2. Exponents of Real Numbers

Exercise 2.1

1. Question

Assuming that *x, y, z* are positive real numbers, simplify each of the following:

(i) $\left(\sqrt{x^{-3}}\right)^5$ (ii) $\sqrt{x^3y^{-2}}$ (iii) $(x^{-2/3}y^{-1/2})^2$ (iv) $(\sqrt{x})^{-2/3}\sqrt{y^4} \div \sqrt{xy^{-1/2}}$ (v) $\sqrt[5]{243x^{10}y^5z^{10}}$ (vi) $\left(\frac{x^{-4}}{y^{-10}}\right)^{5/4}$

Answer

(i) $(\frac{\sqrt{1}}{\sqrt{3}})^5 = (1 / x^{3/2})^5$

$= (1 / x^{3/2 \times 5}) = (1 / x^{15/2})$

(ii) $(\sqrt{x}^3 / y^2) = (x^3 / y^2)^{1/2}$

$= x^{3 \times 1/2} / y^{2 \times 1/2}$

$= x^{3/2} / y$

(iii) $1 / (x^{2/3} y^{1/2})^2$

$= 1 / (x^{2/3 \times 2} y^{1/2 \times 2})$

$= 1 / x^{4/3} y$

(iv) $(x^{1/2})^{-2/3} (y)^2 / (xy^{-1/2})^{1/2}$

$= x^{-1/3}y^2 / (x^{1/2}y^{-1/2 \times 1/2})$

$= (x^{-5/6}) (y^{9/4})$

$= (y^{9/4}) / (x^{5/6})$

(v) $(243x^{10} y^5 z^{10})^{1/5}$

$= (3^5)^{1/5} x^2yz^2$

$= 3x^2yz^2$

(vi) $(y^{10} / x^4)^{5/4}$

$= y^{10 \times 5/4} / x^{4 \times 5/4}$

$= y^{25/2} / x^5$

2. Question

Simplify:

(i) $(16^{-1/5})^{5/2}$ (ii) $\sqrt[3]{(343)^{-2}}$ (iii) $(0.001)^{1/3}$ (iv) $\frac{(25)^{3/2}\times(243)^{3/5}}{(16)^{5/4}\times(8)^{4/3}}$ (v) $\left(\frac{\sqrt{2}}{5}\right)^{8}\div\left(\frac{\sqrt{2}}{5}\right)^{13}$ (vi) $\left(\frac{5^{-1}\times7^{2}}{5^{2}\times7^{-4}}\right)^{7/2}\times\left(\frac{5^{-2}\times7^{3}}{5^{3}\times7^{-5}}\right)^{-5/2}$

Answer

(i) $\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}}$

We know for any non-zero number a,

$(a^m)^n = a^{mn}$

So, $\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}} = 16^{-\frac{1}{2}}$

As we know $4^2 = 16$

Therefore, $\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}} = (4^2)^{-\frac{1}{2}}$

$= 4^{-1}$

As we know for any non-zero number a,

$a^{-1} = 1/a$

So $4^{-1} = 1/4$

(ii) $[(343)^{-2}]^{1/3}$

$(343^{-2})^{\frac{1}{3}}$

We know for any non-zero number a,

$(a^m)^n = a^{mn}$

So, $(343^{-2})^{\frac{1}{3}} = 343^{-\frac{2}{3}}$

As we know $7^3 = 343$

Therefore, $(343^{-2})^{\frac{1}{3}} = (7^3)^{-\frac{2}{3}}$

$= 7^{-2}$

As we know for any non-zero number a,

$a^{-1} = 1/a$

So $7^{-2} = 1/7^2$

$= 1/49$

(iii) $(\frac{1}{1000})^{1/3} = (1 / 10^3)^{1/3}$

$= \frac{1}{10} = 0.1$

(iv) $\frac{(25)^{3/2} \times (243)^{3/5}}{(16)^{5/4} \times (8)^{4/3}}$

We know $25 = 5^2$

$243 = 3^5$

$16 = 2^4$

$8 = 2^3$

So, $\frac{(5^2)^{\frac{3}{2}} \times (3^5)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}} \times (2^3)^{\frac{4}{3}}}$

We know for any non-zero number a,

$(a^m)^n = a^{mn}$

So,

$$= \frac{5^3 \times 3^3}{2^5 \times 2^4}$$

$$= \frac{125 \times 27}{32 \times 16}$$

$$= \frac{3375}{512}$$

(v) $\left(\frac{\sqrt{2}}{5}\right)^{8} \div \left(\frac{\sqrt{2}}{5}\right)^{13}$

We know that for any non-zero number a,

$a^m \div a^n = a^{m-n}$

So,

$$\frac{\left(\frac{\sqrt{2}}{5}\right)^{8}}{\left(\frac{\sqrt{2}}{5}\right)^{13}} = \left(\frac{\sqrt{2}}{5}\right)^{8-13} = \left(\frac{\sqrt{2}}{5}\right)^{-5}$$

As we know for any non-zero number a,

$a^{-1} = 1/a$ $\left(\frac{5}{\sqrt{2}}\right)^{5} = \frac{3125}{4\sqrt{2}}$

(vi) $\left(\frac{5^{-1} \times 7^{2}}{5^{2} \times 7^{-4}}\right)^{7/2} \times \left(\frac{5^{-2} \times 7^{3}}{5^{3} \times 7^{-5}}\right)^{-5/2}$

We know for any non-zero number a,

$(a^m)^n = a^{mn}$

So,

$$\left(\frac{(5^{-1})^{\frac{7}{2}} \times (7^{2})^{\frac{7}{2}}}{(5^{2})^{\frac{7}{2}} \times (7^{-4})^{\frac{7}{2}}}\right) \times \left(\frac{(5^{-2})^{\frac{-5}{2}} \times (7^{3})^{\frac{-5}{2}}}{(5^{3})^{\frac{-5}{2}} \times (7^{-5})^{\frac{-5}{2}}}\right)$$

$$= \left(\frac{5^{-\frac{7}{2}} \times 7^{7}}{5^{7} \times 7^{-14}}\right) \times \left(\frac{5^{5} \times 7^{-\frac{15}{2}}}{5^{-\frac{15}{2}} \times 7^{\frac{25}{2}}}\right)$$

We know for any non-zero number a,

$a^m \times a^n = a^{m+n}$

$$= \left(5^{-\frac{7}{2}-7} \times 7^{7+14}\right) \times \left(5^{5+\frac{15}{2}} \times 7^{-\frac{15}{2}-\frac{25}{2}}\right)$$

$$= \left(5^{\frac{-7-14}{2}} \times 7^{7+14}\right) \times \left(5^{\frac{10+15}{2}} \times 7^{\frac{-15-25}{2}}\right)$$

$$= \left(5^{\frac{-21}{2}} \times 7^{21}\right) \times \left(5^{\frac{25}{2}} \times 7^{\frac{-40}{2}}\right)$$

$$= \left(5^{\frac{-21+25}{2}} \times 7^{21-20}\right)$$

$$= \left(5^{\frac{4}{2}} \times 7^{1}\right)$$

$$= (5^2 \times 7^1)$$

= 25 × 7

= 175

3. Question

Prove that:

(i) $\sqrt{3\times5^{-3}} \div \sqrt[3]{3^{-1}}\sqrt{5} \times \sqrt[6]{3\times5^{6}} = \frac{3}{5}$

(ii) $9^{3/2} - 3\times 5^{0} - \left(\frac{1}{81}\right)^{-1/2} = 15$

(iii) $\left(\frac{1}{4}\right)^{-2} - 3\times8^{2/3} \times4^{0} + \left(\frac{9}{16}\right)^{-1/2} = \frac{16}{3}$

(iv) $\frac{2^{1/2}\times3^{1/3}\times4^{1/4}}{10^{-1/5}\times5^{3/5}} \div \frac{3^{4/3}\times5^{-7/5}}{4^{-3/5}\times6} = 10$

(v) $\sqrt{\frac{1}{4}} + (0.01)^{-1/2} - (27)^{2/3} = \frac{3}{2}$

(vi) $\frac{2^{n}+2^{n-1}}{2^{n+1}-2^{n}} = \frac{3}{2}$

(vii) $\left(\frac{64}{125}\right)^{-2/3} + \frac{1}{\left(\frac{256}{625}\right)^{1/4}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right) = \frac{65}{16}$

(viii) $\frac{3^{-3}\times6^{2}\times\sqrt{98}}{5^{2}\times\sqrt[3]{1/25}\times(15)^{-4/3}\times3^{1/3}} = 28\sqrt{2}$

(ix) $\frac{(0.6)^{0}-(0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^{3}+\left(-\frac{1}{3}\right)^{-1}} = -\frac{3}{2}$

Answer

(i) $(3^{1/2+1/6}.5^{-3/2+1}) / (3^{-1/3}.5^{1/2})$

$=(3^{2/3}.5^{-1/2}) / (3^{-1/3}.5^{1/2})$

$=(3^{2/3 + 1/3}) / (5^{1/2 +1/2})$

$=3/5$

(ii) $(3^2)^{3/2} -3.1 – (1/9^2)^{-1/2}$

$= 3^3 -3 -9$

$=27 -3 -9$

$=27-12$

$=15$

(iii) $2^{(-2)(-2)} -3.8^{2/3} +(3/4)^{-1}$

$=2^4 -3.2^2 + 4/3$

$=16 -12 + 4/3$

$=16/3$

(iv) $[(2.3^{1/3})/(2^{-1/5}\ 5^{2/5})] \times (2^{-1/5}.3)/ (3^{4/3}.5^{7/5})$

$= 2.3^{1/3 +1 -4/3} / 5^{2/5-7/5}$

$= 2.5$

$=10$

(v) $1/2 + 1/(0.01)^{1/2} -3^2$

$=1/2 + 10 – 9$

$=1/2 + 1$

$=3/2$

(vi) $(2^n + 2^{n-1})/\ ) (2^{n+1} - 2^n)$

$=2^n(1 + 2^{-1}\ ) / 2^n (2-1)$

$= [1 + (1/2)]/1$

$=1 + 1/2$

$=3/2$

(vii) $(125/64)^{2/3} + (625/256)^{1/4} + (\ 5/4)$

$=(5/4)^2 + 5/4 + 5/4$

$=25/16 + 5/4 + 5/4$

$=65/16$

(viii) $(3^{-3}.6^2.7(2)^{1/2})/ (5^{4/3}.(15)^{-4/3}.3^{1/3}) = 28(2)^{1/2}$

$(3^{-3}.36.7(2)^{1/2})/ (5^{4/3-4/3}.(3)^{-1})$

$(3^{-2}.36.7(2)^{1/2})/ (5^0)$

$1/9.36.7(2)^{1/2}$

$28\sqrt{2}$

(ix) $\{1- 1/0.1\}/ \{ (3/8)^{-1}(3/2)^3 + (-1/3)^{-1}$

$=1-10/\{ (8/3)(3/2)^3 + (-3)$

$=-9/(3^2-3)$

$= -3/2$

4. Question

If $27^x = \frac{9}{3^x}$, find x.

Answer

We have,

$(27)^x = 9 / 3^x$

$(3^3)^x = 3^2 / 3^x$

$3^{3x} = 3^{2 - x}$

$3x = 2 - x$ {On equating exponents}

$3x + x = 2$

$4x = 2$

$x = \frac{2}{4} = \frac{1}{2}$

Hence, the value of x is $\frac{1}{2}$

5. Question

Find the values of x in each of the following:

(i) $2^{5x} \div 2^x = \sqrt[5]{2^{20}}$

(ii) $(2^3)^4 = (2^2)^x$

(iii) $\left(\frac{3}{5}\right)^x \left(\frac{5}{3}\right)^{2x} = \frac{125}{127}$

(iv) $5^{x-2} \times 3^{2x-3} = 135$

(v) $2^{x-5} \times 5^{x-4} = 5$

(vi) $2^{x-7} \times 5^{x-4} = 1250$

Answer

(i) we have,

$2^{5x} \div 2^x = \sqrt[5]{2^{20}}$

$2^{5x}/2^x = 2^{20/5}$

$2^{5x-x} = 2^4$

$4x = 4$

$x=1$

(ii) We have,

$(2^3)^4 = (2^2)^x$

$2^{3.4} = 2^{2.x}$

$12 = 2x$

$X=6$

(iii) We have,

$\left(\frac{3}{5}\right)^x \left(\frac{5}{3}\right)^{2x} = \frac{125}{127}$

$5^{2x-x}/\ 3^{2x-x} = (5/3)^3$

$5^x/3^x = (5/3)^3$

$(5/3)^x = (5/3)^3$

$x=3$

(iv) We have,

$5^{x-2} \times 3^{2x-3} = 135$

$5^{x-2} \times 3^{2x-3} = 5 \times 27$

$5^{x-2} \times 3^{2x-3} = 5^1 \times 3^3$

x-2 =1 ; 2x-3 =3

x=3 ; x= 3

(v) We have,

$2^{x-5} \times 5^{x-4} = 5$

$2^{x-5} \times 5^{x-4} = 5^1 \times 2^0$

x-5=0 ; x-4 = 1

x= 4 ; x=1 +4 =5

(vi) We have,

$2^{x-7} \times 5^{x-4} = 1250$

$2^{x-7} \times 5^{x-4} = 2^1 \times 5^4$

x – 7=1; x-4 = 4

x= 8; x= 4+4 = 8

CCE - Formative Assessment

1. Question

Write $(625)^{-1/4}$ in decimal form.

Answer

$$\frac{1}{(625)^{\frac{1}{4}}} = \frac{1}{(5^4)^{\frac{1}{4}}} = \frac{1}{5}$$

= 0.2

2. Question

State the product law of exponents.

Answer

The product law of exponent states that while multiplying two parts having same base, you can add the exponents.

3. Question

State the quotient law of exponents.

Answer

The quotient law of exponent states that to divide two exponents with the same base, you keep the base and subtract the powers.

4. Question

State the power law of exponents.

Answer

The power law of exponents states that:

$(a^n)^m = a^{n.m}$

Example: $(2^3)^2 = 2^{3.2}$

$= 2^6 = 64$

5. Question

For any positive real number *x*, find the value of $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$

Answer

$x^{(a - b)(a + b)} \times x^{(b - c)(b + c)} \times x^{(c - a)(c + a)}$

$= x^{a.a - b.b} \times x^{b.b - c.c} \times x^{c.c - a.a}$

$= x^{a.a - b.b + b.b - c.c + c.c - a.a}$

$= x^o = 1$

6. Question

Write the value of $\{5(8^{1/3} + 27^{1/3})^3\}^{1/4}$

Answer

$\{5(8^{1/3} + 27^{1/3})^3\}^{1/4}$

$= \{5 (2 + 3)^3\}^{1/4}$

$= (5^4)^{1/4} = 5$

7. Question

Simplify $[\{(625)^{\frac{1}{2}}\}^{-\frac{1}{4}}]^2$.

Answer

$$[\{(625)^{\frac{1}{2}}\}^{-\frac{1}{4}}]^2$$

$$= 625^{\frac{1}{2} \cdot -\frac{1}{4} \cdot 2}$$

$$= 625^{-\frac{1}{4}} = \frac{1}{5^4}^{\frac{1}{4}}$$

$$= \frac{1}{5}$$

5. Question

For any positive real number *x*, write the value of $\{(x^a)^b\}^{\frac{1}{ab}}\{(x^b)^c\}^{\frac{1}{bc}}\{(x^c)^a\}^{\frac{1}{ca}}$

Answer

$(x)^{ab \times 1/ab} . (x)^{bc . 1/bc} . x^{ca . 1/ca}$

$= x . x . x$

$= x^3$

9. Question

If $(x\text{-}1)^3 = 8$, what is the value of $(x+1)^2$

Answer

$(x - 1)^3 = 8$

$x - 1 = 2$

$x = 3$

$(x + 1)^2 = (3 + 1)^2$

$= 4^2 = 16$

10. Question

If $2^4 \times 4^2 = 16x$, then find the value of *x*.

Answer

$2^4 \times 2^4 = 16x$

$2^8 = 2^4 \times x$

$x = 2^4 = 16$

11. Question

If $3^{x-1}=9$ and $4^{y+2} = 64$, What is the value of $\frac{x}{y}$.

Answer

$3^{x - 1} = 3^2$

$x - 1 = 2$

$x = 3$

$4^{y + 2} = 4^3$

$y + 2 = 3$

$y = 1$

$x / y = 3/1 = 3$

12. Question

Write the value of $\sqrt[3]{7} \times \sqrt[3]{49}$.

Answer

$\sqrt[3]{7} \times \sqrt[3]{49}$

$= (7 . 7^2)^{1/3}$

$= (7)^{3\times\ 1/3}$

$= 7$

13. Question

Write $\left(\frac{1}{9}\right)^{-1/2} \times (64)^{-1/3}$ as a rational number.

Answer

$\left(\frac{1}{9}\right)^{-1/2} \times (64)^{-1/3}$

$= (3^2)^{1/2} \times (1/4^3)^{-1/3}$

$= 3 \times 1/4 = 3/4$

14. Question

Write the value of $\sqrt[3]{125\times27}$.

Answer

$\sqrt[3]{125\times27}$

$= (5^3 \times 3^3)^{1/3}$

$= 5 \times 3$

$= 15$

1. Question

The value of $\{2-3(2-3)^3\}^3$ is

A. 5

B. 125

C. 1/5

D. -125

Answer

$\{2-3(2-3)^3\}^3$

$= \{2 - 3\ (-1)^3\}^3$

$= \{2 + 3\}^3$

$= 5^3 = 125$

2. Question

$(256)^{0.16} \times (256)^{0.09}$

A. 4

B. 16

C. 64

D. 256.25

Answer

$(256)^{0.16} \times (256)^{0.09}$

$= (256)^{\ 0.16 + 0.09}$

$= (256)^{\ 0.25}$

$= 4^4 \times \frac{1}{4} = 4$

3. Question

If $10^{2y} = 25$, then 10^{-y} equals

A. $-\frac{1}{5}$

B. $\frac{1}{50}$

C. $\frac{1}{625}$

D. $\frac{1}{5}$

Answer

$10^{2y} = 25$

$= 10^{y} = x$

$= x^2 = 5^2$

$= x = 5$

$= 1/x = 10^{-y}$

$= 1/5$

4. Question

The value of $x\text{-}y^{x-y}$ when $x = 2$ and $y = -2$ is

A. 18

B. -18

C. 14

D. -14

Answer

$x - y^{x - y}$

$= 2 - (-2)^{(2 + 2)}$

$= 2 - 16 = - 14$

5. Question

The product of the square root of x with the cube root of x is

A. Cube root of the square root of x

B. Sixth root of the fifth power of x

C. Fifth root of the sixth power of x

D. Sixth root of x

Answer

$\sqrt{x} \times \sqrt[3]{x}$

$= x^{1/2} \times x^{1/3}$

$= x^{5/6}$

6. Question

If $9^{x+2} = 240 + 9^{x}$, then $x =$

A. 0.5

B. 0.2

C. 0.4

D. 0.1

Answer

$9^{x+2} = 240 + 9^x$

$9x \times 9^2 = 240 + 9^x$

Let $9^x = y$

$81y = 240 + y$

$80y = 240$

$y = \frac{240}{80}$

$9^x = 3$

$3^{2x} = 3$

$2x = 1$

$x = \frac{1}{2} = 0.5$

7. Question

The seventh root of *x* divided by the eighth root of *x* is

A. *x*

B. $\sqrt{x}$

C. $\sqrt[56]{x}$

D. $\frac{1}{\sqrt[56]{x}}$

Answer

$x^{1/7} / x^{1/8}$

$= (x)^{1/7 - 1/8}$

$= (x)^{1/56}$

$= \sqrt[56]{x}$

8. Question

The square root of 64 divided by the cube root of 64 is

A. 64

B. 2

C. $\frac{1}{2}$

D. $64^{2/3}$

Answer

As 64 can be written as 64 = 2×2×2×2×2×2so 64 = 2^6 $\sqrt{64}=\sqrt{(2^6)}=(2^6)^{1/2}$ = 2^3=8 $\sqrt[3]{64}$ =$(2^6)^{1/3}$ = 2^2=4$\frac{\sqrt{64}}{\sqrt[3]{64}}$ = $\frac{8}{4}$ = 2

9. Question

Which of the following is (are) not equal to $\left\{\left(\frac{5}{6}\right)^{1/5}\right\}^{-1/6}$?

A. $\left(\frac{5}{6}\right)^{\frac{1}{5}\cdot\frac{1}{6}}$

B. $\frac{1}{\left\{\left(\frac{5}{6}\right)^{1/5}\right\}^{1/6}}$

C. $\left(\frac{6}{5}\right)^{1/30}$

D. $\left(\frac{5}{6}\right)^{-1/30}$

Answer

$\left\{\left(\frac{5}{6}\right)^{1/5}\right\}^{-1/6}$

= 1 / $\{(5/6)^{1/5}\}^{1/5}$

= $(5/6)^{-1/30}$

= $(6/5)^{1/30}$

10. Question

When simplified $(x^{-1} + y^{-1})^{-1}$ is equal to

A. xy

B. $x + y$

C. $\frac{xy}{x+y}$

D. $\frac{x+y}{xy}$

Answer

$(x^{-1} + y^{-1})^{-1}$

$= (\frac{1}{x} + \frac{1}{y})^{-1}$

$= (\frac{x+y}{xy})^{-1}$

$= (\frac{xy}{x+y})$

11. Question

If $8^{x+1} = 64$, what is the value of 3^{2x+1}?

A. 1

B. 3

C. 9

D. 27

Answer

$8^{x+1} - 64$

$= 8^{x+1} = 8^2$

On equating powers, we get

$x + 1 = 2$

$x = 1$

$= 3^{2x+1}$

$= 3^3 = 27$

12. Question

If $0 < y < x$, which statement must be true?

A. $\sqrt{x} - \sqrt{y} = \sqrt{x-y}$

B. $\sqrt{x} + \sqrt{x} = \sqrt{2x}$

C. $x\sqrt{y} = y\sqrt{x}$

D. $\sqrt{xy} = \sqrt{x}\sqrt{y}$

Answer

Since, it is the property of square roots.

13. Question

If x is a positive real number and $x^2 = 2$, then $x^3 =$

A. $\sqrt{2}$

B. $2\sqrt{2}$

C. $3\sqrt{2}$

D. 4

Answer

$x^2 = 2$

$x = \sqrt{2}$

$x^3 = (2)^{1/2 \times 3}$

$= 2\sqrt{2}$

14. Question

If $(2^3)^2 = 4^x$, then $3^x =$

A. 3

B. 6

C. 9

D. 27

Answer

$(2^3)^2 = 2^{2x}$

$2x = 6$

$x = 3$

15. Question

If $10^x = 64$, what is the value of $10^{\frac{x}{2}+1}$?

A. 18

B. 42

C. 80

D. 81

Answer

$10^{\frac{x}{2}+1}$ can be written as: $(10^x)^{1/2} \times 10$

$= (64)^{1/2} \times 10$

= 8 × 10

= 80

16. Question

If $\frac{x}{x^{1.5}}$ = $8x^{-1}$ and $x > 0$, then x =

A. $\frac{\sqrt{2}}{4}$

B. $2\sqrt{2}$

C. 4

D. 64

Answer

$$\frac{x}{x^{1.5}} = 8x^{-1}$$

$$\Rightarrow \frac{x}{x^{1.5}} = \frac{8}{x}$$

$$\Rightarrow x^{1+1-1.5} = 8$$

$$\Rightarrow x^{\frac{1}{2}} = 64^{\frac{1}{2}}$$

$$\Rightarrow x = 64$$

17. Question

If $g = t^{2/3} + 4t^{-1/2}$, what is the value of g when $t = 64$?

A. $\frac{31}{2}$

B. $\frac{33}{2}$

C. 16

D. $\frac{257}{16}$

Answer

$g = t^{2/3} + 4t^{-1/2}$

$= (64)^{2/3} + 4\ (64)^{-1/2}$

$= [(64)^{1/3}]^3 + 4\ (\frac{1}{64})^{1/2}$

$= 4^2 + 4\ (\frac{1}{8})$

$= 16 + \frac{1}{2} = \frac{33}{2}$

18. Question

If $x^{-2} = 64$, then $x^{1/3} + x^0 =$

A. 2

B. 3

C. 3/2

D. 2/3

Answer

$(\frac{1}{x})^2 = (8)^2$

$\frac{1}{x} = 8$

$x = \frac{1}{8}$

$x^{1/3} + x^o$

$= (\frac{1}{8})^{1/3} + (\frac{1}{8})^o$

$= \frac{1}{2} + 1 = \frac{3}{2}$

19. Question

If $4^x - 4^{x-1} = 24$, then $(2x)^x$ equals

A. $5\sqrt{5}$

B. $\sqrt{5}$

C. $25\sqrt{5}$

D. 125

Answer

$4^x - 4^{x-1} = 24$

Let 4x = y

$y - \frac{y}{4} = 24$

$4y - y = 96$

$y = 32$

$4^x = 32$

$2^{2x} = 2^5$

$(2x)^x = (2 \times \frac{5}{2})^{5/2}$

$= (5)^{5/2} = 25\sqrt{5}$

20. Question

When simplified $\left(-\frac{1}{27}\right)^{-2/3}$ is

A. 9

B. -9

C. $\frac{1}{9}$

D. $-\frac{1}{9}$

Answer

$(-27)^{2/3}$

$= (3)^{3 \times 2/3}$

$= 9$

21. Question

Which one of the following is not equal to $\left(\sqrt[3]{8}\right)^{-1/2}$?

A. $\left(\sqrt[3]{2}\right)^{-1/2}$

B. $8^{-1/6}$

C. $\frac{1}{(\sqrt[3]{8})^{1/2}}$

D. $\frac{1}{\sqrt{2}}$

Answer

$1 / (8)^{-1/2 \times 1/3}$

$= 2^{-1/2}$

$= \frac{1}{\sqrt{2}}$

22. Question

Which one of the following is not equal to $\left(\frac{100}{9}\right)^{-3/2}$?

A. $\left(\frac{100}{9}\right)^{3/2}$

B. $\frac{1}{\left(\frac{100}{9}\right)^{3/2}}$

C. $\frac{3}{10} \times \frac{3}{10} \times \frac{3}{10}$

D. $\sqrt{\frac{100}{9} \times \frac{100}{9} \times \frac{100}{9}}$

Answer

$1 / (100/9)^{3/2}$

$= (10/3)^{-3/2 \times 2}$

$= \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10}$

23. Question

When simplified $(256)^{-(4^{-3/2})}$ is

A. 8

B. $\frac{1}{8}$

C. 2

D. $\frac{1}{2}$

Answer

$1 / 256^{1/8}$

$= 1/2^{\ 8 \times 1/8}$

$= 1/2$

24. Question

$\frac{5^{n+2} - 6 \times 5^{n+1}}{13 \times 5^n - 2 \times 5^{n+1}}$ is equal to

A. $\frac{5}{3}$

B. $-\frac{5}{3}$

C. $\frac{3}{5}$

D. $-\frac{3}{5}$

Answer

5^n (25 – 30) / 5^n (13 – 10)

= -5 / 3

25. Question

If *a, b, c* are positive real numbers, then $\sqrt{a^{-1}b} \times \sqrt{b^{-1}c} \times \sqrt{c^{-1}a}$ is equal to

A. 1

B. *abc*

C. $\sqrt{abc}$

D. $\frac{1}{abc}$

Answer

$(b/a)^{1/2} \times (c/b)^{1/2} \times (a/c)^{1/2}$

$= (b/a \times c/b \times a/c)^{1/2}$

= 1

26. Question

If $\frac{3^{2x-8}}{225} = \frac{5^3}{5^x}$, then *x* =

A. 2

B. 3

C. 5

D. 4

Answer

$\frac{3^{2x-8}}{225} = \frac{5^3}{5^x}$

$= 5^x \times 3^{2x-8} = 5^5 \times 3^3$Comparing the coefficient of x we get,

= x = 5

27. Question

If $\left(\frac{2}{3}\right)^x \left(\frac{3}{2}\right)^{2x} = \frac{81}{16}$, then *x*=

A. 2

B. 3

C. 4

D. 1

Answer

$(3/2)^{-x}\ (3/2)^{2x} = (3/2)^4$

$= (3/2)^{-x + 2x} = (3/2)^4$

$= -x + 2x = 4$

$= x = 4$

28. Question

The value of $\left\{8^{-4/3} \div 2^{-2}\right\}^{1/2}$ is

A. $\frac{1}{2}$

B. 2

C. $\frac{1}{4}$

D. 4

Answer

$\left\{8^{-4/3} \div 2^{-2}\right\}^{1/2}$

$= \{2^{-4} \div 2^{-2}\}^{1/2}$

$= \{1/16 \times 2^2\}^{1/2}$

$= \frac{1}{2}$

29. Question

If *a, b, c* are positive real numbers, then $\sqrt[5]{3125a^{10}b^5c^{10}}$ is equal to

A. $5a^2bc^2$

B. $25ab^2c$

C. $5a^3bc^3$

D. $125a^2bc^2$

Answer

$(3125a^{10}b^5c^{10})^{1/5}$

$= 5a^2bc^2$

30. Question

The value of $64^{-1/3}\ (64^{1/3} - 64^{2/3})$, is

A. 1

B. $\frac{1}{3}$

C. -3

D. -2

Answer

$64^{-1/3}\ (64^{1/3} - 64^{2/3})$

$= 4^{-1}\ (4 - 4^2)$

$= \frac{1}{4}\ (4 - 16)$

$= \frac{-12}{4} = -3$

31. Question

If $\sqrt{5^n} = 125$, then $5\sqrt[3]{64}$ =

A. 25

B. $\frac{1}{125}$

C. 625

D. 10

Answer

$\sqrt{5^n} = 125$

$5^{n/2} = 5^3$

$n/2 = 3$

$n = 6$

$5\sqrt[3]{64} = 5\ (64)^{1/6}$

$= 5\ (2)^{6/6} = 10$

32. Question

If $(16)^{2x+3} = (64)^{x+3}$, then 4^{2x-2} =

A. 64

B. 256

C. 32

D. 512

Answer

$4^{4x + 6} = 4^{3x + 9}$

$= 4x + 6 = 3x + 9$

$= x = 3$

$4^{2x - 2} = 4^4$

$= 256$

33. Question

If *a, m, n* are positive integers, then $\left\{m\sqrt{n\sqrt{a}}\right\}^{mn}$ is equal to

A. a^{mn}

B. a

C. $a^{m/n}$

D. 1

Answer

$$\left\{m\sqrt{n\sqrt{a}}\right\}^{mn}$$

We know for any non-zero number a,

$a^m \times a^n = a^{m+n}$

$$= \left\{\left(a^{\frac{1}{n}}\right)^{\frac{1}{m}}\right\}^{mn}$$

Again using $(a^m)^n = a^{mn}$ we get, $= \left\{a^{\frac{1}{mn}}\right\}^{mn}$

=a

34. Question

If $2^{-m} \times \frac{1}{2^m} = \frac{1}{4}$, then $\frac{1}{14}\left\{(4^m)^{1/2} + \left(\frac{1}{5^m}\right)^{-1}\right\}$ is equal to

A. $\frac{1}{2}$

B. 2

C. 4

D. $-\frac{1}{4}$

Answer

$2^{-m} \times 1 / 2^m = 1/4$

$= 1/2^m \times 1/2^m = 1/4$

$= 1/4^m = 1/4$

$= m = 1$

$1/14 \{(4^m)^{1/2} + (1/5^m)^{-1}\}$

$= 1/14 \{2 + 5\}$

$= 1/14 \times 7$

$= \frac{1}{2}$

35. Question

If $x = 2$ and $y = 4$, then $\left(\frac{x}{y}\right)^{x-y} + \left(\frac{y}{x}\right)^{y-x} =$

A. 4

B. 8

C. 12

D. 2

Answer

$(2/4)^{2-4} + (4 / 2)^{4-2}$

$= (1/2)^{-2} + 2^2$

$= 2^2 + 2^2$

$= 8$

36. Question

The value of *m* for which $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{-1/3}\right]^{-1/4} = 7^m$, is

A. $-\frac{1}{3}$

B. $\frac{1}{4}$

C. -3

D. 2

Answer

$[\{7^4\}^{-1/3}]^{1/4}$

$= (1/7^4)^{1/3 \times 1/4}$

$= (1/7)^{1/3} = 7^m$

$= 7^{-1/3} = 7^m$

$= m = -1/3$

37. Question

If $\frac{2^{m+n}}{2^{n-m}} = 16$,and $a = 2^{1/10}$, then $\frac{a^{2m+n-p}}{(a^{m-2n+2p})^{-1}} =$

A. 2

B. $\frac{1}{4}$

C. 9

D. $\frac{1}{8}$

Answer

$$\frac{2^{m+n}}{2^{n-m}} = 2^4$$ $2^{m + n - n + m} = 2^4 2^{2m} = 2^4 2m = 4m = 2$Also $a = 2^{1/10}$

$$\frac{a^{2m+n-p}}{\left(a^{m-2n+2p}\right)^{-1}} = a^{2m+n-p} \times a^{m-2n+2p}$$

$= a^{2m + m + n - 2n - p + 2p} = a^{3m - n + p}$

$$= \left(2^{\frac{1}{10}}\right)^{3m-n+p}$$

$$= \left(2^{\frac{1}{10}}\right)^{3(2)-n+p}$$

$$= \left(2^{\frac{1}{10}}\right)^{6-n+p}$$

38. Question

The value of $\{(23+2^2)^{2/3}+(140-19)^{1/2}\}^2$, is

A. 196

B. 289

C. 324

D. 400

Answer

$$[(23+2^2)^{\frac{2}{3}}+(140-19)^{\frac{1}{2}}]^2$$

$$= [27^{\frac{2}{3}}+121^{\frac{1}{2}}]^2$$

$= \{3^2 + 11\}^2$

$= (9 + 11)^2$

$= (20)^2 = 400$

39. Question

If $\sqrt{2^n} = 1024$, then $3^{\frac{n}{4}-4}$ =

A. 3

B. 9

C. 27

D. 81

Answer

$\sqrt{2^n} = 2^{10}$

$2^{n/2} = 2^{10}$

$\frac{n}{2} = 10$

$n = 20$

$= 3^{2\,(n\,/\,4\,-\,4)}$

$= 3^{2\,(20/4\,-\,4)}$

$= 3^2 = 9$

40. Question

If $\frac{3^{5x} \times 81^2 \times 6561}{3^{2x}} = 3^7$, then x =

A. 3

B. -3

C. $\frac{1}{3}$

D. $-\frac{1}{3}$

Answer

$\frac{3^{5x} \times 81^2 \times 6561}{3^{2x}} = 3^7$

$= 3^{5x} \times 3^8 \times 3^8 / 3^{2x} = 3^7$

$= 3^{5x + 16 - 2x} = 3^7$

$= 5x + 16 - 2x = 7$

$= 3x + 16 = 7$

$= 3x = -9$

$x = -3$