

Chapter - 13

Playing with Numbers

Exercise

In each of the questions 1 to 17, out of the four options, only one is correct. Write the correct answer.

1. Generalised form of a four-digit number abdc is

- (a) $1000a + 100b + 10c + d$
- (b) $1000a + 100c + 10b + d$
- (c) $1000a + 100b + 10d + c$
- (d) $a \times b \times c \times d$

Solution:

- (c) $1000a + 100b + 10d + c$

The numbers are expressed as the sum of the product of its digits with the respective place value.

So the generalised form of abdc is $1000a + 100b + 10d + c$

2. Generalised form of a two-digit number xy is

- (a) $x + y$ (b) $10x + y$ (c) $10x - y$ (d) $10y + x$

Solution:

- (b) $10x + y$

The numbers are expressed as the sum of the product of its digits with the respective place value.

So the generalised form of xy is $10x + y$

3. The usual form of $1000a + 10b + c$ is

- (a) abc (b) abco (c) aobc (d) aboc

Solution:

- (c) aobc

The usual form to represent the aobc is $1000a + 100b + 10c$, which is equal to $1000a + 10b + c$

4. Let abc be a three-digit number. Then $abc - cba$ is not divisible by

- (a) 9 (b) 11 (c) 18 (d) 33

Solution:

(c) 18

The general form of abc is $100a+10b+c$
 $(abc - cba) = (100a+10b+c) - (100c+10b+a)$
 $= 99a - 99c$
 $= 99(a - c)$

Now,

$abc - cba$ is divisible by 99, because 99 is the factor of $abc - cba$

So, all the numbers which are the factors of 99 will also be divisible by $abc - cba$

Here, 9, 11 and 33 are the factors of 99.

But 18 is not a factor of 99.

Hence $abc - cba$ is not divisible by 18.

5. The sum of all the numbers formed by the digits x, y and z of the number xyz is divisible by

(a) 11 (b) 33 (c) 37 (d) 74

Solution:

(c) 37

It is known that, three numbers which can be formed by the using the digits x, y and z are xyz, yzx and zxy.

The general form of $xyz = 100x + 10y + z$.

The general form of $yzx = 100y + 10z + x$.

The general form of $zxy = 100z + 10x + y$.

Add the three numbers, we will get

$$xyz + yzx + zxy = (100x + 10y + z) + (100y + 10z + x) + (100z + 10x + y).$$

$$xyz + yzx + zxy = 100x + 10x + x + 100y + 10y + y + 100z + 10z + z$$

$$xyz + yzx + zxy = 111x + 111y + 111z.$$

Now, simplify the expression, we will get

$$xyz + yzx + zxy = 111(x + y + z).$$

Now, it is clear that 111 is the common factor. So,

$xyz + yzx + zxy$ is divisible by 111

Also $xyz + yzx + zxy$ will be divisible by factors of 111.

From the given options, 11, 33, and 74 are not the factors of 111 whereas 37 is the factor of 111.

6. A four-digit number aabb is divisible by 55. Then possible value(s) of b is/are

- (a) 0 and 2 (b) 2 and 5 (c) 0 and 5 (d) 7**

Solution:

(c) 0 and 5

It is known that if a number is divisible by 55, then the number should be divisible by the factors of 55

It means that the number aabb is divisible by 5. By using the divisibility test of 5, it must be 0 or 5.

7. Let abc be a three digit number. Then $abc + bca + cab$ is not divisible by

- (a) $a + b + c$ (b) 3 (c) 37 (d) 9**

Solution:

(d) 9

By simplifying the general form of abc, bca and cab = $abc + bca + cab$
 $= 111(a+b+c)$

Hence, $abc + bca + cab$ is divisible by 111 and also it is divisible by the factors of 111.

Here, 3 and 7 are the factors of 111, and $a+b+c$ is also a factor of $111(a+b+c)$.

But 9 is not the factor of 111.

8. A four-digit number 4ab5 is divisible by 55. Then the value of $b - a$ is

- (a) 0 (b) 1 (c) 4 (d) 5**

Solution:

(b) 1

We know that the four digit number 4ab5 which is divisible by 55 is also divisible by 11 and also the factors of it. By using the divisibility test of 11, the difference between the sum of the alternate digits should be a multiple of 11.

It means that

$$(4+b) - (a+5) = 0, 11, 22, \dots$$

It becomes,

$$b - a - 1 = 0$$

Hence,

$$b - a = 1$$

9. If abc is a three digit number, then the number $abc - a - b - c$ is divisible by

(a) 9 (b) 90 (c) 10 (d) 11

Solution:

(a) 9

We know that the general form of abc is $100a+10b+c$

The given number is

$$abc - a - b - c = 100a + 10b + c - a - b - c$$

By simplifying the above expression,

$$abc - a - b - c = 9(11a+b)$$

Hence,

Then number $abc - a - b - c$ is divisible by 9.

10. A six-digit number is formed by repeating a three-digit number. For example 256256, 678678, etc. Any number of this form is divisible by

(a) 7 only (b) 11 only (c) 13 only (d) 1001

Solution:

(d) 1001

From the given question, the number should be of the form $abcabc$

So the general form of $abcabc$ is $1000000a+100000b+1000c+100a+10b+c$

Now,

$$abcabc = 1000000a+100000b+1000c+100a+10b+c$$

By simplifying the above expression,

$$abcabc = 1001(100a+10b+c)$$

Hence, the six digit number should be divisible by 1001.

11. If the sum of digits of a number is divisible by three, then the number is always divisible by

(a) 2 (b) 3 (c) 6 (d) 9

Solution:

(b) 3

3 is the correct answer because the divisibility test of 3 says that the sum of the digits of a number is divisible by 3, then the number is always divisible by 3.

12. If $x + y + z = 6$ and z is an odd digit, then the three-digit number xyz is

(a) an odd multiple of 3 (b) odd multiple of 6
(c) even multiple of 3 (d) even multiple of 9

Solution:

(a) an odd multiple of 3

We have,

The sum of the digits xyz is given as $x+y+z$ is 6, where z is an odd integer.

The divisibility test of 3, the number xyz is divisible by 3.

Since the last digit is an odd digit, then xyz is an odd multiple of 3.

13. If $5A + B3 = 65$, then the value of A and B is

(a) $A = 2, B = 3$ (b) $A = 3, B = 2$ (c) $A = 2, B = 1$ (d) $A = 1, B = 2$

Solution:

(c) $A = 2, B = 1$

$$\begin{array}{r} 5A \\ + B3 \\ \hline 65 \end{array}$$

In the 1's column $A + 3 = 5$

When A is added with 3, it gives 5

Since A is a digit, it should be between 0 and 9

When you substitute $A = 2$, we get,

$$2 + 3 = 5$$

Similarly, repeat the process for 10's column

Then we will get

$$B = 1$$

Therefore

$$A = 2, \text{ and}$$

$$B = 1$$

14. If $A3 + 8B = 150$, then the value of $A + B$ is

(a) 13 (b) 12 (c) 17 (d) 15

Solution:

(a) 13

We have,

$$A3 + 8B = 150$$

$$\text{Here, } 3 + B = 0,$$

Also,

$3 + B$ is a two-digit number whose unit's digit is zero.

$$3 + B = 10$$

$$B = 7$$

Now, considering ten's column,

$$A + 8 + 1 = 15$$

$$A + 9 = 15$$

$$A = 6$$

Hence,

$$\begin{aligned} A + B &= 6 + 7 \\ &= 13 \end{aligned}$$

15. If $5A \times A = 399$, then the value of A is

- (a) 3 (b) 6 (c) 7 (d) 9

Solution.

(c)

We have,

$$5A \times A = 399$$

Here,

$A \times A = 9$ i.e. $A \times A$ is the number 9 or a number whose unit's digit is 9.

So, the number whose product with itself produces a two-digit number having its unit's digit as 9 is 7.

$$A \times A = 49$$

$$A = 7$$

Now,

$$5 \times A + 4 = 39$$

$$5 \times 7 + 4 = 39$$

So, A satisfies the product.

Hence, the value of A is 7.

16. If $6A \times B = A8B$, then the value of $A - B$ is

- (a) -2 (b) 2 (c) -3 (d) 3

Solution.

(c)

Given,

$$6A \times B = A8B$$

Let,

$$A = 1 \text{ and } B = 3$$

Then,

$$\text{LHS} = 61 \times 3 = 183 \text{ and}$$

$$\text{RHS} = 183$$

Thus, our assumption is true.

$$\begin{aligned}A-6 &= 1-3 \\ &= -2\end{aligned}$$

17. Which of the following numbers is divisible by 99

(a) 913462 (b) 114345 (c) 135792 (d) 3572406

Solution.

(b) 114345

Given a number is divisible by 99.

Now, going through the options, we observe that the number (b) is divisible by 9 and 11 both as the sum of digits of the number is divisible by 9.

Also,

Sum of digits at odd places = Sum of digits at even places.

In questions 18 to 33, fill in the blanks to make the statements true.

18. 3134673 is divisible by 3 and ____.

Solution.

9

3134673 is divisible by 3 and 9 as sum of the digits, $3+1+3+4+6+7+3 = 27$ is divisible by both 3 and 9.

19. $20x3$ is a multiple of 3 if the digit x is ____ or ____ or ____.

Solution:

1,4,7

We know that, if a number is a multiple of 3, then the sum of its digits is again a multiple of 3
So,

$2+0+x+3$ is a multiple of 3.

$x + 5 = 0, 3, 6, 9, 12, 15$

But, x is a digit of the number $20x3$.

x can take values 0, 1, 2, 3, 9.

$x + 5 = 6$ or 9 or 12

Hence,

$x = 1$ or 4 or 7

20. $3x5$ is divisible by 9 if the digit x is ____.

Solution:

1

Since, the number 3×5 is divisible by 9, then the sum of its digits is also divisible by 9.

So,

$3 + x + 5$ is divisible by 9.

$x + 8$ can take values 9, 18, 27, ...

But x is a digit of the number 3×5 ,

So, $x = 1$.

21. The sum of a two-digit number and the number obtained by reversing the digits is always divisible by _____.

Solution:

11

Let ab be any two-digit number, then the number obtained by reversing its digits is ba .

Now,

$$\begin{aligned} ab + ba &= (10a + b) + (10b + a) \\ &= 11a + 11b \\ &= 11(a + b) \end{aligned}$$

Hence, $ab + ba$ is always divisible by 11 and $(a + b)$.

22. The difference of a two-digit number and the number obtained by reversing its digits is always divisible by _____.

Solution:

9

Let ab be any two-digit number,

We have

$$\begin{aligned} ab - ba &= (10a + b) - (10b + a) \\ &= 9a - 9b \\ &= 9(a - b) \end{aligned}$$

Hence, $ab - ba$ is always divisible by 9 and $(a - b)$.

23. The difference of three-digit number and the number obtained by putting the digits in reverse order is always divisible by 9 and _____.

Solution:

11

Let abc be a three-digit number,

Then we have

$$\begin{aligned} abc - cba &= (100a + 10b + c) - (100c + 10b + a) \\ &= (100a - a) + (c - 100c) \\ &= 99a - 99c = 99(a - c) \\ &= 9 \times 11 \times (a - c) \end{aligned}$$

Hence, $abc - cba$ is always divisible by 9, 11 and $(a - c)$.

24. If $\begin{array}{r} 2 \quad B \\ + A \quad B \\ \hline 8 \quad A \end{array}$ then $A = \underline{\hspace{2cm}}$ and $B = \underline{\hspace{2cm}}$.

Solution:

$A = 6$ and $B = 3$

By adding the given digit according to the place value,

$$20 + B + 10A + B = 80 + A$$

$$9A + 2B = 60$$

$$9A = 60 - 2B$$

As, both A and B are whole numbers,

60 - 2B should be divisible by 9

So, the value of B should be 3

Putting $B = 3$ in $9A + 2B = 60$

Simplifying this, we get,

$$A = 6.$$

25. If $\begin{array}{r} A \quad B \\ \times B \\ \hline 9 \quad 6 \end{array}$ then $A = \underline{\hspace{2cm}}$ and $B = \underline{\hspace{2cm}}$.

Solution:

$A = 2$ and $B = 4$

It is given that the units place contains a 6. So, the product of B and B should give a number with 6 in units place. It is possible when $B = 4$ or 6.

When $B = 4$,

$$(10A + B) \times B = 96$$

It becomes $(10A + 4) \times 4 = 96$

$$40A + 16 = 96,$$

By simplifying the equation, it gives $A = 2$.

Similarly,

When $B = 6$,

$$(10A + B) \times (B) = 96$$

By simplifying, it becomes $(10A + 6) \times 6 = 96$

$$60A + 36 = 96, \text{ gives } A = 1.$$

Hence, $A = 2$ $B = 4$ or $A = 1$ $B = 6$

$$\begin{array}{r} \text{B} \quad 1 \\ \times \quad B \\ \hline 4 \quad 9B \end{array}$$

26. If $\begin{array}{r} \text{B} \quad 1 \\ \times \quad B \\ \hline 4 \quad 9B \end{array}$ then $B = \underline{\hspace{2cm}}$.

Solution:

$$B = 7$$

We have, it is clear that the value of B should be greater than 4 and less than 9.

If the B value is less than 4, the solution should have only two digits. If the B value is greater than 9, the solution should have more than 3 digit.

So, the value of B ranges from 4 to 9. The 10's place when multiplied by B gives 49

Hence,

$$B \times B = 49$$

$$B = 7$$

27. 1×35 is divisible by 9 if $x = \underline{\hspace{2cm}}$.

Solution:

0

We know that, when a number is a multiple of 9, then the sum of digits in the number is divisible by 9

So, by adding the digits, we get,

$$x+9 = 9$$

Hence,

$$x = 0$$

28. A four-digit number $abcd$ is divisible by 11, if $d + b = \underline{\hspace{2cm}}$ or $\underline{\hspace{2cm}}$

Solution:

$$a+c \text{ or } b+d$$

Using the divisibility rule of 11, if $abcd$ is divisible by 11,

Then

$$a-b + c-d = 0$$

$$a+ c = b+ d.$$

29. A number is divisible by 11 if the differences between the sum of digits at its odd places and that of digits at the even places is either 0 or divisible by ____.

Solution:

11

Assume that the four digit number $abcd$

Here, the digits at odd places are a and c and in even places are b and d

Hence,

$$(a+c) - (b+d) = 0$$

It follows that, $(a+c) - (b+d)$ is divisible by 11, when $(a+c) - (b+d)$ is not equal to 0.

30. If a 3-digit number abc is divisible by 11, then _____ is either 0 or multiple of 11.

Solution:

$(a+c)-b$

From the divisibility rule of 11, we say that $(a+c)-b$ is either 0 or a multiple of 11.

31. If $A \times 3 = 1A$, then $A =$ _____.

Solution:

5

Using the place values of the numbers, we can write it as:

$$10 + A = 3A$$

$$10 = 2A$$

$$A = 5$$

32. If $B \times B = AB$, then either $A = 2, B = 5$ or $A =$ _____, $B =$ _____.

Solution:

$A= 1$ and $B= 6$

From the given equations, B can take the values between 4 and 9

When

$$B = 4,$$

$$4 \times 4 = 16$$

Then,

$$A=1 \text{ and } B = 6$$

33. If the digit 1 is placed after a 2-digit number whose tens is t and ones digit is u, the new number is _____.

Solution:

tu1

Let, the number is initially $10t+u$ or tu , but after adding 1 to the unit place, tu gets shifted to the one unit higher.

It means that $100t+10u+1$ or $tu1$.

State whether the statements given in questions 34 to 44 are true (T) or false (F):

34. A two-digit number ab is always divisible by 2 if b is an even number.

Solution.

True

By the test of divisibility by 2, we know that a number is divisible by 2, if its unit's digit is even.

35. A three-digit number abc is divisible by 5 if c is an even number.

Solution:

False

By the test of divisibility by 5, we know that if a number is divisible by 5, then its one's digit will be either 0 or 5, so, the numbers ending with the digits 0 or 5 are divisible by 5.

36. A four-digit number $abcd$ is divisible by 4 if ab is divisible by 4.

Solution.

False

As we know that, if a number is divisible by 4, then the number formed by its digits in unit's and ten's place is divisible by 4.

37. A three-digit number abc is divisible by 6 if c is an even number and $a + b + c$ is a multiple of 3.

Solution.

True

If a number is divisible by 6, then it is divisible by both 2 and 3. Since, abc is divisible by 6, it is also divisible by 2 and 3.

Therefore, c is an even number and the sum of digits is divisible by 3, multiple of 3.

38. Number of the form $3N + 2$ will leave remainder 2 when divided by 3.

Solution.

True

Let $x = 3N + 2$.

Then, it can be written as,

$x = (\text{a multiple of } 3) + 2$

x is a number which is 2 more than a multiple of 3

x is a number, which when divided by 3, leaves the remainder 2.

39. Number $7N + 1$ will leave remainder 1 when divided by 7.

Solution.

True

Given,

A number of the form $7N + 1 = x$

Here, we observe that x is a number which is one more than a multiple of 7.

So,

When x is divided by 7, it leaves the remainder 1.

40. If a number a is divisible by b , then it must be divisible by each factor of b .

Solution.

True

Let

$a = 27$, and $b = 9$

Here,

When 27 is divisible by 9, we will get 3

Now, consider the factor of $9 = 1, 3$ and 9

So,

27 is divisible by the factors such as 1 and 3. Hence the statement is true.

41. If $AB \times 4 = 192$, then $A + B = 7$.

Solution:

False

From the given question, the value of B should be either 3 or 8.

If you take the value of B is 3, it should be equal to 19.

Hence, the value of B is 8.

We know that, the value of A should between 0 and 9.

Hence,

$$A \times 4 = 19 - 3$$

$$= 16$$

$$A = \frac{16}{4}$$

$$A = 4$$

Therefore,

$$A = 4 \text{ and } B = 8$$

Hence,

$$A + B = 4 + 8$$

$$= 12$$

42. If $AB + 7C = 102$, where $B \neq 0$, $C \neq 0$, then $A + B + C = 14$.

Solution:

True

From the given number, B+C is either 2 or the two digit number that gives the unit digit as 2.

It is given that $B \neq 0$, $C \neq 0$,

If

B or C = 5 or 7, then A should be 3, then it becomes $A + B + C = 14$

or,

If

B = C = 6, and A = 2, we get $A + B + C = 14$

43. If $213x27$ is divisible by 9, then the value of x is 0.

Solution:

False

If 213×27 is divisible by 9, then the sum of the digits of a number is a multiple of 9.

$$2+1+3+x+2+7 = 15+x$$

Then,

15x must be any multiple of 9 such as 9, 18, 27 ...

Now let,

$$15+x = 18$$

$$x = 18 - 15$$

$$x = 3$$

44. If $N \div 5$ leaves remainder 3 and $N \div 2$ leaves remainder 0, then $N \div 10$ leaves remainder 4.

Solution:

False

Given that,

When N is divided by 5, it leaves the remainder 5.

$$N = 5n+3 \text{ where } n= 0, 1, 2, 3, \dots$$

Similarly, when N is divided by 2, it leaves the remainder 0.

So N is an even Number.

(Using divisibility test rule of 2).

But in $N = 5n+3$, the second term is odd.

So, 5n is an odd number.

When you substitute $n = 1, 3, 5 \dots$ in $5n+3$, we will get 8, 18, 28 ...

Now, if we divide N by 10,

$$N = 10n+8$$

So, when N is divided by 10, it always leaves the remainder 8.

Solve the following :

45. Find the least value that must be given to number a so that the number 91876a2 is divisible by 8.

Solution:

$$a = 3$$

By using the divisibility test of 8, if a number is divisible by 8, then the last three digit of a number should be divisible by 8.

In the given question, the last three digit is 6a2.

The value of 'a' varies from 0 to 9.

If $a = 0$, then $6a2 = 602$ is not divisible by 8.

If $a = 1$, then $6a2 = 612$ is not divisible by 8.

If $a = 2$, then $6a2 = 622$ is not divisible by 8.

If $a = 3$, then $6a2 = 632$ is divisible by 8.

Hence, the least value of 'a' is 3.

$$\begin{array}{r} 1 \ P \\ \times \ P \\ \hline \end{array}$$

46. If $\overline{96}$ where $Q - P = 3$, then find the values of P and Q.

Solution:

We have,

$$Q - P = 3$$

From the question,

$$P \times P = 6$$

Therefore,

$$P = 4 \text{ or } 6$$

If, $P = 4$, $Q = 5$ not suitable for the question.

Hence, $P = 6$ and $Q = 9$.

47. If $1AB + CCA = 697$ and there is no carry-over in addition, find the value of $A + B + C$.

Solution:

According to the question there is no carry over in addition.

$$1 + C = 6$$

$$C = 5$$

Also,

$$A + C = 9$$

$$A = 4$$

Now,

$$B + A = 7$$

$$B = 3$$

$$\text{Hence } A + B + C = 12$$

48. A five-digit number AABAA is divisible by 33. Write all the numbers of this form.

Solution:

The number divisible by 33 is also divisible by 3 and 11.

Sum of its digits is also divisible by 3,

$$A + A + B + A + A = 0, 3, 6, 9,$$

$$4A + B = 0, 3, 6, 9,$$

Sum of its digits is also divisible by 11,

$$(2A + B) - 2A = 0, 11, 22, 33,$$

So,

$$B = 0 \text{ (single digit)}$$

$$A = 3, 6, 9, \dots$$

Hence numbers are 33066, 66066, and 99099.

Find the value of the letters in each of the following questions.

49.

$$\begin{array}{r} A \quad A \\ +A \quad A \\ \hline XA \quad Z \end{array}$$

Solution:

From the 1's column

$$A = 0 \text{ to } 9.$$

From the 10's column

$$A = 5 \text{ to } 9$$

(The $A + A$ is two digit number)

$$A = 5 \text{ to } 8 \text{ is not satisfy the } 10\text{'s column}$$

Hence,

$$A = 9$$

Then,

$$Z = 8 \text{ and}$$

$$X = 1.$$

50.

$$\begin{array}{r} 8 \quad 5 \\ +4 \quad A \\ \hline B \quad C \quad 3 \end{array}$$

Solution:

From the 1's column,

The value of $5 + A$ is a two digit number whose last digit is 3.

Therefore,

$$5 + A = 13$$

$$A = 8$$

From the column 10's,

$$10B + C = 8 + 4 + 1$$

$$10B + C = 13$$

$$= 10 \times 1 + 3$$

Hence,

$$A = 8,$$

$$B = 1 \text{ and}$$

$$C = 3$$

51.

$$\begin{array}{r} B \quad 6 \\ + 8 \quad A \\ \hline C \quad A \quad 2 \end{array}$$

Solution:

From the 1's column,

$$6 + A = 12$$

$$A = 6$$

From the 10's column,

$$CA = B + 8 + 1$$

$$C6 = B + 9$$

From the above equation,

$B + 9$ is a number with unit digit 6.

$$B = 7$$

$$C = 1$$

Hence,

$$A = 6,$$

$$B = 7 \text{ and}$$

$$C = 1.$$

52.

$$\begin{array}{r} 1 \ B \ A \\ + A \ B \ A \\ \hline 8 \ B \ 2 \end{array}$$

Solution:

From the 1's column, $A + A = 12$

$$A = 6$$

From the 10's column,

$$B + B + 1 = B$$

The above condition is fulfilled when $B = 9$.

From the 100's column,

$$A + 1 + 1 = 8$$

$A = 6$ = Satisfy the 1's column value.

So,

$$A = 6,$$

$$B = 9.$$

53.

$$\begin{array}{r} C \ B \ A \\ + C \ B \ A \\ \hline 1 \ A \ 3 \ 0 \end{array}$$

Solution:

From the 1's column,

$$A + A = 0$$

$$A = 0 \text{ or } 5$$

From the 10's column,

$$B + B + 1 = 3$$

$$B = 1, 6$$

From the 100's column,

$$C + C + 1 = 1A$$

$A = 0$ is not satisfy the condition of 10's column.

Therefore,

$$A = 5$$

$$C + C = 15 - 1$$

$$C = 7$$

$B = 1$ is not satisfy the condition of 10's column.

Therefore,

$$B = 6$$

Hence,

$$A = 5,$$

$$B = 6 \text{ and}$$

$$C = 7$$

54.

$$\begin{array}{r} B \quad A \quad A \\ + B \quad A \quad A \\ \hline 3 \quad A \quad 8 \end{array}$$

Solution:

From the 1's column,

$$A + A = 8$$

$$A = 9$$

(because $A + A$ is not a single digit number)

From the 100's column,

$$B + B + 1 = 3$$

$$B = 1$$

Hence,

$$A = 9 \text{ and}$$

$$B = 1$$

55.

$$\begin{array}{r} A \quad 0 \quad 1 \quad B \\ + 1 \quad 0 \quad A \quad B \\ \hline B \quad 1 \quad 0 \quad 8 \end{array}$$

Solution:

From the 1's column, $B + B = 8$

$$B = 9$$

($B + B$ not satisfy the 1000's column)

From the 10's column,

$$A + 1 + 1 = 0$$

$$A = 8$$

Hence,

A = 8 and

B = 9

56.

$$\begin{array}{r} A \quad B \\ \times \quad 6 \\ \hline C \quad 6 \quad 8 \\ \hline \end{array}$$

Solution:

6 x B is a number with unit digit 8.

Therefore,

B = 3 or 8

For B = 3 equation $A \times 6 + 1 = C6$ is not satisfied.

For B = 8 equation $A \times 6 + 1 = C6$ is satisfied.

Hence, A = 7 and B = 8.

57.

$$\begin{array}{r} AB \\ \times AB \\ \hline 6AB \\ \hline \end{array}$$

Solution:

Given,

$$AB \times AB = 6AB$$

... (i)

B x B is a number with unit digit B

So,

B = 1 or 5

The square of a two digit number is a three digit number.

Therefore,

A can be 1, 2 and 3.

If

A = 1, 2, 3 and

B = 1, equation (i) is not valid.

For,

A = 1,

$B = 5$ equation (i) is not valid.

Therefore,

$$A = 2,$$

$B = 5$ equation is valid.

Hence,

$$A = 2,$$

$$B = 5.$$

58.

$$\begin{array}{r} AA \\ \times A \\ \hline CAB \end{array}$$

Solution:

$AA \times A =$ three digit number, that has unit digit B.

Therefore,

A can be 4-9

And

A = 0, 1, 2, 3 give a single digit or a two digit number.

As the ten's digit of the product is A itself.

Therefore,

$A \neq 4, 5, 6, 7$ and 8.

Hence,

$$A = 9,$$

$$B = 1,$$

$$C = 8.$$

59.

$$\begin{array}{r} AB \\ - B7 \\ \hline 45 \end{array}$$

Solution:

In the ones column= $B - 7 = 5$

As,

$$12 - 7 = 5$$

Therefore,

$$B = 2$$

60.

$$\begin{array}{r} 8\ A\ B\ C \\ -\ A\ B\ C\ 5 \\ \hline D\ 4\ 8\ 8 \end{array}$$

Solution:

$$\text{The ones column} = C - 5 = 8$$

We know,

$$13 - 5 = 8$$

Therefore

$$C = 3$$

In the ten's column,

$$B - (C + 1) = 8$$

$$B = 8 + C + 1$$

$$B = 8 + 3 + 1$$

$$B = 12$$

Therefore,

$$B = 2$$

In the hundred's column

$$A - (B + 1) = 4$$

$$A = 4 + B + 1$$

$$A = 4 + 2 + 1$$

$$= 7$$

In the thousand's column,

$$8 - A = D$$

$$8 - 7 = D$$

$$D = 1$$

So,

$$A = 7,$$

$$B = 2,$$

$$C = 3 \text{ and}$$

$$D = 1.$$

61. If $2A7 \div A = 33$, then find the value of A.

Solution:

$$\frac{200 + 10A + 7}{A} = 33$$

$$200 + 10A + 7 = 33A$$

$$207 = 33A - 10A$$

$$207 = 23A$$

$$A = \frac{207}{23}$$

$$A = 9$$

62. 212 x 5 is a multiple of 3 and 11. Find the value of x.

Solution:

As,

212 x 5 is a multiple of 3 $2 + 1 + 2 + x + 5 = 0, 3, 6, 9, 12, 15, 18$

$10 + x = 0, 3, 6, \dots$

$x = 2, 5, 8 \dots$ (i)

Since,

212 x 5 is a multiple of 11,

$(2 + 2 + 5) - (1 + x) = 0, 11, 22, 33$

$8 - x = 0, 11, 22$

$x = 8 \dots$ (ii)

From eqs. (i) and (ii), we have,

$x = 8$

63. Find the value of k where 31k 2 is divisible by 6.

Solution:

As 31k2 is divisible by 6.

31k2 will also be divisible by 2 and 3.

If 31k2 is divided by 3, sum of digit will be multiple of 3.

$3 + 1 + k + 2 = 0, 3, 6, 9, 12, \dots$

$k + 6 = 0, 3, 6, 9, 12$

$k = 0 \text{ or } 3, 6, 9$

64. 1y3y6 is divisible by 11. Find the value of y.

Solution:

It is given that, $y3y6$ is divisible by 11

Therefore, we get,

$$(1 + 3 + 6) - (y + y) = 0, 11, 22$$

$$10 - 2y = 0, 11, 22\ldots$$

$$10 - 2y = 0$$

$$2y = 10$$

$$y = 5$$

65. $756x$ is a multiple of 11, find the value of x .

Solution:

$756x$ is a multiple of 11.

$756x$ is divisible by 11, so $(7 + 6) - (5 + x)$ is a multiple of 11.

$$8 - x = 0$$

$$x = 8.$$

66. A three-digit number $2a3$ is added to the number 326 to give a three-digit number $5b9$ which is divisible by 9. Find the value of $b - a$.

Solution:

$$\begin{array}{r} 2a3 \\ +326 \\ \hline 5b9 \end{array}$$

From 1's column,

$$3 + 6 = 9$$

From 10's Column,

$$2 + 3 = 5$$

Therefore,

$a + 2$ is a single digit number b .

$$a + 2 = b$$

$$b - a = 2.$$

67. Let $E = 3$, $B = 7$ and $A = 4$. Find the other digits in the sum

$$\begin{array}{r}
 \text{B A S E} \\
 + \text{B A L L} \\
 \hline
 \text{G A M E S} \\
 \hline
 \end{array}$$

Solution:

From 1's column,

$$3 + L = S$$

$$S - L = 3 \quad \text{.....(1)}$$

From 10's column,

$$S + L = 3 \quad \text{.....(2)}$$

Solving equation (1) and (2)

$$S = 3 \text{ and}$$

$$L = 0$$

From 100's column,

$$4 + 4 = M$$

$$M = 8$$

From 1000's column,

$$7 + 7 = G4$$

$$G = 1$$

Hence,

$$L = 0,$$

$$M = 8 \text{ and}$$

$$G = 1$$

68. Let D = 3, L = 7 and A = 8. Find the other digits in the sum

$$\begin{array}{r}
 \text{M A D} \\
 + \text{A S} \\
 + \text{A} \\
 \hline
 \text{B U L L} \\
 \hline
 \end{array}$$

Solution:

From the 1's column

3 + S + 8 is a two digit number with unit digit is 7.

Therefore,

$$S = 6$$

From 10's column,

$2A + 1 = 16 + 1 = 7$ and carry 1 From 100's column,
 $M + 1$ is a 2 digit number.

So,

$$M = 9$$

Now,

$$M + 1 = 9 + 1 \\ = 10$$

$$B = 1,$$

$$U = 0$$

Hence,

$$S = 6,$$

$$M = 9,$$

$$B = 1 \text{ and}$$

$$U = 0$$

69. If from a two-digit number, we subtract the number formed by reversing its digits then the result so obtained is a perfect cube. How many such numbers are possible? Write all of them.

Solution:

Let xy is a two digit number.

Then reversing number is yx .

$$xy - yx = (10x + y) - (10y + x)$$

$$xy - yx = 9(x - y)$$

$xy - yx =$ a perfect cube number and multiple of 9.

Therefore,

$$x - y = 3 \quad x = y + 3$$

In above equation, $b = 0$ to 6.

For $b = 0$, $a = 3$ and number is 30

For $b = 1$, $a = 4$ and number is 41

For $b = 2$, $a = 5$ and number is 52

For $b = 3$, $a = 6$ and number is 63

For $b = 4$, $a = 7$ and number is 74

For $b = 5$, $a = 8$ and number is 85

For $b = 6$, $a = 9$ and number is 96

70. Work out the following multiplication.

12345679

× 9

Use the result to answer the following questions.

(a) What will be 12345679×45 ?

(b) What will be 12345679×63 ?

(c) By what number should 12345679 be multiplied to get 888888888?

(d) By what number should 12345679 be multiplied to get 999999999?

Solution:

123456789

× 9

111111111

(a) Solution of the multiplication:

123456789

× 45

555555555

(b) Solution of the multiplication:

123456789

× 63

777777777

(c) Solution of the multiplication:

123456789

× 72

888888888

(d) Solution of the multiplication:

123456789

× 81

999999999

71. Find the value of the letters in each of the following:

$$(i) \begin{array}{r} P \quad Q \\ \times \quad 6 \\ \hline Q \quad Q \quad Q \\ \hline \end{array}$$

$$(ii) \begin{array}{r} 2 \quad L \quad M \\ L \quad M \quad 1 \\ \hline M \quad 1 \quad 8 \\ \hline \end{array}$$

Solution:

(i)

$$\begin{array}{r} PQ \\ \times 6 \\ \hline QQQ \end{array}$$

From the first column,

$$6 \times P + 1 = Q.$$

Therefore,

$$Q = 2, 4, 6 \text{ or } 8$$

Equation $6 \times P + 1$ is satisfied when $Q = 4$.

After solving $6 \times P + 1$

$$P = 7 \text{ and}$$

$$Q = 4$$

(ii)

$$\begin{array}{r} 2LM \\ +LM1 \\ \hline M18 \end{array}$$

From the first column,

$$M + 1 = 8$$

Therefore,

$$M = 7$$

From the second column,

$$L + M = \text{a number with unit digit } 1.$$

So,

$$L = 4.$$

$$\text{Hence } L = 4 \text{ and } M = 7$$

72. If 148101B095 is divisible by 33, find the value of B.

Solution:

Given that the number 148101S095 is divisible by 33, therefore it is also divisible by 3 and 11 both.

Now, the number is divisible by 3, its sum of digits is a multiple of 3.

$1 + 4 + 8 + 1 + 0 + 1 + B + 0 + 9 + 5$ is a multiple of 3.

$29 + B = 0, 3, 6, 9, \dots$

$B = 1, 4, 7 \quad \dots(i)$

Also, given number is divisible by 11,

$(1 + 8 + 0 + B + 9) - (4 + 1 + 1 + 0 + 5) = 0, 11, 22, \dots$

$(18 + B) - 11 = 0, 11, 22$

$B + 7 = 0, 11, 22$

$B + 7 = 11$

$B = 4 \quad \dots(ii)$

From Eqn. (i) and (ii),

$B = 4$

73. If 123123A4 is divisible by 11, find the value of A.

Solution:

Given,

12312344 is divisible by 11, then we have $(1 + 3 + 2 + A) - (2 + 1 + 3 + 4)$ is a multiple of 11.

$(6 + A) - 10 = 0, 11, 22, \dots$

$A - 4 = 0, 11, 22, \dots$

$A - 4 = 0$

[A is a digit of the given number]

$A = 4$

74. If $56x32y$ is divisible by 18, find the least value of y.

Solution:

It is given that, the number $56x32y$ is divisible by 18.

Then, it is also divisible by each factor of 18.

Thus, it is divisible by 2 as well as 3.

Now, the number is divisible by 2, its unit's digit must be an even number that is 0, 2, 4, 6, Therefore, the least value of y is 0.

Again, the number is divisible by 3 also, sum of its digits is a multiple of 3.

$5 + 6 + x + 3 + 2 + y$ is a multiple of 3

$$16 + x + y = 0, 3, 6, 9, \dots$$

$$16 + x = 18$$

$$x = 2,$$

which is the least value of x .