

Chapter - 3
Square Square Root and Cube Cube Root

Exercise

In each of the questions, 1 to 24, write the correct answer from the given four options.

1. 196 is the square of

- (a) 11**
- (b) 12**
- (c) 14**
- (d) 16**

Solution:

Square of 14 = $14 \times 14 = 196$
Hence, the correct option is (C).

2. Which of the following is a square of an even number?

- (a) 144**
- (b) 169**
- (c) 441**
- (d) 625**

Solution:

$144 = 12^2$ is a square of an even number.
Hence, the correct option is (A).

3. A number ending in 9 will have the units place of its square as

- (a) 3**
- (b) 9**
- (c) 1**
- (d) 6**

Solution:

As, $9 \times 9 = 81$
Since, the number ending in 9, will have the unit's place of its square as 1.
Hence, the correct option is (C).

4. Which of the following will have 4 at the units place?

- (a) 14^2**
- (b) 62^2**
- (c) 27^2**

(d) 35^2

Solution:

The unit place of the square of $62 = 2^2 = 4$.
Hence, the correct option is (B).

5. How many natural numbers lie between 5^2 and 6^2 ?

- (a) 9**
- (b) 10**
- (c) 11**
- (d) 12**

Solution:

The natural numbers lying between 5^2 and 6^2 , that is between 25 and 36 are 26, 27, 28, 29, 30, 31, 32, 33, 34 and 35.

Since, the natural numbers lying between 5^2 and 6^2 is 10.
Hence, the correct option is (B).

6. Which of the following cannot be a perfect square?

- (a) 841**
- (b) 529**
- (c) 198**
- (d) All of the above**

Solution:

As, we know that, a number ending with digits 2, 3, 7 or 8 can never be a perfect square. So, 198 can't be written in the form of a perfect square.
Hence, the correct option is (C).

7. The one's digit of the cube of 23 is

- (a) 6**
- (b) 7**
- (c) 3**
- (d) 9**

Solution:

The cube of the numbers ending with digits 3 have 7 at one's digit.
Hence, the correct option is (B).

8. A square board has an area of 144 square units. How long is each side of the board?

- (a) 11 units**

- (b) 12 units
- (c) 13 units
- (d) 14 units

Solution:

Given: area of square board = 144 sq units.

$$\text{So, (Side)}^2 = 144$$

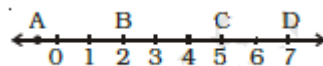
$$(\text{Side})^2 = 12^2$$

$$\text{Side} = 12$$

Hence, the correct option is (B).

9. Which letter best represents the location of $\sqrt{25}$ on a number line?

- (a) A
- (b) B
- (c) C
- (d) D



Solution:

We know that $\sqrt{25}$ can be written as $\sqrt{25} = \sqrt{5^2} = 5$.

Therefore, 5 at letter C represents the best location of $\sqrt{25}$ on number line.

Hence, the correct option is (C).

10. If one member of a Pythagorean triplet is $2m$, then the other two members are

- (a) m, m^2+1
- (b) m^2+1, m^2-1
- (c) m^2, m^2-1
- (d) $m^2, m+1$

Solution:

The other two members of a Pythagorean triplet are m^2+1, m^2-1 .

$$\text{Let } 2m = 4$$

$$\text{So, } m = 2$$

$$\text{Now, } m^2+1 = 2^2 + 1 = 4 + 1 = 5$$

$$\text{And: } m^2-1 = 2^2 - 1 = 4 - 1 = 3$$

$$\text{Now, } 3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$

$$25 = 25$$

Hence, the correct option is (B).

11. The sum of successive odd numbers 1, 3, 5, 7, 9, 11, 13 and 15 is

- (a) 81**
- (b) 64**
- (c) 49**
- (d) 36**

Solution:

As we know that, the sum of first n odd natural numbers is n^2 .

Number of odd numbers (n) = 8

So, the sum of given odd numbers = $n^2 = 8^2 = 64$.

Hence, the correct option is (B).

12. The sum of first n odd natural numbers is

- (a) $2n+1$**
- (b) n^2**
- (c) n^2-1**
- (d) n^2+1**

Solution:

Sum of first n odd natural numbers = $\sum (2n-1) = 2\sum n - n$

$$= \frac{2 \times n(n+1)}{2} - n$$

$$= n(n+1) - n$$

$$= n^2 + n - n$$

$$= n^2$$

Hence, the correct option is (B).

13. Which of the following numbers is a perfect cube?

- (a) 243**
- (b) 216**
- (c) 392**
- (d) 8640**

Solution:

For option (b):

Prime factor of 216 is:

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

Grouping the factors in triplets of equal factors, get:

$$216 = (2 \times 2 \times 2) \times (3 \times 3 \times 3)$$

$$= 2^3 \times 3^3$$

$$= (2 \times 3)^3$$

$$= 6^3$$

Clearly, 216 is a perfect cube.

Hence, the correct option is (B).

14. The hypotenuse of a right triangle with its legs of lengths $3x \times 4x$ is

(a) $5x$

(b) $7x$

(c) $16x$

(d) $25x$

Solution:

Given: the lengths of the legs of right angled triangle are $3x$ and $4x$. So,

$$\text{Hypotenuse} = \sqrt{(3x)^2 + (4x)^2}$$

$$= \sqrt{9x^2 + 16x^2}$$

$$= \sqrt{25x^2}$$

$$= 5x$$

Hence, the correct option is (a).

15. The next two numbers in the number pattern 1, 4, 9, 16, 25 ... are

(a) 35, 48

(b) 36, 49

(c) 36, 48

(d) 35, 49

Solution:

The number pattern can be written as $(1)^2, (2)^2, (3)^2, (4)^2, (5)^2$

Therefore, the next two numbers are $(6)^2$ and $(7)^2$, i.e. 36 and 49.

16. Which among $43^2, 67^2, 52^2, 59^2$ would end with digit 1?

(a) 43^2

(b) 67^2

(c) 52^2

(d) 59^2

Solution:

As we know that the units of the square of a natural number is the unit's digit of the square of the digit at unit's place of the given natural number. So,

Unit's digit of $59^2 = 1$ [As, unit's digit of 9^2 is 1]

Hence, the correct option is (D).

17. A perfect square can never have the following digit in its ones place.

- (a) 1
- (b) 8
- (c) 0
- (d) 6

Solution:

As, we know that a number ending with digit 2,3,7 or 8 can't be perfect square. Since, a perfect square can never have the digit 8 in its one's place.

Hence, the correct option is (B).

18. Which of the following numbers is not a perfect cube?

- (a) 216
- (b) 567
- (c) 125
- (d) 343

Solution:

For option (B):

$$567 = 3 \times 3 \times 3 \times 3 \times 7$$

Clearly, 567 is not a perfect cube

Hence, the correct option is (B).

19. $\sqrt[3]{1000}$ is equal to

- (a) 10
- (b) 100
- (c) 1
- (d) None of these

Solution:

Consider the expression:

$$\sqrt[3]{1000}$$

Now, simplify the above expression as follows:

$$\begin{aligned}\sqrt[3]{1000} &= (10)^{3 \times \frac{1}{3}} \\ &= 10\end{aligned}$$

Hence, the correct option is (A).

20. If m is the square of a natural number n , then n is

- (a) the square of m**
- (b) greater than m**
- (c) equal to m**
- (d) $\sqrt{m}$**

Solution:

According to the question:

$$m = n^2$$

$$\text{So, } n = \sqrt{m}$$

Hence, the correct option is (D).

21. A perfect square number having n digits where n is even will have square root with

- (a) $n+1$ digit**
- (b) $\frac{n}{2}$ digit**
- (c) $\frac{n}{3}$ digit**
- (d) $\frac{n+1}{2}$ digit**

Solution:

As we know that a perfect square number having n digits, where n is even, will have square root with $\frac{n}{2}$ digit.

Hence, the correct option is (B).

22. If m is the cube root of n , then n is

- (a) m^3**
- (b) m**
- (c) $\frac{m}{3}$**
- (d) $\sqrt[3]{m}$**

Solution:

According to the question:

$$m = n^{\frac{1}{3}}$$

$$\text{So, } m^3 = n$$

Hence, the correct option is (A).

23. The value of $\sqrt{248 + \sqrt{52 + \sqrt{144}}}$ is

- (a) 14**
- (b) 12**
- (c) 16**
- (d) 13**

Solution:

Consider the expression:

$$\sqrt{248 + \sqrt{52 + \sqrt{144}}}$$

Now, solve the above expression as follows:

$$\begin{aligned}\sqrt{248 + \sqrt{52 + \sqrt{144}}} &= \sqrt{248 + \sqrt{52 + \sqrt{12^2}}} \\ &= \sqrt{248 + \sqrt{52 + 12}} \\ &= \sqrt{248 + \sqrt{64}} \\ &= \sqrt{248 + \sqrt{8^2}} \\ &= \sqrt{248 + 8} \\ &= \sqrt{256} \\ &= \sqrt{16^2} \\ &= 16\end{aligned}$$

Hence, the correct option is (C).

24. Given that $\sqrt{4096} = 64$, the value of $\sqrt{4096} + \sqrt{40.98}$ is

- (a) 74**
- (b) 60.4**
- (c) 64.4**
- (d) 70.4**

Solution:

Given:

$$\sqrt{4096} = 64$$

Now, the value of $\sqrt{4096} + \sqrt{40.98}$ will be:

$$\begin{aligned}\sqrt{4096} + \sqrt{40.98} &= \sqrt{4096} + \sqrt{\frac{4098}{100}} \\ &= 64 + \frac{64}{10} \\ &= 64 + 6.4 \\ &= 70.4\end{aligned}$$

Hence, the correct option is (D).

In questions 25 to 48, fill in the blanks to make the statements true.

25. There are _____ perfect squares between 1 and 100.

Solution:

The perfect square number between 1 and 100 are 4, 9, 16, 25, 36, 49, 64 and 81.

Hence, there are 8 perfect squares between 1 and 100.

26. There are _____ perfect cubes between 1 and 1000.

Solution:

The perfect square number between 1 and 1000 are 8, 27, 64, 125, 216, 343 and 729.

Hence, there are 8 perfect cubes between 1 and 1000.

27. The units digit in the square of 1294 is _____.

Solution:

The unit's digit in the square of 1294 is 6 as $4 \times 4 = 16$.

28. The square of 500 will have _____ zeroes.

Solution:

According to the question:

$$\begin{aligned} 500 &= (500)^2 \\ &= 500 \times 500 \\ &= 250000 \end{aligned}$$

Hence, the square of 500 have four zeroes.

29. There are _____ natural numbers between n^2 and $(n + 1)^2$.

Solution:

Natural number between n^2 and $(n + 1)^2$ will be calculated as follows:

$$\begin{aligned} &= \left[(n + 1)^2 - n^2 \right] - 1 \\ &= (n^2 + 2n + 1 - n^2) - 1 \\ &= 2n + 1 - 1 \\ &= 2n \end{aligned}$$

Hence, there are 2n natural numbers between n^2 and $(n + 1)^2$.

30. The square root of 24025 will have _____ digits.

Solution:

The number of digits in 24025 is $n = 5$.

So, number of digits in the square of 24025 = $\frac{n+1}{2} = \frac{5+1}{2} = \frac{6}{2} = 3$

Hence, the square root of 24025 will have 3 digits.

31. The square of 5.5 is _____.

Solution:

Square of 5.5 = $(5.5)^2 = 55 \times 5.5 = 30.25$

Hence, the square of 5.5 is 30.25.

32. The square root of 5.3×5.3 is _____.

Solution:

Square root of $5.3 \times 5.3 = \sqrt{(5.3)^2} = 5.3$

Hence, the square root of 5.3×5.3 is 5.3.

33. The cube of 100 will have _____ zeroes.

Solution:

The cube of 100 will be = $100^3 = 100 \times 100 \times 100 = 1000000$

Hence, the cube of 100 will have 6 zeroes.

34. $1m^2 =$ _____ cm^2 .

Solution:

As we know that $1m^2 = 10000 cm^2$. [As, $1m = 100cm$]

35. $1m^3 =$ _____ cm^3 .

Solution:

As we know that $1m^3 = 1000000 cm^3$. [As, $1m = 100cm$]

36. Ones digit in the cube of 38 is _____.

Solution:

Ones digit in the cube of 38 is calculated as:

$$8 \times 8 \times 8 = 512$$

Hence, ones digit in the cube of 38 is 2.

37. The square of 0.7 is _____.

Solution:

The square of 0.7 is $= 0.7 \times 0.7 = 0.49$.

38. The sum of first six odd natural numbers is _____.

Solution:

The first six odd numbers are 1, 3, 5, 7, 9, and 11.

So, sum of first six odd numbers $= 1 + 3 + 5 + 7 + 9 + 11 = 36$

39. The digit at the ones place of 57^2 is _____.

Solution:

The digit at the ones place of 57^2 is $= 57 \times 57 = 3249$

Hence, the digit at the ones place of 57^2 is 9.

40. The sides of a right triangle whose hypotenuse is 17cm are _____ and _____.

Solution:

As we know that for every natural number $m > 1$, $2m$, $m^2 - 1$ and $m^2 + 1$ form a Pythagorean triplet.

So, $m^2 + 1 = (2m)^2 + (m^2 - 1)^2$

Where, $m^2 + 1 = 17$

$$m^2 = 17 - 1$$

$$m^2 = 16$$

$$m = \sqrt{16}$$

$$m = 4$$

$$\text{Then, } 2m = 2 \times 4$$

$$= 8$$

$$\text{And, } m^2 - 1 = 4^2 - 1$$

$$= 16 - 1$$

$$= 15$$

41. $\sqrt{1.96} =$ _____.

Solution:

The value of $\sqrt{1.96}$ is $=\sqrt{(1.4)^2} = 1.4$

42. $(1.2)^3 =$ _____.

Solution:

The value of $(1.2)^3 = 1.2 \times 1.2 \times 1.2 = 1.728$.

43. The cube of an odd number is always an _____ number.

Solution:

As we know that the cube of an odd number is always an odd.

44. The cube root of a number x is denoted by _____.

Solution:

As we know that the cube root of a number x is denoted by $\sqrt[3]{x}$ or $x^{\frac{1}{3}}$.

45. The least number by which 125 be multiplied to make it a perfect square is _____.

Solution:

The least number by which 125 be multiplied to make it a perfect square is 5.

As, $125 \times 5 = 5 \times 5 \times 5 \times 5 = 25 \times 25 = 25^2$

46. The least number by which 72 be multiplied to make it a perfect cube is _____.

Solution:

The least number by which 72 be multiplied to make it a perfect cube is 3.

As, $72 \times 3 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3 = (2 \times 3)^3 = 6^3$

47. The least number by which 72 be divided to make it a perfect cube is _____.

Solution:

The least number by which 72 be divided to make it a perfect cube is 9.

As, $\frac{72}{9} = \frac{9 \times 8}{9} = 8 = 2^3$

48. Cube of a number ending in 7 will end in the digit _____.

Solution:

Cube of a number ending in 7 will end in the digit 3.

In questions 49 to 86, state whether the statements are true (T) or false (F).

49. The square of 86 will have 6 at the units place.

Solution:

The unit's digit of the square of a number $86 = 6 \times 6 = 36$ that is 6.
Hence, the given statement is true.

50. The sum of two perfect squares is a perfect square.

Solution:

Let's take two perfect square 25 and 49.

So, sum of the two perfect square $= 25 + 49 = 74$

Therefore, 74 is not perfect square.

Hence, the given statement is false.

51. The product of two perfect squares is a perfect square.

Solution:

Let's take two perfect square 4 and 9.

So, product of two perfect square is $= 4 \times 9 = 36$

Hence, the given statement is true.

52. There is no square number between 50 and 60.

Solution:

The given statement is **true** because there is no square number between 50 and 60.

53. The square root of 1521 is 31.

Solution:

The square root of 1521 is 39.

Hence, the given statement is false.

54. Each prime factor appears 3 times in its cube.

Solution:

As we know that, a perfect cube can always be expressed as the product of triplets of prime factors.

Hence, the given statement is true.

55. The square of 2.8 is 78.4.

Solution:

The square of 2.8 is $= 2.8^2 = 7.84$

Hence, the given statement is false.

56. The cube of 0.4 is 0.064.

Solution:

The cube of 0.4 is $= 0.4 \times 0.4 \times 0.4 = 0.064$

Hence, the given statement is true.

57. The square root of 0.9 is 0.3.

Solution:

The square root of 0.9 is $= \sqrt{0.9} = 0.9487$

Hence, the given statement is false.

58. The square of every natural number is always greater than the number itself.

Solution:

The given statement is true because,

For example: $-4^2 = 16$

Where, $4 < 16$

59. The cube root of 8000 is 200.

Solution:

The cube root of 8000 is $= \sqrt[3]{8000} = (20)^{\frac{1}{3}} = 20$

Hence, the given statement is false.

60. There are five perfect cubes between 1 and 100.

Solution:

The perfect cube between 1 and 100 are:

$2 \times 2 \times 2 = 8$

$$3 \times 3 \times 3 = 27$$

$$4 \times 4 \times 4 = 64$$

Hence, there are only 3 perfect cubes between 1 and 100.

61. There are 200 natural numbers between 100^2 and 101^2 .

Solution:

As we know that, natural numbers between a and b = $b - a - 1$. So,

$$= 101^2 - 100^2 - 1$$

$$= (101 + 100)(101 - 100) - 1$$

$$= 201 - 1$$

$$= 200$$

Hence, the given statement is true.

62. The sum of first n odd natural numbers is n^2 .

Solution:

$$\text{The sum of odd numbers} = \sum (2n - 1)$$

$$= (2 \times n \times (n + 1))/2 - n$$

$$= n^2 + n - n$$

$$= n^2$$

Hence, the given statement is true.

63. 1000 is a perfect square.

Solution:

1000 is a perfect cube that is $10 \times 10 \times 10 = 1000$

So, it is not a perfect square.

Hence, the given statement is false.

64. A perfect square can have 8 as its unit's digit.

Solution:

A perfect square can never have 8 as its unit's digit.

Hence, the given statement is false.

65. For every natural number m , $(2m-1, 2m^2-2m, 2m^2-2m + 1)$ is a Pythagorean triplet.

Solution:

As we know that for every natural number $m > 1$, $2m$, $m^2 - 1$ and $m^2 + 1$ form a Pythagorean triplet.

Hence, the given statement is false.

66. All numbers of a Pythagorean triplet are odd.**Solution:**

Three natural numbers a , b , c are said to form a Pythagorean triplet if $a^2 + b^2 = c^2$. So,

Pythagorean triplet as, $7^2 = 6^2 + 5^2$

Therefore, 6 is not an odd number.

Hence, the given statement is false.

67. For an integer a , a^3 is always greater than a^2 .**Solution:**

Let us take one negative integer that is -3.

Then, $-3^3 = -3 \times -3 \times -3 = -27$

$-3^2 = -3 \times -3 = 9$

Here, $9 > -27$

Therefore, -3^2 is greater than -3^3 .

Hence, the given statement is false.

68. If x and y are integers such that $x^2 > y^2$, then $x^3 > y^3$.**Solution:**

Let's take negative integers -2 and -3

Then,

$= (-2^2) > (-1^2)$

$= (4) > (1)$

and $= -2^3 < -1^3$

$= -8 < -1$

Hence, the given statement is false.

69. Let x and y be natural numbers. If x divides y , then x^3 divides y^3 .**Solution:**

If x divides $y = \frac{y}{x}$ is a natural number

Then x^3 divides $y^3 = \frac{y^3}{x^3}$ is a natural number

Hence, the given statement is **true**.

70. If a^2 ends in 5, then a^3 ends in 25.

Solution:

Let a^2 be 15^2

So, $15^2 = 225$

Then, $15^3 = 3375$

Hence, the given statement is **false**.

71. If a^2 ends in 9, then a^3 ends in 7.

Solution:

Let a^2 be 7^2

So, $7^2 = 49$

Then, $7^3 = 343$

Hence, the given statement is **false**.

72. The square root of a perfect square of n digits will have $\left(\frac{n+1}{2}\right)$ digits, if n is odd.

Solution:

As we know that if the square has 3 digits, then its square root has i.e. $\left(\frac{3+1}{2}\right)$ digits.

Hence, the given statement is **true**.

73. Square root of a number x is denoted by $\sqrt{x}$.

Solution:

As we know that square root of a number x is denoted by $\sqrt{x}$.

Hence, the given statement is **true**.

74. A number having 7 at its ones place will have 3 at the unit's place of its square.

Solution:

Let's take the number having 7 at its ones place is 47.

So, its square $47^2 = 2209$

Hence, the given statement is false.

75. A number having 7 at its ones place will have 3 at the ones place of its cube.

Solution:

Let's take the number having 7 at its ones place that is 37.

Now, Its cube $37^3 = 50,653$

Another number is 27.

So, its cube $27^3 = 19,683$

Hence, the given statement is true.

76. The cube of a one digit number cannot be a two digit number.

Solution:

The cube of 4 will be $= 4^3 = 4 \times 4 \times 4 = 64$

Hence, the given statement is false.

77. Cube of an even number is odd.

Solution:

The cube of an even number is always an even number.

For example: $2^3 = 8$

$4^3 = 64$

Hence, the given statement is false.

78. Cube of an odd number is even.

Solution:

The cube of odd number is always an odd number.

For example: $3^3 = 27$

$$5^3 = 125$$

Hence, the correct option is **false**.

79. Cube of an even number is even.

Solution:

The cube of an even number is always an even number.

For example: $8^3 = 512$

$$12^3 = 1728$$

Hence, the given statement is **true**.

80. Cube of an odd number is odd.

Solution:

The cube of odd number is always an odd number.

For example: $7^3 = 343$

Hence, the given statement is **true**.

81. 999 is a perfect cube.

Solution:

Factor of 999 = $3 \times 3 \times 3 \times 37 = 3^3 \times 37$

Hence, the given statement is **false**.

82. 363×81 is a perfect cube.

Solution:

Factors of 363×81 are $3 \times 11 \times 11 \times 3 \times 3 \times 3 \times 3$

$$= 3^3 \times 11 \times 11 \times 3 \times 3$$

Since, 363×81 is not a perfect cube

Hence, the given statement is **false**.

83. Cube roots of 8 are +2 and -2.

Solution:

Cube root of 8 is 2 only because $2 \times 2 \times 2 = 8$

Hence, the given statement is **false**.

84. $\sqrt[3]{8+27} = \sqrt[3]{8} + \sqrt[3]{27}$

Solution:

We consider:

$$\begin{aligned}\text{LHS} &= \sqrt[3]{8+27} \\ &= \sqrt[3]{35}\end{aligned}$$

$$\begin{aligned}\text{Then, RHS} &= \sqrt[3]{8} + \sqrt[3]{27} \\ &= 2 + 3 \\ &= 5\end{aligned}$$

Since, $\text{LHS} \neq \text{RHS}$

Hence, the given statement is false.

85. There is no cube root of a negative integer.

Solution:

Let's take a number -27

Cube root of -27 is calculate as follows:

$$\begin{aligned}&= \sqrt[3]{-27} \\ &= \sqrt[3]{(-3) \times (-3) \times (-3)} \\ &= -3\end{aligned}$$

Hence, the given statement is false.

86. Square of a number is positive, so the cube of that number will also be positive.

Solution:

Let's take a number -3

Now, square of the -3 will be $= -3^2 = 9$

Then, cube of the same number $= -3^3 = -27$

Hence, the given statement is false.

Solve the following questions.

87. Write the first five square numbers.

Solution:

The first five square numbers are,

$$1 \times 1 = 1$$

$$2 \times 2 = 4$$

$$3 \times 3 = 9$$

$$4 \times 4 = 16$$

$$5 \times 5 = 25$$

88. Write cubes of first three multiples of 3.

Solution:

The first three multiple of 3 are 3, 6 and 9

Now, cubes of first three multiples of 3 be:

$$3^3 = 3 \times 3 \times 3 = 27$$

$$6^3 = 6 \times 6 \times 6 = 216$$

$$9^3 = 9 \times 9 \times 9 = 729$$

89. Show that 500 is not a perfect square.

Solution:

Factors of 500 are $2 \times 2 \times 5 \times 5 \times 5$

Now, group the factors into pairs, get:

$$\begin{aligned} 500 &= (2 \times 2) \times (5 \times 5 \times 5) \\ &= 2^2 \times 5^3 \end{aligned}$$

So, the prime factors 2 and 5 do not occur in pairs.

Hence, 500 is not a perfect square.

90. Express 81 as the sum of first nine consecutive odd numbers.

Solution:

The first nine consecutive odd numbers are 1, 3, 5, 7, 9, 11, 13, 15 and 17.

We have: 81

As, 81 is a perfect square. So,

$$\begin{aligned} 81 &= (9)^2 \\ &= 1+3+5+7+9+11+13+15+17 \\ &= \text{Sum of first nine consecutive odd numbers.} \end{aligned}$$

91. Using prime factorization, find which of the following perfect squares are.

(a) 484

(b) 11250

(c) 841

(d) 729

Solution:

(a) We have: 484

Let's find the out the factors of 484 by using prime factorization method.

$$\begin{array}{r|l}
 2 & 484 \\
 \hline
 2 & 242 \\
 \hline
 11 & 121 \\
 \hline
 11 & 11 \\
 \hline
 & 1
 \end{array}$$

$$\begin{aligned}
 \text{So, prime factors of } 484 &= 2 \times 2 \times 11 \times 11 \\
 &= 2^2 \times 11^2
 \end{aligned}$$

Hence, 484 is a perfect square.

(b) We have: 11250

Let's find the out the factors of 11250 by using prime factorization method.

$$\begin{array}{r|l}
 2 & 11250 \\
 \hline
 3 & 5625 \\
 \hline
 3 & 1875 \\
 \hline
 5 & 625 \\
 \hline
 5 & 125 \\
 \hline
 5 & 25 \\
 \hline
 5 & 5 \\
 \hline
 & 1
 \end{array}$$

$$\begin{aligned}
 \text{So, prime factors of } 11250 &= 2 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5 \\
 &= 2 \times 3^2 \times 5^2 \times 5^2
 \end{aligned}$$

Hence, 11250 is not a perfect square.

(c) We have: 841

Let's find the out the factors of 841 by using prime factorization method.

$$\begin{array}{r|l}
 29 & 841 \\
 \hline
 29 & 29 \\
 \hline
 & 1
 \end{array}$$

$$\begin{aligned}
 \text{So, prime factors of } 841 &= 29 \times 29 \\
 &= 29^2
 \end{aligned}$$

Hence, 841 is a perfect square.

(d) We have: 729

Let's find the out the factors of 729 by using prime factorization method.

$$\begin{array}{r|l} 3 & 729 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\begin{aligned} \text{So, prime factors of } 729 &= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \\ &= 3^2 \times 3^2 \times 3^2 \end{aligned}$$

Hence, 729 is a perfect square.

92. Using prime factorization, find which of the following are perfect cubes.

(a) 128

(b) 343

(c) 729

(d) 1331

Solution:

(a) We have: 128

Let's find the out the factors of 128 by using prime factorization method.

$$\begin{array}{r|l} 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$$\begin{aligned} \text{So, prime factors of } 128 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^3 \times 2^3 \times 2 \end{aligned}$$

Hence, 128 is not a perfect cube.

(b) We have: 343

Let's find the out the factors of 343 by using prime factorization method.

$$\begin{array}{r|l} 7 & 343 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

So, prime factors of $343 = 7 \times 7 \times 7$
 $= 7^3$

Hence, 343 is a perfect cube.

(c) We have: 729

Let's find the out the factors of 729 by using prime factorization method.

$$\begin{array}{r|l} 3 & 729 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

So, prime factors of $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$
 $= 3^3 \times 3^3$

Hence, 729 is a perfect cube.

(d) We have: 1331

Let's find the out the factors of 1331 by using prime factorization method.

$$\begin{array}{r|l} 11 & 1331 \\ \hline 11 & 121 \\ \hline 11 & 11 \\ \hline & 1 \end{array}$$

So, prime factors of $1331 = 11 \times 11 \times 11$
 $= 11^3$

Hence, 1331 is a perfect cube.

93. Using distributive law, find the squares of

(a) 101

(b) 72

Solution:

(a) By using distributive law 101 can be written as:

$$101 = 100 + 1$$

So,

$$\begin{aligned} 101^2 &= (100 + 1)^2 \\ &= (100 + 1) (100 + 1) \\ &= 100 (100 + 1) + 1 (100 + 1) \\ &= ((100 \times 100) + (100 \times 1)) + ((1 \times 100) + (1 \times 1)) \end{aligned}$$

$$= 10000 + 100 + 100 + 1$$

$$= 10201$$

Hence, the square of the given number i.e. $101^2 = 10201$.

(b) By using distributive law 72 can be written as:

$$72 = 70 + 2$$

So,

$$72^2 = (70 + 2)^2$$

$$= (70 + 2)(70 + 2)$$

$$= 70(70 + 2) + 2(70 + 2)$$

$$= ((70 \times 70) + (70 \times 2)) + ((2 \times 70) + (2 \times 2))$$

$$= 4900 + 140 + 140 + 4$$

$$= 5184$$

Hence, the square of the given number i.e. $72^2 = 5184$.

94. Can a right triangle with sides 6 cm, 10 cm and 8 cm be formed? Give reason.

Solution:

Given: Sides of the right angle triangle are 6 cm, 10 cm and 8 cm.

As we know that, the square one side must be equal to the sum of square of other two sides of right angle triangle.

$$\text{i.e. } a^2 = b^2 + c^2$$

Let $a = 10$ cm, $b = 8$ cm, $c = 6$ cm

So,

$$10^2 = 8^2 + 6^2$$

$$100 = 64 + 36$$

$$100 = 100$$

Hence, right angle triangle is formed by the given sides 6cm, 10cm and 8cm.

95. Write the Pythagorean triplet whose one of the numbers is 4.

Solution:

Given, one of the number in the Pythagorean triplet is 4. So,

$$2m = 4$$

$$m = 2$$

$$\begin{aligned}
 &\text{Then,} \\
 &m^2 + 1 \\
 &= 2^2 + 1 \\
 &= 4 + 1 \\
 &= 5
 \end{aligned}$$

$$\begin{aligned}
 &\text{And,} \\
 &m^2 - 1 \\
 &= 2^2 - 1 \\
 &= 4 - 1 \\
 &= 3
 \end{aligned}$$

Hence, the Pythagorean triplet is 3, 4 and 5.

96. Using prime factorization, find the square roots of
(a) 11025
(b) 4761

Solution:

(a) We have: 11025

Let's find the out the factors of 11025 by using prime factorization method.

$$\begin{array}{r|l}
 3 & 11025 \\
 \hline
 3 & 3675 \\
 \hline
 5 & 1225 \\
 \hline
 5 & 249 \\
 \hline
 7 & 49 \\
 \hline
 7 & 7 \\
 \hline
 & 1
 \end{array}$$

$$\begin{aligned}
 \text{So, prime factors of } 11025 &= 3 \times 3 \times 5 \times 5 \times 7 \times 7 \\
 &= 3^2 \times 5^2 \times 7^2
 \end{aligned}$$

$$\begin{aligned}
 \text{So, square root of } 11025 &= \sqrt{11025} \\
 &= \sqrt{3^2 \times 5^2 \times 7^2} \\
 &= 3 \times 5 \times 7 \\
 &= 105
 \end{aligned}$$

Hence, the Square root of 11025 is 105.

(b) We have: 4761

Let's find the out the factors of 4761 by using prime factorization method.

$$\begin{array}{r|l} 3 & 4761 \\ \hline 3 & 1587 \\ \hline 23 & 529 \\ \hline 23 & 23 \\ \hline & 1 \end{array}$$

$$\begin{aligned} \text{So, prime factors of } 4761 &= 3 \times 3 \times 23 \times 23 \\ &= 3^2 \times 23^2 \end{aligned}$$

$$\begin{aligned} \text{So, square root of } 4761 &= \sqrt{4761} \\ &= \sqrt{3^2 \times 23^2} \\ &= 3 \times 23 \\ &= 69 \end{aligned}$$

Hence, the Square root of 4761 is 69.

97. Using prime factorization, find the cube roots of

(a) 512

(b) 2197

Solution:

(a) We have: 512

Let's find the out the factors of 512 by using prime factorization method.

$$\begin{array}{r|l} 2 & 512 \\ \hline 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$$\begin{aligned} \text{So, prime factors of } 512 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^3 \times 2^3 \times 2^3 \end{aligned}$$

$$\begin{aligned} \text{So, cube root of } 512 &= \sqrt[3]{512} \\ &= \sqrt[3]{2^3 \times 2^3 \times 2^3} \\ &= 2 \times 2 \times 2 \end{aligned}$$

$$= 8$$

Hence, the cube root of 512 is 8.

(b) We have: 2197

Let's find the out the factors of 2197 by using prime factorization method.

$$\begin{array}{r|l} 13 & 2197 \\ \hline 13 & 169 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

So, prime factors of 2197 = $13 \times 13 \times 13$

$$= 13^3$$

So, cube root of 2197 = $\sqrt[3]{2197}$

$$= \sqrt{13^3}$$

$$= 13$$

Hence, the cube root of 2197 is 13.

98. Is 176 a perfect square? If not, find the smallest number by which it should be multiplied to get a perfect square.

Solution:

Let's find the out the factors of 176 by using prime factorisation method.

$$\begin{array}{r|l} 2 & 176 \\ \hline 2 & 88 \\ \hline 2 & 44 \\ \hline 2 & 22 \\ \hline 11 & 11 \\ \hline & 1 \end{array}$$

So, prime factors of 176 = $2 \times 2 \times 2 \times 2 \times 11$
 $= 2^2 \times 2^2 \times 11$

Factor 11 has no pair.

Since, 176 is not a perfect square.

Hence, the smallest number it should be multiplied to get a perfect square is 11.

99. Is 9720 a perfect cube? If not, find the smallest number by which it should be divided to get a perfect cube.

Solution:

Let's find the out the factors of 9720 by using prime factorisation method.

2	9720
2	4860
2	2430
3	1215
3	405
3	135
3	45
3	15
5	5
	1

So, prime factors of 9720 = $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5$
 $= 2^3 \times 3^5 \times 5$

Factors 3 and 4 has no pair.

So, 9720 is not a perfect cube.

Hence, the smallest number it should be divided to get a perfect cube is $3 \times 3 \times 5 = 45$.

100. Write two Pythagorean triplets each having one of the numbers as 5.

Solution:

Given: One of the number in the Pythagorean triplet is 5.

$$m^2 + 1 = 5$$

$$m^2 = 4$$

$$m = 2$$

$$\text{Since, } 2m = 2 \times 2$$

$$m = 4$$

$$\text{And: } = m^2 - 1$$

$$= 2^2 - 1$$

$$= 4 - 1$$

$$= 3$$

Therefore, the Pythagorean triplet is 3, 4 and 5.

Similarly, the other Pythagorean triplet is 5, 12 and 13.

101. By what smallest number should 216 be divided so that the quotient is a perfect square. Also find the square root of the quotient.

Solution:

Let's find the out the factors of 216 by using prime factorisation method.

$$\begin{array}{r|l}
 2 & 216 \\
 \hline
 2 & 108 \\
 \hline
 2 & 54 \\
 \hline
 3 & 27 \\
 \hline
 3 & 9 \\
 \hline
 3 & 3 \\
 \hline
 & 1
 \end{array}$$

$$\begin{aligned}
 \text{Factors of } 216 &= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \\
 &= 2^3 \times 3^3
 \end{aligned}$$

Factors 2 and 3 has no pair.

So, 216 is not a perfect square.

Therefore, the smallest number it should be divided to get a perfect square is $3 \times 2 = 6$.

$$\text{So, } = 216 \div 6$$

$$= 36$$

$$\text{Factors of } 36 = 6 \times 6$$

$$\begin{aligned}
 \text{Square root of } 36 &= \sqrt{36} \\
 &= \sqrt{6 \times 6} \\
 &= 6
 \end{aligned}$$

Hence, 36 is a perfect square and 6 is the square root of 36.

102. By what smallest number should 3600 be multiplied so that the quotient is a perfect cube. Also find the cube root of the quotient.

Solution:

Let's find the out the factors of 3600 by using prime factorisation method.

$$\begin{array}{r|l}
 2 & 3600 \\
 \hline
 2 & 1800 \\
 \hline
 2 & 900 \\
 \hline
 2 & 450 \\
 \hline
 3 & 225 \\
 \hline
 3 & 75 \\
 \hline
 5 & 25 \\
 \hline
 5 & 5 \\
 \hline
 & 1
 \end{array}$$

$$\begin{aligned}\text{Factors of } 3600 &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \\ &= (2 \times 2 \times 2) \times 2 \times 3 \times 3 \times 5 \times 5\end{aligned}$$

Factors 2, 3 and 5 has no pair.

So, 3600 is not a perfect cube.

Therefore, the smallest number it should be multiplied to get a perfect cube is $2 \times 2 \times 3 \times 5 = 60$.

So, the quotient is product of $= 3600 \times 60 = 216000$

$$\begin{aligned}\text{Factors of } 21600 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \\ &= \sqrt[3]{2^3 \times 2^3 \times 3^3 \times 5^3} \\ &= 2 \times 2 \times 3 \times 5 \\ &= 60\end{aligned}$$

Hence, 21600 is a perfect cube and 60 is the cube root of 21600.

103. Find the square root of the following by long division method.

(a) 1369

(b) 5625

Solution:

(a) Let's find the out the square root of 1369 by using long division method.

$$\begin{array}{r} 37 \\ 3 \overline{) 1369} \\ \underline{9} \\ 469 \\ \underline{469} \\ 0 \end{array}$$

Hence, the square root of 1369 by using long division method is 37.

(b) Let's find the out the square root of 5625 by using long division method.

$$\begin{array}{r} 75 \\ 7 \overline{) 5625} \\ \underline{49} \\ 725 \\ \underline{725} \\ 0 \end{array}$$

Hence, the square root of 5625 by using long division method is 75.

104. Find the square root of the following by long division method.

(a) 27.04

(b) 1.44

Solution:

(a) Let's find the out the square root of 27.04 by using long division method.

$$\begin{array}{r}
 \begin{array}{r} 5 \end{array} \begin{array}{r} .2 \end{array} \\
 \begin{array}{r} 5 \end{array} \overline{) 27.04} \\
 \begin{array}{r} 25 \end{array} \begin{array}{r} \vdots \end{array} \\
 \hline
 \begin{array}{r} 102 \end{array} \begin{array}{r} 2 \end{array} \begin{array}{r} 04 \end{array} \\
 \begin{array}{r} 2 \end{array} \begin{array}{r} 04 \end{array} \\
 \hline
 \begin{array}{r} 0 \end{array}
 \end{array}$$

Hence, the square root of 27.04 by using long division method is 5.2.

(b) Let's find the out the square root of 1.44 by using long division method.

$$\begin{array}{r}
 \begin{array}{r} 1 \end{array} \begin{array}{r} .2 \end{array} \\
 \begin{array}{r} 1 \end{array} \overline{) 1.44} \\
 \begin{array}{r} 1 \end{array} \begin{array}{r} \vdots \end{array} \\
 \hline
 \begin{array}{r} 22 \end{array} \begin{array}{r} 44 \end{array} \\
 \begin{array}{r} 44 \end{array} \\
 \hline
 \begin{array}{r} 0 \end{array}
 \end{array}$$

Hence, the square root of 1.44 by using long division method is 1.2.

105. What is the least number that should be subtracted from 1385 to get a perfect square? Also find the square root of the perfect square.

Solution:

Let's find the out the square root of 1385 by using long division method.

$$\begin{array}{r}
 3 7 \\
 \hline
 3 \overline{) 13 85} \\
 \underline{9 } \\
 4 85 \\
 \underline{4 69} \\
 16
 \end{array}$$

From the long division method, the least number that should be subtracted from 1385 to get a perfect square is 16.

$$\text{Now} = 1385 - 16$$

$$= 1369$$

$$\text{So, square root of } 1369 = 37 \times 37$$

$$= \sqrt{37^2}$$

$$= 37$$

So, the square root of a perfect square number 1369 is 37.

Hence, 1369 is a perfect square and 37 is the square root of 1369.

106. What is the least number that should be added to 6200 to make it a perfect square?

Solution:

Let's find the out the square root of 6200 by using long division method.

$$\begin{array}{r}
 7 8 \\
 \hline
 7 \overline{) 62 00} \\
 \underline{49 } \\
 13 00 \\
 \underline{11 84} \\
 1 16
 \end{array}$$

$$\text{So, } 78^2 = 6084$$

By comparing 6084 and 6200, $6084 < 6200$

Then, next perfect square is $79^2 = 6241$

Hence, the least number is $(6241 - 6200) = 41$, which should be added to 6200 to get a perfect square.

107. Find the least number of four digits that is a perfect square.

Solution:

As we know that the least number of four digit is 1000.

Let's find the out the square root of 1000 by using long division method.

$$\begin{array}{r}
 32 \\
 3 \overline{) 1000} \\
 \underline{9} \\
 100 \\
 \underline{9} \\
 100 \\
 \underline{81} \\
 1900 \\
 \underline{1600} \\
 300
 \end{array}$$

Hence, the least number of four digits that is a perfect square = $1000 + 24$
 $= 1024$

108. Find the greatest number of three digits that is a perfect square

Solution:

As we know that the least number of four digit is 999.

Let's find the out the square root of 999 by using long division method.

$$\begin{array}{r}
 31 \\
 3 \overline{) 999} \\
 \underline{9} \\
 99 \\
 \underline{9} \\
 99 \\
 \underline{91} \\
 89
 \end{array}$$

Hence, the least number of four digits that is a perfect square = $999 - 38$
 $= 961$

109. Find the least square number which is exactly divisible by 3, 4, 5, 6 and 8.

Solution:

The LCM of numbers 3, 4, 5, 6 and 8 is calculated as:

2	3, 4, 5, 6, 8
2	3, 2, 5, 3, 4
2	3, 1, 5, 3, 2
3	3, 1, 5, 3, 1
5	1, 1, 5, 1, 1
	1, 1, 1, 1, 1

So, LCM of 3, 4, 5, 6 and 8 = $2 \times 2 \times 2 \times 3 \times 5$
 $= 120$

Factors 2, 3 and 5 has no pair.

So, 120 is not a perfect square.

Therefore, to make it perfect square, 120 must be multiplied with $2 \times 3 \times 5 = 30$

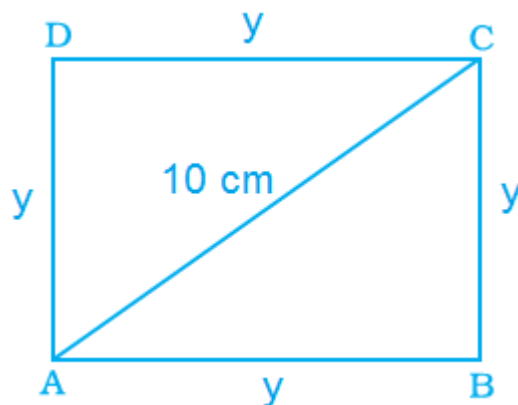
Then: $= 120 \times 30 = 3600$

Hence, 3600 is least square number which is exactly divisible by 3, 4, 5, 6, and 8.

110. Find the length of the side of a square if the length of its diagonal is 10cm.

Solution:

Let ABCD is a square and its Sides are $AB = y$, $BC = y$, $CD = y$, $DA = y$.



Given: Length of Diagonal = $AC = 10$ cm

Now, consider triangle ABC

By the rule of Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$10^2 = y^2 + y^2$$

$$100 = 2y^2$$

$$y^2 = \frac{100}{2}$$

$$y^2 = 50$$

$$y = \sqrt{50} \text{ cm}$$

$$y = 5\sqrt{2} \text{ cm}$$

Hence, the length of the side of square is $\sqrt{50}$ cm or $5\sqrt{2}$ cm .

111. A decimal number is multiplied by itself. If the product is 51.84, find the number.

Solution:

Suppose decimal number be 'x'.

We know that, $x \times x = x^2$

So,

$$x \times x = 51.84$$

$$x^2 = 51.84$$

$$x = \sqrt{51.84}$$

The required number is found out by using long division method.

$$\begin{array}{r} 7 \ .2 \\ 7 \overline{) 51.84} \\ \underline{49} \quad \vdots \\ 2 \ 84 \\ 142 \overline{) 2 \ 84} \\ \underline{2 \ 84} \\ 0 \end{array}$$

Then,

$$x = \sqrt{51.84}$$

$$x = 7.2$$

Hence, 7.2 is the decimal number is multiplied by itself and its product is 51.84.

112. Find the decimal fraction which when multiplied by itself gives 84.64.

Solution:

Suppose decimal fraction be 'a'.

We know that, $x \times x = x^2$

So,

$$x \times x = 84.64$$

$$x^2 = 84.64$$

$$x = \sqrt{84.64}$$

The required number is found out by using long division method.

	9 .2	
9	84 .64	
	81 :	
142	3 64	
	3 64	
	0	

Then,

$$x = \sqrt{84.64}$$

$$x = 9.2$$

Hence, 9.2 is the decimal fraction is multiplied by itself and its product is 84.64.

113. A farmer wants to plough his square field of side 150m. How much area will he have to plough?

Solution:

According to the question,

Length of the side of square field = 150 m

As we know that, area of square = side $\times$ side

So, area of the square field = 150×150

$$= 22,500 \text{ m}^2$$

Hence, the farmer have to plough 22,500 m² area.

114. What will be the number of unit squares on each side of a square graph paper if the total number of unit squares is 256?

Solution:

According to the question,

The total number of unit squares is = 256.

Suppose the number be 'x'.

We know that, $x \times x = x^2$

So,

$$x \times x = 256$$

$$x^2 = 256$$

The required number is found out by using long division method.

$$\begin{array}{r} 16 \\ 16 \overline{) 256} \\ \underline{16} \\ 96 \\ \underline{96} \\ 0 \end{array}$$

$$x = \sqrt{256}$$

$$x = \sqrt{16^2}$$

$$x = 16$$

Hence, the number of unit squares are 16.

115. If one side of a cube is 15m in length, find its volume.

Solution:

According to the question,

Length of one side of the a cube = 15 m

As we know that, volume of cube = (side)³

$$= 15^3$$

$$= 15 \times 15 \times 15$$

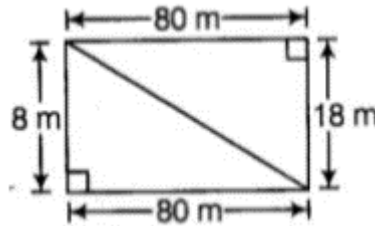
$$= 3375 \text{ m}^3$$

Hence, the volume of cube is 3375 m³.

116. The dimensions of a rectangular field are 80m and 18m. Find the length of its diagonal.

Solution:

According to the question,



Length of rectangular field = 80 m

Breadth of rectangular field = 18 m

Then, length of diagonal = $\sqrt{\text{length}^2 + \text{breadth}^2}$

$$= \sqrt{80^2 + 18^2}$$

$$= \sqrt{6400 + 324}$$

$$= \sqrt{6724}$$

$$= \sqrt{82^2}$$

$$= 82 \text{ m}$$

Hence, the Length of diagonal is 82 m.

117. Find the area of a square field if its perimeter is 96m.

Solution:

Given: perimeter of the square field = 96 m

As we know that, perimeter of square = $4 \times \text{side}$

Then, $96 = 4 \times \text{side}$

$$\text{Side} = \frac{96}{4}$$

$$\text{Side} = 24 \text{ m}$$

So, the length of the side of square = 24 m

Now, area of the square field = $(\text{side})^2$

$$= 24^2$$

$$= 576 \text{ m}^2$$

Hence, the area of the square field is 576m^2 .

118. Find the length of each side of a cube if its volume is 512 cm^3 .

Solution:

Given:

$$\text{Volume of the cube} = 512 \text{ cm}^3$$

$$\text{As we know that, Volume of cube} = \text{side}^3$$

$$512 = \text{side}^3$$

By taking cube root on both the side, get:

$$\sqrt[3]{512} = \text{side}$$

$$\text{Side} = \sqrt[3]{8 \times 8 \times 8}$$

$$\text{Side} = \sqrt[3]{8^3}$$

$$\text{Side} = 8 \text{ cm}$$

Hence, the length of each side of a cube is 8 cm.

119. Three numbers are in the ratio 1:2:3 and the sum of their cubes is 4500. Find the numbers.

Solution:

Suppose the three number be x, 2x, 3x

Given: Sum of cube of three numbers is 4500.

$$x^3 + (2x)^3 + (3x)^3 = 4500$$

$$x^3 + 8x^3 + 27x^3 = 4500$$

$$36x^3 = 4500$$

$$x^3 = \frac{4500}{36}$$

$$x^3 = 125$$

$$x = \sqrt[3]{125}$$

$$x = \sqrt[3]{5 \times 5 \times 5}$$

$$x = \sqrt[3]{5^3}$$

$$x = 5$$

So, the numbers are, $x = 5$,

$$2a = 2 \times 5 = 10$$

$$3a = 3 \times 5 = 15$$

120. How many square meters of carpet will be required for a square room of side 6.5 m to be carpeted?

Solution:

Given: Side of square room = 6.5 m

So, area of square room = 6.5^2

$$= 6.5 \times 6.5$$

$$= 42.25 \text{ m}^2$$

121. Find the side of a square whose area is equal to the area of a rectangle with sides 6.4m and 2.5m.

Solution:

Given,

Length of rectangle = 6.4 m

Breadth of rectangle = 2.5 m

So, area of rectangle = length $\times$ breadth

$$= 6.4 \times 2.5$$

$$= 16 \text{ m}^2$$

And also it's given in the question that area of square is equal to the area of rectangle.

Suppose side of square be 'x'.

Area of square = Area of rectangle

$$x \times x = 16$$

$$x^2 = 16$$

By taking square root on both side

$$x = \sqrt{16}$$

$$x = \sqrt{(4 \times 4)}$$

$$x = \sqrt{4^2}$$

$$x = 4 \text{ m}$$

We know that, area of square = side²

$$4 = \text{side}^2$$

By taking square root on both side

$$\text{Side} = \sqrt{4}$$

$$\text{Side} = \sqrt{2 \times 2}$$

$$\text{Side} = \sqrt{2^2}$$

$$\text{Side} = 2\text{m}$$

Hence, the side of square is 2m.

122. Difference of two perfect cubes is 189. If the cube root of the smaller of the two numbers is 3, find the cube root of the larger number.

Solution:

Given: Difference of two perfect cubes = 189

The cube root of the smaller of the two numbers = 3

So, cube of smaller number = 3^3

$$= 3 \times 3 \times 3$$

$$= 27$$

Suppose the cube root of larger number be x^3 .

So, as per the condition given in the question,

$$x^3 - 27 = 189$$

$$x^3 = 189 + 27$$

$$x^3 = 216$$

By taking cube root on both side,

$$x = \sqrt[3]{216}$$

$$x = \sqrt[3]{6 \times 6 \times 6}$$

$$x = \sqrt[3]{6^3}$$

$$x = 6$$

Hence, the cube root of the larger number is 6.

123. Find the number of plants in each row if 1024 plants are arranged so that number of plants in a row is the same as the number of rows.

Solution:

Given: Total number of plants arranged = 1024

Suppose number of plants in each row be 'x'.

So, as per the condition given in the question,

Total number of plants = $x \times x$

$$1024 = x^2$$

By taking square root on both side,

$$x = \sqrt{1024}$$

$$x = \sqrt{32 \times 32}$$

$$x = \sqrt{32^2}$$

$$x = 32$$

Hence, the number of plants in each row is 32.

124. A hall has a capacity of 2704 seats. If the number of rows is equal to the number of seats in each row, then find the number of seats in each row.

Solution:

Given: Total number of seats = 2704

Suppose number of seats in each row be 'x'.

As per the condition given in the question,

Total number of seats = $a \times a$

$$2704 = x^2$$

By taking square root on both side,

$$x = \sqrt{2704}$$

$$x = \sqrt{52 \times 52}$$

$$x = \sqrt{52^2}$$

$$x = 52$$

Hence, the number of seats in each row is 52.

125. A General wishes to draw up his 7500 soldiers in the form of a square. After arranging, he found out that some of them are left out. How many soldiers were left out?

Solution:

Given: Total number of soldiers = 7500

Let find number of soldiers left out by using long division method:

$$\begin{array}{r} 86 \\ 8 \overline{) 7500} \\ \underline{64} \\ 1100 \\ \underline{996} \\ 104 \end{array}$$

Hence, 104 soldiers were left out.

126. 8649 students were sitting in a lecture room in such a manner that there were as many students in the row as there were rows in the lecture room. How many students were there in each row of the lecture room?

Solution:

Given: Total number of students were sitting in a lecture room = 8649

Suppose number of students in each row be 'x'

According to the question,

Total number of students = $x \times x$

$$8649 = x^2$$

By taking square root on both side,

$$x = \sqrt{8649}$$

$$x = \sqrt{93 \times 93}$$

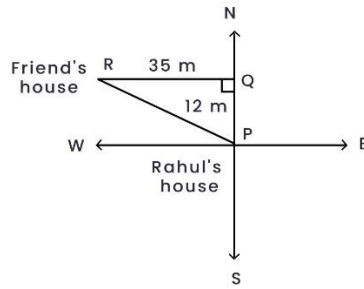
$$x = \sqrt{93^2}$$

$$x = 93$$

Hence, the number of students in each row is 93.

127. Rahul walks 12m north from his house and turns west to walk 35m to reach his friend's house. While returning, he walks diagonally from his friend's house to reach back to his house. What distance did he walk while returning?

Solution:



See the above figure:
Consider the triangle PQR.

As we know that, Pythagoras theorem,

$$PR^2 = PQ^2 + QR^2$$

$$PR^2 = 12^2 + 35^2$$

$$PR^2 = 144 + 1225$$

$$PR^2 = 1369$$

$$PR = \sqrt{1369}$$

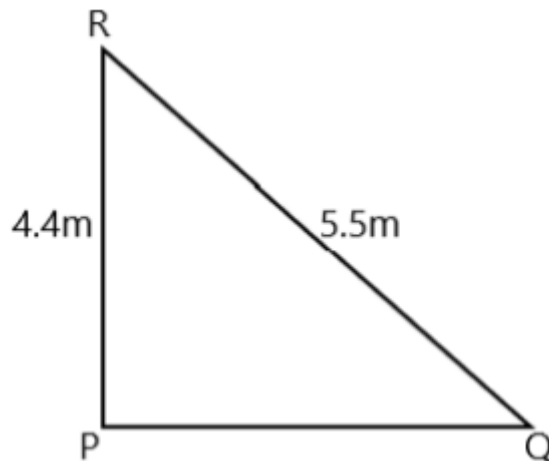
$$PR = 37\text{m}$$

Hence, Rahul walked 37m distance for returning to his house.

128. A 5.5m long ladder is leaned against a wall. The ladder reaches the wall to a height of 4.4m. Find the distance between the wall and the foot of the ladder.

Solution:

Consider the right angle triangle PQR



Given,

Long ladder that is $RQ = 5.5$ m

The ladder reaches the wall to a height, $PR = 4.4$ m

From Pythagoras theorem,

$$RQ^2 = RP^2 + PQ^2$$

$$5.5^2 = 4.4^2 + PQ^2$$

$$PQ^2 = 5.5^2 - 4.4^2$$

$$PQ = \sqrt{5.5^2 - 4.4^2}$$

$$PQ = \sqrt{30.25 - 19.36}$$

$$PQ = \sqrt{10.89}$$

$$\begin{array}{r|l} & 3 \quad .3 \\ 3 & 10 \quad .89 \\ & 9 \quad \vdots \\ \hline 63 & 1 \quad 89 \\ & 1 \quad 89 \\ \hline & 0 \end{array}$$

$$PQ = 3.3$$

Hence, the distance between the wall and the foot of the ladder is 3.3 m.

129. A king wanted to reward his advisor, a wise man of the kingdom. So he asked the wiseman to name his own reward. The wiseman thanked the king but said that he would ask only for some gold coins each day for a month. The coins were to be counted out in a pattern of one coin for the first day, 3 coins for the second day, 5 coins for the third day and so on for 30 days. Without making calculations, find how many coins will the advisor get in that month?

Solution:

According to the question,

Total coins the advisor get at the end = $1 + 3 + 5 + \dots$

The coins order is in the series of odd natural number,

So, the number of terms $(n) = 30$

So, sum of odd natural numbers = n^2

$$= 30^2$$

$$= 900$$

Hence, total coins the advisor get at the end is 900.

130. Find three numbers in the ratio 2:3:5, the sum of whose squares is 608.

Solution:

Suppose the three number be $2x$, $3x$, $5x$.

Given, sum of squares of three numbers is 608

$$(2x)^2 + (3x)^2 + (5x)^2 = 608$$

$$4x^2 + 9x^2 + 25x^2 = 608$$

$$38x^2 = 608$$

$$x^2 = 608/38$$

$$x^2 = 16$$

$$x = \sqrt{16}$$

$$x = \sqrt{4 \times 4}$$

$$x = \sqrt{4^2}$$

$$x = 4$$

Hence, the numbers are, $2x = 2 \times 4 = 8$

$$3x = 3 \times 4 = 12$$

$$5x = 5 \times 4 = 20$$

131. Find the smallest square number divisible by each one of the numbers 8, 9 and 10.

Solution:

Let's find out the LCM of 8, 9 and 10.

2	8, 9, 10
2	4, 9, 5
2	2, 9, 5
3	1, 9, 5
3	1, 3, 5
5	1, 1, 5
	1, 1, 1

So, LCM of 8, 9, and 10 = $2 \times 2 \times 2 \times 3 \times 3 \times 5$
= 360

Now, grouping the factors, $(2 \times 2) \times 2 \times (3 \times 3) \times 5$

Since, 2 and 5 is not able to make their pair.

So, to make it perfect square, 360 must be multiplied with $2 \times 5 = 10$

Then,

$$= 360 \times 10 = 3600$$

Hence, 3600 is least square number which is exactly divisible by 8, 9, and 10.

132. The area of a square plot is $101\frac{1}{400} \text{ m}^2$. Find the length of one side of the plot.

Solution:

According to the question,

$$\text{Area of a square plot} = 101\frac{1}{400} \text{ m}^2 \text{ m}^2 = \frac{40401}{400}$$

As we know that, area of square plot = side²

$$\frac{40401}{400} = \text{side}^2$$

By taking square root on both side, get:

$$\begin{aligned} &= \sqrt{\frac{40401}{400}} \\ &= \sqrt{\frac{201^2}{20^2}} \\ &= \frac{201}{20} \\ &= 10\frac{1}{20} \end{aligned}$$

Hence, the length of one side of the plot is $10\frac{1}{20} \text{ m}$.

133. Find the square root of 324 by the method of repeated subtraction.

Solution:

Let's find out the square root of 324 by using the repeated subtraction method.

Now, we subtract successive odd numbers starting from 1 as:

$$324 - 1 = 323$$

$$323 - 3 = 320$$

$$320 - 5 = 315$$

$$315 - 7 = 308$$

$$308 - 9 = 299$$

$$299 - 11 = 288$$

$$288 - 13 = 275$$

$$275 - 15 = 260$$

$$260 - 17 = 243$$

$$243 - 19 = 224$$

$$224 - 21 = 203$$

$$203 - 23 = 180$$

$$180 - 25 = 155$$

$$155 - 27 = 128$$

$$128 - 29 = 99$$

$$99 - 31 = 68$$

$$68 - 33 = 35$$

$$35 - 35 = 0$$

Here we see 324 reduces to 0 after subtracting 18 odd numbers.

Hence, the square root of 324 is 18.

134. Three numbers are in the ratio 2:3:4. The sum of their cubes is 0.334125. Find the numbers.

Solution:

135. Evaluate $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$

Solution:

Evaluating $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$ as follows:

$$\begin{aligned}
&= \sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064} \\
&= \sqrt[3]{3^3} + \sqrt[3]{(0.2)^3} + \sqrt[3]{(0.4)^3} \\
&= 3 + 0.2 + 0.4 \\
&= 3.6
\end{aligned}$$

Hence, the value of $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$ is equal to 3.6.

136. $\left\{ \left(5^2 + (12^2)^{\frac{1}{2}} \right) \right\}^3$

Solution:

Simplify $\left\{ \left(5^2 + (12^2)^{\frac{1}{2}} \right) \right\}^3$ as follows:

$$\begin{aligned}
\left\{ \left(5^2 + (12^2)^{\frac{1}{2}} \right) \right\}^3 &= \{(25 + 12)\}^3 \\
&= \{37\}^3 \\
&= 37 \times 37 \times 37 \\
&= 50,653
\end{aligned}$$

Hence, the value of $\left\{ \left(5^2 + (12^2)^{\frac{1}{2}} \right) \right\}^3$ is equal to 50,653.

137. $\left\{ \left(6^2 + (8^2)^{\frac{1}{2}} \right) \right\}^3$

Solution:

Simplify $\left\{ \left(6^2 + (8^2)^{\frac{1}{2}} \right) \right\}^3$ as follows:

$$\begin{aligned}
\left\{ \left(6^2 + (8^2)^{\frac{1}{2}} \right) \right\}^3 &= \{(36 + 8)\}^3 \\
&= \{44\}^3 \\
&= 44 \times 44 \times 44 \\
&= 85,184
\end{aligned}$$

Hence, the value of $\left\{ \left(6^2 + (8^2)^{\frac{1}{2}} \right) \right\}^3$ is equal to 85,184.

138. A perfect square number has four digits, none of which is zero. The digits from left to right have values that are: even, even, odd, even. Find the number.

Solution:

Suppose PQRS is perfect square,

Where, P = even, Q = even, R = odd, S = even

Hence, 8836 is the perfect square.

139. Put three different numbers in the circles so that when you add the numbers at the end of each line you always get a perfect square.



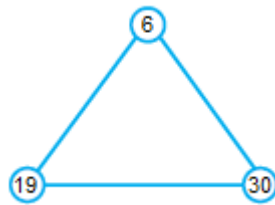
Solution:

6, 19 and 30 are the three numbers in which, when we add the end of each line we always get a perfect square.

$$6 + 19 = 25$$

$$6 + 30 = 36$$

$$19 + 30 = 49$$



140. The perimeters of two squares are 40 and 96 metres respectively. Find the perimeter of another square equal in area to the sum of the first two squares.

Solution:

Given:

Perimeters of first squares = 40m

Perimeter of second square = 96m

Suppose side of first square = x_1 and side of second square = x_2

As we know that, perimeter of first square = $4 \times \text{side}$

So, $40 = 4x_1$

$$x_1 = \frac{40}{4}$$

$$x_1 = 10 \text{ m}$$

So, perimeter of second square = $4 \times \text{side}$

$$96 = 4x_2$$

$$x_2 = \frac{96}{4}$$

$$x_2 = 24 \text{ m}$$

As per the condition given in the question area of another square is equal to the first two squares.

Let us assume y^2 be the area of another square.

So,

$$y^2 = x_1^2 + x_2^2$$

$$y^2 = 10^2 + 24^2$$

$$y^2 = 100 + 576$$

$$y = \sqrt{676}$$

$$y = 26 \text{ m}$$

As we know that Perimeter of another square = $4 \times \text{side}$

So,

$$= 4 \times y$$

$$= 4 \times 26$$

$$= 104 \text{ m}$$

Hence, the perimeter of another square is 104 m.

141. A three digit perfect square is such that if it is viewed upside down, the number seen is also a perfect square. What is the number?

(Hint: The digits 1, 0 and 8 stay the same when viewed upside down, whereas 9 becomes 6 and 6 becomes 9.)

Solution:

A three digit perfect square is such that if it is viewed upside down are 196 and 961.

142. 13 and 31 is a strange pair of numbers such that their squares 169 and 961 are also mirror images of each other. Can you find two other such pairs?

Solution:

Let first pair of numbers are 12 and 21

So, squares of numbers, $12^2 = 144$ and $21^2 = 441$

Again, let second pair of number is 102 and 201

So, squares of numbers, $102^2 = 10404$ and $201^2 = 40401$