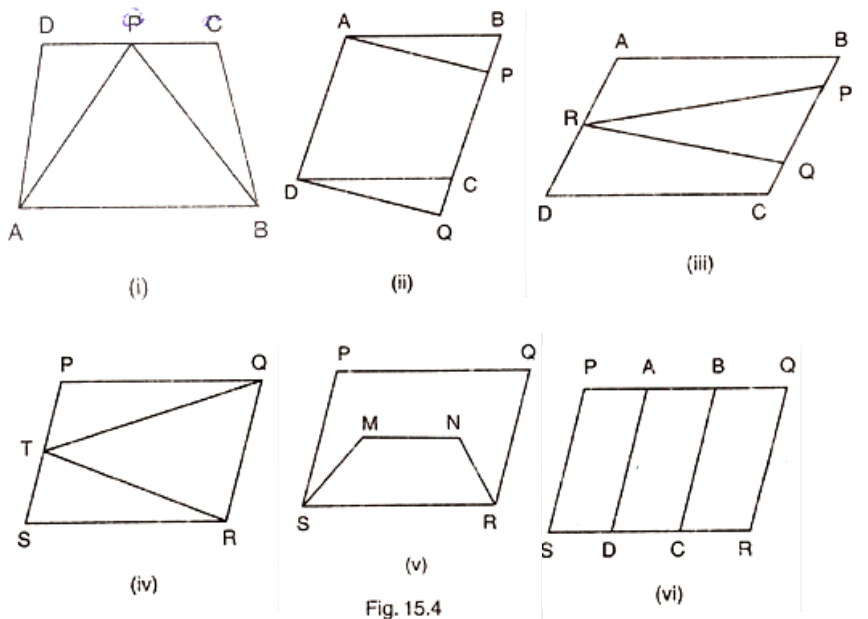


15. Areas of Parallelograms

Exercise 15.1

1. Question

Which of the following figures lie on the same base and between the same parallel. In such a case, write the common base and two parallel:



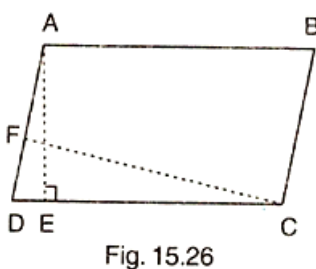
Answer

- (i) $\triangle PCD$ and trapezium ABCD are on the same base CD and between the same parallels AB and DC.
- (ii) Parallelogram ABCD and $\triangle PCD$ are on the same base CD and between the same parallel AD and BQ.
- (iii) Parallelogram ABCD and $\triangle PQR$ are between the same parallels AD and BQ.
- (iv) $\triangle QRT$ and Parallelogram PQRS are on the same base QR and between the same parallels QR and PS.
- (v) Parallelogram PQRS and trapezium SMNR are on the same base SR but they are not between the same parallel.
- (vi) Parallelograms PQRS, AQRD, BCQR are between the same parallels also parallelograms PQRS, BPSC and APSD are between the same parallels.

Exercise 15.2

1. Question

If Fig. 15.26, ABCD is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB=16$ cm, $AE=8$ cm and $CF=10$ cm, find AD.



Answer

Given that,

In a parallelogram ABCD:

$CD = AB = 16$ cm (Opposite sides of parallelogram are equal)

We know that,

Area of parallelogram = Base * Corresponding altitude

Area of parallelogram ABCD:

$$CD * AE = AD * CF$$

$$16 \text{ cm} * 18 \text{ cm} = AD * 10 \text{ cm}$$

$$AD = \frac{16 \times 18}{10}$$

$$AD = 12.8 \text{ cm}$$

2. Question

In Q. No. 1, if AD=6 cm, CF=10 cm and AE=8 cm, find AB.

Answer

$$\text{Area of parallelogram ABCD} = AD * CF \text{ (i)}$$

Again,

$$\text{Area of parallelogram ABCD} = DC * AE \text{ (ii)}$$

From (i) and (ii), we get

$$AD * CF = DC * AE$$

$$6 * 10 = DC * 8$$

$$DC = \frac{60}{8}$$

$$= 7.5 \text{ cm}$$

Therefore,

$$AB = DC = 7.5 \text{ cm (Opposite sides of parallelogram are equal)}$$

3. Question

Let ABCD be a parallelogram of area 124 cm^2 . If E and F are the mid-points of sides AB and CD respectively, then find the area of parallelogram AEFD.

Answer

Given that,

$$\text{Area of parallelogram ABCD} = 124 \text{ cm}^2$$

Construction: Draw AP perpendicular to DC

$$\text{Proof: Area of parallelogram AFED} = DF * AP \text{ (i)}$$

$$\text{Area of parallelogram EBCF} = FC * AP \text{ (ii)}$$

And,

$$DF = FC \text{ (iii) (F is the mid-point of DC)}$$

Compare (i), (ii) and (iii), we get

$$\text{Area of parallelogram AEFD} = \text{Area of parallelogram EBCF}$$

Therefore,

$$\text{Area of parallelogram AEFD} = \frac{\text{Area of parallelogram ABCD}}{2}$$

$$= \frac{124}{2} = 62 \text{ cm}^2$$

4. Question

If ABCD is a parallelogram, then prove that

$$\text{ar}(\Delta ABD) = \text{ar}(\Delta BCD) = \text{ar}(\Delta ABC) = \text{ar}(\Delta ACD) = \frac{1}{2} \text{ar}(\text{||}^{\text{gm}} \text{ABCD})$$

Answer

We know that,

Diagonal of parallelogram divides it into two quadrilaterals.

Since,

AC is the diagonal

Then, Area (ΔABC) = Area (ΔACD)

$$= \frac{1}{2} \text{Area of parallelogram ABCD (i)}$$

Since,

BD is the diagonal

Then, Area (ΔABD) = Area (ΔBCD)

$$= \frac{1}{2} \text{Area of parallelogram ABCD (ii)}$$

Compare (i) and (ii), we get

Therefore,

$$\text{Area}(\Delta ABC) = \text{Area}(\Delta ACD) = \text{Area}(\Delta ABD) = \text{Area}(\Delta BCD) = \frac{1}{2} \text{Area of parallelogram ABCD}$$

Exercise 15.3

1. Question

In Fig. 15.74, compute the area of quadrilateral ABCD.

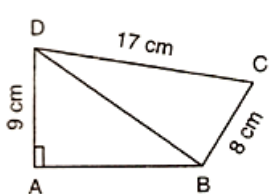


Fig. 15.74

Answer

Given that,

$$DC = 17 \text{ cm}$$

$$AD = 9 \text{ cm}$$

And,

$$BC = 8 \text{ cm}$$

In ΔBCD , we have

$$CD^2 = BD^2 + BC^2$$

$$(17)^2 = BD^2 + (8)^2$$

$$BD^2 = 289 - 64$$

$$= 15$$

In ΔABD , we have

$$BD^2 = AB^2 + AD^2$$

$$(15)^2 = AB^2 + (9)^2$$

$$AB^2 = 225 - 81$$

$$= 144$$

$$= 12$$

Therefore,

$$\text{Area of Quadrilateral } ABCD = \text{Area } (\Delta ABD) + \text{Area } (\Delta BCD)$$

$$\text{Area of quadrilateral } ABCD = \frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 17)$$

$$= 54 + 68$$

$$= 112 \text{ cm}^2$$

2. Question

In Fig. 15.75, PQRS is a square and T and U are respectively, the mid-points of PS and QR. Find the area of ΔOTS if $PQ=8$ cm

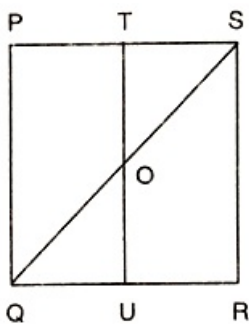


Fig. 15.75

Answer

From the figure,

T and U are the mid points of PS and QR respectively

Therefore,

$$TU \parallel PQ$$

$$TO \parallel PQ$$

Thus,

In ΔPQS and T is the mid-point of PS and $TO \parallel PQ$

Therefore,

$$TO = \frac{1}{2} * PQ$$

$$= 4 \text{ cm}$$

Also,

$$TS = \frac{1}{2} * PS$$

$$= 4 \text{ cm}$$

Therefore,

$$\text{Area } (\Delta OTS) = \frac{1}{2}(TO * TS)$$

$$= \frac{1}{2}(4 * 4)$$

$$= 8 \text{ cm}^2$$

3. Question

Compute the area of trapezium PQRS in Fig. 15.76.

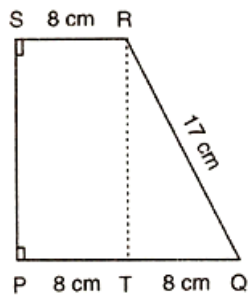


Fig. 15.76

Answer

We have,

Area of trapezium PQRS = Area of rectangle PSRT + Area (Δ QRT)

$$\text{Area of trapezium PQRS} = PT * RT + \frac{1}{2}(QT * RT)$$

$$= 8 * RT + \frac{1}{2}(8 * RT)$$

$$= 12 * RT$$

In Δ QRT, we have

$$QR^2 = QT^2 + RT^2$$

$$RT^2 = QR^2 - QT^2$$

$$RT^2 = (17)^2 - (8)^2$$

$$= 225$$

$$= 15$$

Hence,

$$\text{Area of trapezium PQRS} = 12 * 15$$

$$= 180 \text{ cm}^2$$

4. Question

In Fig. 15.77, $\angle AOB = 90^\circ$, $AC = BC$, $OA = 12 \text{ cm}$ and $OC = 6.5 \text{ cm}$. Find the area of Δ AOB.

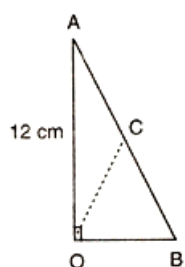


Fig. 15.77

Answer

Since,

The mid-point of the hypotenuse of a right triangle is equidistant from the vertices

Therefore,

$$CA = CB = OC$$

$$CA = CB = 6.5 \text{ cm}$$

$$AB = 13 \text{ cm}$$

In right ($\triangle OAB$)

We have,

$$AB^2 = OB^2 + OA^2$$

$$13^2 = OB^2 + 12^2$$

$$OB = 5 \text{ cm}$$

Therefore,

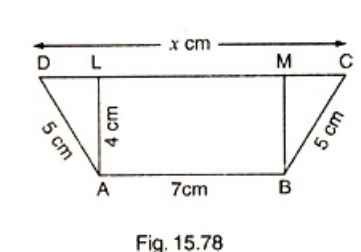
$$\text{Area } (\triangle AOB) = \frac{1}{2}(OA * OB)$$

$$= \frac{1}{2} (12 * 5)$$

$$= 30 \text{ cm}^2$$

5. Question

In Fig. 15.78, ABCD is a trapezium in which $AB=7 \text{ cm}$, $AD=BC=5 \text{ cm}$, $DC=x \text{ cm}$, and distance between AB and DC is 4 cm. Find the value of x and area of trapezium ABCD.



Answer

Draw AL perpendicular to DC

And,

BM perpendicular DC

Then,

$$AL = BM = 4 \text{ cm}$$

And,

$$LM = 7 \text{ cm}$$

In $\triangle ADL$, we have

$$AD^2 = AL^2 + DL^2$$

$$25 = 16 + DL^2$$

$$DL = 3 \text{ cm}$$

Similarly,

$$MC = \sqrt{BC^2 - BM^2}$$

$$= \sqrt{25 - 16}$$

$$= 3 \text{ cm}$$

Therefore,

$$x = CD = CM + ML + LD$$

$$= 3 + 7 + 3$$

$$= 13 \text{ cm}$$

$$\text{Area of trapezium ABCD} = \frac{1}{2} (AB + CD) * AL$$

$$= \frac{1}{2} (7 + 13) * 4$$

$$= 40 \text{ cm}^2$$

6. Question

In Fig. 15.79, OCDE is a rectangle inscribed in a quadrant of a circle of radius 10 cm. If $OE = 2\sqrt{5}$, find the area of the rectangle.

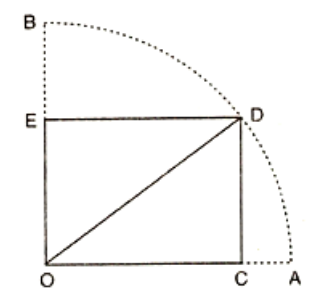


Fig. 15.79

Answer

We have,

$$OD = 10 \text{ cm}$$

And,

$$OE = 2\sqrt{5} \text{ cm}$$

Therefore,

$$OD^2 = OE^2 + DE^2$$

$$DE = \sqrt{OD^2 - OE^2}$$

$$= \sqrt{(10)^2 - (2\sqrt{5})^2}$$

$$= 4\sqrt{5} \text{ cm}$$

Therefore,

$$\text{Area of trapezium OCDE} = OE * DE$$

$$= 2\sqrt{5} * 4\sqrt{5}$$

$$= 40 \text{ cm}^2$$

7. Question

In Fig. 15.80, ABCD is a trapezium in which $AB \parallel DC$. Prove that $\text{ar}(\Delta AOD) = \text{ar}(\Delta BOC)$.

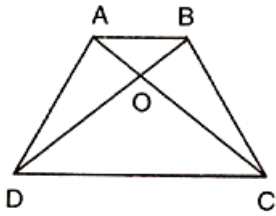


Fig. 15.80

Answer

Given that,

ABCD is a trapezium with $AB \parallel DC$

To prove: Area ($\triangle AOD$) = Area ($\triangle BOC$)

Proof: Since,

$\triangle ABC$ and $\triangle ABD$ are on the same base AB and between the same parallels AB and DC

Therefore,

$$\text{Area}(\triangle ABC) = \text{Area}(\triangle ABD)$$

$$\text{Area}(\triangle ABC) - \text{Area}(\triangle AOB) = \text{Area}(\triangle ABD) - \text{Area}(\triangle AOB)$$

$$\text{Area}(\triangle AOD) = \text{Area}(\triangle BOC)$$

Hence, proved

8. Question

In Fig. 15.81, ABCD and CDEF are parallelograms. Prove that

$$\text{ar}(\triangle ADE) = \text{ar}(\triangle BCF).$$

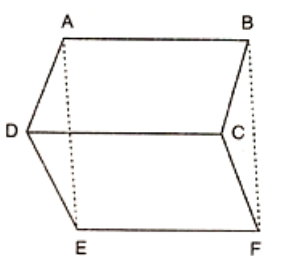


Fig. 15.81

Answer

Given that,

ABCD is a parallelogram

So,

$$AD = BC$$

CDEF is a parallelogram

So,

$$DE = CF$$

ABFE is a parallelogram

So,

$$AE = BF$$

Thus,

In $\triangle ADE$ and $\triangle BCF$, we have

$$AD = BC$$

$$DE = CF$$

And,

$$AE = BF$$

So, by SSS congruence rule, we have

$$\triangle ADE \cong \triangle BCF$$

Therefore,

$$\text{Area}(\triangle ADE) = \text{Area}(\triangle BCF)$$

9. Question

Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that:

$$\text{ar}(\triangle APB) \times \text{ar}(\triangle CPD) = \text{ar}(\triangle APD) \times \text{ar}(\triangle BPC).$$

Answer

Construction: Draw BQ perpendicular to AC

And,

DR perpendicular to AC

Proof: We have,

$$\text{L.H.S} = \text{Area}(\triangle APB) \times \text{Area}(\triangle CPD)$$

$$= \frac{1}{2} (AP \times BQ) \times \frac{1}{2} (PC \times DR)$$

$$= \left(\frac{1}{2} \times PC \times BQ\right) \times \left(\frac{1}{2} \times AP \times DR\right)$$

$$= \text{Area}(\triangle BPC) \times \text{Area}(\triangle APD)$$

$$= \text{R.H.S}$$

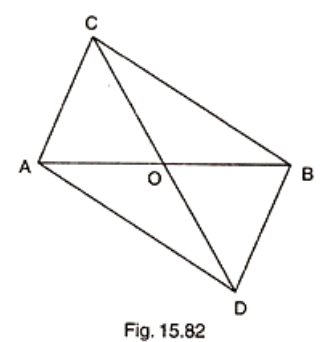
Therefore,

$$\text{L.H.S} = \text{R.H.S}$$

Hence, proved

10. Question

In Fig. 15.82, ABC and ABD are two triangles on the base AB. If line segment CD is bisected by AB at O, Show that $\text{ar}(\triangle ABC) = \text{ar}(\triangle ABD)$.



Answer

Given that,

CD bisected AB at O

To prove: Area ($\triangle ABC$) = Area ($\triangle ABD$)

Construction: CP perpendicular to AB and DQ perpendicular to AB

Proof: Area ($\triangle ABC$) = $\frac{1}{2}$ (AB * CP) (i)

Area ($\triangle ABD$) = $\frac{1}{2}$ (AB * DQ) (ii)

In $\triangle CPO$ and $\triangle DQO$, we have

$\angle CPO = \angle DQO$ (Each 90°)

Given that,

CO = DO

$\angle COP = \angle DOQ$ (Vertically opposite angle)

Then, by AAS congruence rule

$\triangle CPO \cong \triangle DQO$

Therefore,

CP = DQ (By c.p.c.t)

Thus,

Area ($\triangle ABC$) = Area ($\triangle ABD$)

Hence, proved

11. Question

If P is any point in the interior of a parallelogram ABCD, then prove that area of the triangle APB is less than half the area of parallelogram.

Answer

Construction: Draw $DN \perp AB$ and $PM \perp AB$.

Proof: Area of parallelogram ABCD = AB * DN

Area ($\triangle APB$) = $\frac{1}{2}$ (AB * PM)

= AB * PM < AB * DN

= $\frac{1}{2}$ (AB * PM) < $\frac{1}{2}$ (AB * DN)

= Area ($\triangle APB$) < $\frac{1}{2}$ Area of parallelogram ABCD

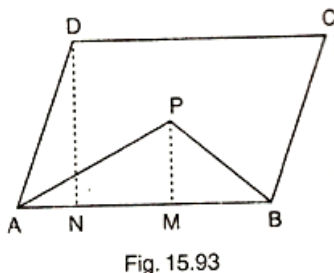


Fig. 15.93

12. Question

If AD is a median of a triangle ABC, then prove that triangles ADB and ADC are equal in area. If G is the mid-point of median AD, prove that $\text{ar}(\triangle BGC) = 2\text{ar}(\triangle AGC)$.

Answer

Construction: Draw $AM \perp BC$

Proof: Since,

AD is the median of $\triangle ABC$

Therefore,

$$BD = DC$$

$$BD \times AM = DC \times AM$$

$$\frac{1}{2}(BD \times AM) = \frac{1}{2}(DC \times AM)$$

$$\text{Area}(\triangle ABD) = \text{Area}(\triangle ACD) \text{ (i)}$$

Now, in $\triangle BGC$

GD is the median

Therefore,

$$\text{Area}(\triangle BGD) = \text{Area}(\triangle CGD) \text{ (ii)}$$

Also,

In $\triangle ACD$, CG is the median

$$\text{Therefore, Area}(\triangle AGC) = \text{Area}(\triangle CGD) \text{ (iii)}$$

From (i), (ii) and (iii) we have

$$\text{Area}(\triangle BGD) = \text{Area}(\triangle AGC)$$

But,

$$\text{Area}(\triangle BGC) = 2 \text{Area}(\triangle BGD)$$

Therefore,

$$\text{Area}(\triangle BGC) = 2 \text{Area}(\triangle AGC)$$

Hence, proved

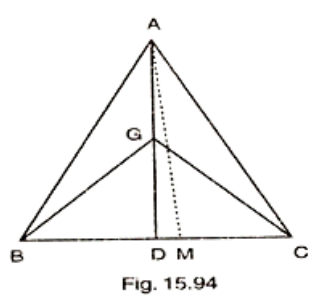


Fig. 15.94

13. Question

A point D is taken on the side BC of a $\triangle ABC$ such that $BD = 2DC$. Prove that

$$\text{ar}(\triangle ABD) = 2\text{ar}(\triangle ADC)$$

Answer

Given that,

In $\triangle ABC$,

We have

$$BD = 2DC$$

To prove: $\text{Area}(\triangle ABD) = 2 \text{Area}(\triangle ADC)$

Construction: Take a point E on BD such that, $BE = ED$

Proof: Since,

$BE = ED$ and,

$BD = 2DC$

Then,

$BE = ED = DC$

Median of the triangle divides it into two equal triangles

Since,

AE and AD are the medians of $\triangle ABD$ and $\triangle AEC$ respectively

Therefore,

$\text{Area}(\triangle ABD) = 2 \text{Area}(\triangle AED)$ (i)

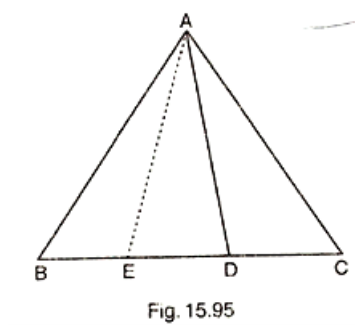
And,

$\text{Area}(\triangle ADC) = \text{Area}(\triangle AED)$ (ii)

Comparing (i) and (ii), we get

$\text{Area}(\triangle ABD) = 2 \text{Area}(\triangle ADC)$

Hence, proved



14. Question

ABCD is a parallelogram whose diagonals intersect at O. If P is any point on BO, prove that

(i) $\text{ar}(\triangle ADO) = \text{ar}(\triangle CDO)$

(ii) $\text{ar}(\triangle ABP) = \text{ar}(\triangle CBP)$

Answer

Given that,

ABCD is a parallelogram

To prove: (i) $\text{Area}(\triangle ADO) = \text{Area}(\triangle CDO)$

(ii) $\text{Area}(\triangle ABP) = \text{Area}(\triangle CBP)$

Proof: We know that,

Diagonals of a parallelogram bisect each other

Therefore,

$AO = OC$ and,

$BO = OD$

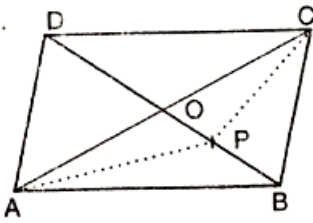


Fig. 15.96

(i) In $\triangle DAC$, DO is a median.

Therefore,

$$\text{Area}(\triangle ADO) = \text{Area}(\triangle CDO)$$

Hence, proved

(ii) In $\triangle BAC$, since BO is a median

Then,

$$\text{Area}(\triangle BAO) = \text{Area}(\triangle BCO) \quad (i)$$

In $\triangle PAC$, since PO is the median

Then,

$$\text{Area}(\triangle PAO) = \text{Area}(\triangle PCO) \quad (ii)$$

Subtract (ii) from (i), we get

$$\text{Area}(\triangle BAO) - \text{Area}(\triangle PAO) = \text{Area}(\triangle BCO) - \text{Area}(\triangle PCO)$$

$$\text{Area}(\triangle ABP) = \text{Area}(\triangle CBP)$$

Hence, proved

15. Question

ABCD is a parallelogram in which BC is produced to E such that $CE=BC$. AE intersects CD at F.

(i) Prove that $\text{ar}(\triangle ADF) = \text{ar}(\triangle ECF)$

(ii) If the area of $\triangle DFB=3 \text{ cm}^2$, find the area of $\text{||}^{\text{gm}} \text{ABCD}$.

Answer

In $\triangle ADF$ and $\triangle ECF$

We have,

$$\angle ADF = \angle ECF$$

$$AD = EC$$

And,

$$\angle DFA = \angle CFA$$

So, by AAS congruence rule,

$$\triangle ADF \cong \triangle ECF$$

$$\text{Area}(\triangle ADF) = \text{Area}(\triangle ECF)$$

$$DF = CF$$

BF is a median in $\triangle BCD$

$$\text{Area}(\triangle BCD) = 2 \text{ Area}(\triangle BDF)$$

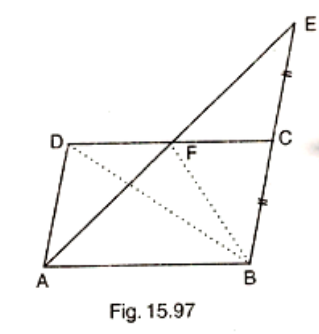
$$\text{Area}(\triangle BCD) = 2 * 3$$

$$= 6\text{cm}^2$$

Hence, Area of parallelogram ABCD = 2 Area (Δ BCD)

$$= 2 * 6$$

$$= 12 \text{ cm}^2$$



16. Question

ABCD is a parallelogram whose diagonals AC and BD intersect at O. A line through O intersects AB at P and DC at Q. Prove that

$$\text{ar}(\Delta \text{POA}) = \text{ar}(\Delta \text{QOC})$$

Answer

In Δ POA and Δ QOC, we have

$$\angle \text{AOP} = \angle \text{COQ} \text{ (Vertically opposite angle)}$$

$$\text{OA} = \text{OC} \text{ (Diagonals of parallelogram bisect each other)}$$

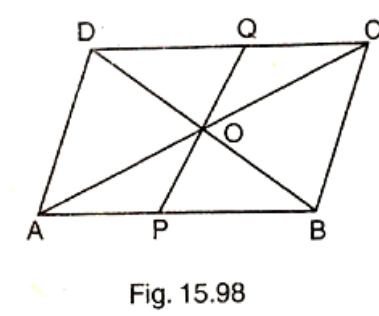
$$\angle \text{PAC} = \angle \text{QCA} \text{ (AB} \parallel \text{DC, alternate angles)}$$

So, by ASA congruence rule, we have

$$\Delta \text{POA} \cong \Delta \text{QOC}$$

$$\text{Area} (\Delta \text{POA}) = \text{Area} (\Delta \text{QOC})$$

Hence, proved



17. Question

ABCD is a parallelogram. E is a point on BA such that BE = 2 EA and F is a point on DC, such that DF=2FC. Prove that AE CF is a parallelogram whose area is one third of the area of parallelogram ABCD.

Answer

Construction: Draw FG perpendicular to AB

Proof: We have,

$$\text{BE} = 2 \text{ EA}$$

And,

$$\text{DF} = 2 \text{ FC}$$

$$AB - AE = 2 AE$$

And,

$$DC - FC = 2 FC$$

$$AB = 3 AE$$

And,

$$DC = 3 FC$$

$$AE = \frac{1}{3} AB \text{ and } FC = \frac{1}{3} DC \text{ (i)}$$

But,

$$AB = DC$$

Then,

$$AE = FC \text{ (Opposite sides of a parallelogram)}$$

Thus,

$$AE \parallel FC \text{ such that } AE = FC$$

Then,

AECF is a parallelogram

$$\text{Now, Area of parallelogram (AECF)} = \frac{1}{3} (AB * FG) \text{ [From (i)]}$$

$$3 \text{ Area of parallelogram AECF} = AB * FG \text{ (ii)}$$

And,

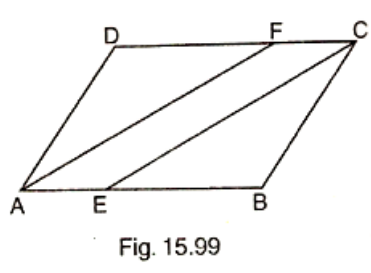
$$\text{Area of parallelogram ABCD} = AB * FG \text{ (iii)}$$

Compare equation (ii) and (iii), we get

$$3 \text{ Area of parallelogram AECF} = \text{Area of parallelogram ABCD}$$

$$\text{Area of parallelogram AECF} = \frac{1}{3} \text{ Area of parallelogram ABCD}$$

Hence, proved



18. Question

In a ΔABC , P and Q are respectively the mid-point of AB and BC and R is the mid-point of AP. Prove that:

$$(i) \text{ ar}(\Delta PBQ) = \text{ar}(\Delta ARC)$$

$$(ii) \text{ ar}(\Delta PQR) = \frac{1}{2} \text{ar}(\Delta ARC)$$

$$(iii) \text{ ar}(\Delta RQC) = \frac{3}{8} \text{ar}(\Delta ABC)$$

Answer

(i) We know that each median of a triangle divides it into two triangles of equal area.

Since,

CR is a median of ΔCAP

Therefore,

$$\text{Area } (\Delta CRA) = \frac{1}{2} \text{Area } (\Delta CAP) \text{ (i)}$$

Also,

CP is a median of ΔCAB

Therefore,

$$\text{Area } (\Delta CAP) = \text{Area } (\Delta CPB) \text{ (ii)}$$

From (i) and (ii), we get

Therefore,

$$\text{Area } (\Delta ARC) = \frac{1}{2} \text{Area } (\Delta CPB) \text{ (iii)}$$

PQ is a median of ΔPBC

Therefore,

$$\text{Area } (\Delta CPB) = 2 \text{Area } (\Delta PQB) \text{ (iv)}$$

From (iii) and (iv), we get

$$\text{Area } (\Delta ARC) = \text{Area } (\Delta PBQ) \text{ (v)}$$

(ii) Since QP and QR medians of ΔQAB and ΔQAP respectively.

$$\text{Area } (\Delta QAP) = \text{Area } (\Delta QBP) \text{ (vi)}$$

And,

$$\text{Area } (\Delta QAP) = 2 \text{Area } (\Delta QRP) \text{ (vii)}$$

From (vi) and (vii), we get

$$\text{Area } (\Delta PRQ) = \frac{1}{2} \text{Area } (\Delta PBQ) \text{ (viii)}$$

From (v) and (viii), we get

$$\text{Area } (\Delta PRQ) = \frac{1}{2} \text{Area } (\Delta ARC) \text{ (ix)}$$

(iii) Since CR is a median of ΔCAP

Therefore,

$$\text{Area } (\Delta ARC) = \frac{1}{2} \text{Area } (\Delta CAP)$$

$$= \frac{1}{2} * \frac{1}{2} \text{Area } (\Delta ABC) \text{ (Therefore, CP is a median of } \Delta ABC)$$

$$= \frac{1}{4} \text{Area } (\Delta ABC) \text{ (x)}$$

Since,

RQ is a median of ΔRBC .

Therefore,

$$\text{Area } (\Delta RQC) = \frac{1}{2} \text{Area } (\Delta RBC)$$

$$= \frac{1}{2} [\text{Area } (\Delta ABC) - \text{Area } (\Delta ARC)]$$

$$= \frac{1}{2} [\text{Area}(\triangle ABC) - \frac{1}{4} \text{Area}(\triangle ABC)]$$

$$= \frac{3}{4} \text{Area}(\triangle ABC)$$

19. Question

ABCD is a parallelogram, G is the point on AB such that AG = 2GB, E is a point of DC such that CE = 2DE and F is the point of BC such that BF = 2FC. Prove that:

$$(i) \text{ar}(\triangle ADGE) = \text{ar}(\triangle GBCE)$$

$$(ii) \text{ar}(\triangle EGB) = \frac{1}{6} \text{ar}(\triangle ABCD)$$

$$(iii) \text{ar}(\triangle EFC) = \frac{1}{2} \text{ar}(\triangle EBF)$$

$$(iv) \text{ar}(\triangle EBG) = \frac{3}{2} \text{ar}(\triangle EFC)$$

Answer

Given: ABCD is a parallelogram in which

$$AG = 2 GB$$

$$CE = 2 DE$$

$$BF = 2 FC$$

(i) Since ABCD is a parallelogram, we have AB $\parallel$ CD and AB = CD

Therefore,

$$BG = \frac{1}{3} AB$$

And,

$$DE = \frac{1}{3} CD = \frac{1}{3} AB$$

Therefore,

$$BG = DE$$

ADEH is a parallelogram (Since, AH is parallel to DE and AD is parallel to HE)

Area of parallelogram ADEH = Area of parallelogram BCIG (i)

(Since, DE = BG and AD = BC parallelogram with corresponding sides equal)

Area ($\triangle HEG$) = Area ($\triangle EGI$) (ii)

(Diagonals of a parallelogram divide it into two equal areas)

From (i) and (ii), we get,

$$\text{Area of parallelogram ADEH} + \text{Area}(\triangle HEG) = \text{Area of parallelogram BCIG} + \text{Area}(\triangle EGI)$$

Therefore,

$$\text{Area of parallelogram ADEG} = \text{Area of parallelogram GBCE}$$

(ii) Height, h of parallelogram ABCD and $\triangle EGB$ is the same

$$\text{Base of } \triangle EGB = \frac{1}{3} AB$$

$$\text{Area of parallelogram ABCD} = h * AB$$

$$\text{Area}(\triangle EGB) = \frac{1}{2} * \frac{1}{3} AB * h$$

$$= \frac{1}{6} (h) * AB$$

$$= \frac{1}{6} * \text{Area of parallelogram ABCD}$$

(iii) Let the distance between EH and CB = x

$$\text{Area } (\triangle EBF) = \frac{1}{2} * BF * x$$

$$= \frac{1}{2} * \frac{2}{3} BC * x$$

$$= \frac{1}{3} * BC * x$$

$$\text{Area } (\triangle EFC) = \frac{1}{2} * CF * x$$

$$= \frac{1}{2} * \frac{1}{3} * BC * x$$

$$= \frac{1}{2} * \text{Area } (\triangle EBF)$$

$$\text{Area } (\triangle EFC) = \frac{1}{2} * \text{Area } (\triangle EBF)$$

(iv) As, it has been proved that

$$\text{Area } (\triangle EGB) = \frac{1}{6} * \text{Area of parallelogram ABCD (iii)}$$

$$\text{Area } (\triangle EFC) = \frac{1}{3} \text{Area } (\triangle EBC)$$

$$\text{Area } (\triangle EFC) = \frac{1}{2} * \frac{1}{3} * CE * EP$$

$$= \frac{1}{2} * \frac{1}{3} * \frac{2}{3} * CD * EP$$

$$= \frac{1}{6} * \frac{2}{3} * \text{Area of parallelogram ABCD}$$

$$\text{Area } (\triangle EFC) = \frac{2}{3} * \text{Area } (\triangle EGB) \text{ [By using (iii)]}$$

$$\text{Area } (\triangle EGB) = \frac{3}{2} \text{Area } (\triangle EFC)$$

20. Question

In Fig. 15.83, $CD \parallel AE$ and $CY \parallel BA$.

(i) Name a triangle equal in area of $\triangle CBX$

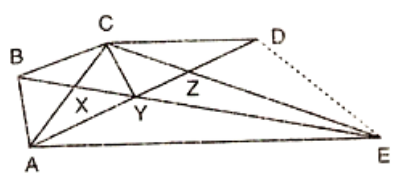


Fig. 15.83

(ii) Prove that $\text{ar}(\triangle ZDE) = \text{ar}(\triangle CZA)$

(iii) Prove that $\text{ar}(\triangle BCYZ) = \text{ar}(\triangle EDZ)$

Answer

(i) $\triangle AYC$ and $\triangle BCY$ are on the same base CY and between the same parallels

$CY \parallel AB$

$$\text{Area } (\triangle AYC) = \text{Area } (\triangle BCY)$$

(Triangles on the same base and between the same parallels are equal in area)

Subtracting ΔCXY from both sides we get,

Area (ΔAYC) - Area (ΔCXY) = Area (ΔBCY) - Area (ΔCXY) (Equals subtracted from equals are equals)

Area (ΔCBX) = Area (ΔAXY)

(ii) Since, ΔACC and ΔADE are on the same base AF and between the same parallels

$CD \parallel AF$

Then,

Area (ΔACE) = Area (ΔADE)

Area (ΔCEA) + Area (ΔAZE) = Area (ΔAZE) + Area (ΔDZE)

Area (ΔCZA) = Area (ΔZDE) (i)

(iii) Since, ΔCBY and ΔCAY are on the same base CY and between the same parallels

$CY \parallel BA$

Then,

Area (ΔCBY) = Area (ΔCAY)

Adding Area (ΔCYG) on both sides we get

Area (ΔCBY) + Area (ΔCYG) = Area (ΔCAY) + Area (ΔCYG)

Area ($\Delta BCYZ$) = Area (ΔCZA) (ii)

Compare (i) and (ii), we get

Area ($\Delta BCZY$) = Area (ΔEDZ)

21. Question

In Fig. 15.84, PSDA is a parallelogram in which $PQ=QR=RS$ and $AP \parallel BQ \parallel CR$. Prove that

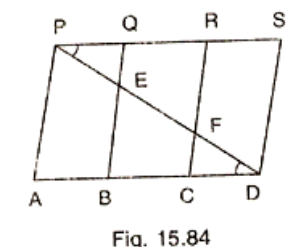


Fig. 15.84

$\text{ar}(\Delta PQE) = \text{ar}(\Delta CFD)$

Answer

Given that,

PSDA is a parallelogram

Since,

$AP \parallel BQ \parallel CR \parallel DS$ and $AD \parallel PS$

Therefore,

$PQ = CD$ (i)

In ΔBED ,

C is the mid-point of BD and $CF \parallel BE$

Therefore,

F is the mid-point of ED

$$EF = PE$$

Similarly,

$$PE = FD \text{ (ii)}$$

In $\triangle PQE$ and $\triangle CFD$, we have

$$PE = FD$$

$$\angle EPQ = \angle FDC \text{ (Alternate angle)}$$

And,

$$PQ = CD$$

So, by SAS theorem, we have

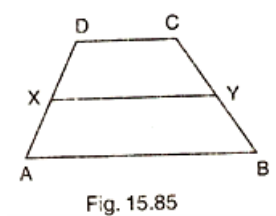
$$\triangle PQE \cong \triangle CFD$$

$$\text{Area } (\triangle PQE) = \text{Area } (\triangle CFD)$$

Hence, proved

22. Question

In Fig. 15.85, ABCD is a trapezium in which $AB \parallel DC$ and $DC = 40$ cm and $AB = 60$ cm. If X and Y are, respectively, the mid points of AD and BC, Prove that



(i) $XY = 50$ cm (ii) DCYX is a trapezium

$$\text{(iii) } \text{ar}(\text{trap. DCYX}) = \frac{9}{11} \text{ar}(\text{trap. XYBA})$$

Answer

(i) Join DY and extend it to meet AB produced at P

$$\angle BYP = \angle CYD \text{ (Vertically opposite angles)}$$

$$\angle DCY = \angle PBY \text{ (Since } DC \parallel AP \text{)}$$

$$BY = CY \text{ (Since Y is the mid-point of BC)}$$

Hence, by A.S.A. congruence rule

$$\triangle BYP \cong \triangle CYD$$

$$DY = YP$$

And,

$$DC = BP$$

Also,

X is the mid-point of AD

Therefore,

$$XY \parallel AP$$

And,

$$XY = \frac{1}{2} AP$$

$$XY = \frac{1}{2} (AB + BP)$$

$$XY = \frac{1}{2} (AB + DC)$$

$$XY = \frac{1}{2} (60 + 40)$$

$$= \frac{1}{2} \times 100$$

$$= 50 \text{ cm}$$

(ii) We have,

$$XY \parallel AP$$

$$XY \parallel AB \text{ and } AB \parallel DC$$

$$XY \parallel DC$$

DCYX is a trapezium.

(iii) Since X and Y are the mid-points of AD and BC respectively

Therefore,

Trapezium DCYX and ABYX are of same height and assuming it as 'h' cm

$$\text{Area (Trapezium DCYX)} = \frac{1}{2} (DC + XY) * h$$

$$= \frac{1}{2} (40 + 50) h$$

$$= 45h \text{ cm}^2$$

$$\text{Area (Trapezium ABYX)} = \frac{1}{2} (AB + XY) * h$$

$$= \frac{1}{2} (60 + 50) * h$$

$$= 55h \text{ cm}^2$$

So,

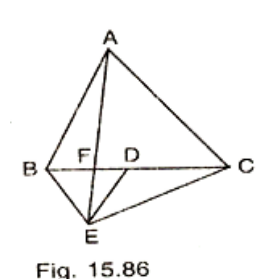
$$\frac{\text{Area of trapezium DCYX}}{\text{Area of trapezium ABYX}} = \frac{45h}{55h}$$

$$= \frac{9}{11}$$

$$\text{Area of trapezium DCYX} = \frac{9}{11} \text{ Area of trapezium ABYX}$$

23. Question

In Fig. 15.86, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. AE intersects BC in F. Prove that



$$(i) \text{ar}(\Delta BDE) = \frac{1}{4} \text{ar}(\Delta ABC)$$

$$(ii) \text{ar}(\Delta BDE) = \frac{1}{2} \text{ar}(\Delta BAE)$$

$$(iii) \text{ar}(\Delta BFE) = \text{ar}(\Delta AFD)$$

$$(iv) \text{ar}(\Delta ABC) = \text{ar}(\Delta BEC)$$

$$(v) \text{ar}(\Delta FED) = \frac{1}{8} \text{ar}(\Delta AFC)$$

$$(vi) \text{ar}(\Delta BFE) = 2\text{ar}(\Delta EFD)$$

Answer

Given that,

ABC and BDF are two equilateral triangles

Let,

$$AB = BC = CA = x$$

Then,

$$BD = \frac{x}{2} = DE = BF$$

(i) We have,

$$\text{Area}(\Delta ABC) = \frac{\sqrt{3}}{4} x^2$$

$$\text{Area}(\Delta BDE) = \frac{\sqrt{3}}{4} \left(\frac{x}{2}\right)^2$$

$$= \frac{1}{4} * \frac{\sqrt{3}}{4} x^2$$

$$\text{Area}(\Delta BDE) = \frac{1}{4} \text{Area}(\Delta ABC)$$

(ii) It is given that triangles ABC and BED are equilateral triangles

$$\angle ACB = \angle DBE = 60^\circ$$

BE || AC (Since, alternate angles are equal)

Triangles BAF and BEC are on the same base BE and between the same parallels BE and AC

Therefore,

$$\text{Area}(\Delta BAE) = \text{Area}(\Delta BEC)$$

$$\text{Area}(\Delta BAE) = 2 \text{Area}(\Delta BDE) \text{ (Therefore, ED is the median)}$$

$$\text{Area}(\Delta BDE) = \frac{1}{2} \text{Area}(\Delta BAE)$$

(iii) Since,

ΔABC and ΔAED are equilateral triangles

Therefore,

$$\angle ABC = 60^\circ \text{ and,}$$

$$\angle BDE = 60^\circ$$

$$\angle ABC = \angle BDE$$

$$AB \parallel DE$$

Triangles BED and AED are on the same base ED and between the same parallels AB and DE

Therefore,

$$\text{Area}(\Delta BED) = \text{Area}(\Delta AED)$$

$$\text{Area } (\triangle BED) - \text{Area } (\triangle EFD) = \text{Area } (\triangle AED) - \text{Area } (\triangle EFD)$$

$$\text{Area } (\triangle BEF) = \text{Area } (\triangle AFD)$$

(iv) Since,

ED is the median of $\triangle BEC$

Therefore,

$$\text{Area } (\triangle BEC) = 2 \text{ Area } (\triangle BDE)$$

$$\text{Area } (\triangle BEC) = 2 * \frac{1}{4} \text{ Area } (\triangle ABC)$$

$$\text{Area } (\triangle BEC) = \frac{1}{2} \text{ Area } (\triangle ABC)$$

$$\text{Area } (\triangle ABC) = 2 \text{ Area } (\triangle BEC)$$

(v) Let h be the height of vertex E, corresponding to the side BD on $\triangle BDE$

Let H be the vertex A, corresponding to the side BC in $\triangle ABC$

From part (i), we have

$$\text{Area } (\triangle BDE) = \frac{1}{4} \text{ Area } (\triangle ABC)$$

$$\frac{1}{2} * BD * h = \frac{1}{4} \left(\frac{1}{2} * BC * H \right)$$

$$BD * h = \frac{1}{4} (2 BD * H)$$

$$h = \frac{1}{2} H \quad (1)$$

From part (iii), we have

$$\text{Area } (\triangle BFC) = \text{Area } (\triangle AFD)$$

$$= \frac{1}{2} * PD * H$$

$$= \frac{1}{2} * FD * 2h$$

$$= 2 \left(\frac{1}{2} * FD * h \right)$$

$$= 2 \text{ Area } (\triangle EFD)$$

$$(vi) \text{Area } (\triangle AFC) = \text{Area } (\triangle AFD) + \text{Area } (\triangle ADC)$$

$$= \text{Area } (\triangle BFE) + \frac{1}{2} \text{Area } (\triangle ABC)$$

[Using part (iii) and AD is the median of Area $\triangle ABC$]

$$= \text{Area } (\triangle BFE) + \frac{1}{2} * 4 \text{ Area } (\triangle BDE) \text{ [Using part (i)]}$$

$$\text{Area } (\triangle BFC) = 2 \text{ Area } (\triangle FED) \quad (2)$$

$$\text{Area } (\triangle BDE) = \text{Area } (\triangle BFE) + \text{Area } (\triangle FED)$$

$$2 \text{ Area } (\triangle FED) + \text{Area } (\triangle FED)$$

$$3 \text{ Area } (\triangle FED) \quad (3)$$

From above equations,

$$\text{Area } (\triangle AFC) = 2 \text{ Area } (\triangle FED) + 2 * 3 \text{ Area } (\triangle FED)$$

$$= \text{Area } (\triangle FED)$$

Hence,

$$\text{Area } (\triangle FED) = \frac{1}{8} \text{Area } (\triangle AFC)$$

24. Question

D is the mid-point of side BC of $\triangle ABC$ and E is the mid-point of BD. If O is the mid-point of AE, prove that $\text{ar}(\triangle BOE) = \frac{1}{8} \text{ar}(\triangle ABC)$

Answer

Join A and D to get AD median

(Median divides the triangle into two triangles of equal area)

Therefore,

$$\text{Area } (\triangle ABD) = \frac{1}{2} \text{Area } (\triangle ABC)$$

Now,

Join A and E to get AE median

Similarly,

We can prove that,

$$\text{Area } (\triangle ABE) = \frac{1}{2} \text{Area } (\triangle ABD)$$

$$\text{Area } (\triangle ABE) = \frac{1}{4} \text{Area } (\triangle ABC) \quad (\text{Area } (\triangle ABD) = \frac{1}{2} \text{Area } (\triangle ABC) \text{ (i)})$$

Join B and O and we get BO median

Now,

$$\text{Area } (\triangle BOE) = \frac{1}{2} \text{Area } (\triangle ABE)$$

$$\text{Area } (\triangle BOE) = \frac{1}{2} * \frac{1}{4} \text{Area } (\triangle ABC)$$

$$\text{Area } (\triangle BOE) = \frac{1}{8} \text{Area } (\triangle ABC)$$

25. Question

In Fig. 15.87, X and Y are the mid-point of AC and AB respectively, $QP \parallel BC$ and CYQ and BXP are straight lines. Prove that:

$$\text{ar}(\triangle ABP) = \text{ar}(\triangle ACQ).$$

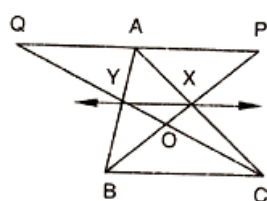


Fig. 15.87

Answer

In $\triangle AXP$ and $\triangle CXB$,

$$\angle PAX = \angle XCB \text{ (Alternative angles } AP \parallel BC)$$

$$AX = CX \text{ (Given)}$$

$$\angle AXP = \angle CXB \text{ (Vertically opposite angles)}$$

$\triangle AXP \cong \triangle CXB$ (By ASA rule)

$AP = BC$ (By c.p.c.t) (i)

Similarly,

$QA = BC$ (ii)

From (i) and (ii), we get

$AP = QA$

Now,

$AP \parallel BC$

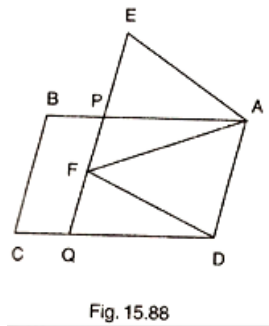
And,

$AP = QA$

Area ($\triangle APB$) = Area ($\triangle ACQ$) (Therefore, Triangles having equal bases and between the same parallels QP and BC)

26. Question

In Fig. 15.88, $ABCD$ and $AEFD$ are two parallelograms. Prove that:



(i) $PE = FQ$

(ii) $\text{ar}(\triangle APE) : \text{ar}(\triangle PFA) = \text{ar}(\triangle QFD) : \text{ar}(\triangle PFD)$

(iii) $\text{ar}(\triangle PEA) = \text{ar}(\triangle QFD)$

Answer

(i) In $\triangle EPA$ and $\triangle FQD$

$\angle PEA = \angle QFD$ (Corresponding angle)

$\angle EPA = \angle FQD$ (Corresponding angle)

$PA = QD$ (Opposite sides of a parallelogram)

Then,

$\triangle EPA \cong \triangle FQD$ (By AAS congruence rule)

Therefore,

$EP = FQ$ (c.p.c.t)

(ii) Since, $\triangle EPA$ and $\triangle FQD$ stand on equal bases PE and FQ lies between the same parallel EQ and AD

Therefore,

Area ($\triangle EPA$) = Area ($\triangle FQD$) (1)

Since,

$\triangle PEA$ and $\triangle PFD$ stand on the same base PF and lie between the same parallel PF and AD

Therefore,

$$\text{Area } (\triangle PFA) = \text{Area } (\triangle PFD) \quad (2)$$

Divide the equation (1) by (2), we get

$$\frac{\text{Area } (\triangle PEA)}{\text{Area } (\triangle PFA)} = \frac{\text{Area } (\triangle QFD)}{\text{Area } (\triangle PFD)}$$

(iii) From (i) part,

$$\triangle EPA \cong \triangle FQD$$

Then,

$$\text{Area } (\triangle EPA) = \text{Area } (\triangle FQD)$$

27. Question

In Fig. 15.89, ABCD is a $\parallel^{\text{gm}}$. O is any point on AC. $PQ \parallel AB$ and $LM \parallel AD$. Prove that:

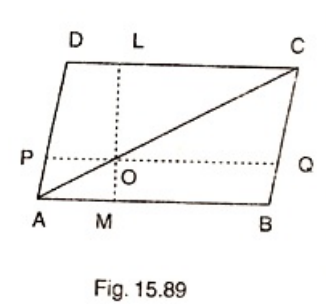


Fig. 15.89

$$\text{ar}(\parallel^{\text{gm}} \text{DLOP}) = \text{ar}(\parallel^{\text{gm}} \text{BMOQ})$$

Answer

Since,

A diagonal of parallelogram divides it into two triangles of equal area

Therefore,

$$\text{Area } (\triangle ADC) = \text{Area } (\triangle ABC)$$

$$\text{Area } (\triangle APO) + \text{Area of parallelogram DLOP} + \text{Area } (\triangle OLC)$$

$$\text{Area } (\triangle AOM) + \text{Area of parallelogram DLOP} + \text{Area } (\triangle OQC) \quad (i)$$

Since,

AO and CO are diagonals of parallelograms AMOP and OQCL respectively

Therefore,

$$\text{Area } (\triangle APO) = \text{Area } (\triangle AMO) \quad (ii)$$

$$\text{Area } (\triangle OLC) = \text{Area } (\triangle OQC) \quad (iii)$$

Subtracting (ii) from (iii), we get

$$\text{Area of parallelogram DLOP} = \text{Area of parallelogram BMOQ}.$$

28. Question

In a $\triangle ABC$, if L and M are points on AB and AC respectively such that $LM \parallel BC$. Prove that:

$$(i) \text{ar}(\triangle LCM) = \text{ar}(\triangle LBM)$$

$$(ii) \text{ar}(\triangle LBC) = \text{ar}(\triangle MBC)$$

$$(iii) \text{ar}(\Delta ABM) = \text{ar}(\Delta ACL)$$

$$(iv) \text{ar}(\Delta LOB) = \text{ar}(\Delta MOC)$$

Answer

(i) Clearly, triangles LMB and LMC are on the same base LM and between the same parallels LM and BC.

Therefore,

$$\text{Area}(\Delta LMB) = \text{Area}(\Delta LMC) \quad (1)$$

(ii) We observe that triangles LBC and MBC are on the same base BC and between the same parallels LM and BC.

Therefore,

$$\text{Area}(\Delta LBC) = \text{Area}(\Delta MBC) \quad (2)$$

(iii) We have,

$$\text{Area}(\Delta LMB) = \text{Area}(\Delta LMC) \quad [\text{From (i)}]$$

$$\text{Area}(\Delta ALM) + \text{Area}(\Delta LMB) = \text{Area}(\Delta ALM) + \text{Area}(\Delta LMC)$$

$$\text{Area}(\Delta ABM) = \text{Area}(\Delta ACL)$$

(iv) We have,

$$\text{Area}(\Delta LBC) = \text{Area}(\Delta MBC) \quad [\text{From (ii)}]$$

$$\text{Area}(\Delta LBC) - \text{Area}(\Delta BOC) = \text{Area}(\Delta MBC) - \text{Area}(\Delta BOC)$$

$$\text{Area}(\Delta LOB) = \text{Area}(\Delta MOC)$$

29. Question

In Fig. 15.90, D and E are two points on BC such that $BD=DE=EC$. Show that $\text{ar}(\Delta ABD) = \text{ar}(\Delta ADE) = \text{ar}(\Delta AEC)$

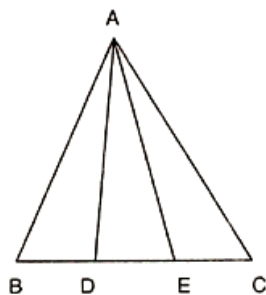


Fig. 15.90

Answer

Draw a line through A parallel to BC

Given that,

$$BD = DE = EC$$

We observed that the triangles ABD and AEC are on the same base and between the same parallels l and BC

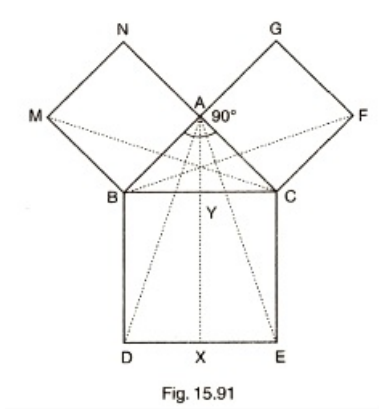
Therefore, their areas are equal

Hence,

$$\text{Area}(\Delta ABD) = \text{Area}(\Delta ADE) = \text{Area}(\Delta AEC)$$

30. Question

In Fig. 15.91, ABC is a right triangle right angled at A, BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment $AX \perp DE$ meets BC at Y. Show that:



(i) $\triangle MBC \cong \triangle ABD$ (ii) $\text{ar}(\text{BYXD}) = 2\text{ar}(\triangle MBC)$

(iii) $\text{ar}(\text{BYXD}) = \text{ar}(\text{ABMN})$

(iv) $\triangle FCB \cong \triangle ACE$

(v) $\text{ar}(\text{CYXE}) = 2\text{ar}(\triangle FCB)$

(vi) $\text{ar}(\text{CYXE}) = \text{ar}(\text{ACFG})$

(vii) $\text{ar}(\text{BCED}) = \text{ar}(\text{ABMN}) + \text{ar}(\text{ACFG})$

Answer

(i) In $\triangle MBC$ and $\triangle ABD$, we have

$$MB = AB$$

$$BC = BD$$

And,

$\angle MBC = \angle ABD$ (Therefore, $\angle MBC$ and $\angle ABC$ are obtained by adding $\angle ABC$ to right angle)

So, by SAS congruence rule, we have

$$\triangle MBC \cong \triangle ABD$$

$$\text{Area}(\triangle MBC) = \text{Area}(\triangle ABD) \quad (1)$$

(ii) Clearly, $\triangle ABC$ and rectangle BYXD are on the same base BD and between the same parallels AX and BD

Therefore,

$$\text{Area}(\triangle ABD) = \frac{1}{2} \text{Area of rectangle BYXD}$$

$$\text{Area of rectangle BYXD} = 2 \text{Area}(\triangle ABD)$$

$$\text{Area of rectangle BYXD} = 2 \text{Area}(\triangle MBC) \quad (2)$$

$$[\text{Therefore, Area}(\triangle ABD) = \text{Area}(\triangle MBC)] \text{ From (1)}$$

(iii) Since,

$\triangle MBC$ and square MBAN are on the same base MB and between the same parallel MB and NC

Therefore,

$$2 \text{Area}(\triangle MBC) = \text{Area of square MBAN} \quad (3)$$

From (2) and (3), we have

$$\text{Area of square MBAN} = \text{Area of rectangle BXYD}$$

(iv) In $\triangle FCB$ and $\triangle ACE$, we have

$$FC = AC$$

$$CB = CE$$

And,

$\angle FCB = \angle ACE$ (Therefore, $\angle FCB$ and $\angle ACE$ are obtained by adding $\angle ACB$ to a right angle)

So, by SAS congruence rule, we have

$$\triangle FCB \cong \triangle ACE$$

(v) We have,

$$\triangle FCB \cong \triangle ACE$$

$$\text{Area}(\triangle FCB) = \text{Area}(\triangle ACE)$$

Clearly,

$\triangle ACE$ and rectangle CYXE are on the same base CE and between the same parallel CE and AX

Therefore,

$$2 \text{ Area}(\triangle ACE) = \text{Area of rectangle CYXE}$$

$$2 \text{ Area}(\triangle FCB) = \text{Area of rectangle CYXE} \quad (4)$$

(vi) Clearly,

$\triangle FCB$ and rectangle FCAG are on the same base FC and between the same parallels FC and BG

Therefore,

$$2 \text{ Area}(\triangle FCB) = \text{Area of rectangle FCAG} \quad (5)$$

From (4) and (5), we get

$$\text{Area of rectangle CYXE} = \text{Area of rectangle ACFG}$$

(vii) Applying Pythagoras theorem in $\triangle ACB$, we have

$$BC^2 = AB^2 + AC^2$$

$$BC * BD = AB * MB + AC * FC$$

$$\text{Area of rectangle BCED} = \text{Area of rectangle ABMN} + \text{Area of rectangle ACFG}$$

CCE - Formative Assessment

1. Question

If ABC and BDE are two equilateral triangles such that D is the mid-point of BC, then find $ar(\triangle ABC) : ar(\triangle BDE)$.

Answer

$\triangle ABC$ and $\triangle BDE$ are equilateral triangles

We know that,

$$\text{Area of equilateral triangle} = \frac{\sqrt{3}}{4} a^2$$

D is the mid-point of BC then,

$$\text{Area}(\triangle BDE) = \frac{\sqrt{3}}{4} * \left(\frac{a}{2}\right)^2$$

$$= \frac{\sqrt{3}}{4} * \frac{a*a}{4}$$

Now,

$$\text{Area}(\triangle ABC) : \text{Area}(\triangle BDE)$$

$$\frac{\sqrt{3}}{4} * a^2 : \frac{\sqrt{3}}{4} * \frac{a*a}{4}$$

$$1 : \frac{1}{4}$$

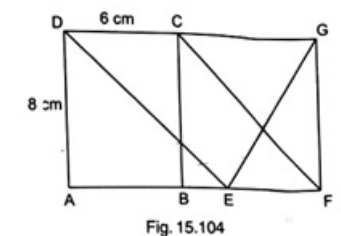
$$4 : 1$$

Hence,

Area ($\triangle ABC$): Area ($\triangle BDE$) is 4: 1

2. Question

In Fig. 15.104, ABCD is a rectangle in which CD = 6 cm, AD = 8 cm. Find the area of parallelogram CDEF.



Answer

Given that,

ABCD is a rectangle

CD = 6 cm

AD = 8 cm

We know that,

Area of parallelogram and rectangle on the same base between the same parallels are equal in area

So,

Area of parallelogram CDEF and rectangle ABCD on the same base and between the same parallels, then

We know that,

Area of parallelogram = Base * Height

Area of rectangle ABCD = Area of parallelogram

= AB * AD

= CD * AD (Therefore, AB = CD)

= 6 * 8

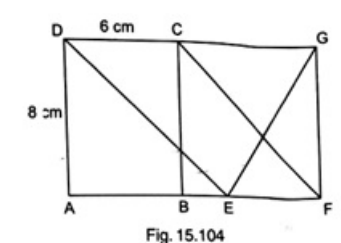
= 48 cm²

Hence,

Area of rectangle ABCD is 48 cm²

3. Question

In Fig. 15.104, find the area of $\triangle GEF$.



Answer

Given that,

ABCD is rectangle

$$CD = 6 \text{ cm}$$

$$AD = 8 \text{ cm}$$

We know that,

If a rectangle and a parallelogram are on the same base and between the same parallels, then the area of a triangle is equal to the half of parallelogram

So, triangle GEF and parallelogram ABCD are on the same base and between same parallels, then we know

$$\text{Area of parallelogram} = \text{Base} * \text{Height}$$

Now,

$$\text{Area } (\triangle GEF) = \frac{1}{2} * \text{Area of ABCD}$$

$$= \frac{1}{2} * AB * AD$$

$$= \frac{1}{2} * 6 * 8$$

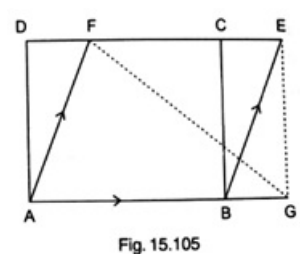
$$= 24 \text{ cm}^2$$

Hence,

$$\text{Area } (\triangle GEF) \text{ is } 24 \text{ cm}^2$$

4. Question

In Fig. 15.105, ABCD is a rectangle with sides AB = 10 cm and AD = 5 cm. Find the area of $\triangle EFG$.



Answer

We know that,

If a triangle and a parallelogram are on the same base and between the same parallel then the area of triangle is equal to the half of the parallelogram

So, triangle EPG and parallelogram ABCD are on the same base and between same parallels. The

We know that,

$$\text{Area of parallelogram} = \text{Base} * \text{Height}$$

Now,

$$\text{Area } (\triangle EPG) = \frac{1}{2} * \text{Area of ABCD}$$

$$= \frac{1}{2} * AB * AD$$

$$= \frac{1}{2} * 10 * 5$$

$$= 25 \text{ cm}^2$$

Hence,

Area ($\triangle EPG$) is 25 cm^2

5. Question

PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm. A is any point on PQ. If PS = 5 cm, then find $ar(\triangle RAS)$.

Answer

Given that,

PQRS is a rectangle

PS = 5 cm

PR = 13 cm

In triangle PSR, by using Pythagoras theorem

$$SR^2 = PR^2 - PS^2$$

$$SR^2 = (13)^2 - (5)^2$$

$$SR^2 = 169 - 25$$

$$SR^2 = 144$$

$$SR = 12 \text{ cm}$$

We have to find the area $\triangle RAS$,

$$\text{Area } (\triangle RAS) = \frac{1}{2} * \text{Base} * \text{Height}$$

$$= \frac{1}{2} * SR * PS$$

$$= \frac{1}{2} * 12 * 5$$

$$= 30 \text{ cm}^2$$

Hence, Area ($\triangle RAS$) is 30 cm^2

6. Question

In square ABCD, P and Q are mid-point of AB and CD respectively. If AB = 8 cm and PQ and BD intersect at O, then find area of $\triangle OPB$.

Answer

Given: ABCD is a square

P and Q are the mid points of AB and CD respectively.

AB = 8cm

PQ and BD intersect at O

Now,

$$AP = BP = \frac{1}{2} AB$$

$$AP = BP = \frac{1}{2} * 8$$

$$= 4 \text{ cm}$$

$$AB = AD = 8 \text{ cm}$$

$$QP \parallel AD$$

Then,

$$AD = QP$$

So,

$$OP = \frac{1}{2}AD$$

$$OP = \frac{1}{2} * 8$$

$$= 4 \text{ cm}$$

Now,

$$\text{Area } (\triangle OPB) = \frac{1}{2} * BP * PO$$

$$= \frac{1}{2} * 4 * 4$$

$$= 8 \text{ cm}^2$$

Hence, Area ($\triangle OPB$) is 8 cm^2

7. Question

ABC is a triangle in which D is the mid-point of BC. E and F are mid-points of DC and AE respectively. If area of $\triangle ABC$ is 16 cm^2 , find the area of $\triangle DEF$.

Answer

Given that,

D, E, F are the mid-points of BC, DC, AE respectively

Let, AD is median of triangle ABC

$$\text{Area } (\triangle ADC) = \frac{1}{2} \text{Area } (\triangle ABC)$$

$$= \frac{1}{2} * 16$$

$$= 8 \text{ cm}^2$$

Now, AE is a median of $\triangle ADC$

$$\text{Area } (\triangle AED) = \frac{1}{2} \text{Area } (\triangle ADC)$$

$$= \frac{1}{2} * 8$$

$$= 4 \text{ cm}^2$$

Again,

DE is the median of $\triangle AED$

$$\text{Area } (\triangle DEF) = \frac{1}{2} \text{Area } (\triangle AED)$$

$$= \frac{1}{2} * 4$$

$$= 2 \text{ cm}^2$$

8. Question

PQRS is a trapezium having PS and QR as parallel sides. A is any point on PQ and B is a point on SR such that $AB \parallel QR$. If area of $\triangle PBQ$ is 17 cm^2 , find the area of $\triangle ASR$.

Answer

Given that,

$$\text{Area } (\triangle PBQ) = 17 \text{ cm}^2$$

PQRS is a trapezium

$$PS \parallel QR$$

A and B are points on PQ and RS respectively

$$AB \parallel QR$$

We know that,

If a triangle and a parallelogram are on the same base and between the same parallels, the area of triangle is equal to half area of parallelogram

Here,

$$\text{Area } (\triangle ABP) = \text{Area } (\triangle ASB) \text{ (i)}$$

$$\text{Area } (\triangle ARQ) = \text{Area } (\triangle ARB) \text{ (ii)}$$

We have to find Area $(\triangle ASR)$,

$$\text{Area } (\triangle ASR) = \text{Area } (\triangle ASB) + \text{Area } (\triangle ARB)$$

$$= \text{Area } (\triangle ABP) + \text{Area } (\triangle ARQ)$$

$$= \text{Area } (\triangle PBQ)$$

$$= 17 \text{ cm}^2$$

Hence,

$$\text{Area } (\triangle ASR) \text{ is } 17 \text{ cm}^2.$$

9. Question

ABCD is a parallelogram. P is the mid-point of AB. BD and CP intersect at Q such that $CO:OP = 3:1$. If $ar(\triangle PBO) = 10 \text{ cm}^2$, Find the area of parallelogram ABCD.

Answer

Given that,

$$CQ:QP = 3:1$$

Let,

$$CQ = 3x$$

$$PQ = x$$

$$\text{Area } (\triangle PBQ) = 10 \text{ cm}^2$$

We know that,

$$\text{Area of triangle} = \frac{1}{2} * \text{Base} * \text{Height}$$

$$\text{Area } (\triangle PBQ) = \frac{1}{2} * x * h$$

$$10 = \frac{1}{2} * x * h$$

$$x * h = 20$$

$$\text{Area } (\triangle BQC) = \frac{1}{2} * 3x * h$$

$$= \frac{1}{2} * 3 * 20$$

$$= 30 \text{ cm}^2$$

Now,

$$\text{Area } (\triangle PCB) = \frac{1}{2} * PB * H = 30 \text{ cm}^2$$

$$PB * H = 60 \text{ cm}^2$$

We have to find area of parallelogram

We know that,

Area of parallelogram = Base * Height

$$\text{Area } (ABCD) = AB * H$$

$$\text{Area } (ABCD) = 2 BP * H$$

$$\text{Area } (ABCD) = 2 (60)$$

$$\text{Area } (ABCD) = 120 \text{ cm}^2$$

Hence,

Area of parallelogram ABCD is 120 cm^2

10. Question

P is any point on base BC of $\triangle ABC$ and D is the mid-point of BC. DE is drawn parallel to PA to meet AC at E. If $\text{ar}(\triangle ABC) = 12 \text{ cm}^2$, then find area of $\triangle EPC$.

Answer

Given that,

$$\text{Area } (\triangle ABC) = 12 \text{ cm}^2$$

D is the mid-point of BC

So,

AD is the median of triangle ABC,

$$\text{Area } (\triangle ABD) = \text{Area } (\triangle ADC) = \frac{1}{2} * \text{Area } (\triangle ABC)$$

$$\text{Area } (\triangle ADB) = \text{Area } (\triangle ADC) = \frac{1}{2} * 12$$

$$= 6 \text{ cm}^2 \text{ (i)}$$

We know that,

Area of triangle between the same parallel and on the same base

$$\text{Area } (\triangle APD) = \text{Area } (\triangle APE)$$

$$\text{Area } (\triangle AMP) + \text{Area } (\triangle PDM) = \text{Area } (\triangle AMP) + \text{Area } (\triangle APE)$$

$$\text{Area } (\triangle PDM) = \text{Area } (\triangle APE) \text{ (ii)}$$

ME is the median of triangle ADC,

$$\text{Area } (\triangle ADC) = \text{Area } (\triangle MCE) + \text{Area } (\triangle APE)$$

$$\text{Area } (\triangle ADC) = \text{Area } (\triangle MCE) + \text{Area } (\triangle PDM) \text{ [From (ii)]}$$

$$\text{Area } (\triangle ADC) = \text{Area } (\triangle PEC)$$

$$6 \text{ cm}^2 = \text{Area } (\triangle PEC) \text{ [From (i)]}$$

Hence,

Area $(\triangle PEC)$ is 6 cm^2 .

1. Question

The opposite sides of a quadrilateral have

- A. No common points
- B. One common point
- C. Two common points
- D. Infinitely many common points

Answer

Since, the two opposite line are joined by two another lines connecting the end points.

2. Question

Two consecutive sides of a quadrilateral have

- A. No common points
- B. One common point
- C. Two common points
- D. Infinitely many common points

Answer

Since, quadrilateral is simple closed figure of four line segments.

3. Question

PQRS is a quadrilateral. PR and QS intersect each other at O. In which of the following cases, PQRS is a parallelogram?

- A. $\angle P=100^\circ$, $\angle Q=80^\circ$, $\angle R=100^\circ$
- B. $\angle P=85^\circ$, $\angle Q=85^\circ$, $\angle R=95^\circ$
- C. PQ=7 cm, QR=7 cm, RS=8 cm, SP=8 cm
- D. OP=6.5 cm, OQ=6.5 cm, OR=5.2 cm, OS=5.2 cm

Answer

Since, the quadrilateral with opposite angles equal is a parallelogram.

4. Question

Which of the following quadrilateral is not a rhombus?

- A. All four sides are equal
- B. Diagonals bisect each other
- C. Diagonals bisect opposite angles
- D. One angle between the diagonals is 60°

Answer

One angle equalling to 60° need not necessarily be a rhombus.

5. Question

Diagonals necessarily bisect opposite angles in a

- A. Rectangle
- B. Parallelogram
- C. Isosceles trapezium
- D. Square

Answer

Each angle measures 45° each after the diagonal bisects them.

6. Question

The two diagonals are equal in a

- A. Parallelogram
- B. Rhombus
- C. Rectangle
- D. Trapezium

Answer

Let ABCD is a rectangle

AC and BD are the diagonals of rectangle

In $\triangle ABC$ and $\triangle BCD$, we have

$AB = CD$ (Opposite sides of rectangle are equal)

$\angle ABC = \angle BCD$ (Each equal to 90°)

$BC = BC$ (Common)

Therefore,

$\triangle ABC \cong \triangle BCD$ (By SAS congruence criterion)

$AC = BD$ (c.p.c.t)

Hence, the diagonals of a rectangle are equal.

7. Question

We get a rhombus by joining the mid-points of the sides of a

- A. Parallelogram
- B. Rhombus
- C. Rectangle
- D. Triangle

Answer

Let ABCD is a rectangle such as $AB = CD$ and $BC = DA$

P, Q, R and S are the mid points of the sides AB, BC, CD and DA respectively

Construction: Join AC and BD

In $\triangle ABC$,

P and Q are the mid-points of AB and BC respectively

Therefore,

$PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ (Mid-point theorem) (i)

Similarly,

In $\triangle ADC$,

$SR \parallel AC$ and $SR = \frac{1}{2} AC$ (Mid-point theorem) (ii)

Clearly, from (i) and (ii)

$PQ \parallel SR$ and $PQ = SR$

Since, in quadrilateral PQRS one pair of opposite sides is equal and parallel to each other, it is a parallelogram.

Therefore,

$PS \parallel QR$ and $PS = QR$ (Opposite sides of a parallelogram) (iii)

In $\triangle ABC$,

Q and R are the mid-points of side BC and CD respectively

Therefore,

$QR \parallel BD$ and $QR = \frac{1}{2} BD$ (Mid-point theorem) (iv)

However, the diagonals of a rectangle are equal

Therefore,

$AC = BD$ (v)

Now, by using equation (i), (ii), (iii), (iv), and (v), we obtain

$PQ = QR = SR = PS$

Therefore, PQRS is a rhombus.

8. Question

The bisectors of any two adjacent angles of a parallelogram intersect at

- A. 30°
- B. 45°
- C. 60°
- D. 90°

Answer

Let, ABCD is a parallelogram

OA and OD are the bisectors of adjacent angles $\angle A$ and $\angle D$

As, ABCD is a parallelogram

Therefore,

$AB \parallel DC$ (Opposite sides of the parallelogram are parallel)

$AB \parallel DC$ and AD is the transversal,

Therefore,

$\angle BAD + \angle CDA = 180^\circ$ (Sum of interior angles on the same side of the transversal is 180°)

$\angle 1 + \angle 2 = 90^\circ$ (AO and DO are angle bisectors $\angle A$ and $\angle D$) (i)

In $\triangle AOD$,

$$\angle 1 + \angle AOD + \angle 2 = 180^\circ$$

$$\angle AOD + 90^\circ = 180^\circ \text{ [From (i)]}$$

$$\angle AOD = 180^\circ - 90^\circ$$

$$= 90^\circ$$

Therefore,

In a parallelogram, the bisectors of the adjacent angles intersect at right angle.

9. Question

The bisectors of the angle of a parallelogram enclose a

- A. Parallelogram
- B. Rhombus
- C. Rectangle
- D. Square

Answer

Let, ABCD is a parallelogram.

AE bisects $\angle BAD$ and BF bisects $\angle ABC$

Also,

CG bisects $\angle BCD$ and DH bisects $\angle ADC$

To prove: LKJI is a rectangle

Proof: $\angle BAD + \angle ABC = 180^\circ$ (Because adjacent angles of a parallelogram are supplementary)

$\triangle ABJ$ is a right triangle

Since its acute interior angles are complementary

Similarly,

In $\triangle CDL$, we get

$$\angle DLC = 90^\circ$$

In $\triangle ADI$, we get

$$\angle AID = 90^\circ$$

Then,

$\angle JIL = 90^\circ$ because $\angle AID$ and $\angle JIL$ are vertical opposite angles

Since three angles of quadrilateral LKJI are right angles, hence 4th angle is also a right angle

Thus LKJI is a rectangle.

10. Question

The figure formed by joining the mid-points of the adjacent sides of a quadrilateral is a

- A. Parallelogram
- B. Rectangle
- C. Square
- D. Rhombus

Answer

Given that,

ABCD is a quadrilateral and P, Q, R and S are the mid points of the sides AB, BC, CD and DA respectively

To prove: PQRS is a parallelogram

Construction: Join A with C

Proof: In $\triangle ABC$,

P and Q are the mid-points of AB and BC respectively

Therefore,

$PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ (Mid-point theorem) (i)

Again,

In $\triangle ACD$,

R and S are mid-points of sides CD and AD respectively

Therefore,

$SR \parallel AC$ and $SR = \frac{1}{2} AC$ (Mid-point theorem) (ii)

From (i) and (ii), we get

$PQ \parallel SR$ and $PQ = SR$

Hence, PQRS is a parallelogram (One pair of opposite sides is parallel and equal)

11. Question

The figure formed by joining the mid-points of the adjacent sides of a rectangle is a

- A. Square
- B. Rhombus
- C. Trapezium
- D. None of these

Answer

Given: ABCD is a rectangle and P, Q, R, S are their midpoints

To Prove: PQRS is a rhombus

Proof: In $\triangle ABC$,

P and Q are the mid points

So, PQ is parallel AC

And,

$PQ = \frac{1}{2} AC$ (The line segment joining the mid points of 2 sides of the triangle is parallel to the third side and half of the third side)

Similarly,

RS is parallel AC

And,

$RS = \frac{1}{2} AC$

Hence, both PQ and RS are parallel to AC and equal to $\frac{1}{2} AC$.

Hence, PQRS is a parallelogram

In triangles APS & BPQ,

AP = BP (P is the mid-point of side AB)

$\angle PAS = \angle PBQ$ (90° each)

AS = BQ (S and Q are the mid points of AD and BC respectively and since opposite sides of a rectangle are equal, so their halves will also be equal)

$\triangle APS \cong \triangle BPQ$ (By SAS congruence rule)

PS = PQ (By c.p.c.t.)

PQRS is a parallelogram in which adjacent sides are equal.

Hence, PQRS is a rhombus.

12. Question

The figure formed by joining the mid-points of the adjacent sides of a rhombus is a

- A. Square
- B. Rectangle
- C. Trapezium
- D. None of these

Answer

To prove: That the quadrilateral formed by joining the mid points of sides of a rhombus is a rectangle.

ABCD is a rhombus P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively.

Construction: Join AC

Proof: In $\triangle ABC$, P and Q are the mid points of AB and BC respectively

Therefore,

$PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ (i) (Mid-point theorem)

Similarly,

$RS \parallel AC$ and $RS = \frac{1}{2} AC$ (ii) (Mid-point theorem)

From (i) and (ii), we get

$PQ \parallel RS$ and $PQ = RS$

Thus, PQRS is a parallelogram (A quadrilateral is a parallelogram, if one pair of opposite sides is parallel and equal)

AB = BC (Given)

Therefore,

$$\frac{1}{2} AB = \frac{1}{2} BC$$

PB = BQ (P and Q are mid points of AB and BC respectively)

In $\triangle PBQ$,

PB = BQ

Therefore,

$\angle BQP = \angle BPQ$ (iii) (Equal sides have equal angles opposite to them)

In $\triangle APS$ and $\triangle CQR$,

$$AP = CQ \left(AB = BC = \frac{1}{2} AB = \frac{1}{2} BC = AP = CQ \right)$$

$$AS = CR \left(AD = CD = \frac{1}{2} AD = \frac{1}{2} CD = AS = CR \right)$$

$PS = RQ$ (Opposite sides of parallelogram are equal)

Therefore,

$\triangle APS \cong \triangle CQR$ (By SSS congruence rule)

$$\angle APS = \angle CQR \text{ (iv) (By c.p.c.t.)}$$

Now,

$$\angle BPQ + \angle SPQ + \angle APS = 180^\circ$$

$$\angle BQP + \angle PQR + \angle CQR = 180^\circ$$

Therefore,

$$\angle BPQ + \angle SPQ + \angle APS = \angle BQP + \angle PQR + \angle CQR$$

$$\angle SPQ = \angle PQR \text{ (v) [From (iii) and (iv)]}$$

$PS \parallel QR$ and PQ is the transversal,

Therefore,

$$\angle SPQ + \angle PQR = 180^\circ \text{ (Sum of adjacent interior angles is } 180^\circ)$$

$$\angle SPQ + \angle SPQ = 180^\circ \text{ [From (v)]}$$

$$2 \angle SPQ = 180^\circ$$

$$\angle SPQ = 90^\circ$$

Thus, PQRS is a parallelogram such that $\angle SPQ = 90^\circ$

Hence, PQRS is a rectangle.

13. Question

The figure formed by joining the mid-points of the adjacent sides of a square is a

- A. Rhombus
- B. Square
- C. Rectangle
- D. Parallelogram

Answer

Let ABCD is a square such that $AB = BC = CD = DA$, $AC = BD$ and P, Q, R and S are the mid points of the sides AB, BC, CD and DA respectively.

In $\triangle ABC$,

P and Q are the mid-points of AB and BC respectively.

Therefore,

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \text{ (Mid-point theorem) (i)}$$

Similarly,

In $\triangle ADC$,

$SR \parallel AC$ and $SR = \frac{1}{2} AC$ (Mid-point theorem) (ii)

Clearly,

$PQ \parallel SR$ and $PQ = SR$

Since, in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other. Hence, it is a parallelogram.

Therefore,

$PS \parallel QR$ and $PS = QR$ (Opposite sides of a parallelogram) (iii)

In $\triangle ABC$,

Q and R are the mid-points of sides BC and CD respectively

Therefore,

$QR \parallel BD$ and $QR = \frac{1}{2} BD$ (Mid-point theorem) (iv)

However, the diagonals of a square are equal

Therefore,

$AC = BD$ (v)

By using equation (i), (ii), (iii), (iv) and (v), we obtain

$PQ = QR = SR = PS$

We know that, diagonals of a square are perpendicular bisector of each other

Therefore,

$\angle AOD = \angle AOB = \angle COD = \angle BOC = 90^\circ$

Now, in quadrilateral EHOS, we have

$SE \parallel OH$

Therefore,

$\angle AOD + \angle AES = 180^\circ$ (Corresponding angle)

$\angle AES = 180^\circ - 90^\circ$

$= 90^\circ$

Again,

$\angle AES + \angle SEO = 180^\circ$ (Linear pair)

$\angle SEO = 180^\circ - 90^\circ$

$= 90^\circ$

Similarly,

$SH \parallel EO$

Therefore,

$\angle AOD + \angle DHS = 180^\circ$ (Corresponding angle)

$\angle DHS = 180^\circ - 90^\circ = 90^\circ$

Again,

$\angle DHS + \angle SHO = 180^\circ$ (Linear pair)

$\angle SHO = 180^\circ - 90^\circ$

$$= 90^\circ$$

Again,

In quadrilateral EHOS, we have

$$\angle SEO = \angle SHO = \angle EOH = 90^\circ$$

Therefore, by angle sum property of quadrilateral in EHOS, we get

$$\angle SEO + \angle SHO + \angle EOH + \angle ESH = 360^\circ$$

$$90^\circ + 90^\circ + 90^\circ + \angle ESH = 360^\circ$$

$$\angle ESH = 90^\circ$$

In the same manner, in quadrilateral EFOP, FGOQ, GHOR, we get

$$\angle HRG = \angle FQG = \angle EPF = 90^\circ$$

Therefore, in quadrilateral PQRS, we have

$$PQ = QR = SR = PS \text{ and } \angle ESH = \angle HRG = \angle FQG = \angle EPF = 90^\circ$$

Hence, PQRS is a square.

14. Question

The figure formed by joining the mid-points of the adjacent sides of a parallelogram is a

- A. Rectangle
- B. Parallelogram
- C. Rhombus
- D. Square

Answer

Let ABCD be a quadrilateral, with points

E, F, G and H the midpoints of

AB, BC, CD, DA respectively.

(I suggest you draw this, and add segments EF, FG, GH, and HE, along with diagonals AC and BD)

$$EF = \frac{1}{2} AB \text{ (Definition of midpoint)}$$

Similarly,

$$BF = \frac{1}{2} BC$$

Thus, triangle BEF is similar to triangle BAC (SAS similarity)

Therefore EF is half the length of diagonal AC, since that's the proportion of the similar triangles.

Similarly, we can show that triangle DHG is similar to triangle DAC,

Therefore,

HG is half the length of diagonal AC

So,

$$EF = HG$$

Similarly,

We can use similar triangles to prove that EH and FG are both half the length of diagonal BD, and therefore equal

This means that both pairs of opposite sides of quadrilateral EFGH are equal, so it is a parallelogram.

15. Question

If one side of a parallelogram is 24° less than twice the smallest angle, then the measure of the largest angle of the parallelogram is

- A. 176°
- B. 68°
- C. 112°
- D. 102°

Answer

Let the small angle be $= x$

Then the second angle $= 2x - 24^\circ$

Since,

Opposite angles are equal the 4 angles will be $x, 2x - 24^\circ, x, 2x - 24^\circ$

So now by angle sum property:

$$x + 2x - 24^\circ + x + 2x - 24^\circ = 360^\circ$$

$$6x - 48^\circ = 360^\circ$$

$$6x = 360^\circ + 48^\circ$$

$$6x = 408^\circ$$

$$x = \frac{408}{6}$$

$$x = 68^\circ$$

Thus, the smallest angle is 68°

The second angle $= 2(68^\circ) - 24^\circ$

$$= 112^\circ$$

16. Question

In a parallelogram ABCD, if $\angle DAB = 75^\circ$ and $\angle DBC = 60^\circ$, then $\angle BDC =$

- A. 75°
- B. 60°
- C. 45°
- D. 55°

Answer

We know that,

The opposite angles of a parallelogram are equal

Therefore,

$$\angle BCD = \angle BAD = 75^\circ$$

Now, in $\triangle BCD$, we have

$$\angle CDB + \angle DBC + \angle BCD = 180^\circ \text{ (Since, sum of the angles of a triangle is } 180^\circ)$$

$$\angle CDB + 60^\circ + 75^\circ = 180^\circ$$

$$\angle CDB + 135^\circ = 180^\circ$$

$$\angle CDB = (180^\circ - 135^\circ) = 45^\circ$$

17. Question

ABCD is a parallelogram and E and F are the centroids of triangles ABD and BCD respectively, then EF=

- A. AE
- B. BE
- C. CE
- D. DE

Answer

Given: ABCD is a parallelogram

E and F are the centroids of triangle ABD and BCD

Since, the diagonals of parallelogram bisect each other

AO is the median of triangle ABD

And,

CO is the median of triangle CBD

$$EO = \frac{1}{3} AO \text{ (Since, centroid divides the median in the ratio 2:1)}$$

Similarly,

$$FO = \frac{1}{3} CO$$

$$EO + FO = \frac{1}{3} AO + \frac{1}{3} CO$$

$$= \frac{1}{3} (AO + CO)$$

$$EF = \frac{1}{3} AC$$

$$AE = \frac{1}{3} AO$$

$$= \frac{2}{3} * \frac{1}{2} AC$$

$$= \frac{1}{3} AC$$

Therefore,

$$EF = AE$$

18. Question

ABCD is a parallelogram M is the mid-point of BD and BM bisects $\angle B$. Then, $\angle AMB =$

- A. 45°
- B. 60°
- C. 90°
- D. 75°

Answer

ABCD is a parallelogram. BD is the diagonal and M is the mid-point of BD.

BD is a bisector of $\angle B$

We know that,

Diagonals of the parallelogram bisect each other

Therefore,

M is the mid-point of AC

$AB \parallel CD$ and BD is the transversal,

Therefore,

$\angle ABD = \angle BDC$ (i) (Alternate interior angle)

$\angle ABD = \angle DBC$ (ii) (Given)

From (i) and (ii), we get

$\angle BDC = \angle DBC$

In $\triangle BCD$,

$\angle BDC = \angle DBC$

$BC = CD$ (iii) (In a triangle, equal angles have equal sides opposite to them)

$AB = CD$ and $BC = AD$ (iv) (Opposite sides of the parallelogram are equal)

From (iii) and (iv), we get

$AB = BC = CD = DA$

Therefore,

ABCD is a rhombus

$\angle AMB = 90^\circ$ (Diagonals of rhombus are perpendicular to each other)

19. Question

ABCD is a parallelogram and E is the mid-point of BC. DE and AB when produced meet at F. Then, $AF =$

A. $\frac{3}{2} AB$

B. $2 AB$

C. $3 AB$

D. $\frac{5}{4} AB$

Answer

ABCD is a parallelogram. E is the midpoint of BC. So, $BE = CE$

DE produced meets the AB produced at F

Consider the triangles CDE and BFE

$BE = CE$ (Given)

$\angle CED = \angle BEF$ (Vertically opposite angles)

$\angle DCE = \angle FBE$ (Alternate angles)

Therefore,

$\triangle CDE \cong \triangle BFE$

So,

$CD = BF$ (c.p.c.t)

But,

$$CD = AB$$

Therefore,

$$AB = BF$$

$$AF = AB + BF$$

$$AF = AB + AB$$

$$AF = 2 AB$$

20. Question

If an angle of a parallelogram is two-third of its adjacent angle, the smallest angle of the parallelogram is

A. 108°

B. 54°

C. 72°

D. 81°

Answer

Since the adjacent angle of a parallelogram are supplementary.

Hence,

$$x + \frac{2}{3}x = 180^\circ$$

$$\frac{5}{3}x = 180^\circ$$

$$x = 108^\circ$$

Now,

$$\frac{2}{3}x = \frac{2}{3} * 108^\circ$$

$$= 72^\circ$$

21. Question

If the degree measures of the angles of quadrilateral are $4x$, $7x$, $9x$ and $10x$, what is the sum of the measures of the smallest angle and largest angle?

A. 140°

B. 150°

C. 168°

D. 180°

Answer

The total must be equal to 360° (Sum of quadrilaterals)

So,

$$4x + 7x + 9x + 10x = 360^\circ$$

$$30x = 360^\circ$$

$$x = 12^\circ$$

Now,

Substitute $x = 12$ into $4x$ (Smallest) + $10x$ (Biggest)

So,

$$(4 * 12) + (10 * 12)$$

$$= 48 + 120$$

$$= 168 \text{ so the answer is C}$$

22. Question

In a quadrilateral ABCD, $\angle A + \angle C$ is 2 times $\angle B + \angle D$. If $\angle A = 140^\circ$ and $\angle D = 60^\circ$, then $\angle B =$

- A. 60°
- B. 80°
- C. 120°
- D. None of these

Answer

Given that,

$$\angle A = 140^\circ$$

$$\angle D = 60^\circ$$

According to question,

$$\angle A + \angle C = 2 (\angle B + \angle D)$$

$$140 + \angle C = 2 (\angle B + 60^\circ)$$

$$\angle B = \frac{1}{2} (\angle C) + 10^\circ \text{ (i)}$$

We know,

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$140^\circ + \frac{1}{2} (\angle C) + 10^\circ + \angle C + 60^\circ = 360^\circ$$

$$\frac{3}{2} \angle C = 150^\circ$$

$$\angle C = 100^\circ$$

$$\angle B = \frac{1}{2} (100^\circ) + 10^\circ$$

$$= 60^\circ$$

23. Question

If the diagonals of a rhombus are 18 cm and 24 cm respectively, then its side is equal to

- A. 16 cm
- B. 15 cm
- C. 20 cm
- D. 17 cm

Answer

ABCD is a rhombus

$$AC = 18 \text{ cm}$$

$$BD = 24 \text{ cm}$$

We have to find the sides of the rhombus

In triangle AOB,

AO = 9cm (Diagonals of a parallelogram bisect each other)

BO = 12cm

AOB is a right - triangle right angled at O (Diagonals of a rhombus are perpendicular to each other)

So,

$AB^2 = AO^2 + BO^2$ (By Pythagoras theorem)

$AB^2 = 9^2 + 12^2$

$AB^2 = 81 + 144$

$AB^2 = 225$

AB = 15cm

In a rhombus, all sides are equal

Thus, each side of the rhombus is 15cm.

24. Question

The diagonals AC and BD of a rectangle ABCD intersect each other at P. If $\angle ABD = 50^\circ$, then $\angle DPC =$

A. 70°

B. 90°

C. 80°

D. 100°

Answer

Given that,

$\angle ABD = \angle ABP = 50^\circ$

$\angle PBC + \angle ABP = 90^\circ$ (Each angle of a rectangle is a right angle)

$\angle PBC = 40^\circ$

Now,

PB = PC (Diagonals of a rectangle are equal and bisect each other)

Therefore,

$\angle BCP = 40^\circ$ (Equal sides has equal angle)

In triangle BPC,

$\angle BPC + \angle PBC + \angle BCP = 180^\circ$ (Angle sum property of a triangle)

$\angle BPC = 100^\circ$

$\angle BPC + \angle DPC = 180^\circ$ (Angles in a straight line)

$\angle DPC = 180^\circ - 100^\circ$

$= 80^\circ$

25. Question

ABCD is a parallelogram in which diagonal AC bisects $\angle BAD$. If $\angle BAC = 35^\circ$, then $\angle ABC =$

A. 70°

B. 110°

C. 90°

D. 120°

Answer

Given: ABCD is a parallelogram

AC is a bisector of angle BAD

$$\angle BAC = 35^\circ$$

$$\angle A = 2 \angle BAC$$

$$\angle A = 2 (35^\circ)$$

$$\angle A = 70^\circ$$

$$\angle A + \angle B = 180^\circ \text{ (Adjacent angles of parallelogram are supplementary)}$$

$$70 + \angle B = 180^\circ$$

$$\angle B = 110^\circ$$

26. Question

In a rhombus ABCD, if $\angle ACB = 40^\circ$, then $\angle ADB =$

A. 70°

B. 45°

C. 50°

D. 60°

Answer

The diagonals in a rhombus are perpendicular,

So,

$$\angle BPC = 90^\circ$$

From triangle BPC,

The sum of angles is 180°

So,

$$\angle CBP = 180^\circ - 40^\circ - 90^\circ$$

$$= 50^\circ$$

Since, triangle ABC is isosceles

We have,

$$AB = BC$$

So,

$$\angle ACB = \angle CAB = 40^\circ$$

Again from triangle APB,

$$\angle PBA = 180^\circ - 40^\circ - 90^\circ$$

$$= 50^\circ$$

Again, triangle ADB is isosceles,

So,

$$\angle ADB = \angle DBA = 50^\circ$$

$$\angle ADB = 50^\circ$$

27. Question

In $\triangle ABC$, $\angle A = 30^\circ$, $\angle B = 40^\circ$ and $\angle C = 110^\circ$. The angles of the triangle formed by joining the mid-points of the sides of this triangle are

- A. $70^\circ, 70^\circ, 40^\circ$
- B. $60^\circ, 40^\circ, 80^\circ$
- C. $30^\circ, 40^\circ, 110^\circ$
- D. $60^\circ, 70^\circ, 50^\circ$

Answer

Since, the triangle formed by joining the mid points of the triangle would be similar to it hence the angles would be equal to the outer triangle's angles.

28. Question

The diagonals of a parallelogram ABCD intersect at O. If $\angle BOC = 90^\circ$ and $\angle BDC = 50^\circ$, then $\angle OAB =$

- A. 40°
- B. 50°
- C. 10°
- D. 90°

Answer

$\angle BOC$ is 90°

So,

$\angle COD$ and $\angle AOB$ all should be 90° by linear pair

$\angle BDC$ is 50° ,

So,

Now as in a parallelogram the opposite sides are equal

We say,

AB parallel to CD

$\angle DCA = 50^\circ$

So,

In triangle COA

$\angle C = 50^\circ$ (Stated above)

$\angle COA = 90^\circ$ (Proved above)

Therefore,

$$90^\circ + 50^\circ + x^\circ = 180^\circ$$

$$x = 40^\circ$$

29. Question

ABCD is a trapezium in which $AB \parallel DC$. M and N are the mid-points of AD and BC respectively, If $AB = 12$ cm, $MN = 14$ cm, then $CD =$

- A. 10 cm
- B. 12 cm

C. 14 cm

D. 16 cm

Answer

Construction: Join A to C

Mark the intersection point of AC and MN as O

Now, M and N are mid points of the non-parallel of a trapezium

Therefore,

$$MN \parallel AB \parallel DC$$

So,

$$MO \parallel BC$$

And,

M is a mid-point of AD

Therefore,

$$MO = \frac{1}{2} BC$$

Similarly,

$$NO = \frac{1}{2} AB$$

Therefore,

$$MN = MO + NO$$

$$= \frac{1}{2} (AB + CD)$$

But,

$$MN = 14 \text{ cm}$$

Hence,

$$\frac{1}{2} (AB + CD) = 14 \text{ cm}$$

$$12 + CD = 28$$

$$CD = 16 \text{ cm}$$

30. Question

Diagonals of a quadrilateral ABCD bisect each other. If $\angle A = 45^\circ$, then $\angle B =$

A. 115°

B. 120°

C. 125°

D. 135°

Answer

Since, diagonals of quadrilateral bisect each other

Hence, it's a parallelogram

We know,

The adjacent angles of parallelogram are supplementary

Therefore,

$$\angle A + \angle B = 180^\circ$$

$$45^\circ + \angle B = 180^\circ$$

$$\angle B = 135^\circ$$

31. Question

P is the mid-point of side BC of a parallelogram ABCD such that $\angle BAP = \angle DAP$. If $AD = 10$ cm, then $CD =$

- A. 5 cm
- B. 6 cm
- C. 8 cm
- D. 10 cm

Answer

Given that,

ABCD is a parallelogram

P is the mid-point of BC

$$\angle DAP = \angle PAB = x$$

$$AD = 10 \text{ cm}$$

To find: The length of CD

$$\angle ABP = 180 - 2x \text{ (Co interior angle of parallelogram)}$$

$$\angle APB = 180^\circ - (180^\circ - 2x + x) = x$$

Therefore,

In triangle ABP,

$$\angle APB = \angle PAB = x$$

Therefore,

$AB = PB$ (In a triangle sides opposite to equal angles are equal in length)

$$CD = AB = PB = \frac{BC}{2} = \frac{AD}{2} = \frac{10}{2} = 5 \text{ cm}$$

32. Question

In $\triangle ABC$, E is the mid-point of median AD such that BE produced meets AC at F. If $AC = 10.5$ cm, then $AF =$

- A. 3 cm
- B. 3.5 cm
- C. 2.5 cm
- D. 5 cm

Answer

Complete the parallelogram ADCP

So the diagonals DP & AC bisect each other at O

Thus O is the midpoint of AC as well as DP (i)

Since ADCP is a parallelogram,

$$AP = DC$$

And,

AP parallel DC

But,

D is mid-point of BC (Given)

AP = BD

And,

AP parallel BD

Hence,

BDPA is also a parallelogram.

So, diagonals AD & BP bisect each other at E (E being given mid-point of AD)

So, BEP is a single straight line intersecting AC at F

In triangle ADP,

E is the mid-point of AD and

O is the midpoint of PD.

Thus, these two medians of triangle ADP intersect at F, which is centroid of triangle ADP

By property of centroid of triangles,

It lies at $\frac{2}{3}$ of the median from vertex

So,

$$AF = \frac{2}{3} AO \text{ (ii)}$$

So,

From (i) and (ii),

$$AF = \frac{2}{3} * \frac{1}{2} * AC$$

$$= \frac{1}{3} AC$$

$$= \frac{10.5}{3}$$

$$= 3.5 \text{ cm}$$