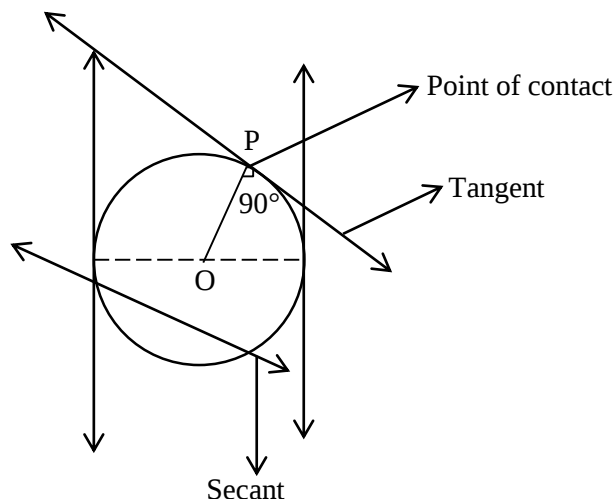


Exercise – 10.1

1. Fill in the blanks

- (i) The common point of tangent and the circle is called point of contact.
- (ii) A circle may have two parallel tangents.
- (iii) A tangent to a circle intersects it in one point.
- (iv) A line intersecting a circle in two points is called a secant.
- (v) The angle between tangent at a point P on circle and radius through the point is 90° .

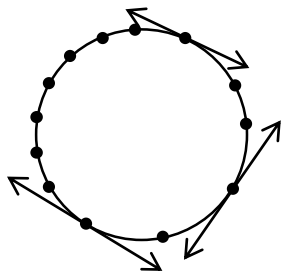
Sol:



2. How many tangents can a circle have?

Sol:

Tangent: A line intersecting circle in one point is called a tangent.

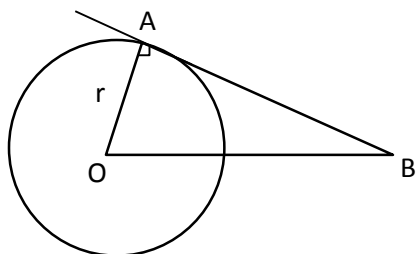


As there are infinite number of points on the circle a circle has many (infinite) tangents.

3. O is the center of a circle of radius 8cm. The tangent at a point A on the circle cuts a line through O at B such that $AB = 15$ cm. Find OB

Sol:

Consider a circle with center O and radius $OA = 8\text{cm} = r$, $AB = 15$ cm.



(AB) tangent is drawn at A (point of contact)

At point of contact, we know that radius and tangent are perpendicular.

In $\triangle OAB$, $\angle OAB = 90^\circ$, By Pythagoras theorem

$$OB^2 = OA^2 + AB^2$$

$$OB = \sqrt{8^2 + 15^2}$$

$$= \sqrt{64 + 225} = \sqrt{229} = 17 \text{ cm}$$

$$\therefore OB = 17 \text{ cm}$$

4. If the tangent at point P to the circle with center O cuts a line through O at Q such that PQ = 24 cm and OQ = 25 cm. Find the radius of circle

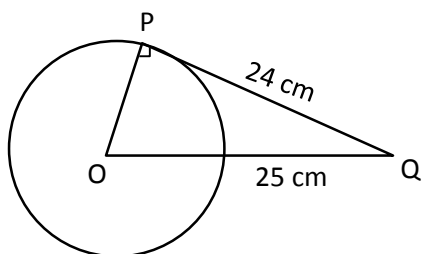
Sol:

Given,

$$PQ = 24 \text{ cm}$$

$$OQ = 25 \text{ cm}$$

$$OP = \text{radius} = ?$$



P is point of contact, At point of contact, tangent and radius are perpendicular to each other

$\therefore \triangle POQ$ is right angled triangle $\angle OPQ = 90^\circ$

By Pythagoras theorem,

$$PQ^2 + OP^2 = OQ^2$$

$$\Rightarrow 24^2 + OP^2 = 25^2$$

$$\Rightarrow OP = \sqrt{25^2 - 24^2} = \sqrt{625 - 576}$$

$$= \sqrt{49} = 7 \text{ cm}$$

$$\therefore OP = \text{radius} = 7 \text{ cm}$$

Exercise – 10.2

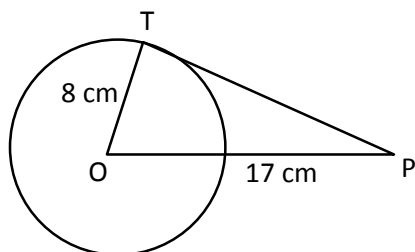
1. If PT is a tangent at T to a circle whose center is O and $OP = 17$ cm, $OT = 8$ cm. Find the length of tangent segment PT.

Sol:

$OT = \text{radius} = 8\text{cm}$

$OP = 17\text{cm}$

$PT = \text{length of tangent} = ?$



T is point of contact. We know that at point of contact tangent and radius are perpendicular.

$\therefore$ OTP is right angled triangle $\angle OTP = 90^\circ$, from Pythagoras theorem $OT^2 + PT^2 = OP^2$

$$8^2 + PT^2 = 17^2$$

$$PT = \sqrt{17^2 - 8^2} = \sqrt{289 - 64}$$

$$= \sqrt{225} = 15\text{cm}$$

$\therefore$ PT = length of tangent = 15 cm.

2. Find the length of a tangent drawn to a circle with radius 5cm, from a point 13 cm from the center of the circle.

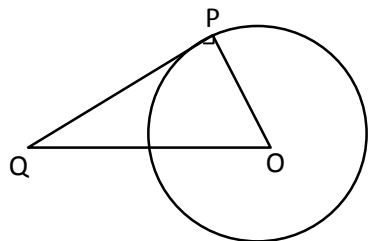
Sol:

Consider a circle with center O.

$OP = \text{radius} = 5$ cm.

A tangent is drawn at point P, such that line through O intersects it at Q, $OQ = 13\text{cm}$.

Length of tangent PQ = ?



At P, we know that tangent and radius are perpendicular.

$\triangle OPQ$ is right angled triangle, $\angle OPQ = 90^\circ$

By pythagoras theorem, $OQ^2 = OP^2 + PQ^2$

$$\Rightarrow 13^2 = 5^2 + PQ^2$$

$$\Rightarrow PQ^2 = 169 - 25 = 144$$

$$\Rightarrow PQ = \sqrt{144} = 12\text{cm}$$

Length of tangent = 12 cm

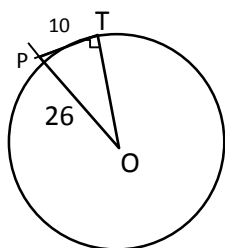
3. A point P is 26 cm away from O of circle and the length PT of the tangent drawn from P to the circle is 10 cm. Find the radius of the circle.

Sol:

Given OP = 26 cm

PT = length of tangent = 10cm

radius = OT = ?



At point of contact, radius and tangent are perpendicular $\angle OTP = 90^\circ$, $\triangle OTP$ is right angled triangle.

By Pythagoras theorem, $OP^2 = OT^2 + PT^2$

$$26^2 = OT^2 + 10^2$$

$$OT^2 = (\sqrt{676 - 100})^2$$

$$OT = \sqrt{576}$$

$$= 24 \text{ cm}$$

OT = length of tangent = 24 cm

4. If from any point on the common chord of two intersecting circles, tangents be drawn to circles, prove that they are equal.

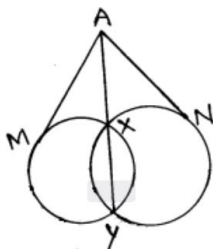
Sol:

Let the two circles intersect at points X and Y.

XY is the common chord.

Suppose 'A' is a point on the common chord and AM and AN be the tangents drawn A to the circle

We need to show that $AM = AN$.



In order to prove the above relation, following property will be used.

“Let PT be a tangent to the circle from an external point P and a secant to the circle through P intersects the circle at points A and B, then $PT^2 = PA \times PB$ ”

Now AM is the tangent and AXY is a secant $\therefore AM^2 = AX \times AY \dots (i)$

AN is a tangent and AXY is a secant $\therefore AN^2 = AX \times AY \dots (ii)$

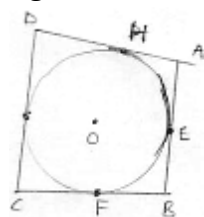
From (i) & (ii), we have $AM^2 = AN^2$

$\therefore AM = AN$

5. If the quadrilateral sides touch the circle prove that sum of pair of opposite sides is equal to the sum of other pair.

Sol:

Consider a quadrilateral ABCD touching circle with center O at points E, F, G and H as in figure.



We know that

The tangents drawn from same external points to the circle are equal in length.

1. Consider tangents from point A [AM $\perp$ AE]

$$AH = AE \dots (i)$$

2. From point B [EB & BF]

$$BF = EB \dots (ii)$$

3. From point C [CF & GC]

$$FC = CG \dots (iii)$$

4. From point D [DG & DH]

$$DH = DG \dots (iv)$$

Adding (i), (ii), (iii), & (iv)

$$(AH + BF + FC + DH) = [(AC + CB) + (CG + DG)]$$

$$\Rightarrow (AH + DH) + (BF + FC) = (AE + EB) + (CG + DG)$$

$$\Rightarrow AD + BC = AB + DC \quad [\text{from fig.}]$$

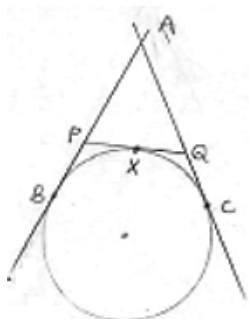
Sum of one pair of opposite sides is equal to other.

6. If AB, AC, PQ are tangents in Fig. and AB = 5cm find the perimeter of $\triangle APQ$.

Sol:

$$\text{Perimeter of } \triangle APQ, (P) = AP + AQ + PQ$$

$$= AP + AQ + (PX + QX)$$



We know that

The two tangents drawn from external point to the circle are equal in length from point A,
 $AB = AC = 5$ cm

From point P, $PX = PB$

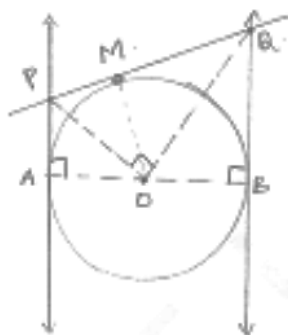
From point Q, $QX = QC$

$$\begin{aligned}\text{Perimeter (P)} &= AP + AQ + (PB + QC) \\ &= (AP + PB) + (AQ + QC) \\ &= AB + AC = 5 + 5 \\ &= 10 \text{ cms.}\end{aligned}$$

7. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at center.

Sol:

Consider circle with center 'O' and has two parallel tangents through A & B at ends of diameter.



Let tangents through M intersects the tangents parallel at P and Q required to prove is that $\angle POQ = 90^\circ$.

From fig. it is clear that ABQP is a quadrilateral

$$\angle A + \angle B = 90^\circ + 90^\circ = 180^\circ \text{ [At point of contact tangent \& radius are perpendicular]}$$

$$\angle A + \angle B + \angle P + \angle Q = 360^\circ \text{ [Angle sum property]}$$

$$\angle P + \angle Q = 360^\circ - 180^\circ = 180^\circ \dots\dots(i)$$

$$\text{At P \& Q } \angle APO = \angle OPQ = \frac{1}{2} \angle P$$

$$\angle BQO = \angle PQO = \frac{1}{2} \angle Q \quad \text{in (i)}$$

$$2\angle OPQ + 2\angle PQO = 180^\circ$$

$$\angle OPQ + \angle PQO = 90^\circ \dots (ii)$$

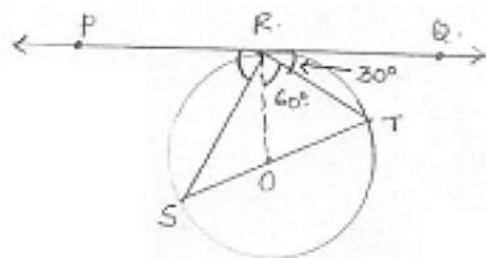
In $\triangle OPQ$, $\angle OPQ + \angle PQO + \angle POQ = 180^\circ$ [Angle sum property]

$$90^\circ + \angle POQ = 180^\circ \text{ [from (ii)]}$$

$$\angle POQ = 180^\circ - 90^\circ = 90^\circ$$

$$\therefore \angle POQ = 90^\circ$$

8. In Fig below, PQ is tangent at point R of the circle with center O. If $\angle TRQ = 30^\circ$. Find $\angle PRS$.



Sol:

Given $\angle TRQ = 30^\circ$.

At point R, $OR \perp RQ$.

$$\angle ORQ = 90^\circ$$

$$\Rightarrow \angle TRQ + \angle ORT = 90^\circ$$

$$\Rightarrow \angle ORT = 90^\circ - 30^\circ = 60^\circ$$

ST is diameter, $\angle SRT = 90^\circ$ [$\because$ Angle in semicircle = 90°]

$$\angle ORT + \angle SRO = 90^\circ$$

$$\angle SRO + \angle PRS = 90^\circ$$

$$\angle PRS = 90^\circ - 30^\circ = 60^\circ$$

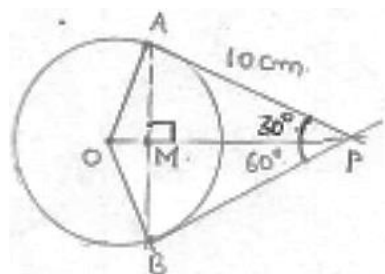
9. If PA and PB are tangents from an outside point P. such that $PA = 10$ cm and $\angle APB = 60^\circ$. Find the length of chord AB.

Sol:

$AP = 10$ cm $\angle APB = 60^\circ$

Represented in the figure

We know that



A line drawn from center to point from where external tangents are drawn divides or bisects the angle made by tangents at that point $\angle APO = \angle OPB = \frac{1}{2} \times 60^\circ = 30^\circ$

The chord AB will be bisected perpendicularly

$$\therefore AB = 2AM$$

In $\triangle AMP$,

$$\sin 30^\circ = \frac{\text{opp.side}}{\text{hypotenuse}} = \frac{AM}{AP}$$

$$AM = AP \sin 30^\circ$$

$$= \frac{AP}{2} = \frac{10}{2} = 5\text{cm}$$

$$AP = 2 AM = 10\text{ cm}$$

---- Method (i)

In $\triangle AMP$, $\angle AMP = 90^\circ$, $\angle APM = 30^\circ$

$$\angle AMP + \angle APM + \angle MAP = 180^\circ$$

$$90^\circ + 30^\circ + \angle MAP = 180^\circ$$

$$\angle MAP = 60^\circ$$

In $\triangle PAB$, $\angle MAP = \angle BAP = 60^\circ$, $\angle APB = 60^\circ$

We also get, $\angle PBA = 60^\circ$

$\therefore \triangle PAB$ is equilateral triangle

$$AB = AP = 10\text{ cm.}$$

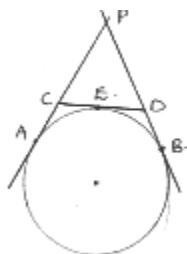
-----Method (ii)

10. From an external point P, tangents PA and PB are drawn to the circle with centre O. If CD is the tangent to the circle at point E and PA = 14 cm. Find the perimeter of ABCD.

Sol:

$$PA = 14\text{ cm}$$

$$\text{Perimeter of } \triangle PCD = PC + PD + CD = PC + PD + CE + ED$$



We know that

The two tangents drawn from external point to the circle are equal in length.

From point P, $PA = PB = 14\text{cm}$

From point C, $CE = CA$

From point D, $DB = ED$

$$\text{Perimeter} = PC + PD + CA + DB$$

$$= (PC + CA) + (PD + DB)$$

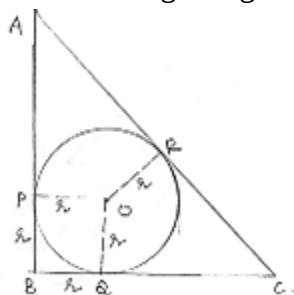
$$= PA + PB = 14 + 14 = 28\text{ cm.}$$

11. In the fig. ABC is right triangle right angled at B such that BC = 6cm and AB = 8cm. Find the radius of its in circle.

Sol:

$$BC = 6\text{cm } AB = 8\text{cm}$$

As ABC is right angled triangle



By Pythagoras theorem

$$AC^2 = AB^2 + BC^2 = 6^2 + 8^2 = 100$$

$$AC = 10\text{ cm}$$

Consider BQOP $\angle B = 90^\circ$,

$\angle BPO = \angle OQB = 90^\circ$ [At point of contact, radius is perpendicular to tangent]

All the angles $= 90^\circ$ & adjacent sides are equal

$\therefore$ BQOP is square $BP = BQ = r$

We know that

The tangents drawn from any external point are equal in length.

$$AP = AR = AB - PB = 8 - r$$

$$QC = RC = BC - BQ = 6 - r$$

$$AC = AR + RC \Rightarrow 10 = 8 - r + 6 - r$$

$$\Rightarrow 10 = 14 - 2r$$

$$\Rightarrow 2r = 4$$

$$\Rightarrow \text{Radius} = 2\text{cm}$$

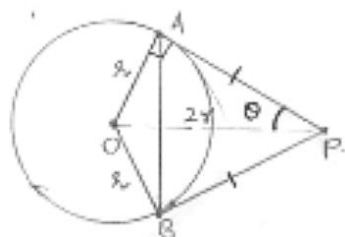
12. From a point P, two tangents PA and PB are drawn to a circle with center O. If OP = diameter of the circle shows that $\triangle APB$ is equilateral.

Sol:

$$OP = 2r$$

Tangents drawn from external point to the circle are equal in length

$$PA = PB$$



At point of contact, tangent is perpendicular to radius.

$$\text{In } \triangle AOP, \sin \theta = \frac{\text{opp.side}}{\text{hypotenuse}} = \frac{r}{2r} = \frac{1}{2}$$

$$\theta = 30^\circ$$

$\angle APB = 120^\circ$, as $PA = PB$ $\angle BAP = \angle ABP = x$.

In $\triangle PAB$, by angle sum property

$$\angle APB + \angle BAP + \angle ABP = 180^\circ$$

$$2x = 120^\circ \Rightarrow x = 60^\circ$$

In this triangle all angles are equal to 60°

$\therefore \triangle APB$ is equilateral.

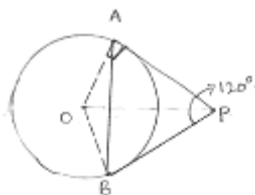
13. Two tangent segments PA and PB are drawn to a circle with center O such that $\angle APB = 120^\circ$. Prove that $OP = 2AP$

Sol:

A + P

OP bisects $\angle APB$

$$\angle APO = \angle OPB = \frac{1}{2} \angle APB = \frac{1}{2} \times 120^\circ = 60^\circ$$



At point A

$$OA \perp AP, \angle OAP = 90^\circ$$

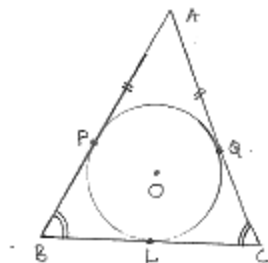
$$\text{In } \triangle OAP, \cos 60^\circ = \frac{AP}{OP}$$

$$\frac{1}{2} = \frac{AP}{OP} \Rightarrow OP = 2AP$$

14. If $\triangle ABC$ is isosceles with $AB = AC$ and C (0, 2) is the in circle of the $\triangle ABC$ touching BC at L , prove that L , bisects BC .

Sol:

Given $\triangle ABC$ is isosceles $AB = AC$



We know that

The tangents from external point to circle are equal in length

From point A, $AP = AQ$

$$\text{But } AB = AC \Rightarrow AP + PB = AQ + QC$$

$$\Rightarrow PB = PC \dots (i)$$

From B, $PB = BL$;(ii) from C, $CL = CQ$ (iii)

From (i), (ii) & (iii)

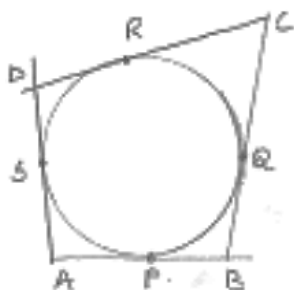
$$BL = CL$$

$\therefore L$ bisects BC .

15. In fig. a circle touches all the four sides of quadrilateral $ABCD$ with $AB = 6\text{cm}$, $BC = 7\text{cm}$, $CD = 4\text{cm}$. Find AD .

Sol:

We know that the tangents drawn from any external point to circle are equal in length.



From A $\rightarrow$ $AS = AP$ (i)

From B $\rightarrow$ $QB = BP$ (ii)

From C $\rightarrow$ $QC = RC$ (iii)

From D $\rightarrow$ $DS = DR$ (iv)

Adding (i), (ii), (iii) & (iv)

$$(AS + QB + QC + DS) = (AB + BP + RC + OR)$$

$$(AS + DS) + (QB + QC) = (AP + BP) + (RC + DR)$$

$$AD + BC = AB + CD$$

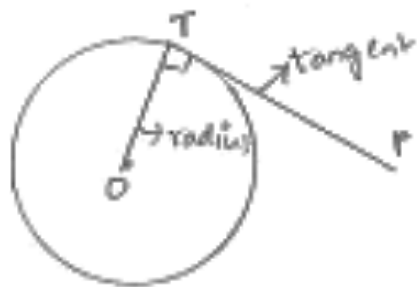
$$\Rightarrow AD + 7 = 6 + 4 \quad AD = 3\text{cm}$$

$$\Rightarrow AD = 10 - 7 = 3\text{cm}$$

16. Prove that the perpendicular at the point of contact to a circle passes through the centre of the circle.

Sol:

We know that



The at point of contact, the tangent is perpendicular to the radius. Radius is line from center to point on circle. Therefore, perpendicular to tangent will pass through center of circle.

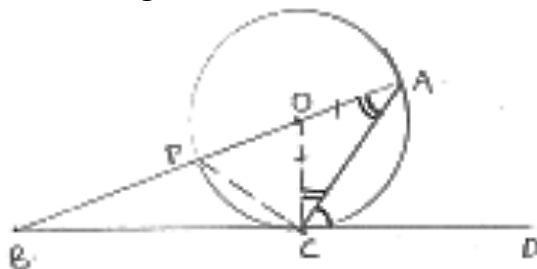
17. In fig., O is the center of the circle and BCD is tangent to it at C. Prove that $\angle BAC + \angle ACD = 90^\circ$

Sol:

Given

O is center of circle

BCD is tangent.



Required to prove: $\angle BAC + \angle ACD = 90^\circ$

Proof: $OA = OC$ [radius]

In $\triangle OAC$, angles opposite to equal sides are equal.

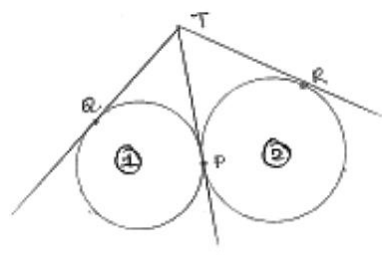
$$\angle OAC = \angle OCA \dots (i)$$

$\angle OCD = 90^\circ$ [tangent is radius are perpendicular at point of contact]

$$\angle ACD + \angle OCA = 90^\circ$$
$$\angle ACD + \angle OAC = 90^\circ \quad [\because \angle OAC = \angle BAC]$$
$$\angle ACD + \angle BAC = 90^\circ \rightarrow \text{Hence proved}$$

18. Two circles touch externally at a point P. From a point T on the tangent at P, tangents TQ and TR are drawn to the circles with points of contact Q and E respectively. Prove that $TQ = TR$.

Sol:



Let the circles be represented by (i) & (ii) respectively

TQ, TP are tangents to (i)

TP, TR are tangents to (ii)

We know that

The tangents drawn from external point to the circle will be equal in length.

For circle (i), $TQ = TP \dots (i)$

For circle (ii), $TP = TR \dots (ii)$

From (i) & (ii) $TQ = TR$

19. In the fig. a circle is inscribed in a quadrilateral ABCD in which $\angle B = 90^\circ$ if $AD = 23\text{cm}$, $AB = 29\text{cm}$ and $DS = 5\text{cm}$, find the radius of the circle.

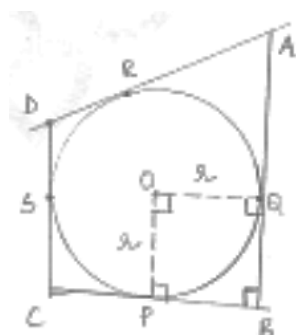
Sol:

Given $AD = 23\text{ cm}$

$AB = 29\text{ cm}$

$\angle B = 90^\circ$

$DS = 5\text{cm}$



From fig in quadrilateral POQB

$\angle OPB = \angle OQB = 90^\circ = \angle B = \angle POQ$

and $PO = OQ$. $\therefore$ POQB is a square $PB = BQ = r$

We know that

Tangents drawn from external point to circle are equal in length.

We know that

Tangents drawn from external point to circle are equal in length.

From A, $AR = AQ \dots (i)$

From B, $PB = QB \dots (ii)$

From C, $PC = CS \dots (iii)$

From D, $DR = DS \dots (iv)$

$$(i) + (ii) + (iv) \Rightarrow AR + DB + DR = AQ + QB + DS$$

$$\Rightarrow (AR + DR) + r = (AQ + QB) + DS$$

$$AD + r = AB + DS$$

$$\Rightarrow 23 + r = 29 + 5$$

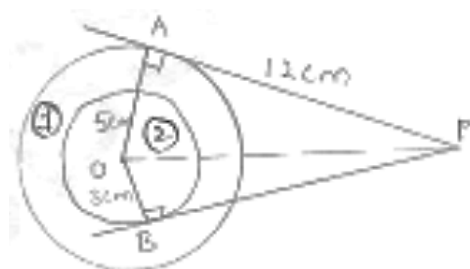
$$\Rightarrow r = 34 - 23 = 11\text{ cm}$$

$$\therefore \text{radius} = 11\text{ cm}$$

20. In fig. there are two concentric circles with Centre O of radii 5cm and 3cm. From an external point P, tangents PA and PB are drawn to these circles if $AP = 12\text{cm}$, find the tangent length of BP.

Sol:

Given



$$OA = 5 \text{ cm}$$

$$OB = 3 \text{ cm}$$

$$AP = 12 \text{ cm}$$

$$BP = ?$$

We know that

At the point of contact, radius is perpendicular to tangent.

For circle 1, $\triangle OAP$ is right triangle

By Pythagoras theorem, $OP^2 = OA^2 + AP^2$

$$\Rightarrow OP^2 = 5^2 + 12^2 = 25 + 144$$

$$= 169$$

$$\Rightarrow OP = \sqrt{169} = 13 \text{ cm}$$

For circle 2, $\triangle OBP$ is right triangle by Pythagoras theorem,

$$OP^2 = OB^2 + BP^2$$

$$13^2 = 3^2 + BP^2$$

$$BP^2 = 169 - 9 = 160$$

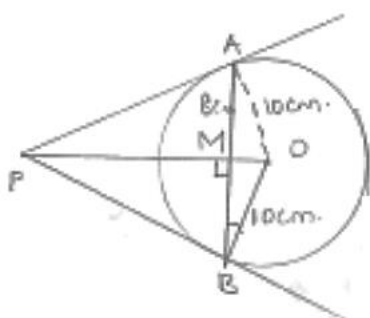
$$BP = \sqrt{160} = 4\sqrt{10} \text{ cm}$$

21. In fig. AB is chord of length 16cm of a circle of radius 10cm. The tangents at A and B intersect at a point P. Find the length of PA.

Sol:

Given length of chord AB = 16cm.

Radius OB = OA = 10 cm.



Let line through Centre to point from where tangents are drawn be intersecting chord AB at M. we know that the line joining Centre to point from where tangents are drawn be intersecting chord AB at M. we know that

The line joining Centre to point from where tangents are drawn bisects the chord joining the points on the circle where tangents intersect the circle.

$$AM = MB = \frac{1}{2}(AB) = \frac{1}{2} \times 16 = 8\text{cm}$$

Consider $\triangle OAM$ from fig. $\angle AMO = 90^\circ$

By Pythagoras theorem, $OA^2 = AM^2 + OM^2$

$$10^2 = 8^2 + OM^2$$

$$OM = \sqrt{100 - 64} = \sqrt{36} = 6\text{cm}$$

In $\triangle AMP$, $\angle AMP = 90^\circ$ by Pythagoras theorem $AP^2 = AM^2 + PM^2$

$$AP^2 = 8^2 + (OP - OM)^2$$

$$PA^2 = 64 + (OP - 6)^2$$

$$(OP - 6)^2 = -64 + PA^2 \dots(i)$$

In $\triangle APO$, $\angle PAO = 90^\circ$ [At point of contact, radius is perpendicular to tangent]

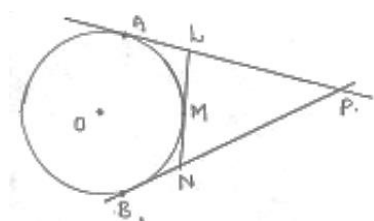
$$OP^2 = OA^2 + PA^2 \quad [\text{Pythagoras theorem}]$$

$$PA^2 = OP^2 - 10^2$$

$$= OP^2 - 100 \dots (ii)$$

22. In figure PA and PB are tangents from an external point P to the circle with centre O. LN touches the circle at M. Prove that $PL + LM = PN + MN$

Sol:



Given

O is Centre of circle

PA and PB are tangents

We know that

The tangents drawn from external point to the circle are equal in length.

From point P, $PA = PB$

$$\Rightarrow PL + AL = PN + NB \dots (i)$$

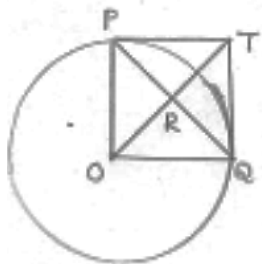
From point L & N, $AL = LM$ and $MN = NB$ } Substitute in (i)

$$PL + Lm = PN + MN$$

$\Rightarrow$ Hence proved.

23. In the fig. $PO \perp QO$. The tangents to the circle at P and Q intersect at a point T. Prove that PQ and OT are right bisectors of each other.

Sol:



Given

$PO \perp OQ$

Consider quadrilateral OQTP.

$\angle POQ = 90^\circ$

$\angle OPT = \angle OQT = 90^\circ$ [At point of contact, tangent and radius are perpendicular]

$\therefore \angle PTO = 90^\circ$

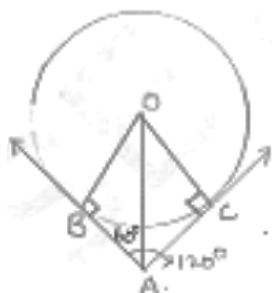
$OP = OQ = \text{radius}$

In this quadrilateral, all the angles are equal and pair of adjacent sides are equal.

$\therefore$ OQTP is a square.

24. In the fig two tangents AB and AC are drawn to a circle O such that $\angle BAC = 120^\circ$. Prove that $OA = 2AB$.

Sol:



Consider Centre O for given circle

$\angle BAC = 120^\circ$

AB and AC are tangents

From the fig.

In $\triangle OBA$, $\angle OBA = 90^\circ$ [radius perpendicular to tangent at point of contact]

$\angle OAB = \angle OAC = \frac{1}{2} \angle BAC = \frac{1}{2} \times 120^\circ = 60^\circ$

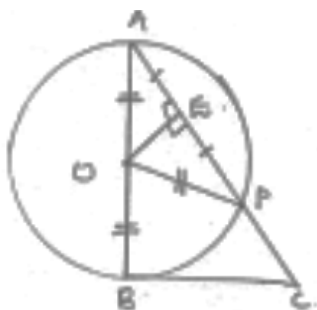
[Line joining Centre to external point from where tangents are drawn bisects angle formed by tangents at that external point]

In $\triangle OBA$, $\cos 60^\circ = \frac{AB}{OA}$

$\frac{1}{2} = \frac{AB}{OA} \Rightarrow OA = 2AB$

25. In the fig. BC is a tangent to the circle with Centre O. OE bisects AP. Prove that $\triangle AEO \sim \triangle ABC$.

Sol:



Given

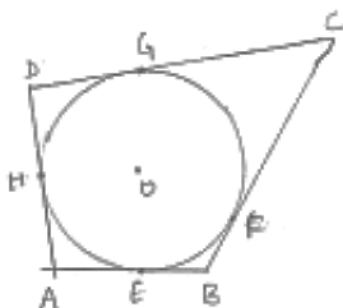
BC is tangent to circle

OE bisects AP, $AE = EP$

Consider $\triangle AOP$

26. The lengths of three consecutive sides of a quadrilateral circumscribing a circle are 4cm, 5cm and 7cm respectively. Determine the length of fourth side.

Sol:



Let us consider a quadrilateral ABCD, $AB = 4\text{cm}$, $BC = 5\text{ cm}$, $CD = 7\text{cm}$, CD as sides circumscribing circle with centre O. and intersecting at points E, F, G, H. as in fig.

We know that the tangents drawn from external point to the circle are equal in length.

From point A, $AE = AH$ (i)

From point B, $BE = BF$ (ii)

From point C, $GC = CE$ (iii)

From point D, $GD = DH$ (iv)

$$(i) + (ii) + (iii) + (iv) \Rightarrow (AE + BE + GC + GD) = (AH + BF + CF + DH)$$

$$\Rightarrow (AE + BE) + (GC + GD) = (AH + DH) + (BF + CF)$$

$$\Rightarrow AB + CD = AD + BC$$

$$\Rightarrow 4 + 7 = 5 + AD$$

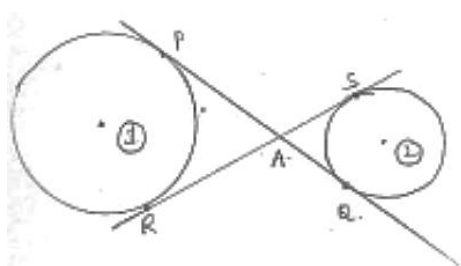
$$\Rightarrow AD = 11 - 5 = 6\text{ cm}$$

Fourth side = 6 cm

27. In fig common tangents PQ and RS to two circles intersect at A. Prove that $PQ = RS$.

Sol:

Consider



Two circles namely (i) & (ii) as shown with common tangents as PQ and RS.

We know that

The tangents from external point to the circle are equal in length.

From A to circle (i) $AP = AR \dots (i)$

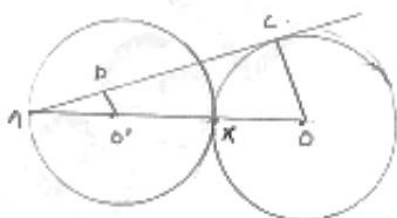
From A to circle (ii), $AQ = AS \dots (ii)$

$(i) + (ii) \Rightarrow AP + AQ = AR + RS$

$\Rightarrow PQ = RS$

28. Equal circles with centers O and O' touch each other at X. OO' produced to meet a circle with Centre O' at A. AC is tangent to the circle whose Centre is a O'D is perpendicular to AC. Find the value of DO'/CO

Sol:



Given circles with centers O and O'

$O'D \perp AC$. Let radius = r

$O'A = O'X = OX = r$

In triangles, $\triangle AO'D$ and $\triangle AOC$

$\angle A = \angle A$ [Common angle]

$\angle ADO' = \angle ACO = 90^\circ$ [$O'D \perp AC$ and at point of contact C, radius $\perp$ tangent]

By A.A similarity $\triangle AO'D \sim \triangle AOC$.

when two triangles are similar then their corresponding sides will be in proportion

By A.A similarity $\triangle AO'D \sim \triangle AOC$

When two triangles are similar then their corresponding sides will be in proportion

$$\frac{AO'}{AO} = \frac{DO'}{CO}$$

$$\Rightarrow \frac{DO'}{CO} = \frac{r}{r+r+r} = \frac{r}{3r} = \frac{1}{3}$$

$$\Rightarrow \frac{DO'}{CO} = \frac{1}{3}$$

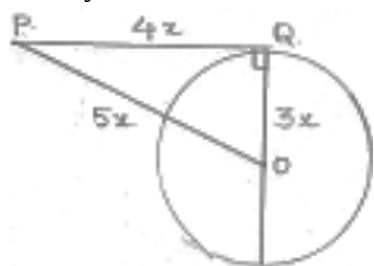
29. In figure $OQ : PQ = 3 : 4$ and perimeter of $\triangle PDQ = 60\text{cm}$. determine PQ , QR and OP .

Sol:

Given $OQ : PQ = 3 : 4$

Let $OQ = 3x$ $PQ = 4x$

$OP = y$



$\angle OQP = 90^\circ$ [since at point of contact, tangent is perpendicular to radius]

In $\triangle OQP$, by Pythagoras theorem

$$OP^2 = OQ^2 + PQ^2$$

$$\Rightarrow y^2 = (3x)^2 + (4x)^2$$

$$\Rightarrow y^2 = 9x^2 + 16x^2 = 25x^2$$

$$\Rightarrow y^2 = \sqrt{25x^2} = 5x$$

$$\text{Perimeter} = OQ + PQ + OP = 3x + 4x + 5x = 12x$$

According to problem perimeter = 60

$$\therefore 12x = 60$$

$$x = \frac{60}{12} = 5\text{cm}$$

$$OQ = 3 \times 5 = 15\text{cm}$$

$$PQ = 4 \times 5 = 20\text{ cm}$$

$$OP = 5 \times 5 = 25\text{cm}$$