow x^2 + [x^2 + 3x + 2] = 154$$

$$\Rightarrow 2x^2 + 3x = 152$$

$$\Rightarrow 2x^2 + 3x - 152 = 0 \quad \dots(i)$$

The value of 'x' can be obtained by the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

here $a = 2, b = 3$ and $c = 152$ from (i)

$$x = \frac{-3 + \sqrt{9 - 4 \times 2 \times -152}}{2 \times 3}$$

$$x = \frac{-3 + \sqrt{9 + 8 \times 152}}{6} = 8, \frac{-19}{2}$$

considering the positive value of x

If $x = 8, x + 1 = 9, x + 2 = 10$

\therefore The three consecutive integers are 8, 9, and 10

13. The product of two successive integral multiples of 5 is 300. Determine the multiples.

Sol:

Given that the product of two successive integral multiples of 5 is 300.

Let the integers be $5x$, and $5(x + 1)$

Then, by the integers be $5x$ and $5(x + 1)$

Then, by the hypothesis, we have

$$5x \cdot 5(x + 1) = 300$$

$$\Rightarrow 25x(x + 1) = 300$$

$$\Rightarrow x^2 + x = 12$$

$$\Rightarrow x^2 + x - 12 = 0$$

$$\Rightarrow x^2 + 4x - 3x - 12 = 0$$

$$\Rightarrow x(x + 4) - 3(x + 4) = 0$$

$$\Rightarrow (x + 4)(x - 3) = 0$$

$$\Rightarrow x = -4 \text{ or } x = 3$$

If $x = -4, 5x = -20, 5(x + 1) = -15$

$x = 3, 5x = 15, 5(x + 1) = 20$

\therefore The two successive integral multiples are 15, 20 or $-15, -20$.

14. The sum of the squares of two numbers as 233 and one of the numbers as 3 less than twice the other number find the numbers.

Sol:

Let the number be x

Then the other number = $2x - 3$

According to the given hypothesis,

$$\Rightarrow x^2 + (2x - 3)^2 = 233$$

$$\Rightarrow x^2 + 4x^2 + 9 - 12x = 233$$

$$\Rightarrow 5x^2 - 12x - 224 = 0 \quad \dots(i)$$

The value of 'x' can be obtained by the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here $a = 5$, $b = 12$ and $c = -224$ from (i)

$$x = \frac{-(-12) + \sqrt{144 + 20 \times 224}}{10} = 8$$

$$x = \frac{-(-12) - \sqrt{144 + 20 \times 224}}{10} = \frac{-28}{5}$$

considering the value of $x = 8$

$$2x - 3 = 16 - 3 = 15$$

\therefore The two numbers are 8 and 15.

15. Find the consecutive even integers whose squares have the sum 340.

Sol:

Let the consecutive even integers be $2x$ and $2x + 2$.

Then according to the given hypothesis,

$$(2x)^2 + (2x + 2)^2 = 340$$

$$\Rightarrow 8x^2 + 8x - 336 = 0$$

$$\Rightarrow x^2 + x - 42 = 0$$

$$\Rightarrow x^2 + 7x - 6x - 42 = 0$$

$$\Rightarrow x(x + 7) - 6(x + 7) = 0$$

$$\Rightarrow (x + 7)(x - 6) = 0$$

$$\Rightarrow x = -7 \text{ or } x = 6$$

Considering, the positive integers of x .

$$\Rightarrow x = 6; 2x = 12 \text{ and } 2x + 2 = 14.$$

\therefore The two consecutive even integers are 12 and 14.

16. The difference of two numbers is 4. If the difference of their reciprocals is $\frac{4}{21}$. Find the numbers.

Sol:

Let the two numbers be x and $x - 4$

Given that the difference of two numbers is 4.

By the given hypothesis, we have $\frac{1}{x-4} - \frac{1}{x} = \frac{4}{21}$

$$\Rightarrow \frac{x-x+4}{x(x-4)} = \frac{4}{21}$$

$$\Rightarrow 84 = 4x(x - 4)$$

$$\Rightarrow x^2 - 4x - 21 = 0$$

$$\Rightarrow x^2 - 7x + 3x - 21 = 0$$

$$\Rightarrow x(x - 7) + 3(x - 7) = 0$$

$$\Rightarrow (x - 7)(x + 3) = 0$$

$$\Rightarrow x = 7 \text{ or } x = -3 \text{ and}$$

$$\text{If } x = -3, x - 4 = -3 - 4 = -7$$

Hence, required numbers are 3, 7 and $-3, -7$

17. Let us find two natural numbers which differ by 3 and whose squares have the sum 117.

Sol:

Let the numbers be x and $x - 3$

By the given hypothesis,

$$x^2 + (x - 3)^2 = 117$$

$$\Rightarrow x^2 + x^2 + 9 - 6x - 117 = 0$$

$$\Rightarrow 2x^2 - 6x - 108 = 0$$

$$\Rightarrow x^2 - 3x - 54 = 0$$

$$\Rightarrow x(x - 9) + 6(x - 9) = 0$$

$$\Rightarrow (x - 9)(x + 6) = 0$$

$$\Rightarrow x = 9 \text{ or } x = -6$$

Considering positive value of x

$$x = 9, x - 3 = 9 - 3 = 6$$

\therefore The two numbers be 9 and 6.

18. The sum of the squares of three consecutive natural numbers as 149. Find the numbers

Sol:

Let the numbers be $x, x + 1$ and $x + 2$ according to the given hypothesis.

$$x^2 + (x + 1)^2 + (x + 2)^2 = 149$$

$$\Rightarrow x^2 + x^2 + 1 + 2x + x^2 + 4 + 4x = 149$$

$$\Rightarrow 3x^2 + 6x + 5 - 149 = 0$$

$$\Rightarrow 3x^2 + x - 144 = 0$$

$$\Rightarrow x^2 + 2x - 48 = 0$$

$$\Rightarrow x(x + 8) - 6(x + 8) = 0$$

$$\Rightarrow (x + 8)(x - 6) = 0$$

$$\Rightarrow x = -8 \text{ or } x = 6$$

Considering the positive value of x

$$x = 6, x + 1 = 7 \text{ and } x + 2 = 8$$

\therefore The three consecutive numbers are 6, 7, 8.

19. Sum of two numbers is 16. The sum of their reciprocals is $\frac{1}{3}$. Find the numbers.

Sol:

Given that the sum of two numbers is 16.

Let the two numbers be x and $16 - x$

By the given hypothesis, we have

$$\Rightarrow \frac{1}{x} + \frac{1}{16-x} = \frac{1}{3}$$

$$\Rightarrow \frac{16-x+x}{x(16-x)} = \frac{1}{3}$$

$$\Rightarrow 48 = 16x - x^2$$

$$\Rightarrow x^2 - 16x + 48 = 0$$

$$\begin{aligned}
 &\Rightarrow x^2 - 12x - 4x + 48 = 0 \\
 &\Rightarrow x(x - 12) - 4(x - 12) = 0 \\
 &\Rightarrow (x - 12)(x - 4) = 0 \\
 &\Rightarrow x = 12 \text{ or } x = 4 \\
 &\therefore \text{The two numbers are 4 and 12.}
 \end{aligned}$$

20. Determine two consecutive multiples of 3, whose product is 270.

Sol:

Let the two consecutive multiples of 3 are $3x$ and $3x + 3$

Given that their product is 270

$$\begin{aligned}
 &\Rightarrow (3x)(3x + 3) = 270 \\
 &\Rightarrow x(3x + 3) = 90 \\
 &\Rightarrow x^2 + x - 30 = 0 \\
 &\Rightarrow x^2 + 6x - 5x - 30 = 0 \\
 &\Rightarrow x(x + 6) - 5(x + 6) = 0 \\
 &\Rightarrow (x + 6)(x - 5) = 0 \\
 &\Rightarrow x = 5 \text{ or } x = -6 \\
 &\text{Considering the positive value of } x. \\
 &\Rightarrow x = 5, 3x = 15 \text{ and } 3x + 3 = 18 \\
 &\therefore \text{The two consecutive multiples of 3 are 15 and 18.}
 \end{aligned}$$

21. The sum of a number and its reciprocal is $\frac{17}{4}$. Find the number.

Sol:

Let the number be 'x'

According to the given hypothesis

$$\begin{aligned}
 x + \frac{1}{x} &= \frac{17}{4} \\
 \Rightarrow \frac{x^2 + 1}{x} &= \frac{17}{4} \\
 \Rightarrow 4(x^2 + 1) &= 17x \\
 \Rightarrow 4x^2 - 17x + 4 &= 0 \\
 \Rightarrow 4x^2 - 16x - x + 4 &= 0 \\
 \Rightarrow 4x(x - 4) - 1(x - 4) &= 0 \\
 \Rightarrow x = \frac{1}{4} \text{ or } x &= 4 \\
 \therefore \text{The value of } x &= 4
 \end{aligned}$$

22. A two-digit number is such that the products of its digits is 8. When 18 is subtracted from the number, the digits interchange their places. Find the number?

Sol:

Let the two digits be x and $x - 2$

Given that the product of their digits is 8.

$$\Rightarrow x(x - 2) = 8$$

$$\Rightarrow x^2 - 2x - 8 = 0$$

$$\Rightarrow x^2 - 4x + 2x - 8 = 0$$

$$\Rightarrow x(x - 4) + 2(x - 4) = 0$$

$$\Rightarrow (x - 4)(x + 2) = 0$$

$$\Rightarrow x = 4 \text{ or } x = -2$$

Considering the positive value $x = 4$, $x - 2 = 2$.

\therefore The two digit number is 42.

23. A two digits number is such that the product of the digits is 12. When 36 is added to the number, the digits inter change their places determine the number

Sol:

Let the tens digit be x

$$\text{Then, the units digit} = \frac{12}{x}$$

$$\therefore \text{Number} = 10x + \frac{12}{x}$$

And, number obtained by interchanging the

$$\text{Digits} = 10 \times \frac{12}{x} + x = \frac{120}{x} + x.$$

$$\Rightarrow 10x + \frac{12}{x} + 36 = \frac{120}{x} + x$$

$$\Rightarrow 9x + \frac{12 - 120}{x} + 36 = 0$$

$$\Rightarrow 9x^2 - 108 + 36x = 0$$

$$\Rightarrow 9(x^2 + 4x - 12) = 0$$

$$\Rightarrow x^2 + 6x - 2x - 12 = 0$$

$$\Rightarrow x(x + 6) - 2(x + 6) = 0$$

$$\Rightarrow (x - 2)(x + 6) = 0 \therefore x = 2 \text{ or } -6$$

But, a digit can never be negative, $80x = 2$

$$\text{Hence, the digit} = 10 \times 2 + \frac{12}{2} = 20 + 6 = 26$$

24. A two digit number is such that the product of the digits is 16. When 54 is subtracted from the number the digits are interchanged. Find the number

Sol:

Let the two digits be:

$$\text{Tens digits be } x \text{ and units } = \frac{16}{x}$$

$$\text{Number} = 10x + \frac{16}{x}$$

$$\text{Number obtained by interchanging} = 10 \times \frac{16}{x} + x$$

$$\Rightarrow \left(10x + \frac{16}{x} \right) - \left(10 \times \frac{16}{x} + x \right) = 54$$

$$\Rightarrow 10x + \frac{16}{7} - \frac{160}{x} + x = 54$$

$$\Rightarrow 10x^2 + 16 - 160 + x^2 = 54x$$

$$\Rightarrow 9x^2 - 54x - 144 = 0$$

$$\Rightarrow x^2 - 6x - 16 = 0$$

$$\Rightarrow x^2 - 8x + 2x - 16 = 0$$

$$\Rightarrow x(x-8) + 2(x-8) = 0$$

$$\Rightarrow (x-8) or x = -2$$

But, a digit can never be negative, hence $x = 8$

$$\text{Hence the required number} = 10 \times 8 + \frac{16}{8} = 82$$

25. Two numbers differ by 3 and their product is 504. Find the number

Sol:

Let the two numbers be x and $x-3$ given that $x(x-3) = 504$

$$\Rightarrow x^2 - 3x - 504 = 0$$

$$\Rightarrow x^2 - 24x + 21x - 504 = 0$$

$$\Rightarrow x(x-24) + 21(x-24) = 0$$

$$\Rightarrow (x-24)(x+21) = 0$$

$$\Rightarrow x = 24 \text{ or } x = 21$$

Case 1: If $x = 24, x = 3 = 21$

Case 2: If $x = 21, x = 3 = 24$

\therefore The two numbers are 21, 24 or -21, -24

26. Two numbers differ by 4 and their product is 192. Find the numbers?

Sol: Let the two numbers be x and $x-4$

Given that their product is 192

$$\Rightarrow x(x-4) = 192$$

$$\Rightarrow x^2 - 4x - 192 = 0$$

$$\Rightarrow x^2 - 16x + 12x - 192 = 0$$

$$\Rightarrow x(x-16) + 12(x-16) = 0$$

$$\Rightarrow (x-16)(x+12) = 0$$

$$\Rightarrow x = 16 \text{ or } x = -12$$

Considering the positive value of x

$$x = 16, \Rightarrow x-4 = 16-4 = 12$$

\therefore The two numbers are 12, 16

27. A two-digit number is 4 times the sum of its digits and twice the product of its digits. Find the numbers

Sol:

Let the digits at tens and units place of the number be x and y respectively then, it is given that $= 10x + y$

$$\Rightarrow 10x + y = 4 \text{ (sum of digits) and } 2xy$$

$$\Rightarrow 10x + y = 4(x + y) \text{ and } 10x + y = 3xy$$

$$\Rightarrow 10x + y = 4x + 4y \text{ and } 10x + y = 3xy$$

$$\Rightarrow 6x - 3y = 0 \text{ and } 10x + y - 3xy = 0$$

$$\Rightarrow y = 2x \text{ and } 10x + 2x = 2xy(2x)$$

$$\Rightarrow 12x = 4x^2$$

$$\Rightarrow 4x^2 - 12x = 0$$

$$\Rightarrow 4x(x-3) = 0$$

$$\Rightarrow 4x = 0 \text{ or } x = 3$$

$$\Rightarrow \text{here we have } y = 2x \Rightarrow 2 \times 3 = 6$$

$$\therefore x = 3 \text{ and } y = 6$$

$$\text{Hence } 10x + y - 10 \times 3 + 6 = 36$$

\therefore The required two digit number is 36

28. The sum of the squares of two positive integers is 208. If the square of the large number is 18 times the smaller. Find the numbers

Sol:

Let the smaller number be x . Then square of a larger number $= 18x$

Also, square of the smaller number $= x^2$

It is given that the sum of the square of the integers is 208.

$$\therefore x^2 + 18x = 208$$

$$\Rightarrow x^2 + 18x - 208 = 0$$

$$\Rightarrow x^2 + 26x - 8x - 208 = 0$$

$$\Rightarrow (x+26)(x-8) = 0 \Rightarrow x = 8 \text{ or } x = -26$$

But, the numbers are positive. Therefore $x = 8$

\therefore square of the larger number $= 18x = 18 \times 8 = 144$

\Rightarrow larger number are 8 and 18.

29. The sum of two numbers is 18. The sum of their reciprocals is $\frac{1}{4}$. Find the numbers

Sol: Let The numbers be x and $18-x$

\Rightarrow according to the given hypothesis

$$\frac{1}{x} + \frac{1}{18-x} = \frac{1}{4}$$

$$\Rightarrow \frac{18-x+x}{x(18-x)} = \frac{1}{4}$$

$$\Rightarrow 72 = 18x - x^2$$

$$\Rightarrow x^2 - 18x - 72 = 0$$

$$\Rightarrow x^2 - 6x - 12x - 72 = 0$$

$$\Rightarrow x(x-6) - 12(x-12) = 0$$

$$\Rightarrow x = 6 \text{ or } x = 12$$

∴ The two numbers are 6, 12

30. The sum of two numbers a and b is 15. and the sum of their reciprocals $\frac{1}{a}$ and $\frac{1}{b}$ is $\frac{3}{10}$.

Find the numbers a and b .

Sol:

Let us assume a number 'x'

$$\text{Such that } \frac{1}{x} + \frac{1}{15-x} = \frac{3}{10}$$

$$\text{Hence } \Rightarrow \frac{15-x+x}{x(15-x)} = \frac{3}{10}$$

$$\Rightarrow 150 = 45x + 3x^2$$

$$\Rightarrow 3x^2 - 45x + 150 = 0$$

$$\Rightarrow x^2 - 15x + 50 = 0$$

$$\Rightarrow x^2 - 10x - 5x + 50 = 0$$

$$\Rightarrow x(x-10) - 5(x-10) = 0$$

$$\Rightarrow (x-10)(x-5) = 0$$

$$\Rightarrow x = 5 \text{ or } x = 10$$

Case i: If $x = a$, $a = 5$ and

$$b = 15 - x, b = 10.$$

Case ii: if $x = 15 + a = 15 + 10 = 5$ $x = a = 10$

$$b = 15 - 10 = 5$$

$$\therefore a=5, b=10 \text{ or } a=10 \text{ and } b=5$$

31. The sum of two numbers is 9. The sum of their reciprocals is $\frac{1}{2}$. Find the numbers.

Sol:

Given that the sum of two numbers is 9 Let the two numbers be x and $9-x$

By the given hypothesis, we have

$$\frac{1}{x} + \frac{1}{9-x} = \frac{1}{2}$$

$$\Rightarrow \frac{9-x+x}{x(9-x)} = \frac{1}{2}$$

$$\Rightarrow 18 = 9x - x^2$$

$$\Rightarrow x^2 - 9x + 18 = 0$$

$$\Rightarrow x^2 - 6x - 3x + 18 = 0$$

$$\Rightarrow x(x-6) - 3(x-6) = 0$$

$$\Rightarrow (x-6)(x-3) = 0$$

$$\Rightarrow x=6 \text{ or } x=3$$

\therefore The two numbers are 3 and 6

32. Three consecutive positive integers are such that the sum of the square of the first and the product of other two is 46. Find the integers.

Sol:

Let the three consecutive positive integers be x , $x+1$ and $x+2$

According to the hypothesis, we have

$$\Rightarrow x^2 + (x+1)(x+2) = 46$$

$$\Rightarrow x^2 + x^2 + 3x + 2 = 46$$

$$\Rightarrow 2x^2 + 3x - 44 = 0$$

$$\Rightarrow 2x^2 - 8x + 11x - 44 = 0$$

$$\Rightarrow 2x(x-4) + 11(x-4) = 0$$

$$\Rightarrow (2x+11)(x-4) = 0$$

$$\Rightarrow x = 4 \text{ or } x = -\frac{11}{2}$$

Considering the positive value of x

$$\Rightarrow x = 4, x + 1 = 5 \text{ and } x + 2 = 6$$

\therefore The three consecutive numbers are 4, 5 and 6.

33. The difference of squares of two numbers is 88. If the larger number is 5 less than twice the smaller number, then find the two numbers

Sol:

Let the smaller number be x . Then, larger number $= 2x - 5$

It is given that the difference of the square of the number is 88.

$$\Rightarrow (2x - 5)^2 - x^2 = 88$$

$$\Rightarrow 4x^2 + 25 - 20x - x^2 = 88$$

$$\Rightarrow 3x^2 - 20x - 63 = 0$$

$$\Rightarrow 3x^2 - 27x + 7x - 63 = 0$$

$$\Rightarrow 3x(x - 9) + 7(x - 9) = 0$$

$$\Rightarrow (x - 9)(3x + 7) = 0$$

$$\therefore x = 9 \text{ or } -\frac{7}{3}$$

As a digit can never be negative, $x = 9$

$$\Rightarrow \text{The numbers} = 2x - 5$$

$$= 2 \times 9 - 5 = 13$$

\therefore Hence, required numbers are 9 and 13

34. The difference of square of two numbers is 180 . the square of the smaller number is 8 times the large numbers find two numbers

Sol:

Let the number be x

By the given hypothesis, we have

$$x^2 - 8x = 180$$

$$\Rightarrow x^2 - 8x - 180 = 0$$

$$\Rightarrow x^2 + 10x + -18x - 180 = 0$$

$$\Rightarrow x(x+10) - 18(x+10) = 0$$

$$\Rightarrow (x+10)(x-18) = 0$$

$$\Rightarrow x = -10 \text{ or } x = 18$$

Case (i): $x = 18$

$$8x = 8 \times 18 = 144$$

$$\therefore \text{Larger number} = \sqrt{144} = \pm 12$$

Case (ii): $x = -10$

Square of larger number $8x = -80$ here no perfect square exist, hence the numbers are 18,12

Exercise 8.8

1. The speed of a boat in still water is 8 km/hr It can go 15 km upstream and 22 km downstream in 5 hours. Find the speed of the stream.

Sol:

Let the speed of the stream be x km/hr

Given that,

Speed of the boat in still water = 8 km/hr

Now,

Speed of the boat in upstream = $(8 - x)$ km/hr

And speed of the boat in downstream = $(8 + x)$ km/hr

$$\text{Time taken for going 15 km upstream} = \frac{15 \text{ km}}{(8-x) \text{ km/hr}} = \frac{15}{8-x} \text{ hours}$$

$$\text{Time taken for going 22 km downstream} = \frac{22 \text{ km}}{(8+x) \text{ km/hr}} = \frac{22}{8+x} \text{ hours}$$

Given that,

Time taken for upstream + downstream = 5 hours

$$\Rightarrow \frac{15}{8-x} \text{ hours} + \frac{22}{8+x} \text{ hours} = 5 \text{ hours}$$

$$\Rightarrow \frac{15}{8-x} + \frac{22}{8+x} = 5$$

$$\Rightarrow \frac{15(8+x) + 22(8-x)}{(8-x)(8+x)} = 5$$

$$\Rightarrow \frac{120 + 15x + 176 - 22x}{8^2 - x^2} = 5$$

$$\Rightarrow 296 - 7x = 5(64 - x^2)$$

$$\Rightarrow 296 - 7x = 320 - 5x^2$$

$$\begin{aligned}
 &\Rightarrow 5x^2 - 7x + 296 - 320 = 0 \\
 &\Rightarrow 5x^2 - 7x - 24 = 0 \quad [5 \times -24 = -120 \Rightarrow -180 = 8 \times -15 - 7 = -15 + 8] \\
 &\Rightarrow 5x^2 - (15 - 8)x - 24 = 0 \\
 &\Rightarrow 5x^2 - 15x + 8x - 24 = 0 \\
 &\Rightarrow 5x(x - 3) + 8(x - 3) = 0 \\
 &\Rightarrow (x - 3)(5x + 8) = 0 \\
 &\Rightarrow x - 3 = 0 \text{ or } 5x + 8 = 0 \\
 &\Rightarrow x = 3 \text{ or } x = \frac{-8}{5}
 \end{aligned}$$

Since, x cannot be a negative value So, $x = 3$

\therefore Speed of the stream is 3 km/hr

2. A passenger train takes 3 hours less for a journey of 360 km, if its speed is increased by 10 km/hr from its usual speed. What is the usual speed?

Sol:

Let the usual speed be x km/hr.

Distance covered in the journey = 360 km

Now,

$$\text{Time taken by the train with the usual speed} = \frac{360 \text{ km}}{x \text{ km/hr}} = \frac{360}{x} \text{ hr}$$

Given that if speed is increased by 10 km/hr, the same train takes 3 hours less.

$$\begin{aligned}
 &\Rightarrow \text{Speed of the train} = (x + 10) \text{ km/hr and time taken by the train after increasing the speed} \\
 &= \frac{360 \text{ km}}{(x+10) \text{ km/hr}} = \frac{360}{x+10} \text{ hr}
 \end{aligned}$$

3. A fast train takes one hour less than a slow train for a journey of 200 km. If the speed of the slow train is 10 km/hr less than that of the fast train, find the speed of the two trains.

Sol:

Let the speed of the slow train be x km/hr.

Given that speed of the slow train is 10 km/hr less than that of fast train

$$\Rightarrow \text{Speed of the fast train} = (x + 10) \text{ km/hr}$$

Total distance covered in the journey = 200 km

$$\text{Time taken by fast train} = \frac{200 \text{ km}}{(x+10) \text{ km/hr}} = \frac{200}{x+10} \text{ hr and}$$

$$\text{Time taken by slow train} = \frac{200 \text{ km}}{x \text{ km/hr}} = \frac{200}{x} \text{ hr}$$

Given that faster train takes 1 hour less than that of slow train

$$\text{i.e., } \frac{800}{x} - \frac{800}{x+10} = 1$$

$$\Rightarrow 800 \left(\frac{1}{x} - \frac{1}{x+10} \right) = 1$$

$$\Rightarrow 800 \left(\frac{x+10-x}{x(x+10)} \right) = 1$$

$$\Rightarrow 800 (10) = x (x + 10) = 1$$

$$\begin{aligned}
 \Rightarrow 8000 &= x^2 + 10x \\
 \Rightarrow x^2 + 10x - 2000 &= 0 \\
 \Rightarrow x^2 + (50 - 40)x + (50x - 40) &= 0 \\
 \Rightarrow (x^2 + 50x - 40x + (50x - 40)) &= 0 \\
 \Rightarrow x(x + 50) - 40(x + 50) &= 0 \\
 \Rightarrow (x + 50)(x - 40) &= 0 \\
 \Rightarrow x + 50 = 0 \text{ or } x - 40 &= 0 \\
 \Rightarrow x = -50 \text{ or } x &= 40
 \end{aligned}$$

Clearly x cannot be a negative volume since it is speed. So, $x = 40$

\therefore Speed of slow train is 40 km/hr

Now,

$$\text{Speed of fast train} = (x + 10) \text{ km/hr} = (40 + 10) \text{ km/hr} = 50 \text{ km/hr}$$

4. A passenger train takes one hour less for a journey of 150 km if its speed is increased by 5 km/hr from its usual speed. Find the usual speed of the train.

Sol:

Let the usual speed of the train be x km/hr

Distance covered in the journey = 150 km

$$\Rightarrow \text{Time taken by the train with usual speed} = \frac{150 \text{ km}}{x \text{ km/hr}} = \frac{150}{x} \text{ hr}$$

Given that, if the speed is increased by 5 km/hr from its usual speed, the train takes one hour less for the same journey.

$$\Rightarrow \text{Speed of the train} = (x + 5) \text{ km/hr}$$

$$\text{Now, time taken by the train after increasing the speed} = \frac{150 \text{ km}}{(x+5) \text{ km/hr}} = \frac{150}{x+5} \text{ hr}$$

$$\text{We have, } \frac{150}{x} - \frac{150}{x+5} = 1$$

$$\Rightarrow 150 \left(\frac{1}{x} - \frac{1}{x+5} \right) = 1$$

$$\Rightarrow 150 \left(\frac{x+5-x}{x(x+5)} \right) = 1$$

$$\Rightarrow 150(5) = x(x + 5)$$

$$\Rightarrow 750 = x^2 + 5x$$

$$\Rightarrow x^2 + 5x - 750 = 0$$

$$\Rightarrow x^2 + 30x - 25x + (30 \times -25) = 0$$

$$\Rightarrow x(x + 30) - 25(x + 30) = 0$$

$$\Rightarrow (x + 30)(x - 25) = 0$$

$$\Rightarrow x = -30 \text{ or } (x - 25) = 0$$

\Rightarrow Since, speed cannot be negative values, so $x = 25$.

\therefore usual speed of the train = 25 km/hr

5. The time taken by a person to cover 150 km was 2.5 hrs more than the time taken in the return journey. If he returned at a speed of 10 km/hr more than the speed of going, what was the speed per hour in each direction?

Sol:

Let the going speed of the person be x km/hr

Given that, the return speed is 10 km/hr more than the going speed

\Rightarrow Return speed of the person = $(x + 10)$ km/hr

Total distance covered = 150 km.

$$\text{Time taken for going} = \frac{150 \text{ km}}{x \text{ km/hr}} = \frac{150}{x} \text{ hr}$$

$$\text{Time taken for returning} = \frac{150 \text{ km}}{(x+10) \text{ km/hr}} = \frac{150}{(x+10)} \text{ hr}$$

Given that, time taken for going is 2.5 hours more than the time for returning

$$\text{i.e. } \frac{150}{x} \text{ hr} - \frac{150}{x+10} \text{ hr} = 2.5 \text{ hr}$$

$$\Rightarrow 150 \left(\frac{1}{x} - \frac{1}{x+10} \right) = \frac{25}{10}$$

$$\Rightarrow 150 \left(\frac{x+10-x}{x(x+10)} \right) = \frac{25}{10}$$

$$\Rightarrow 6(10) = \frac{x(x+10)}{10}$$

$$\Rightarrow 60 \times 10 = x^2 + 10x$$

$$\Rightarrow x^2 + 10x - 600 = 0$$

$$\Rightarrow x^2 + (30 - 20)x + (30 \times -20) = 0$$

$$\Rightarrow x^2 = 30x - 20x + (30 \times -20) = 0$$

$$\Rightarrow x(x + 30) - 20(x + 30) = 0$$

$$\Rightarrow (x + 30)(x - 20) = 0$$

$$\Rightarrow x + 30 = 0 \text{ or } x - 20 = 0$$

$$\Rightarrow x = -30 \text{ or } x = 20$$

Since, speed cannot be negative. So $x = 20$

\therefore speed of the person when going = 20 km/hr

Now, speed of the person when returning = $(x + 10)$ km/hr

$$= (20 + 10) \text{ km/hr}$$

$$= 30 \text{ km/hr}$$

6. A plane left 40 minutes late due to bad weather and in order to reach its destination, 1600 km away in time, it had to increase its speed by 400 km/hr from its usual speed. Find the usual speed of the plane.

Sol:

Let the usual speed of the plane be x km/hr

Total distance travelled = 1600 km

$$\Rightarrow \text{Time taken by the plane with usual speed} = \frac{1600 \text{ km}}{x \text{ km/hr}} = \frac{1600}{x} \text{ hr}$$

Given that, if speed is increased by 400 km/hr, the plane takes 40 minutes less than that of the usual time. Speed of the plane after increasing = $(x + 400)$ km/hr

$$\Rightarrow \text{Time taken by the plane with increasing speed} = \frac{1600 \text{ km}}{(x+400) \frac{\text{km}}{\text{hr}}} = \frac{1600}{x+400} \text{ hr}$$

Now,

$$\frac{1600}{x} \text{ hr} - \frac{1600}{x+400} \text{ hr} = \frac{40}{60} \text{ hr} \quad \left[\because 40 \text{ minutes} = \frac{40}{60} \text{ hr as } 1 \text{ hr} = 60 \text{ min} \right]$$

$$\Rightarrow 1600 \left[\frac{1}{x} - \frac{1}{x+400} \right] = \frac{40}{60}$$

$$\Rightarrow 1600 \left[\frac{x+400-x}{x(x+400)} \right] = \frac{40}{60}$$

$$\Rightarrow 40(400 \times 60) = x(x + 400)$$

$$\Rightarrow x^2 + 400x - 960000 = 0$$

$$\Rightarrow x^2 + (1800 - 800)x + (1800 \times (-800)) = 0$$

$$\Rightarrow (x^2) + 1800x - 800x + (1800 \times -800) = 0$$

$$\Rightarrow x(x + 1800) - 800(x + 1800) = 0$$

$$\Rightarrow (x + 1800)(x - 800) = 0$$

$$\Rightarrow x = -1800 \text{ or } x - 800 = 0$$

$$\Rightarrow x = -1800 \text{ or } x = 800$$

Since, speed cannot be negative. So, $x = 800$

\therefore Usual speed of the plane is 800 km/hr.

7. An aeroplane takes 1 hour less for a journey of 1200 km if its speed is increased by 100 km/hr from its usual speed. Find its usual speed.

Sol:

Let the usual speed of the plane be x km/hr.

Distance covered in the journey = 1800 km

$$\Rightarrow \text{Time taken by the plane with usual speed} = \frac{1200 \text{ km}}{x \frac{\text{km}}{\text{hr}}} = \frac{1200}{x} \text{ hr}$$

Now, speed is increased by 100 km/hr and the time taken is 1 hour less for the same journey.

\Rightarrow Speed of the plane after increased = $(x + 100)$ km/hr and Time taken by plane with

$$\text{increased speed} = \frac{1800 \text{ km}}{(x+100) \frac{\text{km}}{\text{hr}}} = \frac{1800}{x+100} \text{ hr}$$

Now, we have

$$\frac{1800}{x} - \frac{1800}{x+100} = 1$$

$$\Rightarrow 1800 \left(\frac{1}{x} - \frac{1}{x+100} \right) = 1$$

$$\Rightarrow 1800 \left(\frac{x+100-x}{x(x+100)} \right) = 1$$

$$\Rightarrow 1200(100) = x(x + 100)$$

$$\Rightarrow 120000 = x^2 + 100x$$

$$\Rightarrow x^2 + 100x - 120000 = 0$$

$$\Rightarrow x^2 + (400 - 300)x + (400 \times -300) = 0$$

$$\Rightarrow x^2 + 400x - 300x + (400 \times -300) = 0$$

$$\Rightarrow x(x + 400) - 300(x + 400) = 0$$

$$\Rightarrow (x + 400)(x - 300) = 0$$

$$\Rightarrow x + 400 = 0 \text{ or } x - 300 = 0$$

$$\Rightarrow x = -400 \text{ or } x = 300$$

Since, speed cannot be negative so, $x = 300 \therefore$ usual speed of the plane = 300 km/hr

8. A passenger train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. Find the usual speed of the train.

Sol:

Let the usual speed of the train be x km/hr

Distance covered in the journey = 300 km

$$\text{Time taken by the train with usual speed} = \frac{300 \text{ km}}{x \text{ km/hr}} = \frac{300}{x} \text{ hr}$$

Now,

If the speed is increased by 5 km/hr, the train takes 2 hours less for the same journey.

$$\Rightarrow \text{speed of the train after increasing} = (x + 5) \text{ km/hr}$$

$$\text{And time taken by the train after increasing the speed} = \frac{300 \text{ km}}{(x+5) \text{ km/hr}} = \frac{300}{x+5} \text{ hr}$$

We have,

$$\frac{300}{x} \text{ hr} - \frac{300}{x+5} \text{ hr} = 2 \text{ hrs}$$

$$\Rightarrow 300 \left(\frac{1}{x} - \frac{1}{x+5} \right) = 2$$

$$\Rightarrow 300 \left(\frac{x+5-x}{x(x+5)} \right) = 2$$

$$\Rightarrow 300(5) = 2(x(x + 5))$$

$$\Rightarrow 750 = x^2 + 5x$$

$$\Rightarrow x^2 + 5x - 750 = 0$$

$$\Rightarrow x^2 + 30x - 25x + (30 \times -25) = 0$$

$$\Rightarrow (x + 30)(x - 25) = 0$$

$$\Rightarrow x + 30 = 0 \text{ or } x - 25 = 0$$

$$\Rightarrow x = -30 \text{ or } x = 25$$

Since, speed cannot be negative. So $x = 25$

\therefore The usual speed of the train = 25 km/hr.

9. A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more, it would have taken 30 minutes less for the journey. Find the original speed of the train.

Sol:

Let the original speed of the train be x km/hr

Distance covered = 90 km.

$$\Rightarrow \text{Time taken by the train with original speed} = \frac{90 \text{ km}}{x \text{ km/hr}} = \frac{90}{x} \text{ hr}$$

Now, if the speed of the train is increased by 15 km/hr, the train takes 30 minutes less for the same journey

$$\Rightarrow \text{Speed of the train after increasing} = (x + 15) \text{ km/hr and the time taken by the train after increasing the speed} = \frac{90 \text{ km}}{(x+15) \text{ km/hr}} = \frac{90}{x+15} \text{ hr}$$

Now,

$$\frac{90}{x} \text{ hr} - \frac{90}{x+15} \text{ hr} = 30 \text{ min}$$

$$\Rightarrow 90 \text{ hr} \left(\frac{1}{x} - \frac{1}{x+15} \text{ hr} \right) = 30 \text{ min}$$

$$\Rightarrow 90 \left(\frac{1}{x} - \frac{1}{x+15} \right) = \frac{30}{60} \text{ hr} \quad [\because 1 \text{ hr} = 30 \text{ min}]$$

$$\Rightarrow 90 \left(\frac{x+15-x}{x(x+15)} \right) = \frac{1}{2}$$

$$\Rightarrow 90 \times 15 \times 2 = x(x + 15)$$

$$\Rightarrow 2700 = x^2 + 15x$$

$$\Rightarrow x^2 + 15x - 2700 = 0$$

$$\Rightarrow x^2 + (60 - 45)x + (60 \times (-45)) = 0$$

$$\Rightarrow x^2 + 60x - 45x + (60 \times -45) = 0$$

$$\Rightarrow x(x + 60) - 45(x + 60) = 0$$

$$\Rightarrow (x + 60)(x - 45) = 0$$

$$\Rightarrow x + 60 = 0 \text{ or } x - 45 = 0$$

$$\Rightarrow x = -60 \text{ or } x = 45$$

Since, speed cannot be negative. So, $x = 45$

\therefore Original speed of the train = 45 km/hr

10. A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Sol:

Let the speed of the train be x km/hr

Distance covered by the train = 360 km

$$\Rightarrow \text{Time taken by the train with initial speed} = \frac{360 \text{ km}}{x \text{ km/hr}} = \frac{360}{x} \text{ hr}$$

Now, if the speed is 5 km/hr more, the train takes 1 hour less for the same journey.

$$\Rightarrow \text{Speed of the train after increasing the speed} = (x + 5) \text{ km/hr}$$

$$\text{And time taken by the train with increased speed} = \frac{360 \text{ km}}{(x+5) \text{ km}} = \frac{360}{x+5} \text{ hr}$$

Now,

$$\frac{360}{x} \text{ hr} - \frac{360}{x+5} \text{ hr} = 1 \text{ hr}$$

$$\Rightarrow 360 \left(\frac{1}{x} - \frac{1}{x+5} \right) = 1$$

$$\begin{aligned}
 &\Rightarrow 360 \left(\frac{x+5-x}{x(x+5)} \right) = 1 \\
 &\Rightarrow 360 \times 5 = 1 \times x(x+5) \\
 &\Rightarrow x^2 + 5x - 1800 = 0 \\
 &\Rightarrow x^2 + 45x - 40x + (x + 45) = 0 \\
 &\Rightarrow x(x + 45) - 40(x + 45) = 0 \\
 &\Rightarrow (x + 45)(x - 40) = 0 \\
 &\Rightarrow x + 45 = 0 \text{ or } x - 40 = 0 \\
 &\Rightarrow x = -45 \text{ or } x = 40
 \end{aligned}$$

Since, speed is always a positive value i.e. $x = 0 \Rightarrow x = 40$

\therefore The speed of the train = 40 km/hr

11. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/hr more than that of the passenger train, find the average speeds of the two trains.

Sol:

Let the speed of the passenger train be x km/hr

Given that the average speed of the express train is 11 km/hr more than that of passenger train.

\Rightarrow Average speed of express train = $(x + 11)$ km/hr

Now,

$$\text{Time taken by the passenger train} = \frac{138 \text{ km}}{x \text{ km/hr}} = \frac{13}{x} \text{ hr}$$

$$\text{And time taken by the express train} = \frac{132 \text{ km}}{(x+11) \text{ km/hr}} = \frac{132}{x+11} \text{ hr}$$

Given that, express train takes 1 hour less than that of passenger train to reach the destiny.

$$\Rightarrow \frac{132}{x} \text{ hr} - \frac{132}{x+11} \text{ hr} = 1 \text{ hr}$$

$$\Rightarrow 132 \left(\frac{1}{x} - \frac{1}{x+11} \right) = 11$$

$$\Rightarrow 132 \left(\frac{x+11-x}{x(x+11)} \right) = 1$$

$$\Rightarrow 132 \times 11 = x(x + 11) \times 1$$

$$\Rightarrow x^2 - 11x - 1452 = 0$$

$$\Rightarrow x^2 + (44x - 33x) + (44 \times -33) = 0$$

$$\Rightarrow x^2 + 44x - 33x + (44 \times -33) = 0$$

$$\Rightarrow x(x + 4) - 33(x + 44) = 0$$

$$\Rightarrow (x + 44)(x - 33) = 0$$

$$\Rightarrow x + 44 = 0 \text{ or } x - 33 = 0$$

$$\Rightarrow x = -44 \text{ or } x = 33$$

Since, speed cannot be in negative values. So, $x = 33$

\therefore Average speed of the slower train i.e. passenger train = 33 km/hr

And average speed of express train = $(x + 11)$ km/hr = $(33 + 11)$ km/hr = 44 km/hr.

12. An aeroplane left 50 minutes later than its scheduled time, and in order to reach the destination, 1250 km away, in time, it had to increase its speed by 250 km/hr from its usual speed. Find its usual speed.

Sol:

Let the usual speed of the plane be x km/hr

Distance covered by the plane = 1250 km

$$\Rightarrow \text{Time taken by the plane with usual speed} = \frac{1250 \text{ km}}{x \text{ km/hr}} = 1250 \text{ hr}$$

To cover the delay of 50 minutes, the speed of the plane is increased by 250 km/hr

Now,

Speed of the plane after increasing = $(x + 250)$ km/hr and

$$\text{Time taken by the plane with increased speed} = \frac{1250 \text{ km}}{(x+250) \text{ km/hr}} = \frac{1250}{x+250} \text{ hr}$$

From the data we have,

$$\frac{1250}{x} \text{ hr} - \frac{1250}{x+250} \text{ hr} = 50 \text{ min}$$

$$\Rightarrow 1250 \text{ hr} \left(\frac{1}{x} - \frac{1}{x+250} \right) = \frac{50}{60} \text{ hr} [\because 1\text{hr} = 50 \text{ min}]$$

$$\Rightarrow 250 \left(\frac{x+250-x}{x(x+250)} \right) = \frac{1}{6}$$

$$\Rightarrow 250 \times 250 \times 6 = x(x+250) \times 1$$

$$\Rightarrow 375000 = x^2 + 250x$$

$$\Rightarrow x^2 + 250x - 375000 = 0$$

$$\Rightarrow x^2 + (750 - 500)x + (750 \times -500) = 0$$

$$\Rightarrow x^2 + 750x - 500x + (750 \times -500) = 0$$

$$\Rightarrow (x + 750)(x - 500) = 0$$

$$\Rightarrow (x + 750) = 0 \text{ or } x = 500 = 0$$

$$\Rightarrow x = -750 \text{ or } x = 500$$

Since, speed cannot be a negative value. So, $x = 500$

\therefore the usual speed of the plane = 500 km/hr.

Exercise 8.9

1. Ashu is x years old while his mother Mrs Veena is x years old. Five years hence Mrs Veena will be three times old as Ashu. Find their present ages.

Sol:

Given that, Ashu is x years old while his mother Mrs. Veena is x^2 years old.

\Rightarrow Ashu's present age = x years and Mrs. Veena's present age = x^2 years

And also given that, after 5 years Mrs. Veena will be three times old as Ashu.

\Rightarrow Ashu's age after 5 years = $(x + 5)$ years
 And Mrs. Veena's age after 5 years = $(x^2 + 5)$ years

But given that,

$$\Rightarrow (x^2 + 5) = 3(x + 5)$$

$$\Rightarrow x^2 + 5 = 3x + 15$$

$$\Rightarrow x^2 - 3x - 10 = 0$$

$$\Rightarrow x^2 - 5x + 2x - 10 = 0$$

$$\Rightarrow x(x - 5) + 2(x - 5) = 0 \Rightarrow (x - 5)(x + 2) = 0$$

$$\Rightarrow x - 5 = 0 \text{ or } x + 2 = 0$$

$$\Rightarrow x = 5 \text{ or } x = -2$$

Since, age cannot be in negative values. So, $x = 5$ years.

\therefore Present age of Ashu is $x = 5$ years and

Present age of Mrs. Veena is $x^2 \Rightarrow 5^2$ years $\Rightarrow 25$ years.

2. The sum of the ages of a man and his son is 45 years. Five years ago, the product of their ages was four times the man's age at the time. Find their present ages.

Sol:

Let the present age of the son be x years

Given that,

sum of present ages of man and his son is 45 years.

$$\Rightarrow \text{Man's present age} = (45 - x) \text{ years}$$

And also given that,

five years ago, the product of their ages was four times the man's age at the time.

$$\Rightarrow \text{Man's age before 5 years} = (45 - x - 5) \text{ years} = (40 - x) \text{ years}$$

And son's age before 5 years = $(x - 5)$ years

But, given that $(40 - x)(x - 5) = 4(40 - x)$

$$\Rightarrow x - 5 = 4$$

$$\Rightarrow x = 9 \text{ years}$$

$$\Rightarrow \text{Son's present age} \Rightarrow x = 9 \text{ years}$$

$$\text{Now, Man's present age} \Rightarrow (45 - x) \text{ years} = (45 - 9) \text{ years} = 36 \text{ years}$$

\therefore The present ages of man and son are 36 years and 9 years respectively.

3. The product of Shikha's age five years ago and her age 8 years later is 30, her age at both times being given in years. Find her present age.

Sol:

Let the present age of Shikha be x years.

Given that,

The product of her age five years ago and her age 8 years later is 30

Now,

Shika's age five years ago = $(x - 5)$ years

And Shika's age 8 years later = $(x + 8)$ years

Given that.

$$(x - 5)(x + 8) = 30$$

$$\Rightarrow x^2 + 8x - 5x - 40 = 30$$

$$\Rightarrow x^2 + 3x - 70 = 0$$

$$\Rightarrow x^2 + 10x - 7x - 70 = 0$$

$$\Rightarrow x(x + 10) - 7(x + 10) = 0$$

$$\Rightarrow (x + 10)(x - 7) = 0$$

$$\Rightarrow x + 10 = 0 \text{ or } x - 7 = 0$$

$$\Rightarrow x = -10 \text{ or } x = 7$$

Since, age cannot be in negative values, So $x = 7$ years

\therefore The present age of shika is 7 years.

4. The product of Ramu's age (in years) five years ago and his age (in years) nine years later is 15. Determine Ramu's present age.

Sol:

Let the present age of Ramu be a x years

Given that,

The product of his age five years ago and his age y nine years later is 15.

Now, Ramu's age five years ago = $(x - 5)$ years

And Ramu's age nine years later = $(x + 9)$ years

Given that,

$$(x - 5)(x + 9) = 15$$

$$\Rightarrow x^2 + 9x - 5x - 45 = 15$$

$$\Rightarrow x^2 + 4x - 60 = 0$$

$$\Rightarrow x^2 + 10x - 6x - 60 = 0$$

$$\Rightarrow x(x + 10) - 6(x + 10) = 0$$

$$\Rightarrow (x + 10)(x - 6) = 0$$

$$\Rightarrow x + 10 = 0 \text{ or } x - 6 = 0$$

$$\Rightarrow x = -10 \text{ or } x = 6$$

Since, age cannot be in negative values, So $x = 6$ years

\therefore The present age of shika is 6 years.

5. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Sol:

Let the present age of friend 1 be a x years

Given that,

Sum of the ages of two friends = 20 years

\Rightarrow Present age of friend 2 = $(20 - x)$ years

And also given that, four years ago, the product of their age was 48.

\Rightarrow Age of friend 1 before 4 years = $(x - 4)$ years

And age of friend 2 before 4 years = $(20 - x - 4)$ years = $(16 - x)$ years

Given that,

$$(x - 4)(16 - x) = 48$$

$$\Rightarrow 16x - x^2 - 64 + 4x = 48$$

$$\Rightarrow x^2 - 20x + 112 = 0$$

Let D be the discriminant of this quadratic equation.

$$\text{Then, } D = (-20)^2 - 4 \times 112 \times 1 = 400 - 448 = -48 < 0$$

We know that, to have real roots for a quadratic equation that discriminant D must be greater than or equal to 0 i.e. $D \geq 0$

But $D < 0$ in the above. So, above equation does not have real roots

Hence, the given situation is not possible.

6. A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages.

Sol:

Let the age of girls sister be a x years

Given that,

Girl is twice as old as her sister

$$\Rightarrow \text{Girls age} = 2 \times x \text{ years} = 2x \text{ years}$$

Given that, after 4 years, the product of their ages will be 160.

$$\Rightarrow \text{Girls age after 4 years} = (2x + 4) \text{ years}$$

$$\text{And sisters age after 4 years} = (x + 4) \text{ years}$$

Given that,

$$(2x + 4)(x + 4) = 160$$

$$\Rightarrow 2x^2 + 8x + 4x + 16 = 160$$

$$\Rightarrow 2x^2 + 12x - 144 = 0$$

$$\Rightarrow 2(x^2 + 6x - 72) = 0$$

$$\Rightarrow x^2 + 6x - 72 = 0$$

$$\Rightarrow x^2 + 12x - 6x - 72 = 0$$

$$\Rightarrow x(x + 12) - 6(x + 12) = 0$$

$$\Rightarrow (x + 12)(x - 6) = 0$$

$$\Rightarrow x + 12 = 0 \text{ or } x - 6 = 0$$

$$\Rightarrow x = -12 \text{ or } x = 6$$

Since, age cannot be in a negative value.

So, $x = 6$.

\therefore Age of girls sister is $x = 6$ years.

And age of girl is $2x = 2 \times 6$ years = 12 years

Hence, the present ages of girl and her sister are 12 years and 6 years respectively.

7. The sum of the reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Sol:

Let the present age of Rehman be x years.

Now,

Rehman's age 3 years ago = $(x - 3)$ years

And Rehman's age 5 years later = $(x + 5)$ years

Given that,

The sum of reciprocals of Rehman's ages 3 years ago and 5 years later is $\frac{1}{3}$

$$\Rightarrow \frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$$

$$\Rightarrow (2x + 2) \times 3 = 1(x - 3)(x + 5)$$

$$\Rightarrow 6x + 6 = x^2 + 5x - 3x - 15$$

$$\Rightarrow x^2 + 2x - 6x - 15 - 6 = 0$$

$$\Rightarrow x^2 - 4x - 21 = 0$$

$$\Rightarrow x^2 - 7x + 3x - 21 = 0$$

$$\Rightarrow x(x - 7)^2 - 3(x - 7) = 0$$

$$\Rightarrow (x - 7)(x + 3) = 0 \Rightarrow x - 7 = 0 \text{ or } x + 3 = 0$$

$$\Rightarrow x = 7 \text{ or } x = -3$$

Since, age cannot be in negative values. So, $x = 7$ years Hence, the present age of Rehman is 7 years.

Exercise 8.10

1. The hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 5 cm. Find the lengths of these sides.

Sol:

Let the length of the shortest side be x cm

Given that the length of the largest side is 5cm more than that of smaller side

\Rightarrow longest side = $(x + 5)$ cm

And also, given that

Hypotenuse = 25cm

So, let us consider a right angled triangle ABC right angled at B

We have, hypotenuse (AC) = 25 cm

BC = x cm and AB = $(x + 5)$ cm

Since, ABC is a right angled triangle

$$\begin{aligned}
 \text{We have, } (BC)^2 + (AB)^2 &= (AC)^2 \\
 \Rightarrow x^2 \text{ cm}^2 + (x + 5)^2 \text{ cm}^2 &= (25)^2 \text{ cm}^2 \\
 \Rightarrow x^2 + x^2 + 10x + 25 &= 625 \\
 \Rightarrow 2x^2 + 10x - 600 &= 0 \\
 \Rightarrow 2(x^2 + 5x - 300) &= 0 \\
 \Rightarrow x^2 + 5x - 300 &= 0 \\
 \Rightarrow x^2 + 20x - 15x + (20x - 15) &= 0 \\
 \Rightarrow x(x + 20) - 15(x + 20) &= 0 \\
 \Rightarrow (x + 20)(x - 15) &= 0 \\
 \Rightarrow (x + 20) = 0 \text{ or } (x - 15) &= 20
 \end{aligned}$$

2. The hypotenuse of a right triangle is 3.1 cm. If the smaller leg is tripled and the longer leg doubled, new hypotenuse will be 9.1 cm. How long are the legs of the triangle?

Sol:

Using Pythagoras theorem,

$$\begin{aligned}
 (AB)^2 + (BC)^2 &= (AC)^2 \\
 \Rightarrow (2y)^2 \text{ cm}^2 + (3x)^2 \text{ cm}^2 &= (9\sqrt{5})^2 \text{ cm}^2 \\
 \Rightarrow 4y^2 + 9x^2 &= 81 \times 5 \\
 \Rightarrow 4y^2 + 9x^2 &= 405 \\
 \Rightarrow 4(90 - x^2) + 9x^2 &= 405 \quad [\because x^2 + y^2 = 90] \\
 \Rightarrow 4 \times 90 - 4x^2 + 9x^2 &= 405 \\
 \Rightarrow 5x^2 &= 405 - 360 \\
 \Rightarrow 5x^2 &= 45 \\
 \Rightarrow x^2 &= 9 \\
 \Rightarrow x &= \sqrt{3^2} \Rightarrow x = \pm 3
 \end{aligned}$$

Since, x cannot be a negative value. So $x = 3\text{cm}$

We have,

$$\begin{aligned}
 x^2 + y^2 &= 90 \\
 \Rightarrow y^2 &= 90 - (3)^2 \\
 \Rightarrow y^2 &= 90 - 9 \\
 \Rightarrow y^2 &= 81 \Rightarrow y = \sqrt{81} \Rightarrow y = \pm 9
 \end{aligned}$$

Since, y cannot be a negative value. So, $y = 9\text{cm}$

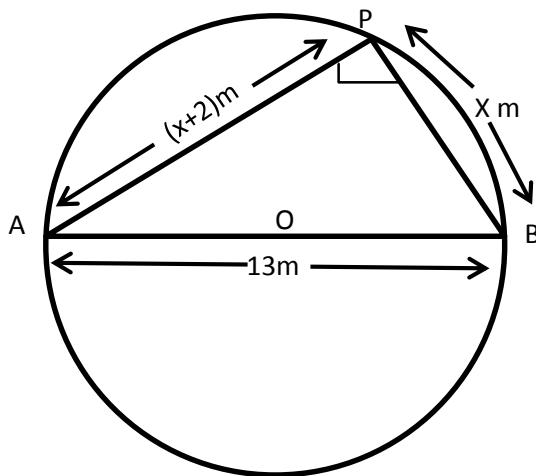
∴ hence, the length of the smaller side is 3 cm and the length of the longer side is 9cm.

3. A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the difference of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?

Sol:

Yes, it is possible to do so as in the given condition

This can be proved as below,



Let P be the required location of the pole such that its distance from gate B is x meter i.e.

$BP = x$ meters and also $AP - BP = 7$ m

$$\Rightarrow AP = BP + 7m = (x + 7)m$$

Since, AB is a diameter and P is a point on the boundary of the semi-circle, ΔAPB is right angled triangle, right angled at P.

Using Pythagoras theorem,

$$(AB)^2 = (AP)^2 + (BP)^2$$

$$\Rightarrow (13)^2 m^2 = (x + 7)^2 m^2 + (x)^2 m^2$$

$$\Rightarrow 169 = x^2 + 14x + 49 + x^2$$

$$\Rightarrow 2x^2 + 14x + 49 - 169 = 0$$

$$\Rightarrow 2x^2 + 14x - 120 = 0$$

$$\Rightarrow 2(x^2 + 7x - 60) = 0$$

$$\Rightarrow x^2 + 7x - 60 = 0$$

$$\Rightarrow x^2 + 12x - 5x - (12 \times -5) = 0$$

$$\Rightarrow x(x + 12) - 5(x + 12) = 0$$

$$\Rightarrow (x + 12)(x - 5) = 0$$

$$\Rightarrow x + 12 = 0 \text{ or } x - 5 = 0$$

$$\Rightarrow x = -12 \text{ or } x = 5$$

Since, x cannot be a negative value, So $x = 5$

$$\Rightarrow BP = 5m$$

$$\text{Now, } AP = (BP + 7)m = (5 + 7)m = 12 \text{ m}$$

\therefore The pole has to be erected at a distance 5 mtrs from the gate B and 12 m from the gate A.

4. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Sol:

120 m, 90 m

Exercise 8.11

1. The perimeter of a rectangular field is 82 m and its area is 400 m². Find the breadth of the rectangle.

Sol:

Let the breadth of the rectangle be x meters

Given that,

Perimeter = 82 m and Area = 400 m²

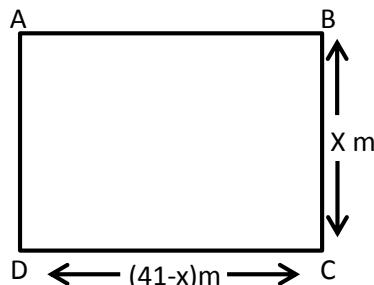
We know that

Perimeter of a rectangle = 2(length + breadth)

$$\Rightarrow 82 = 2(\text{length} + x)$$

$$\Rightarrow 41 = \text{length} + x$$

$$\Rightarrow \text{length} = (41 - x)\text{m}$$



We have

Area of rectangle = length \times breadth

$$\Rightarrow 400 \text{ m}^2 = (41 - x) \text{ m} \times x \text{ m}$$

$$\Rightarrow 400 = 41x - x^2$$

$$\Rightarrow 400x^2 - 41x + 400 = 0$$

$$\Rightarrow x^2 - 25x - 16x + (-25 \times -16) = 0$$

$$\Rightarrow x(x - 25) - 16(x - 25) = 0$$

$$\Rightarrow (x - 25)(x - 16) = 0$$

$$\Rightarrow (x - 25)(x - 16) = 0$$

$$\Rightarrow x - 25 = 0 \text{ or } x - 16 = 0$$

$$\Rightarrow x = 25 \text{ or } x = 16$$

Hence, breadth of the rectangle is 25m or 16m

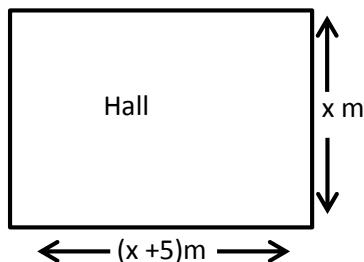
2. The length of a hall is 5 m more than its breadth. If the area of the floor of the hall is 84 m^2 , what are the length and breadth of the hall?

Sol:

Let the breadth of the rectangle (hall) be x meter.

Given that,

Length of the hall is 5m more than its breadth i.e. length = $(x + 5)$ m



And also given that,

Area of the hall = 84 m^2

Since, hall is in the shape of a rectangle,

Area of the rectangular hall = length \times breadth

$$\Rightarrow 84 \text{ m}^2 = xm \times (x + 5)m$$

$$\Rightarrow 84 = x(x + 5)$$

$$\Rightarrow 84 = x^2 + 5x$$

$$\Rightarrow x^2 + 5x - 84 = 0$$

$$\Rightarrow x^2 + 12x - 7x - 84 = 0$$

$$\Rightarrow x(x + 12) - 7(x + 12) = 0$$

$$\Rightarrow (x - 7)(x + 12) = 0$$

$$\Rightarrow x = 7 \text{ m or } x = -12 \text{ m}$$

Since, x cannot be negative. So, breadth of the hall = 7m

Hence, length of the hall = $(x + 5)$ m = $(7 + 5)$ m = 12m.

3. Two squares have sides x cm and $(x + 4)$ cm. The sum of their areas is 656 cm^2 . Find the sides of the squares.

Sol:

Let S_1 and S_2 be two squares.

Let x cm be the side of square S_1 and $(x + 4)$ cm be the side of square S_2 .

We know that,

Area of a square = (Side) 2

$$\Rightarrow \text{Area of square } S_1 = (x)^2 = x^2 \text{ cm}^2$$

$$\Rightarrow \text{Area of square } S_2 = (x + 4)^2 \text{ cm}^2$$

Given that,

$$\text{Area of square } S_1 + \text{Area of square } S_2 = 656 \text{ cm}^2$$

$$\Rightarrow x^2 \text{ cm}^2 + (x + 4) \text{ cm}^2 = 656 \text{ cm}^2$$

$$\begin{aligned}
 \Rightarrow x^2 + x^2 + 8x + 16 &= 656 \\
 \Rightarrow 2x^2 + 8x + 16 - 656 &= 0 \\
 \Rightarrow 2x^2 + 8x - 640 &= 0 \\
 \Rightarrow 2(x^2 + 4x - 320) &= 0 \\
 \Rightarrow x^2 + 4x - 320 &= 0 \\
 \Rightarrow x^2 + 20x - 16x + (20x - 16) &= 0 \\
 \Rightarrow x + 20 &= 0 \text{ or } x - 16 = 0
 \end{aligned}$$

$$\Rightarrow x = -20 \text{ cm or } x = 16 \text{ cm}$$

Since, x cannot be negative. So, x = 16cm

∴ Side of square $S_1 \Rightarrow x = 16\text{cm}$ and

Side of square $S_2 \Rightarrow (x + 4) = (16 + 4)\text{cm} = 20 \text{ cm}$

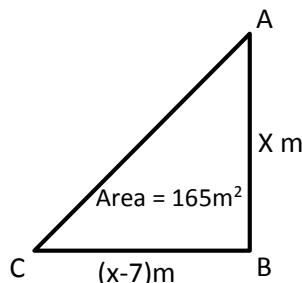
4. The area of a right angled triangle is 165 m^2 . Determine its base and altitude if the latter exceeds the former by 7m.

Sol:

Let the altitude of the right angled triangle be denoted by x meter

Given that altitude exceeds the base of the triangle by 7m.

$$\Rightarrow \text{Base} = (x - 7)\text{m}$$



We know that,

$$\begin{aligned}
 \text{Area of a triangle} &= \frac{1}{2} \times \text{base} \times \text{height} \\
 \Rightarrow 165\text{m}^2 &= \frac{1}{2} \times (x - 7)\text{m} \times x \text{ m} \quad [\because \text{Area} = 165\text{m}^2 \text{ given}] \\
 \Rightarrow 2 \times 165 &= x(x - 7) \\
 \Rightarrow x^2 - 7x &= 330 \\
 \Rightarrow x^2 - 7x - 330 &= 0 \\
 \Rightarrow (x - 22) + 15(x - 22) &= 0 \\
 \Rightarrow (x - 22)(x + 15) &= 0 \\
 \Rightarrow x = 22 \text{ or } x &= -15
 \end{aligned}$$

Since, x cannot be negative. So, x = 22 m

∴ Altitude of the triangle $\Rightarrow x = 22\text{m}$

And base of the triangle $\Rightarrow (x - 7)\text{m} = (22 - 7)\text{m} = 15\text{m}$

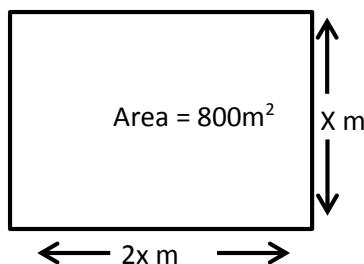
5. Is it possible to design a rectangular mango grove whose length is twice its breadth and the area is 800 m^2 ? If so, find its length and breadth.

Sol:

Let the breadth of the rectangular mango grove be x meter.

Given that length is twice that of breadth \Rightarrow length $= 2 \times x \text{ m} = 2x \text{ m}$

Given that area of the grove is 800 m^2 .



But we know that

Area of a rectangle $=$ length \times breadth

$$\Rightarrow 800 \text{ m}^2 = 2x \text{ m} \times x \text{ m}$$

$$\Rightarrow 2x^2 = 800$$

$$\Rightarrow x^2 = 400$$

$$\Rightarrow x = \sqrt{400} = \sqrt{(20)^2} = \pm 20$$

$$\Rightarrow x = 20 \text{ or } x = -20$$

Since, x cannot be a negative value.

So, $x = 20 \text{ m}$

\therefore Breadth of the mango grove $= 20 \text{ m}$ and length of the mango grove

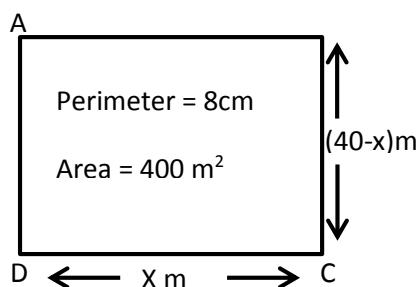
$$= 2x \text{ m} = 2 \times 20 \text{ m} = 40 \text{ m}$$

Yes. It is possible to design a rectangular mango grove whose length is twice its breadth and the area is 800 m^2 .

6. Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2 ? If so, find its length and breadth.

Sol:

To prove the given condition, let us assume that the length of the rectangular park be denoted by $x \text{ m}$.



Given that

Perimeter = 80m and Area = 400m²

We know that,

Perimeter of a rectangle = 2(length + breadth)

$$\Rightarrow 80m = 2(x + \text{breadth})$$

$$\Rightarrow \text{breadth} = \left(\frac{80}{2} - x\right) m$$

$$\Rightarrow \text{breadth} = (40 - x)m$$

And also,

Area of a rectangle = length \times breadth

$$\Rightarrow 400m^2 = x m \times (40 - x)m$$

$$\Rightarrow 400 = x(40 - x)$$

$$\Rightarrow 400 = 40x - x^2$$

$$\Rightarrow x^2 - 40x + 400 = 0$$

$$\Rightarrow x^2 - 2 \times 20 \times x + (20)^2 = 0$$

$$\Rightarrow (x - 20)^2 = 0$$

$$\Rightarrow (x - 20 = 0) \Rightarrow x = 20$$

\therefore length of the rectangular park $\Rightarrow x = 20$ m and breadth of the rectangular park $\Rightarrow (40 - x)m = (40 - 20)m = 20$ m

Yes. It is possible to design a rectangular park of perimeter 80m and area 400m².

7. Sum of the areas of two squares is 640 m². If the difference of their perimeters is 64 m, find the sides of the two squares.

Sol:

Let the two squares be denoted as S_1 and S_2 and let side of squares S_1 be denoted as x meter and that of square S_2 be y m.

Given that,

Difference of their perimeter is 64m.

We know that

Perimeter of a square = 4 \times side

$$\Rightarrow \text{Perimeter of square } S_1 = 4 \times x m = 4x m$$

$$\Rightarrow \text{Perimeter of square } S_2 = 4 \times y m = 4y m$$

Now, difference of perimeter = perimeter of square S_1 – Perimeter of square S_2

$$\Rightarrow 64 m = (4x - 4y)m$$

$$\Rightarrow 64 = 4(x - y)$$

$$\Rightarrow x - y = 16$$

$$\Rightarrow x = y + 16$$

And also,

Given that sum of areas of two squares = 640 m².

We know that,

Area of a square = (Side)²

$$\Rightarrow \text{Area of square } S_1 = x^2 m^2$$

$$\Rightarrow \text{Area of square } S_2 = y^2 m^2$$

Now,

$$\text{Sum of areas of two squares} = \text{Area of square } S_1 + \text{Area of square } S_2$$

$$\Rightarrow 640 m^2 = x^2 m^2 + y^2 m^2$$

$$\Rightarrow 640 = (y + 16)^2 + y^2 \quad [\because x = y + 16]$$

$$\Rightarrow y^2 + 32y + 256 + y^2 = 640$$

$$\Rightarrow 2y^2 + 32y + 256 - 640 = 0$$

$$\Rightarrow 2y^2 + 32y - 384 = 0$$

$$\Rightarrow 2(y^2 + 16y - 192) = 0$$

$$\Rightarrow y^2 + 16y - 192 = 0$$

$$\Rightarrow y^2 + 24y - 8y + (24 \times -8) = 0$$

$$\Rightarrow y(y + 24) - 8(y + 24) = 0$$

$$\Rightarrow (y + 24)(y - 8) = 0$$

$$\Rightarrow y + 24 = 0 \text{ or } y - 8 = 0$$

$$\Rightarrow y = -24 \text{ or } y = 8$$

Since, y cannot be a negative value. So, y = 8m

∴ Side of the square S_2 is y = 8m

And side of the square S_1 is x = (y + 16)m = (8 + 16)m = 24 m

Hence, sides of the two squares is 24m and 8m.

Exercise 8.12

1. A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days, find the time taken by B to finish the work.

Sol:

Let B takes x days to complete the piece of work.

$$\Rightarrow \text{B's one days work} = \frac{1}{x}$$

Now, A takes 10 days less than that of B to finish the same piece of work i.e. $(x - 10)$ days

$$\Rightarrow \text{A's one days work} = \frac{1}{x-10}$$

Given that, both A and B together can finish the same work in 12 days.

$$\Rightarrow (\text{A and B's one days work}) = \frac{1}{12}$$

Now,

$$(\text{A's one days work}) + (\text{B's one days work}) = \frac{1}{x} + \frac{1}{x-10} \text{ and } (\text{A + B's one days work}) = \frac{1}{12}$$

$$\Rightarrow \frac{1}{x} + \frac{1}{x-10} = \frac{1}{12}$$

$$\Rightarrow \frac{x-10+x}{x(x-10)} = \frac{1}{12}$$

$$\Rightarrow (2x - 10) \times 12 = x(x - 10)$$

$$\begin{aligned}
 &\Rightarrow 24x - 120 = x^2 - 10x \\
 &\Rightarrow x^2 - 10x - 24x + 120 = 0 \\
 &\Rightarrow x^2 - 34x + 120 = 0 \\
 &= x^2 - 30x - 4x + (-30 \times -4) = 0 \\
 &\Rightarrow x(x - 30) - 4(x - 30) = 0 \\
 &\Rightarrow (x - 30)(x - 4) = 0 \\
 &\Rightarrow (x - 30) = 0 \text{ or } (x - 4) = 0 \\
 &\Rightarrow x = 30 \text{ or } x = 4
 \end{aligned}$$

We can observe that, the value of x cannot be less than 10.

∴ The time taken by B to finish the work is 30 days.

2. If two pipes function simultaneously, a reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours will the second pipe take to fill the reservoir?

Sol:

Let x be no. of students planned for a picnic

Given that budget for food was Rs 480

$$\Rightarrow \text{Share of each student} = \frac{\text{Total budget}}{\text{No.of students}} = \text{Rs} \frac{480}{x}$$

Given that 8 students failed to go

⇒ No. of students will be $(x - 8)$

Now,

Share of each student will be equal to

$$= \frac{\text{total budget}}{\text{No.of students}} = \text{Rs.} \frac{480}{x-8}$$

Given that if 8 students failed to go, then cost of food for each member increased by Rs. 10.

3. Two water taps together can fill a tank in 9 hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Sol:

Let the time taken by the top of smaller diameter to fill the tank be x hours

$$\Rightarrow \text{Portion of tank filled by smaller pipe in one hour} = \frac{1}{x}$$

Now, larger diameter pipe takes 10 hours less than that of smaller diameter pipe i.e. $(x - 10)$ hours

$$\Rightarrow \text{Portion of tank filled by larger diameter pipe in one hour} = \frac{1}{x-10}$$

Given that,

Two tops together can fill the tank in $9\frac{3}{8}$ hours $\Rightarrow \frac{75}{8}$ hours

Now,

$$\text{Portion of tank filled by both the tops together in one hour} = \frac{1}{75/8} = \frac{8}{75}$$

We have,

Portion of tank filled by smaller pipe in 1 hr + Portion of tank filled by larger pipe in 1 hr.

$$= \frac{1}{x} + \frac{1}{x-10} \Rightarrow \frac{8}{75} = \frac{1}{x} + \frac{1}{x-10}$$

4. Two pipes running together can fill a tank in 11 minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately.

Sol:

Let us take the time taken by the faster pipe to fill the tank as x minutes.

$$\Rightarrow \text{Portion of tank filled by faster pipe in one minute} = \frac{1}{x}$$

Now, time taken by the slower pipe to fill the same tank is 5 minutes more than that of faster pipe i.e. $(x + 5)$ minutes.

$$\Rightarrow \text{Portion of tank filled by slower pipe in one minute} = \frac{1}{x+5}$$

Given that,

The two pipes together can fill the tank in $11 \frac{1}{9}$ minutes $\Rightarrow \frac{100}{9}$ minutes

\Rightarrow portion of tank filled by faster pipe in 1 min + Portions of tank filled by slower pipe in

$$1 \text{ min i.e. } \frac{9}{100} = \frac{1}{x} + \frac{1}{x+5}$$

$$\Rightarrow \frac{9}{100} = \frac{x+5+x}{x(x+5)}$$

Exercise 8.13

1. A piece of cloth costs Rs. 35. If the piece were 4 m longer and each metre costs Rs. one less, the cost would remain unchanged. How long is the piece?

Sol:

Let initial length of the cloth be x m, and cost per each meter of cloth be Rs y

\Rightarrow Total cost of piece of cloth will be length of cloth x cost per each meter

$$\Rightarrow xy$$

$$\text{But given that } xy = \text{Rs. } 35 \Rightarrow y = \text{Rs. } \frac{35}{x}$$

And also,

Given that if the piece were 4m longer and each meter costs Rs. 1 less the cost would remain unchanged.

\Rightarrow Length of the cloth will be $(x + 4)$ m and cost per each meter of cloth will be Rs $(y - 1)$

\Rightarrow Total cost of piece of cloth will be Rs. $(x + 4)(y - 1)$

But,

$$\text{Rs } (x + 4)(y - 1) = \text{Rs } 35$$

$$\Rightarrow xy + 4y - x - 4 = 35$$

$$\begin{aligned}
 & \Rightarrow 35 + 4\left(\frac{35}{2}\right) - x - 4 = 35 & [\because xy = 35 \text{ & } y = \frac{35}{2}] \\
 & \Rightarrow \frac{140-x^2-4x}{x} = 0 \\
 & \Rightarrow x^2 + 4x - 140 = 0 \\
 & \Rightarrow x^2 + 14x - 10x + (14x - 10) = 0 \quad [\because 140 = 14x - 10 = 4x = 14x - 10x] \\
 & \Rightarrow x(x + 14) - 10(x + 14) = 0 \\
 & \Rightarrow (x + 14)(x - 10) = 0 \\
 & \Rightarrow (x + 14) = 0 \text{ or } (x - 10) = 0 \\
 & \Rightarrow x = -14 \text{ or } x = 10
 \end{aligned}$$

Since length of the cloth cannot be in negative integers, the required length of cloth is 10m.

2. Some students planned a picnic. The budget for food was Rs. 480. But eight of these failed to go and thus the cost of food for each member increased by Rs. 10. How many students attended the picnic?

Sol:

Let x be no. of students planned for a picnic

Given that budget for food was Rs 480

$$\Rightarrow \text{Share of each student} = \frac{\text{Total budget}}{\text{No. of students}} = \text{Rs} \frac{480}{x}$$

Given that 8 students foiled to go

\Rightarrow No. of students will be $(x - 8)$

Now,

Share of each student will be equal to

$$= \frac{\text{total budget}}{\text{No. of students}} = \text{Rs.} \frac{480}{x-8}$$

Given that if 8 students failed to go, then cost of food for each member increased by Rs. 10.

3. A dealer sells an article for Rs. 24 and gains as much percent as the cost price of the article. Find the cost price of the article.

Sol:

Let the cost price of the article be Rs x

Given that gain percentage of the article is as much as cost price i.e. x

$$\Rightarrow \text{Selling price} = \text{cost price} + \text{gain}$$

$$\begin{aligned}
 & = \text{Rs } x + \text{cost price} \times \text{gain percentage} \\
 & = \text{Rs } x + \text{Rs } x \times \frac{x}{100} \\
 & = \text{Rs} \left(x + \frac{x^2}{100} \right)
 \end{aligned}$$

Given that selling price = Rs 24

$$\Rightarrow \text{Rs } 24 = \text{Rs} \left(x + \frac{x^2}{100} \right)$$

$$\Rightarrow 24 = x + \frac{x^2}{100}$$

$$\begin{aligned}
 &\Rightarrow \frac{x^2}{100} + x - 24 = 0 \\
 &\Rightarrow x^2 + 100x - 2400 = 0 \\
 &\Rightarrow x^2 + 120x - 20x + (120 \times -80) = 0 \\
 &\Rightarrow x^2(x + 180) - 80(x + 180) = 0 \\
 &\Rightarrow (x + 180)(x - 20) = 0 \\
 &\Rightarrow x + 120 = 0 \text{ or } x - 20 = 0 \\
 &\Rightarrow x = -120 \text{ or } x = 20
 \end{aligned}$$

Since, cost price of the article cannot be negative, the required cost price of the article is Rs 20

$$\begin{aligned}
 &\Rightarrow Rs \frac{480}{x-8} - Rs \frac{480}{x} = Rs 10 \\
 &\Rightarrow \frac{480}{x-8} - \frac{480}{x} = 10 \\
 &\Rightarrow 480 \left(\frac{1}{x-8} - \frac{1}{x} \right) = 10 \\
 &\Rightarrow 48 \left(\frac{x-(x-8)}{x(x-8)} \right) = 1 \\
 &\Rightarrow 48 \left(\frac{8}{x^2-8x} \right) = 1 \\
 &\Rightarrow 48(8) = x^2 - 8x \\
 &\Rightarrow x^2 - 8x - 384 = 0 \\
 &\Rightarrow x^2 - 24x + 16x + (-24 \times 16) = 0 \\
 &\Rightarrow x(x - 24) + 16(x - 24) = 0 \\
 &\Rightarrow (x - 24)(x + 16) = 0 \\
 &\Rightarrow (x - 24) = 0 \text{ or } (x + 16) = 0 \\
 &\Rightarrow x = 24 \text{ or } x = -16
 \end{aligned}$$

Since the value of number of students cannot be negative, the required number of students attended the picnic is 24.

4. Out of a group of swans, $7/2$ times the square root of the total number are playing on the share of a pond. The two remaining ones are swinging in water. Find the total number of swans.

Sol:

Let total number of swans be x

Given that $7/2$ times the square root of the total number of swans are playing on the share of a pond i.e. $\frac{7}{2}\sqrt{x}$ and the two remaining ones are swinging in water

$$\begin{aligned}
 &\Rightarrow \text{Total number of swans } x = \frac{7}{2}\sqrt{x} + 2 \\
 &\Rightarrow x = \frac{7}{2}\sqrt{x} + 2 \quad [\text{Let } \sqrt{x} = y \Rightarrow x = y^2] \\
 &\Rightarrow y^2 = \frac{7}{2}y + 2 \\
 &\Rightarrow y^2 - \frac{7}{2}y - 2 = 0
 \end{aligned}$$

$$\begin{aligned}
 &\Rightarrow 2y^2 - 7y - 4 = 0 \\
 &\Rightarrow 2y^2 - 8y + y - 4 = 0 \\
 &\Rightarrow 2y(y - 4) + 1(y - 4) = 0 \\
 &\Rightarrow (y - 4)(2y + 1) = 0 \\
 &\Rightarrow (y - 4) = 0 \text{ or } (2y + 1) = 0 \\
 &\Rightarrow y = 4 \text{ or } y = \frac{-1}{2} \\
 &\Rightarrow y^2 = 4^2 = 16 \text{ or } y^2 = \left(\frac{-1}{2}\right)^2 = \frac{1}{4}
 \end{aligned}$$

Since, the value of number of swans cannot be a fraction, the required number of swans
 $x = 16$

5. If the list price of a toy is reduced by Rs. 2, a person can buy 2 toys more for Rs. 360. Find the original price of the toy.

Sol:

Let initial list price of the toy be Rs x

Given that total cost of toys = Rs 360

$$\Rightarrow \text{Initially number of toys a person can buy} = \frac{\text{Total cost}}{\text{list price of each toy}} = \frac{\text{Rs } 360}{\text{Rs } x} \Rightarrow \frac{360}{x}$$

Now, if the list price is reduced by Rs 2 i.e. Rs. $(x - 2)$

Number of toys a person can buy is 2 more for Rs 360

$$\Rightarrow \text{Number of toys a person can buy when price is reduced} = \frac{\text{Total cost}}{\text{list price}} = \frac{\text{Rs } 360}{\text{Rs } x-2} = \frac{360}{x-2}$$

Now,

$$\frac{360}{x-2} - \frac{360}{x} = 2$$

$$\Rightarrow 360 \left(\frac{1}{x-2} - \frac{1}{x} \right) = 2$$

$$\Rightarrow 360 \left(\frac{x-(x-2)}{x(x-2)} \right) = 2$$

$$\Rightarrow 360 \left(\frac{x-x+2}{x^2-2x} \right) = 2$$

$$\Rightarrow 360 \left(\frac{2}{x^2-2x} \right) = 2$$

$$\Rightarrow 360 = x^2 - 2x$$

6. Rs. 9000 were divided equally among a certain number of persons. Had there been 20 more persons, each would have got Rs. 160 less. Find the original number of persons.

Sol:

Let the original number of persons be x ,

Total amount to be divided equally is Rs. 9000

$$\Rightarrow \text{Share of each person will be equal to} = \frac{\text{Total amount}}{\text{No.of persons}} = \text{Rs } \frac{900}{x}$$

Given that if there had been 20 more persons

\Rightarrow Final number of persons will be $x + 20$, then each would have got Rs 160 less

Now,

Final share of each person will be equal to $= \frac{\text{Total amount}}{\text{No.of persons}} = \text{Rs} \frac{9000}{x+20}$

We have,

$$\text{Rs} \frac{9000}{x} - \text{Rs} \frac{9000}{x+20} = \text{Rs} 160$$

$$\Rightarrow 9000 \left(\frac{1}{x} - \frac{1}{x+20} \right) = 160$$

$$\Rightarrow 9000 \left(\frac{7+80-x}{x(x+20)} \right) = 160$$

$$\Rightarrow 9000 \left(\frac{20}{x^2+20x} \right) = 160$$

$$\Rightarrow 1125 = x^2 + 20x$$

$$\Rightarrow x^2 + 20x - 1125 = 0$$

$$\Rightarrow x^2 + 45x - 25x + (45 \times -25) = 0$$

$$\Rightarrow x(x + 45) - 25(x + 45) = 0$$

$$\Rightarrow (x + 45)(x - 25) = 0$$

$$\Rightarrow x + 45 = 0 \text{ or } x - 25 = 0$$

$$\Rightarrow x = -45 \text{ or } x = 25$$

Since, share of each person cannot be negative value, the required share of each person is Rs 25.

7. Some students planned a picnic. The budget for food was Rs. 500. But, 5 of them failed to go and thus the cost of food for each member increased by Rs. 5. How many students attended the picnic?

Sol:

Let the number of students planned for the picnic be x

Given budget for food = Rs. 500

$$\Rightarrow \text{Initially share of food for each student} = \frac{\text{total budget}}{\text{no.of students}} = \text{Rs} \frac{500}{x}$$

Given that 5 students failed to go for the picnic

\Rightarrow No. of students attended the picnic will be $(x - 5)$

$$\text{Now, share of food for each student will be equal to} = \frac{\text{total budget}}{\text{no.of students attended}} = \text{Rs} \frac{500}{x-5}$$

Given that, share of food for each student is increased

$$\Rightarrow \text{Rs} \frac{500}{x-5} - \text{Rs} \frac{500}{x} = \text{Rs} 5$$

$$\Rightarrow \frac{500}{x-5} - \frac{500}{x} = 5$$

$$\Rightarrow 500 \left(\frac{1}{x-5} - \frac{1}{x} \right) = 5$$

$$\Rightarrow 500 \left(\frac{s-(x-5)}{x(x-5)} \right) = 5$$

$$\Rightarrow 500 \left(\frac{x-x+5}{x^2-5x} \right) = 5$$

$$\Rightarrow 500 \left(\frac{5}{x^2-5x} \right) = 5$$

$$\begin{aligned}
 \Rightarrow 500 &= x^2 - 5x \\
 \Rightarrow x^2 - 5x - 500 &= 0 \\
 \Rightarrow x^2 - 25x + 20x - 500 &= 0 \\
 \Rightarrow x(x - 25) + 20(x - 25) &= 0 \\
 \Rightarrow (x - 25)(x + 20) &= 0 \\
 \Rightarrow (x - 25) = 0 \text{ or } (x + 20) &= 0 \\
 \Rightarrow x = 25 \text{ or } x = -20
 \end{aligned}$$

Since, the value of x cannot be negative,

$$\Rightarrow x = 25$$

Here, x is the no. of students planned,

Given that 5 students failed to go

$$\Rightarrow \text{No. of students attended the picnic} = x - 5 = 25 - 5 = 20$$

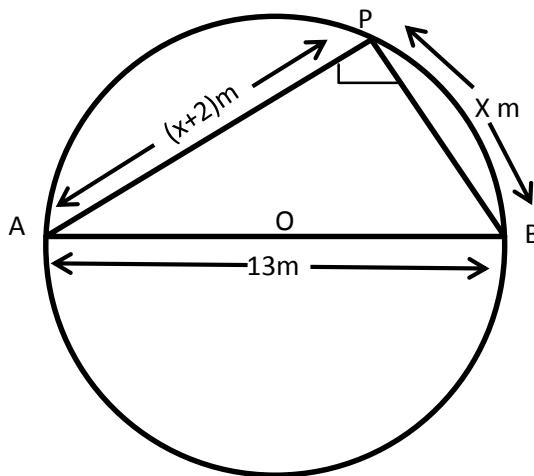
$$\therefore \text{No. of students attended the picnic} = 20$$

8. A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the difference of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?

Sol:

Let P be the required location of the pole such that its distance from gate B is x meters i.e.

$$BP = x \text{ meters and also } AP - BP = 7 \text{ m} \Rightarrow AP = (x + 7) \text{ m}$$



Since, AB is a diameter and P is a point in the semi-circle ΔAPB is right angled at P.

$$\text{Now, } (x + 7)^2 + (x)^2 = (13)^2 \quad [\because AP^2 + BP^2 = AB^2 \text{ and } AB = 13 \text{ m}]$$

$$\Rightarrow x^2 + 14x + 49 + x^2 = 169$$

$$\Rightarrow 2x^2 + 14x + (49 - 169) = 0$$

$$\Rightarrow 2x^2 + 14x - 180 = 0$$

$$\Rightarrow 2(x^2 + 7x - 60) = 0$$

$$\Rightarrow x^2 + 7x - 60 = 0$$

$$\Rightarrow x^2 + 12x - 5x + (12x - 5) = 0$$

$$\Rightarrow x(x + 12) - 5(x + 12) = 0$$

$$\Rightarrow (x + 12)(x - 5) = 0$$

$$\Rightarrow x + 12 = 0 \text{ or } x - 5 = 0$$

$$\Rightarrow x = -12 \text{m or } x = 5 \text{m}$$

Since, BP cannot be in negative value (or) distances cannot be negative values,

The required values of BP and AP are 5m and 12m respectively.

\therefore The pole has to be erected at a distance 5meters from the gate B.

9. In a class test, the sum of the marks obtained by P in Mathematics and science is 28. Had he got 3 marks more in Mathematics and 4 marks less in Science. The product of his marks, would have been 180. Find his marks in the two subjects.

Sol:

Let number of marks obtained by P in mathematics and science be x and y respectively.

Given that sum of these two is 28

$$\Rightarrow x + y = 28 \Rightarrow x = 28 - y$$

Given that if x becomes $(x + 3)$ i.e. marks in mathematics is increased by 3 and y becomes $(y - 4)$ i.e. marks in science is decreased by 4, The product of these two becomes by 4,

$$\Rightarrow (x + 3)(y - 4) = 180$$

$$\Rightarrow (28 - y + 3)(y - 4) = 180 \quad [\because x = 28 - y]$$

$$\Rightarrow (31 - y)(y - 4) = 180$$

$$\Rightarrow 31y - 31 \times 4 - y^2 + 4y = 180$$

$$\Rightarrow 35y - y^2 - 124 = 180$$

$$\Rightarrow y^2 - 35y + 180 + 124 = 0$$

$$\Rightarrow y^2 - 35y + 304 = 0$$

$$\Rightarrow y^2 - 19y - 16y + (-19 \times -16) = 0$$

$$\Rightarrow y(y - 19) - 16(y - 19) = 0$$

$$\Rightarrow (y - 19)(y - 16) = 0$$

$$\Rightarrow y - 19 = 0 \text{ or } y - 16 = 0$$

$$\Rightarrow y = 19 \text{ or } y = 16$$

We have,

$$x + y = 28$$

$$\text{if } y = 19 \Rightarrow x = 28 - y = 28 - 19 = 9 \text{ and}$$

$$\text{if } y = 16 \Rightarrow x = 28 - y = 28 - 16 = 12$$

\therefore Marks in mathematics = 9 and Marks in Science = 19 or

Marks in mathematics = 12 and Marks in Science = 16

10. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210. Find her marks in two subjects.

Sol:

Let marks of shefali in Mathematics and English be x and y respectively.

Given that sum of these two is $30 \Rightarrow x + y = 30 \Rightarrow x = 30 - y$

Given that if x becomes $(x + 2)$ i.e. marks in mathematics is increased by 2 and y becomes $(y - 3)$ i.e. marks in English is decreased by 3,

the product at these two becomes 210

$$\text{i.e. } (x + 2)(y - 3) = 210$$

$$\Rightarrow (30 - y + 2)(y - 3) = 210 \quad [\because x = 30 - y]$$

$$\Rightarrow (32 - y)(y - 3) = 210$$

$$\Rightarrow (32 - y)(y - 3) = 210$$

$$\Rightarrow 32y - 32 \times 3 - y \times 3y = 210$$

$$\Rightarrow 35y - 96 - y^2 = 210$$

$$\Rightarrow y^2 - 35y + 210 + 96 = 0$$

$$\Rightarrow y^2 - 35 + 306 = 0$$

$$\Rightarrow y^2 - 17y - 18y + (-17 \times -18) = 0 \quad [\because 306 = 17 \times 18 = -17 \times -18]$$

$$\Rightarrow y(y - 17) - 18(y - 17) = 0$$

$$\Rightarrow (y - 17)(y - 18) = 0$$

$$\Rightarrow y - 17 = 0 \text{ or } y - 18 = 0$$

$$\Rightarrow y = 17 \text{ or } y = 18$$

We have,

$$x + y = 30$$

$$\text{if } y = 17 \Rightarrow x = 30 - y = 30 - 17 = 13 \text{ and}$$

$$\text{if } y = 18 \Rightarrow x = 30 - y = 30 - 18 = 12$$

\therefore Marks in Mathematics = 13 and marks in English = 17 or

Marks in Mathematics = 12 and marks in English = 18.

11. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs. 90, find the number of articles produced and the cost of each article.

Sol:

Let the number of articles produced on a particular day be x .

Total cost of production on that particular day = Rs 90

$$\text{Given } \Rightarrow \text{Cost of production of each article} = \frac{\text{Total cost of production}}{\text{no.of articles produced}} = \text{Rs } \frac{90}{x}$$

But given that, the cost of production of each article was 3 more than twice the no. of articles produced on that day i.e. Rs $(2x + 3)$

$$\Rightarrow \text{Rs } (2x + 3) = \text{Rs } \frac{90}{x}$$

$$\Rightarrow 2x + 3 = \frac{90}{x}$$

$$\Rightarrow x(2x + 3) = 90 \Rightarrow 2x^2 + 3x - 90 = 0$$

$$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$$

$$\Rightarrow x(2x + 15) - 6(2x + 15) = 0$$

$$\Rightarrow (2x + 15)(x - 6) = 0$$

$$\Rightarrow 2x + 15 = 0 \text{ or } x - 6 = 0$$

$$\Rightarrow x = \frac{-15}{2} \text{ or } x = 6$$

Since, number of articles x cannot be a negative value, the required value of number of articles produced on a particular day x = 6.