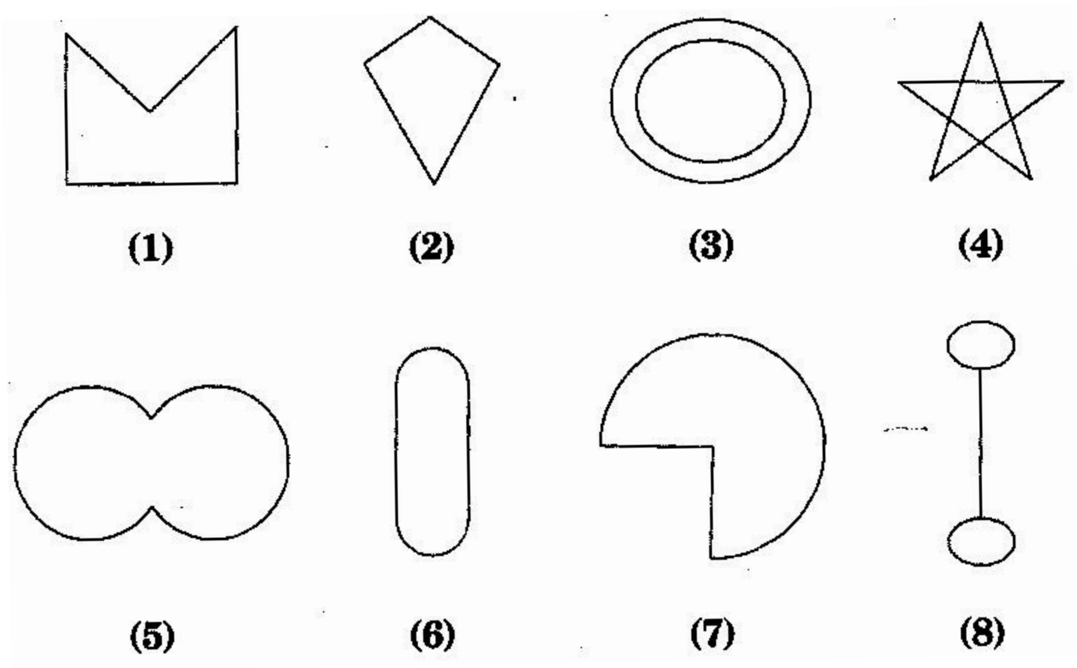
Mathematics

(Chapter – 3) (Understanding Quadrilaterals)
(Class – VIII)

Exercise 3.1

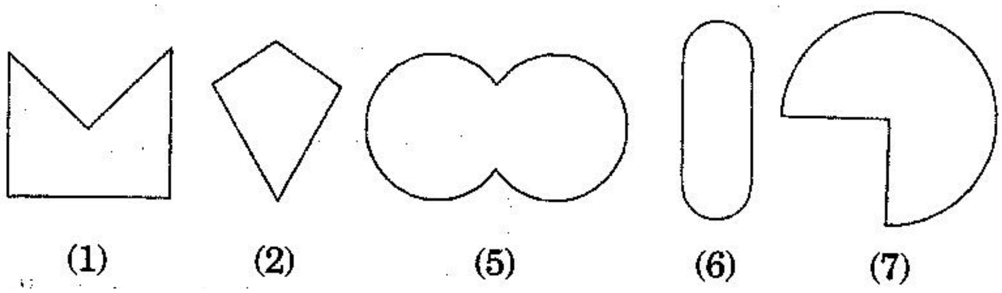
Question 1:
Given here are some figures:



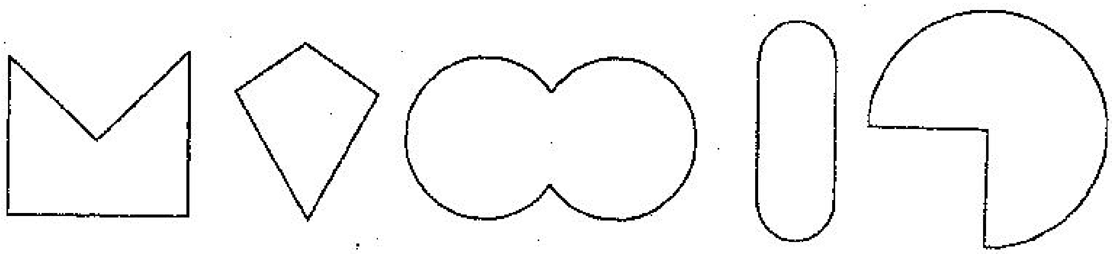
Classify each of them on the basis of the following:
(a) Simple curve
(b) Simple closed curve
(c) Polygon
(d) Convex polygon
(e) Concave polygon

Answer 1:

(a) Simple curve

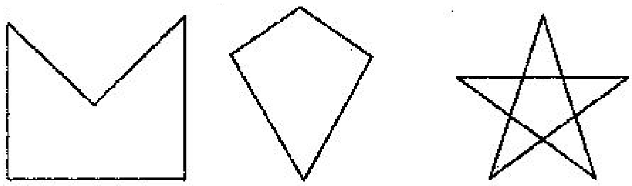


(b) Simple closed curve



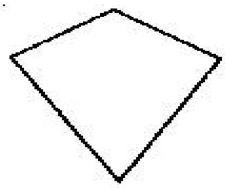
(1) (2) (5) (6) (7)

(c) Polygons



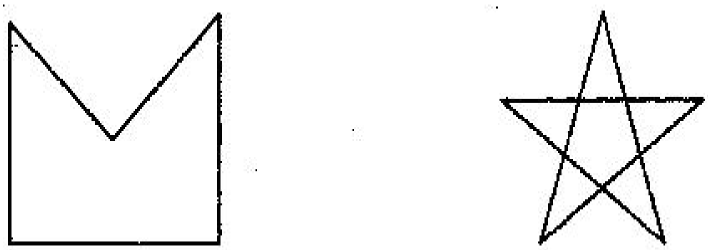
(1) (2) (4)

(d) Convex polygons



(1)

(e) Concave polygon



(1) (4)

Question 2:

How many diagonals does each of the following have?

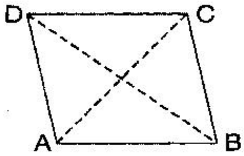
(a) A convex quadrilateral (b) A regular hexagon

(c) A triangle

Answer 2:

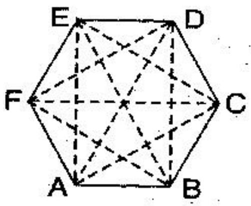
(a) A convex quadrilateral has two diagonals.

Here, AC and BD are two diagonals.



(b) A regular hexagon has 9 diagonals.

Here, diagonals are AD, AE, BD, BE, FC, FB, AC, EC and FD.



(c) A triangle has no diagonal.

Question 3:

What is the sum of the measures of the angles of a convex quadrilateral? Will this property hold if the quadrilateral is not convex? (Make a non-convex quadrilateral and try)

Answer 3:

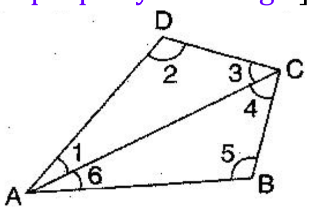
Let ABCD is a convex quadrilateral, then we draw a diagonal AC which divides the quadrilateral in two triangles.

$\angle A + B + \angle C + \angle D = \angle 1 + \angle 6 + \angle 5 + \angle 4 + \angle 3 + \angle 2$

$= (\angle 1 + \angle 2 + \angle 3) + (\angle 4 + \angle 5 + \angle 6)$

$= 180° + 180°$ [By Angle sum property of triangle]

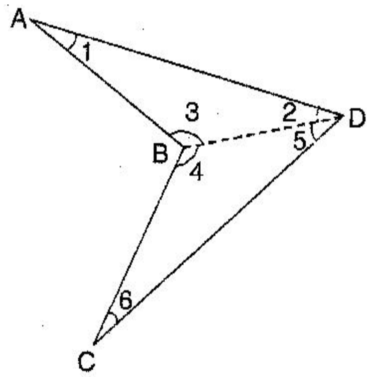
$= 360°$



Hence, the sum of measures of the triangles of a convex quadrilateral is 360°.

Yes, if quadrilateral is not convex then, this property will also be applied.

Let ABCD is a non-convex quadrilateral and join BD, which also divides the quadrilateral in two triangles.



Using angle sum property of triangle,

In $\triangle ABD$, $\angle 1 + \angle 2 + \angle 3 = 180^\circ$ ....(i)

In $\triangle BDC$, $\angle 4 + \angle 5 + \angle 6 = 180^\circ$ ....(i)

Adding eq. (i) and (ii),

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360^\circ$

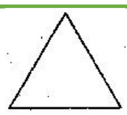
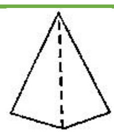
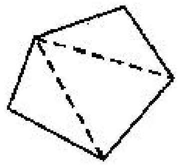
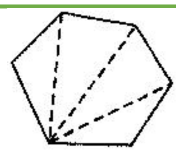
$\Rightarrow \quad \angle 1 + \angle 2 + (\angle 3 + \angle 4) + \angle 5 + \angle 6 = 360^\circ$

$\Rightarrow \quad \angle A + \angle B + \angle C + \angle D = 360^\circ$

Hence proved.

Question 4:

Examine the table. (Each figure is divided into triangles and the sum of the angles deduced from that.)

Figure				
Side	3	4	5	6
Angle sum	$1\times180^\circ = (3-2)\times180^\circ$	$2\times180^\circ = (4-2)\times180^\circ$	$3\times180^\circ = (5-2)\times180^\circ$	$4\times180^\circ = (6-2)\times180^\circ$

What can you say about the angle sum of a convex polygon with number of sides?

Answer 4:

(a) When n = 7, then

Angle sum of a polygon = $(n-2)\times180^\circ = (7-2)\times180^\circ = 5\times180^\circ = 900^\circ$

(b) When n = 8, then

Angle sum of a polygon = $(n-2)\times180^\circ = (8-2)\times180^\circ = 6\times180^\circ = 1080^\circ$

(c) When n = 10, then

Angle sum of a polygon = $(n-2)\times180^\circ = (10-2)\times180^\circ = 8\times180^\circ = 1440^\circ$

(d) When n = n, then

Angle sum of a polygon = $(n-2)\times180^\circ$

Question 5:

What is a regular polygon? State the name of a regular polygon of:

(a) 3 sides
(b) 4 sides
(c) 6 sides

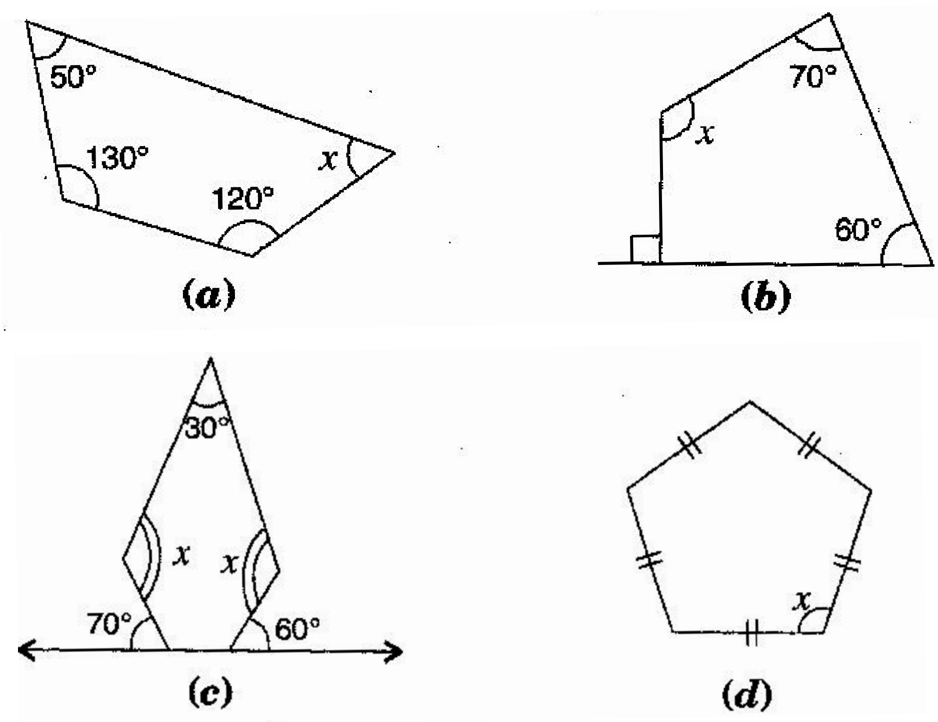
Answer 5:

A regular polygon: A polygon having all sides of equal length and the interior angles of equal size is known as regular polygon.

(i) 3 sides
Polygon having three sides is called a ***triangle.***
(ii) 4 sides
Polygon having four sides is called a ***quadrilateral.***
(iii) 6 sides
Polygon having six sides is called a ***hexagon***.

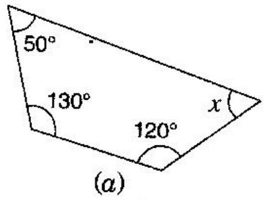
Question 6:

Find the angle measures x in the following figures:



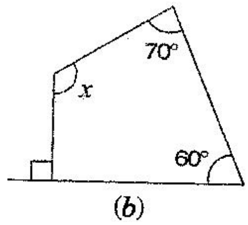
Answer 6:

(a) Using angle sum property of a quadrilateral,



$$50^\circ + 130^\circ + 120^\circ + x = 360^\circ$$
$$\Rightarrow \quad 300^\circ + x = 360^\circ$$
$$\Rightarrow \quad x = 360^\circ - 300^\circ$$
$$\Rightarrow \quad x = 60^\circ$$

(b) Using angle sum property of a quadrilateral,

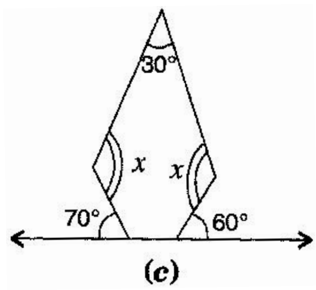


$$90^\circ + 60^\circ + 70^\circ + x = 360^\circ$$
$$\Rightarrow \quad 220^\circ + x = 360^\circ$$
$$\Rightarrow \quad x = 360^\circ - 220^\circ$$
$$\Rightarrow \quad x = 140^\circ$$

(c) First base interior angle = $180^\circ - 70^\circ = 110^\circ$

Second base interior angle = $180^\circ - 60^\circ = 120^\circ$

There are 5 sides, n = 5

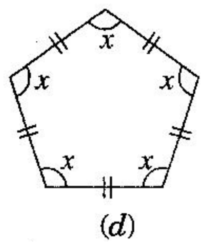


$\therefore$ Angle sum of a polygon = $(n-2)\times 180^\circ$

= $(5-2)\times 180^\circ = 3\times 180^\circ$ = 540°

$$\therefore \quad 30^\circ + x + 110^\circ + 120^\circ + x = 540^\circ$$
$$\Rightarrow \quad 260^\circ + 2x = 540^\circ$$
$$\Rightarrow \quad 2x = 540^\circ - 260^\circ$$
$$\Rightarrow \quad 2x = 280^\circ$$
$$\Rightarrow \quad x = 140^\circ$$

(d) Angle sum of a polygon = $(n-2)\times 180^\circ$



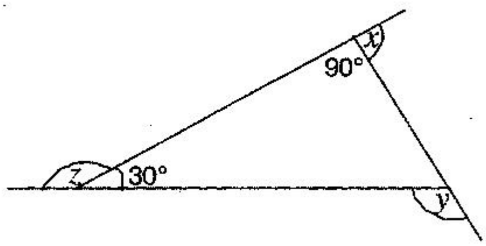
= $(5-2)\times 180^\circ = 3\times 180^\circ$ = 540°

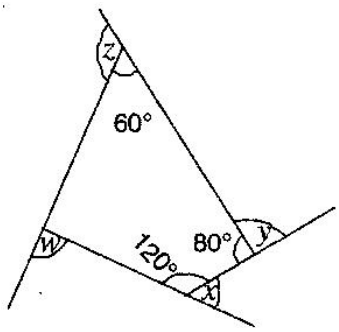
$$\therefore \quad x + x + x + x + x = 540^\circ$$
$$\Rightarrow \quad 5x = 540^\circ$$
$$\Rightarrow \quad x = 108^\circ$$

Hence each interior angle is 108°.

Question 7:

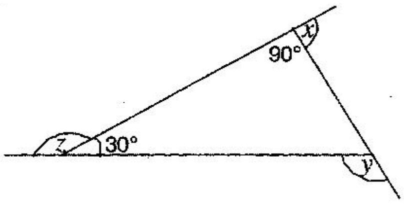
(a) Find $x+y+z$ (b) Find $x+y+z+w$





Answer 7:

(a) Since sum of linear pair angles is 180°.



$\therefore \quad 90° + x = 180°$

$\Rightarrow \quad x = 180° - 90° = 90°$

And $\quad z + 30° = 180°$

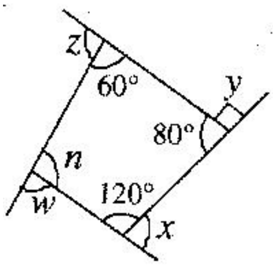
$\Rightarrow \quad z = 180° - 30° = 150°$

Also $\quad y = 90° + 30° = 120°$

[Exterior angle property]

$\therefore \quad x + y + x = 90° + 120° + 150° = 360°$

(b) Using angle sum property of a quadrilateral,



$60° + 80° + 120° + n = 360°$

$\Rightarrow \quad 260° + n = 360°$

$\Rightarrow \quad n = 360° - 260°$

$\Rightarrow \quad n = 100°$

Since sum of linear pair angles is 180°.

$\therefore \quad w + 100 = 180°$ ....(i)

$x + 120° = 180°$ ....(ii)

$y + 80° = 180°$ ....(iii)

$z + 60° = 180°$ ....(iv)

Adding eq. (i), (ii), (iii) and (iv),

$\Rightarrow \quad x + y + z + w + 100° + 120° + 80° + 60° = 180° + 180° + 180° + 180°$

$\Rightarrow \quad x + y + z + w + 360° = 720°$

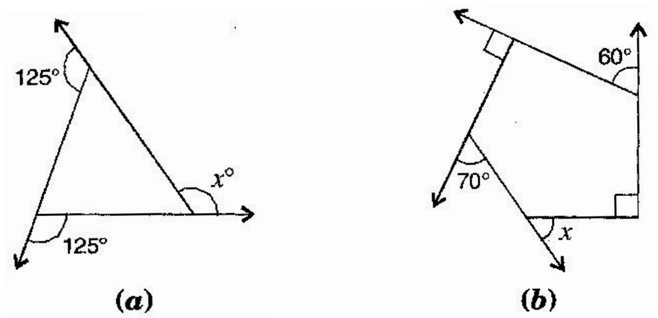
$\Rightarrow \quad x + y + z + w = 720° - 360°$

$\Rightarrow \quad x + y + z + w = 360°$

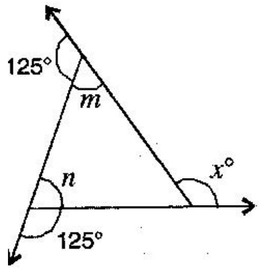
Exercise 3.2

Question 1:

Find x in the following figures:



Answer 1:



(a) Here, $125° + m = 180°$ [Linear pair]

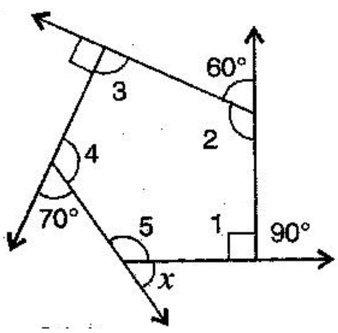
$\Rightarrow$ $m = 180° - 125° = 55°$

and $125° + n = 180°$ [Linear pair]

$\Rightarrow$ $n = 180° - 125° = 55°$

$\because$ Exterior angle $x°$ = Sum of opposite interior angles

$\therefore$ $x° = 55° + 55° = 110°$



(b) Sum of angles of a pentagon = $(n-2) \times 180°$

= $(5-2) \times 180°$

= $3 \times 180° = 540°$

By linear pairs of angles,

$\angle 1 + 90° = 180°$ ....(i)

$\angle 2 + 60° = 180°$ ....(ii)

$\angle 3 + 90° = 180°$ ....(iii)

$\angle 4 + 70° = 180°$ ....(iv)

$\angle 5 + x = 180°$ ....(v)

Adding eq. (i), (ii), (iii), (iv) and (v),

$x + (\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5) + 310° = 900$

$\Rightarrow$ $x + 540° + 310° = 900°$

$\Rightarrow$ $x + 850° = 900°$

$\Rightarrow$ $x = 900° - 850° = 50°$

Question 2:

Find the measure of each exterior angle of a regular polygon of:

(a) 9 sides (b) 15 sides

Answer 2:

(i) Sum of angles of a regular polygon = $(n-2)\times 180°$

$= (9-2)\times 180° = 7\times 180° = 1260°$

Each interior angle = $\frac{\text{Sum of interior angles}}{\text{Number of sides}} = \frac{1260°}{9} = 140°$

Each exterior angle = $180° - 140° = 40°$

(ii) Sum of exterior angles of a regular polygon = $360°$

Each interior angle = $\frac{\text{Sum of interior angles}}{\text{Number of sides}} = \frac{360°}{15} = 24°$

Question 3:

How many sides does a regular polygon have, if the measure of an exterior angle is $24°$?

Answer 3:

Let number of sides be n.

Sum of exterior angles of a regular polygon = $360°$

Number of sides = $\frac{\text{Sum of exterior angles}}{\text{Each interior angle}} = \frac{360°}{24°} = 15$

Hence, the regular polygon has 15 sides.

Question 4:

How many sides does a regular polygon have if each of its interior angles is $165°$?

Answer 4:

Let number of sides be n.

Exterior angle = $180° - 165° = 15°$

Sum of exterior angles of a regular polygon = $360°$

Number of sides = $\frac{\text{Sum of exterior angles}}{\text{Each interior angle}} = \frac{360°}{15°} = 24$

Hence, the regular polygon has 24 sides.

Question 5:

(a) Is it possible to have a regular polygon with of each exterior angle as $22°$?

(b) Can it be an interior angle of a regular polygon? Why?

Answer 5:

(a) No. (Since 22 is not a divisor of $360°$)

(b) No, (Because each exterior angle is $180° - 22° = 158°$, which is not a divisor of $360°$)

Question 6:

(a) What is the minimum interior angle possible for a regular polygon? Why?

(b) What is the maximum exterior angle possible for a regular polygon?

Answer 6:

(a) The equilateral triangle being a regular polygon of 3 sides has the least measure of an interior angle of 60°.

$\because$ Sum of all the angles of a triangle = $180°$

$\therefore$ $x + x + x = 180°$

$\Rightarrow$ $3x = 180°$

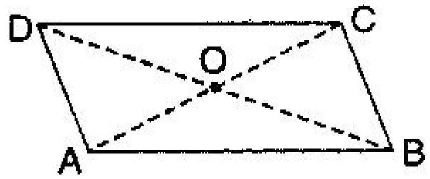
$\Rightarrow$ $x = 60^\circ$

(b) By (a), we can observe that the greatest exterior angle is $180° - 60° = 120°$.

Exercise 3.3

Question 1:

Given a parallelogram ABCD. Complete each statement along with the definition or property used.



(i) AD = ____________

(ii) ∠DCB = ____________

(iii) OC = ____________

(iv) $m\angle DAB + m\angle CDA$ = ____________

Answer 1:

(i) AD = BC [Since opposite sides of a parallelogram are equal]

(ii) ∠DCB = ∠DAB [Since opposite angles of a parallelogram are equal]

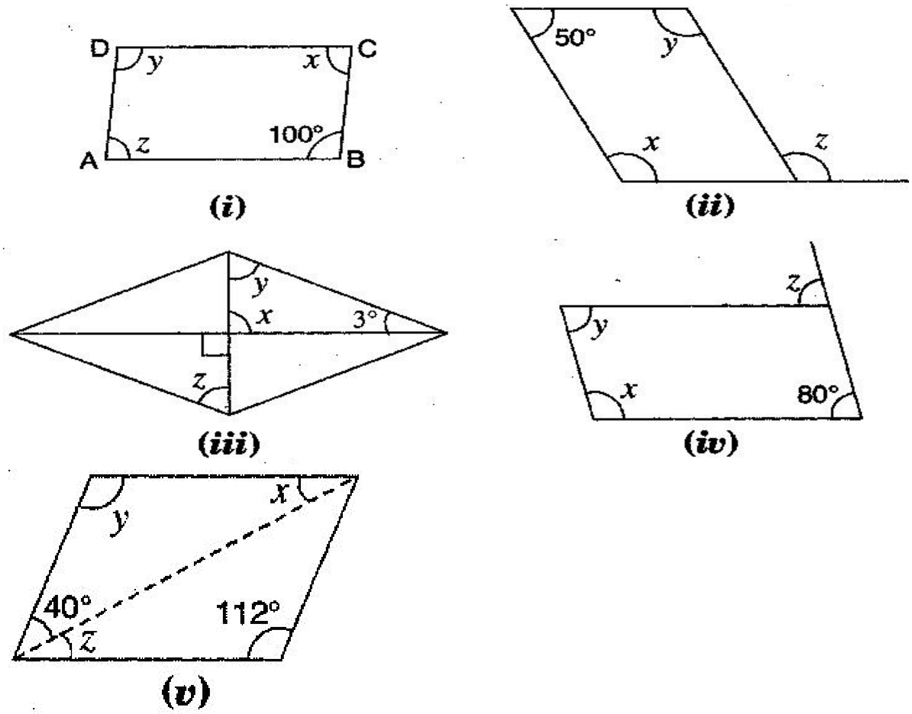
(iii) OC = OA [Since diagonals of a parallelogram bisect each other]

(iv) $m\angle DAB + m\angle CDA = 180°$

[Adjacent angles in a parallelogram are supplementary]

Question 2:

Consider the following parallelograms. Find the values of the unknowns x, y, z.



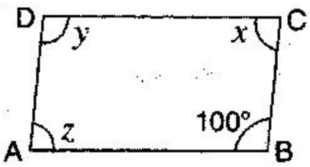
Note: For getting correct answer, read $3^{\circ} = 30^{\circ}$ in figure (iii)

Answer 2:

(i) $\angle B + \angle C = 180^\circ$ [Adjacent angles in a parallelogram are supplementary]

$\Rightarrow \quad 100^\circ + x = 180^\circ$

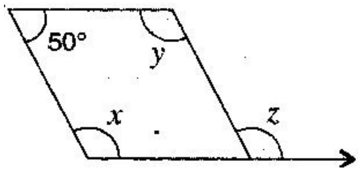
$\Rightarrow \quad x = 180^\circ - 100^\circ = 80^\circ$



and $\quad z = x = 80^\circ$ [Since opposite angles of a parallelogram are equal]

also $\quad y = 100^\circ$ [Since opposite angles of a parallelogram are equal]

(ii) $x + 50^\circ = 180^\circ$ [Adjacent angles in a $||^{gm}$ are supplementary]

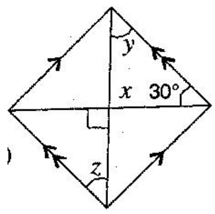


$\Rightarrow \quad x = 180^\circ - 50^\circ = 130^\circ$

$\Rightarrow \quad z = x = 130^\circ$ [Corresponding angles]

(iii) $x = 90^\circ$ [Vertically opposite angles]

$\Rightarrow \quad y + x + 30^\circ = 180^\circ$ [Angle sum property of a triangle]



$\Rightarrow \quad y + 90^\circ + 30^\circ = 180^\circ$

$\Rightarrow \quad y + 120^\circ = 180^\circ$

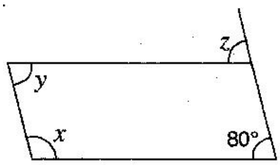
$\Rightarrow \quad y = 180^\circ - 120^\circ = 60^\circ$

$\Rightarrow \quad z = y = 60^\circ$ [Alternate angles]

(iv) $z = 80^\circ$ [Corresponding angles]

$\Rightarrow \quad x + 80^\circ = 180^\circ$ [Adjacent angles in a $||^{gm}$ are supplementary]

$\Rightarrow \quad x = 180^\circ - 80^\circ = 100^\circ$



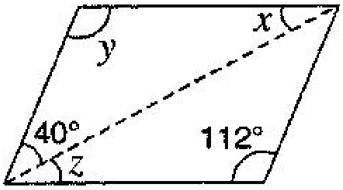
and $\quad y = 80^\circ$ [Opposite angles are equal in a $||^{gm}$]

(v) $y = 112°$ [Opposite angles are equal in a ||gm]

$\Rightarrow \quad 40° + y + x = 180°$ [Angle sum property of a triangle]

$\Rightarrow \quad 40° + 112° + x = 180°$

$\Rightarrow \quad 152° + x = 180°$



$\Rightarrow \quad x = 180° - 152° = 28°$

and $\quad z = x = 28°$ [Alternate angles]

Question 3:

Can a quadrilateral ABCD be a parallelogram, if:

(i) $\angle D + \angle B = 180°$?

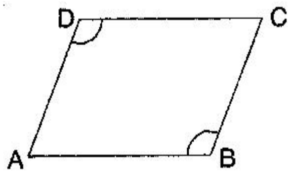
(ii) AB = DC = 8 cm, AD = 4 cm and BC = 4.4 cm?

(iii) $\angle A = 70°$ and $\angle C = 65°$?

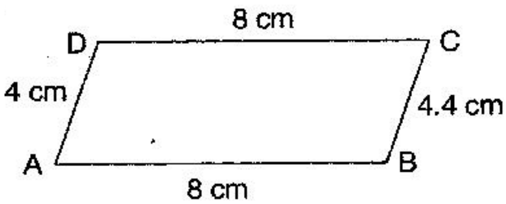
Answer 3:

(i) $\angle D + \angle B = 180°$

It can be, but here, it needs not to be.

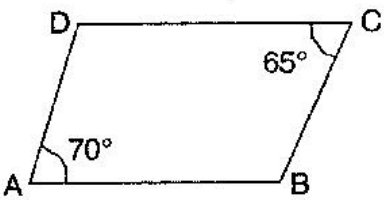


(ii) No, in this case because one pair of opposite sides are equal and another pair of opposite sides are unequal. So, it is not a parallelogram.



(iii) No. $\angle A \neq \angle C$.

Since opposite angles are equal in parallelogram and here opposite angles are not equal in quadrilateral ABCD. Therefore it is not a parallelogram.



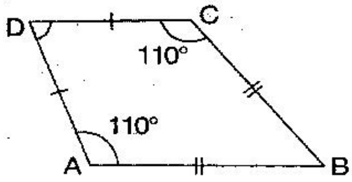
Question 4:

Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measures.

Answer 4:

ABCD is a quadrilateral in which angles $\angle A = \angle C = 110°$.

Therefore, it could be a kite.



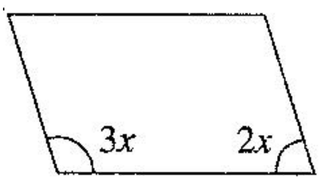
Question 5:

The measure of two adjacent angles of a parallelogram are in the ratio 3:2. Find the measure of each of the angles of the parallelogram.

Answer 5:

Let two adjacent angles be $3x$ and $2x$.

Since the adjacent angles in a parallelogram are supplementary.



$\therefore \quad 3x + 2x = 180°$

$\Rightarrow \quad 5x = 180°$

$\Rightarrow \quad x = \frac{180°}{5} = 36°$

$\therefore$ One angle $= 3x = 3 \times 36° = 108°$

and another angle $= 2x = 2 \times 36° = 72°$

Question 6:

Two adjacent angles of a parallelogram have equal measure. Find the measure of the angles of the parallelogram.

Answer 6:

Let each adjacent angle be x.

Since the adjacent angles in a parallelogram are supplementary.

$\therefore \quad x + x = 180°$

$\Rightarrow \quad 2x = 180°$

$\Rightarrow \quad x = \frac{180^\circ}{2} = 90^\circ$

Hence, each adjacent angle is 90°.

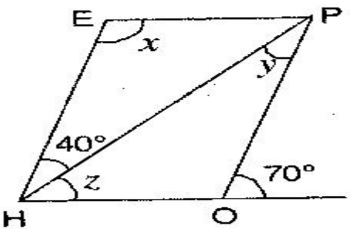
$\therefore \quad x + x + x = 180^\circ$

$\Rightarrow \quad 3x = 180^\circ$

$\Rightarrow \quad x = 60^\circ$

Question 7:

The adjacent figure HOPW is a parallelogram. Find the angle measures x, y and z. State the properties you use to find them.



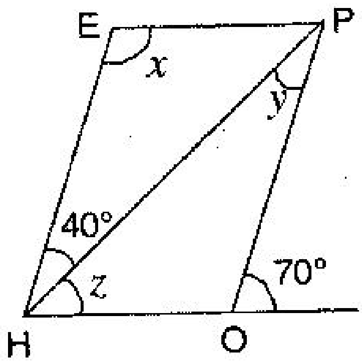
Answer 7:

Here $\angle$ HOP + $70^\circ = 180^\circ$ [Angles of linear pair]

$\angle$ HOP = $180^\circ - 70^\circ = 110^\circ$

and $\angle$ E = $\angle$ HOP [Opposite angles of a ||gm are equal]

$\Rightarrow \quad x = 110^\circ$



$\angle$ PHE = $\angle$ HPO [Alternate angles]

$\therefore \quad y = 40^\circ$

Now $\angle$ EHO = $\angle$ O = 70° [Corresponding angles]

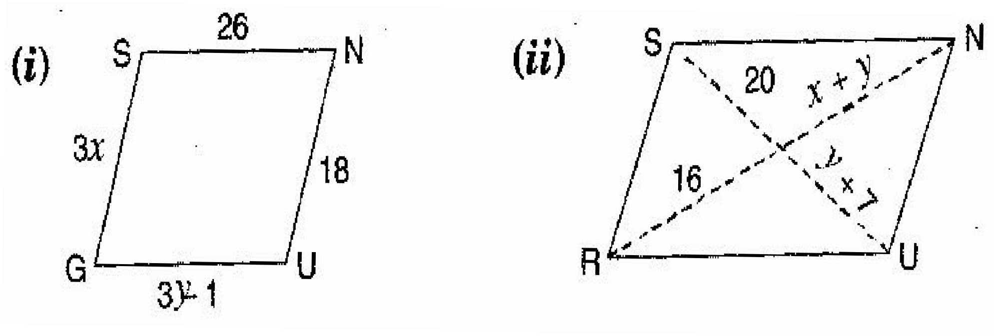
$\Rightarrow \quad 40^\circ + z = 70^\circ$

$\Rightarrow \quad z = 70^\circ - 40^\circ = 30^\circ$

Hence, $x = 110^\circ, y = 40^\circ$ and $z = 30^\circ$

Question 8:

The following figures GUNS and RUNS are parallelograms. Find x and y. (Lengths are in cm)



Answer 8:

(i) In parallelogram GUNS,

GS = UN [Opposite sides of parallelogram are equal]

$\Rightarrow 3x = 18$

$\Rightarrow x = \frac{18}{3} = 6$ cm

Also GU = SN [Opposite sides of parallelogram are equal]

$\Rightarrow 3y - 1 = 26$

$\Rightarrow 3y = 26 + 1$

$\Rightarrow 3y = 27$

$\Rightarrow y = \frac{27}{3} = 9$ cm

Hence, x = 6 cm and y = 9 cm.

(ii) In parallelogram RUNS,

$y + 7 = 20$ [Diagonals of $||^{gm}$ bisects each other]

$\Rightarrow y = 20 - 7 = 13$ cm

and $x + y = 16$

$\Rightarrow x + 13 = 16$

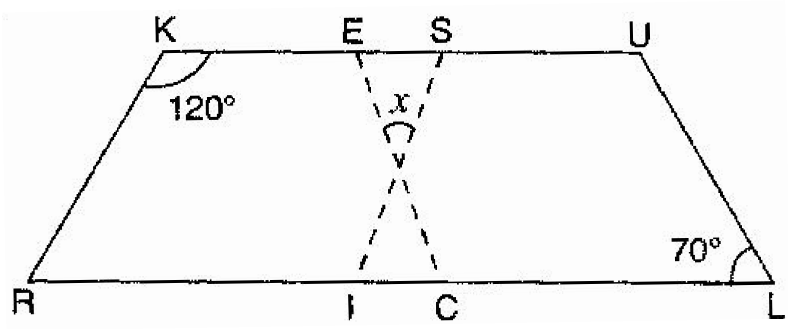
$\Rightarrow x = 16 - 13$

$\Rightarrow x = 3$ cm

Hence, $x = 3$ cm and $y = 13$ cm.

Question 9:

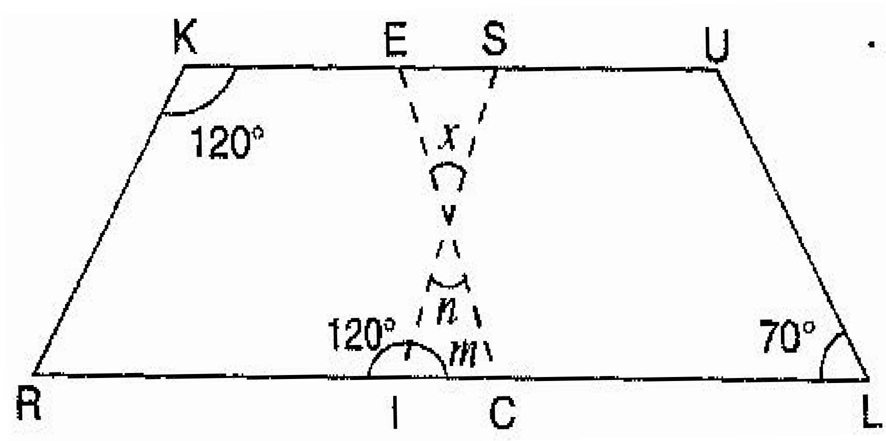
In the figure, both RISK and CLUE are parallelograms. Find the value of x.



Answer 9:

In parallelogram RISK,

$\angle RIS = \angle K = 120°$ [Opposite angles of a ||gm are equal]



$\angle m + 120° = 180°$ [Linear pair]

$\Rightarrow \quad \angle m = 180° - 120° = 60°$

and $\quad \angle ECI = \angle L = 70°$ [Corresponding angles]

$\Rightarrow \quad m + n + \angle ECI = 180°$ [Angle sum property of a triangle]

$\Rightarrow \quad 60° + n + 70° = 180°$

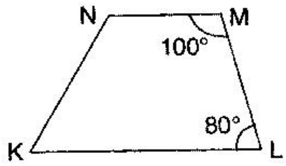
$\Rightarrow \quad 130° + n = 180°$

$\Rightarrow \quad n = 180° - 130° = 50°$

also $\quad x = n = 50°$ [Vertically opposite angles]

Question 10:

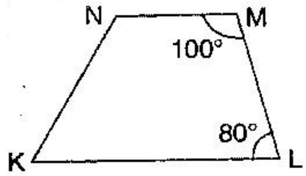
Explain how this figure is a trapezium. Which is its two sides are parallel?



Answer 10:

Here, $\angle M + \angle L = 100° + 80° = 180°$ [Sum of interior opposite angles is $180°$]

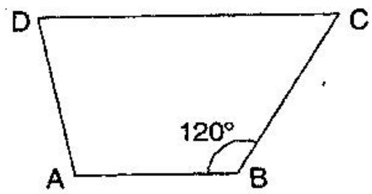
$\therefore$ NM and KL are parallel.



Hence, KLMN is a trapezium.

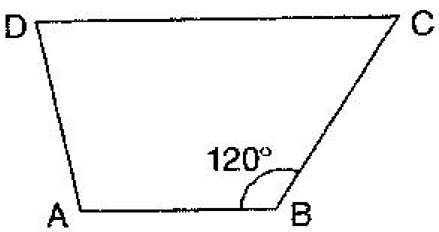
Question 11:

1. Find $m\angle C$ in figure, if $\overline{AB} \parallel \overline{DC}$,



Answer 11:

Here, $\angle B + \angle C = 180°$ [$\because \overline{AB} \parallel \overline{DC}$]

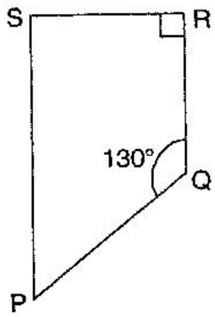


$\therefore$ $120° + m\angle C = 180°$

$\Rightarrow$ $m\angle C = 180° - 120° = 60°$

Question 12:

Find the measure of ∠P and ∠S if $\overline{SP} \,||\, \overline{RQ}$ in given figure.
(If you find $m\angle R$ is there more than one method to find $m\angle P$)



Answer 12:

Here, $\angle P + \angle Q = 180°$ [Sum of co-interior angles is $180°$]

$\Rightarrow \angle P + 130° = 180°$

$\Rightarrow \angle P = 180° - 130°$

$\Rightarrow \angle P = 50°$

$\because \angle R = 90°$ [Given]

$\therefore \angle S + 90° = 180°$

$\Rightarrow \angle S = 180° - 90°$

$\Rightarrow \angle S = 90°$

Yes, one more method is there to find ∠P.

$\angle S + \angle R + \angle Q + \angle P = 360°$ [Angle sum property of quadrilateral]

$\Rightarrow 90° + 90° + 130° + \angle P = 360°$

$\Rightarrow 310° + \angle P = 360°$

$\Rightarrow \angle P = 360° - 310°$

$\Rightarrow \angle P = 50°$

Exercise 3.4

Question 1:

State whether true or false:

(a) All rectangles are squares.
(b) All rhombuses are parallelograms.
(c) All squares are rhombuses and also rectangles.
(d) All squares are not parallelograms.
(e) All kites are rhombuses.
(f) All rhombuses are kites.
(g) All parallelograms are trapeziums.
(h) All squares are trapeziums.

Answer 1:

(a) False.	Since, squares have all sides are equal.
(b) True.	Since, in rhombus, opposite angles are equal and diagonals intersect at mid-point.
(c) True.	Since, squares have the same property of rhombus but not a rectangle.
(d) False.	Since, all squares have the same property of parallelogram.
(e) False.	Since, all kites do not have equal sides.
(f) True.	Since, all rhombuses have equal sides and diagonals bisect each other.
(g) True.	Since, trapezium has only two parallel sides.
(h) True.	Since, all squares have also two parallel lines.

Question 2:

Identify all the quadrilaterals that have:

(a) four sides of equal lengths.
(b) four right angles.

Answer 2:

(a) Rhombus and square have sides of equal length.
(b) Square and rectangle have four right angles.

Question 3:

Explain how a square is:

(i) a quadrilateral

(ii) a parallelogram

(iii) a rhombus

(iv) a rectangle

Answer 3:

(i) A square is a quadrilateral, if it has four unequal lengths of sides.

(ii) A square is a parallelogram, since it contains both pairs of opposite sides equal.

(iii) A square is already a rhombus. Since, it has four equal sides and diagonals bisect at 90° to each other.

(iv) A square is a parallelogram, since having each adjacent angle a right angle and opposite sides are equal.

Question 4:

Name the quadrilateral whose diagonals:

(i) bisect each other.

(ii) are perpendicular bisectors of each other.

(iii) are equal.

Answer 4:

(i) If diagonals of a quadrilateral bisect each other then it is a rhombus, parallelogram, rectangle or square.

(ii) If diagonals of a quadrilateral are perpendicular bisector of each other, then it is a rhombus or square.

(iii) If diagonals are equal, then it is a square or rectangle.

Question 5:

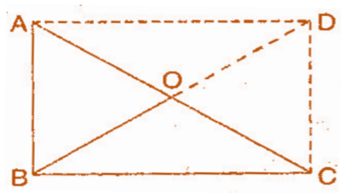
Explain why a rectangle is a convex quadrilateral.

Answer 5:

A rectangle is a convex quadrilateral since its vertex are raised and both of its diagonals lie in its interior.

Question 6:

ABC is a right-angled triangle and O is the mid-point of the side opposite to the right angle. Explain why O is equidistant from A, B and C. (The dotted lines are drawn additionally to help you.)



Answer 6:

Since, two right triangles make a rectangle where O is equidistant point from A, B, C and D because O is the mid-point of the two diagonals of a rectangle.

Since AC and BD are equal diagonals and intersect at mid-point.

So, O is the equidistant from A, B, C and D.