

Exercise 12.1

Question 1:

Evaluate:

(i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-5}$

Answer 1:

$$\begin{aligned} \text{(i)} \quad 3^{-2} &= \frac{1}{3^2} & \left[\because a^{-m} = \frac{1}{a^m} \right] \\ &= \frac{1}{9} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad (-4)^{-2} &= \frac{1}{(-4)^2} & \left[\because a^{-m} = \frac{1}{a^m} \right] \\ &= \frac{1}{16} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad \left(\frac{1}{2}\right)^{-5} &= \left(\frac{2}{1}\right)^5 & \left[\because a^{-m} = \frac{1}{a^m} \right] \\ &= (2)^5 = 32 \end{aligned}$$

Question 2:

Simplify and express the result in power notation with positive exponent:

(i) $(-4)^5 \div (-4)^8$ (ii) $\left(\frac{1}{2^3}\right)^2$ (iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$ (v) $2^{-3} \times (-7)^{-3}$

Answer 2:

$$\begin{aligned} \text{(i)} \quad (-4)^5 \div (-4)^8 &= (-4)^{5-8} & \left[\because a^m \div a^n = a^{m-n} \right] \\ &= (-4)^{-3} = \frac{1}{(-4)^3} & \left[\because a^{-m} = \frac{1}{a^m} \right] \end{aligned}$$

$$\text{(ii)} \quad \left(\frac{1}{2^3}\right)^2 = \frac{1^2}{(2^3)^2} \quad \left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \right]$$

$$= \frac{1}{2^{3 \times 2}} = \frac{1}{2^6}$$

$$\left[\because (a^m)^n = a^{m \times n} \right]$$

$$(iii) \quad (-3)^4 \times \left(\frac{5}{3}\right)^4 = (-3)^4 \times \frac{5^4}{3^4}$$

$$\left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \right]$$

$$= \{(-1)^4 \times 3^4\} \times \frac{5^4}{3^4}$$

$$\left[\because (ab)^m = a^m b^m \right]$$

$$= 3^{4-4} \times 5^4$$

$$\left[\because a^m \div a^n = a^{m-n} \right]$$

$$= 3^0 \times 5^4 = 5^4$$

$$\left[\because a^0 = 1 \right]$$

$$(iv) \quad (3^{-7} \div 3^{-10}) \times 3^{-5} = 3^{-7-(-10)} \times 3^{-5}$$

$$= 3^{-7+10} \times 3^{-5} = 3^3 \times 3^{-5} = 3^{3+(-5)}$$

$$= 3^{-2} = \frac{1}{3^2}$$

$$\left[\because a^m \div a^n = a^{m-n} \right]$$

$$\left[\because a^m \times a^n = a^{m+n} \right]$$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$(v) \quad 2^{-3} \times (-7)^{-3} = \frac{1}{2^3} \times \frac{1}{(-7)^3}$$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= \frac{1}{\{2 \times (-7)\}^3} = \frac{1}{(-14)^3}$$

$$\left[\because (ab)^m = a^m b^m \right]$$

Question 3:

Find the value of:

$$(i) \quad (3^0 + 4^{-1}) \times 2^2$$

$$(ii) \quad (2^{-1} \times 4^{-1}) \div 2^{-2}$$

$$(iii) \quad \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$$

$$(iv) \quad (3^{-1} + 4^{-1} + 5^{-1})^0$$

$$(v) \quad \left\{ \left(\frac{-2}{3}\right)^{-2} \right\}^2$$

 Answer 3:

$$\begin{aligned}
 \text{(i)} \quad (3^0 + 4^{-1}) \times 2^2 &= \left(1 + \frac{1}{4}\right) \times 2^2 \\
 &= \left(\frac{4+1}{4}\right) \times 2^2 = \frac{5}{4} \times 2^2 = \frac{5}{2^2} \times 2^2 \\
 &= 5 \times 2^{2-2} \\
 &= 5 \times 2^0 = 5 \times 1 = 5
 \end{aligned}$$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$\left[\because a^m \div a^n = a^{m-n} \right]$$

$$\left[\because a^0 = 1 \right]$$

$$\begin{aligned}
 \text{(ii)} \quad (2^{-1} \times 4^{-1}) \div 2^{-2} &= \left(\frac{1}{2^1} \times \frac{1}{4^1}\right) \div 2^{-2} \\
 &= \left(\frac{1}{2} \times \frac{1}{2^2}\right) \div 2^{-2} = \frac{1}{2^3} \div 2^{-2} \\
 &= 2^{-3} \div 2^{-2} = 2^{-3-(-2)} = 2^{-3+2} = 2^{-1} \\
 &= \frac{1}{2}
 \end{aligned}$$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$\left[\because a^m \times a^n = a^{m+n} \right]$$

$$\left[\because a^m \div a^n = a^{m-n} \right]$$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$\begin{aligned}
 \text{(iii)} \quad \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} &= (2^{-1})^{-2} + (3^{-1})^{-2} + (4^{-1})^{-2} \\
 &= 2^{-1 \times (-2)} + 3^{-1 \times (-2)} + 4^{-1 \times (-2)} \\
 &= 2^2 + 3^2 + 4^2 = 4 + 9 + 16 = 29
 \end{aligned}$$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$\left[\because (a^m)^n = a^{m \times n} \right]$$

$$\begin{aligned}
 \text{(iv)} \quad (3^{-1} + 4^{-1} + 5^{-1})^0 &= \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right)^0 \\
 &= \left(\frac{20+15+12}{60}\right)^0 = \left(\frac{47}{60}\right)^0 = 1
 \end{aligned}$$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$\left[\because a^0 = 1 \right]$$

$$\begin{aligned}
 \text{(v)} \quad \left\{ \left(\frac{-2}{3}\right)^{-2} \right\}^2 &= \left(\frac{-2}{3}\right)^{-2 \times 2} \\
 &= \left(\frac{-2}{3}\right)^{-4} = \left(\frac{-3}{2}\right)^4 \\
 &= \frac{81}{16}
 \end{aligned}$$

$$\left[\because (a^m)^n = a^{m \times n} \right]$$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

Question 4:

Evaluate:

$$(i) \frac{8^{-1} \times 5^3}{2^{-4}}$$

$$(ii) (5^{-1} \times 2^{-1}) \times 6^{-1}$$

Answer 4:

$$\begin{aligned}(i) \quad \frac{8^{-1} \times 5^3}{2^{-4}} &= \frac{(2^3)^{-1} \times 5^3}{2^{-4}} = \frac{2^{-3} \times 5^3}{2^{-4}} \\ &= 2^{-3-(-4)} \times 5^3 = 2^{-3+4} \times 5^3 \\ &= 2 \times 125 = 250\end{aligned}$$

$$\left[\because (a^m)^n = a^{m \times n} \right]$$

$$\left[\because a^m \div a^n = a^{m-n} \right]$$

$$\begin{aligned}(ii) \quad (5^{-1} \times 2^{-1}) \times 6^{-1} &= \left(\frac{1}{5} \times \frac{1}{2} \right) \times \frac{1}{6} \\ &= \frac{1}{10} \times \frac{1}{6} = \frac{1}{60}\end{aligned}$$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

Question 5:

Find the value of m for which $5^m \div 5^{-3} = 5^5$.

Answer 5:

$$5^m \div 5^{-3} = 5^5$$

$$\Rightarrow 5^{m-(-3)} = 5^5$$

$$\Rightarrow 5^{m+3} = 5^5$$

Comparing exponents both sides, we get

$$\Rightarrow m+3=5$$

$$\Rightarrow m=5-3$$

$$\Rightarrow m=2$$

$$\left[\because a^m \div a^n = a^{m-n} \right]$$

Question 6:

Evaluate:

$$(i) \left\{ \left(\frac{1}{3} \right)^{-1} - \left(\frac{1}{4} \right)^{-1} \right\}^{-1}$$

$$(ii) \left(\frac{5}{8} \right)^{-7} \times \left(\frac{8}{5} \right)^{-4}$$

Answer 6:

$$(i) \left\{ \left(\frac{1}{3} \right)^{-1} - \left(\frac{1}{4} \right)^{-1} \right\} = \left\{ \left(\frac{3}{1} \right)^1 - \left(\frac{4}{1} \right)^1 \right\} \quad \left[\because a^{-m} = \frac{1}{a^m} \right]$$
$$= \{3 - 4\} = -1$$

$$(ii) \left(\frac{5}{8} \right)^{-7} \times \left(\frac{8}{5} \right)^{-4} = \frac{5^{-7}}{8^{-7}} \times \frac{8^{-4}}{5^{-4}}$$
$$= 5^{-7-(-4)} \times 8^{-4-(-7)} \quad \left[\because \left(\frac{a}{b} \right)^m = \frac{a^m}{b^m} \right]$$
$$= 5^{-7+4} \times 8^{-4+7} = 5^{-3} \times 8^3 = \frac{8^3}{5^3} \quad \left[\because a^m \div a^n = a^{m-n} \right]$$
$$= \frac{512}{125} \quad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

Question 7:

Simplify:

$$(i) \frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} \quad (t \neq 0)$$

$$(ii) \frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$$

Answer 7:

$$(i) \frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} = \frac{5^2 \times t^{-4}}{5^{-3} \times 5 \times 2 \times t^{-8}} = \frac{5^{2-(-3)-1} \times t^{-4-(-8)}}{2} \quad \left[\because a^m \div a^n = a^{m-n} \right]$$
$$= \frac{5^{2+3-1} \times t^{-4+8}}{2} = \frac{5^4 \times t^4}{2} = \frac{625}{2} t^4$$

$$\begin{aligned}
 \text{(ii)} \quad \frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}} &= \frac{3^{-5} \times (2 \times 5)^{-5} \times 5^3}{5^{-7} \times (2 \times 3)^{-5}} = \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}} \left[\because (ab)^m = a^m b^m \right] \\
 &= \frac{3^{-5} \times 2^{-5} \times 5^{-5+3}}{5^{-7} \times 2^{-5} \times 3^{-5}} = \frac{3^{-5} \times 2^{-5} \times 5^{-2}}{5^{-7} \times 2^{-5} \times 3^{-5}} \left[\because a^m \times a^n = a^{m+n} \right] \\
 &= 3^{-5-(-5)} \times 2^{-5-(-5)} \times 5^{-2-(-7)} \left[\because a^m \div a^n = a^{m-n} \right] \\
 &= 3^{-5+5} \times 2^{-5+5} \times 5^{-2+7} = 3^0 \times 2^0 \times 5^5 \\
 &= 1 \times 1 \times 3125 \left[\because a^0 = 1 \right] \\
 &= 3125
 \end{aligned}$$

Exercise 12.2

Question 1:

Express the following numbers in standard form:

(i) 0.00000000000085

(ii) 0.000000000000942

(iii) 6020000000000000

(iv) 0.00000000837

(v) 31860000000

Answer 1:

(i) $0.00000000000085 = 0.00000000000085 \times \frac{10^{12}}{10^{12}} = 8.5 \times 10^{-12}$

(ii) $0.000000000000942 = 0.000000000000942 \times \frac{10^{12}}{10^{12}} = 9.42 \times 10^{-12}$

(iii) $6020000000000000 = 6020000000000000 \times \frac{10^{15}}{10^{15}} = 6.02 \times 10^{15}$

(iv) $0.00000000837 = 0.00000000837 \times \frac{10^9}{10^9} = 8.37 \times 10^{-9}$

(v) $31860000000 = 31860000000 \times \frac{10^{10}}{10^{10}} = 3.186 \times 10^{10}$

Question 2:

Express the following numbers in usual form:

(i) 3.02×10^{-6}

(ii) 4.5×10^4

(iii) 3×10^{-8}

(iv) 1.0001×10^9

(v) 5.8×10^{12}

(vi) 3.61492×10^6

Answer 2:

(i) $3.02 \times 10^{-6} = \frac{3.02}{10^6} = 0.00000302$

(ii) $4.5 \times 10^4 = 4.5 \times 10000 = 45000$

(iii) $3 \times 10^{-8} = \frac{3}{10^8} = 0.00000003$

(iv) $1.0001 \times 10^9 = 1000100000$

(v) $5.8 \times 10^{12} = 5.8 \times 1000000000000 = 5800000000000$

(vi) $3.61492 \times 10^6 = 3.61492 \times 1000000 = 3614920$

Question 3:

Express the number appearing in the following statements in standard form:

- (i) 1 micron is equal to $\frac{1}{1000000}$ m.
- (ii) Charge of an electron is 0.000,000,000,000,000,000,16 coulomb.
- (iii) Size of a bacteria is 0.0000005 m.
- (iv) Size of a plant cell is 0.00001275 m.
- (v) Thickness of a thick paper is 0.07 mm.

Answer 3:

- (i) 1 micron = $\frac{1}{1000000} = \frac{1}{10^6} = 1 \times 10^{-6}$ m
- (ii) Charge of an electron is 0.00000000000000000016 coulombs.
 $= 0.00000000000000000016 \times \frac{10^{19}}{10^{19}} = 1.6 \times 10^{-19}$ coulomb
- (iii) Size of bacteria = $0.0000005 = \frac{5}{10000000} = \frac{5}{10^7} = 5 \times 10^{-7}$ m
- (iv) Size of a plant cell is $0.00001275 \text{ m} = 0.00001275 \times \frac{10^5}{10^5} = 1.275 \times 10^{-5} \text{ m}$
- (v) Thickness of a thick paper = $0.07 \text{ mm} = \frac{7}{100} \text{ mm} = \frac{7}{10^2} = 7 \times 10^{-2} \text{ mm}$

Question 4:

In a stack there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack?

Answer 4:

- Thickness of one book = 20 mm
Thickness of 5 books = $20 \times 5 = 100$ mm
Thickness of one paper = 0.016 mm
Thickness of 5 papers = $0.016 \times 5 = 0.08$ mm
Total thickness of a stack = $100 + 0.08$
 $= 100.08 \text{ mm} = 100.08 \times \frac{10^2}{10^2} = 1.0008 \times 10^2 \text{ mm}$