

Exercise 14.1

Question 1:

Find the common factors of the given terms.

(i) $12x, 36$

(iii) $14pq, 28p^2q^2$

(v) $6abc, 24ab^2, 12a^2b$

(vii) $10pq, 20qr, 30rp$

(ii) $2y, 22xy$

(iv) $2x, 3x^2, 4$

(vi) $16x^3, -4x^2, 32x$

(viii) $3x^2y^3, 10x^3y^2, 6x^2y^2z$

Answer 1:

(i) $12x = 2 \times 2 \times 3 \times x$
 $36 = 2 \times 2 \times 3 \times 3$

Hence, the common factors are 2, 2 and 3 = $2 \times 2 \times 3 = 12$

(ii) $2y = 2 \times y$
 $22xy = 2 \times 11 \times x \times y$

Hence, the common factors are 2 and $y = 2 \times y = 2y$

(iii) $14pq = 2 \times 7 \times p \times q$
 $28p^2q^2 = 2 \times 2 \times 7 \times p \times p \times q \times q$

Hence, the common factors are $2 \times 7 \times p \times q = 14pq$

(iv) $2x = 2 \times x \times 1$
 $3x^2 = 3 \times x \times x \times 1$
 $4 = 2 \times 2 \times 1$

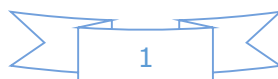
Hence, the common factor is 1.

(v) $6abc = 2 \times 3 \times a \times b \times c$
 $24ab^2 = 2 \times 2 \times 2 \times 3 \times a \times b \times b$
 $12a^2b = 2 \times 2 \times 3 \times a \times a \times b$

Hence, the common factors are $2 \times 3 \times a \times b = 6ab$

(vi) $16x^3 = 2 \times 2 \times 2 \times 2 \times x \times x \times x$
 $-4x^2 = (-1) \times 2 \times 2 \times x \times x$
 $32x = 2 \times 2 \times 2 \times 2 \times 2 \times x$

Hence, the common factors are $2 \times 2 \times x = 4x$



$$\begin{aligned} \text{(vii)} \quad 10pq &= 2 \times 5 \times p \times q \\ 20qr &= 2 \times 2 \times 5 \times q \times r \\ 30rp &= 2 \times 3 \times 5 \times r \times p \end{aligned}$$

Hence, the common factors are $2 \times 5 = 10$

$$\begin{aligned} \text{(viii)} \quad 3x^2y^3 &= 3 \times x \times x \times y \times y \times y \\ 10x^3y^2 &= 2 \times 5 \times x \times x \times x \times y \times y \\ 6x^2y^2z &= 2 \times 3 \times x \times x \times y \times y \times z \end{aligned}$$

Hence, the common factors are $x \times x \times y \times y = x^2y^2$

Question 2:

Factorize the following expressions.

$$\text{(i)} \quad 7x - 42$$

$$\text{(iii)} \quad 7a^2 + 14a$$

$$\text{(v)} \quad 20l^2m + 30alm$$

$$\text{(vii)} \quad 10a^2 - 15b^2 + 20c^2$$

$$\text{(ix)} \quad x^2yz + xy^2z + xyz^2$$

$$\text{(ii)} \quad 6p - 12q$$

$$\text{(iv)} \quad -16z + 20z^3$$

$$\text{(vi)} \quad 5x^2y - 15xy^2$$

$$\text{(viii)} \quad -4a^2 + 4ab - 4ca$$

$$\text{(x)} \quad ax^2y + bxy^2 + cxyz$$

Answer 2:

$$\text{(i)} \quad 7x - 42 = 7 \times x - 2 \times 3 \times 7$$

Taking common factors from each term,

$$= 7(x - 2 \times 3)$$

$$= 7(x - 6)$$

$$\text{(ii)} \quad 6p - 12q = 2 \times 3 \times p - 2 \times 2 \times 3 \times q$$

Taking common factors from each term,

$$= 2 \times 3(p - 2q)$$

$$= 6(p - 2q)$$

$$\text{(iii)} \quad 7a^2 + 14a = 7 \times a \times a + 2 \times 7 \times a$$

Taking common factors from each term,

$$= 7 \times a(a + 2)$$

$$= 7a(a + 2)$$

$$(iv) \quad -16z + 20z^3 = (-1) \times 2 \times 2 \times 2 \times 2 \times z + 2 \times 2 \times 5 \times z \times z \times z$$

Taking common factors from each term,

$$= 2 \times 2 \times z(-2 \times 2 + 5 \times z \times z)$$

$$= 4z(-4 + 5z^2)$$

$$(v) \quad 20l^2m + 30alm = 2 \times 2 \times 5 \times l \times l \times m + 2 \times 3 \times 5 \times a \times l \times m$$

Taking common factors from each term,

$$= 2 \times 5 \times l \times m(2 \times l + 3 \times a)$$

$$= 10lm(2l + 3a)$$

$$(vi) \quad 5x^2y - 15xy^2 = 5 \times x \times x \times y + 3 \times 5 \times x \times y \times y$$

Taking common factors from each term,

$$= 5 \times x \times y(x - 3y)$$

$$= 5xy(x - 3y)$$

$$(vii) \quad 10a^2 - 15b^2 + 20c^2 = 2 \times 5 \times a \times a - 3 \times 5 \times b \times b + 2 \times 2 \times 5 \times c \times c$$

Taking common factors from each term,

$$= 5(2 \times a \times a - 3 \times b \times b + 2 \times 2 \times c \times c)$$

$$= 5(2a^2 - 3b^2 + 4c^2)$$

$$(viii) \quad -4a^2 + 4ab - 4ca = (-1) \times 2 \times 2 \times a \times a + 2 \times 2 \times a \times b - 2 \times 2 \times c \times a$$

Taking common factors from each term,

$$= 2 \times 2 \times a(-a + b - c)$$

$$= 4a(-a + b + c)$$

$$(ix) \quad x^2yz + xy^2z + xyz^2 = x \times x \times y \times z + x \times y \times y \times z + x \times y \times z \times z$$

Taking common factors from each term,

$$= x \times y \times z(x + y + z)$$

$$= xyz(x + y + z)$$

$$(x) \quad ax^2y + bxy^2 + cxyz = a \times x \times x \times y + b \times x \times y \times y + c \times x \times y \times z$$

Taking common factors from each term,

$$= x \times y (a \times x + b \times y + c \times z)$$

$$= xy(ax + by + cz)$$

Question 3:

Factorize:

$$(i) \quad x^2 + xy + 8x + 8y$$

$$(iii) \quad ax + bx - ay - by$$

$$(v) \quad z - 7 + 7xy - xyz$$

$$(ii) \quad 15xy - 6x + 5y - 2$$

$$(iv) \quad 15pq + 15 + 9q + 25p$$

Answer 3:

$$(i) \quad x^2 + xy + 8x + 8y = x(x + y) + 8(x + y) \\ = (x + y)(x + 8)$$

$$(ii) \quad 15xy - 6x + 5y - 2 = 3x(5y - 2) + 1(5y - 2) \\ = (5y - 2)(3x + 1)$$

$$(iii) \quad ax + bx - ay - by = (ax + bx) - (ay + by) \\ = x(a + b) - y(a + b) \\ = (a + b)(x - y)$$

$$(iv) \quad 15pq + 15 + 9q + 25p = 15pq + 25p + 9q + 15 \\ = 5p(3q + 5) + 3(3q + 5) \\ = (3q + 5)(5p + 3)$$

$$(v) \quad z - 7 + 7xy - xyz = 7xy - 7 - xyz + z \\ = 7(xy - 1) - z(xy - 1) \\ = (xy - 1)(7 - z) = (-1)(1 - xy)(-1)(z - 7) \\ = (1 - xy)(z - 7)$$

Exercise 14.2

Question 1:

Factorize the following expressions:

(i) $a^2 + 8a + 16$

(ii) $p^2 - 10p + 25$

(iii) $25m^2 + 30m + 9$

(iv) $49y^2 + 84yz + 36z^2$

(v) $4x^2 - 8x + 4$

(vi) $121b^2 - 88bc + 16c^2$

(vii) $(l + m)^2 - 4lm$ [Hint: Expand $(l + m)^2$ first]

(viii) $a^4 + 2a^2b^2 + b^4$

Answer 1:

(i) $a^2 + 8a + 16 = a^2 + (4 + 4)a + 4 \times 4$

Using identity $x^2 + (a + b)x + ab = (x + a)(x + b)$,

Here $x = a, a = 4$ and $b = 4$

$$a^2 + 8a + 16 = (a + 4)(a + 4) = (a + 4)^2$$

(ii) $p^2 - 10p + 25 = p^2 + (-5 - 5)p + (-5)(-5)$

Using identity $x^2 + (a + b)x + ab = (x + a)(x + b)$,

Here $x = p, a = -5$ and $b = -5$

$$p^2 - 10p + 25 = (p - 5)(p - 5) = (p - 5)^2$$

(iii) $25m^2 + 30m + 9 = (5m)^2 + 2 \times 5m \times 3 + (3)^2$

Using identity $a^2 + 2ab + b^2 = (a + b)^2$, here $a = 5m, b = 3$

$$25m^2 + 30m + 9 = (5m + 3)^2$$

(iv) $49y^2 + 84yz + 36z^2 = (7y)^2 + 2 \times 7y \times 6z + (6z)^2$

Using identity $a^2 + 2ab + b^2 = (a + b)^2$, here $a = 7y, b = 6z$

$$49y^2 + 84yz + 36z^2 = (7y + 6z)^2$$

$$(v) \quad 4x^2 - 8x + 4 = (2x)^2 - 2 \times 2x \times 2 + (2)^2$$

Using identity $a^2 - 2ab + b^2 = (a - b)^2$, here $a = 2x, b = 2$

$$4x^2 - 8x + 4 = (2x - 2)^2 = (2)^2 (x - 1)^2 = 4(x - 1)^2$$

$$(vi) \quad 121b^2 - 88bc + 16c^2 = (11b)^2 - 2 \times 11b \times 4c + (4c)^2$$

Using identity $a^2 - 2ab + b^2 = (a - b)^2$, here $a = 11b, b = 4c$

$$121b^2 - 88bc + 16c^2 = (11b - 4c)^2$$

$$(vii) \quad \begin{aligned} (l + m)^2 - 4lm &= l^2 + 2 \times l \times m + m^2 - 4lm & \left[\because (a + b)^2 = a^2 + 2ab + b^2 \right] \\ &= l^2 + 2lm + m^2 - 4lm \\ &= l^2 - 2lm + m^2 \\ &= (l - m)^2 & \left[\because (a - b)^2 = a^2 - 2ab + b^2 \right] \end{aligned}$$

$$(viii) \quad \begin{aligned} a^4 + 2a^2b^2 + b^4 &= (a^2)^2 + 2 \times a^2 \times b^2 + (b^2)^2 \\ &= (a^2 + b^2)^2 & \left[\because (a + b)^2 = a^2 + 2ab + b^2 \right] \end{aligned}$$

Question 2:

Factorize:

$$(i) \quad 4p^2 - 9q^2$$

$$(ii) \quad 63a^2 - 112b^2$$

$$(iii) \quad 49x^2 - 36$$

$$(iv) \quad 16x^5 - 144x^2$$

$$(v) \quad (l + m)^2 - (l - m)^2$$

$$(vi) \quad 9x^2y^2 - 16$$

$$(vii) \quad (x^2 - 2xy + y^2) - z^2$$

$$(viii) \quad 25a^2 - 4b^2 + 28bc - 49c^2$$

Answer 2:

$$(i) \quad \begin{aligned} 4p^2 - 9q^2 &= (2p)^2 - (3q)^2 \\ &= (2p - 3q)(2p + 3q) & \left[\because a^2 - b^2 = (a - b)(a + b) \right] \end{aligned}$$

$$(ii) \quad \begin{aligned} 63a^2 - 112b^2 &= 7(9a^2 - 16b^2) = 7[(3a)^2 - (4b)^2] \\ &= 7(3a - 4b)(3a + 4b) & \left[\because a^2 - b^2 = (a - b)(a + b) \right] \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad 49x^2 - 36 &= (7x)^2 - (6)^2 \\
 &= (7x - 6)(7x + 6) \quad \left[\because a^2 - b^2 = (a - b)(a + b) \right]
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad 16x^5 - 144x^3 &= 16x^3(x^2 - 9) \\
 &= 16x^3[(x)^2 - (3)^2] \\
 &= 16x^3(x - 3)(x + 3) \quad \left[\because a^2 - b^2 = (a - b)(a + b) \right]
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad (l + m)^2 - (l - m)^2 &= [(l + m) + (l - m)][(l + m) - (l - m)] \\
 &\quad \left[\because a^2 - b^2 = (a - b)(a + b) \right] \\
 &= (l + m + l - m)(l + m - l + m) \\
 &= (2m)(2l) = 4lm
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad 9x^2y^2 - 16 &= (3xy)^2 - (4)^2 \\
 &= (3xy - 4)(3xy + 4) \quad \left[\because a^2 - b^2 = (a - b)(a + b) \right]
 \end{aligned}$$

$$\begin{aligned}
 \text{(vii)} \quad (x^2 - 2xy + y^2) - z^2 &= (x - y)^2 - z^2 \\
 &\quad \left[\because (a - b)^2 = a^2 - 2ab + b^2 \right] \\
 &= (x - y - z)(x - y + z) \quad \left[\because a^2 - b^2 = (a - b)(a + b) \right]
 \end{aligned}$$

$$\begin{aligned}
 \text{(viii)} \quad 25a^2 - 4b^2 + 28bc - 49c^2 &= 25a^2 - (4b^2 - 28bc + 49c^2) \\
 &= 25a^2 - [(2b)^2 - 2 \times 2b \times 7c + (7c)^2] \\
 &= 25a^2 - (2b - 7c)^2 \quad \left[\because (a - b)^2 = a^2 - 2ab + b^2 \right] \\
 &= (5a)^2 - (2b - 7c)^2 \\
 &= [5a - (2b - 7c)][5a + (2b - 7c)] \quad \left[\because a^2 - b^2 = (a - b)(a + b) \right] \\
 &= (5a - 2b + 7c)(5a + 2b - 7c)
 \end{aligned}$$

Question 3:

Factorize the expressions:

(i) $ax^2 + bx$

(ii) $7p^2 + 21q^2$

(iii) $2x^3 + 2xy^2 + 2xz^2$

(iv) $am^2 + bm^2 + bn^2 + an^2$

(v) $(lm + l) + m + 1$

(vi) $y(y + z) + 9(y + z)$

(vii) $5y^2 - 20y - 8z + 2yz$

(viii) $10ab + 4a + 5b + 2$

(ix) $6xy - 4y + 6 - 9x$

Answer 3:

(i) $ax^2 + bx = x(ax + b)$

(ii) $7p^2 + 21q^2 = 7(p^2 + 3q^2)$

(iii) $2x^3 + 2xy^2 + 2xz^2 = 2x(x^2 + y^2 + z^2)$

(iv) $am^2 + bm^2 + bn^2 + an^2 = m^2(a + b) + n^2(a + b)$
 $= (a + b)(m^2 + n^2)$

(v) $(lm + l) + m + 1 = l(m + 1) + 1(m + 1) = (m + 1)(l + 1)$

(vi) $y(y + z) + 9(y + z) = (y + z)(y + 9)$

(vii) $5y^2 - 20y - 8z + 2yz = 5y^2 - 20y + 2yz - 8z$
 $= 5y(y - 4) + 2z(y - 4)$
 $= (y - 4)(5y + 2z)$

(viii) $10ab + 4a + 5b + 2 = 2a(5b + 2) + 1(5b + 2)$
 $= (5b + 2)(2a + 1)$

(ix) $6xy - 4y + 6 - 9x = 6xy - 9x - 4y + 6$
 $= 3x(2y - 3) - 2(2y - 3)$
 $= (2y - 3)(3x - 2)$

Question 4:

Factorize:

(i) $a^4 - b^4$

(ii) $p^4 - 81$

(iii) $x^4 - (y+z)^4$

(iv) $x^4 - (x-z)^4$

(v) $a^4 - 2a^2b^2 + b^4$

Answer 4:

(i)
$$\begin{aligned} a^4 - b^4 &= (a^2)^2 - (b^2)^2 \\ &= (a^2 - b^2)(a^2 + b^2) & [\because a^2 - b^2 - (a-b)(a+b)] \\ &= (a-b)(a+b)(a^2 + b^2) & [\because a^2 - b^2 - (a-b)(a+b)] \end{aligned}$$

(ii)
$$\begin{aligned} p^4 - 81 &= (p^2)^2 - (9)^2 \\ &= (p^2 - 9)(p^2 + 9) & [\because a^2 - b^2 - (a-b)(a+b)] \\ &= (p^2 - 3^2)(p^2 + 9) \\ &= (p-3)(p+3)(p^2 + 9) & [\because a^2 - b^2 - (a-b)(a+b)] \end{aligned}$$

(iii)
$$\begin{aligned} x^4 - (y+z)^4 &= (x^2)^2 - [(y+z)^2]^2 \\ &= [x^2 - (y+z)^2][x^2 + (y+z)^2] & [\because a^2 - b^2 - (a-b)(a+b)] \\ &= [x - (y+z)][x + (y+z)][x^2 + (y+z)^2] & [\because a^2 - b^2 - (a-b)(a+b)] \\ &= (x-y+z)(x+y+z)[x^2 + (y+z)^2] \end{aligned}$$

(iv)
$$\begin{aligned} x^4 - (x-z)^4 &= (x^2)^2 - [(x-z)^2]^2 \\ &= [x^2 - (x-z)^2][x^2 + (x-z)^2] & [\because a^2 - b^2 - (a-b)(a+b)] \\ &= [x - (x-z)][x + (x-z)][x^2 + (x-z)^2] & [\because a^2 - b^2 - (a-b)(a+b)] \\ &= (x-x+z)(x+x-z)(x^2 + x^2 - 2xz + z^2) & [\because (a-b)^2 = a^2 - 2ab + b^2] \\ &= x(2x+z)(2x^2 - 2xz + z^2) \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad a^4 - 2a^2b^2 + b^4 &= (a^2)^2 - 2a^2b^2 + (b^2)^2 \\
 &= (a^2 - b^2)^2 & \left[\because (a-b)^2 = a^2 - 2ab + b^2 \right] \\
 &= [(a-b)(a+b)]^2 & \left[\because a^2 - b^2 = (a-b)(a+b) \right] \\
 &= (a-b)^2 (a+b)^2 & \left[\because (xy)^m = x^m \cdot y^m \right]
 \end{aligned}$$

Question 5:

Factorize the following expressions:

$$\text{(i)} \quad p^2 + 6p + 8$$

$$\text{(ii)} \quad q^2 - 10q + 21$$

$$\text{(iii)} \quad p^2 + 6p - 16$$

Answer 5:

$$\begin{aligned}
 \text{(i)} \quad p^2 + 6p + 8 &= p^2 + (4+2)p + 4 \times 2 \\
 &= p^2 + 4p + 2p + 4 \times 2 \\
 &= p(p+4) + 2(p+4) \\
 &= (p+4)(p+2)
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad q^2 - 10q + 21 &= q^2 - (7+3)q + 7 \times 3 \\
 &= q^2 - 7q - 3q + 7 \times 3 \\
 &= q(q-7) - 3(q-7) \\
 &= (q-7)(q-3)
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad p^2 + 6p - 16 &= p^2 + (8-2)p - 8 \times 2 \\
 &= p^2 + 8p - 2p - 8 \times 2 \\
 &= p(p+8) - 2(p+8) \\
 &= (p+8)(p-2)
 \end{aligned}$$

Exercise 14.3

Question 1:

Carry out the following divisions:

(i) $28x^4 \div 56x$

(ii) $-36y^3 \div 9y^2$

(iii) $66pq^2r^3 \div 11qr^2$

(iv) $34x^3y^3z^3 \div 51xy^2z^3$

(v) $12a^8b^8 \div (-6a^6b^4)$

Answer 1:

$$\begin{aligned} \text{(i)} \quad 28x^4 \div 56x &= \frac{28x^4}{56x} = \frac{28}{56} \times \frac{x^4}{x} \\ &= \frac{1}{2}x^3 \end{aligned}$$

$$\left[\because x^m \div x^n = x^{m-n} \right]$$

$$\begin{aligned} \text{(ii)} \quad -36y^3 \div 9y^2 &= \frac{-36y^3}{9y^2} = \frac{-36}{9} \times \frac{y^3}{y^2} \\ &= -4y \end{aligned}$$

$$\left[\because x^m \div x^n = x^{m-n} \right]$$

$$\begin{aligned} \text{(iii)} \quad 66pq^2r^3 \div 11qr^2 &= \frac{66pq^2r^3}{11qr^2} = \frac{66}{11} \times \frac{pq^2r^3}{qr^2} \\ &= 6pqr \end{aligned}$$

$$\left[\because x^m \div x^n = x^{m-n} \right]$$

$$\begin{aligned} \text{(iv)} \quad 34x^3y^3z^3 \div 51xy^2z^3 &= \frac{34x^3y^3z^3}{51xy^2z^3} = \frac{34}{51} \times \frac{x^3y^3z^3}{xy^2z^3} \\ &= \frac{2}{3}x^2y \end{aligned}$$

$$\left[\because x^m \div x^n = x^{m-n} \right]$$

$$\begin{aligned} \text{(v)} \quad 12a^8b^8 \div (-6a^6b^4) &= \frac{12a^8b^8}{-6a^6b^4} = \frac{12}{-6} \times \frac{a^8b^8}{a^6b^4} \\ &= -2a^2b^4 \end{aligned}$$

$$\left[\because x^m \div x^n = x^{m-n} \right]$$

Question 2:

Divide the given polynomial by the given monomial:

- (i) $(5x^2 - 6x) \div 3x$ (ii) $(3y^8 - 4y^6 + 5y^4) \div y^4$
(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$
(iv) $(x^3 + 2x^2 + 3x) \div 2x$ (v) $(p^3q^6 - p^6q^3) \div p^3q^3$

Answer 2:

$$\begin{aligned} \text{(i)} \quad (5x^2 - 6x) \div 3x &= \frac{5x^2 - 6x}{3x} \\ &= \frac{5x^2}{3x} - \frac{6x}{3x} = \frac{5}{3}x - 2 = \frac{1}{3}(5x - 6) \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad (3y^8 - 4y^6 + 5y^4) \div y^4 &= \frac{3y^8 - 4y^6 + 5y^4}{y^4} \\ &= \frac{3y^8}{y^4} - \frac{4y^6}{y^4} + \frac{5y^4}{y^4} = 3y^4 - 4y^2 + 5 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad 8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2 &= \frac{8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3)}{4x^2y^2z^2} \\ &= \frac{8x^3y^2z^2}{4x^2y^2z^2} + \frac{8x^2y^3z^2}{4x^2y^2z^2} + \frac{8x^2y^2z^3}{4x^2y^2z^2} = 2x + 2y + 2z \\ &= 2(x + y + z) \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad (x^3 + 2x^2 + 3x) \div 2x &= \frac{x^3 + 2x^2 + 3x}{2x} \\ &= \frac{x^3}{2x} + \frac{2x^2}{2x} + \frac{3x}{2x} = \frac{x^2}{2} + \frac{2x}{2} + \frac{3}{2} = \frac{1}{2}(x^2 + 2x + 3) \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad (p^3q^6 - p^6q^3) \div p^3q^3 &= \frac{p^3q^6 - p^6q^3}{p^3q^3} \\ &= \frac{p^3q^6}{p^3q^3} - \frac{p^6q^3}{p^3q^3} = q^3 - p^3 \end{aligned}$$

Question 3:

Work out the following divisions:

(i) $(10x - 25) \div 5$

(ii) $(10x - 25) \div (2x - 5)$

(iii) $10y(6y + 21) \div 5(2y + 7)$

(iv) $9x^2y^2(3z - 24) \div 27xy(z - 8)$

(v) $96abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6)$

Answer 3:

(i)
$$(10x - 25) \div 5 = \frac{10x - 25}{5}$$
$$= \frac{5(2x - 5)}{5} = 2x - 5$$

(ii)
$$(10x - 25) \div (2x - 5) = \frac{10x - 25}{(2x - 5)}$$
$$= \frac{5(2x - 5)}{(2x - 5)} = 5$$

(iii)
$$10y(6y + 21) \div 5(2y + 7) = \frac{10y(6y + 21)}{5(2y + 7)}$$
$$= \frac{2 \times 5 \times y \times 3(2y + 7)}{5(2y + 7)} = 2 \times y \times 3 = 6y$$

(iv)
$$9x^2y^2(3z - 24) \div 27xy(z - 8) = \frac{9x^2y^2(3z - 24)}{27xy(z - 8)}$$
$$= \frac{9}{27} \times \frac{xy \times xy \times 3(z - 8)}{xy(z - 8)} = xy$$

(v)
$$96abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6) = \frac{96abc(3a - 12)(5b - 30)}{144(a - 4)(b - 6)}$$
$$= \frac{12 \times 4 \times 2 \times abc \times 3(a - 4) \times 5(b - 6)}{12 \times 4 \times 3(a - 4)(b - 6)} = 10abc$$

Question 4:

Divide as directed:

(i) $5(2x+1)(3x+5) \div (2x+1)$

(ii) $26xy(x+5)(y-4) \div 13x(y-4)$

(iii) $52pqr(p+q)(q+r)(r+p) \div 104pq(q+r)(r+p)$

(iv) $20(y+4)(y^2+5y+3) \div 5(y+4)$

(v) $x(x+1)(x+2)(x+3) \div x(x+1)$

Answer 4:

(i) $5(2x+1)(3x+5) \div (2x+1) = \frac{5(2x+1)(3x+5)}{(2x+1)}$
 $= 5(3x+5)$

(ii) $26xy(x+5)(y-4) \div 13x(y-4) = \frac{26xy(x+5)(y-4)}{13x(y-4)}$
 $= \frac{13 \times 2 \times xy(x+5)(y-4)}{13x(y-4)} = 2y(x+5)$

(iii) $52pqr(p+q)(q+r)(r+p) \div 104pq(q+r)(r+p) = \frac{52pqr(p+q)(q+r)(r+p)}{104pq(q+r)(r+p)}$
 $= \frac{52pqr(p+q)(q+r)(r+p)}{52 \times 2 \times pq(q+r)(r+p)} = \frac{1}{2}r(p+q)$

(iv) $20(y+4)(y^2+5y+3) \div 5(y+4) = \frac{20(y+4)(y^2+5y+3)}{5(y+4)}$
 $= 4(y^2+5y+3)$

(v) $x(x+1)(x+2)(x+3) \div x(x+1) = \frac{x(x+1)(x+2)(x+3)}{x(x+1)}$
 $= (x+2)(x+3)$

Question 5:

Factorize the expressions and divide them as directed:

(i) $(y^2 + 7y + 10) \div (y + 5)$

(ii) $(m^2 - 14m - 32) \div (m + 2)$

(iii) $(5p^2 - 25p + 20) \div (p - 1)$

(iv) $4yz(z^2 + 6z - 16) \div 2y(z + 8)$

(v) $5pq(p^2 - q^2) \div 2p(p + q)$

(vi) $12xy(9x^2 - 16y^2) \div 4xy(3x + 4y)$

(vii) $39y^3(50y^2 - 98) \div 26y^2(5y + 7)$

Answer 5:

(i)
$$\begin{aligned}(y^2 + 7y + 10) \div (y + 5) &= \frac{y^2 + 7y + 10}{(y + 5)} \\&= \frac{y^2 + (2 + 5)y + 2 \times 5}{(y + 5)} = \frac{y^2 + 2y + 5y + 2 \times 5}{(y + 5)} \\&= \frac{(y + 2)(y + 5)}{(y + 5)} \quad \left[\because x^2 + (a + b)x + ab = (x + a)(x + b) \right] \\&= y + 2\end{aligned}$$

(ii)
$$\begin{aligned}(m^2 - 14m + 32) \div (m + 2) &= \frac{m^2 - 14m + 32}{(m + 2)} \\&= \frac{m^2 + (-16 + 2)m + (-16) \times 2}{(m + 2)} \\&= \frac{(m - 16)(m + 2)}{(m + 2)} \quad \left[\because x^2 + (a + b)x + ab = (x + a)(x + b) \right] \\&= (m - 16)\end{aligned}$$

(iii)
$$\begin{aligned}(5p^2 - 25p + 20) \div (p - 1) &= \frac{5p^2 - 25p + 20}{(p - 1)} \\&= \frac{5p^2 - 20p - 5p + 20}{(p - 1)}\end{aligned}$$

$$\begin{aligned}
 &= \frac{5p(p-4) - 5(p-4)}{(p-1)} \\
 &= \frac{(5p-5)(p-4)}{(p-1)} = \frac{5(p-1)(p-4)}{(p-1)} \\
 &= 5(p-4)
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad 4yz(z^2 + 6z - 16) \div 2y(z+8) &= \frac{4yz(z^2 + 6z - 16)}{2y(z+8)} \\
 &= \frac{4yz[z^2 + (8-2)z + 8 \times (-2)]}{2y(z+8)} \\
 &= \frac{4yz(z-2)(z+8)}{2y(z+8)} \quad \left[\because x^2 + (a+b)x + ab = (x+a)(x+b) \right] \\
 &= 2z(z-2)
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad 5pq(p^2 - q^2) \div 2p(p+q) &= \frac{5pq(p^2 - q^2)}{2p(p+q)} \\
 &= \frac{5pq(p-q)(p+q)}{2p(p+q)} \quad \left[\because a^2 - b^2 = (a-b)(a+b) \right] \\
 &= \frac{5}{2}q(p-q)
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad 12xy(9x^2 - 16y^2) \div 4xy(3x+4y) &= \frac{12xy(9x^2 - 16y^2)}{4xy(3x+4y)} \\
 &= \frac{12xy[(3x)^2 - (4y)^2]}{4xy(3x+4y)} \\
 &= \frac{12xy(3x-4y)(3x+4y)}{4xy(3x+4y)} \quad \left[\because a^2 - b^2 = (a-b)(a+b) \right] \\
 &= 3(3x-4y)
 \end{aligned}$$

$$\begin{aligned}
 \text{(vii)} \quad 39y^3(50y^2 - 98) \div 26y^2(5y + 7) &= \frac{39y^3(50y^2 - 98)}{26y^2(5y + 7)} \\
 &= \frac{39y^3 \times 2(25y^2 - 49)}{26y^2(5y + 7)} \\
 &= \frac{39y^2 \times 2[(5y)^2 - (7)^2]}{26y^2(5y + 7)} \\
 &= \frac{39y^2 \times 2(5y - 7)(5y + 7)}{26y^2(5y + 7)} \quad \left[\because a^2 - b^2 = (a - b)(a + b) \right] \\
 &= 3y(5y - 7)
 \end{aligned}$$

Exercise 14.4

Question 1:

Find and correct the errors in the following mathematical statement: $4(x-5) = 4x-5$

Answer 1:

$$\text{L.H.S.} = 4(x-5) = 4x-20 \neq \text{R.H.S.}$$

Hence, the correct mathematical statements is $4(x-5) = 4x-20$.

Question 2:

Find and correct the errors in the following mathematical statement: $x(3x+2) = 3x^2 + 2$

Answer 2:

$$\text{L.H.S.} = x(3x+2) = 3x^2 + 2x \neq \text{R.H.S.}$$

Hence, the correct mathematical statements is $x(3x+2) = 3x^2 + 2x$.

Question 3:

Find and correct the errors in the following mathematical statement: $2x+3y = 5xy$

Answer 3:

$$\text{L.H.S.} = 2x+3y \neq \text{R.H.S.}$$

Hence, the correct mathematical statements is $2x+3y = 2x+3y$.

Question 4:

Find and correct the errors in the following mathematical statement: $x+2x+3x = 5x$

Answer 4:

$$\text{L.H.S.} = x+2x+3x = 6x \neq \text{R.H.S.}$$

Hence, the correct mathematical statements is $x+2x+3x = 6x$.

Question 5:

Find and correct the errors in the following mathematical statement: $5y + 2y + y - 7y = 0$

 Answer 5:

$$\text{L.H.S.} = 5y + 2y + y - 7y = 8y - 7y = y \neq \text{R.H.S.}$$

Hence, the correct mathematical statements is $5y + 2y + y - 7y = y$.

Question 6:

Find and correct the errors in the following mathematical statement: $3x + 2x = 5x^2$

 Answer 6:

$$\text{L.H.S.} = 3x + 2x = 5x \neq \text{R.H.S.}$$

Hence, the correct mathematical statements is $3x + 2x = 5x$.

Question 7:

Find and correct the errors in the following mathematical statement:

$$(2x)^2 + 4(2x) + 7 = 2x^2 + 8x + 7$$

 Answer 7:

$$\text{L.H.S.} = (2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7 \neq \text{R.H.S.}$$

Hence, the correct mathematical statements is $(2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7$.

Question 8:

Find and correct the errors in the following mathematical statement:

$$(2x)^2 + 5x = 4x + 5x = 9x$$

 Answer 8:

$$\text{L.H.S.} = (2x)^2 + 5x = 4x^2 + 5x \neq \text{R.H.S.}$$

Hence, the correct mathematical statements is $(2x)^2 + 5x = 4x^2 + 5x$.

Question 9:

Find and correct the errors in the following mathematical statement:

$$(3x+2)^2 = 3x^2 + 6x + 4$$

 Answer 9:

$$\text{L.H.S.} = (3x+2)^2 = (3x)^2 + 2 \times 3x \times 2 + (2)^2 = 9x^2 + 12x + 4 \neq \text{R.H.S.}$$

Hence, the correct mathematical statements is $(3x+2)^2 = 9x^2 + 12x + 4$.

Question 10:

Find and correct the errors in the following mathematical statements:

Substituting $x = -3$ in:

(a) $x^2 + 5x + 4$ gives $(-3)^2 + 5(-3) + 4 = 9 + 2 + 4 = 15$

(b) $x^2 - 5x + 4$ gives $(-3)^2 - 5(-3) + 4 = 9 - 15 + 4 = -2$

(c) $x^2 + 5x$ gives $(-3)^2 + 5(-3) = -9 - 15 = -24$

 Answer 10:

(a) L.H.S. = $x^2 + 5x + 4$

Putting $x = -3$ in given expression,

$$= (-3)^2 + 5(-3) + 4 = 9 - 15 + 4 = -2 \neq \text{R.H.S.}$$

Hence, $x^2 + 5x + 4$ gives $(-3)^2 + 5(-3) + 4 = 9 - 15 + 4 = -2$.

(b) L.H.S. = $x^2 - 5x + 4$

Putting $x = -3$ in given expression,

$$= (-3)^2 - 5(-3) + 4 = 9 + 15 + 4 = 28 \neq \text{R.H.S.}$$

Hence, $x^2 - 5x + 4$ gives $(-3)^2 - 5(-3) + 4 = 9 + 15 + 4 = 28$.

(c) L.H.S. = $x^2 + 5x$

Putting $x = -3$ in given expression,

$$= (-3)^2 + 5(-3) = 9 - 15 = -6 \neq \text{R.H.S.}$$

Hence, $x^2 + 5x$ gives $(-3)^2 + 5(-3) = 9 - 15 = -6$.

Question 11:

Find and correct the errors in the following mathematical statement: $(y-3)^2 = y^2 - 9$

Answer 11:

$$\begin{aligned} \text{L.H.S.} &= (y-3)^2 = y^2 - 2 \times y \times 3 + (3)^2 & \left[\because (a-b)^2 = a^2 - 2ab + b^2 \right] \\ &= y^2 - 6y + 9 \neq \text{R.H.S.} \end{aligned}$$

Hence, the correct statements is $(y-3)^2 = y^2 - 6y + 9$.

Question 12:

Find and correct the errors in the following mathematical statement: $(z+5)^2 = z^2 + 25$

Answer 12:

$$\begin{aligned} \text{L.H.S.} &= (z+5)^2 = z^2 + 2 \times z \times 5 + (5)^2 & \left[\because (a+b)^2 = a^2 + 2ab + b^2 \right] \\ &= z^2 + 10z + 25 \end{aligned}$$

Hence, the correct statement is $(z+5)^2 = z^2 + 10z + 25$.

Question 13:

Find and correct the errors in the following mathematical statement:

$$(2a+3b)(a-b) = 2a^2 - 3b^2$$

Answer 13:

$$\begin{aligned} \text{L.H.S.} &= (2a+3b)(a-b) = 2a(a-b) + 3b(a-b) \\ &= 2a^2 - 2ab + 3ab - 3b^2 = 2a^2 + ab - 3b^2 \neq \text{R.H.S.} \end{aligned}$$

Hence, the correct statement is $(2a+3b)(a-b) = 2a^2 + ab - 3b^2$.

Question 14:

Find and correct the errors in the following mathematical statement:

$$(a+b)(a+2) = a^2 + 8$$

 Answer14:

$$\begin{aligned}\text{L.H.S.} &= (a+4)(a+2) = a(a+2) + 4(a+2) \\ &= a^2 + 2a + 4a + 8 = a^2 + 6a + 8 \neq \text{R.H.S.}\end{aligned}$$

Hence, the correct statement is $(a+4)(a+2) = a^2 + 6a + 8$.

Question 15:

Find and correct the errors in the following mathematical statement:

$$(a-4)(a-2) = a^2 - 8$$

 Answer 15:

$$\begin{aligned}\text{L.H.S.} &= (a-4)(a-2) = a(a-2) - 4(a-2) \\ &= a^2 - 2a - 4a + 8 = a^2 - 6a + 8 \neq \text{R.H.S.}\end{aligned}$$

Hence, the correct statement is $(a-4)(a-2) = a^2 - 6a + 8$.

Question 16:

Find and correct the errors in the following mathematical statement: $\frac{3x^2}{3x^2} = 0$

 Answer 16:

$$\text{L.H.S.} = \frac{3x^2}{3x^2} = \frac{1}{1} = 1 \neq \text{R.H.S.}$$

Hence, the correct statement is $\frac{3x^2}{3x^2} = 1$.

Question 17:

Find and correct the errors in the following mathematical statement: $\frac{3x^2+1}{3x^2} = 1+1 = 2$

 Answer 17:

$$\begin{aligned}\text{L.H.S.} &= \frac{3x^2+1}{3x^2} = \frac{3x^2}{3x^2} + \frac{1}{3x^2} \\ &= 1 + \frac{1}{3x^2} \neq \text{R.H.S.}\end{aligned}$$

Hence, the correct statement is $\frac{3x^2+1}{3x^2} = 1 + \frac{1}{3x^2}$.

Question 18:

Find and correct the errors in the following mathematical statement: $\frac{3x}{3x+2} = \frac{1}{2}$

 Answer 18:

$$\text{L.H.S.} = \frac{3x}{3x+2} \neq \text{R.H.S.}$$

Hence, the correct statement is $\frac{3x}{3x+2} = \frac{3x}{3x+2}$.

Question 19:

Find and correct the errors in the following mathematical statement: $\frac{3}{4x+3} = \frac{1}{4x}$

 Answer 19:

$$\text{L.H.S.} = \frac{3}{4x+3} \neq \text{R.H.S.}$$

Hence, the correct statement is $\frac{3}{4x+3} = \frac{3}{4x+3}$.

Question 20:

Find and correct the errors in the following mathematical statement: $\frac{4x+5}{4x} = 5$

 Answer 20:

$$\text{L.H.S.} = \frac{4x+5}{4x} = \frac{4x}{4x} + \frac{5}{4x} = 1 + \frac{5}{4x} \neq \text{R.H.S.}$$

Hence, the correct statement is $\frac{4x+5}{4x} = 1 + \frac{5}{4x}$.

Question 21:

Find and correct the errors in the following mathematical statement: $\frac{7x+5}{5} = 7x$

 Answer 21:

$$\text{L.H.S.} = \frac{7x+5}{5} = \frac{7x}{5} + \frac{5}{5} = \frac{7x}{5} + 1 \neq \text{R.H.S.}$$

Hence, the correct statement is $\frac{7x+5}{5} = \frac{7x}{5} + 1$.