

16. Circles

Exercise 16.1

1. Question

Fill in the blanks:

- (i) All points lying inside/outside a circle are calledpoints/.....points.
- (ii) Circles having the same centre and different radii are called Circles.
- (iii) A point whose distance from the centre of a circle is greater than its radius lies in of the circle.
- (iv) A continuous piece of a circle is of the circle.
- (v) The longest chord of a circle is a of the circle.
- (vi) An arc is awhen its ends are the ends of a diameter.
- (vii) Segment of a circle is the region between an arc andof the circle.
- (viii) A circle divides the plane, on which it lies, inparts.

Answer

- (i) Interior/exterior
- (ii) Concentric
- (iii) Exterior
- (iv) Arc
- (v) Diameter
- (vi) Semi-circle
- (vii) Centre
- (viii) Three

2. Question

Write the truth value (T/F) of the following with suitable reasons:

- (i) A circle is a plane figure.
- (ii) Line segment joining the centre to any point on the circle is a radius of the circle.
- (iii) If a circle is divided into three equal arcs each is a major arc.
- (iv) A circle has only finite number of equal chords.

- (v) A chord of a circle, which is twice as long as its radius is a diameter of the circle.
- (vi) Sector is the region between the chord and its corresponding arc.
- (vii) The degree measure of an arc is the complement of the central angle containing the arc.
- (viii) The degree measure of a semi-circle is 180° .

Answer

- (i) True: Because it is a one dimensional figure
- (ii) True: Since, line segment joining the centre to any point on the circle is a radius of the circle
- (iii) True: Because each arc measures equal
- (iv) False: Since, a circle has only infinite number of equal chords
- (v) True: Because, radius equal to $\frac{1}{2}$ times of its diameter
- (vi) True: Yes, sector is the region between the chord and its corresponding arc
- (vii) False: The degree measure of an arc is half of the central angle containing the arc
- (viii) True: Yes, The degree measure of a semi-circle is 180°

Exercise 16.2

1. Question

The radius of a circle is 8 cm and the length of one of its chords is 12 cm. Find the distance of the chord from the centre.

Answer

Given that,

Radius of circle (OA) = 8 cm

Chord (AB) = 12 cm

Draw OC perpendicular to AB

We know that,

The perpendicular from centre to chord bisects the chord

Therefore,

$$AC = BC = \frac{12}{2}$$

$$= 6 \text{ cm}$$

Now,

In $\triangle OCA$, by using Pythagoras theorem

$$AC^2 + OC^2 = OA^2$$

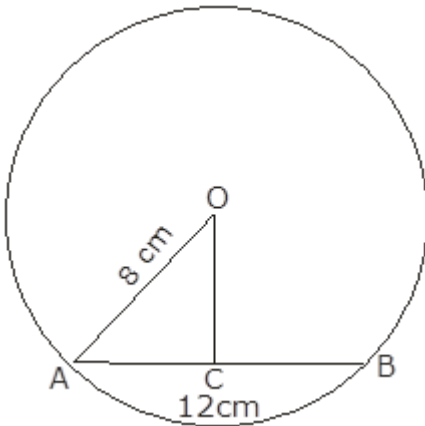
$$6^2 + OC^2 = 8^2$$

$$36 + OC^2 = 64$$

$$OC^2 = 64 - 36$$

$$OC^2 = 28$$

$$OC = 5.291 \text{ cm}$$



2. Question

Find the length of a chord which is at a distance of 5 cm from the centre of a circle of radius 10 cm.

Answer

Given that,

Distance (OC) = 5 cm

Radius of circle (OA) = 10 cm

In $\triangle OCA$, by using Pythagoras theorem

$$AC^2 + OC^2 = OA^2$$

$$AC^2 + 5^2 = 10^2$$

$$AC^2 = 100 - 25$$

$$AC^2 = 75$$

$$AC = 8.66 \text{ cm}$$

We know that,

The perpendicular from centre to chord bisects the chord

Therefore,

$$AC = BC = 8.66 \text{ cm}$$

Then,

$$\text{Chord AB} = 8.66 + 8.66$$

$$= 17.32 \text{ cm}$$

3. Question

Find the length of a chord which is at a distance of 4 cm from the centre of the circle of radius 6 cm.

Answer

Radius of circle (OA) = 6 cm

Distance (OC) = 4 cm

In $\triangle OCA$, by using Pythagoras theorem

$$AC^2 + OC^2 = OA^2$$

$$AC^2 + 4^2 = 6^2$$

$$AC^2 = 36 - 16$$

$$AC^2 = 20$$

$$AC = 4.47 \text{ cm}$$

We know that,

The perpendicular distance from centre to chord bisects the chord

$$AC = BC = 4.47 \text{ cm}$$

Then,

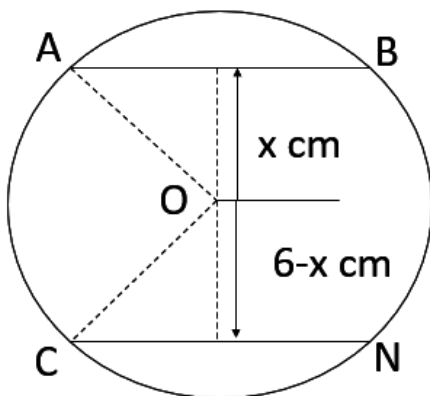
$$AB = 4.47 + 4.47$$

$$= 8.94 \text{ cm}$$

4. Question

Two chords AB , CD of lengths 5 cm, 11 cm respectively of a circle are parallel. If the distance between AB and CD is 3 cm, find the radius of the circle.

Answer



Let r be the radius of the given circle and its center be O . Draw $OM \perp AB$ and $ON \perp CD$. Since, OM perpendicular AB , ON perpendicular CD .

and $AB \parallel CD$

Therefore, points M , O and N are collinear.

So, $MN = 6\text{cm}$

Let, $OM = x\text{ cm}$.

Then, $ON = (6 - x)\text{cm}$.

Join OA and OC .

Then $OA = OC = r$

As the perpendicular from the centre to a chord of the circle bisects the chord.

$$\therefore AM = BM = \frac{1}{2} AB$$

$$= \frac{1}{2} \times 5 = 2.5\text{cm}$$

$$CN = DN = \frac{1}{2} CD$$

$$= \frac{1}{2} \times 11 = 5.5\text{cm}$$

In right triangles OAM and OCN , we have,

$$OA^2 = OM^2 + AM^2 \text{ and } OC^2 = ON^2 + CN^2$$

$$r^2 = x^2 + \left(\frac{5}{2}\right)^2 \dots\dots (i)$$

$$r^2 = (6 - x)^2 + \left(\frac{11}{2}\right)^2 \dots\dots (ii)$$

From (i) and (ii), we have

$$x^2 + \left(\frac{5}{2}\right)^2 = (6 - x)^2 + \left(\frac{11}{2}\right)^2$$

$$x^2 + \frac{25}{4} = (6 - x)^2 + \frac{121}{4}$$

$$\Rightarrow 4x^2 + 25 = 144 + 4x^2 - 48x + 121$$

$$\Rightarrow 48x = 240$$

$$\Rightarrow x = 240/48$$

$$\Rightarrow x = 5$$

Putting the value of x in equation (i), we get

$$r^2 = 5^2 + (5/2)^2$$

$$\Rightarrow r^2 = 25 + 25/4$$

$$\Rightarrow r^2 = 125/4$$

$$\Rightarrow r = 5\sqrt{5}/2 \text{ cm}$$

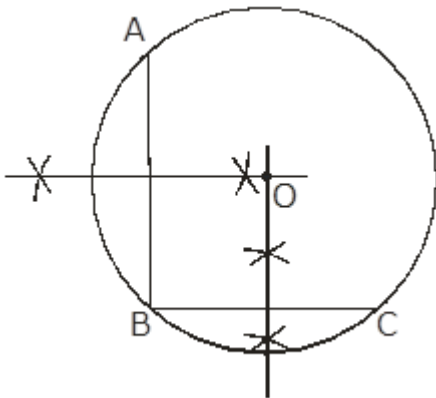
5. Question

Give a method to find the centre of a given circle.

Answer

Steps of construction:

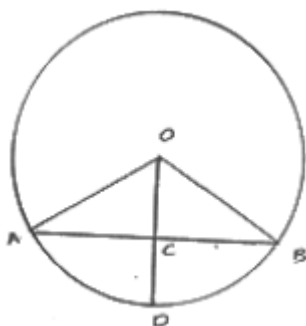
- (i) Take three points A, B and C on the given circle
- (ii) Join AB and BC
- (iii) Draw the perpendicular bisectors of chord AB and BC which intersect each other at O
- (iv) Point O will be required circle because we know that perpendicular bisector of chord always passes through centre.



6. Question

Prove that the line joining the mid-point of a chord to the centre of the circle passes through the mid-point of the corresponding minor arc.

Answer



Given that,

C is the mid-point of chord AB

To prove: D is the mid-point of arc AB

Proof: In $\triangle OAC$ and $\triangle OBC$,

OA = OB (Radius of circle)

AC = OC (Common)

AC = BC (C is the mid-point of AB)

Then,

$\triangle OAC \cong \triangle OBC$ (By SSS congruence rule)

$\angle AOC = \angle BOC$ (By c.p.c.t)

$m(\widehat{AD}) = m(\widehat{BD})$

$\widehat{AD} \cong \widehat{BD}$

Here, D is the mid-point of arc AB.

7. Question

Prove that a diameter of a circle which bisects a chord of the circle also bisects the angle subtended by the chord at the centre of the circle.

Answer

Given that,

PQ is a diameter of circle which bisects chord AB to C

To prove: PQ bisects $\angle AOB$

Proof: In $\triangle AOC$ and $\triangle BOC$,

OA = OB (Radius of circle)

OC = OC (Common)

AC = BC (Given)

Then,

$\triangle AOC \cong \triangle BOC$ (By SSS congruence rule)

$\angle AOC = \angle BOC$ (By c.p.c.t)

Hence, PQ bisects $\angle AOB$.

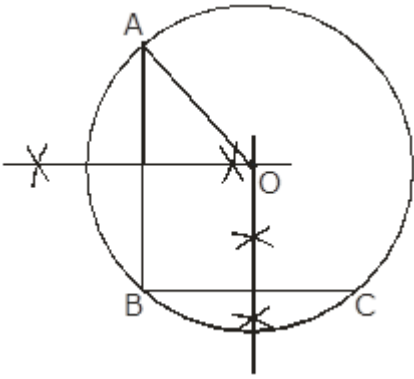
8. Question

Given an arc of a circle, show how to complete the circle.

Answer

Steps of construction:

- (i) Take three points A, B and C on the given arc
- (ii) Join AB and BC
- (iii) Draw the perpendicular bisectors of chord AB and BC which intersect each other at point O. Then, O will be the required centre of the required circle.
- (iv) Join OA
- (v) With centre O and radius OA, complete the circle



9. Question

Prove that two different circles cannot intersect each other at more than two points.

Answer

Suppose two circles intersect in three points A, B and C. Then A, B, C are non-collinear. So, a unique circle passes through these three points. This is contradiction to the fact that two given circles are passing through A, B, C. Hence, two circles cannot intersect each other at more than two points.

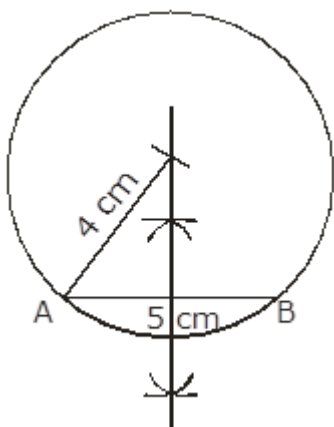
10. Question

A line segment AB is length 5 cm. Draw a circle of radius 4 cm passing through A and B. Can you draw a circle of radius 2 cm passing through A and B? Give reason in support of your answer.

Answer

- (i) Draw a line segment AB of 5 cm
- (ii) Draw the perpendicular bisector of AB
- (iii) With centre A and radius of 4 cm draw an arc which intersects the perpendicular bisector at point O. O will be the required centre.
- (iv) Join OA
- (v) With centre O and radius OA draw a circle.

No, we cannot draw a circle of radius 2 cm passing through A and B because when we draw an arc of radius 2 cm with centre A, the arc will not intersect the perpendicular bisector and we will not find the centre.



11. Question

An equilateral triangle of side 9 cm is inscribed in a circle. Find the radius of the circle.

Answer

Let ABC be an equilateral triangle of side 9 cm

Let, AD be one of its medians and G be the centroids of the triangle ABC

Then,

$$AG: GD = 2: 1$$

We know that,

In an equilateral triangle centroid coincides with the circumcentre

Therefore,

G is the centre of the circumference with circum radius GA

Also, G is the centre and GD is perpendicular to BC

Therefore,

In right triangle ADB, we have

$$AB^2 = AD^2 + DB^2$$

$$9^2 = AD^2 + DB^2$$

$$AD = \frac{9\sqrt{3}}{2} \text{ cm}$$

Therefore,

$$\text{Radius} = AG = \frac{2}{3} AD$$

$$= 3\sqrt{3} \text{ cm}$$

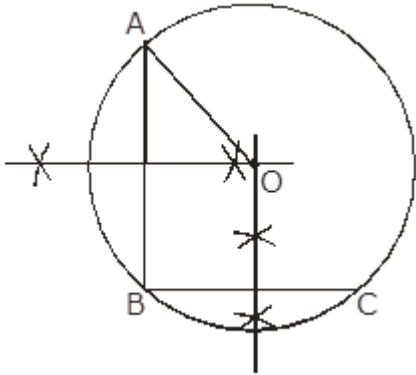
12. Question

Given an arc of a circle, complete the circle.

Answer

Steps of construction:

- (i) Take three points A, B, C on the given arc
- (ii) Join AB and BC
- (iii) Draw the perpendicular bisectors of chords AB and BC which intersect each other at point O. Then, O will be the required centre of the required circle.
- (iv) Join OA
- (v) With centre O and radius OA, complete the circle

**13. Question**

Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

Answer

Each pair of circles have 0, 1 or 2 points in common. The maximum number of points in common is 2.

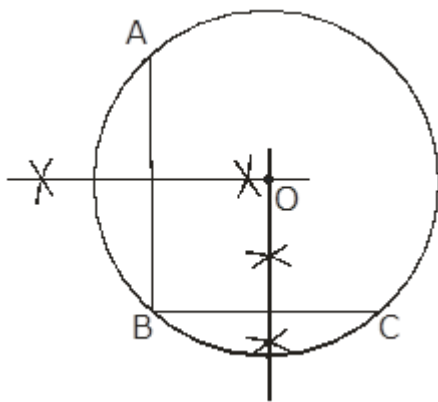
14. Question

Suppose you are given a circle. Give a construction to find its centre.

Answer

Steps of construction:

- (i) Take three points A, B and C in the given circle.
- (ii) Join AB and BC
- (iii) Draw the perpendicular bisector of chord AB and BC which intersect each other at O
- (iv) Point O will be the required centre of the circle because we know that, the perpendicular bisector of the chord always passes through the centre



15. Question

Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are opposite side of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Answer

Draw OM perpendicular to AB and ON perpendicular to CD

Join OB and OC

$$BM = \frac{AB}{2} = \frac{5}{2} \text{ (Perpendicular from centre bisects the chord)}$$

$$ND = \frac{CD}{2} = \frac{11}{2}$$

Let,

ON be x so OM will be $(6 - x)$

In $\triangle MOB$,

$$OM^2 + MB^2 = OB^2$$

$$(6 - x)^2 + \left(\frac{5}{2}\right)^2 = OB^2$$

$$36 + x^2 - 12x + \frac{25}{4} = OB^2 \text{ (i)}$$

In $\triangle NOD$,

$$ON^2 + ND^2 = OD^2$$

$$x^2 + \left(\frac{11}{2}\right)^2 = OD^2$$

$$x^2 + \frac{121}{4} = OD^2 \text{ (ii)}$$

We have,

$OB = OD$ (Radii of same circle)

So from (i) and (ii), we get

$$36 + x^2 + 2x + \frac{25}{4} = x^2 + \frac{121}{4}$$

$$12x = 36 + \frac{25}{4} - \frac{121}{4}$$

$$= \frac{144+25-121}{4} = \frac{48}{4}$$

$$= 12$$

From (ii), we get

$$(1)^2 + \left(\frac{121}{4}\right) = OD^2$$

$$OD^2 = 1 + \frac{121}{4}$$

$$= \frac{125}{4}$$

$$OD = \frac{5\sqrt{5}}{2}$$

So, radius of circle is found to be $\frac{5}{2}\sqrt{5}$ cm

16. Question

The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance of 4 cm from the centre, what is the distance of the other chord from the centre?

Answer

Distance of smaller chord AB from centre of circle = 4 cm

$$OM = 4 \text{ cm}$$

$$MB = \frac{AB}{2} = \frac{6}{2}$$

$$= 3 \text{ cm}$$

In $\triangle OMB$,

$$OM^2 + MB^2 = OB^2$$

$$(4)^2 + (3)^2 = OB^2$$

$$16 + 9 = OB^2$$

$$OB^2 = 25$$

$$OB = 5 \text{ cm}$$

In $\triangle OND$,

$$OD = OB = 5 \text{ cm (Radii of same circle)}$$

$$ND = \frac{CD}{2} = \frac{8}{2}$$

$$= 4 \text{ cm}$$

$$ON^2 + ND^2 = OD^2$$

$$ON^2 + (4)^2 = (5)^2$$

$$ON^2 = 25 - 16$$

$$= 9$$

$$ON = 3$$

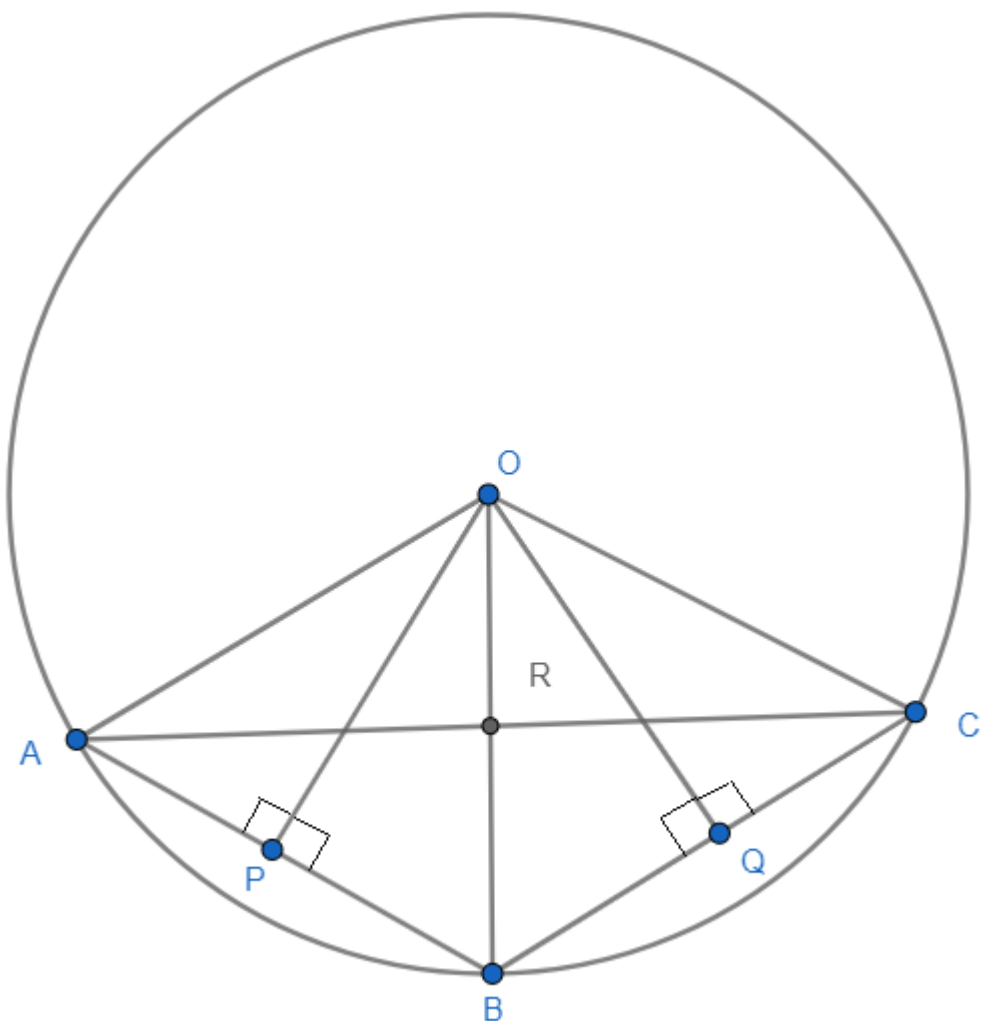
SO, distance of bigger chord from circle is 3 cm.

Exercise 16.3

1. Question

Three girls Ishita, Isha and Nisha are playing a game by standing on a circle of radius 20 m drawn in a park. Ishita throws a ball to Isha, Isha to Nisha and Nisha to Ishita. If the distance between Ishita and Isha and between Isha and Nisha is 24 m each, what is distance between Ishita and Nisha?

Answer



Let A be the position of Ishita, B be the position of Isha and C be the position of Nisha
 Given $AB = BC = 24$ m $OA = OB = OC = 20$ m [Radii of circle]
 Draw perpendiculars OP and OQ on AB and BC respectively
 $AP = PB = 12$ m
 In right $\triangle OPA$, $OP^2 + AP^2 = OA^2$
 $OP^2 + (12)^2 = (20)^2$
 $OP^2 = 256$ sq m

Therefore, $OP = 16$ m
 From the figure, OABC is a kite since $OA = OC$ and $AB = BC$. Recall that the diagonals of a kite are perpendicular and the diagonal common to both the isosceles triangles is bisected by another diagonal. Therefore, $\angle ARB = 90^\circ$ and $AR = RC$
 Area of $\triangle OAB$

$$= \frac{1}{2} \times OP \times AB$$

$$= \frac{1}{2} \times 16 \times 24 = 192 \text{ sq m}$$

$$\text{Also area of } \triangle OAB = \frac{1}{2} \times OB \times AR$$

$$\text{Hence, } \frac{1}{2} \times OB \times AR = 192$$

$$\frac{1}{2} \times 20 \times AR = 192$$

Therefore, $AR = 19.2$ m But $AC = 2AR = 2(19.2) = 38.4$ m Thus the distance between Ishita and Nisha is 38.4 m

2. Question

A circular park of radius 40 m is situated in a colony. Three boys Ankur, Amit and Anand are sitting at equal distance on its boundary each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone.

Answer

Given that,

$$AB = BC = CA$$

So, ABC is an equilateral triangle

$$OA = 40 \text{ cm (Radius)}$$

Medians of equilateral triangle pass through the circumference (O) of the equilateral triangle ABC

We also know that,

Median intersects each other at 2: 1 as AD is the median of equilateral triangle ABC, we can write:

$$\frac{OA}{OD} = \frac{2}{1}$$

$$\frac{40}{OD} = \frac{2}{1}$$

$$OD = 20 \text{ m}$$

Therefore,

$$AO = OA + OD$$

$$= 40 + 20$$

$$= 60 \text{ m}$$

In $\triangle ADC$,

By using Pythagoras theorem

$$AC^2 = AO^2 + DC^2$$

$$AC^2 = (60)^2 + \left(\frac{AC}{2}\right)^2$$

$$AC^2 = 3600 + \frac{AC \cdot AC}{4}$$

$$\frac{3}{4} AC^2 = 3600$$

$$AC^2 = 4800$$

$$AC = 40\sqrt{3} \text{ m}$$

So, length of string of each phone will be $40\sqrt{3}$ m

Exercise 16.4

1. Question

In Fig. 16.120, O is the centre of the circle. If $\angle APB = 50^\circ$, find $\angle AOB$ and $\angle OAB$.

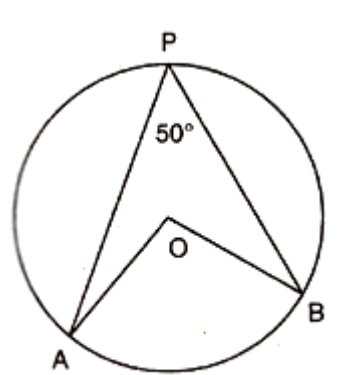


Fig. 16.120

Answer

$$\angle APB = 50^\circ \text{ (Given)}$$

By degree measure theorem,

$$\angle AOB = \angle APB$$

$$\angle APB = 2 \times 50$$

$$= 100^\circ$$

Since,

$$OA = OB \text{ (Radii)}$$

Hence,

$$\angle OAB = \angle OBA \text{ (Angle opposite to equal sides are equal)}$$

Let,

$$\angle OAB = x$$

In Triangle OAB,

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$x + x + 100^\circ = 180^\circ$$

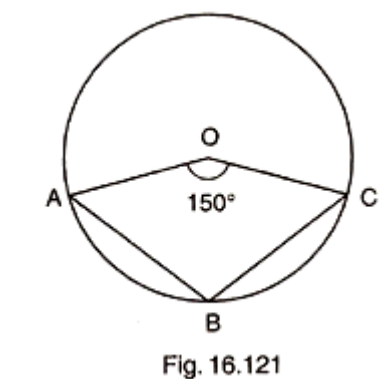
$$2x = 80^\circ$$

$$x = 40^\circ$$

$$\angle OAB = \angle OBA = 40^\circ$$

2. Question

In Fig. 16.121, it is given that O is the centre of the circle and $\angle AOC = 150^\circ$. Find $\angle ABC$.



Answer

We have,

$$\angle AOC = 150^\circ$$

Therefore,

$$\angle AOC + \text{Reflex } \angle AOC = 360^\circ$$

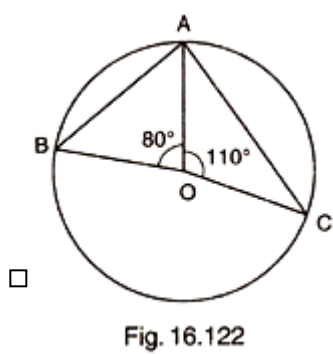
$$\text{Reflex } \angle AOC = 210^\circ$$

$$2 \angle ABC = 210^\circ \text{ (By degree measure theorem)}$$

$$\angle ABC = 105^\circ$$

3. Question

In Fig. 16.122, O is the centre of the circle. Find $\angle BAC$.



Answer

We have,

$$\angle AOB = 80^\circ$$

$$\angle AOC = 110^\circ$$

$$\angle AOB + \angle AOC + \angle BOC = 360^\circ \text{ (Complete angle)}$$

$$80^\circ + 110^\circ + \angle BOC = 360^\circ$$

$$\angle BOC = 170^\circ$$

By degree measure theorem,

$$\angle BOC = 2 \angle BAC$$

$$170^\circ = 2 \angle BAC$$

$$\angle BAC = 85^\circ$$

4. Question

If O is the centre of the circle, find the value of x in each of the following figures :

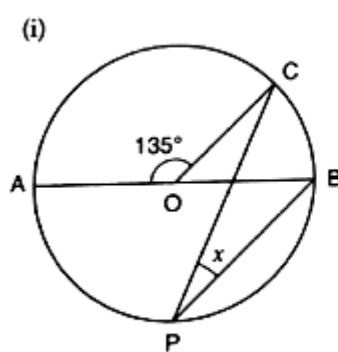


Fig. 16.123

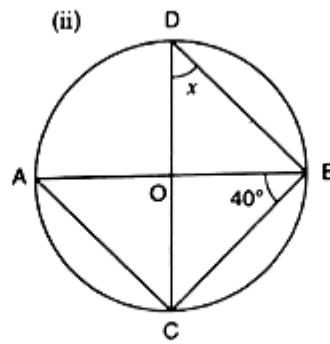


Fig. 16.124

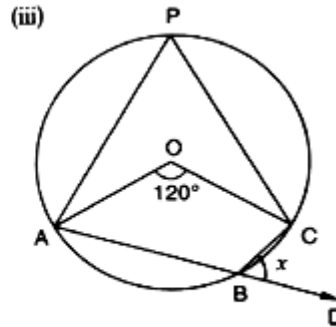


Fig. 16.125

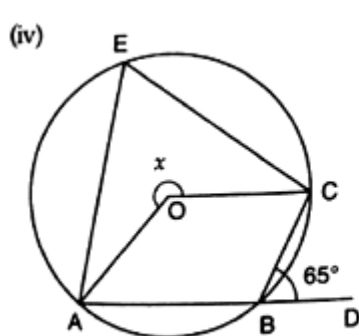


Fig. 16.126

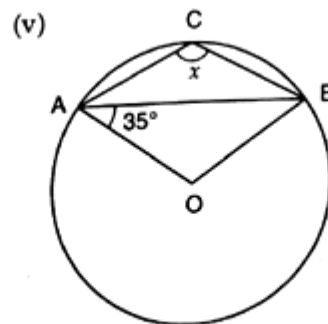


Fig. 16.127

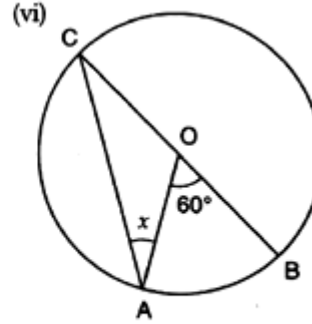


Fig. 16.128

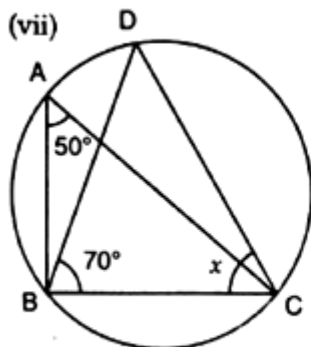


Fig. 16.129

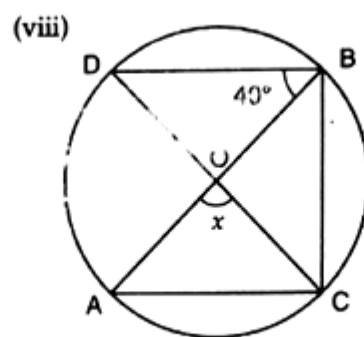


Fig. 16.130

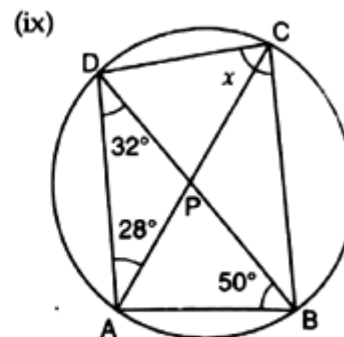
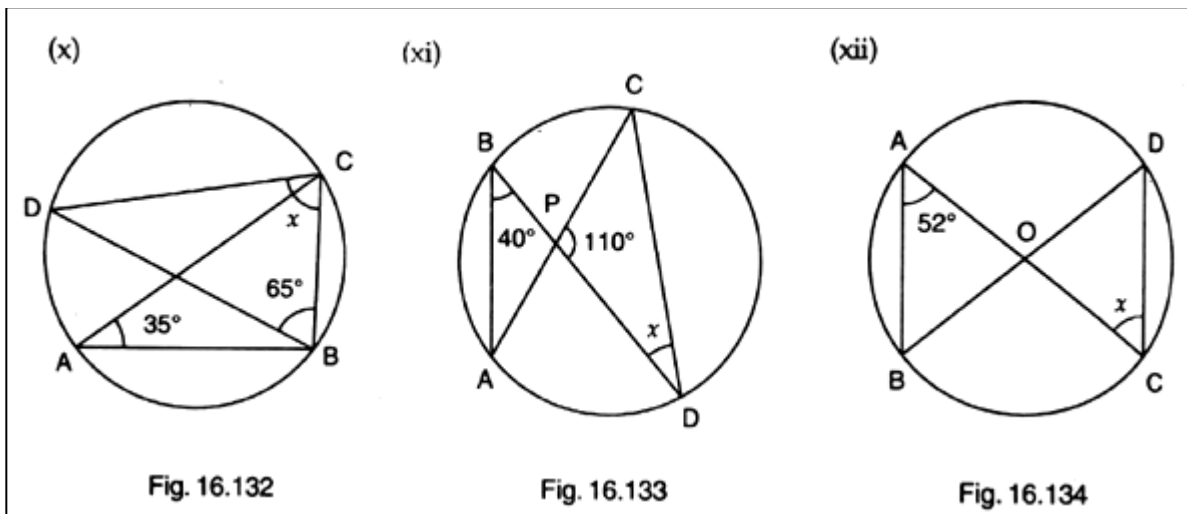


Fig. 16.131



Answer

(i) $\angle AOC = 135^\circ$

Therefore,

$$\angle AOC + \angle BOC = 180^\circ \text{ (Linear pair)}$$

$$135^\circ + \angle BOC = 180^\circ$$

$$\angle BOC = 45^\circ$$

By degree measure theorem,

$$\angle BOC = 2 \angle COB$$

$$45^\circ = 2x$$

$$x = 22 \frac{1}{2}^\circ$$

(ii) We have,

$$\angle ABC = 40^\circ$$

$$\angle ACB = 90^\circ \text{ (Angle in semi-circle)}$$

In triangle ABC, by angle sum property

$$\angle CAB + \angle ACB + \angle ABC = 180^\circ$$

$$\angle CAB + 90^\circ + 40^\circ = 180^\circ$$

$$\angle CAB = 50^\circ$$

Now,

$$\angle COB = \angle CAB \text{ (Angle on same segment)}$$

$$x = 50^\circ$$

(iii) We have,

$$\angle AOC = 120^\circ$$

By degree measure theorem,

$$\angle AOC = 2 \angle APC$$

$$120^\circ = 2 \angle APC$$

$$\angle APC = 60^\circ$$

Therefore,

$$\angle APC + \angle ABC = 180^\circ \text{ (Opposite angles of cyclic quadrilateral)}$$

$$60^\circ + \angle ABC = 180^\circ$$

$$\angle ABC = 120^\circ$$

$$\angle ABC + \angle DBC = 180^\circ \text{ (Linear pair)}$$

$$120^\circ + x = 180^\circ$$

$$x = 60^\circ$$

(iv) We have,

$$\angle CBD = 65^\circ$$

Therefore,

$$\angle ABC + \angle CBD = 180^\circ \text{ (Linear pair)}$$

$$\angle ABC + 65^\circ = 180^\circ$$

$$\angle ABC = 115^\circ$$

Therefore,

$$\text{Reflex } \angle AOC = 2 \angle ABC \text{ (By degree measure theorem)}$$

$$x = 2 * 115^\circ$$

$$= 230^\circ$$

(v) We have,

$$\angle OAB = 35^\circ$$

Then,

$$\angle OBA = \angle OAB = 35^\circ \text{ (Angle opposite to equal sides are equal)}$$

In triangle AOB, by angle sum property

$$\angle AOB + \angle OAB + \angle OBA = 180^\circ$$

$$\angle AOB + 35^\circ + 35^\circ = 180^\circ$$

$$\angle AOB = 110^\circ$$

Therefore,

$$\angle AOB + \text{Reflex } \angle AOB = 360^\circ \text{ (Complete angle)}$$

$$110^\circ + \text{Reflex } \angle AOB = 360^\circ$$

$$\text{Reflex } \angle AOB = 250^\circ$$

By degree measure theorem,

$$\text{Reflex } \angle AOB = 2 \angle ACB$$

$$250^\circ = 2x$$

$$x = 125^\circ$$

(vi) We have,

$$\angle AOB = 60^\circ$$

By degree measure theorem,

$$\angle AOB = 2 \angle ACB$$

$$60^\circ = 2 \angle ACB$$

$$\angle ACB = 30^\circ$$

$$x = 30^\circ$$

(vii) We have,

$$\angle BAC = 50^\circ$$

$$\angle DBC = 70^\circ$$

Therefore,

$$\angle BDC = \angle BAC = 50^\circ \text{ (Angles on same segment)}$$

In triangle BDC, by angle sum property

$$\angle BDC + \angle BCD + \angle DBC = 180^\circ$$

$$50^\circ + x + 70^\circ = 180^\circ$$

$$120^\circ + x = 180^\circ$$

$$x = 60^\circ$$

(viii) We have,

$$\angle DBO = 40^\circ$$

$$\angle DBC = 90^\circ \text{ (Angle in semi-circle)}$$

Therefore,

$$\angle DBO + \angle OBC = 90^\circ$$

$$40^\circ + \angle OBC = 90^\circ$$

$$\angle OBC = 50^\circ$$

By degree measure theorem,

$$\angle AOC = 2 \angle OBC$$

$$x = 2 * 50^\circ$$

$$x = 100^\circ$$

(ix) In triangle DAB, by angle sum property

$$\angle ADB + \angle DAB + \angle ABD = 180^\circ$$

$$32^\circ + \angle DAB + 50^\circ = 180^\circ$$

$$\angle DAB = 98^\circ$$

Now,

$$\angle DAB + \angle DCB = 180^\circ \text{ (Opposite angle of cyclic quadrilateral)}$$

$$98^\circ + x = 180^\circ$$

$$x = 180^\circ - 98^\circ$$

$$= 82^\circ$$

(x) We have,

$$\angle BAC = 35^\circ$$

$$\angle BDC = \angle BAC = 35^\circ \text{ (Angle on same segment)}$$

In triangle BCD, by angle sum property

$$\angle BDC + \angle BCD + \angle DBC = 180^\circ$$

$$30^\circ + x + 65^\circ = 180^\circ$$

$$x = 80^\circ$$

(xi) We have,

$$\angle ABD = 40^\circ$$

$$\angle ACD = \angle ABD = 40^\circ \text{ (Angle on same segment)}$$

In triangle PCD, by angle sum property

$$\angle PCD + \angle CPD + \angle PDC = 180^\circ$$

$$40^\circ + 110^\circ + x = 180^\circ$$

$$x = 30^\circ$$

(xii) Given that,

$$\angle BAC = 52^\circ$$

$$\angle BDC = \angle BAC = 52^\circ \text{ (Angle on same segment)}$$

Since, $OD = OC$

Then,

$$\angle ODC = \angle OCD \text{ (Opposite angles to equal radii)}$$

$$x = 52^\circ$$

5. Question

O is the circumcentre of the triangle ABC and OD is perpendicular on BC . Prove that $\angle BOD = \angle A$

Answer

Given that,

O is the circumcentre of triangle ABC and OD perpendicular BC

To prove: $\angle BOD = \angle A$

Proof: In triangle OBD and triangle OCD , we have

$$\angle ODB = \angle ODC \text{ (Each } 90^\circ)$$

$$OB = OC \text{ (Radii)}$$

$$OD = OD \text{ (Common)}$$

By R.H.S rule,

$$\triangle ODB \cong \triangle ODC$$

$$\angle BOD = \angle COD \text{ (By c.p.c.t) (i)}$$

By degree measure theorem,

$$\angle BOC = 2 \angle BAC$$

$$2 \angle BOD = 2 \angle BAC \text{ [From (i)]}$$

$$\angle BOD = \angle BAC$$

Hence, proved

6. Question

In Fig. 16.135, O is the centre of the circle, BO is the bisector of $\angle ABC$. Show that $AB = AC$.

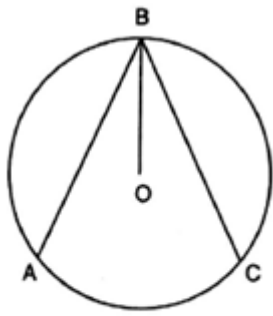


Fig. 16.135

Answer

Given that,

BO is the bisector of $\angle ABC$

To prove: $AB = BC$

Proof: $\angle ABO = \angle CBO$ (BO bisector of $\angle ABC$) (i)

$OB = OA$ (Radii)

Therefore,

$\angle ABO = \angle DAB$ (Opposite angle to equal sides are equal) (ii)

$OB = OC$ (Radii)

Therefore,

$\angle CBO = \angle OCB$ (Opposite angles to equal sides are equal) (iii)

Compare (i), (ii) and (iii)

$\angle OAB = \angle OCB$ (iv)

In triangle OAB and OCB, we have

$\angle OAB = \angle OCB$ [From (iv)]

$\angle OBA = \angle OBC$ (Given)

$OB = OB$ (Common)

By AAS congruence rule

$\triangle OAB \cong \triangle OCB$

$AB = BC$ (By c.p.c.t)

Hence, proved

7. Question

In Fig. 16. 136, O is the centre of the circle, prove that $\angle x = \angle y + \angle z$.

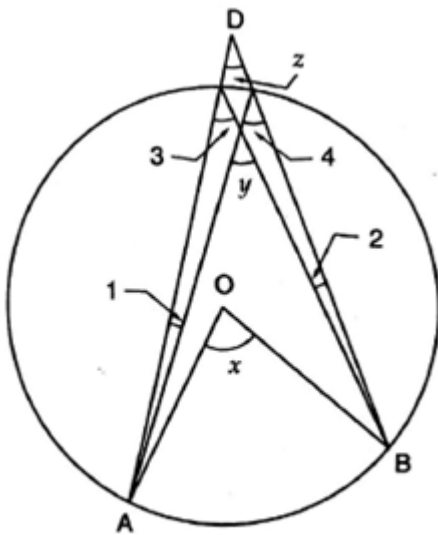


Fig. 16.136

Answer

We have,

$$\angle 3 = \angle 4 \text{ (Angle on same segment)}$$

By degree measure theorem,

$$\angle x = 2 \angle 3$$

$$\angle x = \angle 3 + \angle 3$$

$$\angle x = \angle 3 + \angle 4 \text{ (i) (Therefore, } \angle 3 = \angle 4 \text{)}$$

But,

$$\angle y = \angle 3 + \angle 1 \text{ (By exterior angle property)}$$

$$\angle 3 = \angle y - \angle 1 \text{ (ii)}$$

From (i) and (ii),

$$\angle x = \angle y - \angle 1 + \angle 4$$

$$\angle x = \angle y + \angle 4 - \angle 1$$

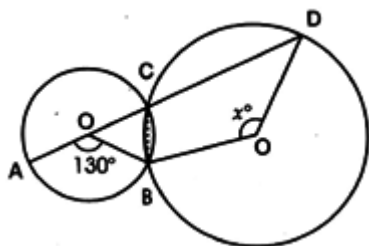
$$\angle x = \angle y + \angle z + \angle 1 - \angle 1 \text{ (By exterior angle property)}$$

$$\angle x = \angle y + \angle z$$

Hence, proved

8. Question

In Fig. 16.137, O and O' are centres of two circles intersecting at B and C . ACD is a straight line, find x .



Answer

By degree measure theorem,

$$\angle AOB = 2 \angle ACB$$

$$130^\circ = 2 \angle ACB$$

$$\angle ACB = 65^\circ$$

Therefore,

$$\angle ACB + \angle BCD = 180^\circ \text{ (Linear pair)}$$

$$65^\circ + \angle BCD = 180^\circ$$

$$\angle BCD = 115^\circ$$

By degree measure theorem,

$$\text{Reflex } \angle BOD = 2 \angle BCD$$

$$\text{Reflex } \angle BOD = 2 * 115^\circ$$

$$= 230^\circ$$

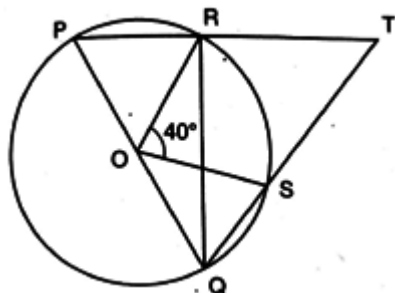
$$\angle BOD + \text{Reflex } \angle BOD = 360^\circ \text{ (Complete angle)}$$

$$230^\circ + x = 360^\circ$$

$$x = 130^\circ$$

9. Question

In Fig. 16.138, O is the centre of a circle and PQ is a diameter. If $\angle ROS = 40^\circ$, find $\angle RTS$.



Answer

Since,

PQ is diameter

Then,

$$\angle PRQ = 90^\circ \text{ (Angle in semi-circle)}$$

Therefore,

$$\angle PRQ + \angle TRQ = 180^\circ \text{ (Linear pair)}$$

$$90^\circ + \angle TRQ = 180^\circ$$

$$\angle TRQ = 90^\circ$$

By degree measure theorem,

$$\angle ROS = 2 \angle RQS$$

$$\angle RQS = 20^\circ$$

In triangle RQT, we have

$$\angle RQT + \angle QRT + \angle RTS = 180^\circ \text{ (By angle sum property)}$$

$$20^\circ + 90^\circ + \angle RTS = 180^\circ$$

$$\angle RTS = 70^\circ$$

10. Question

In Fig. 16.139, if $\angle ACB = 40^\circ$, $\angle DPB = 120^\circ$, find $\angle CBD$.

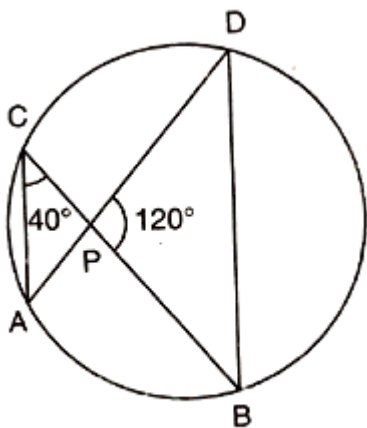


Fig. 16.139

Answer

We have,

$$\angle ACB = 40^\circ$$

$$\angle DPB = 120^\circ$$

$\angle ADB = \angle ACB = 40^\circ$ (Angle on same segment)

In triangle PDB, by angle sum property

$$\angle PDB + \angle PBD + \angle BPD = 180^\circ$$

$$40^\circ + \angle PBD + 120^\circ = 180^\circ$$

$$\angle PBD = 20^\circ$$

Therefore,

$$\angle CBD = 20^\circ$$

11. Question

A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chords at a point on the minor arc and also at a point on the major arc.

Answer

We have,

Radius OA = Chord AB

$$OA = OB = AB$$

Then, triangle OAB is an equilateral triangle

Therefore,

$$\angle AOB = 60^\circ \text{ (Angle of an equilateral triangle)}$$

By degree measure theorem,

$$\angle AOB = 2 \angle APB$$

$$60^\circ = 2 \angle APB$$

$$\angle APB = 30^\circ$$

Now,

$$\angle APB + \angle AQB = 180^\circ \text{ (Opposite angle of cyclic quadrilateral)}$$

$$30^\circ + \angle AQB = 180^\circ$$

$$\angle AQB = 150^\circ$$

Therefore,

$$\text{Angle by chord AB at minor arc} = 150^\circ$$

$$\text{And, by major arc} = 30^\circ$$

Exercise 16.5

1. Question

In Fig. 16.176, ΔABC is an equilateral triangle. Find $m\angle BEC$.

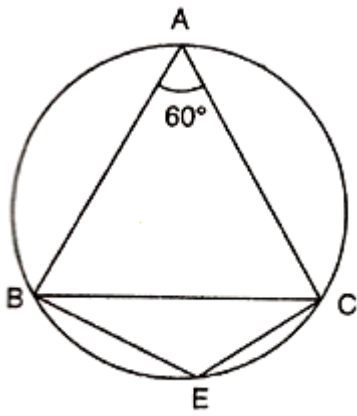


Fig. 16.176

Answer

Since,

Triangle ABC is an equilateral triangle

$$\angle BAC = 60^\circ$$

$$\angle BAC + \angle BEC = 180^\circ \text{ (Opposite angles of quadrilateral)}$$

$$60^\circ + \angle BEC = 180^\circ$$

$$\angle BEC = 120^\circ$$

2. Question

In Fig. 16.177, ΔPQR is an isosceles triangle with $PQ=PR$ and $m\angle PQR=35^\circ$. Find $m\angle QSR$ and $m\angle QTR$.

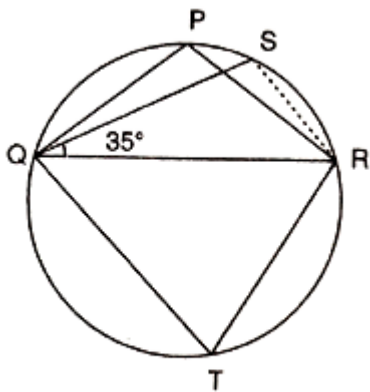


Fig. 16.177

Answer

We have,

$$\angle PQR = 35^\circ$$

$$\angle PQR + \angle PRQ = 35^\circ \text{ (Angle opposite to equal sides)}$$

In triangle PQR, by angle sum property

$$\angle P + \angle Q + \angle R = 180^\circ$$

$$\angle P + 35^\circ + 35^\circ = 180^\circ$$

$$\angle P = 110^\circ$$

Now,

$$\angle QSR + \angle QTR = 180^\circ$$

$$110^\circ + \angle QTR = 180^\circ$$

$$\angle QTR = 70^\circ$$

3. Question

In Fig. 16.178, O is the centre of the circle. If $\angle BOD = 160^\circ$, find the values of x and y .

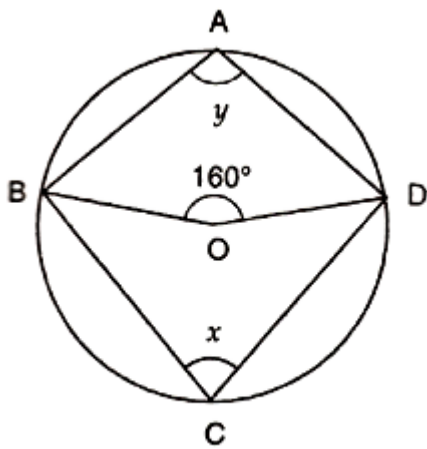


Fig. 16.178

Answer

Given that,

O is the centre of the circle

We have,

$$\angle BOD = 160^\circ$$

By degree measure theorem,

$$\angle BOD = 2 \angle BCD$$

$$160^\circ = 2x$$

$$x = 80^\circ$$

Therefore,

$$\angle BAD + \angle BCD = 180^\circ \text{ (Opposite angles of Cyclic quadrilateral)}$$

$$y + x = 180^\circ$$

$$y + 80^\circ = 180^\circ$$

$$y = 100^\circ$$

4. Question

In Fig. 16.179 $ABCD$ is a cyclic quadrilateral. If $\angle BCD = 100^\circ$ and $\angle ABD = 70^\circ$, find $\angle ADB$.

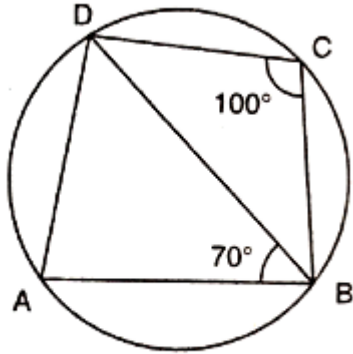


Fig. 16.179

Answer

We have,

$$\angle BCD = 100^\circ$$

$$\angle ABD = 70^\circ$$

Therefore,

$$\angle DAB + \angle BCD = 180^\circ \text{ (Opposite angles of cyclic quadrilateral)}$$

$$\angle DAB + 100^\circ = 180^\circ$$

$$\angle DAB = 180^\circ - 100^\circ$$

$$= 80^\circ$$

In triangle DAB , by angle sum property

$$\angle ADB + \angle DAB + \angle ABD = 180^\circ$$

$$\angle ABD + 80^\circ + 70^\circ = 180^\circ$$

$$\angle ABD = 30^\circ$$

5. Question

If $ABCD$ is a cyclic quadrilateral in which $AD \parallel BC$ (fig. 16.180). Prove that $\angle B = \angle C$.

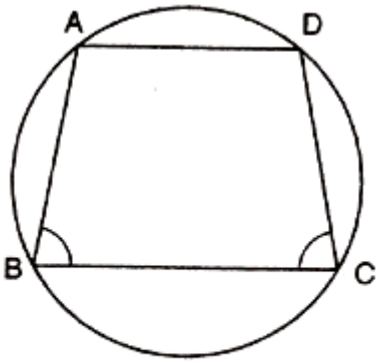


Fig. 16.180

Answer

Since, ABCD is a cyclic quadrilateral with $AD \parallel BC$

Then,

$$\angle A + \angle C = 180^\circ \text{ (i) (Opposite angles of cyclic quadrilateral)}$$

And,

$$\angle A + \angle B = 180^\circ \text{ (ii) (Co. interior angles)}$$

Comparing (i) and (ii), we get

$$\angle B = \angle C$$

Hence, proved

6. Question

In Fig. 16.181, O is the centre of the circle. Find $\angle CBD$.

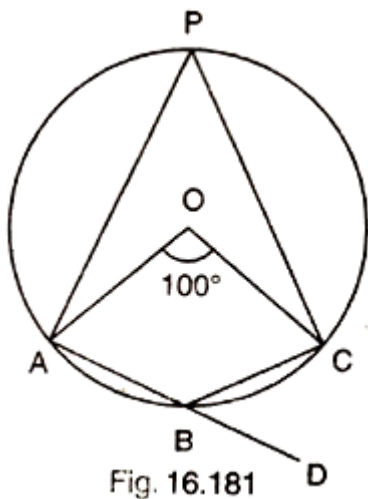


Fig. 16.181

Answer

Given that,

$$\angle BOC = 100^\circ$$

By degree measure theorem,

$$\angle AOC = 2 \angle APC$$

$$100^\circ = 2 \angle APC$$

$$\angle APC = 50^\circ$$

Therefore,

$$\angle APC + \angle ABC = 180^\circ \text{ (Opposite angles of a cyclic quadrilateral)}$$

$$50^\circ + \angle ABC = 180^\circ$$

$$\angle ABC = 130^\circ$$

Therefore,

$$\angle ABC + \angle CBD = 180^\circ \text{ (Linear pair)}$$

$$130^\circ + \angle CBD = 180^\circ$$

$$\angle CBD = 50^\circ$$

7. Question

In Fig. 16.182, AB and CD are diameters of a circle with centre O . If $\angle OBD = 50^\circ$, find $\angle AOC$.

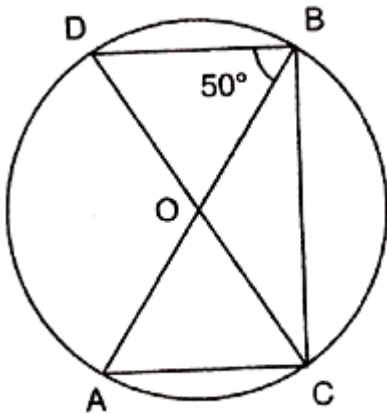


Fig. 16.182

Answer

Given that,

$$\angle OBD = 50^\circ$$

Since,

AB and CD are the diameters of the circles and O is the centre of the circle

Therefore,

$$\angle PBC = 90^\circ \text{ (Angle in the semi-circle)}$$

$$\angle OBD + \angle DBC = 90^\circ$$

$$50^\circ + \angle DBC = 90^\circ$$

$$\angle DBC = 40^\circ$$

By degree measure theorem,

$$\angle AOC = 2 \angle ABC$$

$$\angle AOC = 2 * 40^\circ$$

$$= 80^\circ$$

8. Question

On a semi-circle with AB as diameter, a point C is taken, so that $m(\angle CAB) = 30^\circ$. Find $m(\angle ACB)$ and $m(\angle ABC)$.

Answer

We have,

$$\angle CAB = 30^\circ$$

$$\angle ACB = 90^\circ \text{ (Angle in semi-circle)}$$

IN triangle ABC , by angle sum property

$$\angle CAB + \angle ACB + \angle ABC = 180^\circ$$

$$30^\circ + 90^\circ + \angle ABC = 180^\circ$$

$$\angle ABC = 60^\circ$$

9. Question

In a cyclic quadrilateral $ABCD$ if $AB \parallel CD$ and $B=70^\circ$, find the remaining angles.

Answer

Given that,

$$\angle B = 70^\circ$$

Since, $ABCD$ is a cyclic quadrilateral

Then,

$$\angle B + \angle D = 180^\circ$$

$$70^\circ + \angle D = 180^\circ$$

$$\angle D = 110^\circ$$

Since, $AB \parallel DC$

Then,

$$\angle B + \angle C = 180^\circ \text{ (Co. interior angle)}$$

$$70^{\circ} + \angle C = 180^{\circ}$$

$$\angle C = 110^{\circ}$$

Now,

$$\angle A + \angle C = 180^{\circ} \text{ (Opposite angles of cyclic quadrilateral)}$$

$$\angle A + 110^{\circ} = 180^{\circ}$$

$$\angle A = 70^{\circ}$$

10. Question

In a cyclic quadrilateral $ABCD$, if $m \angle A = 3(m \angle C)$. Find $m \angle A$.

Answer

WE have,

$$\angle A = 3 \angle C$$

$$\text{Let, } \angle C = x$$

Therefore,

$$\angle A + \angle C = 180^{\circ} \text{ (Opposite angles of cyclic quadrilateral)}$$

$$3x + x = 180^{\circ}$$

$$4x = 180^{\circ}$$

$$x = 45^{\circ}$$

$$\angle A = 3x$$

$$= 3 * 45^{\circ}$$

$$= 135^{\circ}$$

Therefore,

$$\angle A = 135^{\circ}$$

11. Question

In Fig. 16.183, O is the centre of the circle $\angle DAB = 50^{\circ}$. Calculate the values of x and y .

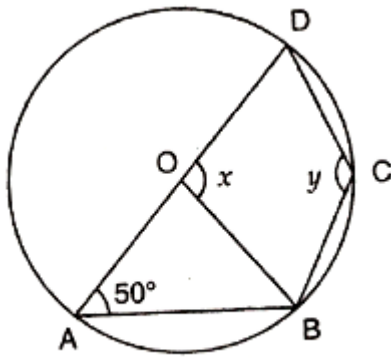


Fig. 16.183

Answer

We have,

$$\angle DAB = 50^\circ$$

By degree measure theorem

$$\angle BOD = 2 \angle BAD$$

$$x = 2 * 50^\circ$$

$$= 100^\circ$$

Since, ABCD is a cyclic quadrilateral

Then,

$$\angle A + \angle C = 180^\circ$$

$$50^\circ + y = 180^\circ$$

$$y = 130^\circ$$

12. Question

In Fig. 16.184, if $\angle BAC = 60^\circ$ and $\angle BCA = 20^\circ$, find $\angle ADC$.

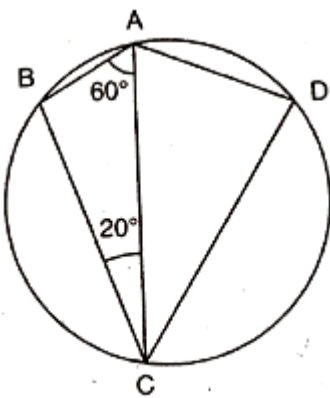


Fig. 16.184

Answer

By using angle sum property in triangle ABC,

$$\angle B = 180^\circ - (60^\circ + 20^\circ)$$

$$= 100^\circ$$

In cyclic quadrilateral ABCD, we have

$$\angle B + \angle D = 180^\circ$$

$$\angle D = 180^\circ - 100^\circ$$

$$= 80^\circ$$

13. Question

In Fig. 16.185, if ABC is an equilateral triangle. Find $\angle BDC$ and $\angle BEC$

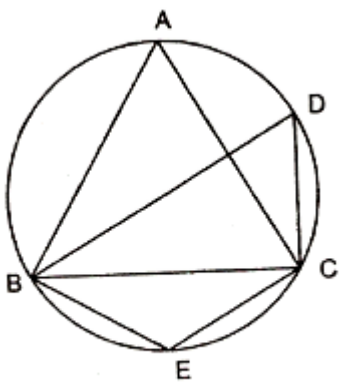


Fig. 16.185

Answer

Since, ABC is an equilateral triangle

Then,

$$\angle BAC = 60^\circ$$

Therefore,

$$\angle BDC = \angle BAC = 60^\circ \text{ (Angles in the same segment)}$$

Since, quadrilateral ABEC is a cyclic quadrilateral

Then,

$$\angle BAC + \angle BEC = 180^\circ$$

$$60^\circ + \angle BEC = 180^\circ$$

$$\angle BEC = 180^\circ - 60^\circ$$

$$= 120^\circ$$

14. Question

In Fig. 16.186, O is the centre of the circle. If $\angle CEA = 30^\circ$, find the values of x , y and z .

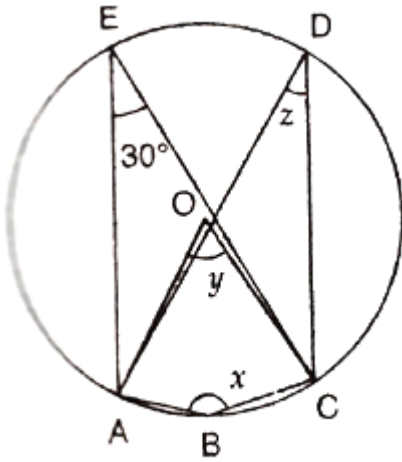


Fig. 16.186

Answer

We have,

$$\angle AEC = 30^\circ$$

Since, quadrilateral ABCE is a cyclic quadrilateral

Then,

$$\angle BAC + \angle AEC = 180^\circ$$

$$x + 30^\circ = 180^\circ$$

$$x = 150^\circ$$

By degree measure theorem,

$$\angle AOC = 2 \angle AEC$$

$$y = 2 * 30^\circ$$

$$= 60^\circ$$

Therefore,

$$\angle ADC = \angle AEC \text{ (Angles in same segment)}$$

$$z = 30^\circ$$

15. Question

In Fig. 16.187, $\angle BAD = 78^\circ$, $\angle DCF = x^\circ$ and $\angle DEF = y^\circ$. Find the values of x and y .

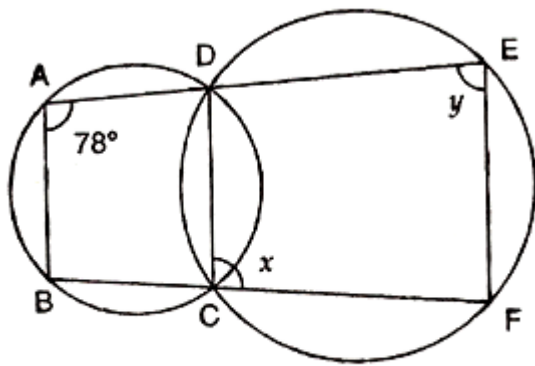


Fig. 16.187

Answer

We have,

$$\angle BAD = 78^\circ$$

$$\angle DCF = x^\circ$$

$$\angle DEF = y^\circ$$

Since, ABCD is a cyclic quadrilateral

$$\angle BAD + \angle BCD = 180^\circ$$

$$78^\circ + \angle BCD = 180^\circ$$

$$\angle BCD = 102^\circ$$

Now,

$$\angle BCD + \angle DCF = 180^\circ \text{ (Linear pair)}$$

$$102^\circ = x - 180^\circ$$

$$x = 78^\circ$$

Since,

DCEF is a cyclic quadrilateral

$$x + y = 180^\circ$$

$$78^\circ + y = 180^\circ$$

$$y = 102^\circ$$

16. Question

In a cyclic quadrilateral ABCD, if $\angle A - \angle C = 60^\circ$, prove that the smaller of two is 60° .

Answer

WE have,

$$\angle A - \angle C = 60^\circ \text{ (i)}$$

Since, ABCD is a cyclic quadrilateral

Then,

$$\angle A + \angle C = 180^\circ \text{ (ii)}$$

Adding (i) and (ii), we get

$$\angle A - \angle C + \angle A + \angle C = 60^\circ + 180^\circ$$

$$2 \angle A = 240^\circ$$

$$\angle A = 120^\circ$$

Put value of $\angle A$ in (ii), we get

$$120^\circ + \angle C = 180^\circ$$

$$\angle C = 60^\circ$$

17. Question

In Fig. 16.188, ABCD is a cyclic quadrilateral. Find the value of x .

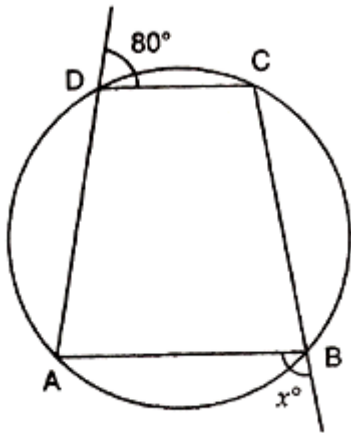


Fig. 16.188

Answer

$$\angle FDC + \angle CDA = 180^\circ \text{ (Linear pair)}$$

$$80^\circ + \angle CDA = 180^\circ$$

$$\angle CDA = 100^\circ$$

Since, ABCD is a cyclic quadrilateral

$$\angle ADC + \angle ABC = 180^\circ$$

$$100^\circ + \angle ABC = 180^\circ$$

$$\angle ABC = 80^\circ$$

Now,

$$\angle ABC + \angle ABF = 180^\circ \text{ (Linear pair)}$$

$$80^\circ + x = 180^\circ$$

$$x = 180^\circ - 80^\circ$$

$$= 100^\circ$$

18. Question

$ABCD$ is a cyclic quadrilateral in which:

(i) $BC \parallel AD$, $\angle ADC = 110^\circ$ and $\angle BAC = 50^\circ$. Find $\angle DAC$.

(ii) $\angle DBC = 80^\circ$ and $\angle BAC = 40^\circ$ Find $\angle BCD$.

(iii) $\angle BCD = 100^\circ$ and $\angle ABD = 70^\circ$. Find $\angle ADB$.

Answer

(i) Since, $ABCD$ is a cyclic quadrilateral

Then,

$$\angle ABC + \angle ADC = 180^\circ$$

$$\angle ABC + 110^\circ = 180^\circ$$

$$\angle ABC = 70^\circ$$

Since, $AD \parallel BC$

Then,

$$\angle DAB + \angle ABC = 180^\circ \text{ (Co. interior angle)}$$

$$\angle DAC + 50^\circ + 70^\circ = 180^\circ$$

$$\angle DAC = 180^\circ - 120^\circ$$

$$= 60^\circ$$

(ii) $\angle BAC = \angle BDC = 40^\circ$ (Angle in the same segment)

In $\triangle BDC$, by angle sum property

$$\angle DBC + \angle BCD + \angle BDC = 180^\circ$$

$$80^\circ + \angle BCD + 40^\circ = 180^\circ$$

$$\angle BCD = 60^\circ$$

(iii) Given that,

Quadrilateral $ABCD$ is a cyclic quadrilateral

Then,

$$\angle BAD + \angle BCD = 180^\circ$$

$$\angle BAD = 80^\circ$$

In triangle ABD, by angle sum property

$$\angle ABD + \angle ADB + \angle BAD = 180^\circ$$

$$70^\circ + \angle ADB + 80^\circ = 180^\circ$$

$$\angle ADB = 30^\circ$$

19. Question

Prove that the perpendicular bisectors of the sides of a cyclic quadrilateral are concurrent.

Answer

Let ABCD be a cyclic quadrilateral and let O be the centre of the corresponding circle

Then, each side of the quadrilateral ABCD is a chord of the circle and the perpendicular bisector of a chord always passes through the centre of the circle

So, right bisectors of the sides of the quadrilateral ABCD will pass through the centre O of the corresponding circle.

20. Question

Prove that the centre of the circle circumscribing the cyclic rectangle ABCD is the point of intersection of its diagonals.

Answer

Let O be the circle circumscribing the cyclic rectangle ABCD.

Since, $\angle ABC = 90^\circ$ and AC is the chord of the circle. Similarly, BD is a diameter

Hence, point of intersection of AC and BD is the centre of the circle.

21. Question

Prove that the circles described on the four sides of a rhombus as diameters, pass through the point of intersection of its diagonals.

Answer

Let ABCD be a rhombus such that its diagonals AC and BD intersect at O

Since, the diagonals of a rhombus intersect at right angle

Therefore,

$$\angle ACB = \angle BOC = \angle COD = \angle DOA = 90^\circ$$

Now,

$$\angle AOB = 90^\circ = \text{circle described on AB as diameter will pass through O}$$

Similarly, all the circles described on BC, AD and CD as diameter pass through O.

22. Question

If the two sides of a pair of opposite sides of a cyclic quadrilateral are equal, prove that its diagonals are equal.

Answer

Given that,

ABCD is cyclic quadrilateral in which $AB = DC$

To prove: $AC = BD$

Proof: In $\triangle PAB$ and $\triangle PDC$,

$AB = DC$ (Given)

$\angle BAP = \angle CDP$ (Angles in the same segment)

$\angle PBA = \angle PCD$ (Angles in the same segment)

Then,

$\triangle PAB = \triangle PDC$ (i) (By c.p.c.t)

$PA = PD$ (ii) (By c.p.c.t)

Adding (i) and (ii), we get

$PA + PC = PD + PB$

$AC = BD$

23. Question

ABCD is a cyclic quadrilateral in which BA and CD when produced meet in E and $EA = ED$. Prove that:

(i) $AD \parallel BC$ (ii) $EB = EC$

Answer

Given that, ABCD is a cyclic quadrilateral in which

(i) Since,

$EA = ED$

Then,

$\angle EAD = \angle EDA$ (i) (Opposite angles to equal sides)

Since, ABCD is a cyclic quadrilateral

Then,

$\angle ABC + \angle ADC = 180^\circ$

But,

$$\angle ABC + \angle EBC = 180^\circ \text{ (Linear pair)}$$

Then,

$$\angle ADC = \angle EBC \text{ (ii)}$$

Compare (i) and (ii), we get

$$\angle EAD = \angle EBC \text{ (iii)}$$

Since, corresponding angles are equal

Then,

$$BC \parallel AD$$

(ii) From (iii), we have

$$\angle EAD = \angle EBC$$

Similarly,

$$\angle EDA = \angle ECB \text{ (iv)}$$

Compare equations (i), (iii) and (iv), we get

$$\angle EBC = \angle ECB$$

$$EB = EC \text{ (Opposite angles to equal sides)}$$

24. Question

Circles are described on the sides of a triangle as diameters. Prove that the circles on any two sides intersect each other on the third side (or third side produced).

Answer

Since,

AB is a diameter

Then,

$$\angle ADB = 90^\circ \text{ (i) (Angle in semi-circle)}$$

Since,'

AC is a diameter

Then,

$$\angle ADC = 90^\circ \text{ (ii) (Angle in semi-circle)}$$

Adding (i) and (ii), we get

$$\angle ADB + \angle ADC = 90^\circ + 90^\circ$$

$$\angle BDC = 180^\circ$$

Then, BDC is a line

Hence, the circles on any two sides intersect each other on the third side.

25. Question

Prove that the angle in a segment shorter than a semicircle is greater than a right angle.

Answer

Given that,

$\angle ACB$ is an angle in minor segment

To prove: $\angle ACB > 90^\circ$

Proof: By degree measure theorem

Reflex $\angle AOB = 2 \angle ACB$

And,

Reflex $\angle AOB > 180^\circ$

Then,

$2 \angle ACB > 180^\circ$

$\angle ACB > \frac{180}{2}$

$\angle ACB > 90^\circ$

Hence, proved

26. Question

Prove that the angle in a segment greater than a semi-circle is less than a right angle.

Answer

Given that,

$\angle ACB$ is an angle in major segment

To prove: $\angle ACB < 90^\circ$

Proof: By degree measure theorem,

$\angle AOB = 2 \angle ACB$

And,

$\angle AOB < 180^\circ$

Then,

$2 \angle ACB < 180^\circ$

$\angle ACB < 90^\circ$

Hence, proved

27. Question

$ABCD$ is a cyclic trapezium with $AD \parallel BC$. If $\angle B = 70^\circ$, determine other three angles of the trapezium.

Answer

Given that,

$ABCD$ is a cyclic trapezium with $AD \parallel BC$ and $\angle B = 70^\circ$

Since, $ABCD$ is a quadrilateral

Then,

$$\angle B + \angle D = 180^\circ$$

$$70^\circ + \angle D = 180^\circ$$

$$\angle D = 110^\circ$$

Since, $AD \parallel BC$

Then,

$$\angle A + \angle B = 180^\circ \text{ (Co. interior angle)}$$

$$\angle A + 70^\circ = 180^\circ$$

$$\angle A = 110^\circ$$

Since, $ABCD$ is a cyclic quadrilateral

$$\text{Then, } \angle A + \angle C = 180^\circ$$

$$110^\circ + \angle C = 180^\circ$$

$$\angle C = 70^\circ$$

28. Question

Prove that the line segment joining the mid-point of the hypotenuse of a right triangle to its opposite vertex is half of the hypotenuse.

Answer

Let, triangle ABC be a right angle triangle at $\angle B$

Let P be the mid-point of hypotenuse AC

Draw a circle with centre P and AC as diameter

Since,

$$\angle ABC = 90^\circ$$

Therefore, the circle passes through B

Therefore,

$BP = \text{Radius}$

Also,

$AP = CP = \text{Radius}$

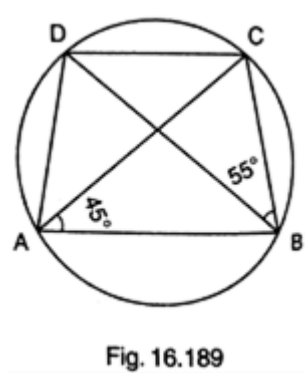
Therefore,

$AP = BP = CP$

Hence, $BP = \frac{1}{2} AC$

29. Question

In Fig. 16.189, $ABCD$ is a cyclic quadrilateral in which AC and BD are its diagonals. If $\angle DBC = 55^\circ$ and $\angle BAC = 45^\circ$, find $\angle BCD$.



Answer

Since angles on the same segment of a circle are equal

Therefore,

$$\angle CAD = \angle DBC = 55^\circ$$

$$\angle DAB = \angle CAD + \angle BAC$$

$$= 55^\circ + 45^\circ$$

$$= 100^\circ$$

But,

$$\angle DAB + \angle BCD = 180^\circ \text{ (Opposite angles of a cyclic quadrilateral)}$$

Therefore,

$$\angle BCD = 180^\circ - 100^\circ$$

$$\angle BCD = 80^\circ$$

CCE - Formative Assessment

1. Question

In Fig. 16.193, two circles intersect at A and B . The centre of the smaller circle is O and it lies on the circumference of the larger circle. If $\angle APB = 70^\circ$, find $\angle ACB$.

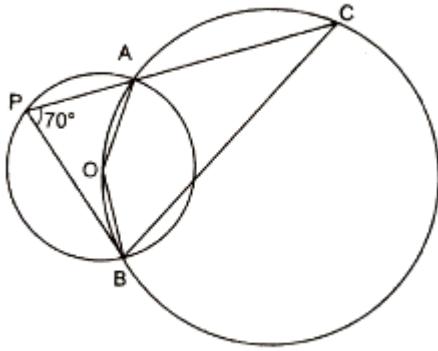


Fig. 16.193

Answer

O is the centre of the smaller circle.

$$\angle APB = 70^\circ$$

By degree measure theorem,

$$\angle AOB = 2 \angle APB$$

$$\angle AOB = 2 \times 70^\circ$$

$$= 140^\circ$$

Therefore,

$AOBC$ is a cyclic quadrilateral

$$\angle ACB + \angle AOB = 180^\circ$$

$$\angle ACB + 140^\circ = 180^\circ$$

$$\angle ACB = 40^\circ$$

2. Question

In Fig. 16.194, two congruent circles with centres O and O' intersect at A and B . If $\angle AO'B = 50^\circ$, then find $\angle APB$.

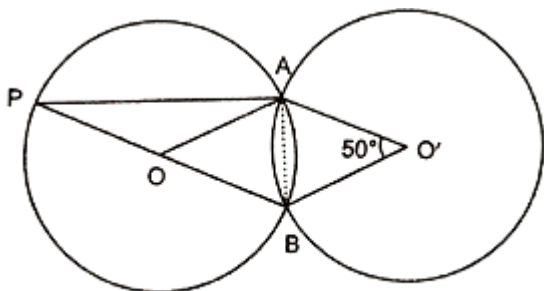


Fig. 16.194

Answer

$$\angle AO'B = 50^\circ$$

Since, both the triangle are congruent so their corresponding angles are equal.

$$\angle AOB = \angle AO'B = 50^\circ$$

Now,

$$\angle APB = \frac{\angle AOB}{2}$$

$$\angle APB = \frac{50}{2}$$

$$= 25^\circ$$

3. Question

In Fig. 16.195, $ABCD$ is a cyclic quadrilateral in which $\angle BAD = 75^\circ$, $\angle ABD = 58^\circ$ and $\angle ADC = 77^\circ$, AC and BD intersect at P . Then, find $\angle DPC$.

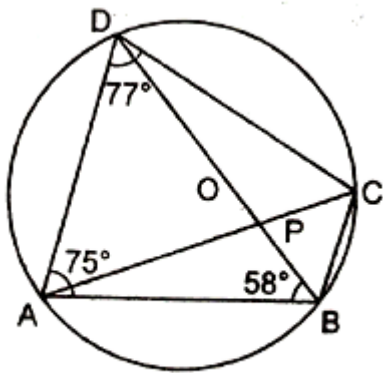


Fig. 16.195

Answer

$$\angle DBA = \angle DCA = 58^\circ \text{ (Angles on the same segment)}$$

In triangle DCA

$$\angle DCA + \angle CDA + \angle DAC = 180^\circ$$

$$58^\circ + 77^\circ + \angle DAC = 180^\circ$$

$$\angle DAC = 45^\circ$$

$$\angle DPC = 180^\circ - 58^\circ - 30^\circ$$

$$= 92^\circ$$

4. Question

In Fig. 16.196, if $\angle AOB = 80^\circ$ and $\angle ABC = 30^\circ$, then find $\angle CAO$.

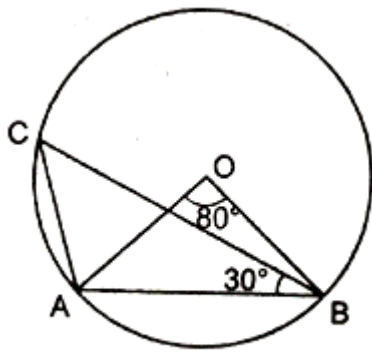


Fig. 16.196

Answer

$$2 \angle OAB = 100^\circ$$

$$\angle OAB = 50^\circ$$

Therefore,

$$\angle OAB = \angle OBA = 50^\circ$$

$$\angle AOB = 2 \angle BCA \text{ (Angle subtended by any point on circle)}$$

$$80^\circ = 2 \angle BCA$$

$$\angle BCA = 40^\circ$$

Now,

In triangle ABC

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + 30^\circ + 40^\circ = 180^\circ$$

$$\angle A = 110^\circ$$

$$\angle CAB = \angle CAO + \angle OAB$$

$$110^\circ = \angle CAO + 50^\circ$$

$$\angle CAO = 60^\circ$$

5. Question

In Fig. 16.196, if O is the circumcentre of ΔABC , then find the value of $\angle OBC + \angle BAC$.

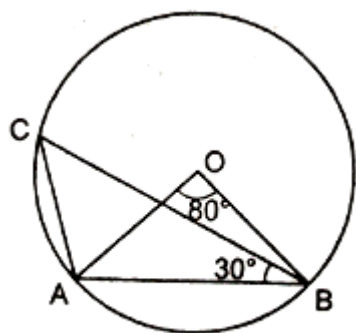


Fig. 16.196

Answer

$$\angle OBC + \angle CBA = \angle OBA$$

$$\angle OBC + 30^\circ = 50^\circ$$

$$\angle OBC = 20^\circ$$

$$\angle OBC + \angle BAC = \angle OBC + \angle CAB$$

$$= 20^\circ + 110^\circ$$

$$= 130^\circ$$

6. Question

In Fig. 16.197, AOC is a diameter of the circle and arc $AXB = \frac{1}{2}$ arc BYC . Find $\angle BOC$.

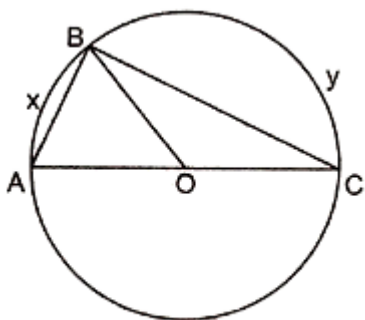


Fig. 16.197

Answer

Given that,

$$\text{Arc } AXB = \frac{1}{2} \text{ Arc } BYC \quad (i)$$

Since,

Arc AXBYC is the arc equal to half circumference

And,

Angle subtended by half circumference at centre is 180°

$$\text{Arc } AXBYC = \text{Arc } AXB + \text{Arc } BYC$$

$$\text{Arc AXBYC} = \frac{1}{2} \text{Arc BYC} + \text{Arc BYC}$$

$$\text{Arc AXBYC} = \frac{2}{3} \text{Arc AXBYC}$$

Now,

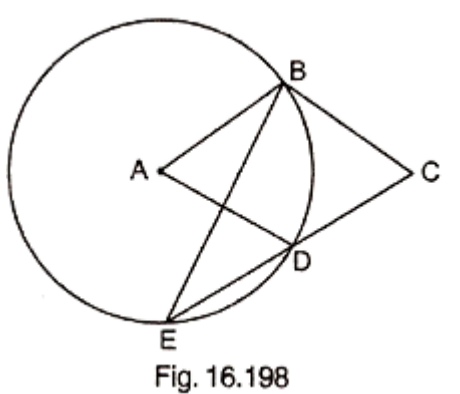
$$\angle BOC = \frac{2}{3} \angle AOC$$

$$\angle BOC = \frac{2}{3} * 180^\circ$$

$$\angle BOC = 120^\circ$$

7. Question

In Fig. 16.198, A is the centre of the circle. $ABCD$ is a parallelogram and CDE is a straight line. Find $\angle BCD : \angle ABE$



Answer

Given that,

A is the centre of the circle, then

$$AB = AD$$

$ABCD$ is a parallelogram, then

$$AD \parallel BC, AB \parallel CD$$

CDE is a straight line, then

$$AB \parallel CE$$

Let,

$$\angle BEC = \angle ABE = x' \text{ (Alternate angle)}$$

We know that,

The angle subtended by an arc of a circle at the centre double the angle subtended by it at any point on the remaining part of circle

$$\angle BAD = 2 \angle BEC$$

$$\angle BAD = 2x'$$

In a rhombus opposite angles are equal to each other

$$\angle BAD = \angle BCD = 2x'$$

Now, we have to find

$$\frac{\angle BCD}{\angle ABE} = \frac{2x'}{x'}$$

$$\frac{\angle BCD}{\angle ABE} = \frac{2}{1}$$

$$\frac{\angle BCD}{\angle ABE} = \frac{2x'}{x'}$$

Hence,

$$\angle BCD : \angle ABE \text{ is } 2 : 1$$

8. Question

In Fig. 16.199, AB is a diameter of the circle such that $\angle A = 35^\circ$ and $\angle Q = 25^\circ$, find $\angle PBR$.

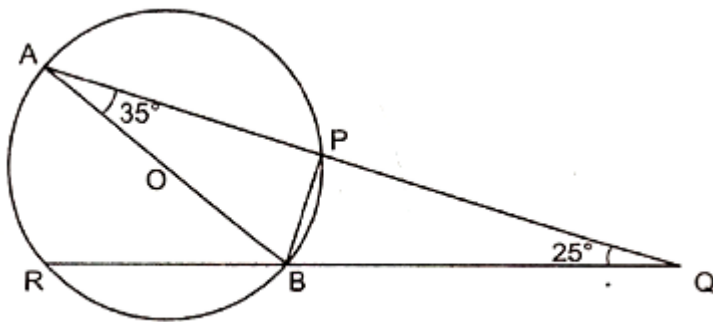


Fig. 16.199

Answer

In triangle ABQ,

$$\angle ABQ + \angle AQB + \angle BAQ = 180^\circ$$

$$\angle ABQ + 25^\circ + 35^\circ = 180^\circ$$

$$\angle ABQ = 120^\circ$$

$$\angle APB = 90^\circ \text{ (Angle in the semi-circle)}$$

In triangle APB,

$$\angle APB + \angle PBA + \angle PAB = 180^\circ$$

$$90^\circ + \angle PBA + 35^\circ = 180^\circ$$

$$\angle PBA = 55^\circ$$

Now,

$$\angle PBR = \angle PBA + \angle PBR$$

$$\angle PBR = 55^\circ + (180^\circ - 120^\circ)$$

$$\angle PBR = 115^\circ$$

Thus,

$$\angle PBR = 115^\circ$$

9. Question

In Fig. 16.200, P and Q are centres of two circles intersecting at B and C . ACD is a straight line. Then, $\angle BQD =$

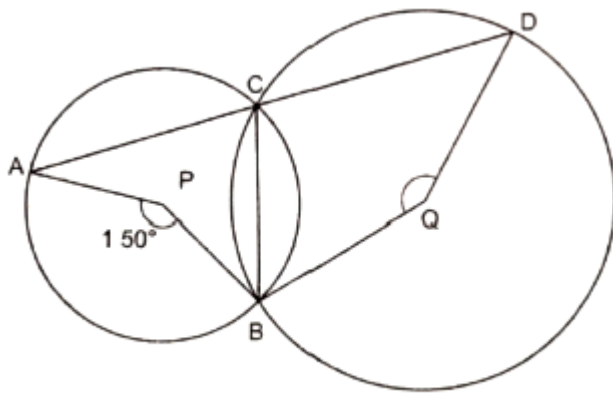


Fig. 16.200

Answer

We know that,

$$\angle ACB = \frac{\angle APB}{2}$$

$$\angle ACB = \frac{150}{2}$$

$$\angle ACB = 75^\circ$$

Since,

ACD is a straight line, so

$$\angle ACB + \angle BCD = 180^\circ$$

$$75^\circ + \angle BCD = 180^\circ$$

$$\angle BCD = 180^\circ - 75^\circ$$

$$= 105^\circ$$

Now,

$$\angle BCD = \frac{1}{2} \text{ Reflex } \angle BQD$$

$$105^\circ = \frac{1}{2} (360^\circ - \angle BQD)$$

$$210^\circ = 360^\circ - \angle BQD$$

$$\angle BQD = 360^\circ - 210^\circ$$

Therefore,

$$\angle BQD = 150^\circ$$

10. Question

In Fig. 16.201, $ABCD$ is a quadrilateral inscribed in a circle with centre O . CD is produced to E such that $\angle AED = 95^\circ$ and $\angle OBA = 30^\circ$. Find $\angle OAC$.

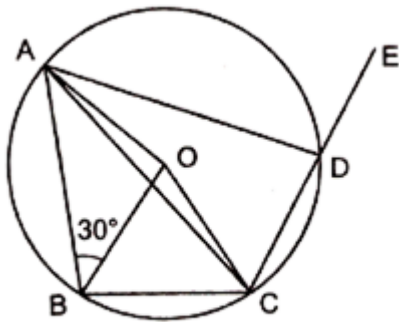


Fig. 16.201

Answer

$$\angle ADE = 95^\circ \text{ (Given)}$$

Since,

$$OA = OB, \text{ so}$$

$$\angle OAB = \angle OBA$$

$$\angle OAB = 30^\circ$$

$$\angle ADE + \angle ADC = 180^\circ \text{ (Linear pair)}$$

$$95^\circ + \angle ADC = 180^\circ$$

$$\angle ADC = 85^\circ$$

We know that,

$$\angle ADC = 2 \angle AOC$$

$$\angle ADC = 2 * 85^\circ$$

$$\angle ADC = 170^\circ$$

Since,

$$AO = OC \text{ (Radii of circle)}$$

$$\angle OAC = \angle OCA \text{ (Sides opposite to equal angle) (i)}$$

In triangle OAC,

$$\angle OAC + \angle OCA + \angle AOC = 180^\circ$$

$$\angle OAC + \angle OAC + 170^\circ = 180^\circ \text{ [From (i)]}$$

$$2 \angle OAC = 10^\circ$$

$$\angle OAC = 5^\circ$$

Thus,

$$\angle OAC = 5^\circ$$

1. Question

If the length of a chord of a circle is 16 cm and is at a distance of 15 cm from the centre of the circle, then the radius of the circle is

A. 15 cm

B. 16 cm

C. 17 cm

D. 34 cm

Answer

Let AB be the chord of length 16cm.

Given that,

Distance from centre to the chord AB is OC = 15 cm

Now,

$$OC \perp AB$$

Therefore,

AC = CB (Since perpendicular drawn from centre of the circle bisects the chord)

Therefore,

$$AC = CB = 8 \text{ cm}$$

In right $\triangle OCA$,

$$OA^2 = AC^2 + OC^2$$

$$= 8^2 + 15^2$$

$$= 225 + 64$$

$$= 289$$

$$OA = 17 \text{ cm}$$

Thus, the radius of the circle is 17 cm

2. Question

The radius of a circle is 6 cm. The perpendicular distance from the centre of the circle to the chord which is 8 cm in length, is

- A. $\sqrt{5}$ cm
- B. $2\sqrt{5}$ cm
- C. $2\sqrt{7}$ cm
- D. $\sqrt{7}$ cm

Answer

Let, O be the centre of the circle with chord AB = 8cm

And,

OC be the perpendicular bisector of AC

$$AO = 6\text{cm}$$

$$AC = 4\text{cm}$$

In $\triangle AOC$,

$$OA^2 = AC^2 + OC^2$$

$$6^2 = 4^2 + OC^2$$

$$OC^2 = 20$$

$$OC = 2\sqrt{5}$$

3. Question

If O is the centre of a circle of radius r and AB is chord of the circle at a distance $r/2$ from O, then $\angle BAO =$

- A. 60°
- B. 45°
- C. 30°
- D. 15°

Answer

Let, O be the centre of the circle and r be the radius

$$\sin A = \frac{OA}{AC}$$

$$= \frac{r}{2r}$$

$$= \frac{1}{2}$$

$$\sin A = \frac{1}{2}$$

$$\sin A = \sin 30^\circ$$

$$A = 30^\circ$$

Therefore,

$$\angle BAO = \angle CAO = 30^\circ$$

4. Question

$ABCD$ is a cyclic quadrilateral such that $\angle ADB = 30^\circ$ and $\angle DCA = 80^\circ$, then $\angle DAB =$

A. 70°

B. 100°

C. 125°

D. 150°

Answer

$ABCD$ is a cyclic quadrilateral

$$\angle ADB = 30^\circ$$

$$\angle DCA = 80^\circ$$

$$\angle ADB = \angle ACB = 30^\circ \text{ (Angle on the same segment)}$$

Now,

$$\angle BCD = \angle ACB + \angle DCA$$

$$= 30^\circ + 80^\circ$$

$$= 110^\circ$$

$$\angle OAB + \angle BCD = 180^\circ$$

$$\angle OAB + 110^\circ = 180^\circ$$

$$\angle OAB = 70^\circ$$

5. Question

A chord of length 14 cm is at a distance of 6 cm from the centre of a circle. The length of another chord at a distance of 2 cm from the centre of the circle is

A. 12 cm

B. 14 cm

C. 16 cm

D. 18 cm

Answer

Let AB and CD be two chords of the circle.

Draw OM perpendicular to AB and ON = CD

$$AB = 14 \text{ cm}$$

$$OM = 6 \text{ cm}$$

$$ON = 2 \text{ cm}$$

Let,

$$CD = x$$

In $\triangle AOM$,

$$AO^2 = AM^2 + OM^2$$

$$= 7^2 + 6^2$$

$$AO^2 = 85 \text{ (i)}$$

In $\triangle CON$,

$$CO^2 = ON^2 + CN^2$$

$$CO^2 = 4 + \frac{x \cdot x}{4} \text{ (ii)}$$

We Know,

$$AO = CO$$

$$AO^2 = CO^2$$

$$85 = 4 + \frac{x \cdot x}{4}$$

$$x^2 = 324$$

$$x = 18 \text{ cm}$$

6. Question

One chord of a circle is known to be 10 cm. The radius of this circle must be

A. 5 cm

B. Greater than 5 cm

C. Greater than or equal to 5 cm

D. Less than 5 cm

Answer

It must be greater than 5cm.

7. Question

ABC is a triangle with B as right angle, $AC=5$ cm and $AB = 4$ cm. A circle is drawn with A as centre and AC as radius. The length of the chord of this circle passing through C and B is

- A. 3 cm
- B. 4 cm
- C. 5 cm
- D. 6 cm

Answer

Given: $AC = \text{radius} = 5$ cm

$AB = 4$ cm

DC is a chord passing B and C

In $\triangle ABC$

$$AC^2 = AB^2 + BC^2$$

$$BC^2 = 9$$

$$BC = 3 \text{ cm}$$

$$CD = 2 BC$$

$$= 6 \text{ cm}$$

8. Question

If AB , BC and CD are equal chords of a circle with O as a centre and AD diameter, then $\angle AOB =$

- A. 60°
- B. 90°
- C. 120°
- D. None of these

Answer

We can't say that,

$$\angle AOB = 60^\circ, 90^\circ \text{ or } 120^\circ$$

So, angle AOB is none of these.

9. Question

Let C be the mid-point of an arc AB of a circle such that $m\widehat{AB} = 183^\circ$. If the region bounded by the arc ACB and line segment AB is denoted by S , then the centre O of the circle lies

- A. In the interior of S
- B. In the exterior of S
- C. On the segment AB
- D. On AB and bisects AB

Answer

The centre O lies in the interior of S

10. Question

In a circle, the major arc is 3 times the minor arc. The corresponding central angles and the degree measures of two arcs are

- A. 90° and 270°
- B. 90° and 90°
- C. 270° and 90°
- D. 60° and 210°

Answer

Arc $ACB = 3$ arc AB (Given)

Central angle = 270°

Degree measures of the two arcs are 90°

11. Question

If A and B are two points on a circle such that $m(\widehat{AB}) = 260^\circ$. A possible value for the angle subtended by arc BA at a point on the circle is

- A. 100°
- B. 75°
- C. 50°
- D. 25°

Answer

Arc $AB = 260^\circ$ (Given)

Let a point C on the circle

We Know that,

An angle subtended by an arc at the centre of the circle is double the angle subtended at any point on the circle.

$$\angle ACB = \frac{1}{2} \angle AOB$$

$$\angle ACB = \frac{1}{2} * 100$$

$$= 50^\circ$$

12. Question

An equilateral triangle ABC is inscribed in a circle with centre O . The measures of $\angle BOC$ is

- A. 30°
- B. 60°
- C. 90°
- D. 120°

Answer

Given that O is the centre of circle.

Triangle ABC is an equilateral triangle

$$\angle A = \angle B = \angle C = 60^\circ$$

We Know that,

An angle subtended by an arc at the centre of the circle is double the angle subtended at any point on the circle.

$$\angle BOC = 2 \angle BAC$$

$$\angle BOC = 2 \angle A$$

$$\angle BOC = 120^\circ$$

13. Question

In a circle with centre O , AB and CD are two diameters perpendicular to each other. The length of chord AC is

- A. $2AB$
- B. $\sqrt{2}$
- C. $\frac{1}{2}AB$
- D. $\frac{1}{\sqrt{2}}AB$

Answer

Given: O is the centre of circle

AB and CD are diameters of the circle

$$AO = BO = CO = DO \text{ (Radius of the circle)}$$

In right angle $\triangle AOC$,

$$\cos A = \frac{AM}{OA}$$

$$\cos A = \frac{\frac{1}{2}AC}{\frac{1}{2}AB}$$

$$\cos A = \frac{AC}{AB} \quad (i)$$

$$\angle OMA = 90^\circ$$

$$\angle AOM = \angle MAO = 45^\circ$$

Using value of angle A in (i)

$$\cos 45^\circ = \frac{AC}{AB}$$

$$\frac{1}{\sqrt{2}} = \frac{AC}{AB}$$

$$AC = \frac{1}{\sqrt{2}} AB$$

14. Question

Two equal circles of radius r intersect such that each passes through the centre of the other. The length of the common chord of the circles is

- A. $\sqrt{r}$
- B. $\sqrt{2} r$
- C. $\sqrt{3} r$
- D. $\frac{\sqrt{3}}{2} r$

Answer

Let O and O' be the centre of two circles

OA and O'A = Radius of the circles

AB be the common chord of both the circles

OM perpendicular to AB

And,

O'M perpendicular to AB

$\triangle AOO'$ is an equilateral triangle.

AM = Altitude of $\triangle AOO'$

$$\text{Height of } \triangle AOO' = \frac{\sqrt{3}}{2} r$$

$$AB = 2 AM$$

$$= 2 \frac{\sqrt{3}}{2} r$$

$$= \sqrt{3} r$$

15. Question

If AB is a chord of a circle, P and Q are the two points on the circle different from A and B , then

- A. $\angle APB = \angle AQB$
- B. $\angle APB + \angle AQB = 180^\circ$ or $\angle APB = \angle AQB$
- C. $\angle APB + \angle AQB = 90^\circ$
- D. $\angle APB + \angle AQB = 180^\circ$

Answer

AB is a chord of circle P and Q are two points on circle

$\angle APB = \angle AQB$ (Angles on the same segment)

16. Question

If two diameters of a circle intersect each other at right angles, then quadrilateral formed by joining their end points is a

- A. Rhombus
- B. Rectangle
- C. Parallelogram
- D. Square

Answer

Let AB and CD are two diameters of circle

$$\angle AOD = \angle BOD = \angle BOC = \angle AOC = 90^\circ$$

AB and CD are diagonals of quadrilateral $ABCD$

They intersect each other at right angles

And

$$AB = BC = CD = DA$$

We know that,

Sides of a square are equal and diagonals intersect at 90°

Therefore, $ABCD$ is a square

17. Question

If ABC is an arc of a circle and $\angle ABC = 135^\circ$, then the ratio of arc $\widehat{ABC}$ to the circumference is

A. 1 : 4

B. 3 : 4

C. 3 : 8

D. 1 : 2

Answer

$$\angle ABC = 135^\circ$$

ABC is an arc

$$\text{Circumference} = 360^\circ$$

$$\text{Arc} = 135^\circ$$

$$\frac{\text{Arc } ABC}{\text{Circumference}} = \frac{135}{360}$$

$$= \frac{3}{8}$$

18. Question

The chord of a circle is equal to its radius. The angle subtended by this chord at the minor arc of the circle is

A. 60°

B. 75°

C. 120°

D. 150°

Answer

Let AB be the chord of circle equal to radius r

$$OA = OB = r \text{ (Radii)}$$

Therefore,

$$OA = OB = AB$$

OBC is equilateral triangle

$$\text{Each angle} = 60^\circ$$

Hence,

$$\text{Angle subtended by AB at minor arc} = \frac{1}{2} (\text{Reflex } \angle AOD)$$

$$= \frac{1}{2} (360^\circ - 60^\circ)$$

$$= 150^\circ$$

19. Question

$PQRS$ is a cyclic quadrilateral such that PR is a diameter of the circle. If $\angle QPR=67^\circ$ and $\angle SPR=72^\circ$, then $\angle QRS=$

A. 41°

B. 23°

C. 67°

D. 18°

Answer

Given that,

$PQRS$ is a cyclic quadrilateral

$$\angle QPR = 67^\circ$$

$$\angle SPR = 72^\circ$$

$$\angle SPQ = \angle QPR + \angle SPR$$

$$= 67^\circ + 72^\circ$$

$$= 139^\circ$$

$$\angle SPQ + \angle QRS = 180^\circ \text{ (Opposite angles of cyclic quadrilateral)}$$

$$139^\circ + \angle QRS = 180^\circ$$

$$\angle QRS = 41^\circ$$

20. Question

If A, B, C are three points on a circle with centre O such that $\angle AOB=90^\circ$ and $\angle BOC=120^\circ$, then $\angle ABC=$

A. 60°

B. 75°

C. 90°

D. 135°

Answer

$$\angle BOC = 120^\circ$$

$$\angle AOC = \angle AOB + \angle BOC$$

$$= 90^\circ + 120^\circ$$

$$= 210^\circ$$

Now,

$$\angle ABC = \frac{1}{2} * (\text{Reflex } \angle AOC)$$

$$= \frac{1}{2} (360^\circ - 210^\circ)$$

$$= 75^\circ$$

21. Question

AB and CD are two parallel chords of a circle with centre O such that $AB=6$ cm and $CD= 12$ cm. The chords are on the same side of the centre and the distance between them is 3 cm. The radius of the circle is

A. 6 cm

B. $5\sqrt{2}$ cm

C. 7 cm

D. $3\sqrt{5}$ cm

Answer

Given that,

$AB \parallel CD$ (Chords on same side of centre)

$AO = CO$ (Radii)

OL and OM perpendicular bisector of CD and AB respectively

$CL = LD = 6$ cm

$AM = MB = 3$ cm

$LM = 3$ cm (Given)

In $\triangle COL$,

$$CO^2 = OL^2 + 6^2 \text{ (i)}$$

In AOM ,

$$AO^2 = AM^2 + OM^2$$

$$= 3^2 + (OL + LM)^2$$

$$= 9 + OL^2 + 9 + 6OL$$

$$OL^2 = AO^2 - 18 - 6OL \text{ (ii)}$$

Using (ii) in (i),

$$OL = 3 \text{ cm}$$

Putting OL in (i),

$$AO^2 = \sqrt{45}$$

$$AO = 3\sqrt{5}$$

22. Question

In a circle of radius 17 cm, two parallel chords are drawn on opposite side of a diameter. The distance between the chords is 23 cm. If the length of one chord is 16 cm, then the length of the other is

- A. 34 cm
- B. 15 cm
- C. 23 cm
- D. 30 cm

Answer

Given that,

$AB \parallel CD$ (Chords on opposite side of centre)

$DO = BO$ (Radii)

OL and OM perpendicular bisector of CD and AB respectively

$$LM = 23 \text{ cm}$$

$$AB = 16 \text{ cm}$$

In $\triangle OLB$,

$$OB^2 = OL^2 + LB^2$$

$$OL^2 = 225$$

$$OL = 15 \text{ cm}$$

$$OM = LM - OL$$

$$= 8 \text{ cm}$$

In $\triangle OMD$,

$$OD^2 = OM^2 + MD^2$$

$$MD^2 = 225$$

$$MD = 15 \text{ cm}$$

Now,

$$CD = 2 MD = 30 \text{ cm}$$

23. Question

The greatest chord of a circle is called its

- A. Radius

- B. Secant
- C. Diameter
- D. None of these

Answer

The largest chord in any circle is its diameter.

24. Question

Angle formed in minor segment of a circle is

- A. Acute
- B. Obtuse
- C. Right angle
- D. None of these

Answer

The minor segment in a circle always forms an obtuse angle.

25. Question

Number of circles that can be drawn through three non-collinear points is

- A. 1
- B. 0
- C. 2
- D. 3

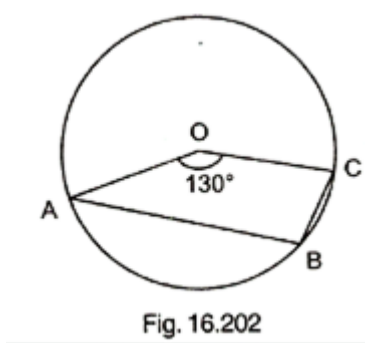
Answer

Only and only a single circle can be drawn passing through any three non collinear points.

26. Question

In Fig. 16.202, O is the centre of the circle such that $\angle AOC = 130^\circ$, then $\angle ABC =$

- A. 130°
- B. 115°
- C. 65°
- D. 165°



Answer

We have,

$$\angle AOC = 130^\circ$$

$$\angle ABC = \frac{1}{2} * (\text{Reflex of } AOC)$$

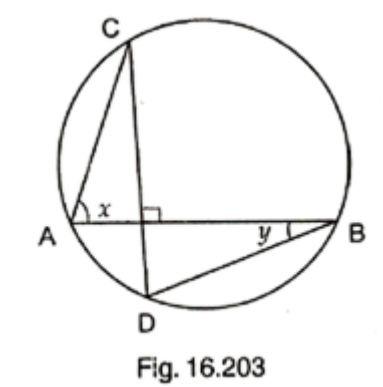
$$= \frac{1}{2} * (360^\circ - 130^\circ)$$

$$= \frac{1}{2} * 230$$

$$= 115^\circ$$

27. Question

In Fig. 16.203, if chords AB and CD of the circle intersect each other at right angles, then $x + y =$



A. 45°

B. 60°

C. 75°

D. 90°

Answer

Given: AB and CD are two chords of the circle.

$$\angle APC = 90^\circ$$

$$\angle ACP = \angle PBD = y \text{ (Angles on the same segment)}$$

In $\triangle ACP$,

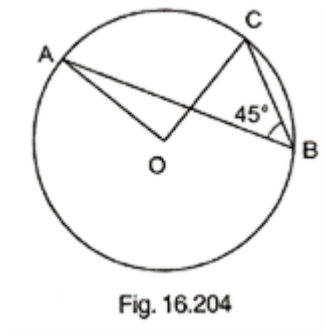
$$\angle ACP + \angle APC + \angle PAC = 180^\circ$$

$$y + 90^\circ + y = 180^\circ$$

$$x + y = 90^\circ$$

28. Question

In Fig. 16.204, if $\angle ABC = 45^\circ$, then $\angle AOC =$



A. 45°

B. 60°

C. 75°

D. 90°

Answer

We know that,

An angle subtended by an arc at the centre of the circle is double the angle subtended at any point on the circle

$$\angle AOC = 2 \angle ABC$$

$$= 2 * 45^\circ$$

$$= 90^\circ$$

29. Question

In Fig. 16.205, chords AD and BC intersect each other at right angles at a point P . If $\angle DAB = 35^\circ$, then $\angle ADC =$

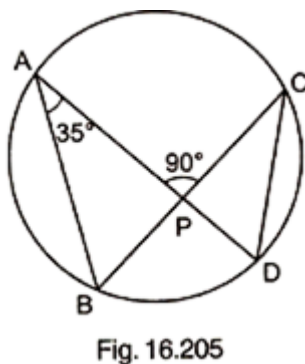


Fig. 16.205

- A. 35°
- B. 45°
- C. 55°
- D. 65°

Answer

Given that,

Chords AD and BC intersect at right angles,

$$\angle DAB = 35^\circ$$

$$\angle APC = 90^\circ$$

$$\angle APC + \angle CPD = 180^\circ$$

$$90^\circ + \angle CPD = 180^\circ$$

$$\angle CPD = 90^\circ$$

$$\angle DAB = \angle PCD = 35^\circ \text{ (Angles on the same segment)}$$

In triangle PCD,

$$\angle PCD + \angle PDC + \angle CPD = 180^\circ$$

$$35^\circ + \angle PDC + 90^\circ = 180^\circ$$

$$\angle PDC = 45^\circ$$

$$\angle ADC = 45^\circ$$

30. Question

In Fig. 16.206, O is the centre of the circle and $\angle BDC = 42^\circ$. The measure of $\angle ACB$ is

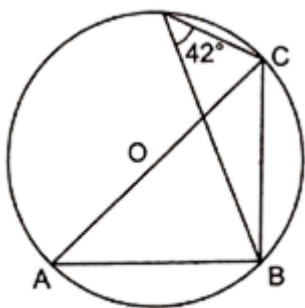


Fig. 16.206

- A. 42°
- B. 48°
- C. 58°
- D. 52°

Answer

$$\angle BDC = 42^\circ$$

$$\angle ABC = 90^\circ \text{ (Angle in a semi-circle)}$$

In $\triangle ABC$,

$$\angle ABC + \angle BAC = 42^\circ \text{ (Angles on the same segment)}$$

$$90^\circ + 42^\circ + \angle ACB = 180^\circ$$

$$\angle ACB = 48^\circ$$