3. Rationalisation

Exercise 3.1

1. Question

Simplify each of the following :

(i) $\sqrt[3]{4} \times \sqrt[3]{16}$ (ii) $\frac{\sqrt[4]{1250}}{\sqrt[4]{2}}$

Answer

(i) $\sqrt[3]{4}\times\sqrt[3]{16}$

$=\sqrt[3]{4\times16} = \sqrt[3]{64}$

$=\sqrt[3]{4^3} = (4^3)^{\frac{1}{3}} = 4$

(ii) $\frac{\sqrt[4]{1250}}{\sqrt[4]{2}} = \sqrt[4]{\frac{1250}{2}}$

$= \sqrt[4]{\frac{625\times2}{2}} = \sqrt[4]{625}$

$=\sqrt[4]{5^4} = (5^4)^{\frac{1}{4}} = 5$

2. Question

Simplify the following expressions :

(i) $(4+\sqrt{7})$ $(3+\sqrt{2})$

(ii) $(3+\sqrt{3})$ $(5 - \sqrt{2})$

(iii) $(\sqrt{5} -2)$ $(\sqrt{3}-\sqrt{5})$

Answer

(i) $(4+\sqrt{7})(3+\sqrt{2})$

$= 4\times3 + 4\times\sqrt{2} + \sqrt{7}\times3 + \sqrt{7}\times\sqrt{2}$

$= 12 + 4\sqrt{2} + 3\sqrt{7} + \sqrt{14}$

(ii) $(3+\sqrt{3})(5-\sqrt{2})$

$= 3\times5 + 3\times(-\sqrt{2}) + \sqrt{3}\times5 + \sqrt{3}\times(-\sqrt{2})$

$= 15 - 3\sqrt{2} + 5\sqrt{3} - \sqrt{3} \times 2$

$= 15 - 3\sqrt{2} + 5\sqrt{3} - \sqrt{6}$

(iii) $(\sqrt{5} - 2)(\sqrt{3} - \sqrt{5})$

$= \sqrt{5} \times \sqrt{3} + \sqrt{5} \times (-\sqrt{5}) + (-2) \times \sqrt{3} + (-2) \times (-\sqrt{5})$

$= \sqrt{5 \times 3} - \sqrt{5 \times 5} - 2\sqrt{3} + 2\sqrt{5}$

$= \sqrt{15} - \sqrt{5^2} - 2\sqrt{3} + 2\sqrt{5}$

$= \sqrt{15} - 5 - 2\sqrt{3} + 2\sqrt{5}$

3. Question

Simplify the following expressions :

(i) $(11 + \sqrt{11})(11 - \sqrt{11})$

(ii) $(5 + \sqrt{7})(5 - \sqrt{7})$

(iii) $(\sqrt{8} - \sqrt{2})(\sqrt{8} + \sqrt{2})$

(iv) $(3 + \sqrt{3})(3 - \sqrt{3})$

(v) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

Answer

(i) $(11 + \sqrt{11})(11 - \sqrt{11}) = (11)^2 - (\sqrt{11})^2$

Because $(a + b)(a - b) = a^2 - b^2$

= 121 - 11 = 110

(ii) $(5 + \sqrt{7})(5 - \sqrt{7}) = (5)^2 - (\sqrt{7})^2$

= 25 – 7 = 18

(iii) $(\sqrt{8} - \sqrt{2})(\sqrt{8} + \sqrt{2}) = (\sqrt{8})^2 - (\sqrt{2})^2$

= 8 – 2 = 6

(iv) $(3 + \sqrt{3})(3 - \sqrt{3}) = (3)^2 - (\sqrt{3})^2$

= 9 – 3 = 6

(v) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2$

= 5 – 2 = 3

4. Question

Simplify the following expressions:

(i) $(\sqrt{3}+\sqrt{7})^2$ (ii) $(\sqrt{5}-\sqrt{3})^2$ (iii) $(2\sqrt{5}+3\sqrt{2})^2$

Answer

(i) $\left(\sqrt{3}+\sqrt{7}\right)^2 = \left(\sqrt{3}\right)^2 + 2\times\sqrt{3}\times\sqrt{7}+\left(\sqrt{7}\right)^2$

Because: $(a+b)^2 = (a)^2 + 2\times\sqrt{a}\times\sqrt{b}+(b)^2$

= $3+2\sqrt{3\times7}+ 7$

= $10+2\sqrt{21}$

(ii) $\left(\sqrt{5}-\sqrt{3}\right)^2 = \left(\sqrt{5}\right)^2 - 2\times\sqrt{5}\times\sqrt{3}+\left(\sqrt{3}\right)^2$

$(a)^2 - 2\times\sqrt{a}\times\sqrt{b}+ (b)^2$

$= 5 - 2\sqrt{5\times3}+3$

$= 8 - 2\sqrt{15}$

(iii) $\left(2\sqrt{5}+3\sqrt{2}\right)^2 = \left(2\sqrt{5}\right)^2 + 2\left(2\sqrt{5}\right)\times\left(3\sqrt{2}\right)+\left(3\sqrt{2}\right)^2$

= $2^2\times\sqrt{5}^2 + 2\times2\times3\times\sqrt{5\times2}+ 3^2\times\sqrt{2}^2$

= $4\times5+12\sqrt{5\times2}+9\times2$

= $20+12\sqrt{10}+18$

= $38+12\sqrt{10}$

Exercise 3.2

1. Question

Rationalise the denominator of each of the following (i-vii) :

(i) $\frac{3}{\sqrt{5}}$ (ii) $\frac{3}{2\sqrt{5}}$ (iii) $\frac{1}{\sqrt{12}}$ (iv) $\frac{\sqrt{2}}{\sqrt{5}}$ (v) $\frac{\sqrt{3}+1}{\sqrt{2}}$ (vi) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$ (vii) $\frac{3\sqrt{2}}{\sqrt{5}}$

Answer

(i) As there is √5 in the denominator and we know that √5 x √5 = 5

So, multiply numerator and denominator by √5,

$$\frac{3}{\sqrt{5}} = \frac{3\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}} = \frac{3\sqrt{5}}{5} = \frac{3}{5}\sqrt{5}$$

(ii) $\frac{3}{2\sqrt{5}}\times\frac{2\sqrt{5}}{2\sqrt{5}} = \frac{3\times2\sqrt{5}}{\left(2\sqrt{5}\right)^2} = \frac{6\sqrt{5}}{20} = \frac{3}{10}\sqrt{5}$

$$\frac{1}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}} = \frac{\sqrt{12}}{12}$$

(iii)

$$= \frac{\sqrt{3}\sqrt{4}}{12}$$

$$= \frac{2\sqrt{3}}{12}$$

$$= \frac{\sqrt{3}}{6}$$

(iv) $\frac{\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{2}\times\sqrt{5}}{(\sqrt{5})^2} = \frac{1}{5}\sqrt{10}$

(v) $\frac{\sqrt{3}+1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{2}$

(vi) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}+\sqrt{5}}{3}$

(vii) $\frac{3\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{10}}{5}$

2. Question

Find the value to three places of decimals of each of the following. It is given that

$\sqrt{2}$ = 1.414, $\sqrt{3}$ = 1.732, $\sqrt{5}$ =2.236 and $\sqrt{10}$ = 3.162.

(i) $\frac{2}{\sqrt{3}}$ (ii) $\frac{3}{\sqrt{10}}$ (iii) $\frac{\sqrt{5}+1}{\sqrt{2}}$ (iv) $\frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}$

(v) $\frac{2+\sqrt{3}}{3}$ (vi) $\frac{\sqrt{2}-1}{\sqrt{5}}$

Answer

(i) Given that √2 = 1.414, √3=1.732, √5 = 2.236 and √10= 3.162

So we have,

$\frac{2}{\sqrt{3}}$ Rationalising factor of denominator is √3

$$\frac{2}{\sqrt{3}} = \frac{2\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\frac{2\times1.732}{3} = \frac{3.464}{3}$$

=1.15466667 = 1.54

(ii) we have $\frac{3}{\sqrt{10}}$ rationalisation factor of denominator is √10

$$\frac{3}{\sqrt{10}} = \frac{3\times\sqrt{10}}{\sqrt{10}\times\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\frac{3\times3.162}{10} = 0.9486$$

(iii) we have $\frac{\sqrt{5}+1}{\sqrt{2}}$ rationalisation factor of denominator is √2

$$= \frac{\sqrt{5}+1}{\sqrt{2}} = \frac{\sqrt{5}+1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{(\sqrt{5}+1)\sqrt{2}}{(\sqrt{2})^2} = \frac{\sqrt{5}\times\sqrt{2}+1\times\sqrt{2}}{2}$$

$$= \frac{\sqrt{5\times2}+\sqrt{2}}{2} = \frac{\sqrt{10}+\sqrt{2}}{2}$$

$$= \frac{3.162+1.414}{2} = \frac{4.576}{2} = 2.288$$

(iv) we have $\frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}$ rationalisation factor of denominator is √2

$$= \frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}} = \frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{(\sqrt{10}+\sqrt{15})\sqrt{2}}{(\sqrt{2})^2}$$

$$= \frac{\sqrt{10}\times\sqrt{2}+\sqrt{15}\times\sqrt{2}}{2} = \frac{\sqrt{10\times2}+\sqrt{15\times2}}{2}$$

$$= \frac{\sqrt{20}+\sqrt{30}}{2} = \frac{\sqrt{2\times10}+\sqrt{3\times10}}{2}$$

$$= \frac{\sqrt{2}\times\sqrt{10}+\sqrt{3}\times\sqrt{10}}{2} = \frac{1.414\times3.162+1.732\times3.162}{2}$$

$$= \frac{4.471068+5.476584}{2} = \frac{9.947652}{2}$$

$$= 4.973826 = 4.973$$

(v) We have $\frac{2+\sqrt{3}}{3}$

$$= \frac{2+\sqrt{3}}{3} = \frac{2+1.732}{3} = \frac{3.732}{3} = 1.244$$

(vi) We have $\frac{\sqrt{2}-1}{\sqrt{5}}$ rationalising factor of denominator is √5

$$= \frac{\sqrt{2}-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{(\sqrt{2}\times\sqrt{5}) - (1\times\sqrt{5})}{(\sqrt{5})^2}$$

$$= \frac{\sqrt{2\times5} - 1\sqrt{5}}{5} = \frac{\sqrt{10}-\sqrt{5}}{5}$$

$$= \frac{3.162 - 2.236}{5} = \frac{0.926}{5}$$

$$= 0.185$$

3. Question

Express each one of the following with rational denominator:

(i) $\frac{1}{3+\sqrt{2}}$ (ii) $\frac{1}{\sqrt{6}-\sqrt{5}}$ (iii) $\frac{16}{\sqrt{41}-5}$

(iv) $\frac{30}{5\sqrt{3}-3\sqrt{5}}$ (v) $\frac{1}{2\sqrt{5}-\sqrt{3}}$ (vi) $\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$ (vii) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$ (viii) $\frac{3\sqrt{2}+1}{2\sqrt{5}-3}$ (ix) $\frac{b^2}{\sqrt{a^2+b^2}+a}$

Answer

(i) we have $\frac{1}{3+\sqrt{2}}$ rationalizing factor of the denominator is 3-√2

$$= \frac{1}{3+\sqrt{2}} = \frac{1}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$$

$$= \frac{3-\sqrt{2}}{(3)^2 - (\sqrt{2})^2}$$

$$because\ (a+b)(a-b) = (a)^2 - (b)^2$$

$$= \frac{3-\sqrt{2}}{9-2} = \frac{3-\sqrt{2}}{7}$$

(ii) we have $\frac{1}{\sqrt{6}-\sqrt{5}}$ rationalizing factor of the denominator is $\sqrt{6}+\sqrt{5}$

$$= \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}$$

$$= \frac{\sqrt{6}+\sqrt{5}}{(\sqrt{6})^2 - (\sqrt{5})^2} = \frac{\sqrt{6}+\sqrt{5}}{6-5} = \frac{\sqrt{6}+\sqrt{5}}{1}$$

$$= \sqrt{6}+\sqrt{5}$$

(iii) we have $\frac{16}{\sqrt{41}-5}$ rationalizing factor of the denominator is $\sqrt{41}+5$

$= \frac{16}{\sqrt{41}-5} \times \frac{\sqrt{41}+5}{\sqrt{41}+5}$

$= \frac{16 \times (\sqrt{41}+5)}{(\sqrt{41}-5)(\sqrt{41}+5)} = \frac{16\sqrt{41}+5}{(\sqrt{41})^2-(5)^2}$

$= \frac{16\sqrt{41}+5}{41-25} = \frac{16\sqrt{41}+5}{16} = \sqrt{41}+5$

(iv) we have $\frac{30}{5\sqrt{3}-3\sqrt{5}}$ to rationalize factor of $5\sqrt{3}-3\sqrt{5}\ is\ 5\sqrt{3}+3\sqrt{5}$

$= \frac{30}{5\sqrt{3}-3\sqrt{5}} \times \frac{5\sqrt{3}+3\sqrt{5}}{5\sqrt{3}+3\sqrt{5}} = \frac{3(5\sqrt{3}+3\sqrt{5})}{(5\sqrt{3})^2-(3\sqrt{5})^2}$

$= \frac{30(5\sqrt{3}+3\sqrt{5})}{5^2(\sqrt{3})^2-3^2(\sqrt{5})^2} = \frac{30(5\sqrt{3}+3\sqrt{5})}{25\times 3-9\times 5}$

$= \frac{30(5\sqrt{3}+3\sqrt{5})}{75-45} = \frac{30(5\sqrt{3}+3\sqrt{5})}{30}$

$=5\sqrt{3}+3\sqrt{5}$

(v) we have $\frac{1}{2\sqrt{5}-\sqrt{3}}$ to rationalize factor of $2\sqrt{5}-\sqrt{3}\ is\ 2\sqrt{5}+\sqrt{3}$

$= \frac{1}{2\sqrt{5}-\sqrt{3}} \times \frac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}+\sqrt{3}} = \frac{2\sqrt{5}+\sqrt{3}}{(2\sqrt{5})^2-(\sqrt{3})^2}$

$= \frac{2\sqrt{5}+\sqrt{3}}{2^2(\sqrt{5})^2-(\sqrt{3})^2} = \frac{2\sqrt{5}+\sqrt{3}}{4\times 5-3}$

$= \frac{2\sqrt{5}+\sqrt{3}}{20-3} = \frac{2\sqrt{5}+\sqrt{3}}{17}$

(vi) we have $\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$ to rationalize factor of $2\sqrt{2}-\sqrt{3}\ is\ 2\sqrt{2}+\sqrt{3}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}} \times \frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}} = \frac{(\sqrt{3}+1)(2\sqrt{2}+\sqrt{3})}{(2\sqrt{2})^2-(\sqrt{3})^2}$

$= \frac{\sqrt{3}\times 2\sqrt{2}+2\sqrt{2}+\sqrt{3}\times\sqrt{3}+\sqrt{3}}{4\times 2-3}$

$= \frac{2\sqrt{2\times 3}+2\sqrt{2}+3+\sqrt{3}}{8-3}$

$= \frac{2\sqrt{6}+2\sqrt{2}+3+\sqrt{3}}{5}$

(vii) we have $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$to rationalize factor of $6+4\sqrt{2}\ is\ 6-4\sqrt{2}$

$$\frac{6-4\sqrt{2}}{6+4\sqrt{2}} = \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}}$$

$$= \frac{(6-4\sqrt{2})^2}{(6)^2-(4\sqrt{2})^2}$$

Because; $(a+b)(a-b) = a^2 - b^2$

$$(a-b)(a+b) = (a-b)^2$$

$$so, \frac{\left(6-4\sqrt{2}\right)^2}{6^2-\left(4\sqrt{2}\right)^2}$$

$$= \frac{6^2 - 2\times 6\times 4\sqrt{2} + \left(4\sqrt{2}\right)}{36 - 4^2\left(\sqrt{2}\right)^2}$$

$$= \frac{36 - 48\sqrt{2} + 32}{36 - 32} = \frac{68 - 48\sqrt{2}}{4}$$

$$= \frac{4\left(17 - 12\sqrt{2}\right)}{4} = 17 - 12\sqrt{2}$$

(viii) we have $\frac{3\sqrt{2}+1}{2\sqrt{5}-3}$to rationalize factor of $2\sqrt{5}-3\ is\ 2\sqrt{5}+3$

$$= \frac{3\sqrt{2}+1}{2\sqrt{5}-3} \times \frac{2\sqrt{5}+3}{2\sqrt{5}+3} = \frac{\left(3\sqrt{2}+1\right)\left(2\sqrt{5}+3\right)}{\left(2\sqrt{5}-3\right)\left(2\sqrt{5}+3\right)}$$

$$= \frac{3\sqrt{2}\times 2\sqrt{5} + 3\sqrt{2}\times 3 + 1\times 2\sqrt{5} + 1\times 3}{\left(2\sqrt{2}\right)^2 - 3^2}$$

$$= \frac{6\sqrt{10} + 9\sqrt{2} + 2\sqrt{5} + 3}{20 - 9}$$

$$= \frac{6\sqrt{10} + 9\sqrt{2} + 2\sqrt{5} + 3}{11}$$

(ix) we have $\frac{b^2}{\sqrt{a^2+b^2+a}}$to rationalize factor of $\sqrt{a^2+b^2}+a\ is \sqrt{a^2+b^2}-a$

$$= \frac{b^2}{\sqrt{a^2+b^2}+a} \times \frac{\sqrt{a^2+b^2}-a}{\sqrt{a^2+b^2}-a} = \frac{b^2\left(\sqrt{a^2+b^2}-a\right)}{(\sqrt{a^2+b^2})^2-(a)^2}$$

$$= \frac{b^2\left(\sqrt{a^2+b^2}\right)}{a^2+b^2-a^2} = \frac{b^2(\sqrt{a^2+b^2}-a^2}{b^2}$$

$= (\sqrt{a^2 + b^2} - a^2)$

4. Question

Rationalies the denominator and simplify :

(i) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ (ii) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}$ (iii) $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$

(iv) $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$ (v) $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

(vi) $\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$

Answer

i) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{3+2-2\sqrt{6}}{3-2} = 5 - 2\sqrt{6}.$

ii) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{35+14\sqrt{3}-20\sqrt{3}-24}{49-48} = 11 - 6\sqrt{3}.$

iii) $\frac{1+\sqrt{2}}{3-2\sqrt{2}} = \frac{1+\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{3+3\sqrt{2}+2\sqrt{2}+4}{9-8} = 7 + 5\sqrt{2}.$

iv) $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} = \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} \times \frac{3\sqrt{5}+2\sqrt{6}}{3\sqrt{5}+2\sqrt{6}} = \frac{6\sqrt{30}-15+4\sqrt{36}-2\sqrt{30}}{45-24} = \frac{4\sqrt{30}+9}{21}.$

v) $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} = \frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} \times \frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}} = \frac{48+20\sqrt{6}-12\sqrt{6}-30}{48-18} = \frac{18+8\sqrt{6}}{30} = \frac{9+4\sqrt{6}}{15}.$

vi) $\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} = \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} \times \frac{2\sqrt{2}-3\sqrt{3}}{2\sqrt{2}-3\sqrt{3}} = \frac{4\sqrt{6}-2\sqrt{10}-18+3\sqrt{15}}{8-27} = \frac{18+2\sqrt{10}-4\sqrt{6}-3\sqrt{15}}{19}$

5. Question

Simplify :

(i) $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$ (ii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

(iii) $\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$

(iv) $\frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2-\sqrt{5}}$

(v) $\frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}}$

Answer

i) $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}} = \frac{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}-2\sqrt{3})}{18-12} + \frac{2\sqrt{3}(\sqrt{3}+\sqrt{2})}{3-2}$

$= \frac{30-12\sqrt{6}}{6} + (6+2\sqrt{6}) = (5 - 2\sqrt{6} + 6 + 2\sqrt{6} = 11$

ii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{[(\sqrt{5}+\sqrt{3})(\sqrt{5}+\sqrt{3})+(\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})]}{5-3}=\frac{8+2\sqrt{15}+8-2\sqrt{15}}{2}=8.$

iii) $\frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}$ rationalising factors of denominators are $3-\sqrt{5}\ and\ 3+\sqrt{5}$

$$=\frac{7+3\sqrt{5}}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}$$

$$=\frac{(7+3\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}-\frac{(7-3\sqrt{5})(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$$

$$\frac{7\times3+7\times(-\sqrt{5})+3\sqrt{5}+3+3\sqrt{5}\times(-\sqrt{5})}{3^2-(\sqrt{5})^2}-\frac{7\times3+7\times\sqrt{5}+(-3\sqrt{5})\times3+(-3\sqrt{5})\times\sqrt{5}}{3^2-(\sqrt{5})^2}$$

$$=\frac{21-7\sqrt{5}+9\sqrt{5}-3\times5}{9-5}-\frac{21+7\sqrt{5}-9\sqrt{5}-3\times5}{9-5}$$

$$=\frac{21-15+2\sqrt{5}}{4}-\frac{21-15-2\sqrt{5}}{4}$$

$$=\frac{6+2\sqrt{5}}{4}-\frac{6-2\sqrt{5}}{4}$$

$$=\frac{6+2\sqrt{5}-(6-2\sqrt{5})}{4}$$

$$=\frac{6+2\sqrt{5}-6+2\sqrt{5}}{4}$$

$$=\frac{4\sqrt{5}}{4}=\sqrt{5}$$

(iv) $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2-\sqrt{5}}$

Rationalising factor for $2+\sqrt{3}\ is\ 2-\sqrt{3}$

For $\sqrt{5}-\sqrt{3}\ is\ \sqrt{5}+\sqrt{3}$ and

For $2-\sqrt{5}\ is\ 2+\sqrt{5}$

$$=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\frac{1}{2-\sqrt{5}}\times\frac{2+\sqrt{5}}{2+\sqrt{5}}$$

$$=\frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2}+\frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}+\frac{2+\sqrt{5}}{2^2-(\sqrt{5})^2}$$

$$= \frac{2-\sqrt{3}}{4-3} + \frac{2\sqrt{5}+2\sqrt{3}}{5-3} + \frac{2+\sqrt{5}}{4-5}$$

$$= \frac{2-\sqrt{3}}{1} + \frac{2\sqrt{5}+2\sqrt{3}}{2} + \frac{2+\sqrt{5}}{-1}$$

$$= 2-\sqrt{3} + 2\frac{\left(\sqrt{5}+\sqrt{3}\right)}{2} - \left(2+\sqrt{3}\right)$$

$$= 2-\sqrt{3}+\sqrt{5}+\sqrt{3}-2-\sqrt{3} = 0$$

(v) $\frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}}$

Rationalising factors for denominators are,

For $\sqrt{5}+\sqrt{3}\ is\ \sqrt{5}-\sqrt{3}$

For $\sqrt{3}+\sqrt{2}\ is\ \sqrt{3}-\sqrt{2}$ and

For $\sqrt{5}+\sqrt{2}\ is\ \sqrt{5}-\sqrt{2}$

$$= \frac{2}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{2\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2} + \frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2} - \frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{2\left(\sqrt{5}-\sqrt{3}\right)}{5-3} + \frac{\sqrt{3}-\sqrt{2}}{3-2} - \frac{3\left(\sqrt{5}-\sqrt{2}\right)}{5-2}$$

$$= \frac{2\left(\sqrt{5}-\sqrt{3}\right)}{2} + \frac{\sqrt{3}-\sqrt{2}}{1} - \frac{3\left(\sqrt{5}-\sqrt{2}\right)}{3}$$

$$= \sqrt{5}-\sqrt{3}+\sqrt{3}-\sqrt{2}-\sqrt{5}+\sqrt{2} = 0$$

6. Question

In each of the following determine rational numbers *a* and *b*.

(i) $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$ (ii) $\frac{4+\sqrt{2}}{2+\sqrt{2}} = a - \sqrt{b}$

(iii) $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$ (iv) $\frac{5+3\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$

(v) $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a - b\sqrt{77}$

(vi) $\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a + b\sqrt{5}$

Answer

(i) $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$

Given,

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$$

Rationalising factor for denominator is $\sqrt{3}-1$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{(\sqrt{3})^2 - 2\sqrt{3}\times 1 + (1)^2}{3-2} = \frac{3 - 2\sqrt{3} + 1}{2}$$

$$= \frac{4 - 2\sqrt{3}}{2} = \frac{2(2-\sqrt{3})}{2} = 2 - \sqrt{3}$$

$$we\ have, \frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$$

$$= 2 - \sqrt{3} = a - b\sqrt{3} \ = \ 2 - (1)\sqrt{3} = a - b\sqrt{3}$$

On equating rational and irrational parts,

We get a = 2 and b = 1

(ii) $\frac{4+\sqrt{2}}{2+\sqrt{2}} = a - \sqrt{b}$ rationalising factor for the denominator is $2 - \sqrt{2}$

$$= \frac{4+\sqrt{2}}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}} = \frac{4\times 2 + \sqrt{2}\times 2 + 4\times(-\sqrt{2}) + \sqrt{2}\times(-\sqrt{2})}{2^2 - (\sqrt{2})^2}$$

$$= \frac{8 + 2\sqrt{2} - 4\sqrt{2} - \sqrt{2}}{4-2} = \frac{6 - 2\sqrt{2}}{2}$$

$$= \frac{2(3-\sqrt{2})}{2} = 3 - \sqrt{2}$$

We have $\frac{4+\sqrt{2}}{2+\sqrt{2}} = a - \sqrt{b} = 3 - \sqrt{2} = a - \sqrt{b}$

On equating rational and irrational parts we get,

a=3 and b=2

(iii) $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$

Rationalising factor for the denominator is $3+\sqrt{2}$

$$\frac{3+\sqrt{2}}{3-\sqrt{2}}\times\frac{3+\sqrt{2}}{3+\sqrt{2}}=\frac{\left(3+\sqrt{2}\right)^2}{3^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{3^2+2\times3\times\sqrt{2}+\left(\sqrt{2}\right)^2}{9-2}=\frac{9+6\sqrt{2}+2}{7}$$

$$=\frac{11+6\sqrt{2}}{7}=\frac{11}{7}+\frac{6}{7}\sqrt{2}$$

$$we\ have\ \frac{3+\sqrt{2}}{3-\sqrt{2}}=a+b\sqrt{2}$$

On equating rational and irrational parts we get,

$$a=\frac{11}{7}, and\ b=\frac{6}{7}$$

(iv) $\frac{5+3\sqrt{3}}{7+4\sqrt{3}}=a+b\sqrt{3}$ given,

Rationalising factor for denominator is $7-4\sqrt{3}$

$$=\frac{5+3\sqrt{3}}{7+4\sqrt{3}}\times\frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$=\frac{5\times7+5\times\left(-4\sqrt{3}\right)+3\sqrt{3}\times7+3\sqrt{3}\times\left(-4\sqrt{3}\right)}{(7)^2-\left(4\sqrt{3}\right)^2}$$

$$=\frac{35-20\sqrt{3}+21\sqrt{3}-12\times3}{49-48}$$

$$=\frac{35-36+\sqrt{3}}{1}=\frac{\sqrt{3}-1}{1}=\sqrt{3}-1$$

We have $\frac{5+3\sqrt{3}}{7+4\sqrt{3}}=a+b\sqrt{3}$

$$\sqrt{3}-1=a+b\sqrt{3}$$

On equating rational and irrational parts we get,

a= -1 and b=1

(v) $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}=a-b\sqrt{77}$ given,

$$= \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} \times \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}-\sqrt{7}} = \frac{\left(\sqrt{11}-\sqrt{7}\right)^2}{\left(\sqrt{11}\right)^2-\left(\sqrt{7}\right)^2}$$

$$= \frac{\left(\sqrt{11}\right)^2 - 2\sqrt{11}\times\sqrt{7} + \left(\sqrt{7}\right)^2}{11-7} = \frac{11 - 2\sqrt{11\times7}+7}{4}$$

$$= \frac{18-2\sqrt{77}}{4} = \frac{2\left(9-\sqrt{77}\right)}{4}$$

$$= \frac{9-\sqrt{77}}{2} = \frac{9}{2} - \frac{\sqrt{77}}{2}$$

We have $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a - b\sqrt{77}$

$$= \frac{9}{2} - \frac{\sqrt{77}}{2} = a - b\sqrt{77}$$

$$= \frac{9}{2} - \frac{1}{2}\sqrt{77} = a - b\sqrt{77}$$

On equating rational and irrational parts we get

$$a = \frac{9}{2}, b = \frac{1}{2}$$

(vi) $\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a + b\sqrt{5}$ given,

$$= \frac{4+3\sqrt{5}}{4-3\sqrt{5}} = \frac{4+3\sqrt{5}}{4-3\sqrt{5}} \times \frac{4+3\sqrt{5}}{4+3\sqrt{5}}$$

$$= \frac{\left(4+3\sqrt{5}\right)^2}{4^2-\left(3\sqrt{5}\right)^2} = \frac{4^2+2\times4\times3\sqrt{5}+\left(3\sqrt{5}\right)^2}{16-3^2\left(\sqrt{5}\right)^2}$$

$$= \frac{16+24\sqrt{5}+45}{16-45} = \frac{61+24\sqrt{5}}{-29} = \frac{-\left(61+24\sqrt{5}\right)}{29}$$

$$= \frac{-61}{29} - \frac{24}{29}\sqrt{5}$$

We have $\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a + b\sqrt{5}$

On equating rational and irrational parts we have,

$$a = \frac{-61}{29} \text{ and } b = \frac{-24}{29}$$

7. Question

If $x = 2+\sqrt{3}$, find the value of $x^3 + \frac{1}{x^3}$.

Answer

Given $x = 2+\sqrt{3}$ and given to find the value of $x^3 + \frac{1}{x^3}$

We have $x = 2+\sqrt{3}$

$$= \frac{1}{x} = \frac{1}{2+\sqrt{3}}$$

$$rationalising\ factor\ for\ denominator\ is\ 2-\sqrt{3}$$

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{2-\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1} = 2-\sqrt{3}$$

$$\therefore \frac{1}{x} = 2-\sqrt{3}$$

$$and\ also, \left(x+\frac{1}{x}\right) = 2+\sqrt{3}+2-\sqrt{3}$$

$$= 2+2 = 4$$

$$\therefore \left(x+\frac{1}{x}\right) = 4\ equation\ (i)$$

We know that ,

$$x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)\left(x^2 - x\times\frac{1}{x} + \frac{1}{x^2}\right)$$

$$= \left(x+\frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$$

$$= \left(x+\frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 2 - 2 - 1\right)$$

$$= \left(x+\frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 2\times x\times\frac{1}{x} - 3\right)$$

$$= x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 2\times x\times\frac{1}{x} - 3\right)$$

$$= \left(x+\frac{1}{x}\right)\left(\left(x+\frac{1}{x}\right)^2 - 3\right)$$

By putting $\left(x+\frac{1}{x}\right) = 4$ we get

$$= x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)\left(\left(x+\frac{1}{x}\right)^2 - 3\right)$$

$$= (4)(4^2 - 3)$$

$$= 4(16 - 3)$$

$$= 4(13) = 52$$

∴ The value of $x^3 + \frac{1}{x^3}$ is 52.

8. Question

If x = 3+ $\sqrt{8}$, find the value of $x^2 + \frac{1}{x^2}$.

Answer

Given that $x = 3 + \sqrt{8}$

And given to find the value of $x^2 + \frac{1}{x^2}$

We have $x = 3 + \sqrt{8}$

The rationalising factor for denominator is $3 - \sqrt{8}$

$$= \frac{1}{x} = \frac{1}{3+\sqrt{8}} \times \frac{3-\sqrt{8}}{3-\sqrt{8}}$$

$$= \frac{3-\sqrt{8}}{3^2-(\sqrt{8})^2} = \frac{3-\sqrt{8}}{9-8} = \frac{3-\sqrt{8}}{1} = 3 - \sqrt{8}$$

$$\therefore \frac{1}{x} = 3 - \sqrt{8}$$

Also, $\left(x+\frac{1}{x}\right) = 3 + \sqrt{8} + 3 - \sqrt{8} = 3 + 3 = 6$

$$\therefore \left(x+\frac{1}{x}\right) = 6$$

We know that,

$$= x^2 + \frac{1}{x^2} = \left(x^2 + \frac{1}{x^2}\right)^2 - 2$$

$$\textit{by putting } x + \frac{1}{x} = 6 \textit{ in the above we get,}$$

$$x^2 + \frac{1}{x^2} = (6)^2 - 2$$

= 36 - 2 = 34

∴ The value of $x^2 + \frac{1}{x^2}$ is 34.

9. Question

Find the value of $\frac{6}{\sqrt{5}-\sqrt{3}}$, it being given that $\sqrt{3}$ = 1.732 and $\sqrt{5}$ =2.236

Answer

$\frac{6}{\sqrt{5}+\sqrt{3}}$ Rationalising factor for the denominator is $\sqrt{5}+\sqrt{3}$

$$= \frac{6}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{6(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$= \frac{6(\sqrt{5}+\sqrt{3})}{5-3} = \frac{6(\sqrt{5}+\sqrt{3})}{2} = 3(\sqrt{5}+\sqrt{3})$$

We have $\sqrt{3} = 1.732, \sqrt{5} = 2.236$

$$\frac{6}{\sqrt{5}-\sqrt{3}} = 3(2.236 + 1.732)$$

= 3(3.968)

= 11.904

$$\therefore \textit{value of } \frac{6}{\sqrt{5}-\sqrt{3}} \textit{ is } 11.904$$

10. Question

Find the values of each of the following correct to three places of decimals, it being given that $\sqrt{2}$ = 1.414, $\sqrt{3}$ = 1.732, $\sqrt{5}$ =2.236, $\sqrt{6}$ = 2.4495 and $\sqrt{10}$ = 3.162.

(i) $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$ (ii) $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$

Answer

(i) We have $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$ rationalising factor for denominator is $3-2\sqrt{5}$

$$= \frac{3-\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$

$$= \frac{3\times3 + 3\times(-2\sqrt{5}) + (-\sqrt{5})(3) + (-\sqrt{5})(-2\sqrt{5})}{3^2 - (2\sqrt{5})^2}$$

$$= \frac{9 - 6\sqrt{5} - 3\sqrt{5} + 2\times5}{9-20} = \frac{9 + 10 - 9\sqrt{5}}{-11}$$

$$= \frac{19-9\sqrt{5}}{-11} = \frac{9\sqrt{5}-19}{11}$$

We have $\sqrt{5} = 2.236$

$$= \frac{3-\sqrt{5}}{3+2\sqrt{5}} = \frac{9(2.236)-19}{11} = \frac{20.124-19}{11}$$

$$= \frac{1.124}{11} = 0.102181818$$

$$= 0.102$$

$$= \textit{the value of } \frac{3-\sqrt{5}}{3+2\sqrt{5}} = 0.102$$

(ii) $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$ by putting the value of $\sqrt{2}$ in the equation we get,

$$= \frac{1+\sqrt{2}}{3-2\sqrt{2}} = \frac{1+1.414.}{3-2\times 1.414} = \frac{2.414}{3-2.8284}$$

$$= \frac{2.4142}{0.1716} = 14.0687$$

$$= 14.068$$

$$= 14.070$$

11. Question

If $x = \frac{\sqrt{3}+1}{2}$, find the value of $4x^3+2x^2-8x+7$.

Answer

Given $x = \frac{\sqrt{3}+1}{2}$ and given to find the value of $4x^3 + 2x^2 - 8x + 7$

$$x = \frac{\sqrt{3}+1}{2}$$

$$= 2x = \sqrt{3}+1$$

$$= (2x-1) = \sqrt{3}$$

Squaring on both the sides we get,

$$= (2x-1)^2 = \left(\sqrt{3}\right)^2$$

$$= (2x)^2 - 2\times 2x\times 1 + (1)^2 = 3$$

$= 4x^2 - 4x + 1 = 3$

$= 4x^2 - 4x + 1 - 3 = 0$

$= 4x^2 - 4x - 2 = 0$

$= 2(2x^2 - 2x - 1) = 0$

$= 2x^2 - 2x - 1 = 0$

Now take $4x^3 + 2x^2 - 8x + 7$

$= 2x\ (2x^2 - 2x - 1) + 4x^2 + 2x + 2x^2 - 8x + 7$

$= 2x\ (2x^2 - 2x - 1) + 6x^2 - 6x + 7$

$= 2x(0) + 3(2x^2 - 2x - 1) + 7 + 3$

$= 0 + 3(0) + 10 = 10$

The value of $4x^3 + 2x^2 - 8x + 7$ is 10.

CCE - Formative Assessment

1. Question

Write the value of (2+ $\sqrt{3}$) (2- $\sqrt{3}$).

Answer

(2+√3) (2-√3)

= $(2)^2$ - $(\sqrt{3})^2$ [(a+b) (a-b) = $a^2 - b^2$]

= 4 – 3 = 1.

2. Question

Write the reciprocal of 5 + $\sqrt{2}$.

Answer

Reciprocal of 5 + √2 = 1/ (5 + √2)

$= \frac{1}{5+\sqrt{2}} = \frac{1}{5+\sqrt{2}} \times \frac{5-\sqrt{2}}{5-\sqrt{2}} = \frac{5-\sqrt{2}}{25-2} = \frac{5-\sqrt{2}}{23}$

3. Question

Write the rationalisation factor of 7-3 $\sqrt{5}$.

Answer

Rationalizing factor of 7- 3√5

$= \frac{1}{7-3\sqrt{5}} = 7 + 3\sqrt{5}.$

4. Question

If $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ = x+$y\sqrt{3}$, find the values of x and y.

Answer

Given, $\frac{\sqrt{3}-1}{\sqrt{3}+1} = x + y\sqrt{3}$

$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{3-1} = \frac{4-2\sqrt{3}}{2} = 2 - \sqrt{3}.$

So, x= 2 , y = -1

5. Question

If x = $\sqrt{2}$ -1, then write the value of $\frac{1}{x}$.

Answer

Given . x = √2-1

$$= \frac{1}{x} = \frac{1}{\sqrt{2}-1} = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}+1}{2-1} = \sqrt{2}+1.$$

6. Question

Simplify $\sqrt{3+2\sqrt{2}}$.

Answer

Consider $\sqrt{(3+2\sqrt{2})}$,

$$\sqrt{(3+2\sqrt{2})} = \sqrt{(2+1+2\sqrt{2})}$$

$$= \sqrt{((\sqrt{2})^2 + (1)^2 + 2\times 1\times \sqrt{2})}$$

As we know, $(a+b)^2 = a^2 + b^2 + 2ab$

$$= \sqrt{(\sqrt{2}+1)^2}$$

$$= \sqrt{2}+1$$

7. Question

Simplify $\sqrt{3-2\sqrt{2}}$.

Answer

$\sqrt{(3-2\sqrt{2})} = \sqrt{(\sqrt{2})^2 + (1)^2 - 2\times \sqrt{2}\times 1} = \sqrt{(\sqrt{2} - 1)^2} = \sqrt{2} - 1.$

8. Question

If $a = \sqrt{2} + 1$, then find the value of $a - \frac{1}{a}$.

Answer

Given , a = $\sqrt{2} + 1$

$= \frac{1}{a} = \frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = (\sqrt{2} - 1)$

$= a - \left(\frac{1}{a}\right) = \sqrt{2} + 1 - (\sqrt{2} - 1) = 2.$

9. Question

If $x = 2 + \sqrt{3}$, find the value of $x + \frac{1}{x}$.

Answer

Given, x = $2+\sqrt{3}$

$= \frac{1}{x} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$

$= x + \frac{1}{x} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4.$

10. Question

Write the rationalisation factor of $\sqrt{5} - 2$.

Answer

Rationalizing factor of $\sqrt{5} - 2$

$= \frac{1}{\sqrt{5}-2} = \sqrt{5} + 2$

11. Question

If $x = 3 + 2\sqrt{2}$, then find the value of $\sqrt{x} - \frac{1}{\sqrt{x}}$.

Answer

Given x = $3 + 2\sqrt{2}$

$= \sqrt{x} = \sqrt{3 + 2\sqrt{2}} = \sqrt{(\sqrt{2} + 1)^2}$

$= \sqrt{x} = \sqrt{2} + 1$

$= \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{1} = \sqrt{2} - 1$

So, $\sqrt{x} - 1/\sqrt{x} = \sqrt{2}+1 - (\sqrt{2} - 1)$

$= 1 + 1 = 2.$

1. Question

$\sqrt{10} \times \sqrt{15}$ is equal to

A. $5\sqrt{6}$

B. $6\sqrt{5}$

C. $\sqrt{30}$

D. $\sqrt{25}$

Answer

$\sqrt{10} \times \sqrt{15} = (\sqrt{5}\times\sqrt{2}) \times (\sqrt{5}\times\sqrt{3})$

$= 5(\sqrt{6})$

2. Question

$\sqrt[5]{6} \times \sqrt[5]{6}$ is equal to

A. $\sqrt[5]{36}$

B. $\sqrt[5]{6\times 0}$

C. $\sqrt[5]{6}$

D. $\sqrt[5]{12}$

Answer

$^5\sqrt{6} \times {}^5\sqrt{6} = (6)^{1/5} \times (6)^{1/5} = (36)^{1/5}$

$= {}^5\sqrt{36}$

3. Question

The rationalisation factor of $\sqrt{3}$ is

A. $-\sqrt{3}$

B. $\frac{1}{\sqrt{3}}$

C. $2\sqrt{3}$

D. $-2\sqrt{3}$

Answer

Rationalisation factor of $\sqrt{3} = 1/\sqrt{3}$

4. Question

The rationalisation factor of $2+\sqrt{3}$ is

A. $2-\sqrt{3}$

B. $2+\sqrt{3}$

C. $\sqrt{2}-3$

D. $\sqrt{3}-2$

Answer

Rationalisation factor of 2+√3 = 1/2+√3 = 2-√3

5. Question

If $x = \sqrt{5}+2$, then $x - \frac{1}{x}$ equals

A. $2\sqrt{5}$

B. 4

C. 2

D. $\sqrt{5}$

Answer

Given x = √5+2

$$= \frac{1}{x} = \frac{1}{\sqrt{5}+2} = \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{\sqrt{5}-2}{5-4} = \sqrt{5}-2$$

$so, x - \frac{1}{x}$ = √5 + 2 – (√5 – 2) = 4

6. Question

If $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$, then

A. $a = 2$, $b = 1$

B. $a = 2$, $b = -1$

C. $a = -2$, $b = 1$

D. $a = b = 1$

Answer

Given $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{4-2\sqrt{3}}{2} = 2 - \sqrt{3}$$

So, a = 2 , b = 1.

7. Question

The simplest rationalising of $\sqrt[3]{500}$ is

A. $\sqrt[3]{2}$

B. $\sqrt[3]{5}$

C. $\sqrt{3}$

D. none of these

Answer

$\sqrt[3]{500} = \sqrt[3]{(125 \times 4)} = 5 \times \sqrt[3]{4}$

8. Question

The simplest rationalising factor of $\sqrt{3} + \sqrt{5}$ is

A. $\sqrt{3}$ -5

B. 3- $\sqrt{5}$

C. $\sqrt{3} - \sqrt{5}$

D. $\sqrt{3} + \sqrt{5}$

Answer

Simplest rationalizing factor of $\sqrt{3} + \sqrt{5}$

$1/(\sqrt{3}+\sqrt{5}) = \sqrt{3}-\sqrt{5}$

9. Question

The simplest rationalising factor of $2\sqrt{5} - \sqrt{3}$ is

A. $2\sqrt{5} + 3$

B. $2\sqrt{5}$ +

C. $\sqrt{5} + \sqrt{3}$

D. $\sqrt{5} - \sqrt{3}$

Answer

Simplest rationalizing factor of $2\sqrt{5} - \sqrt{3}$

$= 1/(2\sqrt{5} - \sqrt{3})$

$= 2\sqrt{5} + \sqrt{3}$

10. Question

If $x = \frac{2}{3+\sqrt{7}}$, then $(x-3)^2 =$

A. 1

B. 3

C. 6

D. 7

Answer

Given X = 2/(3+√7)

$= \left(\frac{2}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}}\right) = \frac{2(3-\sqrt{7})}{9-7} = 3 - \sqrt{7}$

= $(x - 3)^2$ = $(3 - \sqrt{7} - 3)^2$ = $\sqrt{7}^2$ = 7

11. Question

If x = 7+4$\sqrt{3}$ and *xy*=1, then = $\frac{1}{x^2}+\frac{1}{y^2}$

A. 64

B. 134

C. 194

D. 1/49

Answer

Given . x = 7+4√3 , xy = 1

Y = 1/x = 1/7+ 4√3 = 7-4√3

Y^2 = $1/x^2$ = 49 + 48 - 56√3 = 97 - 56√3

Similarly, x = 1/y

= x^2 = $1/y^2$ = $(7 + 4\sqrt{3})^2$ = 49 + 48 + 56√3 = 97+ 56√3

So, $1/x^2 + 1/y^2$ = 97 + 56√3 + 97 − 56√3 = 194

12. Question

If *x* + $\sqrt{15}$ = 4, then $x+\frac{1}{x}$ =

A. 2

B. 4

C. 8

D. 1

Answer

Given x +√15 = 4

$X = 4 - \sqrt{15}$

$1/x = 1/(4 - \sqrt{15}) = (4 + \sqrt{15}) / 16 - 15 = 4 + \sqrt{15}$

So, $x + 1/x = 4 - \sqrt{15} + 4 + \sqrt{15} = 8$

13. Question

If $x = \sqrt[3]{2+\sqrt{3}}$, then $x^3 + \frac{1}{x^3}$ =

A. 2

B. 4

C. 8

D. 9

Answer

Given $x = \sqrt[3]{2+\sqrt{3}}$

$= x^3 = 2 + \sqrt{3}$

Similarly, $1/x^3 = 2 - \sqrt{3}$

$X^3 + 1/x^3 = 2 + \sqrt{3} + 2 - \sqrt{3} = 4.$

14. Question

If $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$, then $x + y + xy$ =

A. 9

B. 5

C. 17

D. 7

Answer

Given $x = \sqrt{5} + \sqrt{3} / \sqrt{5} - \sqrt{3}$, $y = \sqrt{5} - \sqrt{3} / \sqrt{5} + \sqrt{3}$

$X = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{8+2\sqrt{15}}{2} = 4 + \sqrt{15}$

$Y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{8-2\sqrt{15}}{2} = 4 - \sqrt{15}$

$Xy = 4^2 - \sqrt{15}^2 = 16 - 15 = 1$

So,

$X + y + xy = 4 + \sqrt{15} + 4 - \sqrt{15} + 1 = 9.$

15. Question

If $x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then $x^2 + xy + y^2 =$

A. 101

B. 99

C. 98

D. 102

Answer

Given x = $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, y = $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

X= $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = 5 - 2\sqrt{6}$

$X^2 = (5 - 2\sqrt{6})^2 = 25 + 24 - 20\sqrt{6}) = 49 - 20\sqrt{6}$

Similarily y = $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = 5 + 2\sqrt{6}$

$Y^2 = (5 + 2\sqrt{6})^2 = 49 + 20\sqrt{6}$

Xy = $(5 - 2\sqrt{6})(5 + 2\sqrt{6}) = 25 - 24 = 1$

So, $x^2 + xy + y^2 = 49 - 20\sqrt{6} + 1 + 49 + 20\sqrt{6} = 99$.

16. Question

The value of $\sqrt{3-2\sqrt{2}}$ is

A. $\sqrt{2}$ -1

B. $\sqrt{2}$ +1

C. $\sqrt{3}$ - $\sqrt{2}$

D. $\sqrt{3}$ + $\sqrt{2}$

Answer

$$\sqrt{3-2\sqrt{2}}$$

(try to break the terms in form of $(a+b)^2$ or $(a - b)^2$)

$\sqrt{(\sqrt{2})^2 + 1^2 - 2 \times \sqrt{2} \times 1)} = \sqrt{(\sqrt{2}-1)^2} = \sqrt{2} - 1$.

17. Question

The value of $\sqrt{3-2\sqrt{2}}$ is

A. $\sqrt{3}$ - $\sqrt{2}$

B. $\sqrt{3}+\sqrt{2}$

C. $\sqrt{5}$ **+** $\sqrt{6}$

D. none of these

Answer

$$\sqrt{3-2\sqrt{2}}$$

(try to break the terms in form of $(a+b)^2$ or $(a - b)^2$)

$\sqrt{(\sqrt{2})^2 + 1^2 - 2 \times \sqrt{2} \times 1} = \sqrt{(\sqrt{2}-1)^2} = \sqrt{2} - 1$.

18. Question

If $\sqrt{2}$ =1.4142, then $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is equal to

A. 0.1718

B. 5.8282

C. 0.4142

D. 2.4142

Answer

Given $\sqrt{2} = 1.4142$

$\sqrt{(\sqrt{2}-1)/\sqrt{2}+1} = \sqrt{(\sqrt{2}-1)^2} = \sqrt{2} - 1 = 1.4142 - 1 = 0 .4142$

19. Question

If $\sqrt{2}$ =1.414, then the value of $\sqrt{6}-\sqrt{3}$ upto three place of decimal is

A. 0.235

B. 0.707

C. 1.414

D. 0.471

Answer

Given , $\sqrt{2} = 1.414$

$\sqrt{6} - \sqrt{3} = \sqrt{2} \times \sqrt{3} - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1) = 1.732 (1.414 - 1) = 1.732 \times 0.414 = 0.707$

20. Question

The positive square of 7 + $\sqrt{48}$ is

A. 7 + 2 $\sqrt{3}$

B. $7+\sqrt{3}$

C. $2+\sqrt{3}$

D. $3+\sqrt{2}$

Answer

$7 + \sqrt{48}$

$= 7 + \sqrt{(16\times3)} = 7 + 4\sqrt{3}$ (try to break it in form of $(a+b)^2$)

$= (2)^2 + (\sqrt{3})^2 + 2\times2\times\sqrt{3} = (2+\sqrt{3})^2 = (2+\sqrt{3})(2+\sqrt{3})$.

21. Question

$\frac{1}{\sqrt{9}-\sqrt{8}}$ is equal to

A. $3 + 2\sqrt{2}$

B. $\frac{1}{3+2\sqrt{2}}$

C. $3-2\sqrt{2}$

D. $\frac{3}{2}-\sqrt{2}$

Answer

$1/\sqrt{9}-\sqrt{8}$

$= 1/(\sqrt{9} - \sqrt{8}) \times (\sqrt{9} + \sqrt{8}) / (\sqrt{9}+\sqrt{8})$

$= \sqrt{9} + \sqrt{8} = 3 + 2\sqrt{2}$

22. Question

The value of $\frac{\sqrt{48}+\sqrt{32}}{\sqrt{27}+\sqrt{18}}$ is

A. $\frac{4}{3}$

B. 4

C. 3

D. $\frac{3}{4}$

Answer

$\sqrt{48} + \sqrt{32} / \sqrt{27} + \sqrt{18}$

$= 4\sqrt{3} + 4\sqrt{2} / 3\sqrt{3} + 3\sqrt{2} = (4\sqrt{3}+4\sqrt{2})/(3\sqrt{3}+3\sqrt{2}) \times (3\sqrt{3}- 3\sqrt{2})/ (3\sqrt{3}-3\sqrt{2})$

$=(36 + 12\sqrt{6} - 12\sqrt{6} -24) / (27-18) = 12/9 = 4/3$

23. Question

If $x = \sqrt{6} + \sqrt{5}$, then $x^2 + \frac{1}{x^2}$ -2 =

A. $2\sqrt{6}$

B. $2\sqrt{5}$

C. 24

D. 20

Answer

Given x= $\sqrt{6} + \sqrt{5}$

$X^2 = 11 + 2\sqrt{11}$

$1/x^2 = 11 - 2\sqrt{11}$

So, $x^2 + 1/x^2 - 2 = 11 + 2\sqrt{11} + 11 - 2\sqrt{11} - 2 = 22 - 2 = 20$.

24. Question

If $\sqrt{13 - a\sqrt{10}} = \sqrt{8} + \sqrt{5}$, then a =

A. -5

B. -6

C. -4

D. -2

Answer

$\sqrt{(13 - a\sqrt{10})} = \sqrt{8} + \sqrt{5}$

Squaring both side,..

$= 13 - a\sqrt{10} = 8 + 5 + 2\times\sqrt{8}\times\sqrt{5}$

$= 13 - a\sqrt{10} = 13 + 2\sqrt{40}$

$= - a\sqrt{10} = 4\sqrt{10}$

$= a = -4$

25. Question

If $= \frac{5-\sqrt{3}}{2+\sqrt{3}} = x + y\sqrt{3}$, then

A. $x = 13$, $y = -7$

B. $x = -13$, $y = 7$

C. $x = -13$, $y = -7$

D. $x = 13$, $y = 7$

Answer

$5 - \sqrt{3}/ 2+\sqrt{3} = x + y\sqrt{3}$

$= (5-\sqrt{3}) / (2+ \sqrt{3}) \times (2 -\sqrt{3}) / (2 -\sqrt{3})$

$= (10 - 5\sqrt{3} - 2\sqrt{3} +3)/ (4-3)$

$= 10 - 7\sqrt{3} +3$

$= 13 - 7\sqrt{3} = x + y\sqrt{3}$

So , $x= 13$, $y = -7$