Chapter - 7
Algebraic Expressions, Identities and Factorization

Exercise

In questions 1 to 33, there are four options out of which one is correct. Write the correct answer.

1. The product of a monomial and a binomial is a
(a) Monomial (b) Binomial (c) Trinomial (d) None of these

Solution:
Let binomial = x + y and monomial = 2x
So, the product of a monomial and a binomial = $2x\times(x+y)$

$$=2x^2+2xy$$

Hence, the product of a monomial and a binomial is a binomial.

2. In a polynomial, the exponents of the variables are always
(a) Integers (b) Positive integers (c) Non-negative integers (d) Non-positive integers

Solution:
As we know that in a polynomial, the exponents of the variables are always positive integers.

3. Which of the following is correct?
(a) $(a-b)^2=a^2+2ab-b^2$ (b) $(a-b)^2=a^2-2ab+b^2$
(c) $(a-b)^2=a^2-b^2$ (d) $(a+b)^2=a^2+2ab-b^2$

Solution:
According to the question:

$$(a-b)^2=(a-b)(a-b)$$
$$=a(a-b)-b(a-b)$$
$$=a\times a-a\times b-b\times a+b\times b$$
$$=a^2-ab-ab+b^2 \quad [\text{As } a\times b=b\times a]$$
$$=a^2-2ab+b^2$$

And:

$$
\begin{aligned}
(a+b)^2 &= (a+b)(a+b)\\
&= a(a+b)+b(a+b)\\
&= a\times a + a\times b + b\times a + b\times b\\
&= a^2+ab+ab+b^2 \qquad\qquad [\text{As } a\times b = b\times a]\\
&= a^2+2ab+b^2
\end{aligned}
$$

Hence, the correct option is (b).

4. The sum of –7pq and 2pq is

(a) –9pq (b) 9pq (c) 5pq (d) – 5pq

Solution:

The sum of –7pq and 2pq is calculated as:

$= -7pq + 2pq$

$= pq(-7+2)$

$= -5pq$

Hence, the correct option is (d).

5. If we subtract $-3x^2y^2$ from x^2y^2, then we get

(a) $-4x^2y^2$ (b) $-2x^2y^2$ (c) $2x^2y^2$ (d) $4x^2y^2$

Solution:

To subtract $-3x^2y^2$ from x^2y^2 as follows:

$x^2y^2 - (-3x^2y^2) = x^2y^2 + 3x^2y^2$

$= (1+3)\, x^2y^2$

$= 4x^2y^2$

Hence, the correct option is (d).

6. Like term as $4m^3n^2$ is

(a) $4m^2n^2$ (b) $-6m^3n^2$ (c) $6pm^3n^2$ (d) $4m^3n$

Solution:

The like term as $4m^3n^2$ is $-6m^3n^2$ because it contains the same literal factor m^3n^2.

7. Which of the following is a binomial?

(a) 7 × a + a (b) $6a^2 + 7b + 2c$ (c) 4a × 3b × 2c (d) $6(a^2 + b)$

Solution:

As we know that binomials are algebraic expressions consisting of two unlike terms.

From option (d);

$6\left(a^2+b\right)=6a^2+6b$

Hence, the correct option is (d).

8. Sum of a –b +ab, b+c–bc and c – a – ac is
(a)2c+ab–ac–bc (b)2c–ab–ac–bc (c)2c+ab+ac+bc (d)2c–ab+ac+bc

Solution:
Sum of a – b + ab, b + c – bc and c – a – ac is calculated as follows:

$=\left(a-b+ab\right)+\left(b+c-bc\right)+\left(c-a-ac\right)$

$=a-b+ab+b+c-bc+c-a-ac$

$=2c+ab-ac-bc$

Hence, the correct option is (a)

9. Product of the following monomials 4p, – 7q^3, –7pq is
(a) 196 p^{2}q^4 (b) 196 pq^4 (c) – 196 p^{2}q^4 (d) 196 p^{2}q^3

Solution:
Product of the following monomials 4p, – 7q^3, –7pq is calculated as follows:

$=4p\times\left(-7q^3\right)\times\left(-7pq\right)$

$=4\times\left(-7\right)\times\left(-7\right)\times p\times q^3\times pq$

$=196p^2q^4$

Hence, the correct option is (a).

10. Area of a rectangle with length 4ab and breadth 6b^2 is
(a) 24a^{2}b^2 (b) 24ab^3 (c) 24ab^2 (d) 24ab

Solution:
The formula of area of a rectangle = Length $\times$Breadth

$=4ab\times 6b^2$

$=24ab^3$

Hence, the correct option is (b).

11. Volume of a rectangular box (cuboid) with length = 2ab, breadth = 3ac and height = 2ac is
(a) 12a^{3}bc^2 (b) 12a^{3}bc (c) 12a^{2}bc (d) 2ab +3ac + 2ac

Solution:
The formula of volume of a cuboid = Length$\times$Breadth$\times$Height

$= 2ab \times 3ac \times 2ab$

$= (2\times 3\times 2)\times ab\times ac\times ac$

$= 12a^3bc^2$

Hence, the correct option is (a).

12. Product of $6a^2 - 7b + 5ab$ and $2ab$ is
(a) $12a^3b - 14ab^2 + 10ab$ (b) $12a^3b - 14ab^2 + 10a^2b^2$
(c) $6a^2 - 7b + 7ab$ (d) $12a^2b - 7ab^2 + 10ab$

Solution:

Product of $6a^2 - 7b + 5ab$ and $2ab$ is calculated as follows:

Required product = $2ab\times(6a^2 - 7b + 5ab)$

$= 2ab\times(6a^2 - 7b + 5ab)$

$= 2ab\times 6a^2 + 2ab\times(-7b) + 2ab\times 5ab$

$= 12a^3b - 14ab^2 + 10a^2b^2$

Hence, the correct option is (b).

13. Square of $3x - 4y$ is
(a) $9x^2 - 16y^2$ (b) $6x^2 - 8y^2$ (c) $9x^2 + 16y^2 + 24xy$ (d) $9x^2 + 16y^2 - 24xy$

Solution:

Square of $3x - 4y$ is:

$3x - 4y = (3x - 4y)^2$ [Now, use the identity: $(a-b)^2 = a^2 - 2ab + b^2$]

So,

$(3x-4y) = (3x)^2 - 2\times 3x\times 4y + (4y)^2$

$= 9x^2 - 24xy + 16y^2$

Hence, the correct option is (d).

14. Which of the following are like terms?
(a) $5xyz^2$, $-3xy^2z$ (b) $-5xyz^2$, $7xyz^2$ (c) $5xyz^2$, $5x^2yz$ (d) $5xyz^2$, $x^2y^2z^2$

Solution:

As we know that the terms having same algebraic (literal) factors are called like term.

So, the term $-5xyz^2$, $7xyz^2$ are like terms.

Hence, the correct option is (b).

15. Coefficient of y in the term $\frac{-y}{3}$ is

(a) – 1 (b) – 3 (c) $\frac{-1}{3}$ (d) $\frac{1}{3}$

Solution:

The term $-\frac{y}{3}$ can be written as $-\frac{1}{3}\times y$.

Therefore, the coefficient of y is $-\frac{1}{3}$.

Hence, the correct option is (c).

16. $a^2 - b^2$ is equal to

(a) $(a - b)^2$ (b) (a – b) (a – b) (c) (a + b) (a – b) (d) (a + b) (a + b)

Solution:

The standard identity of $a^2 - b^2$ is equal to:

$a^2 - b^2 = (a+b)(a-b)$

Hence, the correct option is (b).

17. Common factor of 17abc, $34ab^2$, $51a^2b$ is

(a) 17abc (b) 17ab (c) 17ac (d) $17a^2b^2c$

Solution:

Common factor of 17abc, $34ab^2$, $51a^2b$ is calculated as follows:

$17abc = 17\times a\times b\times c$

$34ab^2 = 2\times 17\times a\times b\times b$

$51a^2b = 3\times 17\times a\times b\times c$

So, common factor is $17\times a\times b = 17ab$.

Hence, the correct option is (b).

18. Square of 9x – 7xy is

(a) $81x^2 + 49x^2y^2$ (b) $81x^2 - 49x^2y^2$

(c) $81x^2 + 49x^2y^2 - 126x^2y$ (d) $81x^2 + 49x^2y^2 - 63x^2y$

Solution:

Square of 9x – 7xy is $(9x-7xy)^2$.

Now, $(9x-7xy)^2 = (9x)^2 - 2\times 9x\times 7xy + (7xy)^2$ [Using the identity: $(a-b)^2 = a^2 - 2ab + b^2$]

$= 81x^2 - 126x^2y + 49x^2y^2$

$= 81x^2 + 49x^2y^2 - 126x^2y$

Hence, the correct option is (c).

19. Factorised form of 23xy – 46x + 54y – 108 is
(a) (23x + 54) (y – 2) **(b) (23x + 54y) (y – 2)**
(c) (23xy + 54y) (– 46x – 108) **(d) (23x + 54) (y + 2)**

Solution:
Consider the expression:
Now, factorised 23xy – 46x + 54y – 108 as follows:
$= 23x(y-2)+54(y-2)$ [Taking common out in I and II expressions]
$=(y-2)(23x+54)$ [Taking (y-2) common]
$=(23x+54)(y-2)$
Hence, the correct option is (a).

20. Factorised form of r^2 –10r+21 is
(a) (r–1)(r–4) **(b) (r–7) (r–3)** **(c) (r–7) (r+3)** **(d) (r+7) (r+3)**

Solution:
Consider the expression:
$r^2 - 10r + 21$
Now, factorised the above expression as follows:
$r^2 - 10r + 21 = r^2 - 7r - 3r + 21$ [By splitting the middle term, So that the product of their numerical coefficient is equal constant term]
$= r(r-7) - 3(r-7)$
$= (r-7)(r-3)$
Hence, the correct option is (b).

21. Factorised form of p^2 – 17p – 38 is
(a) (p–19)(p+2) **(b) (p–19)(p–2)** **(c) (p+19)(p+2)** **(d) (p+19)(p – 2)**

Solution:
Consider the expression:
$p^2 - 17p - 38$
Now, factorised the above expression as follows:
$p^2 - 17p - 38 = p^2 - 19p + 2p - 38$ [By splitting the middle term, So that the product of their numerical coefficient is equal constant term]
$= p(p-19) + 2(p-19)$
$= (p-19)(p+2)$
Hence, the correct option is (a).

22. On dividing $57p^2qr$ by 114pq, we get

(a) $\frac{1}{4}pr$ (b) $\frac{3}{4}pr$ (c) $\frac{1}{2}pr$ (d) 2pr

Solution:

According to the question:

$$\frac{57p^2qr}{114pq} = \frac{57 \times p \times p \times q \times r}{114 \times p \times q}$$

$$= \frac{57}{114}pr$$

$$= \frac{1}{2}pr$$

Hence, the correct option is (c).

23. On dividing $p(4p^2 – 16)$ by $4p(p – 2)$, we get

(a) 2p + 4 (b) 2p – 4 (c) p + 2 (d) p – 2

Solution:

According to the question:

$$\frac{p(4p^2-16)}{4p(p-2)} = \frac{4p(p^2-4)}{4p(p-2)}$$

$$= \frac{(p^2-2^2)}{(p-2)}$$

$$= \frac{(p-2)(p+2)}{(p-2)} \qquad \left[As\ (a^2-b^2)=(a+b)(a-b)\right]$$

$$= p+2$$

Hence, the correct option is (c).

24. The common factor of 3ab and 2cd is

(a) 1 (b) – 1 (c) a (d) c

Solution:

There is no common factor of 3ab and 2cd except 1.

Hence, the correct option is (a).

25. An irreducible factor of $24x^2y^2$ is

(a) x^2 (b) y^2 (c) x (d) 24x

Solution:

As we know that an irreducible factor is a factor which can't be expressed further as a product of factors. So,

$24x^2y^2 = 2\times 2\times 2\times 2\times 3\times x\times x\times y\times y$

Therefore, an irreducible factor is x.

Hence, the correct option is (c).

26. Number of factors of $(a + b)^2$ is

(a) 4 (b) 3 (c) 2 (d) 1

Solution:

$(a + b)^2$ can be written as (a+b)(a+b).

Therefore, the number of factor is 2.

Hence, the correct option is (c).

27. The factorised form of 3x – 24 is

(a) 3x × 24 (b) 3 (x – 8) (c) 24 (x – 3) (d) 3(x – 12)

Solution:

The factorised form of 3x – 24 is = 3(x-8)

Hence, the correct option is (b).

28. The factors of $x^2 – 4$ are

(a) (x–2), (x–2) (b) (x+2), (x–2) (c) (x+2), (x+2) (d) (x–4), (x–4)

Solution:

The factors of $x^2 – 4$ are:

$$x^2 - 4 = x^2 - 2^2$$

$$= (x+2)(x-2) \qquad [\text{As } (a^2 - b^2) = (a+b)(a-b)]$$

Hence, the correct option is (b).

29. The value of $(– 27x^2y) \div (–9xy)$ is

(a) 3xy (b) – 3xy (c) – 3x (d) 3x

Solution:

The value of $(– 27x^2y) \div (–9xy)$ is calculated as follows:

$$\frac{-27x^2y}{-9xy} = \frac{-3\times 9\times x\times x\times y}{-9xy}$$

$$= 3x$$

Hence, the correct option is (d).

30. The value of $(2x^2 + 4) \div 2$ is

(a) $2x^2 + 2$ (b) $x^2 + 2$ (c) $x^2 + 4$ (d) $2x^2 + 4$

Solution:

The value of $(2x^2 + 4) \div 2$ is $= \frac{(2x^2+4)}{2}$

$= \frac{2(x^2+2)}{2}$

$= x^2 + 2$

Hence, the correct option is (b).

31. The value of $(3x^3 + 9x^2 + 27x) \div 3x$ is
(a) $x^2 + 9 + 27x$ (b) $3x^3 + 3x^2 + 27x$ (c) $3x^3 + 9x^2 + 9$ (d) $x^2 + 3x + 9$

Solution:

The value of $(3x^3 + 9x^2 + 27x) \div 3x$ is $= \frac{3x^3+9x^2+27x}{3x}$

$= \frac{3x^3+9x^2+27x}{3x}$

$= \frac{3x(x^2+3x+9)}{3x}$

$= x^2 + 3x + 9$

Hence, the correct option is (d).

32. The value of $(a + b)^2 + (a – b)^2$ is
(a) $2a + 2b$ (b) $2a – 2b$ (c) $2a^2 + 2b^2$ (d) $2a^2 – 2b^2$

Solution:

The value of $(a + b)^2 + (a – b)^2$ is calculated as follows:

$(a + b)^2 + (a – b)^2 = a^2 + b^2 + 2ab + a^2 + b^2 - 2ab$

$= 2a^2 + 2b^2$

Hence, the correct option is (c).

33. The value of $(a + b)^2 – (a – b)^2$ is
(a) 4ab (b) – 4ab (c) $2a^2 + 2b^2$ (d) $2a^2 – 2b^2$

Solution:

The value of $(a + b)^2 – (a – b)^2$ is calculated as follows:

$$(a + b)^2 - (a - b)^2 = a^2 + b^2 + 2ab - (a^2 + b^2 - 2ab)$$
$$= a^2 + b^2 + 2ab - a^2 - b^2 + 2ab$$
$$= 4ab$$

Hence, the correct option is (a).

In questions 34 to 58, fill in the blanks to make the statements true:

34. The product of two terms with like signs is a _________ term.

Solution:

The product of two terms with like signs is a positive term.
For example: the product of 2x is 3y is,
$= 2x \times 3y$
$= 6xy$

35. The product of two terms with unlike signs is a ________ term.

Solution:

The product of two terms with unlike signs is a negative term.
For example: the product of -2x is 3y is,
$= -2x \times 3y$
$= -6xy$

36. a (b + c) = ax ____ × ax _____.

Solution:

a(b + c) = $a \times \underline{b} + a \times \underline{c}$ [By using left distribution law]
$= ab + ac$

37. (a – b) _________ = $a^2 - 2ab + b^2$

Solution:

As we know that: (a – b) $\underline{(a - b)} = (a - b)^2$ = $a^2 - 2ab + b^2$

38. $a^2 - b^2$ = (a + b) __________.

Solution:

As we know that: $a^2 - b^2$ = (a + b) $\underline{(a - b)}$

39. $(a – b)^2$ + ____________ = $a^2 – b^2$

Solution:

As we know that: $\left(a-b\right)^2 = a^2 + b^2 - 2ab$

Now, adding $2ab - 2b^2$ both sides in the above identity, get:

$\left(a-b\right)^2 + 2ab - 2b^2 = a^2 + b^2 - 2ab + 2ab - 2b^2$

$\left(a-b\right)^2 + \underline{2ab - 2b^2} = a^2 - b^2$

40. $(a + b)^2 – 2ab$ = ___________ + ____________

Solution:

$(a + b)^2 – 2ab = a^2 + b^2 + 2ab - 2ab$ [Using the identity: $\left(a-b\right)^2 = a^2 + b^2 - 2ab$]

$= \underline{a^2} + \underline{b^2}$

41. (x + a) (x + b) = x^2 + (a + b) x + ________.

Solution:

(x + a) (x + b) = $x^2 + bx + ax + ab$

$= x^2 + \left(a+b\right)x + \underline{ab}$

42. The product of two polynomials is a ________.

Solution:

As the product of two polynomials is again a <u>polynomial</u>.

43. Common factor of ax^2 + bx is __________.

Solution:

Common factor of ax^2 + bx is <u>x (ax + b)</u>. [Taking x as a common]

44. Factorised form of 18mn + 10mnp is ________.

Solution:

Factorised form of 18mn + 10mnp is <u>2mn (9 + 5p)</u>. [Taking 2mn as common]

45. Factorised form of $4y^2$ – 12y + 9 is ________.

Solution:

Consider the expression:

$4y^2 - 12y + 9$

Now, factorised form of $4y^2 - 12y + 9$ will be calculated by using the splitting the middle term as follows:

$$4y^2 - 12y + 9 = (2y)^2 - 2 \times 2y \times 3 + 3^2$$

$$= (2y-3)^2 \qquad [\text{As}(a-b)^2 = a^2 - 2ab + b^2]$$

$$= (2y-3)(2y-3)$$

Hence, Factorised form of $4y^2 - 12y + 9$ is $(2y-3)(2y-3)$.

46. $38x^3y^2z \div 19xy^2$ is equal to _________.

Solution:

Consider the expression:

$38x^3y^2z \div 19xy^2$

Now, simplify the above expression as follows:

$$\frac{38x^3y^2z}{19xy^2} = \frac{2 \times 19 \times x \times x^2y^2z}{19xy^2}$$

$$= 2x^2z$$

Hence, $38x^3y^2z \div 19xy^2$ is equal to $2x^2z$.

47. Volume of a rectangular box with length 2x, breadth 3y and height 4z is _________.

Solution:

The formula of the volume of the rectangular box = Length x Breadth x Height

So, the volume of the rectangular box is:

= 2x x 3y x 4z

= (2 x 3 x 4) xyz

= 24 xyz

Hence, volume of a rectangular box with length 2x, breadth 3y and height 4z is 24xyz.

48. $67^2 - 37^2 = (67 - 37) \times$ ________ = _________.

Solution:

$$67^2 - 37^2 = (67-37)(67+37) \qquad [\text{As, } a^2 - b^2 = (a-b)(a+b)]$$

$$= 30 \times 104$$

$$= 3120$$

Hence, $67^2 - 37^2 = (67-37)\underline{(67+37)} = \underline{3120}$.

49. $103^2 - 102^2 =$ ________ $\times (103 - 102) =$ _________.

Solution:

$103^2 - 102^2 = (103+102)(103-102)$ **[As,** $a^2 - b^2 = (a-b)(a+b)$]

$= 205 \times 1$

$= 205$

Hence, $103^2 - 102^2 = (103+102)(103-102) = 205$.

50. Area of a rectangular plot with sides $4x^2$ and $3y^2$ is __________.

Solution:

The formula of the area of rectangle = Length $\times$ Breadth

So, area of a rectangular plot = $4x^2 \times 3y^2$

$= 4 \times 3x^2y^2$

$= 12x^2y^2$

Hence, area of a rectangular plot with sides $4x^2$ and $3y^2$ is $\underline{12x^2y^2}$.

51. Volume of a rectangular box with l = b = h = 2x is _________.

Solution:

The formula of the volume of the rectangular box is = $l \times b \times h$

$= 2x \times 2x \times 2x$

$= 8x^3$

Hence, volume of a rectangular box with l = b = h = 2x is $\underline{8x^3}$.

52. The coefficient in – 37abc is __________.

Solution:

As we know that the constant term involved in term of an algebraic expression is called the numerical coefficient of that term.

Hence, the coefficient in – 37abc is -37.

53. Number of terms in the expression $a^2 + bc \times d$ is ________.

Solution:

The expression $a^2 + bc \times d$ can be written as $a^2 + bcd$.

Hence, number of terms in the expression $a^2 + bc \times d$ is 2.

54. The sum of areas of two squares with sides 4a and 4b is _______.

Solution:

As we know that: Area of a square = $(\text{Side})^2$

So, area of the square whose one side is 4a = $(4a)^2 = 16a^2$

And are of the square with side 4b = $(4b)^2 = 16b^2$

Hence, the sum of areas = $16a^2 + 16b^2 = \underline{16(a^2 + b^2)}$.

55. The common factor method of factorisation for a polynomial is based on ___________ property.

Solution:

The common factor method of factorisation for a polynomial is based on Distributive property.

56. The side of the square of area $9y^2$ is __________.

Solution:

As we know that: Area of a square = $(\text{Side})^2$

$9y^2 = (\text{Side})^2$

Side = $\sqrt{9y^2}$

$= 3y$

Hence, the side of the square of area $9y^2$ is $\underline{3y}$.

57. On simplification $\frac{3x+3}{3}$ = _________.

Solution:

On simplification $\frac{3x+3}{3} = \frac{3x}{3} + \frac{3}{3} = \underline{x+1}$.

58. The factorisation of 2x + 4y is __________.

Solution:

The factorisation of 2x + 4y is 2(x + 2y).

In questions 59 to 80, state whether the statements are True (T) or False (F):

59. $(a + b)^2 = a^2 + b^2$

Solution:
As we know that $(a+b)^2 = a^2 + b^2 + 2ab$.
Hence, the given statement is false.

60. (a – b)^2 = a^2 – b^2

Solution:
As we know that $(a+b)^2 = a^2 + b^2 - 2ab$.
Hence, the given statement is false.

61. (a + b) (a – b) = a^2 – b^2

Solution:
As we know that: $(a+b)(a-b) = a^2 - b^2$.
Hence, the given statement is false.

62. The product of two negative terms is a negative term.

Solution:
As we know that the product of two negative terms is always a positive term, i.e. (-) x (-) = (+).
Hence, the given statement is false.

63. The product of one negative and one positive term is a negative term.

Solution:
As we know that when we multiply a negative term by a positive term, the resultant will be a negative term, i-e. (-) x (+) = (-).
Hence, the given statement is true.

64. The coefficient of the term – 6x^{2}y^2 is – 6.

Solution:
As we can see that the coefficient of the term – $6x^2y^2$ is – 6.
Hence, the given statement is true.

65. p^{2}q + q^{2}r + r^{2}q is a binomial.

Solution:
As we can see that the given expression contains three unlike terms, so it is a trinomial.

Hence, the given statement is false.

66. The factors of $a^2 – 2ab + b^2$ are (a + b) and (a + b).

Solution:

As we know that: $(a-b)^2 = a^2 - 2ab + b^2 = (a+b)(a-b)$.

Hence, the given statement is false.

67. h is a factor of 2π (h + r).

Solution:

As we can see that the given expression has only two factor 2π and (h + r).

Hence, the given statement is false.

68. Some of the factors of $\frac{n^2}{2} + \frac{n}{2}$ are $\frac{1}{2}$ n and (n + 1).

Solution:

The factor of $\frac{n^2}{2} + \frac{n}{2}$ is calculated as:

$$\frac{n^2}{2} + \frac{n}{2} = \frac{1}{2}n(n+1)$$

So, the factors of $\frac{n^2}{2} + \frac{n}{2}$ are $\frac{1}{2}$ n and (n + 1).

Hence, the given statement is false.

69. An equation is true for all values of its variables.

Solution:

As equation is true only for some values of its variables, that is 2x – 4= 0 is true, only for x =2.

Hence, the given statement is false.

70. $x^2 + (a + b)x + ab = (a + b) (x + ab)$

Solution:

$x^2 + (a + b)x$ + ab can be written as $(x+a)(x+b)$.

Hence, the given statement is false.

71. Common factor of $11pq^2$, $121p^2q^3$, $1331p^2q$ is $11p^2q^2$.

Solution:

Common factor of following term is calculated as follows:

$11pq^2 = 11\times p\times q\times q$

$121p^2q^3 = 11\times 11\times p\times p\times q\times q\times q$

$1331p^2q = 11\times 11\times 11\times p\times p\times q$

So, the common factor of the following term is $11p^2q^2$.

Hence, the given statement is false.

72. Common factor of $12a^2b^2 + 4ab^2 – 32$ is 4.

Solution:

Common factor of following expression is calculated as follows:

$12a^2b^2 + 4ab^2 – 32 = 4(3a^2b^2 + ab^2 – 8)$

So, the common factor is 4.

Hence, the given statement is true.

73. Factorisation of $– 3a^2 + 3ab + 3ac$ is $3a (–a – b – c)$.

Solution:

Factorisation of $– 3a^2 + 3ab + 3ac$ is calculated as follows:

$– 3a^2 + 3ab + 3ac = 3a(-a+b+c)$

So, the factor of $– 3a^2 + 3ab + 3ac$ is $3a(-a+b+c)$.

Hence, the given statement is false.

74. Factorised form of $p^2 + 30p + 216$ is $(p + 18) (p – 12)$.

Solution:

Factorised form of $p^2 + 30p + 216$ by using the splitting the middle term is calculated as follows:

$p^2 + 30p + 216 = p^2 + 12p+18p + 216$

$= p (p+12) +18(p+12)$

$= (p+18) (p+12)$

So, the factorised form of $p^2 + 30p + 216$ is p+18) (p+12).

Hence the given statement is false.

75. The difference of the squares of two consecutive numbers is their sum.

Solution:

Suppose n and n+1 be any two consecutive numbers.

So, their sum = n + n + 1 = 2n +1

Now, the difference of their square:

$$(n+1)^2 - n^2 = n^2 + 1 + 2n - n^2 \quad [\text{As, } (a+b)^2 = a^2 + 2ab + b^2]$$
$$= 2n + 1$$
Hence, the given statement is true.

76. abc + bca + cab is a monomial.

Solution:
Since, abc + bca + cab =abc + abc + abc =3abc
So, the given expression is monomial.
Hence, the given statement is true.

77. On dividing $\frac{p}{3}$ by $\frac{3}{p}$, the quotient is 9.

Solution:
When $\frac{p}{3}$ dividing by $\frac{3}{p}$, get:

$$\frac{\frac{p}{3}}{\frac{3}{p}} = \frac{p}{3} \times \frac{p}{3}$$
$$= \frac{p^2}{9}$$

So, the quotient is $\frac{1}{9}p^2$.
Hence, the given statement is false.

78. The value of p for $51^2 - 49^2 = 100p$ is 2.

Solution:
The value of p is calculated as follows:
$51^2 - 49^2 = 100p$
$$(51+49)(51-49) = 110p \quad [\text{As } a^2 - b^2 = (a+b)(a-b)]$$
$$100 \times 2 = 100p$$
$$p = 2$$
So, the value of p is 2.
Hence, the given statement is true.

79. (9x – 51) ÷ 9 is x – 51.

Solution:
Consider the expression:

(9x – 51) ÷ 9

Now, simplify the above expression as follows:

$$\frac{9x-51}{9}=\frac{9x}{9}-\frac{51}{9}$$

$$=x-\frac{51}{9}$$

Hence, the given statement is false.

80. The value of (a + 1) (a – 1) (a^2 + 1) is a^4 – 1.

Solution:

The value of (a + 1) (a – 1) (a^2 + 1) is calculated as follows:

$$(a+1)\ (a-1)\ (a^2+1)=(a^2-1)(a^2+1)$$ [Using the identity: (a+b)(a-b)= a^2-b^2]

$$=(a^2)^2-1^2$$ [Again using the same identity]

$$=a^4-1$$

81. Add:

(i) $7a^2bc$, – $3abc^2$, $3a^2bc$, $2abc^2$

(ii) 9ax, + 3by – cz, – 5by + ax + 3cz

(iii) $xy^2z^2 + 3x^2y^2z - 4x^2yz^2$, – $9x^2y^2z + 3xy^2z^2 + x^2yz^2$

(iv) $5x^2 - 3xy + 4y^2 - 9$, $7y^2 + 5xy - 2x^2 + 13$

(v) $2p^4 - 3p^3 + p^2 - 5p + 7$, $-3p^4 - 7p^3 - 3p^2 - p - 12$

(vi) 3a (a – b + c), 2b (a – b + c)

(vii) 3a (2b + 5c), 3c (2a + 2b)

Solution:

(i) Adding $7a^2bc$, – $3abc^2$, $3a^2bc$, $2abc^2$ as follows:

$$7a^2bc+(-3abc^2)+3a^2bc+2abc^2=7a^2bc-3abc^2+3a^2bc+2abc^2$$

$$=(7a^2bc+3a^2bc)+(-3abc^2+2abc^2)$$ [Grouping like terms]

$$=10a^2bc+(-abc^2)$$

$$=10ax-2by+2cz$$

(ii) Adding *9ax*, + *3by* – *cz*, – *5by* + *ax* + *3cz* as follows:

$$(9ax+3by-cz)+(-5by+ax+3cz)=9ax+3by-cz-5by+ax+3cz$$

$$=(9ax+ax)+(3by-5by)+(-cz+3cz)$$ [Grouping like terms]

$$=10ax-2by+2cz$$

(iii) Adding $xy^2z^2 + 3x^2y^2z - 4x^2yz^2$, – $9x^2y^2z + 3xy^2z^2 + x^2yz^2$ as follows:

$$xy^2z^2 + 3x^2y^2z - 4x^2yz^2 + \left(-9x^2y^2z + 3xy^2z^2 + x^2yz^2\right) = xy^2z^2 + 3x^2y^2z - 4x^2yz^2 - 9x^2y^2z + 3xy^2z^2 + x^2yz^2$$

$= \left(xy^2z^2 + 3xy^2z^2\right) + \left(3x^2y^2z - 9x^2y^2z\right) + \left(-4x^2yz^2 + x^2yz^2\right)$ [Grouping like terms]

$= 4xy^2z^2 - 6x^2y^2z - 3x^2yz^2$

(iv) Adding $5x^2 - 3xy + 4y^2 - 9$, $7y^2 + 5xy - 2x^2 + 13$ as follows:

$$5x^2 - 3xy + 4y^2 - 9 + 7y^2 + 5xy - 2x^2 + 13 = 5x^2 - 3xy + 4y^2 - 9 + 7y^2 + 5xy - 2x^2 + 13$$

$$= \left(5x^2 - 2x^2\right) + \left(-3xy + 5xy\right) + \left(4y^2 + 7y^2\right) + \left(-9 + 13\right)$$

[Grouping like terms]

$$= 3x^2 - 2xy - 11y^2 + 4$$

(v) Adding $2p^4 - 3p^3 + p^2 - 5p + 7$, $-3p^4 - 7p^3 - 3p^2 - p - 12$ as follows:

$$2p^4 - 3p^3 + p^2 - 5p + 7 + \left(-3p^4 - 7p^3 - 3p^2 - p - 12\right) = 2p^4 - 3p^3 + p^2 - 5p + 7 - 3p^4 - 7p^3 - 3p^2 - p - 12$$

$= \left(2p^4 - 3p^4\right) + \left(-3p^3 - 7p^3\right) + \left(p^2 - 3p^2\right) + \left(-5p - p\right) + \left(7 - 12\right)$ [Grouping like terms]

$= -p^4 - 10p^3 - 2p^2 - 6p - 5$

(vi) Adding $3a\ (a - b + c)$, $2b\ (a - b + c)$ as follows:

$$3a\left(a - b + c\right) + 2b\left(a - b + c\right) = \left(3a^2 - 3ab + 3ac\right) + \left(2ab - 2b^2 + 2bc\right)$$

$= 3a^2 - 3ab + 2ab + 3ac + 2bc - 2b^2$ [Grouping like terms]

$$= 3a^2 - ab + 3ac + 2bc - 2b^2$$

(vii) Adding 3a (2b + 5c), 3c (2a + 2b) as follows:

$$3a\left(2b + 5c\right) + 3c\left(2a + 2b\right) = \left(6ab + 15ac\right) + \left(6ac + 6bc\right)$$

$$= 6ab + 15ac + 6ac + 6bc$$

$$= 6ab + 21ac + 6bc$$

82. Subtract:

(i) $5a^2b^2c^2$ from $-7a^2b^2c^2$

(ii) $6x^2 - 4xy + 5y^2$ from $8y^2 + 6xy - 3x^2$

(iii) $2ab^2c^2 + 4a^2b^2c - 5a^2bc^2$ from $-10a^2b^2c + 4ab^2c^2 + 2a^2bc^2$

(iv) $3t^4 - 4t^3 + 2t^2 - 6t + 6$ from $-4t^4 + 8t^3 - 4t^2 - 2t + 11$

(v) 2ab + 5bc – 7ac from 5ab – 2bc – 2ac + 10abc

(vi) 7p (3q + 7p) from 8p (2p – 7q)

(vii) $-3p^2 + 3pq + 3px$ from 3p (– p – a – r)

Solution:

(i) Subtracting $5a^2b^2c^2$ from $-7a^2b^2c^2$ as follows:

$$-7a^2b^2c^2 - 5a^2b^2c^2 = -12a^2b^2c^2$$

(ii) Subtracting $6x^2 - 4xy + 5y^2$ from $8y^2 + 6xy - 3x^2$ as follows:

$$8y^2+6xy-3x^2-\left(6x^2-4xy+5y^2\right)=8y^2+6xy-3x^2-6x^2+4xy-5y^2$$
$$=\left(8y^2-5y^2\right)+\left(6xy+4xy\right)-\left(3x^2+6x^2\right)$$
$$=3y^2+10xy-9x^2$$

(iii) Subtracting $2ab^2c^2 + 4a^2b^2c - 5a^2bc^2$ from $-10a^2b^2c + 4ab^2c^2 + 2a^2bc^2$ as follows:

$$\left[\begin{array}{l}-10a^2b^2c+4ab^2c^2+2a^2bc^2-\left(2ab^2c^2+4a^2b^2c-5a^2bc^2\right)=-10a^2b^2c+4ab^2c^2+\\2a^2bc^2-2ab^2c^2-4a^2b^2c+5a^2bc^2\end{array}\right]$$
$$=\left(-10a^2b^2c-4a^2b^2c\right)+\left(4ab^2c^2-2ab^2c^2\right)+\left(2a^2bc^2+5a^2bc^2\right)$$
$$=-14a^2b^2c+2ab^2c^2+7a^2bc^2$$

(iv) Subtracting $3t^4 - 4t^3 + 2t^2 - 6t + 6$ from $-4t^4 + 8t^3 - 4t^2 - 2t + 11$ as follows:

$$-4t^4+8t^3-4t^2-2t+11-\left(3t^4-4t^3+2t^2-6t+6\right)=\left[\begin{array}{l}-4t^2+8t^3-4t^2-2t\\+11-3t^4+4t^3-2t^2+6t-6\end{array}\right]$$
$$=\left(-4t^4-3t^4\right)+\left(8t^3+4t^3\right)+\left(-4t^2-2t^2\right)+\left(-2t+6t\right)+\left(11-6\right)$$
$$=7t^4+12t^3-6t^2+4t+5$$

(v) Subtracting $2ab + 5bc - 7ac$ from $5ab - 2bc - 2ac + 10abc$ as follows:

$$5ab-2bc-2ac+10abc-\left(2ab+5bc-7ac\right)=5ab-2bc-2ac+10abc-2ab-5bc+7ac$$
$$=\left(5ab-2ab\right)+\left(-2bc-5bc\right)+\left(-2ac+7ac\right)+10abc \quad \text{[Grouping like terms]}$$
$$=3ab-7bc+5ac+10abc$$

(vi) Subtracting $7p(3q + 7p)$ from $8p(2p - 7q)$ as follows:

$$8p\left(2p-7q\right)-7p\left(3q+7p\right)=16p^2-56pq-21pq-49p^2$$
$$=\left(16p^2-49p^2\right)+\left(-56pq-21pq\right) \quad \text{[Grouping like terms]}$$
$$=-33p^2-77pq$$

(vii) Subtracting $-3p^2 + 3pq + 3px$ from $3p(-p - a - r)$ as follows:

$$3p\left(-p-a-r\right)-\left(-3p^2p^2+3pq+3px\right)=-3p^2-3ap-3pr+3p^2-3pq-3px$$
$$=\left(-3p^2+3p^2\right)-3ap-3pr-3pq-3px \quad \text{[Grouping like terms]}$$
$$=-3ap-3pr-3pq-3px$$

83. Multiply the following:

(i) $-7pq^2r^3$, $-13p^3q^2r$

(ii) $3x^2y^2z^2$, 17xyz
(iii) $15xy^2$, $17yz^2$
(iv) $–5a^2bc$, 11ab, $13abc^2$
(v) $–3x^2y$, (5y – xy)
(vi) abc, (bc + ca)
(vii) 7pqr, (p – q + r)
(viii) $x^2y^2z^2$, (xy – yz + zx)
(ix) (p + 6), (q – 7)
(x) 6mn, 0mn
(xi) a, a^5, a^6
(xii) –7st, –1, – $13st^2$
(xiii) b^3, $3b^2$, $7ab^5$
(xiv) $-\frac{100}{9}rs$; $\frac{3}{4}r^3 s^2$
(xv) $(a^2 - b^2)$, $(a^2 + b^2)$
(xvi) (ab + c), (ab + c)
(xvii) (pq – 2r), (pq – 2r)
(xviii) $\left(\frac{3}{4}x - \frac{4}{3}y\right)$, $\left(\frac{2}{3}x + \frac{3}{2}y\right)$
(xix) $\frac{3}{2}p^2 + \frac{2}{3}q^2$, $(2p^2 - 3q^2)$
(xx) $(x^2 - 5x + 6)$, (2x + 7)
(xxi) $(3x^2 + 4x - 8)$, $(2x^2 - 4x + 3)$
(xxii) (2x – 2y – 3), (x + y + 5)

Solution:

(i) Multiplying $-7pq^2r^3$ to $13p^3q^2r$ as follows:

$$-7pq^2r^3 \times 13p^3q^2r = (-7)\times(-13)\,p^4q^4r^4$$
$$= 91p^4q^4r^4$$

(ii) Multiplying $3x^2y^2z^2$ to 17xyz as follows:

$$3x^2y^2z^2 \times 17xyz = (3\times 17)\,x^2y^2z^2 \times xyz$$
$$= 51x^3y^3z^3$$

(iii) Multiplying $15xy^2$ to $17yz^2$ as follows:

$$15xy^2 \times 17yz^2 = 15xy^2 \times 17yz^2$$
$$= (15\times 17)\,xy^2 \times yz^2$$
$$= 255xy^3z^2$$

(iv) Multiplying $-5a^2bc$, 11ab, and $13abc^2$ as follows:

$$-5a^2bc \times 11ab \times 13abc^2 = (-5 \times 11 \times 13)\,a^2bc \times ab \times abc^2$$
$$= -715a^4b^3c^3$$

(v) Multiplying $-3x^2y$ to $(5y - xy)$ as follows:

$$-3x^2y \times (5y\ -\ xy) = -3x^2y \times 5y + 3x^2y \times xy$$
$$= -15x^2y^2 + 3x^3y^2$$

(vi) Multiplying abc to (bc + ca) as follows:

$$abc \times (bc + ca) = abc \times bc + abc \times ca$$
$$= ab^2c^2 + a^2bc^2$$

(vii) Multiplying 7pqr to (p – q + r) as follows:

$$7pqr \times (p - q + r) = 7pqr \times p - 7pqr \times q + 7pqr \times r$$
$$= 7p^2qr - 7pq^2r + 7pqr^2$$

(viii) Multiplying $x^2y^2z^2$ to (xy – yz + zx) as follows:

$$x^2y^2z^2 \times (xy - yz + zx) = x^2y^2z^2 \times xy - x^2y^2z^2 \times yz + x^2y^2z^2 \times zx$$
$$= x^3y^3z^2 - x^2y^3z^3 + x^3y^2z^3$$

(ix) Multiplying (p + 6) to (q – 7) as follows:

$$(p+6) \times (q-7) = p(q-7) + 6(q-7)$$
$$= pq - 7p + 6q - 42$$

(x) Multiplying 6mn to 0mn as follows:

$$6mn \times 0mn = (6 \times 0)\,mn$$
$$= 0m^2n^2$$
$$= 0$$

(xi) Multiplying a, a^5 and a^6 as follows:

$$a \times a^5 \times a^6 = a^{1+5+6}$$
$$= a^{12}$$

(xii) Multiplying $-7\text{st}, -1$ and -13st^2 as follows:

$$-7\text{st} \times -1 \times -13\text{st}^2 = [-7 \times (-1) \times (-13)]\,st \times (st^2)$$
$$= -91s^2t^3$$

(xiii) Multiplying $\text{b}^3, 3\text{b}^2$ and 7ab^5 as follows:

$$b^3 \times 3b^2 \times 7ab^5 = (1\times 3\times 7) b^3 \times b^2 \times ab^5$$
$$= 21ab^{10}$$

(xiv) Multiplying $-\frac{100}{9}$rs to $\frac{3}{4}r^3s^2$ as follows:

$$-\frac{100}{9}rs \times \frac{3}{4}r^3s^2 = \left(\frac{-100}{9}\times\frac{3}{4}\right) rs \times r^3s^2$$
$$= -\frac{25}{3}r^4s^3$$

(xv) Multiplying $(a^2 - b^2)$ to $(a^2 + b^2)$ as follows:

$$(a^2 - b^2)(a^2 + b^2) = a^2(a^2 + b^2) - b^2(a^2 + b^2)$$
$$= a^4 + a^2b^2 - b^2a^2 - b^4$$
$$= a^4 - b^4$$

(xvi) Multiplying (ab + c) to (ab + c) as follows:

$$(ab + c)(ab + c) = ab(ab + c) + c(ab + c)$$
$$= a^2b^2 + abc + cab + c^2$$
$$= a^2b^2 + 2abc + c^2$$

(xvii) Multiplying (pq – 2r) to (pq – 2r) as follows:

$$(pq - 2r)(pq - 2r) = pq(pq - 2r) - 2r(pq - 2r)$$
$$= p^2q^2 - 2pqr - 2rpq + 4r^2$$
$$= p^2q^2 - 4pqr + 4r^2$$

(xviii) Multiplying $\left(\frac{3}{4}x - \frac{4}{3}y\right)$ to $\left(\frac{2}{3}x + \frac{3}{2}y\right)$ as follows:

$$\left(\frac{3}{4}x-\frac{4}{3}y\right) \text{to} \left(\frac{2}{3}x+\frac{3}{2}y\right)=\frac{3}{4}x\left(\frac{2}{3}x+\frac{3}{2}y\right)-\frac{4}{3}y\left(\frac{2}{3}x+\frac{3}{2}y\right)$$

$$=\frac{3}{4}\times\frac{2}{3}x^2+\frac{3}{4}\times\frac{3}{2}xy-\frac{4}{3}\times\frac{2}{3}yx-\frac{4}{3}\times\frac{3}{2}y^2$$

$$=\frac{1}{2}x^2+\frac{9}{8}xy-\frac{8}{9}xy-2y^2$$

$$=\frac{1}{2}x^2+\left(\frac{9}{8}-\frac{8}{9}\right)xy-2y^2$$

$$=\frac{1}{2}x^2+\left(\frac{81-64}{72}\right)xy-2y^2$$

$$=\frac{1}{2}x^2+\frac{17}{72}xy-2y^2$$

(xix) Multiplying $\left(\frac{3}{2}p^2+\frac{2}{3}q^2\right)$ to $\left(2p^2-3q^2\right)$ as follows:

$$\left(\frac{3}{2}p^2+\frac{2}{3}q^2\right)\times\left(2p^2-3q^2\right)=\frac{3}{2}p^2\left(2p^2-3q^2\right)+\frac{2}{3}q^2\left(2p^2-3q^2\right)$$

$$=\frac{3}{2}p^2\times 2p^2-\frac{9}{2}p^2q^2+\frac{4}{3}q^2p^2-2q^4$$

$$=3p^4+\left(\frac{4}{3}-\frac{9}{2}\right)p^2q^2-2q^4$$

$$=3p^4+\left(\frac{8-27}{6}\right)p^2q^2-2q^4$$

$$=3p^4-\frac{19}{6}p^2q^2-2q^4$$

(xx) Multiplying $\left(x^2-5x+6\right)$ to $\left(2x+7\right)$ as follows:

$$\left(x^2-5x+6\right)\left(2x+7\right)=x^2\left(2x+7\right)-5x\left(2x+7\right)+6\left(2x+7\right)$$

$$=2x^3+7x^2-10x^2-35x+12x+42$$

$$=2x^3-3x^2-23x+42$$

(xxi) Multiplying $\left(3x^2+4x-8\right)$ to $\left(2x^2-4x+3\right)$ as follows:

$$\left(3x^2+4x-8\right)\times\left(2x^2-4x+3\right)=3x^2\left(2x^2-4x+3\right)+4x\left(2x^2-4x+3\right)-8\left(2x^2-4x+3\right)$$

$$=6x^4-12x^3+9x^2+8x^3-16x^2+12x-16x^2+32x-24$$

$$=6x^4-12x^3+8x^3+9x^2-16x^2-16x^2+12x+32x-24$$

$$=6x^4-4x^3-23x^2+44x-24$$

(xxii) Multiplying $(2x-2y-3)$ to $(x+y+5)$ as follows:

$$(2x-2y-3)\times(x+y+5)=2x(x+y+5)-2y(x+y+5)-3(x+y+5)$$
$$=2x^2+2xy+10x-2yx-2y^2-10y-3x-3y-15$$
$$=2x^2+2xy-2yx+10x-3x-2y^2-10y-3y-15$$
$$=2x^2+7x-13y-2y^2-15$$

84. Simplify

(i) $(3x+2y)^2+(3x-2y)^2$

(ii) $(3x+2y)^2-(3x-2y)^2$

(iii) $\left(\frac{7}{9}a+\frac{9}{7}b\right)^2-ab$

(iv) $\left(\frac{3}{4}x-\frac{4}{3}y\right)^2+2xy$

(v) $(1.5p+1.2q)^2-(1.5p-1.2q)^2$

(vi) $(2.5m+1.5q)^2+(2.5m-1.5q)^2$

(vii) $(x^2-4)+(x^2+4)+16$

(viii) $(ab-c)^2+2abc$

(ix) $(a-b)(a^2+b^2+ab)-(a+b)(a^2+b^2-ab)$

(x) $(b^2-49)(b+7)+343$

(xi) $(4.5a+1.5b)^2+(4.5b+1.5a)^2$

(xii) $(pq-qr)^2+4pq^2r$

(xiii) $(s^2t+tq^2)^2-(2stq)^2$

Solution:

(i) Consider the expression:

$(3x+2y)^2+(3x-2y)^2$

Now, simplify the above expression as follows:

$$(3x+2y)^2+(3x-2y)^2=(3x)^2+(2y)^2+2\times3x\times2y+(3x)^2+(2y)^2-2\times3x\times2y$$

[Using the identity: $(a+b)^2=a^2+b^2+2ab$ and $(a-b)^2=a^2+b^2-2ab$]

$$=9x^2+4y^2+12xy+9x^2+4y^2-12xy$$
$$=(9x^2+9x^2)+(4y^2+4y^2)+12xy-12xy$$
$$=18x^2+8y^2$$

(ii) Consider the expression:

$(3x+2y)^2-(3x-2y)^2$

Now, simplify the above expression as follows:

$$(3x+2y)^2-(3x-2y)^2=\left[(3x+2y)+(3x-2y)\right]\left[(3x+2y)-(3x-2y)\right]$$

[Using the identity: $a^2 - b^2 = (a-b)(a+b)$]

$$= (3x+2y+3x-2y)(3x+2y-3x+2y)$$
$$= 6x \times 4y$$
$$= 24xy$$

(iii) Consider the expression:

$\left(\frac{7}{9}a + \frac{9}{7}b\right)^2 - ab$

Now, simplify the above expression as follows:

$\left(\frac{7}{9}a\right)^2 + \left(\frac{9}{7}b\right)^2 + 2\times\frac{7}{9}a\times\frac{9}{7}b - ab$ [Using the identity: $(a+b)^2 = a^2 + b^2 + 2ab$]

$$= \frac{49}{81}a^2 + \frac{81}{49}b^2 + 2ab - ab$$
$$= \frac{49}{81}a^2 + ab + \frac{81}{49}b^2$$

(iv) Consider the expression:

$\left(\frac{3}{4}x - \frac{4}{3}y\right)^2 + 2xy$

Now, simplify the above expression as follows:

$\left(\frac{3}{4}x - \frac{4}{3}y\right)^2 + 2xy = \left(\frac{3}{4}x\right)^2 + \left(\frac{4}{3}y\right)^2 - 2\times\frac{3}{4}x\times\frac{4}{3}y + 2xy$ [Using the identity: $(a-b)^2 = a^2 + b^2 - 2ab$]

$$= \frac{9}{16}x^2 + \frac{16}{9}y^2 - 2xy + 2xy$$
$$= \frac{9}{16}x^2 + \frac{16}{9}y^2$$

(v) Consider the expression:

$(1.5p + 1.2q)^2 - (1.5p - 1.2q)^2$

Now, simplify the above expression as follows:

$(1.5p + 1.2q)^2 - (1.5p - 1.2q)^2 = \left[(1.5p+1.2q)+(1.5p-1.2q)\right]\left[(1.5p+1.2q)-(1.5p-1.2q)\right]$

[Using the identity: $a^2 - b^2 = (a+b)(a-b)$]

$$= \left[(1.5p+1.5p)+(1.2q-1.2q)\right]\left[(1.5p-1.5p)+(1.2q+1.2q)\right]$$
$$= 3p \times 2.4q$$
$$= 7.2pq$$

(vi) Consider the expression:

$(2.5m + 1.5q)^2 + (2.5m - 1.5q)^2$

Now, simplify the above expression as follows:

$$(2.5m+1.5q)^2+(2.5m-1.5q)^2=(2.5m)^2+(1.5q)^2+2\times 2.5m\times 1.5q+(2.5m)^2+(1.5q)^2-2\times(2.5m)\times(1.5q)$$

[Using the identity: $(a+b)^2=a^2+b^2+2ab$ and $(a-b)^2=a^2+b^2-2ab$]

$$=6.25m^2+2.25q^2+6.25m^2+2.25q^2$$

$$=(6.25+6.25)m^2+(2.25+2.25)q^2$$

$$=12.5m^2+4.5q^2$$

(vii) Consider the expression:
$(x^2 – 4) + (x^2 + 4) + 16$
Now, simplify the above expression as follows:

$$(x^2-4)+(x^2+4)+16=x^2-4+x^2+4+16$$

$$=2x^2+16$$

(viii) Consider the expression:
$(ab – c)^2 + 2abc$
Now, simplify the above expression as follows:

$=(ab)^2+c^2-2abc+2abc$ [Using the identity: $(a-b)^2=a^2+b^2-2ab$]

$=a^2b^2+c^2$

(ix) Consider the expression:
$(a – b) (a^2 + b^2 + ab) – (a + b) (a^2 + b^2 – ab)$
Now, solve the above expression as follows:

$$\left\{\begin{matrix}(a-b)(a^2+b^2+ab)-(a+b)(a^2+b^2-ab)=a(a^2+b^2+ab)-b(a^2+b^2+ab)\\ -a(a^2+b^2-ab)-b(a^2+b^2-ab)\end{matrix}\right\}$$

$$=a^3+ab^2+a^2b-ba^2-b^3-ab^2-a^3-ab^2+a^2b-ba^2-b^3+ab^2$$

$$=(a^3-a^3)+(-b^3-b^3)+(ab^2-ab^2)+(a^2b-a^2b+a^2b-a^2b)$$

$$=0-2b^3+0+0+0$$

$$=-2b^3$$

(x) Consider the expression:
$(b^2 – 49) (b + 7) + 343$
Now, solve the above expression as follows:

$$(b^2-49)(b+7)+343=b^2(b+7)-49(b+7)+343$$

$$=b^3+7b^2-49b-343+343$$

$$=b^3-49b+7b^2$$

(xi) Consider the expression:

$(4.5a + 1.5b)^2 + (4.5b + 1.5a)^2$

Now, simplify the above expression as follows:

$$\left\{\begin{matrix}(4.5a + 1.5b)^2 + (4.5b + 1.5a)^2 = (4.5a)^2 + (1.5b)^2 + 2 \times 4.5a \times 1.5b \\ + (4.5b)^2 + (1.5a)^2 + 2 \times 4.5b \times 1.5a\end{matrix}\right\}$$

[Using the identity: $(a+b)^2 = a^2 + b^2 + 2ab$]

$$= 20.25a^2 + 2.25b^2 + 13.5ab + 20.25b^2 + 2.25a^2 + 13.5ab$$

$$= 40.5a^2 + 4.5b^2 + 27ab$$

(xii) Consider the expression:

$(pq - qr)^2 + 4pq^2r$

Now, simplify the above expression as follows:

$$(pq - qr)^2 + 4pq^2r = p^2q^2 + q^2r^2 - 2pq^2r + 4pq^2r$$

[Using the identity: $(a-b)^2 = a^2 + b^2 - 2ab$]

$$= p^2q^2 + q^2r^2 + 2pqr$$

(xiii) Consider the expression:

$(s^2t + tq^2)^2 - (2stq)^2$

Now, solve the above expression as follows:

$$(s^2t + tq^2)^2 - (2stq)^2 = (s^2t)^2 + (tq^2)^2 + 2 \times s^2t \times tq^2 - 4s^2t^2q^2$$

[Using the identity: $(a+b)^2 = a^2 + b^2 + 2ab$]

$$= s^4t^2 + t^2q^4 + 2s^2t^2q^2 - 4s^2t^2q^2$$

$$= s^4t^2 + t^2q^4 - 2s^2t^2q^2$$

85. Expand the following, using suitable identities.

(i) $(xy + yz)^2$

(ii) $(x^2y - xy^2)^2$

(iii) $\left(\frac{4}{5}a + \frac{5}{4}b\right)^2$

(iv) $\left(\frac{2}{4}x - \frac{3}{2}y\right)^2$

(v) $\left(\frac{4}{5}p + \frac{5}{3}\right)^2$

(vi) (x + 3) (x + 7)

(vii) (2x + 9) (2x – 7)

(viii) $\left(\frac{4x}{5} + \frac{y}{4}\right)\left(\frac{4x}{5} + \frac{3y}{4}\right)$

(ix) $\left(\frac{2x}{3} - \frac{2}{3}\right)\left(\frac{2x}{3} + \frac{2a}{3}\right)$

(x) (2x – 5y) (2x – 5y)

(xi) $\left(\frac{2a}{3} + \frac{b}{3}\right)\left(\frac{2a}{3} - \frac{b}{3}\right)$

(xii) $(x^2 + y^2)(x^2 - y^2)$

(xiii) $(a^2 + b^2)^2$

(xiv) $(7x + 5)^2$

(xv) $(0.9p - 0.5q)^2$

(xvi) $x^2y^2 = (xy)^2$

Solution:

(i) Consider the expression:

$(xy + yz)^2$

Now, simplify the above expression as follows:

$(xy+yz)^2 = (xy)^2 + (yz)^2 + 2\times xy\times yz$ [Using the identity: $(a+b)^2 = a^2+b^2+2ab$]

$$= x^2y^2 + y^2z^2 + 2xy^2z$$

(ii) Consider the expression:

$(x^2y - xy^2)^2$

Now, simplify the above expression as follows:

$\left(x^2y - xy^2\right)^2 = (xy)^2 + (yz)^2 + 2\times xy\times yz$ [Using the identity:

$(a-b)^2 = a^2+b^2-2ab$]

$$= x^2y^2 + y^2z^2 + 2xy^2z$$

(iii) Consider the expression:

$$\left(\frac{4}{5}a + \frac{5}{4}b\right)^2$$

Now, simplify the above expression as follows:

$$\left(\frac{4}{5}a + \frac{5}{4}b\right)^2 = \left(\frac{4}{5}a\right)^2 + \left(\frac{5}{4}b\right)^2 + 2\times\frac{4}{5}a\times\frac{5}{4}b$$

[Using the identity: $(a+b)^2 = a^2+b^2+2ab$]

$$= \frac{16}{25}a^2 + \frac{25}{16}b^2 + 2ab$$

(iv) Consider the expression:

$$\left(\frac{2}{4}x - \frac{3}{2}y\right)^2$$

Now, simplify the above expression as follows:

$$\left(\frac{2}{4}x-\frac{3}{2}y\right)^2=\left(\frac{2}{3}x\right)^2+\left(\frac{3}{2}y\right)^2-2\times\frac{2}{3}x\times\frac{3}{2}y$$

[Using the identity: $(a-b)^2=a^2+b^2-2ab$]

$$=\frac{4}{9}x^2+\frac{9}{4}y^2-2xy$$

(v) Consider the expression:

$$\left(\frac{4}{5}p+\frac{5}{3}q\right)^2$$

Now, simplify the above expression as follows:

$$\left(\frac{4}{5}p+\frac{5}{3}q\right)^2=\left(\frac{4}{5}p\right)^2+\left(\frac{5}{3}q\right)^2+2\times\frac{4}{5}p\times\frac{5}{3}q$$

[Using the identity: $(a+b)^2=a^2+b^2+2ab$]

$$=\frac{16}{25}p^2+\frac{25}{9}q^2+\frac{8}{3}pq$$

(vi) Consider the expression:

$$(x+3)(x+7)$$

Now, simplify the above expression as follows:

$$(x+3)(x+7)=x^2+(3+7)x+3\times7$$

[Using the identity: $(x+a)(x+b)=x^2+(a+b)x+ab$]

$$=x^2+10x+21$$

(vii) Consider the expression:

$$(2x+9)(2x-7)$$

Now, simplify the above expression as follows:

$$(2x+9)(2x-7)=(2x+9)\left[2x+(-7)\right]$$

$$=\left(2x^2\right)+\left[9+(-7)\right]2x+9\times(-7)$$

[Using the identity: $(x+a)(x+b)=x^2+(a+b)x+ab$]

$$=4x^2+4x-63$$

(viii) Consider the expression:

$$\left(\frac{4x}{5}+\frac{y}{4}\right)\left(\frac{4x}{5}+\frac{3y}{4}\right)$$

Now, simplify the above expression as follows:

$$\left(\frac{4x}{5}+\frac{y}{4}\right)\left(\frac{4x}{5}+\frac{3y}{4}\right)=\left(\frac{4x}{5}\right)^2+\left(\frac{y}{4}+\frac{3y}{4}\right)\frac{4x}{5}+\frac{y}{4}\times\frac{3y}{4}$$

[Using the identity: $(x+a)(x+b)=x^2+(a+b)x+ab$]

$$=\frac{16}{25}x^2+\frac{4xy}{5}+\frac{3y^2}{16}$$

(ix) Consider the expression:

$$\left(\frac{2x}{3}-\frac{2}{3}\right)\left(\frac{2x}{3}+\frac{2a}{3}\right)$$

Now, simplify the above expression as follows:

$$\left(\frac{2x}{3}-\frac{2}{3}\right)\left(\frac{2x}{3}+\frac{2a}{3}\right)=\left(\frac{2x}{3}\right)^2+\left(\frac{-2}{3}+\frac{2a}{3}\right)\frac{2x}{3}+\left(\frac{-2}{3}\times\frac{2a}{3}\right)$$

[Using the identity: $(x+a)(x+b)=x^2+(a+b)x+ab$]

$$=\frac{4x^2}{9}+\frac{2a-2}{3}\times\frac{2}{3}x-\frac{4}{9}a$$

$$=\frac{4x^2}{9}+\frac{4}{9}(a-1)x-\frac{4}{9}a$$

(x) Consider the expression:

$$(2x-5y)(2x-5y)$$

Now, simplify the above expression as follows:

$$(2x-5y)(2x-5y)=(2x-5y)^2$$

$=(4x)^2+(5y)^2-2\times 2x\times 5y$ [Using the identity: $(a-b)^2=a^2+b^2-2ab$]

$$=16x^2+25y^2-20xy$$

(xi) Consider the expression:

$$\left(\frac{2a}{3}+\frac{b}{3}\right)\left(\frac{2a}{3}-\frac{b}{3}\right)$$

Now, simplify the above expression as follows:

$$\left(\frac{2a}{3}+\frac{b}{3}\right)\left(\frac{2a}{3}-\frac{b}{3}\right)=\left(\frac{2a}{3}\right)^2-\left(\frac{b}{3}\right)^2$$

$=\frac{4}{9}a^2-\frac{1}{9}b^2$ [Using the identity: $(a+b)(a-b)=a^2-b^2$]

(xii) Consider the expression:

$$(x^2+y^2)(x^2-y^2)$$

Now, simplify the above expression as follows:

$(x^2+y^2)(x^2-y^2)=(x^2)^2-(y^2)^2$ [Using the identity: $(a+b)(a-b)=a^2-b^2$]

$$=x^4-y^4$$

(xiii) Consider the expression:

$\left(a^2+b^2\right)^2$

Now, simplify the above expression as follows:

$\left(a^2+b^2\right)^2=\left(a^2\right)^2+\left(b^2\right)^2+2a^2\times b^2$

$=a^4+b^4+2a^2b^2$ [Using the identity: $(a+b)^2=a^2+b^2+2ab$]

(xiv) Consider the expression:

$(7x+5)^2$

Now, simplify the above expression as follows:

$(7x+5)^2=(7x)^2+5^2+2\times 7x\times 5$

$=49x^2+25+70x$ [Using the identity: $(a+b)^2=a^2+b^2+2ab$]

(xv) Consider the expression:

$(0.9p-0.5q)^2$

Now, simplify the above expression as follows:

$(0.9p-0.5q)^2=(0.9p)^2+(0.5q)^2-2\times 0.9p\times 0.5q$

[Using the identity: $(a-b)^2=a^2+b^2-2ab$]

$=0.81p^2+0.25q^2-0.9pq$

(xvi) It is equation not exponents.

86. Using suitable identities, evaluate the following.

(i) $(52)^2$ **(ii) $(49)^2$**
(iii) $(103)^2$ **(iv) $(98)^2$**
(v) $(1005)^2$ **(vi) $(995)^2$**
(vii) 47 × 53 **(viii) 52 × 53**
(ix) 105 × 95 **(x) 104 × 97**
(xi) 101 × 103 **(xii) 98 × 103**
(xiii) $(9.9)^2$ **(xiv) 9.8 × 10.2**
(xv) 10.1 × 10.2 **(xvi) $(35.4)^2 – (14.6)^2$**
(xvii) $(69.3)^2 – (30.7)^2$ **(xviii) $(9.7)^2 – (0.3)^2$**
(xix) $(132)^2 – (68)^2$ **(xx) $(339)^2 – (161)^2$**
(xxi) $(729)^2 – (271)^2$

Solution:

(i) $(52)^2=(50+2)^2$

$=(50)^2+(2)^2+2\times 50\times 2$ [Using the identity: $(a+b)^2=a^2+b^2+2ab$]

$= 2500 + 4 + 200$
$= 2704$

(ii) $(49)^2 = (50-1)^2$
$= (50)^2 + (1)^2 - 2\times 50\times 2$ [Using the identity: $(a-b)^2 = a^2 + b^2 - 2ab$]
$= 2500 + 1 - 100$
$= 2401$

(iii) $(103)^2 = (100+3)^2$
$= (100)^2 + (3)^2 + 2\times 100\times 3$ [Using the identity: $(a+b)^2 = a^2 + b^2 + 2ab$]
$= 10000 + 9 + 600$
$= 10609$

(iv) $(98)^2 = (100-2)^2$
$= (100)^2 + (2)^2 - 2\times 100\times 2$ [Using the identity: $(a-b)^2 = a^2 + b^2 + 2ab$]
$= 10000 + 4 - 400$
$= 9604$

(v) $(1005)^2 = (1000+5)^2$
$= (1000)^2 + (5)^2 + 2\times 1000\times 5$ [Using the identity: $(a+b)^2 = a^2 + b^2 + 2ab$]
$= 1000000 + 25 + 10000$
$= 1010025$

(vi) $(995)^2 = (1000-5)^2$
$= (1000)^2 + (5)^2 - 2\times 1000\times 5$ [Using the identity: $(a-b)^2 = a^2 + b^2 - 2ab$]
$= 1000000 + 25 - 10000$
$= 990025$

(vii) $47\times 53 = (50-3)(50+3)$
$= (50)^2 - (3)^2$ [Using the identity: $(a-b)(a+b) = a^2 - b^2$]
$= 2500 - 9$
$= 2491$

(viii) $52\times 53 = (50+2)(50+3)$
$= (50)^2 + (2+3)\times 50 + 2\times 2$ [Using the identity: $(x+b)(x+b) = x^2 + (a+b)x + ab$]

$= 2500 + 250 + 6$

$= 2756$

(ix) $105 \times 95 = (100+5)(100-5)$

$= (100)^2 - (5)^2$ [Using the identity: $(a+b)(a-b) = a^2 - b^2$]

$= 10000 - 25$

$= 9975$

(x) $104 \times 97 = (100+4)(100-3)$

$= (100)^2 + (4-3) \times 100 + 4 \times (-3)$ [Using the identity: $(x+b)(x+b) = x^2 + (a+b)x + ab$]

$= 10000 + 100 - 12$

$= 10088$

(xi) $101 \times 103 = (100+1)(100+3)$

$= (100)^2 + (1+3) \times 100 + 3 \times 1$ [Using the identity: $(x+b)(x+b) = x^2 + (a+b)x + ab$]

$= 10000 + 400 + 3$

$= 10403$

(xii) $98 \times 103 = (100-2)(100+3)$

$= (100)^2 + (-2+3) \times 100 + (-2) \times 3$ [Using the identity: $(x+b)(x+b) = x^2 + (a+b)x + ab$]

$= 10000 + 100 - 6$

$= 10094$

(xiii) $(9.9)^2 = (10-0.1)^2$

$= (10)^2 + (0.1)^2 - 2 \times 10 \times 0.1$ [Using the identity: $(a-b)^2 = a^2 + b^2 - 2ab$]

$= 100 + 0.01 - 2$

$= 98.01$

(xiv) $(9.8) \times (10.2) = (10-0.2)(10+0.2)$

$= (10)^2 - (0.2)^2$ [Using the identity: $(a+b)(a-b) = a^2 - b^2$]

$= 100 - 0.04$

$= 99.96$

(xv) $(10.1) \times 10.2 = (10+0.1)(10+0.2)$

$= (10)^2 + (0.1+0.2) \times 10 + 0.1 \times 0.2$ [Using the identity: $(x+b)(x+b) = x^2 + (a+b)x + ab$]

$= 100 + 0.3 \times 10 + 0.02$

$= 103.02$

(xvi) $(35.4)^2 - (14.6)^2 = (35.4 + 14.6)(35.4 - 14.6)$

$= 50 \times 20.8$ [Using the identity: $(a+b)(a-b) = a^2 - b^2$]

$= 1040$

(xvii) $(68.3)^2 - (30.7)^2 = (69.3 + 30.7)(69.3 - 30.7)$

$= 100 \times 38.6$ [Using the identity: $(a+b)(a-b) = a^2 - b^2$]

$= 3860$

(xviii) $(9.7)^2 - (0.3)^2 = (9.7 + 0.3)(9.7 - 0.3)$

$= 10 \times 9.4$ [Using the identity: $a^2 - b^2 = (a+b)(a-b)$]

$= 94$

(xix) $(132)^2 - (68)^2 = (132 + 68)(132 - 68)$

$= 200 \times 64$ [Using the identity: $a^2 - b^2 = (a+b)(a-b)$]

$= 12800$

(xx) $(339)^2 - (161)^2 = (339 + 161)(339 - 161)$

$= 500 \times 178$ [Using the identity: $a^2 - b^2 = (a+b)(a-b)$]

$= 89000$

87. Write the greatest common factor in each of the following terms.
(i) – $18a^2$, 108a
(ii) $3x^2y$, $18xy^2$, – 6xy
(iii) 2xy, $-y^2$, $2x^2y$
(iv) l^2m^2n, lm^2n^2, l^2mn^2
(v) 21pqr, $-7p^2q^2r^2$, $49p^2qr$
(vi) qrxy, pryz, rxyz
(vii) $3x^3y^2z$, $-6xy^3z^2$, $12x^2yz^3$
(viii) $63p^2a^2r^2s$, – $9pq^2r^2s^2$, $15p^2qr^2s^2$, – $60p^2a^2rs^2$
(ix) $13x^2y$, 169xy
(x) $11x^2$, $12y^2$

Solution:

(i) Factor of – $18a^2$ and 108a will be:

$-18a^2 = -8 \times a \times a$

$108a = 18 \times 10 \times a$

So, the greatest common factor is 18a.

(ii) Factor of $3x^2y$, $18xy^2$, $-6xy$ will be:

$3x^2y = -3 \times x \times x \times y$

$18xy^2 = 3 \times 6 \times x \times y \times y$

$-6xy = -1 \times 3 \times 2 \times x \times y$

So, the greatest common factor is 3xy.

(iii) Factor of 2xy, $-y^2$, $2x^2y$ will be:

$2xy = 2 \times x \times y$

$-y^2 = -y \times y$

$2x^2y = 2 \times x \times x \times y$

So, the greatest common factor is y.

(iv) Factor of l^2m^2n, lm^2n^2, l^2mn^2 will be:

$l^2m^2n = l \times l \times m \times m \times n$

$lm^2n^2 = l \times m \times m \times n \times n$

$l^2mn^2 = l \times l \times m \times n \times n$

So, the greatest common factor is lmn.

(v) Factor of 21pqr, $-7p^2q^2r^2$, $49p^2qr$ will be:

$21pqr = 7 \times 3 \times p \times q \times r$

$-7p^2q^2r^2 = -7 \times p \times p \times q \times q \times r \times r$

$49p^2qr = 7 \times 7 \times p \times p \times q \times r$

So, the greatest common factor is 7pqr.

(vi) Factor of qrxy, pryz, rxyz will be:

$qrxy = q \times r \times x \times y$

$pryz = p \times r \times y \times z$

$rxyz = r \times x \times y \times z$

So, the greatest common factor is ry.

(vii) Factor of $3x^3y^2z$, $-6xy^3z^2$, $12x^2yz^3$ will be:

$3x^3y^2z = 3 \times x \times x \times x \times y \times y \times z$

$-6xy^3z^2 = -3 \times 2 \times x \times y \times y \times y \times z \times z$

$12x^2yz^3 = 3 \times 4 \times x \times x \times y \times z \times z \times z$

So, the greatest common factor is 3xyz.

(viii) Factor of $63p^2a^2r^2s$, $-9pq^2r^2s^2$, $15p^2qr^2s^2$, $-60p^2a^2rs^2$ will be:

$$63p^2a^2r^2s = 3\times 3\times 7\times p\times p\times a\times a\times r\times r\times s$$

$$-9pq^2r^2s^2 = -3\times 3\times p\times q\times q\times r\times r\times s\times s$$

$$15p^2qr^2s^2 = 3\times 5\times p\times p\times q\times r\times r\times s\times s$$

$$-60p^2a^2rs^2 = -2\times 2\times 3\times 5\times p\times p\times a\times a\times r\times s\times s$$

So, the greatest common factor is 3prs.

(ix) Factor of $13x^2y$, 169xy will be:

$$13x^2y = 13\times x\times x\times y$$

$$169xy = 13\times 13\times x\times y$$

So, the greatest common factor is 13xy.

(x) We have $11x^2, 12y^2$

There is no common factor between x^2 and y^2.

So, the greatest common factor is 1.

88. Factorise the following expressions.

(i) 6ab + 12bc

(ii) –xy – ay

(iii) $ax^3 - bx^2 + cx$

(iv) $l^2m^2n - lm^2n^2 - l^2mn^2$

(v) $3pqr - 6p^2q^2r^2 - 15r^2$

(vi) $x^3y^2 + x^2y^3 - xy^4 + xy$

(vii) $4xy^2 - 10x^2y + 16x^2y^2 + 2xy$

(viii) $2a^3 - 3a^2b + 5ab^2 - ab$

(ix) $63p^2q^2r^2s - 9pq^2r^2s^2 + 15p^2qr^2s^2 - 60p^2q^2rs^2$

(x) $24x^2yz^3 - 6xy^3z^2 + 15x^2y^2z - 5xyz$

(xi) $a^3 + a^2 + a + 1$

(xii) lx + my + mx + ly

(xiii) $a^3x - x^4 + a^2x^2 - ax^3$

(xiv) $2x^2 - 2y + 4xy - x$

(xv) $y^2 + 8zx - 2xy - 4yz$

(xvi) $ax^2y - bxyz - ax^2z + bxy^2$

(xvii) $a^2b + a^2c + ab + ac + b^2c + c^2b$

(xviii) $2ax^2 + 4axy + 3bx^2 + 2ay^2 + 6bxy + 3by^2$

Solution:

(i) Consider the expression:

6ab + 12bc

Now, simplify the above expression as follows:

$$6ab+12bc=6ab+6\times 2bc$$
$$=6b\left(a+2c\right)$$

(ii) Consider the expression:

–xy – ay

Now, simplify the above expression as follows:

$$-xy-ay=-y\left(x+a\right)$$

(iii) Consider the expression:

$ax^3 - bx^2 + cx$

Now, simplify the above expression as follows:

$$ax^3-bx^2+cx=x\left(ax^2-bx+c\right)$$

(iv) Consider the expression:

$l^2m^2n - lm^2n^2 - l^2mn^2$

Now, simplify the above expression as follows:

$$l^2m^2n-lm^2n^2-l^2mn^2=lmn\left(lm-mn-\ln\right)$$

(v) Consider the expression:

$3pqr - 6p^2q^2r^2 - 15r^2$

Now, simplify the above expression as follows:

$$3pqr-6p^2q^2r^2-15r^2=3pqr-3\times 2p^2q^2r^2-3\times 5r^2$$
$$=3r\left(pq-2p^2q^2r-5r\right)$$

(vi) Consider the expression:

$x^3y^2 + x^2y^3 - xy^4 + xy$

Now, simplify the above expression as follows:

$$x^3y^2+x^2y^3-xy^4+xy=xy\left(x^2y+xy^2-y^3+1\right)$$

(vii) Consider the expression:

$4xy^2 - 10x^2y + 16x^2y^2 + 2xy$

Now, simplify the above expression as follows:

$$4xy^2-10x^2y+16x^2y^2+2xy=2\times 2xy^2-2\times 5\times x^2y+2\times 8\times x^2y^2+2xy$$
$$=2xy\left(2y-5x+8xy+1\right)$$

(viii) Consider the expression:

$2a^3 - 3a^2b + 5ab^2 - ab$

Now, simplify the above expression as follows:

$$2a^3-3a^2b+5ab^2-ab=a\left(2a^2-3ab+5b^2-b\right)$$

(ix) Consider the expression:

$63p^2q^2r^2s - 9pq^2r^2s^2 + 15p^2qr^2s^2 - 60p^2q^2rs^2$

Now, simplify the above expression as follows:

$$\left[63p^2q^2r^2s - 9pq^2r^2s^2 + 15p^2qr^2s^2 - 60p^2q^2rs^2 = 3\times 21p^2q^2r^2s - 3\times 3pq^2r^2s^2 + 3\times 5p^2qr^2s^2 - 3\times 20p^2q^2rs^2\right]$$

$$= 3pqrs\left(21pqr - 3qrs + 5prs - 20pqs\right)$$

(x) Consider the expression:

$24x^2yz^3 - 6xy^3z^2 + 15x^2y^2z - 5xyz$

Now, simplify the above expression as follows:

$$24x^2yz^3 - 6xy^3z^2 + 15x^2y^2z - 5xyz = xyz\left(24xz^2 - 6y^2z + 15xy - 5\right)$$

(xi) Consider the expression:

$a^3 + a^2 + a + 1$

Now, simplify the above expression as follows:

$$a^3 + a^2 + a + 1 = a^2\left(a+1\right) + 1\left(a+1\right)$$

$$= \left(a+1\right)\left(a^2+1\right)$$

(xii) Consider the expression:

$lx + my + mx + ly$

Now, simplify the above expression as follows:

$$lx + my + mx + ly = lx + mx + my + ly$$

$$= x\left(l+m\right) + y\left(m+l\right)$$

$$= \left(l+m\right)\left(x+y\right)$$

(xiii) Consider the expression:

$a^3x - x^4 + a^2x^2 - ax^3$

Now, simplify the above expression as follows:

$$a^3x - x^4 + a^2x^2 - ax^3 = x\left(a^3 - x^3 + a^2x - ax^2\right)$$

$$= x\left(a^3 + a^2x - x^3 - ax^2\right)$$

$$= x\left[a^2\left(a+x\right) - x^2\left(x+a\right)\right]$$

$$= x\left[\left(x+a\right)\left(a^2-x^2\right)\right]$$

$$= x\left(a^2-x^2\right)\left(a+x\right)$$

(xiv) Consider the expression:

$2x^2 - 2y + 4xy - x$

Now, simplify the above expression as follows:

$$2x^2 - 2y + 4xy - x = 2x^2 - x - 2y + 4xy$$
$$= x(2x-1) - 2y(1-2x)$$
$$= x(2x-1) + 2y(2x-1)$$
$$= (2x-1)(x+2y)$$

(xv) Consider the expression:
$y^2 + 8zx - 2xy - 4yz$
Now, simplify the above expression as follows:
$$y^2 + 8zx - 2xy - 4yz = y^2 - 2xy + 8zx - 4yz$$
$$= y(y-2x) - 4z(y-2x)$$
$$= (y-2x)(y-4z)$$

(xvi) Consider the expression:
$ax^2y - bxyz - ax^2z + bxy^2$
Now, simplify the above expression as follows:
$$ax^2y - bxyz - ax^2z + bxy^2 = x\left(axy - byz - axz + by^2\right)$$
$$= x\left(axy - axz - byz + by^2\right)$$
$$= x\left[ax(y-z) + by(-z+y)\right]$$
$$= x\left[(ax+by)(y-z)\right]$$

(xvii) Consider the expression:
$a^2b + a^2c + ab + ac + b^2c + c^2b$
Now, simplify the above expression as follows:
$$a^2b + a^2c + ab + ac + b^2c + c^2b = \left(a^2b + ab + b^2c\right) + \left(a^2c + ac + c^2b\right)$$
$$= b\left(a^2 + a + bc\right) + c\left(a^2 + a + bc\right)$$
$$= \left(a^2 + a + bc\right)(b+c)$$

(xviii) Consider the expression:
$2ax^2 + 4axy + 3bx^2 + 2ay^2 + 6bxy + 3by^2$
Now, simplify the above expression as follows:
$$2ax^2 + 4axy + 3bx^2 + 2ay^2 + 6bxy + 3by^2 = \left(2ax^2 + 2ay^2 + 4axy\right) + \left(3bx^2 + 3by^2 + 6bxy\right)$$
$$= 2a\left(x^2 + y^2 + 2xy\right) + 3b\left(x^2 + y^2 + 2xy\right)$$
$$= (2a+3b)(x+y)^2$$

89. Factorise the following, using the identity $a^2 + 2ab + b^2 = (a + b)^2$
(i) $x^2 + 6x + 9$

(ii) x^2 + 12x + 36
(iii) x^2 + 14x + 49
(iv) x^2 + 2x + 1
(v) $4x^2$ + 4x + 1
(vi) a^2x^2 + 2ax + 1
(vii) a^2x^2 + 2abx + b^2
(viii) a^2x^2 + 2abxy + b^2y^2
(ix) $4x^2$ + 12x + 9
(x) $16x^2$ + 40x +25
(xi) $9x^2$ + 24x + 16
(xii) $9x^2$ + 30x + 25
(xiii) $2x^3$ + $24x^2$ + 72x
(xiv) a^2x^3 + $2abx^2$ + b^2x
(xv) $4x^4$ + $12x^3$ + $9x^2$
(xvi) (x2/4) + 2x + 4
(xvii) $9x^2$ + 2xy + (y2/9)

Solution:

(i) $x^2+6x+9=x^2+2\times3\times x\times3^2$ [As $a^2+2ab+b^2=(a+b)^2$]

$=(x+3)^2$

$=(x+3)(x+3)$

(ii) $x^2+12x+36=x^2+2\times6\times x+6^2$ [As $a^2+2ab+b^2=(a+b)^2$]

$=(x+6)^2$

$=(x+6)(x+6)$

(iii) $x^2+14x+49=x^2+2\times7\times x+7^2$ [As $a^2+2ab+b^2=(a+b)^2$]

$=(x+7)^2$

$=(x+7)(x+7)$

(iv) $x^2+2x+1=x^2+2\times1\times x+1^2$ [As $a^2+2ab+b^2=(a+b)^2$]

$=(x+1)^2$

$=(x+1)(x+1)$

(v) $4x^2+4x+1=(2x)^2+2\times2x\times1+1^2$ [As $a^2+2ab+b^2=(a+b)^2$]

$$=(2x+1)^2$$
$$=(2x+1)(2x+1)$$

(vi) $a^2x^2+2ax+1=(ax)^2+2\times ax\times 1+1^2$ [As $a^2+2ab+b^2=(a+b)^2$]

$$=(ax+1)^2$$
$$=(ax+1)(ax+1)$$

(vii) $a^2x^2+2abc+b^2=(ax)^2+2\times ax\times b+b^2$ [As $a^2+2ab+b^2=(a+b)^2$]

$$=(ax+b)^2$$
$$=(ax+b)(ax+b)$$

(viii) $a^2x^2+2abxy+b^2y^2=(ax)^2+2\times ax\times by+(by)^2$

$$=(ax+by)^2$$
$$=(ax+by)(ax+by)$$

(ix) $4x^2+12x+9=(2x)^2+2\times 2x\times 3+3^2$

$$=(2x+3)^2$$
$$=(2x+3)(2x+3)$$

(x) $16x^2+40x+25=(4x)^2+2\times 4x\times 5+5^2$

$$=(4x+5)^2$$
$$=(4x+5)(4x+5)$$

(xi) $9x^2+24x+16=(3x)^2+2\times 3x\times 4+4^2$

$$=(3x+4)^2$$
$$=(3x+4)(3x+4)$$

(xii) $9x^2+30x+25=(3x)^2+2\times 3x\times 5+5^2$

$$=(3x+5)^2$$
$$=(3x+5)(3x+5)$$

(xiii) $2x^3+24x^2+72x=2x\left(x^2+12x+36\right)$

$$= 2x\left(x^2 + 2\times 6\times x + 6^2\right)$$
$$= 2x\left(x+6\right)^2$$
$$= 2x\left(x+6\right)\left(x+6\right)$$

(xiv) $a^2x^3 + 2abx^2 + b^2x = x\left(a^2x^2 + 2abx + b^2\right)$

$$= x\left[\left(ax\right)^2 + 2\times ax\times b + b^2\right]$$
$$= x\left[\left(ax\right)^2 + 2\times ax\times b + b^2\right]$$
$$= x\left(ax+b\right)^2$$
$$= x\left(ax+b\right)\left(ax+b\right)$$

(xv) $4x^4 + 12x^3 + 9x^2 = x^2\left(4x^2 + 12x + 9\right)$

$$= x^2\left[\left(2x\right)^2 + 2\times 2x\times 3 + 3^2\right]$$
$$= x^2\left(2x+3\right)^2$$
$$= x^2\left(2x+3\right)\left(2x+3\right)$$

(xvi) $\frac{x^2}{4} + 2x + 4 = \frac{x^2}{4} + 2\times\frac{x}{2}\times 2 + 2^2$

$$= \left(\frac{x}{2}+2\right)^2$$
$$= \left(\frac{x}{2}+2\right)\left(\frac{x}{2}+2\right)$$

(xvii) $9x^2 + 2xy + \frac{y^2}{9} = \left(3x\right)^2 + 2\times 3x\times\frac{y}{3} + \left(\frac{y}{3}\right)^2$

$$= \left(3x+\frac{y}{3}\right)^2$$
$$= \left(3x+\frac{y}{3}\right)\left(3x+\frac{y}{3}\right)$$

90. Factorise the following, using the identity $a^2 - 2ab + b^2 = (a - b)^2$.

(i) $x^2 - 8x + 16$

(ii) $x^2 - 10x + 25$

(iii) $y^2 - 14y + 49$

(iv) $p^2 - 2p + 1$

(v) $4a^2 – 4ab + b^2$
(vi) $p^2y^2 – 2py + 1$
(vii) $a^2y^2 – 2aby + b^2$
(viii) $9x^2 – 12x + 4$
(ix) $4y^2 – 12y + 9$
(x) (x2/4) – 2x + 4
(xi) $a^2y^3 – 2aby^2 + b^2y$
(xii) $9y^2$ – 4xy + (4x2/9)

Solution:

(i) $x^2-8x+16=x^2-2\times x\times 4+4^2$
$$=(x-4)^2$$
$$=(x-4)(x-4)$$

(ii) $x^2-10x+25=x^2-2\times x\times 5+5^2$
$$=(x-5)^2$$
$$=(x-5)(x-5)$$

(iii) $y^2-14y+49=y^2-2\times y\times 7+7^2$
$$=(y-7)^2$$
$$=(y-7)(y-7)$$

(iv) $p^2-2p+1=p^2-2\times p\times 1+1^2$
$$=(p-1)^2$$
$$=(p-1)(p-1)$$

(v) $4a^2-4ab+b^2=(2a)^2-2\times 2a\times b+b^2$
$$=(2a-b)^2$$
$$=(2a-b)(2a-b)$$

(vi) $p^2y^2-2py+1=(py)^2-2\times py\times 1+1^2$
$$=(py-1)^2$$
$$=(py-1)(py-1)$$

(vii) $a^2y^2-2aby+b^2=(ay)^2-2\times ay\times b+b^2$

$$= (ay - b)^2$$
$$= (ay - b)(ay - b)$$

(viii) $9x^2 - 12x + 4 = (3x)^2 - 2 \times 3x \times 2 + 2^2$
$$= (3x - 2)^2$$
$$= (3x - 2)(3x - 2)$$

(ix) $4y^2 - 12y + 9 = (2y)^2 - 2 \times 2y \times 3 + 3^2$
$$= (2y - 3)^2$$
$$= (2y - 3)(2y - 3)$$

(x) $\frac{x^2}{4} - 2x + 4 = \left(\frac{x}{2}\right)^2 - 2 \times \frac{x}{2} \times 2 + 2^2$
$$= \left(\frac{x}{2} - 2\right)^2$$
$$= \left(\frac{x}{2} - 2\right)\left(\frac{x}{2} - 2\right)$$

(xi) $a^2y^3 - 2aby^2 + b^2y = y\left(a^2y^2 - 2aby + b^2\right)$
$$= y\left[(ay)^2 - 2 \times ay \times b + b^2\right]$$
$$= y(ay - b)^2$$
$$= y(ay - b)(ay - b)$$

(xii) $9y^2 - 4xy + \frac{4x^2}{9} = (3y)^2 - 2 \times 3y \times \frac{2}{3}x + \left(\frac{2}{3}x\right)^2$
$$= \left(3y - \frac{2}{3}x\right)^2$$
$$= \left(3y - \frac{2x}{3}\right)\left(3y - \frac{2x}{3}\right)$$

91. Factorise the following.
(i) $x^2 + 15x + 26$
(ii) $x^2 + 9x + 20$
(iii) $y^2 + 18x + 65$

(iv) p^2 + 14p + 13
(v) y^2 + 4y – 21
(vi) y^2 – 2y – 15
(vii) 18 + 11x + x^2
(viii) x^2 – 10x + 21
(ix) x^2 = 17x + 60
(x) x^2 + 4x – 77
(xi) y^2 + 7y + 12
(xii) p^2 – 13p – 30
(xiii) a^2 – 16p – 80

Solution:

(i) $x^2+15x+26=x^2+2x+13x+2\times13$
$=x(x+2)+13(x+2)$
$=(x+2)(x+13)$

(ii) $x^2+9x+20=x^2+5x+4x+5\times4$
$=x(x+5)+4(x+5)$
$=(x+5)(x+4)$

(iii) $y^2+18y+65=y^2+13y+5y+5\times13$
$=y(y+13)+5(y+13)$
$=(y+13)(y+5)$

(iv) $p^2+14p+13=p^2+13p+p+13\times1$
$=p(p+13)+1(p+13)$
$=(p+13)(p+1)$

(v) $y^2+4y-21=y^2+(7-3)y-21$
$=y^2+7y-3y-21$
$=y(y+7)-3(y+7)$
$=(y+7)(y-3)$

(vi) $y^2-2y-15=y^2+(3-5)y-15$

$$= y^2 + 3y - 5y - 15$$
$$= y(y+3) - 5(y+3)$$
$$= (y+3)(y-5)$$

(vii) $18 + 11x + x^2 = x^2 + 11x + 18$
$$= x^2 + (9+2)x + 18$$
$$= x^2 + 9x + 2x + 18$$
$$= x(x+9) + 2(x+9)$$
$$= (x+9)(x+2)$$

(viii) $x^2 - 10x + 21 = x^2 - (7+3)x + 21$
$$= x^2 - 7x - 3x + 21$$
$$= x(x-7) - 3(x-7)$$
$$= (x-7)(x-3)$$

(ix) $x^2 - 17x + 60 = x^2 - (12+5)x + 60$
$$= x^2 - 12x - 5x + 60$$
$$= x(x-12) - 5(x-12)$$
$$= (x-12)(x-5)$$

(x) $x^2 + 4x - 77 = x^2 + (11-7)x - 77$
$$= x^2 + 11x - 7x - 77$$
$$= x(x+11) - 7(x+11)$$
$$= (x+11)(x-7)$$

(xi) $y^2 + 7y + 12 = y^2 + (4+3)y + 12$
$$= y^2 + 4y + 3y + 12$$
$$= y(y+4) + 3(y+4)$$
$$= (y+4)(y+3)$$

(xii) $p^2 - 13p - 30 = p^2 - (15-2)p - 30$
$$= p^2 - 15p + 2p - 30$$
$$= p(p-15) + 2(p-15)$$
$$= (p-15)(p+2)$$

(xiii) $p^2 - 16p - 80 = p^2 - (20-4)p - 80$

$$= p^2 - 20p + 4p - 80$$

$$= p(p-20) + 4(p-20)$$

$$= (p-20)(p+4)$$

92. Factorise the following using the identity $a^2 - b^2 = (a + b)(a - b)$.

(i) $x^2 - 9$

(ii) $4x^2 - 25y^2$

(iii) $4x^2 - 49y^2$

(iv) $3a^2b^3 - 27a^4b$

(v) $28ay^2 - 175ax^2$

(vi) $9x^2 - 1$

(vii) $25ax^2 - 25a$

(viii) $(x^2/9) - (y^2/25)$

(ix) $(2p^2/25) - 32q^2$

(x) $49x^2 - 36y^2$

(xi) $y^3 - \frac{y}{9}$

(xii) $(x^2/25) - 625$

(xiii) $(x^2/8) - (y^2/18)$

(xiv) $(4x^2/9) - (9y^2/16)$

(xv) $(x^3y/9) - (xy^3/16)$

(xvi) $1331x^3y - 11y^3x$

(xvii) $\frac{1}{36}a^2b^2 - \frac{16}{49}b^2c^2$

(xviii) $a^4 - (a - b)^4$

(xix) $x^4 - 1$

(xx) $y^4 - 625$

(xxi) $p^5 - 16p$

(xxii) $16x^4 - 81$

(xxiii) $x^4 - y^4$

(xxiv) $y^4 - 81$

(xxv) $16x^4 - 625y^4$

(xxvi) $(a - b)^2 - (b - c)^2$

(xxvii) $(x + y)^4 - (x - y)^4$

(xxviii) $x^4 - y^4 + x^2 - y^2$

(xxix) $8a^3 - 2a$

(xxx) $x^2 - (y^2/100)$

(xxxi) $9x^2 - (3y + z)^2$

Solution:

(i) $x^2 - 9 = x^2 - 3^2$

$= (x-3)(x+3)$

(ii) $4x^2 - 25y^2 = (2x)^2 - (5y)^2$

$= (2x-5y)(2x+5y)$

(iii) $4x^2 - 49y^2 = (2x)^2 - (7y)^2$

$= (2x-7y)(2x+7y)$

(iv) $3a^2b^3 - 27a^4b = 3a^2b(b^2 - 9a^2)$

$= 3a^2b\left[b^2 - (3a)^2\right]$

$= 3a^2b(b+3a)(b-3a)$

(v) $28ay^2 - 175ax^2 = 7a(4y^2 - 25x^2)$

$= 7a\left[(2y)^2 - (5x)^2\right]$

$= 7a(2y-5x)(2y+5x)$

(vi) $9x^2 - 1 = (3x)^2 - 1^2$

$= (3x-1)(3x+1)$

(vii) $25ax^2 - 25a = 25a(x^2 - 1^2)$

$= 25a(x-1)(x+1)$

(viii) $\frac{x^2}{9} - \frac{y^2}{25} = \left(\frac{x}{3}\right)^2 - \left(\frac{y}{3}\right)^2$

$= \left(\frac{x}{3} - \frac{y}{5}\right)\left(\frac{x}{3} + \frac{y}{5}\right)$

(ix) $\frac{2p^2}{25} - 32q^2 = 2\left(\frac{p^2}{25} - 16q^2\right)$

$$= 2\left[\left(\frac{p}{5}\right)^2 - (4q)^2\right]$$

$$= 2\left(\frac{p}{5} + 4q\right)\left(\frac{p}{5} - 4q\right)$$

(x) $49x^2 - 36y^2 = (7x)^2 - (6y)^2$

$$= (7x - 6y)(7x + 6y)$$

(xi) $y^3 - \frac{y}{9} = y\left(y^2 - \frac{1}{9}\right)$

$$= y\left[y^2 - \left(\frac{1}{3}\right)^2\right]$$

$$= y\left(y + \frac{1}{3}\right)\left(y - \frac{1}{3}\right)$$

(xii) $\frac{x^2}{25} - 625 = \left(\frac{x}{5}\right)^2 - (25)^2$

$$= \left(\frac{x}{5} - 25\right)\left(\frac{x}{5} + 25\right)$$

(xiii) $\frac{x^2}{8} - \frac{y^2}{18} = \frac{1}{2}\left(\frac{x^2}{4} - \frac{y^2}{9}\right)$

$$= \frac{1}{2}\left[\left(\frac{x}{2}\right)^2 - \left(\frac{y}{3}\right)^2\right]$$

$$= \frac{1}{2}\left(\frac{x}{2} + \frac{y}{3}\right)\left(\frac{x}{2} - \frac{y}{3}\right)$$

(xiv) $\frac{4x^2}{9} - \frac{9y^2}{16} = \left(\frac{2x}{3}\right)^2 - \left(\frac{3y}{4}\right)^2$

$$= \left(\frac{2x}{3} + \frac{3y}{4}\right)\left(\frac{2x}{3} - \frac{3y}{4}\right)$$

(xv) $\frac{x^3y}{9} - \frac{xy^3}{16} = xy\left(\frac{x^2}{9} - \frac{y^2}{16}\right)$

$$= xy\left[\left(\frac{x}{3}\right)^2 - \left(\frac{y}{4}\right)^2\right]$$

$$= xy\left(\frac{x}{3} + \frac{y}{4}\right)\left(\frac{x}{3} - \frac{y}{4}\right)$$

(xvi) $1331x^3y - 11y^3x = (11)^3 x^3y - 11y^3x$

$$= 11xy\left(11^2x^2 - y^2\right)$$

$$= 11xy\left[(11x)^2 - y^2\right]$$

$$= 11xy(11x + y)(11x - y)$$

(xvii) $\frac{1}{36}a^2b^2 - \frac{16}{49}b^2c^2 = \left(\frac{ab}{6}\right)^2 - \left(\frac{4bc}{7}\right)^2$

$$= \left(\frac{ab}{6} + \frac{4bc}{7}\right)\left(\frac{ab}{6} - \frac{4bc}{7}\right)$$

$$= b^2\left(\frac{a}{6} + \frac{4c}{7}\right)\left(\frac{a}{6} - \frac{4c}{7}\right)$$

(xviii) $a^4 - (a-b)^4 = \left(a^2\right)^2 - \left[(a-b)^2\right]^2$

$$= \left[a^2 + (a-b)^2\right]\left[a^2 - (a-b)^2\right]$$

$$= \left[a^2 + a^2 + b^2 - 2ab\right]\left[a^2 - \left(a^2 + b^2 - 2ab\right)\right]$$

$$= \left[2a^2 + b^2 - 2ab\right]\left[-b^2 + 2ab\right]$$

$$= \left(2a^2 + b^2 - 2ab\right)\left(2ab - b^2\right)$$

(xix) $x^4 - 1 = \left(x^2\right)^2 - 1$

$$= \left(x^2 + 1\right)\left(x^2 - 1\right)$$

$$= \left(x^2 + 1\right)(x+1)(x-1)$$

(xx) $y^4 - 625 = \left(y^2\right)^2 - (25)^2$

$$= \left(y^2 + 25\right)\left(y^2 - 25\right)$$

$$= \left(y^2 + 25\right)\left(y^2 - 5^2\right)$$

$$= \left(y^2 + 25\right)(y+5)(y-5)$$

(xxi) $p^5-16p=p\left(p^4-16\right)$

$$=p\left[\left(p^2\right)^2-4^2\right]$$

$$=p\left(p^2+4\right)\left(p^2-4\right)$$

$$=p\left(p^2+4\right)\left(p^2-2^2\right)$$

$$=p\left(p^2+4\right)\left(p+2\right)\left(p-2\right)$$

(xxii) $16x^4-81=\left(4x^2\right)^2-9^2$

$$=\left(4x^2+9\right)\left(4x^2-9\right)$$

$$=\left(4x^2+9\right)\left[\left(2x\right)^2-3^2\right]$$

$$=\left(4x^2+9\right)\left(2x+3\right)\left(2x-3\right)$$

(xxiii) $x^4-y^4=\left(x^2\right)^2-\left(y^2\right)^2$

$$=\left(x^2+y^2\right)\left(x^2-y^2\right)$$

$$=\left(x^2+y^2\right)\left(x+y\right)\left(x-y\right)$$

(xxiv) $y^4-81=\left(y^2\right)^2-\left(9\right)^2$

$$=\left(y^2+9\right)\left[\left(y\right)^2-\left(3\right)^2\right]$$

$$=\left(y^2+9\right)\left(y+3\right)\left(y-3\right)$$

(xxv) $16x^4-625y^4=\left(4x^2\right)^2-\left(25y^2\right)^2$

$$=\left(4x^2+25y^2\right)\left(4x^2-25y^2\right)$$

$$=\left(4x^2+25y^2\right)\left[\left(2x\right)^2-\left(5y\right)^2\right]$$

$$=\left(4x^2+25y^2\right)\left(2x+5y\right)\left(2x-5y\right)$$

(xxvi) $\left(a-b\right)^2-\left(b-c\right)^2=\left(a-b+b-c\right)\left(a-b-b+c\right)\left(a-c\right)\left(a-2b+c\right)$

(xxvii) $\left(x+y\right)^4-\left(x-y\right)^4=\left[\left(x+y\right)^2\right]^2-\left[\left(x-y\right)^2\right]^2$

$$=\left[(x+y)^2+(x-y)^2\right]\left[(x+y)^2-(x-y)^2\right]$$
$$=\left(x^2+y^2+2xy+x^2+y^2-2xy\right)(x+y+x-y)(x+y-x+y)$$
$$=\left(2x^2+2y^2\right)(2x)(2y)$$
$$=2\left(x^2+y^2\right)(2x)(2y)$$
$$=8xy\left(x^2+y^2\right)$$

(xxix) $8a^3-2a=2a\left(4a^2-1\right)$

$$=2a\left[(2a)^2-(1)^2\right]$$
$$=2a(2a+1)(2a-1)$$

(xxx) $x^2-\dfrac{y^2}{100}=x^2-x-\left(\dfrac{y}{10}\right)^2$

$$=\left(x+\frac{y}{10}\right)\left(x-\frac{y}{10}\right)$$

(xxxi) $9x^2-(3y+z)^2=(3x)^2-(3y+z)$

$$=(3x+3y+z)(3x-3y-z)$$

93. The following expressions are the areas of rectangles. Find the possible lengths and breadths of these rectangles.

(i) $x^2 - 6x + 8$ (ii) $x^2 - 3x + 2$

(iii) $x^2 - 7x + 10$ (iv) $x^2 + 19x - 20$ (v) $x^2 + 9x + 20$

Solution:

(i) Consider the expression:

$x^2 - 6x + 8$

To find the possible length and breadth of the rectangle we have to factorise the given expression as follows:

$$x^2-6x+8=x^2-(4+2)x+8$$
$$=x^2-4x-2x+8$$
$$=x(x-4)-2(x-4)$$
$$=(x-4)(x-2)$$

So, area of rectangle = Length $\times$ Breadth.

Hence, the possible length and breadth are (x-4) and (x-2).

(ii) Consider the expression:

$x^2 - 3x + 2$

To find the possible length and breadth of the rectangle we have to factorise the given expression as follows:

$$\begin{aligned} x^2 - 3x + 2 &= x^2 - (2+1)x + 2 \\ &= x^2 - 2x - x + 2 \\ &= x(x-2) - 1(x-2) \\ &= (x-2)(x-1) \end{aligned}$$

So, area of rectangle = Length $\times$ Breadth.

Hence, the possible length and breadth are (x-2) and (x-1).

(iii) Consider the expression:

$x^2 - 7x + 10$

To find the possible length and breadth of the rectangle we have to factorise the given expression as follows:

$$\begin{aligned} x^2 - 7x + 10 &= x^2 - (5+2)x + 10 \\ &= x^2 - 5x - 2x + 10 \\ &= x(x-5) - 2(x-5) \\ &= (x-5)(x-2) \end{aligned}$$

So, area of rectangle = Length $\times$ Breadth.

Hence, the possible length and breadth are (x-5) and (x-2).

(iv) Consider the expression:

$x^2 + 19x - 20$

To find the possible length and breadth of the rectangle we have to factorise the given expression as follows:

$$\begin{aligned} x^2 + 19x - 20 &= x^2 + (20-1)x - 20 \\ &= x^2 + 20x - x - 20 \\ &= x(x+20) - 1(x+20) \\ &= (x+20)(x-1) \end{aligned}$$

So, area of rectangle = Length $\times$ Breadth.

Hence, the possible length and breadth are (x+20) and (x-1).

(v) Consider the expression:

$x^2 + 9x + 20$

To find the possible length and breadth of the rectangle we have to factorise the given expression as follows:

$$x^2+9x+20=x^2+(5+4)x+20$$
$$=x^2+5x+4x+20$$
$$=x(x+5)+4(x+5)$$
$$=(x+5)(x+4)$$

So, area of rectangle = Length $\times$ Breadth.
Hence, the possible length and breadth are (x+5) and (x+4).

94. Carry out the following divisions:
(i) $51x^3y^2z \div 17xyz$ **(ii) $76x^3yz^3 \div 19x^2y^2$**
(iii) $17ab^2c^3 \div (-abc^2)$ **(iv) $-121p^3q^3r^3 \div (-11xy^2z^3)$**

Solution:

(i) Consider the expression:
$51x^3y^2z \div 17xyz$
Simplify the above expression as follows:
$$\frac{51x^3y^2z}{17xyz}=\frac{17\times 3\times x\times x\times x\times y\times y\times z}{17\times x\times y\times z}$$
$$=3x^2y$$

(ii) Consider the expression:
$76x^3yz^3 \div 19x^2y^2$
Simplify the above expression as follows:
$$\frac{76x^3yz^3}{19x^2y^2}=\frac{4\times 19\times x\times x\times x\times y\times z\times z\times z}{19\times x\times x\times y\times y}$$
$$=\frac{4xz^3}{y}$$

(iii) Consider the expression:
$17ab^2c^3 \div (-abc^2)$
Simplify the above expression as follows:
$$\frac{17ab^2c^3}{-abc^2}=\frac{17\times a\times b\times b\times c\times c\times c}{-a\times b\times c\times c}$$
$$=-17bc$$

(iv) Consider the expression:
$-121p^3q^3r^3 \div (-11xy^2z^3)$
Simplify the above expression as follows:

$$\frac{-12p^3q^3r^3}{-11xy^2z^3} = \frac{-11\times 11\times p\times p\times p\times q\times q\times q\times r\times r\times r}{-11\times x\times y\times y\times z\times z\times z}$$

$$= \frac{11p^3q^3r^3}{xy^2z^3}$$

95. Perform the following divisions:

(i) $(3pqr - 6p^2q^2r^2) \div 3pq$ **(ii) $(ax^3 - bx^2 + cx) \div (-dx)$**

(iii) $(x^3y^3 + x^2y^3 - xy^4 + xy) \div xy$ **(iv) $(-qrxy + pryz - rxyz) \div (-xyz)$**

Solution:

(i) Consider the expression:

$(3pqr - 6p^2q^2r^2) \div 3pq$

Now, simplify the above expression as follows:

$$\frac{3pqr - 6p^2q^2r^2}{3pq} = \frac{3pqr}{3pq}$$

$$= \frac{3pqr}{3pq} - \frac{6p^2q^2r^2}{3pq}$$

$$= r - \frac{2\times 3\times p\times p\times q\times q\times r\times r}{3\times p\times q}$$

$$= r - 2pqr^2$$

(ii) Consider the expression:

$(ax^3 - bx^2 + cx) \div (-dx)$

Now, simplify the above expression as follows:

$$\frac{ax^3 - bx^2 + cx}{-dx} = \frac{ax^3}{-dx} + \frac{bx^2}{dx} + \frac{cx}{-dx}$$

$$= \frac{a\times x\times x\times x}{-d\times x} + \frac{b\times x\times x}{-d\times x} + \frac{c\times x}{-d\times x}$$

$$= -\frac{a}{d}x^2 + \frac{d}{d}x - \frac{c}{d}$$

(iii) Consider the expression:

$(x^3y^3 + x^2y^3 - xy^4 + xy) \div xy$

Now, simplify the above expression as follows:

$$\frac{x^3y^3 + x^2y^3 - xy^4 + xy}{xy} = \frac{x^3y^3}{xy} + \frac{x^2y^3}{xy} - \frac{xy^4}{xy} + \frac{xy}{xy}$$

$$= \frac{x\times x\times x\times y\times y\times y}{x\times y} + \frac{x\times x\times y\times y\times y}{x\times y} - \frac{x\times y\times y\times y\times y}{x\times y} + \frac{x\times y}{x\times y}$$

$$= x^2y^2 + xy^2 - y^3 + 1$$

(iv) Consider the expression:

(– qrxy + pryz – rxyz) ÷ (– xyz)

Now, simplify the above expression as follows:

$$\frac{-qrxy+pryz-rxyz}{-xyz}=\frac{-qrxy}{-xyz}+\frac{pryz}{-xyz}-\frac{rxyz}{-xyz}$$

$$=\frac{qr}{z}-\frac{pr}{x}+r$$

96. Factorise the expressions and divide them as directed:

(i) $(x^2 - 22x + 117) \div (x - 13)$

(ii) $(x^3 + x^2 - 132x) \div x(x - 11)$

(iii) $(2x^3 - 12x^2 + 16x) \div (x - 2)(x - 4)$

(iv) $(9x^2 - 4) \div (3x + 2)$

(v) $(3x^2 - 48) \div (x - 4)$

(vi) $(x^4 - 16) \div x^3 + 2x^2 + 4x + 8$

(vii) $(3x^4 - 1875) \div (3x^2 - 75)$

Solution:

(i) Consider the expression:

$(x^2 - 22x + 117) \div (x - 13)$

Now, simplify the above expression as follows:

$$\frac{x^2-22x+117}{x-13}=\frac{x^2-13x-9x+117}{x-13}$$

$$=\frac{x(x-13)-9(x-13)}{x-13}$$

$$=\frac{(x-13)(x-9)}{x-13}$$

$$=x-9$$

(ii) Consider the expression:

$(x^3 + x^2 - 132x) \div x(x - 11)$

Now, simplify the above expression as follows:

$$\frac{x^3+x^2-132x}{x(x-11)}=\frac{x(x^2+x-132)}{x(x-11)}$$

$$=\frac{x^2+12x-11x-132}{x-11}$$

$$=x+12$$

(iii) Consider the expression:

$(2x^3 - 12x^2 + 16x) \div (x - 2)(x - 4)$

Now, simplify the above expression as follows:

$$\frac{2x^3 - 12x^2 + 16x}{(x-2)(x-4)} = \frac{2x(x^2 - 6x + 8)}{(x-2)(x-4)}$$

$$= \frac{2x(x^2 - 4x - 2x + 8)}{(x-2)(x-4)}$$

$$= \frac{2x[x(x-4) - 2(x-4)]}{(x-2)(x-4)}$$

$$= \frac{2x(x-4)(x-2)}{(x-2)(x-4)}$$

$$= 2x$$

(iv) Consider the expression:

$(9x^2 - 4) \div (3x + 2)$

Now, simplify the above expression as follows:

$$\frac{9x^2 - 4}{3x + 2} = \frac{(3x)^2 - (2)^2}{3x + 2}$$

$$= \frac{(3x+2)(3x-2)}{3x+2}$$

$$= 3x - 2$$

(v) Consider the expression:

$(3x^2 - 48) \div (x - 4)$

Now, simplify the above expression as follows:

$$\frac{3x^2 - 48}{x - 4} = \frac{3(x^2 - 16)}{x - 4}$$

$$= \frac{3(x^2 - 4^2)}{x - 4}$$

$$= \frac{3(x+4)(x-4)}{x-4}$$

$$= 3(x + 4)$$

(vi) Consider the expression:

$(x^4 - 16) \div x^3 + 2x^2 + 4x + 8$

Now, simplify the above expression as follows:

$$\frac{x^4-16}{x^3+2x^2+4x+8}=\frac{\left(x^2\right)^2-4^2}{x^2\left(x+2\right)+4\left(x+2\right)}$$
$$=\frac{\left(x^2+4\right)\left(x^2-4\right)}{\left(x^2+4\right)\left(x+2\right)}$$
$$=\frac{x^2-2^2}{x+2}$$
$$=\frac{\left(x+2\right)\left(x-2\right)}{x+2}$$
$$=x-2$$

(vii) Consider the expression:
$(3x^4 – 1875) \div (3x^2 – 75)$
Now, simplify the above expression as follows:

$$\frac{3x^4-1875}{3x^2-75}=\frac{x^4-625}{x^2-25}$$
$$=\frac{\left(x^2\right)^2-\left(25\right)^2}{x^2-25}$$
$$=\frac{\left(x^2+25\right)\left(x^2-25\right)}{\left(x^2-25\right)}$$
$$=x^2+25$$

97. The area of a square is given by $4x^2 + 12xy + 9y^2$. Find the side of the square.

Solution:

Given:
Area of square = $4x^2 + 12xy + 9y^2$
Now, the sides of square will be calculated as follows:
$4x^2 + 12xy + 9y^2 = \text{side}^2$ [As, area of square = side^2]
Side = $\left(2x\right)^2+2\times2x\times3y+\left(3y\right)^2$
Side = $\left(2x+3y\right)^2$
Side = 2x+3y
Hence, the side of the given square is (2x+3y).

98. The area of a square is $9x^2 + 24xy + 16y^2$. Find the side of the square.

Solution:

Given:
Area of square = $9x^2 + 24xy + 16y^2$
Now, the sides of square will be calculated as follows:
$9x^2 + 24xy + 16y^2 = \text{side}^2$ [As, area of square = side^2]
$\text{Side}^2 = (3x)^2 + 2\times 3x\times 4y + (4y)^2$
$\text{Side}^2 = (3x+4y)^2$
Side = 3x+4y
Hence, the side of the given square is (3x+4y).

99. The area of a rectangle is $x^2 + 7x + 12$. If its breadth is (x + 3), then find its length.

Solution:
Given:
Area of rectangle = $x^2 + 7x + 12$
Now, the length of rectangle will be calculated as follows:
Length × Breadth = $x^2 + 7x + 12$ [As, area of rectangle = Length × Breadth]
Length × (x+3) = $x^2 + 4x + 3x + 12$
Length × (x+3) = $x(x+4) + 3(x+4)$
Length × (x+3) = $(x+3)(x+4)$
Length = x+4
Hence, the length of the given rectangle is x+4.

100. The curved surface area of a cylinder is 2π (y^2 – 7y + 12) and its radius is (y – 3). Find the height of the cylinder (C.S.A. of cylinder = 2πrh).

Solution:
Given:
Curved surface area of a cylinder = 2π (y^2 – 7y + 12) and radius of cylinder = y – 3.
Now, the height of the cylinder will be calculated as follows:
Curved surface area of a cylinder = 2π (y^2 – 7y + 12) [As, Curved surface area of a cylinder = $2\pi rh$]
$2\pi\times(y-3)\times h = 2\pi(y^2-7y+12)$
$2\pi\times(y-3)\times h = 2\pi(y^2-4y-3y+12)$
$2\pi\times(y-3)\times h = 2\pi(y(y-4)-3(y-4))$
$2\pi\times(y-3)\times h = 2\pi(y-3)(y-4)$
$h = y-4$
Hence, the height of the cylinder is y - 4.

101. The area of a circle is given by the expression πx^2 + 6πx + 9π. Find the radius of the circle.

Solution:

Given:

Area of a circle = πx^2 + 6πx + 9π.

Now, the radius of the circle(r) will be calculated as follows:

Area of a circle = πx^2 + 6πx + 9π [As, area of a circle= πr^2]

$\pi r^2 = \pi x^2 + 6\pi x + 9\pi$

$\pi r^2 = \pi\left(x^2 + 6x + 9\right)$

$\pi r^2 = \pi x\left(x+3\right) + 3\left(x+3\right)$

$\pi r^2 = \pi\left(x+3\right)\left(x+3\right)$

$\pi r^2 = \pi\left(x+3\right)^2$

$r^2 = \left(x+3\right)^2$

$r = x+3$

Hence, the radius of the circle is x+3.

102. The sum of first n natural numbers is given by the expression (n2/2) + $\frac{n}{2}$. Factorise this expression.

Solution:

As we know that the sum of first n natural numbers = $\frac{n^2}{2} + \frac{n}{2}$

Factorisation of given expression = $\frac{1}{2}\left(n^2 + n\right) = \frac{1}{2}n\left(n+1\right)$

103. The sum of (x + 5) observations is x^4 – 625. Find the mean of the observations.

Solution:

Given: The sum of (x + 5) observations is x^4 – 625.

As we know that the mean of the n observations $x_1, x_2, ...x_n$ is $\frac{x_1 + x_2 + ... + x_n}{n}$.

So, the mean of (x+5) observations = $\frac{\text{Sum of } \left(x+5\right) \text{ observations}}{x+5}$

$$= \frac{x^4 - 625}{x + 5}$$

$$= \frac{\left(x^2\right)^2 - (25)^2}{x + 5}$$

$$= \frac{\left(x^2 + 25\right)\left(x^2 - 25\right)}{x + 5}$$

$$= \frac{\left(x^2 + 25\right)\left[(x)^2 - (5)^2\right]}{x + 5}$$

$$= \frac{\left(x^2 + 25\right)(x + 5)(x - 5)}{x + 5}$$

$$= \left(x^2 + 25\right)(x - 5)$$

104. The height of a triangle is $x^4 + y^4$ and its base is 14xy. Find the area of the triangle.

Solution:

Give: The height of a triangle and its base are $x^4 + y^4$ and 14xy, respectively.

As we know that the area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times 14xy \times \left(x^4 + y^4\right)$$

$$= 7xy\left(x^4 + y^4\right)$$

105. The cost of a chocolate is Rs (x + y) and Rohit bought (x + y) chocolates. Find the total amount paid by him in terms of x. If x = 10, find the amount paid by him.

Solution:

Given: cost of a chocolate = Rs. x+ 4
Rohit bought (x+4) chocolates.
So, the cost of (x+4) chocolates = Cost of one chocolate $\times$ Number of chocolates

$$= (x + 4)(x + 4)$$

$$= (x + 4)^2$$

The total amount paid by Rohit = Rs. $x^2 + 8x + 16$
Therefore, if x = 10. Then, the amount paid by Rohit =
$10^2 + 8 \times 10 + 16 = 100 + 80 + 16 = \text{Rs.}196$.

106. The base of a parallelogram is (2x + 3 units) and the corresponding height is (2x – 3 units). Find the area of the parallelogram in terms of x. What will be the area of parallelogram of x = 30 units?

Solution:

Given: the base and the corresponding height of a parallelogram are (2x+3) units and (2x-3) units, respectively.

Area of a parallelogram = Base $\times$Height

$$=(2x+3)\times(2x-3)$$

$$=(2x)^2-(3)^2$$

$$=(4x^2-9) \text{ sq units}$$

Therefore, if x = 10. Then, the area of the parallelogram = $4\times10^2-9=400-9=391$ sq units

107. The radius of a circle is 7ab – 7bc – 14ac. Find the circumference of the circle. ($\pi = \frac{22}{7}$)

Solution:

Given: Radius of the circle = 7ab – 7bc – 14ac = r

As we know that the circumference of the circle = $2\pi r$

$$=2\times\frac{22}{7}\times(7ab-7bc-14ac)$$

$$=\frac{44}{7}\times7(ab-bc-2ac)$$

$$=44[ab-c(b+2a)]$$

108. If p + q = 12 and pq = 22, then find $p^2 + q^2$.

Solution:

Given: p+q =12 and pq = 22

Now, the value of p^2+q^2 will be calculated as follows:

$(p+q)^2 = p^2+q^2+2pq$ [Using the identity: $(a+b)^2=a^2+b^2+2ab$]

$12^2 = p^2+q^2+2\times22$

$p^2+q^2=12^2-44$

$p^2+q^2=144-44$

$p^2+q^2=100$

109. If a + b = 25 and $a^2 + b^2$ = 225, then find ab.

Solution:

Given: a+ b =25 and a^2+b^2+2ab .

$(25)^2=225+2ab$

$2ab=25^2-225$

$2ab=625-225$

$2ab=400$

$ab=\frac{400}{2}$

$ab=200$

110. If x – y = 13 and xy = 28, then find x^2 + y^2.

Solution:

Given: x –y = 13 and xy = 28

Since, $(x-y)^2=x^2+y^2-2xy$ [Using the identity: $(a-b)^2=a^2+b^2-2ab$]

$(13)^2=x^2+y^2-2\times 28$

$x^2+y^2=13^2+56$

$x^2+y^2=169+56$

$x^2+y^2=225$

111. If m – n = 16 and m^2 + n^2 = 400, then find mn.

Solution:

Given: m –n =16n and $m^2+n^2=400$

Since, $(m-n)^2=m^2+n^2=2mn$ [Using the identity: $(a-b)^2=a^2+b^2-2ab$]

$(16)^2=400-2mn$

$2mn=400-2mn$

$2mn=400-(16)^2$

$2mn=400-256$

$2mn=144$

$mn=\frac{144}{2}$

$mn=72$

112. If a^2 + b^2 = 74 and ab = 35, then find a + b.

Solution:

Given: $a^2+b^2=74$ and $ab=35$.

Since, $(a+b)^2=a^2+b^2+2ab$ [Using the identity, $(a+b)^2=a^2+b^2+2ab$]

$(a+b)^2=74+2\times35$

$(a+b)^2=74+70$

$(a+b)^2=144$

$a+b=\sqrt{144}$

$a+b=14$

113. Verify the following:

(i) (ab + bc) (ab – bc) + (bc + ca) (bc – ca) + (ca + ab) (ca – ab) = 0

(ii) (a + b + c) ($a^2+b^2+c^2$ – ab – bc – ca) = $a^3+b^3+c^3$ – 3abc

(iii) (p – q) (p^2 + pq + q^2) = p^3-q^3

(iv) (m + n) (m^2 – mn + n^2) = m^3+n^3

(v) (a + b) (a + b) (a + b) = $a^3+3a^2b+3ab^2+b^3$

(vi) (a – b) (a – b) (a – b) = $a^3-3a^2b+3ab^2-b^3$

(vii) (a^2-b^2) (a^2+b^2) + (b^2-c^2) (b^2+c^2) + (c^2-a^2) + (c^2+a^2) = 0

(viii) $(5x+8)^2-160x=(5x-8)^2$

(ix) $(7p-13q)^2+364pq=(7p+13q)^2$

(x) $\left(\frac{3p}{7}+\frac{7}{6p}\right)^2-\left(\frac{3}{7}p+\frac{7}{6p}\right)^2=2$

Solution:

(i) Taking LHS = (ab + bc) (ab – bc) +(bc+ca)(bc-ca)+(ca+ab)(ca-ab)

$=\left[(ab)^2-(bc)^2\right]+\left[(bc)^2-(ca)^2\right]+\left[(ca)^2-(ab)^2\right]$

[Using the identity: (a+b)(a-b)=a^2-b^2]

$=a^2b^2-b^2c^2+b^2c^2-c^2a^2+c^2a^2-a^2b^2$

$=0$

$=RHS$

Hence, verified.

(ii) Taking LHS = (a+b+c)$\left(a^2+b^2+c^2-ab-bc-ca\right)$

$= a\left(a^2 + b^2 + c^2 - ab - bc - ca\right) + b\left(a^2 + b^2 + c^2 - ab - bc - ca\right) + c\left(a^2 + b^2 + c^2 - ab - bc - ca\right)$

$= a^3 + ab^2 + ac^2 - a^2b - abc - a^2c + ba^2 + b^3 + bc^2 - b^2a - b^2c - bca + ca^2 + cb^2 + c^3 - cab - c^2b - c^2a$

$= a^3 + b^3 + c^3 - 3abc$

$= RHS$

Hence, verified.

(iii) Taking LHS = (p – q) $\left(p^2 + pq + q^2\right)$

$= p\left(p^2 + pq + q^2\right) - q\left(p^2 + pq + q^2\right)$

$= p^3 + p^2q + pq^2 - qp^2 - pq^2 - q^3$

$= p^3 - q^3$

$= RHS$

Hence, verified.

(iv) Taking LHS $=\left(m + n\right)\left(m^2 - mn + n^2\right)$

$= m\left(m^2 - mn + n^2\right) + n\left(m^2 - mn + n^2\right)$

$= m^3 - m^2n + mn^2 + nm^2 - mn^2 + n^3$

$= m^3 + n^3$

$= RHS$

Hence, verified.

(v) Taking LHS = $\left(a + b\right)\left(a + b\right)\left(a + b\right)$

$= \left(a + b\right)\left(a + b\right)^2$

$= \left(a + b\right)\left(a^2 + b^2 + 2ab\right)$

$= a\left(a^2 + 2ab + b^2\right) + b\left(a^2 + 2ab + b^2\right)$

$= a^3 + 2a^2b + ab^2 + ba^2 + 2ab^2 + b^3$

$= a^3 + 3a^2b + 3ab^2 + b^3$

$= RHS$

Hence, verified.

(vi) Taking LHS $=\left(a - b\right)\left(a - b\right)\left(a - b\right)$

$=(a-b)(a-b)^2$

$=(a-b)(a^2-2ab+b^2)$

$=a(a^2-2ab+b^2)+b(a^2-2ab+b^2)$

$=a^3-2a^2b+ab^2-ba^2+2ab^2-b^3$

$=a^3-3a^2b+3ab^2-b^3$

$=RHS$

Hence, verified.

(vii) Taking LHS $=(a^2-b^2)(a^2+b^2)+(b^2-c^2)(b^2+c^2)+(c^2-a^2)(c^2+a^2)$

$=(a^4-b^4+b^4-c^4+c^4-a^4)$

$=0$

$=RHS$

Hence, verified.

(viii) Taking LHS $=(5x+8)^2-160x$

$=(5x)^2+8^2+2\times5x\times8-160x$

$=(5x)^2+(8)^2+80x-160x$

$=(5x)^2+(8)^2-80x$

$=(5x)^2+(8)^2-2\times5x\times8$

$=(5x-8)$

$=RHS$

Hence, verified.

(ix) Taking LHS $=(7p-13q)^2+364pq$

$=(7p)^2+(13q)^2-2\times7p\times13q+364pq$

$=(7p)^2+(13q)^2-182pq+364pq$

$=(7p)^2+(13q)^2+182pq$

$=(7p)^2+(13q)^2+2\times7p\times13q$

$=(7p+13q)^2$

$=RHS$

Hence, verified.

(x) Taking LHS $=\left(\frac{3p}{7}+\frac{7}{6p}\right)^2-\left(\frac{3p}{7}-\frac{7}{6p}\right)^2$

$$=\left[\left(\frac{3p}{7}+\frac{7}{6p}\right)+\left(\frac{3p}{7}-\frac{7}{6p}\right)\right]\left[\left(\frac{3p}{7}+\frac{7}{6p}\right)-\left[\frac{3p}{7}-\frac{7}{6p}\right]\right]$$

$$=\left(\frac{3p}{7}+\frac{7}{6p}+\frac{3p}{7}-\frac{7}{6p}\right)\left(\frac{3p}{7}+\frac{7}{6p}-\frac{3p}{7}+\frac{7}{6p}\right)$$

$$=\frac{6p}{7}\times\frac{14}{6p}$$

$$=2$$

$$=RHS$$

Hence, verified.

114. Find the value of a, if
(i) 8a = $35^2 - 27^2$
(ii) 9a = $76^2 - 67^2$
(iii) pqa = $(3p + q)^2 - (3p - q)^2$
(iv) $pq^2a = (4pq + 3q)^2 - (4pq - 3q)^2$

Solution:

(i) Consider the equation:

8a = $35^2 - 27^2$

Now, the value of a will be calculated as follows:

$8a=(35+27)(35-27)$ [Using the identity: $a^2-b^2=(a+b)(a-b)$]

$8a=62\times 8$

$a=\frac{62\times 8}{8}$

$a=62$

Hence, the value of a is 62.

(ii) Consider the equation:

9a = $76^2 - 67^2$

Now, the value of a will be calculated as follows:

$9a=(76+67)(76-67)$ [Using the identity: $a^2-b^2=(a+b)(a-b)$]

9a = 143×9

$a=\frac{143\times 9}{9}$

a =143

Hence, the value of a is 143.

(iii) Consider the equation:

pqa = $(3p + q)^2 - (3p - q)^2$

Now, the value of a will be calculated as follows:

$$pqa = (3p+q)^2 - (3p-q)^2$$
$$pqa = [(3p+q)+(3p-q)][(3p+q)-(3p-q)]$$
$$pqa = [(3p+q+3p-q)][3p+q-3p+q]$$
$$pqa = 6p \times 2q$$
$$a = \frac{6p \times 2q}{pq}$$
$$a = \frac{(6\times 2)\,pq}{pq}$$
$$a = 12$$

(iv) Consider the equation:
$pq^2a = (4pq + 3q)^2 - (4pq - 3q)^2$
Now, the value of a will be calculated as follows:

$$pq^2a = (4pq+3q)^2 - (4pq-3q)^2$$
$$= [(4pq+3q)+(4pq-3q)][(4pq+3q)-(4pq-3q)]$$
$$= (4pq+3q+4pq-3q)(4pq+3q-4pq+3q)$$
$$= 8pq \times 6q$$
$$pq^2a = 48pq^2$$
$$a = \frac{48pq^2}{pq^2}$$
$$a = 48$$

115. What should be added to 4c (– a + b + c) to obtain 3a (a + b + c) – 2b (a – b + c)?

Solution:

Let *x* be added to the given expression to 4c (– a + b + c) to obtain 3a (a + b + c) – 2b (a – b + c).

$$x + 4c(-a+b+c) = 3a(a+b+c) - 2b(a-b+c)$$
$$x = 3a(a+b+c) - 2b(a-b+c) - 4c(-a+b+c)$$
$$= 3a^2 + 3ab + 3ac - 2ba + 2b^2 - 2bc + 4ca - 4cb - 4c^2$$
$$x = 3a^2 + ab + 7ac + 2b^2 - 6bc - 4c^2$$

116. Subtract b (b^2 + b – 7) + 5 from $3b^2$ – 8 and find the value of expression obtained for b = – 3.

Solution:

According to the question:

Required difference = $\left(3b^2-8\right)-\left[b\left(b^2+b-7\right)+5\right]$

$=3b^2-8-b^3-b^2+7b-5$

$=-b^3+2b^2+7b-13$

Now, if b = -3

The value of above expression = $-\left(-3\right)^2+2\left(-3\right)^2+7\left(-3\right)-13$

$=-\left(-27\right)+2\times9-21-13$

$=27+18-21-13$

$=45-34$

$=11$

117. If $x - \frac{1}{x} = 7$ then find the value of $x^2 + \frac{1}{x^2}$

Solution:

Given: $x - \frac{1}{x} = 7$

Now, the value of $x^2 + \frac{1}{x^2}$ will be calculated as follows:

$$\left(x-\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}-2\times x\times\frac{1}{x}$$

$$7^2 = x^2+\frac{1}{x^2}-2$$

$$x^2+\frac{1}{x^2}=49+2$$

$$x^2+\frac{1}{x^2}=51$$

Hence, the value of $x^2 + \frac{1}{x^2}$ is 51.

118. Factorise $x^2 + \frac{1}{x^2} + 2 - 3x - \frac{3}{x}$

Solution:

Consider the expression:

$x^2 + \frac{1}{x^2} + 2 - 3x - \frac{3}{x}$

Now, factorise the above expression as follows:

$$x^2+\frac{1}{x^2}+2-3x-\frac{3}{x}=x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x}-3\left(x+\frac{1}{x}\right)$$

$$=\left(x+\frac{1}{x}\right)^2-3\left(x+\frac{1}{x}\right)$$

$$=\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}-3\right)$$

119. Factorise $p^4 + q^4 + p^2q^2$.

Solution:

Consider the expression:

$p^4 + q^4 + p^2q^2$

Now, factorise the above expression as follows:

$$p^4+q^4+p^2q^2=p^4+q^4+2p^2q^2-2p^2q^2+p^2q^2$$

$$=p^4+q^4+2p^2q^2-p^2q^2$$

$$=\left[\left(p^2\right)^2+\left(q^2\right)^2+2p^2q^2\right]-p^2q^2$$

$$=\left(p^2+q^2\right)-\left(pq\right)^2$$

$$=\left(p^2+q^2+pq\right)\left(p^2+q^2-pq\right)$$

120. Find the value of

(i) $\frac{6.25\times6.25-1.75\times1.75}{4.5}$

(ii) $\frac{198\times198-102\times102}{96}$

Solution:

(i) $\frac{6.25\times6.25-1.75\times1.75}{4.5}=\frac{(6.25)^2-(1.75)^2}{4.5}$

$$=\frac{(6.25+1.75)(6.25-1.75)}{4.5}$$

$$=\frac{8\times4.5}{4.5}$$

$$=8$$

(ii) $\frac{198\times198-102\times102}{96}=\frac{(198)^2-(102)^2}{96}$

$$= \frac{(198+102)(198-102)}{96}$$
$$= \frac{300 \times 96}{96}$$
$$= 300$$

121. The product of two expressions is $x^5 + x^3 + x$. If one of them is $x^2 + x + 1$, find the other.

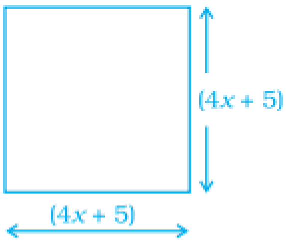
Solution:

Given: The expression $x^5 + x^3 + x$ has two product where one of them is $x^2 + x + 1$.
Let other expression is A. So, according to the question,

$$A \times (x^2 + x + 1) = x^5 + x^3 + x$$
$$A = \frac{x(x^4 + x^2 + 1)}{x^2 + x + 1}$$
$$A = \frac{x[x^4 + 2x^2 + 1 - x^2]}{x^2 + x + 1}$$
$$A = \frac{x(x^4 + 2x^2 + 1 - x^2)}{x^2 + x + 1}$$
$$A = \frac{x[(x^4 + 2x^2 + 1) - x^2]}{x^2 + x + 1}$$
$$A = \frac{x[(x^2 + 1)^2 - x^2]}{x^2 + x + 1}$$
$$A = \frac{x(x^2 + 1 + x)(x^2 + 1 - x)}{x^2 + x + 1}$$
$$A = x(x^2 + 1 - x)$$

Hence, the another expression is $x(x^2 + 1 - x)$.

122. Find the length of the side of the given square if area of the square is 625 square units and then find the value of x.



Solution:

Given: A square having length of a side (4x+5) units and are is 625 sq units.

As, area of a square = $(\text{Side})^2$

$(4x+5)^2 = 625$

$(4x+5)^2 = 25^2$

$4x+5 = 25$

$4x = 25-5$

$4x = 20$

$x = 5$

Hence, side = 4x+5 $= 4\times5+5 = 25$ units .

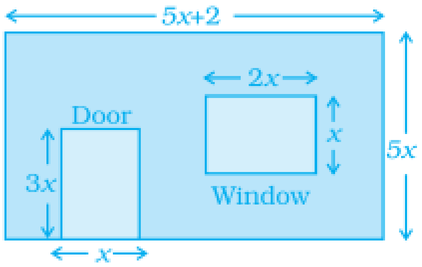
123. Take suitable number of cards given in the adjoining diagram [G(x × x) representing x^2, R (x × 1) representing x and Y (1 × 1) representing 1] to factorise the following expressions, by arranging the cards in the form of rectangles: (i) $2x^2 + 6x + 4$ (ii) $x^2 + 4x + 4$. Factorise $2x^2 + 6x + 4$ by using the figure.

Calculate the area of figure.

Solution:

The given information is incomplete for solution of this question.

124. The figure shows the dimensions of a wall having a window and a door of a room. Write an algebraic expression for the area of the wall to be painted.



Solution:

Given: A wall of dimension $5x\times(5x+2)$ having a window and a door of dimension $(2x\times x)$ and $(3x\times x)$, respectively.

Now, area of the window = $2x\times x = 2x^2$ sq units

Area of the door $= 3x\times x = 3x^2$ sq units

And area of wall = $(5x+2)\times 5x = 25x^2 + 10x$ sq units

So, area of the required part of the wall to be painted = Area of the wall – (Area of the window + Area of the door)

$$= 25x^2 + 10x - (2x^2 + 3x^2)$$
$$= 25x^2 + 10x - 5x^2$$
$$= 20x^2 + 10x$$
$$= 2\times 2\times 5\times x\times x + 2\times 5\times x$$
$$= 2\times 5\times x(2x+1)$$
$$= 10x(2x+1) \text{ sq units}$$

125. Match the expressions of column I with that of column II:

Column I	**Column II**
(1) $(21x + 13y)^2$	**(a) $441x^2 - 169y^2$**
(2) $(21x - 13y)^2$	**(b) $441x^2 + 169y^2 + 546xy$**
(3) $(21x - 13y)(21x + 13y)$	**(c) $441x^2 + 169y^2 - 546xy$**
	(d) $441x^2 - 169y^2 + 546xy$

Solution:

(i) $(21x+13y) = (21x)^2 + (13y)^2 + 2\times 21x\times 13y$

[Using the identity: $(a+b)^2 = a^2 + b^2 + 2ab$]

$$= 441x^2 + 169y^2 + 546xy$$

(ii) $(21x-13y)^2 = (21x)^2 + (13y)^2 - 2\times 21x\times 13y$

[Using the identity: $(a-b)^2 = a^2 + b^2 - 2ab$]

$$= 441x^2 + 169y^2 - 546xy$$

(iii) $(21x-13y)(21x+13y) = (21x)^2 - (13y)^2$

[Using the identity: $(a-b)(a+b) = a^2 - b^2$]

$$= 441x^2 - 169y^2$$

Hence, $(i)\to(b), (ii)\to(c), (iii)\to(a)$