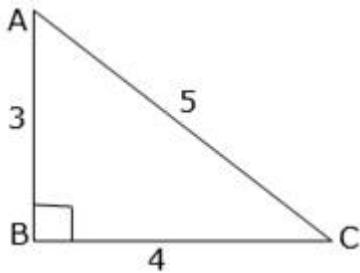


4. Trigonometric Ratios and Identities

Exercise 4.1

1. Question

From the given figure, find the value of the following:



- (i) $\sin C$
- (ii) $\sin A$
- (iii) $\cos C$
- (iv) $\cos A$
- (v) $\tan C$
- (vi) $\tan A$

Answer

- (i) $\sin C$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

So, here $\theta = C$

Side opposite to $\angle C = AB = 3$

Hypotenuse = $AC = 5$

$$\text{So, } \sin C = \frac{AB}{AC} = \frac{3}{5}$$

- (ii) $\sin A$

So, here $\theta = A$

The side opposite to $\angle A = BC = 4$

Hypotenuse = $AC = 5$

$$\text{So, } \sin A = \frac{BC}{AC} = \frac{4}{5}$$

(iii) $\cos C$

We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

So, here $\theta = C$

Side adjacent to $\angle C = BC = 4$

Hypotenuse = $AC = 5$

$$\text{So, } \cos C = \frac{BC}{AC} = \frac{4}{5}$$

(iv) $\cos A$

Here, $\theta = A$

Side adjacent to $\angle A = AB = 3$

Hypotenuse = $AC = 5$

$$\text{So, } \cos A = \frac{AB}{AC} = \frac{3}{5}$$

(v) $\tan C$

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

So, here $\theta = C$

Side opposite to $\angle C = AB = 3$

Side adjacent to $\angle C = BC = 4$

$$\text{So, } \tan C = \frac{AB}{BC} = \frac{3}{4}$$

(vi) $\tan A$

here $\theta = A$

Side opposite to $\angle A = BC = 4$

Side adjacent to $\angle A = AB = 3$

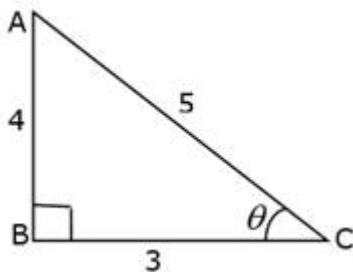
$$\text{So, } \tan A = \frac{AB}{BC} = \frac{4}{3}$$

2. Question

From the given figure, find the value of :

(i) $\tan \theta$

(ii) $\cos \theta$



Answer

(i) $\tan \theta$

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

Side opposite to $\theta = AB = 4$

Side adjacent to $\theta = BC = 3$

$$\text{So, } \tan \theta = \frac{AB}{BC} = \frac{4}{3}$$

(ii) $\cos \theta$

We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to $\theta = BC = 3$

Hypotenuse = $AC = 5$

$$\text{So, } \cos \theta = \frac{BC}{AC} = \frac{3}{5}$$

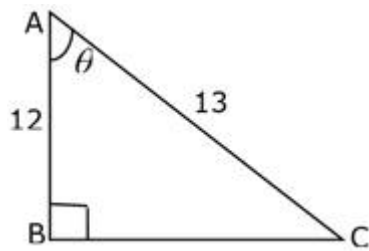
3. Question

From the given figure, find the value of

(i) $\sin \theta$

(ii) $\tan \theta$

(iii) $\tan A - \cot C$



Answer

(i) $\sin \theta$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to $\theta = BC = ?$

Hypotenuse = $AC = 13$

Firstly we have to find the value of BC .

So, we can find the value of BC with the help of Pythagoras theorem.

According to Pythagoras theorem,

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (12)^2 + (BC)^2 = (13)^2$$

$$\Rightarrow 144 + (BC)^2 = 169$$

$$\Rightarrow (BC)^2 = 169 - 144$$

$$\Rightarrow (BC)^2 = 25$$

$$\Rightarrow BC = \sqrt{25}$$

$$\Rightarrow BC = \pm 5$$

But side BC can't be negative. So, $BC = 5$

Now, $BC = 5$ and $AC = 13$

$$\text{So, } \sin \theta = \frac{BC}{AC} = \frac{5}{13}$$

(ii) $\tan \theta$

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

Side opposite to $\theta = BC = 5$

Side adjacent to $\theta = AB = 12$

$$\text{So, } \tan \theta = \frac{BC}{AB} = \frac{5}{12}$$

(iii) $\tan A - \cot C$

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

and

$$\cot \theta = \frac{\text{side adjacent to angle } \theta}{\text{side opposite to angle } \theta}$$

$\tan A$

Here, $\theta = A$

Side opposite to $\angle A = BC = 5$

Side adjacent to $\angle A = AB = 12$

$$\text{So, } \tan A = \frac{BC}{AB} = \frac{5}{12}$$

$\cot C$

Here, $\theta = C$

Side adjacent to $\angle C = BC = 5$

Side opposite to $\angle C = AB = 12$

$$\text{So, } \cot C = \frac{BC}{AB} = \frac{5}{12}$$

$$\text{So, } \tan A - \cot C = \frac{5}{12} - \frac{5}{12} = 0$$

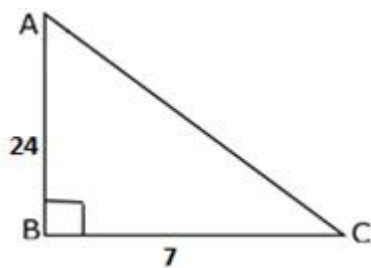
4 A. Question

In $\triangle ABC$, right angled at B, $AB = 24$ cm, $BC = 7$ cm. Determine

a. $\sin A$, $\cos A$

b. $\sin C$, $\cos C$

Answer



(i)

(a) $\sin A$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

So, here $\theta = A$

Side opposite to $\angle A = BC = 7$

Hypotenuse = $AC = ?$

Firstly we have to find the value of AC .

So, we can find the value of AC with the help of Pythagoras theorem.

According to Pythagoras theorem,

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (24)^2 + (7)^2 = (AC)^2$$

$$\Rightarrow 576 + 49 = (AC)^2$$

$$\Rightarrow (AC)^2 = 625$$

$$\Rightarrow AC = \sqrt{625}$$

$$\Rightarrow AC = \pm 25$$

But side AC can't be negative. So, $AC = 25\text{cm}$

Now, $BC = 7$ and $AC = 25$

$$\text{So, } \sin A = \frac{BC}{AC} = \frac{7}{25}$$

$\cos A$

We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

So, here $\theta = A$

Side adjacent to $\angle A = AB = 24$

Hypotenuse = $AC = 25$

$$\text{So, } \cos A = \frac{AB}{AC} = \frac{24}{25}$$

(b) $\sin C$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

So, here $\theta = C$

The side opposite to $\angle C = AB = 24$

Hypotenuse = $AC = 25$

$$\text{So, } \sin C = \frac{AB}{AC} = \frac{24}{25}$$

$\cos C$

We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

So, here $\theta = C$

Side adjacent to $\angle C = BC = 7$

Hypotenuse = $AC = 25$

$$\text{So, } \cos C = \frac{BC}{AC} = \frac{7}{25}$$

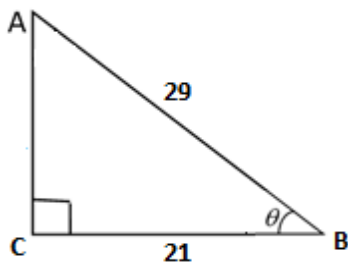
4 B. Question

Consider $\triangle ACB$, right angled at C , in which $AB = 29$ units, $BC = 21$ units and $\angle ABC = \theta$. Determine the values of

a. $\cos^2 \theta + \sin^2 \theta$

b. $\cos^2 \theta - \sin^2 \theta$

Answer



(a) $\cos^2 \theta + \sin^2 \theta$

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem.

According to Pythagoras theorem,

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

$$\Rightarrow (AC)^2 + (BC)^2 = (AB)^2$$

$$\Rightarrow (AC)^2 + (21)^2 = (29)^2$$

$$\Rightarrow (AC)^2 = (29)^2 - (21)^2$$

$$\text{Using the identity } a^2 - b^2 = (a+b)(a-b)$$

$$\Rightarrow (AC)^2 = (29-21)(29+21)$$

$$\Rightarrow (AC)^2 = (8)(50)$$

$$\Rightarrow (AC)^2 = 400$$

$$\Rightarrow AC = \sqrt{400}$$

$$\Rightarrow AC = \pm 20$$

But side AC can't be negative. So, $AC = 20$ units

Now, we will find the $\sin \theta$ and $\cos \theta$

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

In $\triangle ACB$, Side opposite to angle $\theta = AC = 20$

and Hypotenuse = $AB = 29$

$$\text{So, } \sin \theta = \frac{AC}{AB} = \frac{20}{29}$$

Now, We know that

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

In $\triangle ACB$, Side adjacent to angle $\theta = BC = 21$

and Hypotenuse = $AB = 29$

$$\text{So, } \cos \theta = \frac{BC}{AB} = \frac{21}{29}$$

$$\text{So, } \cos^2 \theta + \sin^2 \theta = \left(\frac{21}{29}\right)^2 + \left(\frac{20}{29}\right)^2$$

$$= \frac{441 + 400}{29 \times 29}$$

$$= \frac{841}{841}$$

$$= 1$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$(b) \cos^2 \theta - \sin^2 \theta$$

Putting values, we get

$$\cos^2 \theta - \sin^2 \theta = \left(\frac{21}{29}\right)^2 - \left(\frac{20}{29}\right)^2$$

$$= \frac{441 - 400}{29 \times 29}$$

$$= \frac{41}{841}$$

4 C. Question

In $\triangle ABC$, $\angle A$ is a right angle, then find the values of $\sin B$, $\cos C$ and $\tan B$ in each of the following :

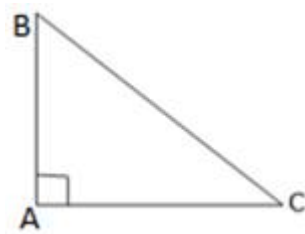
a. $AB = 12, AC = 5, BC = 13$

b. $AB = 20, AC = 21, BC = 29$

c. $BC = \sqrt{2}, AB = AC = 1$

Answer

Given that $\angle A$ is a right angle.



(a) $AB = 12, AC = 5, BC = 13$

To Find : $\sin B$, $\cos C$ and $\tan B$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Here, $\theta = B$

Side opposite to angle $B = AC = 5$

Hypotenuse = $BC = 13$

$$\text{So, } \sin B = \frac{AC}{BC} = \frac{5}{13}$$

Now, $\cos C$

We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Here, $\theta = C$

Side adjacent to angle $C = AC = 5$

Hypotenuse = BC = 13

$$\text{So, } \cos C = \frac{AC}{BC} = \frac{5}{13}$$

Now, $\tan B$

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

Here, $\theta = B$

The side opposite to angle B = AC = 5

The side adjacent to angle B = AB = 12

$$\text{So, } \tan B = \frac{AC}{AB} = \frac{5}{12}$$

(b) AB = 20, AC = 21, BC = 29

To Find: $\sin B$, $\cos C$ and $\tan B$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Here, $\theta = B$

The side opposite to angle B = AC = 21

Hypotenuse = BC = 29

$$\text{So, } \sin B = \frac{AC}{BC} = \frac{21}{29}$$

Now, $\cos C$

We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Here, $\theta = C$

Side adjacent to angle C = AC = 21

Hypotenuse = BC = 29

$$\text{So, } \cos C = \frac{AC}{BC} = \frac{21}{29}$$

Now, $\tan B$

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

Here, $\theta = B$

The side opposite to angle $B = AC = 21$

The side adjacent to angle $B = AB = 20$

$$\text{So, } \tan B = \frac{AC}{AB} = \frac{21}{20}$$

(c) $BC = \sqrt{2}$, $AB = AC = 1$

To Find: $\sin B$, $\cos C$ and $\tan B$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Here, $\theta = B$

The side opposite to angle $B = AC = 1$

Hypotenuse = $BC = \sqrt{2}$

$$\text{So, } \sin B = \frac{AC}{BC} = \frac{1}{\sqrt{2}}$$

Now, $\cos C$

We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Here, $\theta = C$

Side adjacent to angle $C = AC = 1$

Hypotenuse = $BC = \sqrt{2}$

$$\text{So, } \cos C = \frac{AC}{BC} = \frac{1}{\sqrt{2}}$$

Now, $\tan B$

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

Here, $\theta = B$

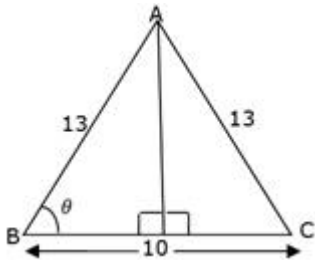
The side opposite to angle B = AC = 1

The side adjacent to angle B = AB = 1

$$\text{So, } \tan B = \frac{AC}{AB} = \frac{1}{1} = 1$$

5 A. Question

Find the value of the following : (a) $\sin \theta$ (b) $\cos \theta$ (c) $\tan \theta$ from the figures given below :



Answer

Firstly, we give the name to the midpoint of BC i.e. M

$$BC = BM + MC = 2BM \text{ or } 2MC$$

$$\Rightarrow BM = 5 \text{ and } MC = 5$$

Now, we have to find the value of AM, and we can find out with the help of Pythagoras theorem.

So, In $\triangle AMB$

$$\Rightarrow (AM)^2 + (BM)^2 = (AB)^2$$

$$\Rightarrow (AM)^2 + (5)^2 = (13)^2$$

$$\Rightarrow (AM)^2 = (13)^2 - (5)^2$$

Using the identity $a^2 - b^2 = (a+b)(a-b)$

$$\Rightarrow (AM)^2 = (13-5)(13+5)$$

$$\Rightarrow (AM)^2 = (8)(18)$$

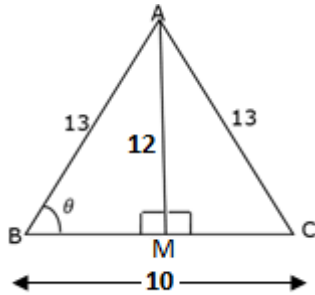
$$\Rightarrow (AM)^2 = 144$$

$$\Rightarrow AM = \sqrt{144}$$

$$\Rightarrow AM = \pm 12$$

But side AM can't be negative. So, $AM = 12$

a. $\sin \theta$



We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

In $\triangle AMB$

Side opposite to $\theta = AM = 12$

Hypotenuse = $AB = 13$

$$\text{So, } \sin \theta = \frac{AM}{AB} = \frac{12}{13}$$

$$\text{So, } \sin \theta = \frac{12}{13}$$

b. $\cos \theta$

$$\text{We know that, } \cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

In $\triangle AMB$

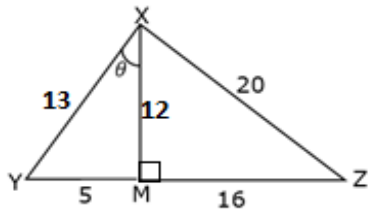
The side adjacent to $\theta = BM = 5$

Hypotenuse = $AB = 13$

$$\text{So, } \cos \theta = \frac{BM}{AB} = \frac{5}{13}$$

$$\text{So, } \cos \theta = \frac{5}{13}$$

c. $\tan \theta$



We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

In $\triangle AMB$

Side opposite to $\theta = AM = 12$

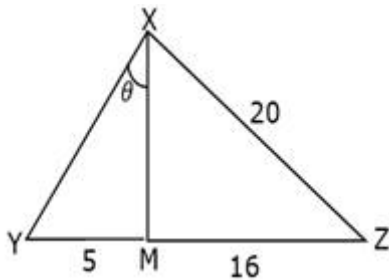
The side adjacent to $\theta = BM = 5$

$$\text{So, } \tan \theta = \frac{AM}{BM} = \frac{12}{5}$$

$$\text{So, } \tan \theta = \frac{12}{5}$$

5 B. Question

Find the value of the following : (a) $\sin \theta$ (b) $\cos \theta$ (c) $\tan \theta$ from the figures given below :



Answer

Firstly, we have to find the value of XM and we can find out with the help of Pythagoras theorem

So, In $\triangle XMZ$

$$\Rightarrow (XM)^2 + (MZ)^2 = (XZ)^2$$

$$\Rightarrow (XM)^2 + (16)^2 = (20)^2$$

$$\Rightarrow (XM)^2 = (20)^2 - (16)^2$$

Using the identity $a^2 - b^2 = (a+b)(a-b)$

$$\Rightarrow (XM)^2 = (20-16)(20+16)$$

$$\Rightarrow (XM)^2 = (4)(36)$$

$$\Rightarrow (XM)^2 = 144$$

$$\Rightarrow XM = \sqrt{144}$$

$$\Rightarrow XM = \pm 12$$

But side XM can't be negative. So, $XM = 12$

Now, In $\triangle XMY$ we have the value of XM and MY but we don't have the value of XY.

So, again we apply the Pythagoras theorem in $\triangle XMY$

$$\Rightarrow (XM)^2 + (MY)^2 = (XY)^2$$

$$\Rightarrow (12)^2 + (5)^2 = (XY)^2$$

$$\Rightarrow 144 + 25 = (XY)^2$$

$$\Rightarrow (XY)^2 = 169$$

$$\Rightarrow XY = \sqrt{169}$$

$$\Rightarrow XY = \pm 13$$

But side XY can't be negative. So, $XY = 13$

a. $\sin \theta$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

In $\triangle XMY$

Side opposite to $\theta = MY = 5$

Hypotenuse = $XY = 13$

$$\text{So, } \sin \theta = \frac{MY}{XY} = \frac{5}{13}$$

b. $\cos \theta$

We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

In $\triangle XMY$

Side adjacent to $\theta = XM = 12$

Hypotenuse = $XY = 13$

$$\text{So, } \cos \theta = \frac{XM}{XY} = \frac{12}{13}$$

c. $\tan \theta$

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

In $\triangle XMY$

The side opposite to $\theta = MY = 5$

Side adjacent to $\theta = XM = 12$

$$\text{So, } \tan \theta = \frac{MY}{XM} = \frac{5}{12}$$

6. Question

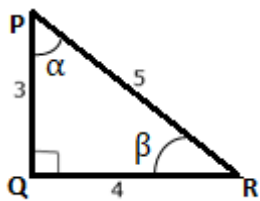
In $\triangle PQR$, $\angle Q$ is a right angle $PQ = 3$, $QR = 4$. If $\angle P = \alpha$ and $\angle R = \beta$, then find the values of

(i) $\sin \alpha$ (ii) $\cos \alpha$

(iii) $\tan \alpha$ (iv) $\sin \beta$

(v) $\cos \beta$ (vi) $\tan \beta$

Answer



Given : $PQ = 3$, $QR = 4$

$$\Rightarrow (PQ)^2 + (QR)^2 = (PR)^2$$

$$\Rightarrow (3)^2 + (4)^2 = (PR)^2$$

$$\Rightarrow 9 + 16 = (PR)^2$$

$$\Rightarrow (PR)^2 = 25$$

$$\Rightarrow PR = \sqrt{25}$$

$$\Rightarrow PR = \pm 5$$

But side PR can't be negative. So, $PR = 5$

(i) $\sin \alpha$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Here, $\theta = \alpha$

The side opposite to angle $\alpha = QR = 4$

Hypotenuse = $PR = 5$

$$\text{So, } \sin \alpha = \frac{4}{5}$$

(ii) $\cos \alpha$

We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Here, $\theta = \alpha$

The side adjacent to angle $\alpha = PQ = 3$

Hypotenuse = $PR = 5$

$$\text{So, } \cos \alpha = \frac{3}{5}$$

(iii) $\tan \alpha$

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

Here, $\theta = \alpha$

Side opposite to angle $\alpha = QR = 4$

Side adjacent to angle $\alpha = PQ = 3$

$$\text{So, } \tan \alpha = \frac{4}{3}$$

(iv) $\sin \beta$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Here, $\theta = \beta$

The side opposite to angle $\beta = PQ = 3$

Hypotenuse = PR = 5

$$\text{So, } \sin \beta = \frac{3}{5}$$

(v) $\cos \beta$

We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Here, $\theta = \beta$

Side adjacent to angle $\beta = QR = 4$

Hypotenuse = PR = 5

$$\text{So, } \cos \beta = \frac{4}{5}$$

(vi) $\tan \beta$

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

Here, $\theta = \beta$

Side opposite to angle $\beta = PQ = 3$

Side adjacent to angle $\beta = QR = 4$

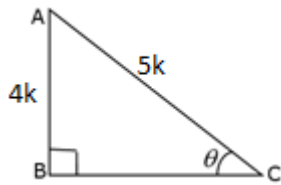
$$\text{So, } \tan \beta = \frac{3}{4}$$

7 A. Question

If $\sin \theta = \frac{4}{5}$, then find the values of $\cos \theta$ and $\tan \theta$.

Answer

Given: $\sin \theta = \frac{4}{5}$



We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

$$\text{Or } \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin \theta = \frac{4}{5} \Rightarrow \frac{P}{H} = \frac{4}{5} \Rightarrow \frac{AB}{AC} = \frac{4}{5}$$

Let,

$$\text{Perpendicular} = AB = 4k$$

$$\text{and Hypotenuse} = AC = 5k$$

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

In right angled ΔABC , we have

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (4k)^2 + (BC)^2 = (5k)^2$$

$$\Rightarrow 16k^2 + (BC)^2 = 25k^2$$

$$\Rightarrow (BC)^2 = 25k^2 - 16k^2$$

$$\Rightarrow (BC)^2 = 9k^2$$

$$\Rightarrow BC = \sqrt{9k^2}$$

$$\Rightarrow BC = \pm 3k$$

But side BC can't be negative. So, $BC = 3k$

Now, we have to find the value of $\cos \theta$ and $\tan \theta$

We know that,

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}}$$

The side adjacent to angle θ or base = BC = 3k

Hypotenuse = AC = 5k

$$\text{So, } \cos \theta = \frac{3k}{5k} = \frac{3}{5}$$

Now,

$$\text{We know that, } \tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

Perpendicular = AB = 4k

Base = BC = 3k

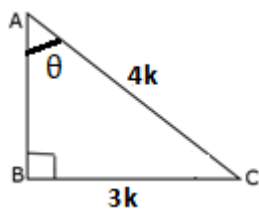
$$\text{So, } \tan \theta = \frac{4k}{3k} = \frac{4}{3}$$

7 B. Question

If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.

Answer

$$\text{Given: } \sin A = \frac{3}{4}$$



We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

$$\text{Or } \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin \theta = \frac{3}{4} \Rightarrow \frac{P}{H} = \frac{3}{4} \Rightarrow \frac{BC}{AC} = \frac{3}{4}$$

Let,

Side opposite to angle θ = BC = 3k

and Hypotenuse = AC = 4k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (AB)^2 + (3k)^2 = (4k)^2$$

$$\Rightarrow (AB)^2 + 9k^2 = 16k^2$$

$$\Rightarrow (AB)^2 = 16k^2 - 9k^2$$

$$\Rightarrow (AB)^2 = 7k^2$$

$$\Rightarrow AB = k\sqrt{7}$$

$$\text{So, } AB = k\sqrt{7}$$

Now, we have to find the value of $\cos A$ and $\tan A$

We know that,

$$\cos \theta = \frac{\text{Side adjacent to angle } \theta}{\text{hypotenuse}}$$

Here, $\theta = A$

The side adjacent to angle A = AB = $k\sqrt{7}$

Hypotenuse = AC = 4k

$$\text{So, } \cos A = \frac{k\sqrt{7}}{4k} = \frac{\sqrt{7}}{4}$$

Now,

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

The side opposite to angle A = BC = 3k

The side adjacent to angle A = AB = $k\sqrt{7}$

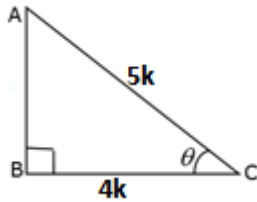
$$\text{So, } \tan A = \frac{3k}{k\sqrt{7}} = \frac{3}{\sqrt{7}}$$

8. Question

If $\sin \theta = \frac{3}{5}$, then find the values $\cos \theta$ and $\tan \theta$.

Answer

Given: $\sin \theta = \frac{3}{5}$



We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

$$\text{Or } \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin \theta = \frac{3}{5} \Rightarrow \frac{P}{H} = \frac{3}{5} \Rightarrow \frac{AB}{AC} = \frac{3}{5}$$

Let,

Perpendicular = AB = 3k

and Hypotenuse = AC = 5k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (3k)^2 + (BC)^2 = (5k)^2$$

$$\Rightarrow 9k^2 + (BC)^2 = 25k^2$$

$$\Rightarrow (BC)^2 = 25k^2 - 9k^2$$

$$\Rightarrow (BC)^2 = 16k^2$$

$$\Rightarrow BC = \sqrt{16k^2}$$

$$\Rightarrow BC = \pm 4k$$

But side BC can't be negative. So, $BC = 4k$

Now, we have to find the value of $\cos \theta$ and $\tan \theta$

We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

The side adjacent to angle $\theta = BC = 4k$

Hypotenuse = $AC = 5k$

$$\text{So, } \cos \theta = \frac{4k}{5k} = \frac{4}{5}$$

Now, $\tan \theta$

$$\text{We know that, } \tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

Perpendicular = $AB = 3k$

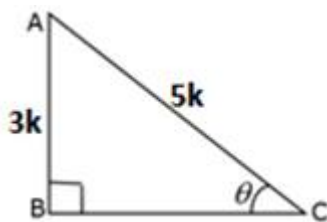
Base = $BC = 4k$

$$\text{So, } \tan \theta = \frac{3k}{4k} = \frac{3}{4}$$

9. Question

If $\cos \theta = \frac{4}{5}$, then find the value of $\tan \theta$.

Answer



We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

$$\text{Or } \cos \theta = \frac{\text{base}}{\text{Hypotenuse}}$$

$$\cos \theta = \frac{4}{5} \Rightarrow \frac{B}{H} = \frac{4}{5} \Rightarrow \frac{BC}{AC} = \frac{4}{5}$$

Let,

Base = BC = 4k

Hypotenuse = AC = 5k

Where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (AB)^2 + (4k)^2 = (5k)^2$$

$$\Rightarrow (AB)^2 + 16k^2 = 25k^2$$

$$\Rightarrow (AB)^2 = 25k^2 - 16k^2$$

$$\Rightarrow (AB)^2 = 9k^2$$

$$\Rightarrow AB = \sqrt{9k^2}$$

$$\Rightarrow AB = \pm 3k$$

But side AB can't be negative. So, AB = 3k

Now, we have to find $\tan \theta$

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

Side opposite to angle θ = BC = 4k

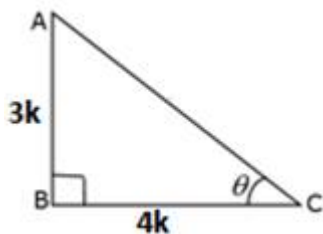
Side adjacent to angle θ = AB = 3k

$$\text{So, } \tan \theta = \frac{4k}{3k} = \frac{4}{3}$$

10 A. Question

If $\tan \theta = \frac{3}{4}$, then find the values of $\cos \theta$ and $\sin \theta$.

Answer



We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

$$\text{Or } \tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan \theta = \frac{3}{4} \Rightarrow \frac{P}{B} = \frac{3}{4} \Rightarrow \frac{AB}{BC} = \frac{3}{4}$$

Let,

The side opposite to angle θ = AB = 3k

The side adjacent to angle θ = BC = 4k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (3k)^2 + (4k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 9k^2 + 16k^2$$

$$\Rightarrow (AC)^2 = 25k^2$$

$$\Rightarrow AC = \sqrt{25k^2}$$

$$\Rightarrow AC = \pm 5k$$

But side AC can't be negative. So, AC = 5k

Now, we will find the sin θ and cos θ

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle θ = AB = 3k

and Hypotenuse = AC = 5k

$$\text{So, } \sin \theta = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

Now, We know that

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle $\theta = BC = 4k$

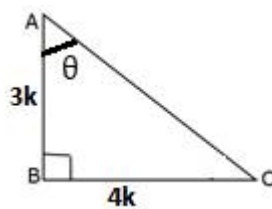
and Hypotenuse = $AC = 5k$

$$\text{So, } \cos \theta = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

10 B. Question

If $\tan A = 4/3$. Find the other trigonometric ratios of the angle A.

Answer



We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

$$\text{Or } \tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

Here, $\theta = A$

$$\tan A = \frac{4}{3} \Rightarrow \frac{P}{B} = \frac{4}{3} \Rightarrow \frac{BC}{AB} = \frac{4}{3}$$

Let,

The side opposite to angle A = $BC = 4k$

The side adjacent to angle A = $AB = 3k$

where k is any positive integer

Firstly we have to find the value of AC .

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (3k)^2 + (4k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 9k^2 + 16k^2$$

$$\Rightarrow (AC)^2 = 25 k^2$$

$$\Rightarrow AC = \sqrt{25 k^2}$$

$$\Rightarrow AC = \pm 5k$$

But side AC can't be negative. So, $AC = 5k$

Now, we will find the $\sin A$ and $\cos A$

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle $A = BC = 4k$

and Hypotenuse = $AC = 5k$

$$\text{So, } \sin A = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

Now, We know that

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle $A = AB = 3k$

and Hypotenuse = $AC = 5k$

$$\text{So, } \cos A = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

Now, we find other trigonometric ratios

$$\operatorname{Cosec} A = \frac{1}{\sin A}$$

$$= \frac{1}{\frac{4}{5}}$$

$$= \frac{5}{4}$$

$$\sec A = \frac{1}{\cos A}$$

$$= \frac{1}{\frac{3}{5}}$$

$$= \frac{5}{3}$$

$$\cot A = \frac{1}{\tan A}$$

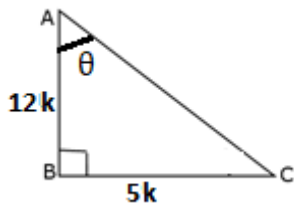
$$= \frac{1}{\frac{4}{3}}$$

$$= \frac{3}{4}$$

11. Question

If $\cot \theta = \frac{12}{5}$, then find the value of $\sin \theta$.

Answer



We know that,

$$\cot \theta = \frac{\text{side adjacent to angle } \theta}{\text{side opposite to angle } \theta}$$

$$\text{Or } \cot \theta = \frac{\text{base}}{\text{perpendicular}}$$

$$\cot \theta = \frac{12}{5} \Rightarrow \frac{B}{P} = \frac{12}{5} \Rightarrow \frac{AB}{BC} = \frac{12}{5}$$

Let,

Side adjacent to angle θ = AB = 12k

The side opposite to angle θ = BC = 5k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (12k)^2 + (5k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 144k^2 + 25k^2$$

$$\Rightarrow (AC)^2 = 169k^2$$

$$\Rightarrow AC = \sqrt{169k^2}$$

$$\Rightarrow AC = \pm 13k$$

But side AC can't be negative. So, $AC = 13k$

Now, we will find the $\sin \theta$

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle $\theta = BC = 5k$

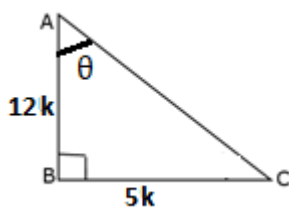
and Hypotenuse = $AC = 13k$

$$\text{So, } \sin \theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

12. Question

If $\tan \theta = \frac{5}{12}$, then find the value of $\cos \theta$.

Answer



We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

$$\text{Or } \tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan \theta = \frac{5}{12} \Rightarrow \frac{P}{B} = \frac{5}{12} \Rightarrow \frac{BC}{AB} = \frac{5}{12}$$

Let,

The side opposite to angle θ = BC = 5k

The side adjacent to angle θ = AB = 12k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (12k)^2 + (5k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 144 k^2 + 25 k^2$$

$$\Rightarrow (AC)^2 = 169 k^2$$

$$\Rightarrow AC = \sqrt{169 k^2}$$

$$\Rightarrow AC = \pm 13k$$

But side AC can't be negative. So, AC = 13k

Now, We know that

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle θ = AB = 12k

and Hypotenuse = AC = 13k

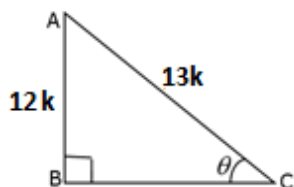
$$\text{So, } \cos \theta = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

13. Question

If $\sin \theta = \frac{12}{13}$, then find the value of $\cos \theta$ and $\tan \theta$.

Answer

$$\text{Given: } \sin \theta = \frac{12}{13}$$



We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

$$\text{Or } \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin \theta = \frac{12}{13} \Rightarrow \frac{P}{H} = \frac{12}{13} \Rightarrow \frac{AB}{AC} = \frac{12}{13}$$

Let,

Side opposite to angle $\theta = 12k$

and Hypotenuse = $13k$

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (12k)^2 + (BC)^2 = (13)^2$$

$$\Rightarrow 144 k^2 + (BC)^2 = 169 k^2$$

$$\Rightarrow (BC)^2 = 169 k^2 - 144 k^2$$

$$\Rightarrow (BC)^2 = 25 k^2$$

$$\Rightarrow BC = \sqrt{25 k^2}$$

$$\Rightarrow BC = \pm 5k$$

But side BC can't be negative. So, $BC = 5k$

Now, we have to find the value of $\cos \theta$ and $\tan \theta$

We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle $\theta = BC = 5k$

Hypotenuse = $AC = 13k$

$$\text{So, } \cos \theta = \frac{5k}{13k} = \frac{5}{13}$$

Now, $\tan \theta$

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

side opposite to angle $\theta = AB = 12k$

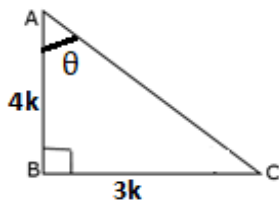
Side adjacent to angle $\theta = BC = 5k$

$$\text{So, } \tan \theta = \frac{12k}{5k} = \frac{12}{5}$$

14. Question

If $\tan \theta = 0.75$, then find the value of $\sin \theta$.

Answer



We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

$$\text{Or } \tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

Given: $\tan \theta = 0.75$

$$\Rightarrow \tan \theta = \frac{75}{100} = \frac{3}{4}$$

$$\tan \theta = \frac{3}{4} \Rightarrow \frac{P}{B} = \frac{3}{4} \Rightarrow \frac{BC}{AB} = \frac{3}{4}$$

Let,

The side opposite to angle $\theta = BC = 3k$

The side adjacent to angle $\theta = AB = 4k$

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (4k)^2 + (3k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 16k^2 + 9k^2$$

$$\Rightarrow (AC)^2 = 25k^2$$

$$\Rightarrow AC = \sqrt{25k^2}$$

$$\Rightarrow AC = \pm 5k$$

But side AC can't be negative. So, $AC = 5k$

Now, we will find the $\sin \theta$

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle $\theta = BC = 3k$

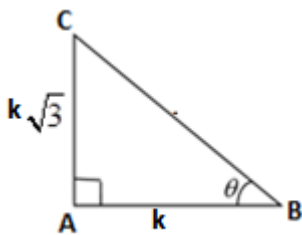
and Hypotenuse = $AC = 5k$

$$\text{So, } \sin \theta = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

15. Question

If $\tan B = \sqrt{3}$, then find the values of $\sin B$ and $\cos B$.

Answer



We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

$$\text{Or } \tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

Given: $\tan B = \sqrt{3}$

$$\Rightarrow \tan B = \frac{\sqrt{3}}{1}$$

$$\tan B = \frac{\sqrt{3}}{1} \Rightarrow \frac{P}{B} = \frac{\sqrt{3}}{1} \Rightarrow \frac{AC}{AB} = \frac{\sqrt{3}}{1}$$

Let,

Side opposite to angle B = AC = $\sqrt{3}k$

The side adjacent to angle B = AB = $1k$

where k is any positive integer

Firstly we have to find the value of BC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (AC)^2 = (BC)^2$$

$$\Rightarrow (1k)^2 + (\sqrt{3}k)^2 = (BC)^2$$

$$\Rightarrow (BC)^2 = 1k^2 + 3k^2$$

$$\Rightarrow (BC)^2 = 4k^2$$

$$\Rightarrow BC = \sqrt{4k^2}$$

$$\Rightarrow BC = \pm 2k$$

But side BC can't be negative. So, $BC = 2k$

Now, we will find the $\sin B$ and $\cos B$

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle B = AC = $k\sqrt{3}$

and Hypotenuse = BC = $2k$

$$\text{So, } \sin B = \frac{AC}{BC} = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$$

Now, we know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

The side adjacent to angle B = AB = $1k$

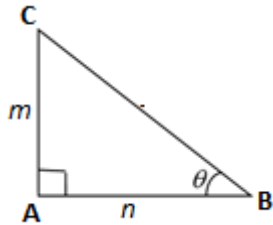
Hypotenuse = BC = $2k$

$$\text{So, } \cos B = \frac{AB}{BC} = \frac{1k}{2k} = \frac{1}{2}$$

16. Question

If $\tan \theta = \frac{m}{n}$, then find the values of $\cos \theta$ and $\sin \theta$.

Answer



We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

$$\text{Or } \tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\text{Here, } \tan \theta = \frac{m}{n}$$

So, Side opposite to angle θ = AC = m

The side adjacent to angle θ = AB = n

Firstly we have to find the value of BC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (AC)^2 = (BC)^2$$

$$\Rightarrow (n)^2 + (m)^2 = (BC)^2$$

$$\Rightarrow (BC)^2 = m^2 + n^2$$

$$\Rightarrow BC = \sqrt{m^2 + n^2}$$

$$\text{So, } BC = \sqrt{(m^2 + n^2)}$$

Now, we will find the $\sin B$ and $\cos B$

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle θ = AC = m

and Hypotenuse = BC = $\sqrt{(m^2 + n^2)}$

$$\text{So, } \sin \theta = \frac{AC}{BC} = \frac{m}{\sqrt{m^2+n^2}}$$

Now, we know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle $\theta = AB = n$

Hypotenuse = $BC = \sqrt{(m^2 + n^2)}$

$$\text{So, } \cos \theta = \frac{AB}{BC} = \frac{n}{\sqrt{m^2+n^2}}$$

17. Question

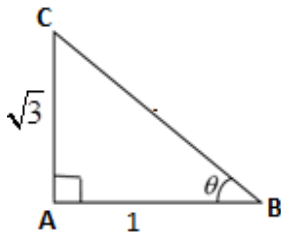
If $\sin \theta = \sqrt{3} \cos \theta$, then find the values of $\cos \theta$ and $\sin \theta$.

Answer

Given : $\sin \theta = \sqrt{3} \cos \theta$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = \sqrt{3}$$

$$\Rightarrow \tan \theta = \sqrt{3}$$



We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

$$\text{Or } \tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

and $\tan \theta = \sqrt{3}$

$$\Rightarrow \tan \theta = \frac{\sqrt{3}}{1}$$

$$\tan \theta = \frac{\sqrt{3}}{1} \Rightarrow \frac{P}{B} = \frac{\sqrt{3}}{1} \Rightarrow \frac{AC}{AB} = \frac{\sqrt{3}}{1}$$

Let,

The side opposite to angle $\theta = AC = k\sqrt{3}$

The side adjacent to angle $\theta = AB = 1k$

where k is any positive integer

Firstly we have to find the value of BC .

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (AC)^2 = (BC)^2$$

$$\Rightarrow (1k)^2 + (k\sqrt{3})^2 = (BC)^2$$

$$\Rightarrow (BC)^2 = 1k^2 + 3k^2$$

$$\Rightarrow (BC)^2 = 4k^2$$

$$\Rightarrow BC = \sqrt{4k^2}$$

$$\Rightarrow BC = \pm 2k$$

But side BC can't be negative. So, $BC = 2k$

Now, we will find the $\sin \theta$ and $\cos \theta$

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle $\theta = AC = k\sqrt{3}$

and Hypotenuse = $BC = 2k$

$$\text{So, } \sin \theta = \frac{AC}{BC} = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$$

Now, we know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

The side adjacent to angle $\theta = AB = 1k$

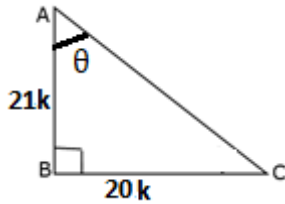
Hypotenuse = $BC = 2k$

$$\text{So, } \cos \theta = \frac{AB}{BC} = \frac{1k}{2k} = \frac{1}{2}$$

18 A. Question

If $\cot \theta = \frac{21}{20}$, then find the values of $\cos \theta$ and $\sin \theta$.

Answer



We know that,

$$\cot \theta = \frac{\text{side adjacent to angle } \theta}{\text{side opposite to angle } \theta}$$

$$\text{Or } \cot \theta = \frac{\text{base}}{\text{perpendicular}}$$

$$\cot \theta = \frac{21}{20} \Rightarrow \frac{B}{P} = \frac{21}{20} \Rightarrow \frac{AB}{BC} = \frac{21}{20}$$

Let,

Side adjacent to angle θ = AB = 21k

The side opposite to angle θ = BC = 20k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (21k)^2 + (20k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 441 k^2 + 400 k^2$$

$$\Rightarrow (AC)^2 = 841 k^2$$

$$\Rightarrow AC = \sqrt{841 k^2}$$

$$\Rightarrow AC = \pm 29k$$

But side AC can't be negative. So, AC = 29k

Now, we will find the $\sin \theta$

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle $\theta = BC = 20k$

and Hypotenuse = $AC = 29k$

$$\text{So, } \sin \theta = \frac{BC}{AC} = \frac{20k}{29k} = \frac{20}{29}$$

Now, we know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle $\theta = AB = 21k$

Hypotenuse = $AC = 29k$

$$\text{So, } \cos \theta = \frac{AB}{AC} = \frac{21k}{29k} = \frac{21}{29}$$

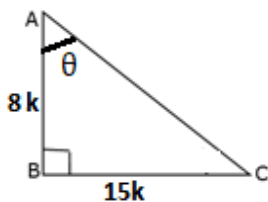
18 B. Question

If $15 \cot A = 18$, find $\sin A$ and $\sec A$.

Answer

Given: $15 \cot A = 8$

$$\Rightarrow \cot A = \frac{8}{15}$$



And we know that,

$$\cot \theta = \frac{\text{side adjacent to angle } \theta}{\text{side opposite to angle } \theta}$$

$$\text{Or } \cot \theta = \frac{\text{base}}{\text{perpendicular}}$$

$$\cot A = \frac{8}{15} \Rightarrow \frac{B}{P} = \frac{8}{15} \Rightarrow \frac{BC}{AC} = \frac{8}{15}$$

Let,

Side adjacent to angle A = AB = 8k

The side opposite to angle A = BC = 15k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (8k)^2 + (15k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 64 k^2 + 225 k^2$$

$$\Rightarrow (AC)^2 = 289 k^2$$

$$\Rightarrow AC = \sqrt{289 k^2}$$

$$\Rightarrow AC = \pm 17k$$

But side AC can't be negative. So, AC = 17k

Now, we will find the sin θ

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle θ = BC = 15k

and Hypotenuse = AC = 17k

$$\text{So, } \sin \theta = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$$

Now, we know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

The side adjacent to angle θ = AB = 8k

Hypotenuse = AC = 17k

$$\text{So, } \cos \theta = \frac{AB}{AC} = \frac{8k}{17k} = \frac{8}{17}$$

$$\therefore \sec \theta = \frac{1}{\cos \theta}$$

$$= \frac{1}{\frac{8}{17}}$$

$$= \frac{17}{8}$$

19. Question

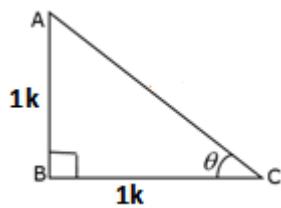
If $\sin \theta = \cos \theta$ and $0^\circ < \theta < 90^\circ$, then find the values of $\sin \theta$ and $\cos \theta$.

Answer

Given: $\sin \theta = \cos \theta$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = 1$$

$$\Rightarrow \tan \theta = 1$$



$$\tan \theta = \frac{1}{1} \Rightarrow \frac{P}{B} = \frac{1}{1} \Rightarrow \frac{AB}{BC} = \frac{1}{1}$$

Let,

Side opposite to angle $\theta = AB = 1k$

The side adjacent to angle $\theta = BC = 1k$

where k is any positive integer

Firstly we have to find the value of AC .

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (1k)^2 + (1k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 1k^2 + 1k^2$$

$$\Rightarrow (AC)^2 = 2k^2$$

$$\Rightarrow AC = \sqrt{2k^2}$$

$$\Rightarrow AC = k\sqrt{2}$$

$$\text{So, } AC = k\sqrt{2}$$

Now, we will find the $\sin \theta$

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle $\theta = AB = 1k$

and Hypotenuse = $AC = k\sqrt{2}$

$$\text{So, } \sin \theta = \frac{AB}{AC} = \frac{1k}{k\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Now, we know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

The side adjacent to angle $\theta = BC = 1k$

Hypotenuse = $AC = k\sqrt{2}$

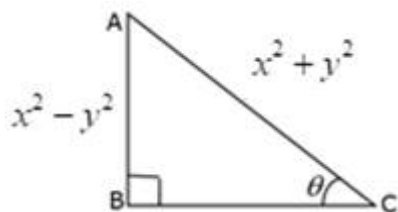
$$\text{So, } \cos \theta = \frac{BC}{AC} = \frac{1k}{k\sqrt{2}} = \frac{1}{\sqrt{2}}$$

20. Question

If $\sin \theta = \frac{x^2 - y^2}{x^2 + y^2}$, then find the values of $\cos \theta$ and $\frac{1}{\tan \theta}$.

Answer

$$\sin \theta = \frac{x^2 - y^2}{x^2 + y^2}$$



We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

$$\text{Or, } \sin \theta = \frac{\text{Perpendicular}}{\text{hypotenuse}}$$

$$\sin \theta = \frac{x^2 - y^2}{x^2 + y^2} \Rightarrow \frac{P}{H} = \frac{x^2 - y^2}{x^2 + y^2} \Rightarrow \frac{AB}{AC} = \frac{x^2 - y^2}{x^2 + y^2}$$

Let,

Side opposite to angle $\theta = AB = x^2 - y^2$

and Hypotenuse = $AC = x^2 + y^2$

In right angled ΔABC , we have

$$(AB)^2 + (BC)^2 = (AC)^2 \text{ [by using Pythagoras theorem]}$$

$$\Rightarrow (x^2 - y^2)^2 + (BC)^2 = (x^2 + y^2)^2$$

$$\Rightarrow (BC)^2 = (x^2 + y^2)^2 - (x^2 - y^2)^2$$

$$\text{Using the identity, } a^2 - b^2 = (a+b)(a-b)$$

$$\Rightarrow (BC)^2 = [(x^2 + y^2 + x^2 - y^2)][x^2 + y^2 - (x^2 - y^2)]$$

$$\Rightarrow (BC)^2 = (2x^2)(2y^2)$$

$$\Rightarrow (BC)^2 = (4x^2y^2)$$

$$\Rightarrow BC = \sqrt{4x^2y^2}$$

$$\Rightarrow BC = \pm 2xy$$

$$\Rightarrow BC = 2xy \text{ [taking positive square root since, side cannot be negative]}$$

$$\therefore \cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{2xy}{x^2 + y^2}$$

$$\text{and } \tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AB}{BC} = \frac{x^2 - y^2}{2xy}$$

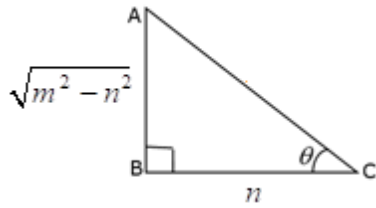
$$\text{So, } \frac{1}{\tan \theta} = \frac{1}{\frac{x^2 - y^2}{2xy}} = \frac{2xy}{x^2 - y^2}$$

21. Question

If $\tan \theta = \frac{\sqrt{m^2 - n^2}}{n}$, then find the values of $\sin \theta$ and $\cos \theta$.

Answer

$$\text{Given: } \tan \theta = \frac{\sqrt{m^2 - n^2}}{n}$$



We know that,

$$\tan \theta = \frac{\sqrt{m^2 - n^2}}{n} \Rightarrow \frac{P}{B} = \frac{\sqrt{m^2 - n^2}}{n} \Rightarrow \frac{AB}{BC} = \frac{\sqrt{m^2 - n^2}}{n}$$

Let,

$$AB = \sqrt{(m^2 - n^2)} \text{ and } BC = n$$

In right angled ΔABC , we have

$$(AB)^2 + (BC)^2 = (AC)^2 \text{ [by using Pythagoras theorem]}$$

$$\Rightarrow (\sqrt{(m^2 - n^2)})^2 + (n)^2 = (AC)^2$$

$$\Rightarrow m^2 - n^2 + n^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = (m^2)$$

$$\Rightarrow AC = \sqrt{m^2}$$

$$\Rightarrow AC = \pm m$$

$$\Rightarrow AC = m \text{ [taking positive square root since, side cannot be negative]}$$

Now, we have to find the value of $\cos \theta$ and $\sin \theta$

We, know that

$$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$= \frac{BC}{AC} = \frac{n}{m}$$

and

$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

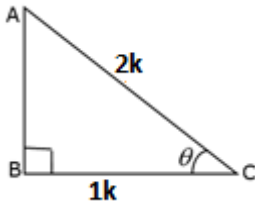
$$= \frac{AB}{AC} = \frac{\sqrt{m^2 - n^2}}{m}$$

22 A. Question

If $\sec \theta = 2$, then find the values of other t-ratios of angle θ .

Answer

Given: $\sec \theta = 2$



We know that,

$$\sec \theta = \frac{\text{hypotenuse}}{\text{base}}$$

$$\sec \theta = \frac{2}{1} \Rightarrow \frac{H}{B} = \frac{2}{1} \Rightarrow \frac{AC}{BC} = \frac{2}{1}$$

Let,

$$BC = 1k \text{ and } AC = 2k$$

where, k is any positive integer.

In right angled $\triangle ABC$, we have

$$(AB)^2 + (BC)^2 = (AC)^2 \text{ [by using Pythagoras theorem]}$$

$$\Rightarrow (AB)^2 + (1k)^2 = (2k)^2$$

$$\Rightarrow (AB)^2 + k^2 = 4k^2$$

$$\Rightarrow (AB)^2 = 4k^2 - k^2$$

$$\Rightarrow (AB)^2 = 3k^2$$

$$\Rightarrow AB = k\sqrt{3}$$

Now, we have to find the value of other trigonometric ratios.

We, know that

$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$= \frac{BC}{AC} = \frac{1k}{2k} = \frac{1}{2}$$

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$= \frac{AB}{BC} = \frac{k\sqrt{3}}{1k} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

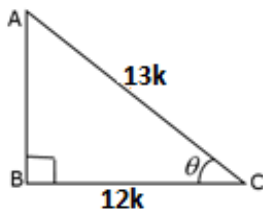
$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{\sqrt{3}}$$

22 B. Question

Given $\sec \theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Answer



$$\text{Given: } \sec \theta = \frac{13}{12}$$

We know that,

$$\sec \theta = \frac{\text{hypotenuse}}{\text{base}}$$

$$\sec \theta = \frac{13}{12} \Rightarrow \frac{H}{B} = \frac{13}{12} \Rightarrow \frac{AC}{BC} = \frac{13}{12}$$

Let,

$$BC = 12k \text{ and } AC = 13k$$

where, k is any positive integer.

In right angled ΔABC , we have

$$(AB)^2 + (BC)^2 = (AC)^2 \text{ [by using Pythagoras theorem]}$$

$$\Rightarrow (AB)^2 + (12k)^2 = (13k)^2$$

$$\Rightarrow (AB)^2 + 144k^2 = 169k^2$$

$$\Rightarrow (AB)^2 = 169k^2 - 144k^2$$

$$\Rightarrow (AB)^2 = 25k^2$$

$$\Rightarrow AB = \sqrt{25k^2}$$

$$\Rightarrow AB = \pm 5k \text{ [taking positive square root since, side cannot be negative]}$$

Now, we have to find the value of other trigonometric ratios.

We, know that

$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$= \frac{BC}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$= \frac{AB}{BC} = \frac{5k}{12k} = \frac{5}{12}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{5}{13}} = \frac{13}{5}$$

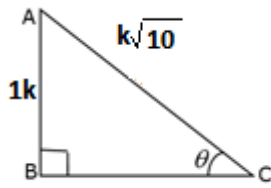
$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{5}{12}} = \frac{12}{5}$$

23. Question

If $\operatorname{cosec} \theta = \sqrt{10}$, then find the values of other t-ratios of angle θ .

Answer

Given: $\operatorname{cosec} \theta = \sqrt{10}$



We know that,

$$\operatorname{cosec} \theta = \frac{\text{hypotenuse}}{\text{perpendicular}}$$

$$\operatorname{cosec} \theta = \frac{\sqrt{10}}{1} \Rightarrow \frac{H}{P} = \frac{\sqrt{10}}{1} \Rightarrow \frac{AC}{AB} = \frac{\sqrt{10}}{1}$$

Let,

$$AB = 1k \text{ and } AC = k\sqrt{10}$$

where, k is any positive integer.

In right angled $\triangle ABC$, we have

$$(AB)^2 + (BC)^2 = (AC)^2 \text{ [by using Pythagoras theorem]}$$

$$\Rightarrow (1k)^2 + (BC)^2 = (k\sqrt{10})^2$$

$$\Rightarrow (BC)^2 = 10k^2 - k^2$$

$$\Rightarrow (BC)^2 = 9k^2$$

$$\Rightarrow BC = \sqrt{9k^2}$$

$$\Rightarrow BC = \pm 3k \text{ [taking positive square root since, side cannot be negative]}$$

Now, we have to find the value of other trigonometric ratios.

We, know that

$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{1k}{k\sqrt{10}} = \frac{1}{\sqrt{10}}$$

$$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$= \frac{BC}{AC} = \frac{3k}{k\sqrt{10}} = \frac{3}{\sqrt{10}}$$

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$= \frac{AB}{BC} = \frac{1k}{3k} = \frac{1}{3}$$

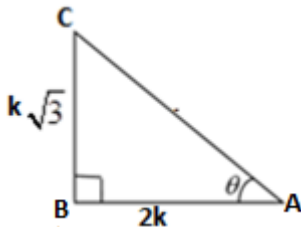
$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{3}{\sqrt{10}}} = \frac{\sqrt{10}}{3}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{1}{3}} = 3$$

24 A. Question

If $\tan A = \frac{\sqrt{3}}{2}$, then find the values of $\sin A + \cos A$.

Answer



We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

$$\text{Or } \tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\text{Given: } \tan A = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \tan A = \frac{\sqrt{3}}{2}$$

$$\tan A = \frac{\sqrt{3}}{2} \Rightarrow \frac{P}{B} = \frac{\sqrt{3}}{2} \Rightarrow \frac{BC}{AB} = \frac{\sqrt{3}}{2}$$

Let,

Side opposite to angle A = BC = $k\sqrt{3}$

Side adjacent to angle A = AB = $2k$

where, k is any positive integer

Firstly we have to find the value of BC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (2k)^2 + (\sqrt{3}k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 4k^2 + 3k^2$$

$$\Rightarrow (AC)^2 = 7k^2$$

$$\Rightarrow AC = \sqrt{7}k$$

$$\Rightarrow AC = k\sqrt{7}$$

$$\text{So, } AC = k\sqrt{7}$$

Now, we will find the $\sin A$ and $\cos A$

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle A = BC = $k\sqrt{3}$

and Hypotenuse = AC = $k\sqrt{7}$

$$\text{So, } \sin A = \frac{BC}{AC} = \frac{k\sqrt{3}}{k\sqrt{7}} = \frac{\sqrt{3}}{\sqrt{7}}$$

Now, we know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle A = AB = $2k$

Hypotenuse = AC = $k\sqrt{7}$

$$\text{So, } \cos A = \frac{AB}{AC} = \frac{2k}{k\sqrt{7}} = \frac{2}{\sqrt{7}}$$

Now, we have to find $\sin A + \cos A$

Putting values of $\sin A$ and $\cos A$, we get

$$\sin A + \cos A = \frac{\sqrt{3}}{\sqrt{7}} + \frac{2}{\sqrt{7}} = \frac{\sqrt{3} + 2}{\sqrt{7}}$$

24 B. Question

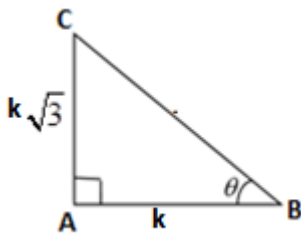
If $\sin \theta = \sqrt{3} \cos \theta$, find the value of $\cos \theta - \sin \theta$.

Answer

Given: $\sin \theta = \sqrt{3} \cos \theta$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = \sqrt{3}$$

$$\Rightarrow \tan \theta = \sqrt{3}$$



We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

$$\text{Or } \tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

Given: $\tan \theta = \sqrt{3}$

$$\Rightarrow \tan \theta = \frac{\sqrt{3}}{1}$$

$$\tan \theta = \frac{\sqrt{3}}{1} \Rightarrow \frac{P}{B} = \frac{\sqrt{3}}{1} \Rightarrow \frac{AC}{AB} = \frac{\sqrt{3}}{1}$$

Let,

Side opposite to angle $\theta = AC = \sqrt{3}k$

Side adjacent to angle $\theta = AB = 1k$

where k is any positive integer

Firstly we have to find the value of BC .

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (AC)^2 = (BC)^2$$

$$\Rightarrow (1k)^2 + (\sqrt{3}k)^2 = (BC)^2$$

$$\Rightarrow (BC)^2 = 1k^2 + 3k^2$$

$$\Rightarrow (BC)^2 = 4k^2$$

$$\Rightarrow BC = \sqrt{4}k$$

$$\Rightarrow BC = \pm 2k$$

But side BC can't be negative. So, $BC = 2k$

Now, we will find the $\sin B$ and $\cos B$

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle $\theta = AC = k\sqrt{3}$

and Hypotenuse = $BC = 2k$

$$\text{So, } \sin \theta = \frac{AC}{BC} = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$$

Now, we know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

The side adjacent to angle $\theta = AB = 1k$

Hypotenuse = $BC = 2k$

$$\text{So, } \cos \theta = \frac{AB}{BC} = \frac{1k}{2k} = \frac{1}{2}$$

Now, we have to find the value of $\cos \theta - \sin \theta$

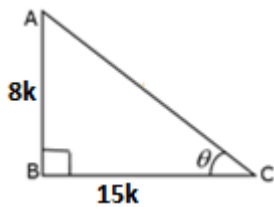
Putting the values of $\sin \theta$ and $\cos \theta$, we get

$$\cos \theta - \sin \theta = \frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1 - \sqrt{3}}{2}$$

24 C. Question

If $\tan \theta = \frac{8}{15}$, find the value of $1 + \cos^2 \theta$.

Answer



We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

$$\text{Or } \tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan \theta = \frac{8}{15} \Rightarrow \frac{P}{B} = \frac{8}{15} \Rightarrow \frac{AB}{BC} = \frac{8}{15}$$

Let,

Side opposite to angle θ = AB = 8k

Side adjacent to angle θ = BC = 15k

where, k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (8k)^2 + (15k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 64k^2 + 225k^2$$

$$\Rightarrow (AC)^2 = 289 k^2$$

$$\Rightarrow AC = \sqrt{289 k^2}$$

$$\Rightarrow AC = \pm 17k$$

But side AC can't be negative. So, AC = 17k

Now, we will find the $\cos \theta$

We know that

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle θ = BC = 15k

and Hypotenuse = AC = 17k

$$\text{So, } \cos \theta = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$$

Now, we have to find the value of $1 + \cos^2 \theta$

Putting the value of $\cos \theta$, we get

$$1 + \cos^2 \theta = 1 + \left(\frac{15}{17}\right)^2$$

$$= 1 + \frac{225}{289}$$

$$= \frac{289 + 225}{289}$$

$$= \frac{514}{289}$$

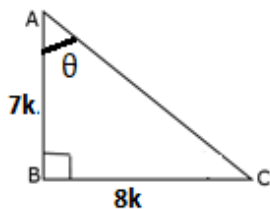
25. Question

If $\cot \theta = \frac{7}{8}$, evaluate

(i) $\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$

(ii) $\cot^2 \theta$

Answer



Given: $\cot \theta = \frac{7}{8}$

We know that,

$$\cot \theta = \frac{\text{side adjacent to angle } \theta}{\text{side opposite to angle } \theta}$$

$$\text{Or } \cot \theta = \frac{\text{base}}{\text{perpendicular}}$$

$$\cot \theta = \frac{7}{8} \Rightarrow \frac{B}{P} = \frac{7}{8} \Rightarrow \frac{AB}{BC} = \frac{7}{8}$$

Let,

Side adjacent to angle θ = AB = 7k

Side opposite to angle θ = BC = 8k

where, k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (7k)^2 + (8k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 49k^2 + 64k^2$$

$$\Rightarrow (AC)^2 = 113k^2$$

$$\Rightarrow AC = \sqrt{113k^2}$$

$$\Rightarrow AC = k\sqrt{113}$$

$$\therefore \sin \theta = \frac{P}{H} = \frac{BC}{AC} = \frac{8k}{k\sqrt{113}} = \frac{8}{\sqrt{113}}$$

$$\text{and } \cos \theta = \frac{B}{H} = \frac{AB}{AC} = \frac{7k}{k\sqrt{113}} = \frac{7}{\sqrt{113}}$$

$$(i) \frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$$

We know that,

$$(a+b)(a-b) = (a^2 - b^2)$$

So, using this identity, we get

$$= \frac{(1)^2 - (\sin \theta)^2}{(1)^2 - (\cos \theta)^2}$$

$$= \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta}$$

$$= \frac{1 - \left(\frac{8}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2}$$

$$= \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}}$$

$$= \frac{\frac{113 - 64}{113}}{\frac{113 - 49}{113}}$$

$$= \frac{49}{64}$$

$$(ii) \cot^2 \theta$$

$$\text{Given } \cot \theta = \frac{7}{8}$$

$$= \left(\frac{7}{8}\right)^2$$

$$= \frac{49}{64}$$

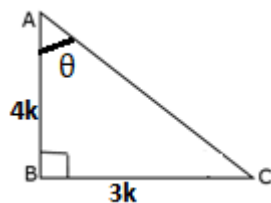
26 A. Question

If $3 \cot A = 4$, check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Answer

Given: $3 \cot A = 4$

$$\Rightarrow \cot A = \frac{4}{3}$$



We know that,

$$\cot \theta = \frac{\text{side adjacent to angle } \theta}{\text{side opposite to angle } \theta}$$

$$\text{Or } \cot \theta = \frac{\text{base}}{\text{perpendicular}}$$

$$\cot A = \frac{4}{3} \Rightarrow \frac{B}{P} = \frac{4}{3} \Rightarrow \frac{AB}{BC} = \frac{4}{3}$$

Let,

Side adjacent to angle A = AB = 4k

The side opposite to angle A = BC = 3k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (4k)^2 + (3k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 16k^2 + 9k^2$$

$$\Rightarrow (AC)^2 = 25k^2$$

$$\Rightarrow AC = \sqrt{25k^2}$$

$$\Rightarrow AC = \pm 5k \text{ [taking positive square root since, side cannot be negative]}$$

$$\therefore \tan A = \frac{1}{\cot A} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

$$\sin A = \frac{P}{H} = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

$$\text{and } \cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

$$\text{Now, LHS} = \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2}$$

$$= \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}}$$

$$\begin{aligned}
&= \frac{\frac{16-9}{16}}{\frac{16+9}{16}} \\
&= \frac{7}{25} \dots (i)
\end{aligned}$$

And RHS = $\cos^2 A - \sin^2 A$

$$\begin{aligned}
&= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 \\
&= \frac{16}{25} - \frac{9}{25} \\
&= \frac{7}{25} \dots (ii)
\end{aligned}$$

From Eqs. (i) and (ii) LHS = RHS

Hence Proved

26 B. Question

In a right triangle ABC, right angled at B, if $\tan A = 1$, then verify that $2 \sin A \cos A = 1$.

Answer

$$\tan A = 1$$

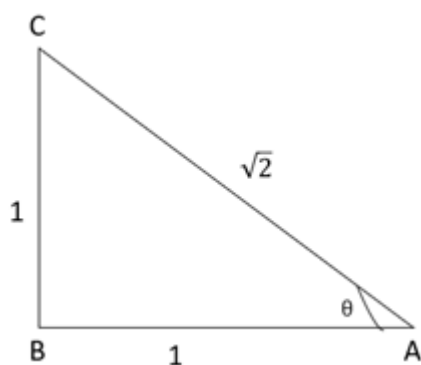
As we know

$$\tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

Now construct a right angle triangle right angled at B such that

$$\angle BAC = \theta$$

Hence perpendicular = BC = 1 and base = AB = 1



By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = (1)^2 + (1)^2$$

$$\Rightarrow AC^2 = 2$$

$$\Rightarrow AC = \sqrt{2}$$

As,

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} \text{ and } \cos\theta = \frac{\text{base}}{\text{hypotenuse}}$$

$$\Rightarrow \sin\theta = \frac{1}{\sqrt{2}} \text{ and } \cos\theta = \frac{1}{\sqrt{2}}$$

Hence,

$$2 \sin A \cos A = 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow 2 \sin A \cos A = 2 \times \frac{1}{2}$$

$$\Rightarrow 2 \sin A \cos A = 1$$

$$= \text{R.H.S}$$

Hence proved.

27. Question

If $4\sin^2 \theta = 3$ and $0^\circ < \theta < 90^\circ$, find the value of $1 + \cos \theta$.

Answer

$$4\sin^2 \theta = 3$$

$$\Rightarrow \sin^2 \theta = \frac{3}{4}$$

$$\Rightarrow \sin \theta = \pm \frac{\sqrt{3}}{2}$$

But it is given $0^\circ < \theta < 90^\circ$

$$\text{So, } \sin \theta = \frac{\sqrt{3}}{2}$$

$$\sin \theta = \frac{\sqrt{3}}{2} \Rightarrow \frac{P}{H} = \frac{\sqrt{3}}{2}$$

Let, $P = k\sqrt{3}$ and $H = 2k$

In right angled ΔABC , we have

$$B^2 + P^2 = H^2$$

$$\Rightarrow B^2 + (k\sqrt{3})^2 = (2k)^2$$

$$\Rightarrow B^2 + 3k^2 = 4k^2$$

$$\Rightarrow B^2 = 4k^2 - 3k^2$$

$$\Rightarrow B^2 = k^2$$

$$\Rightarrow B = \pm k$$

$\Rightarrow B = k$ [taking positive square root since, side cannot be negative]

$$\therefore \cos \theta = \frac{B}{H} = \frac{k}{2k} = \frac{1}{2}$$

$$\text{So, } 1 + \cos \theta = 1 + \frac{1}{2} = \frac{2+1}{2} = \frac{3}{2}$$

28. Question

If $\tan \theta = \frac{p}{q}$, find the value of $\frac{p \sin \theta - q \cos \theta}{p \sin \theta + q \cos \theta}$.

Answer

$$\text{Given: } \tan \theta = \frac{p}{q}$$

$$\text{Now, } \frac{p \sin \theta - q \cos \theta}{p \sin \theta + q \cos \theta}$$

$$\Rightarrow \frac{\cos \theta \left(\frac{p \sin \theta}{\cos \theta} - q \right)}{\cos \theta \left(\frac{p \sin \theta}{\cos \theta} + q \right)}$$

$$\Rightarrow \frac{p \tan \theta - q}{p \tan \theta + q} \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$\Rightarrow \frac{p \left(\frac{p}{q} \right) - q}{p \left(\frac{p}{q} \right) + q}$$

$$\Rightarrow \frac{\frac{p^2 - q^2}{q}}{\frac{p^2 + q^2}{q}}$$

$$\Rightarrow \frac{p^2 - q^2}{p^2 + q^2}$$

$$\therefore \frac{p \sin \theta - q \cos \theta}{p \sin \theta + q \cos \theta} = \frac{p^2 - q^2}{p^2 + q^2}$$

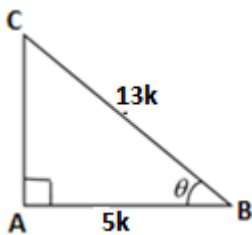
29. Question

If $13 \cos \theta = 5$, $\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta}$.

Answer

Given: $13 \cos \theta = 5$

$$\Rightarrow \cos \theta = \frac{5}{13}$$



We know that,

$$\cos \theta = \frac{\text{Base}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{5}{13} \Rightarrow \frac{B}{H} = \frac{5}{13}$$

Let $AB = 5k$ and $BC = 13k$

In right angled $\triangle ABC$, we have

$$B^2 + P^2 = H^2$$

$$\Rightarrow (5k)^2 + P^2 = (13k)^2$$

$$\Rightarrow P^2 + 25k^2 = 169k^2$$

$$\Rightarrow P^2 = 169k^2 - 25k^2$$

$$\Rightarrow P^2 = 144k^2$$

$$\Rightarrow P = \sqrt{144k^2}$$

$$\Rightarrow P = \pm 12k$$

$$\Rightarrow P = 12k \text{ [taking positive square root since, side cannot be negative]}$$

$$\therefore \sin \theta = \frac{P}{H} = \frac{12}{13}$$

$$\text{Now, } \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta}$$

$$= \frac{\frac{12}{13} + \frac{5}{13}}{\frac{12}{13} - \frac{5}{13}}$$

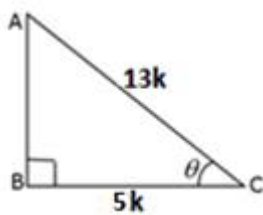
$$= \frac{17}{7}$$

30. Question

$$\text{If } \sec \theta = \frac{13}{5}, \text{ show that } \frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta} = 3.$$

Answer

$$\text{Given: } \sec \theta = \frac{13}{5}$$



We know that,

$$\sec \theta = \frac{\text{hypotenuse}}{\text{base}}$$

$$\sec \theta = \frac{13}{5} \Rightarrow \frac{H}{B} = \frac{13}{5} \Rightarrow \frac{AC}{BC} = \frac{13}{5}$$

Let,

$$BC = 5k \text{ and } AC = 13k$$

where, k is any positive integer.

In right angled ΔABC , we have

$$(AB)^2 + (BC)^2 = (AC)^2 \text{ [by using Pythagoras theorem]}$$

$$\Rightarrow (AB)^2 + (5k)^2 = (13k)^2$$

$$\Rightarrow (AB)^2 + 25k^2 = 169k^2$$

$$\Rightarrow (AB)^2 = 169k^2 - 25k^2$$

$$\Rightarrow (AB)^2 = 144k^2$$

$$\Rightarrow AB = \sqrt{144k^2}$$

$$\Rightarrow AB = \pm 12k \text{ [taking positive square root since, side cannot be negative]}$$

Now, we have to find the value of other trigonometric ratios.

We, know that

$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

$$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$= \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\text{Now, LHS} = \frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta}$$

$$= \frac{2 \left(\frac{12}{13} \right) - 3 \left(\frac{5}{13} \right)}{4 \left(\frac{12}{13} \right) - 9 \left(\frac{5}{13} \right)}$$

$$= \frac{24 - 15}{48 - 45}$$

$$= \frac{9}{3}$$

$$= 3 = \text{RHS}$$

Hence Proved

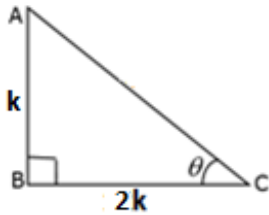
31. Question

If $2 \tan \theta = 1$, find the value of $\frac{3 \cos \theta + \sin \theta}{2 \cos \theta - \sin \theta}$.

Answer

Given: $2 \tan \theta = 1$

$$\Rightarrow \tan \theta = \frac{1}{2}$$



We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

$$\text{Or } \tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan \theta = \frac{1}{2} \Rightarrow \frac{P}{B} = \frac{1}{2} \Rightarrow \frac{AB}{BC} = \frac{1}{2}$$

Let,

Side opposite to angle $\theta = AB = 1k$

Side adjacent to angle $\theta = BC = 2k$

where, k is any positive integer

Firstly we have to find the value of AC .

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (k)^2 + (2k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = k^2 + 4k^2$$

$$\Rightarrow (AC)^2 = 5k^2$$

$$\Rightarrow AC = \sqrt{5k^2}$$

$$\Rightarrow AC = \pm k\sqrt{5}$$

But side AC can't be negative. So, $AC = k\sqrt{5}$

Now, we will find the $\sin \theta$ and $\cos \theta$

We know that

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle $\theta = BC = 2k$

and Hypotenuse = $AC = k\sqrt{5}$

$$\text{So, } \cos \theta = \frac{BC}{AC} = \frac{2k}{k\sqrt{5}} = \frac{2}{\sqrt{5}}$$

$$\text{And } \sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle $\theta = AB = 1k$

And Hypotenuse = $AC = k\sqrt{5}$

$$\text{So, } \sin \theta = \frac{AB}{AC} = \frac{1k}{k\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$\text{Now, } \frac{3 \cos \theta + \sin \theta}{2 \cos \theta - \sin \theta}$$

$$= \frac{3 \left(\frac{2}{\sqrt{5}} \right) + \frac{1}{\sqrt{5}}}{2 \left(\frac{2}{\sqrt{5}} \right) - \frac{1}{\sqrt{5}}}$$

$$= \frac{6 + 1}{4 - 1}$$

$$= \frac{7}{3}$$

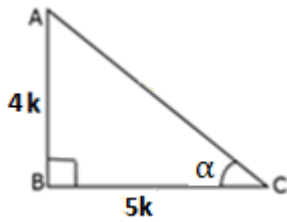
32. Question

$$\text{If } 5 \tan \alpha = 4, \text{ show that } \frac{5 \sin \alpha - 3 \cos \alpha}{5 \sin \alpha + 2 \cos \alpha} = \frac{1}{6}.$$

Answer

Given: $5 \tan \alpha = 4$

$$\Rightarrow \tan \alpha = \frac{4}{5}$$



We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

$$\text{Or } \tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan \alpha = \frac{4}{5} \Rightarrow \frac{P}{B} = \frac{4}{5} \Rightarrow \frac{AB}{BC} = \frac{4}{5}$$

Let,

The side opposite to angle α = AB = 4k

The side adjacent to angle α = BC = 5k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (4k)^2 + (5k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 16k^2 + 25k^2$$

$$\Rightarrow (AC)^2 = 41k^2$$

$$\Rightarrow AC = \sqrt{41k^2}$$

$$\Rightarrow AC = \pm k\sqrt{41}$$

But side AC can't be negative. So, $AC = k\sqrt{41}$

Now, we will find the $\sin \alpha$ and $\cos \alpha$

We know that

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle α = BC = 5k

and Hypotenuse = AC = $k\sqrt{41}$

$$\text{So, } \cos \alpha = \frac{BC}{AC} = \frac{5k}{k\sqrt{41}} = \frac{5}{\sqrt{41}}$$

$$\text{And } \sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle α = AB = 4k

And Hypotenuse = AC = $k\sqrt{5}$

$$\text{So, } \sin \alpha = \frac{AB}{AC} = \frac{4k}{k\sqrt{41}} = \frac{4}{\sqrt{41}}$$

$$\text{Now, LHS} = \frac{5 \sin \alpha - 3 \cos \alpha}{5 \sin \alpha + 2 \cos \alpha}$$

$$= \frac{5 \left(\frac{4}{\sqrt{41}} \right) - 3 \left(\frac{5}{\sqrt{41}} \right)}{5 \left(\frac{4}{\sqrt{41}} \right) + 2 \left(\frac{5}{\sqrt{41}} \right)}$$

$$= \frac{20 - 15}{20 + 10}$$

$$= \frac{5}{30}$$

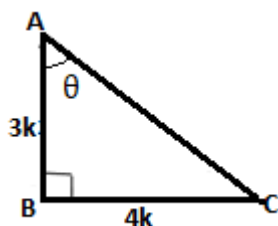
$$= \frac{1}{6} = \text{RHS}$$

Hence Proved

33. Question

If $\cot \theta = \frac{3}{4}$, prove that $\sqrt{\frac{\sec \theta + \operatorname{cosec} \theta}{\sec \theta - \operatorname{cosec} \theta}} = \sqrt{7}$.

Answer



We know that,

$$\cot \theta = \frac{\text{side adjacent to angle } \theta}{\text{side opposite to angle } \theta}$$

$$\text{Or } \cot \theta = \frac{\text{base}}{\text{perpendicular}}$$

$$\cot \theta = \frac{3}{4} \Rightarrow \frac{B}{P} = \frac{3}{4} \Rightarrow \frac{AB}{BC} = \frac{3}{4}$$

Let,

Side adjacent to angle θ = AB = 3k

The side opposite to angle θ = BC = 4k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (3k)^2 + (4k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 9k^2 + 16k^2$$

$$\Rightarrow (AC)^2 = 25k^2$$

$$\Rightarrow AC = \sqrt{25k^2}$$

$$\Rightarrow AC = \pm 5k$$

But side AC can't be negative. So, AC = 5k

Now, we will find the sin θ

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle θ = BC = 4k

and Hypotenuse = AC = 5k

$$\text{So, } \sin \theta = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

Now, we know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

The side adjacent to angle $\theta = AB = 3k$

Hypotenuse = $AC = 5k$

$$\text{So, } \cos \theta = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

$$\therefore \sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$$

And

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$$

$$\text{Now, LHS} = \sqrt{\frac{\sec \theta + \operatorname{cosec} \theta}{\sec \theta - \operatorname{cosec} \theta}}$$

$$= \sqrt{\frac{\left(\frac{5}{3}\right) + \left(\frac{5}{4}\right)}{\left(\frac{5}{3}\right) - \left(\frac{5}{4}\right)}}$$

$$= \sqrt{\frac{\left(\frac{20+15}{12}\right)}{\left(\frac{20-15}{12}\right)}}$$

$$= \sqrt{\frac{35}{5}}$$

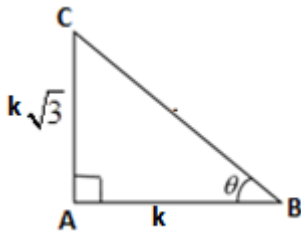
$$= \sqrt{7} = \text{RHS}$$

Hence Proved

34. Question

$$\text{If } \cot \theta = \frac{1}{\sqrt{3}}, \text{ verify that: } \frac{1 - \cos^2 \theta}{2 - \sin^2 \theta} = \frac{3}{5}.$$

Answer



We know that,

$$\cot \theta = \frac{\text{side adjacent to angle } \theta}{\text{side opposite to angle } \theta}$$

$$\text{Or } \cot \theta = \frac{\text{base}}{\text{perpendicular}}$$

$$\cot \theta = \frac{1}{\sqrt{3}} \Rightarrow \frac{B}{P} = \frac{1}{\sqrt{3}} \Rightarrow \frac{AB}{AC} = \frac{1}{\sqrt{3}}$$

Let,

Side adjacent to angle θ = AB = 1k

Side opposite to angle θ = AC = $k\sqrt{3}$

where, k is any positive integer

Firstly we have to find the value of BC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (AC)^2 = (BC)^2$$

$$\Rightarrow (1k)^2 + (\sqrt{3}k)^2 = (BC)^2$$

$$\Rightarrow (BC)^2 = 1k^2 + 3k^2$$

$$\Rightarrow (BC)^2 = 4k^2$$

$$\Rightarrow BC = \sqrt{4k^2}$$

$$\Rightarrow BC = \pm 2k$$

But side BC can't be negative. So, BC = 2k

Now, we will find the $\sin \theta$ and $\cos \theta$

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle θ = AC = $k\sqrt{3}$

and Hypotenuse = BC = 2k

$$\text{So, } \sin \theta = \frac{AC}{BC} = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$$

Now, we know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle θ = AB = 1k

Hypotenuse = BC = 2k

$$\text{So, } \cos \theta = \frac{AB}{BC} = \frac{1k}{2k} = \frac{1}{2}$$

$$\text{Now, LHS} = \frac{1 - \cos^2 \theta}{2 - \sin^2 \theta}$$

$$= \frac{1 - \left(\frac{1}{2}\right)^2}{2 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1 - \frac{1}{4}}{2 - \frac{3}{4}}$$

$$= \frac{\frac{4 - 1}{4}}{\frac{8 - 3}{4}}$$

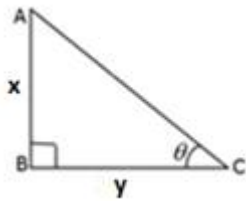
$$= \frac{3}{5} = \text{RHS}$$

Hence Proved

35. Question

If $\tan \theta = \frac{x}{y}$, find the value of $x \sin \theta + y \cos \theta$.

Answer



We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

$$\text{Or } \tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan \theta = \frac{x}{y} \Rightarrow \frac{P}{B} = \frac{x}{y} \Rightarrow \frac{AB}{BC} = \frac{x}{y}$$

Let,

Side opposite to angle θ = AB = x

Side adjacent to angle θ = BC = y

where, k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (x)^2 + (y)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = x^2 + y^2$$

$$\Rightarrow AC = \sqrt{(x^2 + y^2)}$$

Now, we will find the $\sin \theta$ and $\cos \theta$

We know that

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle θ = BC = y

and Hypotenuse = AC = $\sqrt{(x^2 + y^2)}$

$$\text{So, } \cos \theta = \frac{BC}{AC} = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\text{And } \sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle θ = AB = x

And Hypotenuse = AC = $\sqrt{x^2 + y^2}$

$$\text{So, } \sin \theta = \frac{AB}{AC} = \frac{x}{\sqrt{x^2 + y^2}}$$

Now, $x \sin \theta + y \cos \theta$

$$= x \left(\frac{x}{\sqrt{x^2 + y^2}} \right) + y \left(\frac{y}{\sqrt{x^2 + y^2}} \right)$$

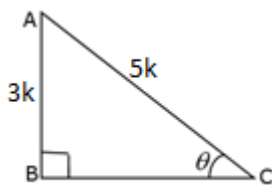
$$= \frac{x^2 + y^2}{\sqrt{x^2 + y^2}}$$

$$= \sqrt{x^2 + y^2}$$

36. Question

If $\sin \theta = \frac{3}{5}$, find the value of $\tan^2 \theta + \sin \theta \cos \theta + \cot \theta$.

Answer



$$\text{Given: } \sin \theta = \frac{3}{5}$$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

$$\text{Or } \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin \theta = \frac{3}{5} \Rightarrow \frac{P}{H} = \frac{3}{5} \Rightarrow \frac{AB}{AC} = \frac{3}{5}$$

Let,

$$\text{Perpendicular} = AB = 3k$$

and Hypotenuse = AC = 5k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

In right angled ΔABC , we have

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (3k)^2 + (BC)^2 = (5k)^2$$

$$\Rightarrow 9k^2 + (BC)^2 = 25k^2$$

$$\Rightarrow (BC)^2 = 25k^2 - 9k^2$$

$$\Rightarrow (BC)^2 = 16k^2$$

$$\Rightarrow BC = \sqrt{16k^2}$$

$$\Rightarrow BC = \pm 4k$$

But side BC can't be negative. So, $BC = 4k$

Now, we have to find the value of $\cos \theta$ and $\tan \theta$

We know that,

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}}$$

The side adjacent to angle θ or base = $BC = 4k$

Hypotenuse = $AC = 5k$

$$\text{So, } \cos \theta = \frac{4k}{5k} = \frac{4}{5}$$

Now,

We know that,

$$\tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

Perpendicular = $AB = 3k$

Base = $BC = 4k$

$$\text{So, } \tan \theta = \frac{3k}{4k} = \frac{3}{4}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

Now, $\tan^2 \theta + \sin \theta \cos \theta + \cot \theta$

$$= \left(\frac{3}{4}\right)^2 + \left(\frac{3}{5}\right)\left(\frac{4}{5}\right) + \left(\frac{4}{3}\right)$$

$$= \left(\frac{9}{16}\right) + \left(\frac{12}{25}\right) + \left(\frac{4}{3}\right)$$

$$= \frac{675 + 576 + 1600}{16 \times 25 \times 3}$$

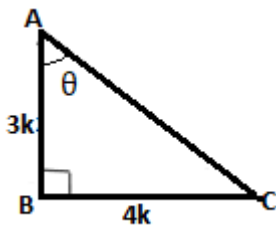
$$= \frac{2851}{1200}$$

37. Question

If $4 \cot \theta = 3$, show that $\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} = 7$.

Answer

Given: $\cot \theta = \frac{3}{4}$



We know that,

$$\cot \theta = \frac{\text{side adjacent to angle } \theta}{\text{side opposite to angle } \theta}$$

$$\text{Or } \cot \theta = \frac{\text{base}}{\text{perpendicular}}$$

$$\cot \theta = \frac{3}{4} \Rightarrow \frac{B}{P} = \frac{3}{4} \Rightarrow \frac{AB}{BC} = \frac{3}{4}$$

Let,

Side adjacent to angle θ = AB = 3k

The side opposite to angle θ = BC = 4k

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (3k)^2 + (4k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 9k^2 + 16k^2$$

$$\Rightarrow (AC)^2 = 25k^2$$

$$\Rightarrow AC = \sqrt{25k^2}$$

$$\Rightarrow AC = \pm 5k$$

But side AC can't be negative. So, $AC = 5k$

Now, we will find the $\sin \theta$

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle $\theta = BC = 4k$

and Hypotenuse = $AC = 5k$

$$\text{So, } \sin \theta = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

Now, we know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle $\theta = AB = 3k$

Hypotenuse = $AC = 5k$

$$\text{So, } \cos \theta = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

$$\text{Now, LHS} = \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta}$$

$$= \frac{\frac{4}{5} + \frac{3}{5}}{\frac{4}{5} - \frac{3}{5}}$$

$$= \frac{7}{1}$$

$$= 7 = \text{RHS}$$

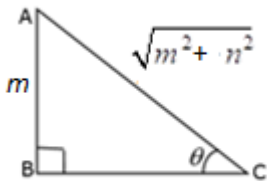
Hence Proved

38. Question

If $\sin \theta = \frac{m}{\sqrt{m^2 + n^2}}$, prove that $m \sin \theta + n \cos \theta = \sqrt{m^2 + n^2}$

Answer

$$\text{Given: } \sin \theta = \frac{m}{\sqrt{m^2 + n^2}}$$



We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

$$\text{Or } \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin \theta = \frac{m}{\sqrt{m^2 + n^2}} \Rightarrow \frac{P}{H} = \frac{m}{\sqrt{m^2 + n^2}} \Rightarrow \frac{AB}{AC} = \frac{m}{\sqrt{m^2 + n^2}}$$

Let,

$$\text{Perpendicular} = AB = m$$

$$\text{and Hypotenuse} = AC = \sqrt{m^2 + n^2}$$

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

In right angled ΔABC , we have

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (m)^2 + (BC)^2 = (\sqrt{m^2 + n^2})^2$$

$$\Rightarrow m^2 + (BC)^2 = m^2 + n^2$$

$$\Rightarrow (BC)^2 = m^2 + n^2 - m^2$$

$$\Rightarrow (BC)^2 = n^2$$

$$\Rightarrow BC = \sqrt{n^2}$$

$$\Rightarrow BC = \pm n$$

But side BC can't be negative. So, $BC = n$

Now, we have to find the value of $\cos \theta$ and $\tan \theta$

We know that,

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}}$$

Side adjacent to angle θ or base = $BC = n$

Hypotenuse = $AC = \sqrt{m^2 + n^2}$

$$\text{So, } \cos \theta = \frac{n}{\sqrt{m^2 + n^2}}$$

Now, $LHS = m \sin \theta + n \cos \theta$

$$= m \left(\frac{m}{\sqrt{m^2 + n^2}} \right) + n \left(\frac{n}{\sqrt{m^2 + n^2}} \right)$$

$$= \frac{m^2 + n^2}{\sqrt{m^2 + n^2}}$$

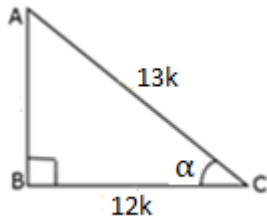
$$= \sqrt{m^2 + n^2} = RHS$$

Hence Proved

39. Question

If $\cos \alpha = \frac{12}{13}$, show that $\sin \alpha (1 - \tan \alpha) = \frac{35}{156}$

Answer



We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

$$\text{Or } \cos \theta = \frac{\text{base}}{\text{Hypotenuse}}$$

$$\cos \alpha = \frac{12}{13} \Rightarrow \frac{B}{H} = \frac{12}{13} \Rightarrow \frac{BC}{AC} = \frac{12}{13}$$

Let,

$$\text{Base} = BC = 12k$$

$$\text{Hypotenuse} = AC = 13k$$

Where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (AB)^2 + (12k)^2 = (13k)^2$$

$$\Rightarrow (AB)^2 + 144k^2 = 169k^2$$

$$\Rightarrow (AB)^2 = 169k^2 - 144k^2$$

$$\Rightarrow (AB)^2 = 25k^2$$

$$\Rightarrow AB = \sqrt{25k^2}$$

$$\Rightarrow AB = \pm 5k$$

But side AB can't be negative. So, $AB = 5k$

Now, we have to find $\sin \alpha$ and $\tan \alpha$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle $\alpha = AB = 5k$

And Hypotenuse = $AC = 13k$

$$\text{So, } \sin \alpha = \frac{5k}{13k} = \frac{5}{13}$$

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

Side opposite to angle $\alpha = AB = 5k$

Side adjacent to angle $\alpha = BC = 12k$

$$\text{So, } \tan \alpha = \frac{5k}{12k} = \frac{5}{12}$$

Now, LHS = $\sin \alpha (1 - \tan \alpha)$

$$= \frac{5}{13} \left(1 - \frac{5}{12} \right)$$

$$= \frac{5}{13} \left(\frac{12 - 5}{12} \right)$$

$$= \frac{35}{156} = \text{RHS}$$

Hence Proved

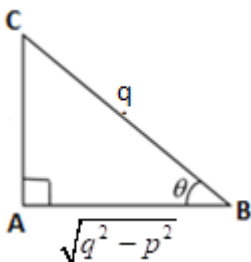
40. Question

If $q \cos \theta = \sqrt{q^2 - p^2}$, prove that $q \sin \theta = p$.

Answer

Given : $q \cos \theta = \sqrt{q^2 - p^2}$

$$\Rightarrow \cos \theta = \frac{\sqrt{q^2 - p^2}}{q}$$



We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

$$\text{Or } \cos \theta = \frac{\text{base}}{\text{Hypotenuse}}$$

$$\cos \theta = \frac{\sqrt{q^2 - p^2}}{q} \Rightarrow \frac{B}{H} = \frac{\sqrt{q^2 - p^2}}{q} \Rightarrow \frac{BC}{AC} = \frac{\sqrt{q^2 - p^2}}{q}$$

Let,

$$\text{Base} = BC = \sqrt{(q^2 - p^2)}$$

$$\text{Hypotenuse} = AC = q$$

Where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (AB)^2 + (\sqrt{(q^2 - p^2)})^2 = (q)^2$$

$$\Rightarrow (AB)^2 + (q^2 - p^2) = q^2$$

$$\Rightarrow (AB)^2 = q^2 - q^2 + p^2$$

$$\Rightarrow (AB)^2 = p^2$$

$$\Rightarrow AB = \sqrt{p^2}$$

$$\Rightarrow AB = \pm p$$

But side AB can't be negative. So, $AB = p$

Now, we have to find $\sin \theta$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

The side opposite to angle $\theta = AB = p$

And Hypotenuse = AC = q

$$\text{So, } \sin \theta = \left(\frac{p}{q} \right)$$

Now, LHS = $q \sin \theta$

$$= q \left(\frac{p}{q} \right)$$

$$= q = \text{RHS}$$

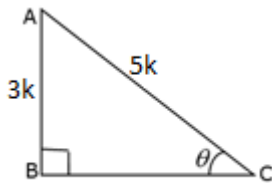
Hence Proved

41. Question

If $\sin \theta = \frac{3}{5}$, show that : $\frac{\cos \theta - \frac{1}{\tan \theta}}{2 \cot \theta} = -\frac{1}{5}$

Answer

Given: $\sin \theta = \frac{3}{5}$



We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

$$\text{Or } \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin \theta = \frac{3}{5} \Rightarrow \frac{P}{H} = \frac{3}{5} \Rightarrow \frac{AB}{AC} = \frac{3}{5}$$

Let,

$$\text{Perpendicular} = AB = 3k$$

$$\text{and Hypotenuse} = AC = 5k$$

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

In right angled ΔABC , we have

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (3k)^2 + (BC)^2 = (5k)^2$$

$$\Rightarrow 9k^2 + (BC)^2 = 25k^2$$

$$\Rightarrow (BC)^2 = 25k^2 - 9k^2$$

$$\Rightarrow (BC)^2 = 16k^2$$

$$\Rightarrow BC = \sqrt{16k^2}$$

$$\Rightarrow BC = \pm 4k$$

But side BC can't be negative. So, $BC = 4k$

Now, we have to find the value of $\cos \theta$ and $\tan \theta$

We know that,

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}}$$

The side adjacent to angle θ or base = $BC = 4k$

Hypotenuse = $AC = 5k$

$$\text{So, } \cos \theta = \frac{4k}{5k} = \frac{4}{5}$$

Now,

We know that,

$$\tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

Perpendicular = $AB = 3k$

Base = $BC = 4k$

$$\text{So, } \tan \theta = \frac{3k}{4k} = \frac{3}{4}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

$$\text{Now, LHS} = \frac{\cos \theta - \frac{1}{\tan \theta}}{2 \cot \theta}$$

$$= \frac{\left(\frac{4}{5}\right) - \left(\frac{1}{\frac{3}{4}}\right)}{2 \left(\frac{4}{3}\right)}$$

$$= \frac{\left(\frac{4}{5}\right) - \left(\frac{4}{3}\right)}{\left(\frac{8}{3}\right)}$$

$$= \frac{\frac{12 - 20}{15}}{\left(\frac{8}{3}\right)}$$

$$= \frac{\left(-\frac{8}{15}\right)}{\left(\frac{8}{3}\right)}$$

$$= -\frac{1}{5} = \text{RHS}$$

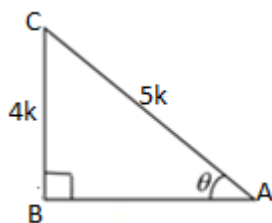
Hence Proved

42 A. Question

Find the value of

$\cos A \sin B + \sin A \cos B$, if $\sin A = \frac{4}{5}$ and $\cos B = \frac{12}{13}$.

Answer



Given: $\sin A = \frac{4}{5}$ and $\cos B = \frac{12}{13}$

To find: $\cos A \sin B + \sin A \cos B$

As, we have the value of $\sin A$ and $\cos B$ but we don't have the value of $\cos A$ and $\sin B$

So, First we find the value of $\cos A$ and $\sin B$

$$\sin A = \frac{4}{5}$$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

$$\text{Or } \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin A = \frac{4}{5} \Rightarrow \frac{P}{H} = \frac{4}{5}$$

Let,

Side opposite to angle A = 4k

and Hypotenuse = 5k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$$\Rightarrow (P)^2 + (B)^2 = (H)^2$$

$$\Rightarrow (4k)^2 + (B)^2 = (5)^2$$

$$\Rightarrow 16 k^2 + (B)^2 = 25 k^2$$

$$\Rightarrow (B)^2 = 25 k^2 - 16 k^2$$

$$\Rightarrow (B)^2 = 9 k^2$$

$$\Rightarrow B = \sqrt{9 k^2}$$

$$\Rightarrow B = \pm 3k \text{ [taking positive square root since, side cannot be negative]}$$

So, Base = 3k

Now, we have to find the value of cos A

We know that,

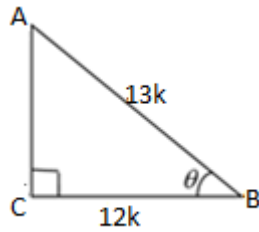
$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle A = 3k

Hypotenuse = 5k

$$\text{So, } \cos A = \frac{3k}{5k} = \frac{3}{5}$$

Now, we have to find the sin B



We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

$$\cos B = \frac{12}{13} \Rightarrow \frac{B}{H} = \frac{12}{13}$$

Let,

Side adjacent to angle B = 12k

Hypotenuse = 13k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$$\Rightarrow (B)^2 + (P)^2 = (H)^2$$

$$\Rightarrow (12k)^2 + (P)^2 = (13)^2$$

$$\Rightarrow 144 k^2 + (P)^2 = 169 k^2$$

$$\Rightarrow (P)^2 = 169 k^2 - 144 k^2$$

$$\Rightarrow (P)^2 = 25 k^2$$

$$\Rightarrow P = \sqrt{25 k^2}$$

$$\Rightarrow P = \pm 5k \text{ [taking positive square root since, side cannot be negative]}$$

So, Perpendicular = 5k

Now, we have to find the value of sin B

We know that,

$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin B = \frac{P}{H} = \frac{5k}{13k} = \frac{5}{13}$$

Now, $\cos A \sin B + \sin A \cos B$

Putting the values of $\sin A$, $\sin B$, $\cos A$ and $\cos B$, we get

$$\Rightarrow \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$$

$$\Rightarrow \frac{15 + 48}{5 \times 13}$$

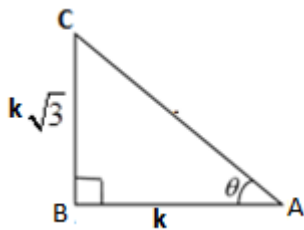
$$\Rightarrow \frac{63}{65}$$

42 B. Question

Find the value of

$\sin A \cdot \cos B - \cos A \cdot \sin B$, if $\tan A = \sqrt{3}$ and $\sin B = 1/2$.

Answer



Given: $\tan A = \sqrt{3}$ and $\sin B = \frac{1}{2}$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

$$\text{Or } \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin B = \frac{1}{2} \Rightarrow \frac{P}{H} = \frac{1}{2}$$

Let,

Side opposite to angle $\theta = 1k$

and Hypotenuse = $2k$

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$$\Rightarrow (AC)^2 + (BC)^2 = (AB)^2$$

$$\Rightarrow (1k)^2 + (BC)^2 = (2k)^2$$

$$\Rightarrow k^2 + (BC)^2 = 4k^2$$

$$\Rightarrow (BC)^2 = 4k^2 - k^2$$

$$\Rightarrow (BC)^2 = 3k^2$$

$$\Rightarrow BC = \sqrt{3k^2}$$

$$\Rightarrow BC = k\sqrt{3}$$

$$\text{So, } BC = k\sqrt{3}$$

Now, we have to find the value of $\cos B$

We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

The side adjacent to angle B = BC = $k\sqrt{3}$

Hypotenuse = AB = $2k$

$$\text{So, } \cos B = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$$

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

$$\text{Or } \tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

Given: $\tan A = \sqrt{3}$

$$\Rightarrow \tan A = \frac{\sqrt{3}}{1}$$

$$\tan A = \frac{\sqrt{3}}{1} \Rightarrow \frac{P}{B} = \frac{\sqrt{3}}{1} \Rightarrow \frac{BC}{AB} = \frac{\sqrt{3}}{1}$$

Let,

The side opposite to angle A = BC = $\sqrt{3}k$

The side adjacent to angle A = AB = $1k$

where k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (1k)^2 + (\sqrt{3}k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 1k^2 + 3k^2$$

$$\Rightarrow (AC)^2 = 4k^2$$

$$\Rightarrow AC = \sqrt{4k^2}$$

$$\Rightarrow AC = \pm 2k$$

But side AC can't be negative. So, $AC = 2k$

Now, we will find the sin A and cos A

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle A = BC = $k\sqrt{3}$

and Hypotenuse = AC = $2k$

$$\text{So, } \sin A = \frac{BC}{AC} = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$$

Now, we know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

The side adjacent to angle A = AB = $1k$

Hypotenuse = AC = $2k$

$$\text{So, } \cos A = \frac{AB}{AC} = \frac{1k}{2k} = \frac{1}{2}$$

Now, $\sin A \cdot \cos B - \cos A \cdot \sin B$

Putting the values of sin A, sin B cos A and Cos B, we get

$$\Rightarrow \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$\Rightarrow \frac{3}{4} - \frac{1}{4}$$

$$\Rightarrow \frac{2}{4}$$

$$= \frac{1}{2}$$

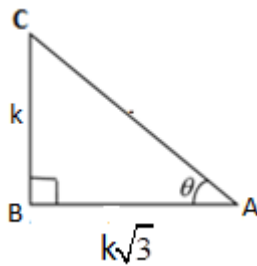
42 C. Question

Find the value of

$\sin A \cdot \cos B + \cos A \cdot \sin B$ if $\tan A = \frac{1}{\sqrt{3}}$ and $\tan B = \sqrt{3}$.

Answer

Given:



$$\tan A = \frac{1}{\sqrt{3}}$$

$$\tan A = \frac{1}{\sqrt{3}} \Rightarrow \frac{P}{B} = \frac{1}{\sqrt{3}}$$

Let,

Side opposite to angle A = BC = 1k

Side adjacent to angle A = AB = $k\sqrt{3}$

where, k is any positive integer

Firstly we have to find the value of BC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (\sqrt{3}k)^2 + (1k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 1 k^2 + 3 k^2$$

$$\Rightarrow (AC)^2 = 4 k^2$$

$$\Rightarrow AC = \sqrt{4} k$$

$$\Rightarrow AC = \pm 2k$$

But side AC can't be negative. So, $AC = 2k$

Now, we will find the $\sin A$ and $\cos A$

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle $A = BC = k$

and Hypotenuse = $AC = 2k$

$$\text{So, } \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

Now, we know that,

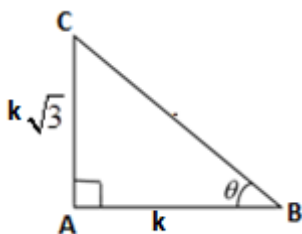
$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle $A = AB = k\sqrt{3}$

Hypotenuse = $AC = 2k$

$$\text{So, } \cos A = \frac{AB}{AC} = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$$

Now,



Given: $\tan B = \sqrt{3}$

$$\Rightarrow \tan B = \frac{\sqrt{3}}{1}$$

$$\tan B = \frac{\sqrt{3}}{1} \Rightarrow \frac{P}{B} = \frac{\sqrt{3}}{1} \Rightarrow \frac{AC}{AB} = \frac{\sqrt{3}}{1}$$

Let,

Side opposite to angle B = AC = $\sqrt{3}k$

Side adjacent to angle B = AB = $1k$

where, k is any positive integer

Firstly we have to find the value of BC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (AC)^2 = (BC)^2$$

$$\Rightarrow (1k)^2 + (\sqrt{3}k)^2 = (BC)^2$$

$$\Rightarrow (BC)^2 = 1k^2 + 3k^2$$

$$\Rightarrow (BC)^2 = 4k^2$$

$$\Rightarrow BC = \sqrt{4k^2}$$

$$\Rightarrow BC = \pm 2k$$

But side BC can't be negative. So, $BC = 2k$

Now, we will find the $\sin B$ and $\cos B$

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle B = AC = $k\sqrt{3}$

and Hypotenuse = BC = $2k$

$$\text{So, } \sin B = \frac{AC}{BC} = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$$

Now, we know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle B = AB = $1k$

Hypotenuse = BC = $2k$

$$\text{So, } \cos B = \frac{AB}{BC} = \frac{1k}{2k} = \frac{1}{2}$$

Now, $\sin A \cdot \cos B + \cos A \cdot \sin B$

Putting the values of $\sin A$, $\sin B$, $\cos A$ and $\cos B$, we get

$$\Rightarrow \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \frac{1}{4} + \frac{3}{4}$$

$$\Rightarrow \frac{4}{4}$$

$$=1$$

42 D. Question

Find the value of

$$\frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}, \text{ if } \sin A = \frac{1}{\sqrt{2}} \text{ and } \cos B = \frac{\sqrt{3}}{2}$$

Answer

$$\text{Given: } \sin A = \frac{1}{\sqrt{2}} \text{ and } \cos B = \frac{\sqrt{3}}{2}$$

$$\sin A = \frac{1}{\sqrt{2}}$$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

$$\text{Or } \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin A = \frac{1}{\sqrt{2}} \Rightarrow \frac{P}{H} = \frac{1}{\sqrt{2}}$$

Let,

Side opposite to angle A = k

and Hypotenuse = $k\sqrt{2}$

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$$\Rightarrow (P)^2 + (B)^2 = (H)^2$$

$$\Rightarrow (k)^2 + (B)^2 = (k\sqrt{2})^2$$

$$\Rightarrow k^2 + (B)^2 = 2k^2$$

$$\Rightarrow (B)^2 = 2k^2 - k^2$$

$$\Rightarrow (B)^2 = k^2$$

$$\Rightarrow B = \sqrt{k^2}$$

$$\Rightarrow B = \pm k \text{ [taking positive square root since, side cannot be negative]}$$

So, Base = k

Now, we have to find the value of tan A

We know that,

$$\tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\text{So, } \tan A = \frac{k}{k} = 1$$

Now, we have to find the tan B

We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

$$\cos B = \frac{\sqrt{3}}{2} \Rightarrow \frac{B}{H} = \frac{\sqrt{3}}{2}$$

Let,

Side adjacent to angle B = $k\sqrt{3}$

Hypotenuse = $2k$

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$$\Rightarrow (B)^2 + (P)^2 = (H)^2$$

$$\Rightarrow (k\sqrt{3})^2 + (P)^2 = (2k)^2$$

$$\Rightarrow 3k^2 + (P)^2 = 4k^2$$

$$\Rightarrow (P)^2 = 4k^2 - 3k^2$$

$$\Rightarrow (P)^2 = k^2$$

$$\Rightarrow P = \sqrt{k^2}$$

$$\Rightarrow P = \pm k \text{ [taking positive square root since, side cannot be negative]}$$

So, Perpendicular = k

Now, we have to find the value of sin B

We know that,

$$\tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\text{So, } \tan B = \frac{k}{k\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\text{Now, } \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \frac{(1) + \left(\frac{1}{\sqrt{3}}\right)}{1 - (1)\left(\frac{1}{\sqrt{3}}\right)}$$

$$\Rightarrow \frac{\frac{\sqrt{3} + 1}{\sqrt{3}}}{\frac{\sqrt{3} - 1}{\sqrt{3}}}$$

$$\Rightarrow \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

Now, multiply and divide by the conjugate of $\sqrt{3} - 1$, we get

$$\Rightarrow \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$\Rightarrow \frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2 - (1)^2} [\because (a-b)(a+b) = (a^2 - b^2)]$$

$$\Rightarrow \frac{3 + 1 + 2\sqrt{3}}{3 - 1}$$

$$\Rightarrow \frac{4 + 2\sqrt{3}}{2}$$

$$\Rightarrow 2 + \sqrt{3}$$

42 E. Question

Find the value of

$\sec A \cdot \tan A + \tan^2 A - \operatorname{cosec} A$, if $\tan A = 2$

Answer

Given: $\tan A = 2 \Rightarrow \tan^2 A = 4$

We know that, $\sec^2 A = 1 + \tan^2 A$

$$\Rightarrow \sec^2 A = 1 + 4$$

$$\Rightarrow \sec^2 A = 5$$

$$\Rightarrow \sec A = \sqrt{5}$$

$$\Rightarrow \cos A = \frac{1}{\sqrt{5}}$$

Now, we know that $\tan A = \frac{\sin A}{\cos A}$

$$\Rightarrow 2 = \frac{\sin A}{\frac{1}{\sqrt{5}}}$$

$$\Rightarrow 2 = \sqrt{5} \sin A$$

$$\Rightarrow \sin A = \frac{2}{\sqrt{5}}$$

$$\Rightarrow \operatorname{cosec} A = \frac{\sqrt{5}}{2}$$

Now, putting all the values in the given equation, we get

$\sec A \cdot \tan A + \tan^2 A - \operatorname{cosec} A$

$$\Rightarrow (\sqrt{5})(2) + (4) - \left(\frac{\sqrt{5}}{2}\right)$$

$$\Rightarrow \frac{4\sqrt{5} + 8 - \sqrt{5}}{2}$$

$$\Rightarrow \frac{3\sqrt{5} + 8}{2}$$

42 F. Question

Find the value of

$$\frac{1}{\tan A} + \frac{\sin A}{1 + \cos A}, \text{ if } \operatorname{cosec} A = 2$$

Answer

Given: $\operatorname{cosec} A = 2$

Now, we have to find $\frac{1}{\tan A} + \frac{\sin A}{1 + \cos A}$

First, we simplify the above given trigonometry equation, we get

$$\frac{1}{\frac{\sin A}{\cos A}} + \frac{\sin A}{1 + \cos A}$$

$$\Rightarrow \frac{\cos A}{\sin A} + \frac{\sin A}{1 + \cos A}$$

Taking the LCM, we get

$$\Rightarrow \frac{\cos A(1 + \cos A) + \sin A(\sin A)}{(\sin A)(1 + \cos A)}$$

$$\Rightarrow \frac{\cos A + \cos^2 A + \sin^2 A}{\sin A(1 + \cos A)} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$\Rightarrow \frac{\cos A + 1}{\sin A(1 + \cos A)}$$

$$\Rightarrow \frac{1}{\sin A} [\because \operatorname{cosec} \theta = \frac{1}{\sin \theta}]$$

$$\Rightarrow \operatorname{cosec} A$$

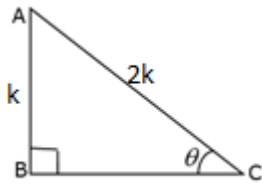
$$\Rightarrow 2$$

43 A. Question

If $\sin B = \frac{1}{2}$, prove that : $3 \cos B - 4 \cos^3 B = 0$

Answer

Given: $\sin B = \frac{1}{2}$



We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

$$\text{Or } \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin B = \frac{1}{2} \Rightarrow \frac{P}{H} = \frac{1}{2} \Rightarrow \frac{AB}{AC} = \frac{1}{2}$$

Let,

Perpendicular = AB = k

and Hypotenuse = AC = 2k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

In right angled ΔABC , we have

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (k)^2 + (BC)^2 = (2k)^2$$

$$\Rightarrow k^2 + (BC)^2 = 4k^2$$

$$\Rightarrow (BC)^2 = 4k^2 - k^2$$

$$\Rightarrow (BC)^2 = 3k^2$$

$$\Rightarrow BC = \sqrt{3k^2}$$

$$\Rightarrow BC = k\sqrt{3}$$

So, $BC = k\sqrt{3}$

Now, we have to find the value of $\cos B$

We know that,

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}}$$

Side adjacent to angle B or base = BC = $k\sqrt{3}$

Hypotenuse = AC = $2k$

$$\text{So, } \cos B = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$$

Now, LHS = $3 \cos B - 4 \cos^3 B$

$$\Rightarrow 3 \left(\frac{\sqrt{3}}{2} \right) - 4 \left(\frac{\sqrt{3}}{2} \right)^3$$

$$\Rightarrow \frac{3\sqrt{3}}{2} - 4 \left(\frac{3\sqrt{3}}{8} \right)$$

$$\Rightarrow \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$$

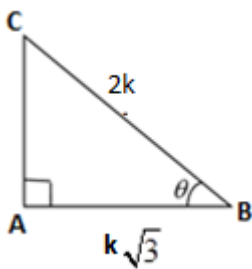
=RHS

Hence Proved

43 B. Question

If $\cos \theta = \frac{\sqrt{3}}{2}$, prove that: $3 \sin \theta - 4 \sin^3 \theta = 1$.

Answer



We know that,

$$\cos \theta = \frac{\text{Base}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\sqrt{3}}{2} \Rightarrow \frac{B}{H} = \frac{\sqrt{3}}{2}$$

Let $AB = k\sqrt{3}$ and $BC = 2k$

In right angled ΔABC , we have

$$B^2 + P^2 = H^2$$

$$\Rightarrow (k\sqrt{3})^2 + P^2 = (2k)^2$$

$$\Rightarrow P^2 + 3k^2 = 4k^2$$

$$\Rightarrow P^2 = 4k^2 - 3k^2$$

$$\Rightarrow P^2 = k^2$$

$$\Rightarrow P = \sqrt{k^2}$$

$$\Rightarrow P = \pm k$$

$$\Rightarrow P = k \text{ [taking positive square root since, side cannot be negative]}$$

Now,

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

$$\text{Or } \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\therefore \sin \theta = \frac{P}{H} = \frac{k}{2k} = \frac{1}{2}$$

$$\text{Now, LHS} = 3\sin \theta - 4\sin^3 \theta$$

$$= 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$$

$$\Rightarrow \frac{3}{2} - \frac{4}{8}$$

$$\Rightarrow \frac{3}{2} - \frac{1}{2}$$

$$\Rightarrow \frac{3-1}{2}$$

$$\Rightarrow \frac{2}{2}$$

$$\Rightarrow 1 = \text{RHS}$$

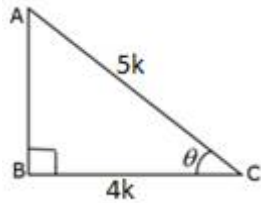
Hence Proved

43 C. Question

If $\sec \theta = \frac{5}{4}$, prove that : $\frac{\tan \theta}{1 + \tan^2 \theta} = \frac{\sin \theta}{\sec \theta}$

Answer

Given: $\sec \theta = \frac{5}{4}$



We know that,

$$\sec \theta = \frac{\text{hypotenuse}}{\text{base}}$$

$$\sec \theta = \frac{5}{4} \Rightarrow \frac{H}{B} = \frac{5}{4} \Rightarrow \frac{AC}{BC} = \frac{5}{4}$$

Let,

$$BC = 4k \text{ and } AC = 5k$$

where, k is any positive integer.

In right angled ΔABC , we have

$$(AB)^2 + (BC)^2 = (AC)^2 \text{ [by using Pythagoras theorem]}$$

$$\Rightarrow (AB)^2 + (4k)^2 = (5k)^2$$

$$\Rightarrow (AB)^2 + 16k^2 = 25k^2$$

$$\Rightarrow (AB)^2 = 25k^2 - 16k^2$$

$$\Rightarrow (AB)^2 = 9k^2$$

$$\Rightarrow AB = \sqrt{9k^2}$$

$$\Rightarrow AB = \pm 3k \text{ [taking positive square root since, side cannot be negative]}$$

Now, we have to find the value of other trigonometric ratios.

We, know that

$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

$$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$= \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$= \frac{AB}{BC} = \frac{3k}{4k} = \frac{3}{4}$$

$$\text{Now, LHS} = \frac{\tan \theta}{1 + \tan^2 \theta}$$

$$= \frac{\left(\frac{3}{4}\right)}{1 + \left(\frac{3}{4}\right)^2}$$

$$= \frac{\left(\frac{3}{4}\right)}{1 + \left(\frac{9}{16}\right)}$$

$$= \frac{\left(\frac{3}{4}\right)}{\left(\frac{16 + 9}{16}\right)}$$

$$= \frac{\left(\frac{3}{4}\right)}{\left(\frac{25}{16}\right)}$$

$$= \frac{3}{4} \times \frac{16}{25}$$

$$= \frac{12}{25}$$

$$\text{Now, RHS} = \frac{\sin \theta}{\sec \theta}$$

$$= \frac{\left(\frac{3}{5}\right)}{\left(\frac{5}{4}\right)}$$

$$= \frac{3}{5} \times \frac{4}{5}$$

$$= \frac{12}{25}$$

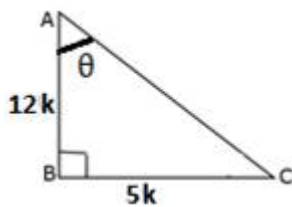
$$\therefore \text{LHS} = \text{RHS}$$

Hence Proved

43 D. Question

$$\cot B = \frac{12}{5}, \text{ prove that : } \tan^2 B - \sin^2 B = \sin^4 B \sec^2 B.$$

Answer



We know that,

$$\cot \theta = \frac{\text{side adjacent to angle } \theta}{\text{side opposite to angle } \theta}$$

$$\text{Or } \cot \theta = \frac{\text{base}}{\text{perpendicular}}$$

$$\cot B = \frac{12}{5} \Rightarrow \frac{B}{P} = \frac{12}{5} \Rightarrow \frac{AB}{BC} = \frac{12}{5}$$

Let,

Side adjacent to angle B = AB = 12k

Side opposite to angle B = BC = 5k

where, k is any positive integer

Firstly we have to find the value of AC.

So, we can find the value of AC with the help of Pythagoras theorem

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (12k)^2 + (5k)^2 = (AC)^2$$

$$\Rightarrow (AC)^2 = 144 k^2 + 25 k^2$$

$$\Rightarrow (AC)^2 = 169 k^2$$

$$\Rightarrow AC = \sqrt{169 k^2}$$

$$\Rightarrow AC = \pm 13k$$

But side AC can't be negative. So, $AC = 13k$

Now, we will find the $\sin \theta$

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

Side opposite to angle B = BC = 5k

and Hypotenuse = AC = 13k

$$\text{So, } \sin B = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

Now, we know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle B = AB = 12k

Hypotenuse = AC = 13k

$$\text{So, } \cos B = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

$$\tan B = \frac{P}{B} = \frac{BC}{AB} = \frac{5}{12}$$

$$\sec B = \frac{1}{\cos B} = \frac{1}{\frac{12}{13}} = \frac{13}{12}$$

Now, LHS = $\tan^2 B - \sin^2 B$

$$= \left(\frac{5}{12}\right)^2 - \left(\frac{5}{13}\right)^2$$

$$= \frac{25}{144} - \frac{25}{169}$$

$$= \frac{4225 - 3600}{144 \times 169}$$

$$= \frac{625}{144 \times 169}$$

$$= \frac{625}{24336}$$

Now, RHS = $\sin^4 B \sec^2 B$

$$= \left(\frac{5}{13}\right)^4 \left(\frac{13}{12}\right)^2$$

$$= \left(\frac{5}{13}\right)^2 \left(\frac{5}{13}\right)^2 \left(\frac{13}{12}\right)^2$$

$$= \frac{625}{144 \times 169}$$

$$= \frac{625}{24336}$$

Now, LHS = RHS

Hence Proved

44. Question

If $\cos \theta = \frac{q}{\sqrt{p^2 + q^2}}$, prove that $\left(\frac{\sqrt{p^2 + q^2}}{p} + \frac{q}{p} \right)^2 = \frac{\sqrt{p^2 + q^2} + q}{\sqrt{p^2 + q^2} - q}$

Answer

$$\text{Given: } \cos \theta = \frac{q}{\sqrt{p^2 + q^2}}$$

Now, squaring both the sides, we get

$$= \cos^2 \theta = \frac{q^2}{p^2 + q^2}$$

$$\Rightarrow p^2 + q^2 = \frac{q^2}{\cos^2 \theta}$$

$$\Rightarrow p^2 = \frac{q^2}{\cos^2 \theta} - q^2$$

$$\Rightarrow p^2 = \frac{q^2 - q^2 \cos^2 \theta}{\cos^2 \theta}$$

$$\Rightarrow p^2 = \frac{q^2(1 - \cos^2 \theta)}{\cos^2 \theta}$$

$$\Rightarrow p^2 = \frac{q^2 \sin^2 \theta}{\cos^2 \theta}$$

$$\Rightarrow p^2 = q^2 \tan^2 \theta \dots (1)$$

$$\text{Now, solving LHS} = \left(\frac{\sqrt{p^2 + q^2}}{p} + \frac{q}{p} \right)^2$$

Putting the value of p^2 in the above equation, we get

$$= \left(\frac{\sqrt{q^2 \tan^2 \theta + q^2}}{p} + \frac{q}{p} \right)^2$$

$$\Rightarrow \left(\frac{\sqrt{q^2 (\tan^2 \theta + 1)}}{p} + \frac{q}{p} \right)^2$$

$$\Rightarrow \left(\frac{\sqrt{q^2 \sec^2 \theta}}{p} + \frac{q}{p} \right)^2 \quad [\because 1 + \tan^2 \theta = \sec^2 \theta]$$

$$\Rightarrow \left(\frac{q \sec \theta}{p} + \frac{q}{p} \right)^2$$

$$\Rightarrow \frac{q^2 (\sec \theta + 1)^2}{p^2}$$

$$\Rightarrow \frac{q^2 (\sec \theta + 1)^2}{q^2 \tan^2 \theta} \quad (\text{from Eq. (1)})$$

$$\Rightarrow \frac{(\sec \theta + 1)^2}{(\sec^2 \theta - 1)}$$

$$\Rightarrow \frac{(\sec \theta + 1)(\sec \theta + 1)}{(\sec \theta - 1)(\sec \theta + 1)} \quad [\because (a + b)(a - b) = (a^2 - b^2)]$$

$$\Rightarrow \frac{\sec \theta + 1}{\sec \theta - 1}$$

Now, we solve the RHS

$$= \frac{\sqrt{p^2 + q^2} + q}{\sqrt{p^2 + q^2} - q}$$

$$= \frac{\sqrt{q^2 \tan^2 \theta + q^2} + q}{\sqrt{q^2 \tan^2 \theta + q^2} - q}$$

$$\begin{aligned}
&= \frac{\sqrt{q^2(\tan^2\theta + 1)} + q}{\sqrt{q^2(\tan^2\theta + 1)} - q} \\
&= \frac{\sqrt{q^2\sec^2\theta} + q}{\sqrt{q^2\sec^2\theta} - q} [\because 1 + \tan^2\theta = \sec^2\theta] \\
&= \frac{q\sec\theta + q}{q\sec\theta - q} \\
&\Rightarrow \frac{\sec\theta + 1}{\sec\theta - 1}
\end{aligned}$$

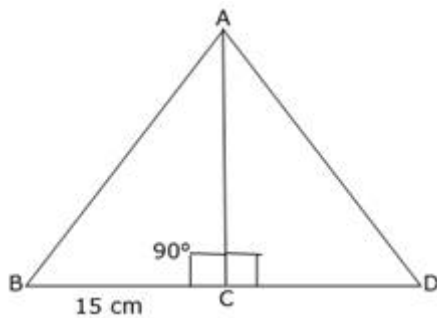
$\therefore \text{LHS} = \text{RHS}$

Hence Proved

45. Question

In the given figure, $BC = 15 \text{ cm}$ and $\sin B = 4/5$, show that

$$\tan^2 B - \frac{1}{\cos^2 B} = -1$$



Answer

Given: $BC = 15 \text{ cm}$ and $\sin B = \frac{4}{5}$

We know that,

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

$$\text{Or } \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin B = \frac{4}{5} \Rightarrow \frac{P}{H} = \frac{4}{5} \Rightarrow \frac{AC}{AB} = \frac{4}{5}$$

Let,

Side opposite to angle $B = 4k$

and Hypotenuse = 5k

where, k is any positive integer

So, by Pythagoras theorem, we can find the third side of a triangle

$$\Rightarrow (AC)^2 + (BC)^2 = (AB)^2$$

$$\Rightarrow (4k)^2 + (BC)^2 = (5)^2$$

$$\Rightarrow 16k^2 + (BC)^2 = 25k^2$$

$$\Rightarrow (BC)^2 = 25 k^2 - 16 k^2$$

$$\Rightarrow (BC)^2 = 9 k^2$$

$$\Rightarrow BC = \sqrt{9 k^2}$$

$$\Rightarrow BC = \pm 3k$$

But side BC can't be negative. So, $BC = 3k$

Now, we have to find the value of $\cos B$ and $\tan B$

We know that,

$$\cos \theta = \frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$$

Side adjacent to angle B = $BC = 3k$

Hypotenuse = $AB = 5k$

$$\text{So, } \cos B = \frac{3k}{5k} = \frac{3}{5}$$

Now, $\tan B$

We know that,

$$\tan \theta = \frac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$$

side opposite to angle B = $AC = 4k$

Side adjacent to angle B = $BC = 3k$

$$\text{So, } \tan B = \frac{4k}{3k} = \frac{4}{3}$$

$$\text{Now, } \tan^2 B = \frac{1}{\cos^2 B}$$

$$= \left(\frac{4}{3}\right)^2 - \left(\frac{1}{\frac{3}{5}}\right)^2$$

$$= \frac{16}{9} - \frac{25}{9}$$

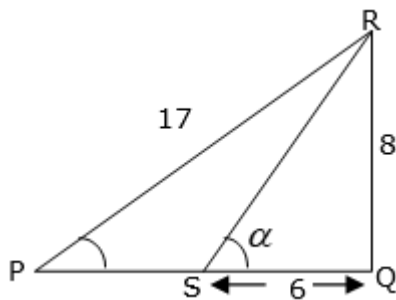
$$= \frac{-9}{9}$$

$$= -1 = \text{RHS}$$

Hence Proved

46. Question

In the given figure, find $3 \tan \theta - 2 \sin \alpha + 4 \cos \alpha$.



Answer

First of all, we find the value of RS

In right angled ΔRQS , we have

$$(RQ)^2 + (QS)^2 = (RS)^2$$

$$\Rightarrow (8)^2 + (6)^2 = (RS)^2$$

$$\Rightarrow 64 + 36 = (RS)^2$$

$$\Rightarrow RS = \sqrt{100}$$

$$\Rightarrow RS = \pm 10 \text{ [taking positive square root, since side cannot be negative]}$$

$$\Rightarrow RS = 10$$

$$\therefore \sin \alpha = \frac{P}{H} = \frac{8}{10} = \frac{4}{5}$$

$$\cos \alpha = \frac{B}{H} = \frac{6}{10} = \frac{3}{5}$$

Now, we find the value of QP

In right angled ΔRQP

$$(RQ)^2 + (QP)^2 = (RP)^2$$

$$\Rightarrow (8)^2 + (QP)^2 = (17)^2$$

$$\Rightarrow 64 + (QP)^2 = 289$$

$$\Rightarrow (QP)^2 = 289 - 64$$

$$\Rightarrow (QP)^2 = 225$$

$$\Rightarrow QP = \sqrt{225}$$

$$\Rightarrow QP = \pm 15 \text{ [taking positive square root, since side cannot be negative]}$$

$$\Rightarrow QP = 15$$

$$\tan \theta = \frac{P}{B} = \frac{8}{15}$$

Now, $3 \tan \theta - 2 \sin \alpha + 4 \cos \alpha$

$$\Rightarrow 3 \left(\frac{8}{15} \right) - 2 \left(\frac{4}{5} \right) + 4 \left(\frac{3}{5} \right)$$

$$\Rightarrow \frac{24}{15} - \frac{8}{5} + \frac{12}{5}$$

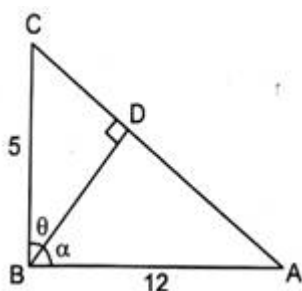
$$\Rightarrow \frac{24 - 24 + 36}{15}$$

$$\Rightarrow \frac{36}{15}$$

$$\Rightarrow \frac{12}{5}$$

47. Question

In the given figure ΔABC is right angled at B and BD is perpendicular to AC. Find (i) $\cos \theta$, (ii) $\cot \alpha$.



Answer

Firstly, we find the value of AC

In right angled ΔABC

$$(AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (12)^2 + (5)^2 = (AC)^2$$

$$\Rightarrow 144 + 25 = (AC)^2$$

$$\Rightarrow (AC)^2 = 169$$

$$\Rightarrow AC = \sqrt{169}$$

$$\Rightarrow AC = \pm 13$$

$$\Rightarrow AC = 13 \text{ [taking positive square root since, side cannot be negative]}$$

$$(i) \cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{12}{13}$$

$$(ii) \cot \alpha = \frac{\text{Base}}{\text{Perpendicular}} = \frac{12}{5}$$

48. Question

If $5 \sin^2 \theta + \cos^2 \theta = 2$, find the value of $\sin \theta$.

Answer

$$\text{Given: } 5 \sin^2 \theta + \cos^2 \theta = 2$$

$$\Rightarrow 5 \sin^2 \theta + (1 - \sin^2 \theta) = 2 \text{ [}\because \sin^2 \theta + \cos^2 \theta = 1\text{]}$$

$$\Rightarrow 5 \sin^2 \theta + 1 - \sin^2 \theta = 2$$

$$\Rightarrow 4 \sin^2 \theta = 2 - 1$$

$$\Rightarrow 4 \sin^2 \theta = 1$$

$$\Rightarrow \sin^2 \theta = \frac{1}{4}$$

$$\Rightarrow \sin \theta = \sqrt{\frac{1}{4}}$$

$$\Rightarrow \sin \theta = \pm \frac{1}{2}$$

49. Question

If $7 \sin^2 \theta + 3 \cos^2 \theta = 4$, find the value of $\tan \theta$.

Answer

$$\text{Given : } 7 \sin^2 \theta + 3 \cos^2 \theta = 4$$

$$\Rightarrow 7 \sin^2 \theta + 3(1 - \sin^2 \theta) = 4 \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$\Rightarrow 7 \sin^2 \theta + 3 - 3 \sin^2 \theta = 4$$

$$\Rightarrow 4 \sin^2 \theta = 4 - 3$$

$$\Rightarrow 4 \sin^2 \theta = 1$$

$$\Rightarrow \sin^2 \theta = \frac{1}{4}$$

$$\Rightarrow \sin \theta = \sqrt{\frac{1}{4}}$$

$$\Rightarrow \sin \theta = \pm \frac{1}{2}$$

Put the value of $\sin^2 \theta = \frac{1}{4}$ in given equation, we get

$$\Rightarrow 7 \left(\frac{1}{2}\right)^2 + 3 \cos^2 \theta = 4$$

$$\Rightarrow \frac{7}{4} + 3 \cos^2 \theta = 4$$

$$\Rightarrow 3 \cos^2 \theta = 4 - \frac{7}{4}$$

$$\Rightarrow 3 \cos^2 \theta = \frac{16-7}{4}$$

$$\Rightarrow 3 \cos^2 \theta = \frac{9}{4}$$

$$\Rightarrow \cos^2 \theta = \frac{3}{4}$$

$$\Rightarrow \cos \theta = \sqrt{\frac{3}{4}}$$

$$\Rightarrow \cos \theta = \pm \frac{\sqrt{3}}{2}$$

Now, we know that $\tan \theta = \frac{\sin \theta}{\cos \theta}$

$$\Rightarrow \tan \theta = \frac{\left(\pm \frac{1}{2}\right)}{\left(\pm \frac{\sqrt{3}}{2}\right)}$$

$$\Rightarrow \tan \theta = \pm \frac{1}{\sqrt{3}}$$

50. Question

If $4 \cos \theta + 3 \sin \theta = 5$, find the value of $\tan \theta$.

Answer

Given : $4 \cos \theta + 3 \sin \theta = 5$

Squaring both the sides, we get

$$\Rightarrow (4 \cos \theta + 3 \sin \theta)^2 = 25$$

$$\Rightarrow 16 \cos^2 \theta + 9 \sin^2 \theta + 2(4 \cos \theta)(3 \sin \theta) = 25 \quad [\because (a + b)^2 = a^2 + b^2 + 2ab]$$

$$\Rightarrow 16 \cos^2 \theta + 9 \sin^2 \theta + 24 \cos \theta \sin \theta = 25$$

Divide by $\cos^2 \theta$, we get

$$\Rightarrow \frac{16 \cos^2 \theta}{\cos^2 \theta} + \frac{9 \sin^2 \theta}{\cos^2 \theta} + \frac{24 \cos \theta \sin \theta}{\cos^2 \theta} = \frac{25}{\cos^2 \theta}$$

$$\Rightarrow 16 + 9 \tan^2 \theta + 24 \tan \theta = 25 \sec^2 \theta$$

$$\Rightarrow 16 + 9 \tan^2 \theta + 24 \tan \theta = 25(1 + \tan^2 \theta) \quad [\because 1 + \tan^2 \theta = \sec^2 \theta]$$

$$\Rightarrow 16 + 9 \tan^2 \theta + 24 \tan \theta = 25 + 25 \tan^2 \theta$$

$$\Rightarrow 16 \tan^2 \theta - 24 \tan \theta + 9 = 0$$

$$\Rightarrow 16 \tan^2 \theta - 12 \tan \theta - 12 \tan \theta + 9 = 0$$

$$\Rightarrow 4 \tan \theta (4 \tan \theta - 3) - 3(4 \tan \theta - 3) = 0$$

$$\Rightarrow (4 \tan \theta - 3)^2 = 0$$

$$\Rightarrow \tan \theta = \frac{3}{4}$$

51. Question

If $7 \sin A + 24 \cos A = 25$, find the value of $\tan A$.

Answer

Given : $7 \sin A + 24 \cos A = 25$

Squaring both the sides, we get

$$\Rightarrow (7 \sin A + 24 \cos A)^2 = 625$$

$$\Rightarrow 49 \sin^2 A + 576 \cos^2 A + 2(7 \sin A)(24 \cos A) = 625 \quad [\because (a + b)^2 = a^2 + b^2 + 2ab]$$

$$\Rightarrow 49 \sin^2 A + 576 \cos^2 A + 336 \cos A \sin A = 625$$

Divide by $\cos^2 \theta$, we get

$$\Rightarrow \frac{49 \sin^2 A}{\cos^2 A} + \frac{576 \cos^2 A}{\cos^2 A} + \frac{336 \cos A \sin A}{\cos^2 A} = \frac{625}{\cos^2 A}$$

$$\Rightarrow 49 \tan^2 A + 576 + 336 \tan A = 625 \sec^2 A$$

$$\Rightarrow 49 \tan^2 A + 576 + 336 \tan A = 625(1 + \tan^2 A) \quad [\because 1 + \tan^2 \theta = \sec^2 \theta]$$

$$\Rightarrow 49 \tan^2 A + 576 + 336 \tan A = 625 + 625 \tan^2 A$$

$$\Rightarrow 576 \tan^2 A - 336 \tan A + 49 = 0$$

$$\Rightarrow 576 \tan^2 A - 168 \tan A - 168 \tan A + 49 = 0$$

$$\Rightarrow 24 \tan \theta (24 \tan A - 7) - 7(24 \tan A - 7) = 0$$

$$\Rightarrow (24 \tan A - 7)^2 = 0$$

$$\Rightarrow \tan A = \frac{7}{24}$$

52. Question

If $9 \sin \theta + 40 \cos \theta = 41$, find the value of $\cos \theta$ and $\operatorname{cosec} \theta$

Answer

Given: $9 \sin \theta + 40 \cos \theta = 41$

$$\Rightarrow 9 \sin \theta = 41 - 40 \cos \theta \quad \dots(i)$$

Squaring both sides, we get

$$\Rightarrow 81\sin^2 \theta = 1681 + 1600 \cos^2 \theta - 2(41)(40\cos \theta) [\because (a - b)^2 = a^2 + b^2 - 2ab]$$

$$\Rightarrow 81(1 - \cos^2 \theta) = 1681 + 1600 \cos^2 \theta - 3280\cos \theta$$

$$\Rightarrow 81 - 81\cos^2 \theta = 1681 + 1600\cos^2 \theta - 3280 \cos \theta$$

$$\Rightarrow 1681\cos^2 \theta - 3280\cos \theta + 1600 = 0$$

$$\Rightarrow (41)^2 \cos^2 \theta - 2(41)(40\cos \theta) + (40)^2 = 0$$

$$\Rightarrow (41\cos \theta - 40)^2 = 0$$

$$\Rightarrow \cos \theta = \frac{40}{41}$$

Now, putting the value of $\cos \theta$ in Eq. (i), we get

$$\Rightarrow 9\sin \theta = 41 - 40 \left(\frac{40}{41} \right)$$

$$\Rightarrow 9\sin \theta = \left(\frac{1681 - 1600}{41} \right)$$

$$\Rightarrow \sin \theta = \left(\frac{81}{41 \times 9} \right)$$

$$\Rightarrow \frac{1}{\operatorname{cosec} \theta} = \left(\frac{9}{41} \right)$$

$$\Rightarrow \operatorname{cosec} \theta = \frac{41}{9}$$

53. Question

If $\tan A + \sec A = 3$, find the value of $\sin A$.

Answer

$$\tan A + \sec A = 3$$

$$\Rightarrow \tan A = 3 - \sec A$$

Squaring both the sides, we get

$$\Rightarrow \tan^2 A = (3 - \sec A)^2$$

$$\Rightarrow \tan^2 A = 9 + \sec^2 A - 6\sec A$$

$$\Rightarrow \sec^2 A - 1 = 9 + \sec^2 A - 6\sec A [\because 1 + \tan^2 A = \sec^2 A]$$

$$\Rightarrow -1 - 9 = -6\sec A$$

$$\Rightarrow -10 = -6\sec A$$

$$\Rightarrow \sec A = \frac{10}{6}$$

$$\Rightarrow \frac{1}{\cos A} = \frac{5}{3} \left[\because \sec A = \frac{1}{\cos A} \right]$$

$$\Rightarrow \cos A = \frac{3}{5}$$

$$\text{Now, } \tan A + \sec A = 3$$

$$\Rightarrow \frac{\sin A}{\cos A} + \frac{1}{\cos A} = 3 \left[\because \tan A = \frac{\sin A}{\cos A} \right]$$

$$\Rightarrow \frac{\sin A}{\cos A} = \frac{3\cos A - 1}{\cos A}$$

$$\Rightarrow \sin A = 3\cos A - 1$$

$$\Rightarrow \sin A = 3\left(\frac{3}{5}\right) - 1$$

$$\Rightarrow \sin A = \left(\frac{9-5}{5}\right)$$

$$\Rightarrow \sin A = \left(\frac{4}{5}\right)$$

54. Question

If $\operatorname{cosec} A + \cot A = 5$, find the value of $\cos A$.

Answer

$$\operatorname{cosec} A + \cot A = 5$$

$$\Rightarrow \cot A = 5 - \operatorname{cosec} A$$

Squaring both the sides, we get

$$\Rightarrow \cot^2 A = (5 - \operatorname{cosec} A)^2$$

$$\Rightarrow \cot^2 A = 25 + \operatorname{cosec}^2 A - 10\operatorname{cosec} A$$

$$\Rightarrow \operatorname{cosec}^2 A - 1 = 25 + \operatorname{cosec}^2 A - 10\operatorname{cosec} A \left[\because 1 + \cot^2 A = \operatorname{cosec}^2 A \right]$$

$$\Rightarrow -1 - 25 = -10\operatorname{cosec} A$$

$$\Rightarrow -26 = -10 \operatorname{cosec} A$$

$$\Rightarrow \operatorname{cosec} A = \frac{26}{10}$$

$$\Rightarrow \frac{1}{\sin A} = \frac{13}{5} \left[\because \operatorname{cosec} A = \frac{1}{\sin A} \right]$$

$$\Rightarrow \sin A = \frac{5}{13}$$

$$\text{Now, } \operatorname{cosec} A + \cot A = 5$$

$$\Rightarrow \frac{1}{\sin A} + \frac{\cos A}{\sin A} = 5 \left[\because \cot A = \frac{\cos A}{\sin A} \right]$$

$$\Rightarrow \frac{13}{5} + \frac{\cos A}{\frac{5}{13}} = 5$$

$$\Rightarrow \frac{13}{5} + \frac{13 \cos A}{5} = 5$$

$$\Rightarrow \frac{13 \cos A}{5} = 5 - \frac{13}{5}$$

$$\Rightarrow \frac{13 \cos A}{5} = \frac{25 - 13}{5}$$

$$\Rightarrow \cos A = \frac{12}{13}$$

55. Question

$$\text{If } \tan \theta + \sec \theta = x, \text{ show that } \sin \theta = \frac{x^2 - 1}{x^2 + 1}$$

Answer

$$\tan \theta + \sec \theta = x$$

$$\Rightarrow \tan \theta = x - \sec \theta$$

Squaring both sides, we get

$$\Rightarrow \tan^2 \theta = (x - \sec \theta)^2$$

$$\Rightarrow \tan^2 \theta = x^2 + \sec^2 \theta - 2x \sec \theta$$

$$\Rightarrow \sec^2 \theta - 1 = x^2 + \sec^2 \theta - 2x \sec \theta \left[\because 1 + \tan^2 A = \sec^2 A \right]$$

$$\Rightarrow -1 - x^2 = -2x \sec \theta$$

$$\Rightarrow \sec \theta = \frac{1 + x^2}{2x}$$

Now,

$$\tan \theta = x - \sec \theta$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = x - \sec \theta$$

$$\Rightarrow \sin \theta \left(\frac{1 + x^2}{2x} \right) = x - \left(\frac{1 + x^2}{2x} \right)$$

$$\Rightarrow \sin \theta \left(\frac{1 + x^2}{2x} \right) = \left(\frac{2x^2 - 1 + x^2}{2x} \right)$$

$$\Rightarrow \sin \theta \left(\frac{1 + x^2}{2x} \right) = \left(\frac{x^2 - 1}{2x} \right)$$

$$\Rightarrow \sin \theta = \frac{x^2 - 1}{x^2 + 1} = \text{RHS}$$

Hence Proved

56. Question

If $\cos \theta + \sin \theta = 1$, prove that $\cos \theta - \sin \theta = \pm 1$

Answer

Using the formula,

$$(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$$

$$\Rightarrow (\cos \theta + \sin \theta)^2 + (\cos \theta - \sin \theta)^2 = 2(\cos^2 \theta + \sin^2 \theta)$$

$$\Rightarrow 1 + (\cos \theta - \sin \theta)^2 = 2(1)$$

$$\Rightarrow (\cos \theta - \sin \theta)^2 = 2 - 1$$

$$\Rightarrow (\cos \theta - \sin \theta)^2 = 1$$

$$\Rightarrow (\cos \theta - \sin \theta) = \sqrt{1}$$

$$\Rightarrow (\cos \theta - \sin \theta) = \pm 1$$

Exercise 4.2

1. Question

Find the value of the following :

(i) $\sin 30^\circ + \cos 60^\circ$

(ii) $\sin^2 45^\circ + \cos^2 45^\circ$

(iii) $\sin 30^\circ + \cos 60^\circ - \tan 45^\circ$

(iv) $\sqrt{1 + \tan^2 60^\circ}$

(v) $\tan 60^\circ \times \cos 30^\circ$

Answer

(i) $\sin 30^\circ + \cos 60^\circ$

We know that,

$$\sin(30^\circ) = \frac{1}{2}$$

$$\cos(60^\circ) = \frac{1}{2}$$

So, $\sin(30^\circ) + \cos(60^\circ)$

$$= \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)$$

$$= 1$$

(ii) $\sin^2 45^\circ + \cos^2 45^\circ$

We know that,

$$\sin(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

So, $\sin^2 45^\circ + \cos^2 45^\circ$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)$$

$$= 1$$

$$(iii) \sin 30^\circ + \cos 60^\circ - \tan 45^\circ$$

$$\sin(30^\circ) = \frac{1}{2}$$

$$\cos(60^\circ) = \frac{1}{2}$$

$$\tan(45^\circ) = 1$$

$$\text{So, } \sin 30^\circ + \cos 60^\circ - \tan 45^\circ$$

$$= \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) - 1$$

$$= \frac{1 + 1 - 2}{2}$$

$$= 0$$

$$(iv) \sqrt{1 + \tan^2 60^\circ}$$

We know that

$$\tan(60^\circ) = \sqrt{3}$$

So,

$$= \sqrt{1 + \tan^2 60^\circ}$$

$$= \sqrt{1 + (\sqrt{3})^2}$$

$$= \sqrt{1 + 3}$$

$$= \sqrt{4}$$

$$= 2$$

$$(v) \tan 60^\circ \times \cos 30^\circ$$

$$\tan(60^\circ) = \sqrt{3}$$

$$\cos(30^\circ) = \frac{\sqrt{3}}{2}$$

So,

$$\tan 60^\circ \times \cos 30^\circ$$

$$= \sqrt{3} \times \frac{\sqrt{3}}{2}$$

$$= \frac{3}{2}$$

2. Question

If $\theta = 45^\circ$, find the value of

$$(i) \tan^2 \theta + \frac{1}{\sin^2 \theta}$$

$$(ii) \cos^2 \theta - \sin^2 \theta$$

Answer

$$(i) \tan^2 \theta + \frac{1}{\sin^2 \theta}$$

Given $\theta = 45^\circ$

We know that,

$$\tan(45^\circ) = 1$$

$$\sin(45^\circ) = \frac{1}{\sqrt{2}}$$

$$= (1)^2 + \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= 1 + 2$$

$$= 3$$

$$(ii) \cos^2 \theta - \sin^2 \theta$$

Given $\theta = 45^\circ$

$$\sin(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

So, $\cos^2 45^\circ - \sin^2 45^\circ$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 0$$

3 A. Question

Find the numerical value of the following :

$$\sin 45^\circ \cdot \cos 45^\circ - \sin^2 30^\circ.$$

Answer

We know that,

$$\sin(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\sin(30^\circ) = \frac{1}{2}$$

Now, putting the values

$$= \left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)^2$$

$$= \left(\frac{1}{2}\right) - \left(\frac{1}{4}\right)$$

$$= \left(\frac{1}{4}\right)$$

3 B. Question

Find the numerical value of the following :

$$\frac{\tan 60^\circ}{\sin 60^\circ + \cos 60^\circ}$$

Answer

We know that,

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(60^\circ) = \frac{1}{2}$$

$$\tan(60^\circ) = \sqrt{3}$$

Now putting the values;

$$= \frac{\sqrt{3}}{\frac{\sqrt{3}}{2} + \frac{1}{2}}$$

$$= \frac{\sqrt{3}}{\frac{1 + \sqrt{3}}{2}}$$

$$= \sqrt{3} \times \frac{2}{1 + \sqrt{3}}$$

$$= \frac{2\sqrt{3}}{1 + \sqrt{3}}$$

Multiplying and dividing by the conjugate of $(1 + \sqrt{3})$

$$= \frac{2\sqrt{3}}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}}$$

$$= \frac{2\sqrt{3} - 6}{(1)^2 - (\sqrt{3})^2} [\because (a)^2 - (b)^2 = (a+b)(a-b)]$$

$$= \frac{2\sqrt{3} - 6}{-2}$$

Multiplying and dividing by (-2)

$$= 3 - \sqrt{3}$$

3 C. Question

Find the numerical value of the following :

$$\frac{\tan 60^\circ}{\sin 60^\circ + \cos 30^\circ}$$

Answer

We know that,

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\tan(60^\circ) = \sqrt{3}$$

Now putting the values;

$$= \frac{\sqrt{3}}{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}$$

$$= \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 1$$

3 D. Question

Find the numerical value of the following :

$$\frac{4}{\sin^2 60^\circ} + \frac{3}{\cos^2 60^\circ}$$

Answer

We know that

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(60^\circ) = \frac{1}{2}$$

Now putting the value, we get

$$= \frac{4}{\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{3}{\left(\frac{1}{2}\right)^2}$$

$$= 4 \times \left(\frac{2}{\sqrt{3}}\right)^2 + 3 \times (2)^2$$

$$= 4 \left(\frac{4}{3}\right) + 3 \times 4$$

$$= \frac{16}{3} + 12$$

$$= \frac{16 + 36}{3}$$

$$= \frac{52}{3}$$

3 E. Question

Find the numerical value of the following :

$$\sin^2 60^\circ - \cos^2 60^\circ$$

Answer

We know that,

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(60^\circ) = \frac{1}{2}$$

Now putting the value;

$$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$= \left(\frac{3}{4}\right) - \left(\frac{1}{4}\right)$$

$$= \frac{1}{2}$$

3 F. Question

Find the numerical value of the following :

$$4\sin^2 30^\circ + 3 \tan 30^\circ - 8 \sin 45^\circ \cos 45^\circ$$

Answer

We know that,

$$\sin (30^\circ) = \frac{1}{2}$$

$$\tan (30^\circ) = \frac{1}{\sqrt{3}}$$

$$\sin(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

Now putting the value, we get

$$= 4 \times \left(\frac{1}{2}\right)^2 + 3 \times \left(\frac{1}{\sqrt{3}}\right)^2 - 8 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$= 4 \times \frac{1}{4} + 3 \times \frac{1}{3} - 8 \times \frac{1}{2}$$

$$= 1 + 1 - 4$$

$$= -2$$

3 G. Question

Find the numerical value of the following :

$$2\sin^2 30^\circ - 3\cos^2 45^\circ + \tan^2 60^\circ$$

Answer

We know that,

$$\sin(30^\circ) = \frac{1}{2}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\tan(60^\circ) = \sqrt{3}$$

Now putting the value;

$$= 2 \times \left(\frac{1}{2}\right)^2 - 3 \times \left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2$$

$$= 2 \times \frac{1}{4} - 3 \times \frac{1}{2} + 3$$

$$= \frac{1}{2} - \frac{3}{2} + 3$$

$$= \frac{1 - 3 + 6}{2}$$

$$= \frac{4}{2}$$

$$= 2$$

3 H. Question

Find the numerical value of the following :

$$\sin 90^\circ + \cos 0^\circ + \sin 30^\circ + \cos 60^\circ$$

Answer

We know that,

$$\sin(90^\circ) = 1$$

$$\cos(0^\circ) = 1$$

$$\sin(30^\circ) = \frac{1}{2}$$

$$\cos(60^\circ) = \frac{1}{2}$$

Now putting the value;

$$= 1 + 1 + \frac{1}{2} + \frac{1}{2}$$

$$= \frac{2 + 2 + 1 + 1}{2}$$

$$= \frac{6}{2}$$

$$= 3$$

3 I. Question

Find the numerical value of the following :

$$\sin 90^\circ - \cos 0^\circ + \tan 0^\circ + \tan 45^\circ$$

Answer

We know that

$$\sin (90^\circ) = 1$$

$$\cos (0^\circ) = 1$$

$$\tan(0^\circ) = 0$$

$$\tan(45^\circ) = 1$$

Now putting the value, we get

$$= 1 - 1 + 0 + 1$$

$$= 1$$

3 J. Question

Find the numerical value of the following :

$$\cos^2 0^\circ + \tan^2 \frac{\pi}{4} + \sin^2 \frac{\pi}{4}, \text{ where } \pi = 180^\circ$$

Answer

We know that

$$\cos (0^\circ) = 1$$

$$\tan(45^\circ) = 1 \left[\frac{\pi}{4} = \frac{180^\circ}{4} = 45^\circ \right]$$

$$\sin(45^\circ) = \frac{1}{\sqrt{2}} \left[\frac{\pi}{4} = \frac{180^\circ}{4} = 45^\circ \right]$$

Now putting the values;

$$= (1)^2 + (1)^2 + \left(\frac{1}{\sqrt{2}} \right)^2$$

$$= 1 + 1 + \frac{1}{2}$$

$$= \frac{2 + 2 + 1}{2}$$

$$= \frac{5}{2}$$

3 K. Question

Find the numerical value of the following :

$$\frac{\cos 60^\circ}{\sin^2 45^\circ} - 3 \cot 45^\circ + 2 \sin 90^\circ$$

Answer

We know that,

$$\cos(60^\circ) = \frac{1}{2}$$

$$\sin(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\cot(45^\circ) = 1$$

$$\sin(90^\circ) = 1$$

Now putting the values, we get

$$= \frac{\frac{1}{2}}{\left(\frac{1}{\sqrt{2}} \right)^2} - 3(1) + 2(1)$$

$$= 1-3+2$$

$$=0$$

3 L. Question

Find the numerical value of the following :

$$\frac{4}{\tan^2 60^\circ} + \frac{1}{\cos^2 30^\circ} - \sin^2 45^\circ$$

Answer

We can write the above equation as:

$$= 4 \cot^2 60^\circ + \sec^2 30^\circ - \sin^2 45^\circ \dots (a) \left[\because \cos \theta = \frac{1}{\sec \theta} \text{ and } \tan \theta = \frac{1}{\cot \theta} \right]$$

$$\sin(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\cot(60^\circ) = \frac{1}{\sqrt{3}}$$

$$\sec(30^\circ) = \frac{2}{\sqrt{3}}$$

Now putting the values in (a);

$$= 4 \left(\frac{1}{\sqrt{3}} \right)^2 + \left(\frac{2}{\sqrt{3}} \right)^2 - \left(\frac{1}{\sqrt{2}} \right)^2$$

$$= 4 \times \frac{1}{3} + \frac{4}{3} - \frac{1}{2}$$

$$= \frac{8 + 8 - 3}{6}$$

$$= \frac{13}{6}$$

3 M. Question

Find the numerical value of the following :

$$\cos 60^\circ \cdot \cos 30^\circ - \sin 60^\circ \cdot \sin 30^\circ$$

Answer

We know that,

$$\cos(60^\circ) = \frac{1}{2}$$

$$\cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin(30^\circ) = \frac{1}{2}$$

Now putting the values, we get

$$= \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

$$= 0$$

3 N. Question

Find the numerical value of the following :

$$\frac{4(\sin^2 60^\circ - \cos^2 45^\circ)}{\tan^2 30^\circ + \cos^2 90^\circ}$$

Answer

We know that,

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\cos(90^\circ) = 0$$

Now putting the values;

$$= 4 \times \frac{\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2}{\left(\frac{1}{\sqrt{3}}\right)^2 - (0)^2}$$

$$= 4 \times \frac{\frac{3}{4} - \frac{1}{2}}{\frac{1}{3}}$$

$$= 4 \times \frac{1}{4} \times 3$$

$$= 3$$

4 A. Question

Evaluate the following :

$$\sin 30^\circ \cdot \cos 45^\circ + \cos 30^\circ \cdot \sin 45^\circ$$

Answer

We know that,

$$\sin(30^\circ) = \frac{1}{2}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin(45^\circ) = \frac{1}{\sqrt{2}}$$

Now putting the values, we get

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{1 + \sqrt{3}}{2\sqrt{2}}$$

4 B. Question

Evaluate the following :

$$\operatorname{cosec}^2 30^\circ \cdot \tan^2 45^\circ - \sec^2 60^\circ$$

Answer

We know that

$$\operatorname{cosec} (30^\circ) = 2$$

$$\tan(45^\circ) = 1$$

$$\sec (60^\circ) = 2$$

Now putting the values;

$$= (2)^2 \times (1)^2 - (2)^2$$

$$= 4 - 4$$

$$= 0$$

4 C. Question

Evaluate the following :

$$2\sin^2 30^\circ \cdot \tan 60^\circ - 3\cos^2 60^\circ \cdot \sec^2 30^\circ$$

Answer

We know that

$$\sin (30^\circ) = \frac{1}{2}$$

$$\tan (60^\circ) = \sqrt{3}$$

$$\cos(60^\circ) = \frac{1}{2}$$

$$\sec (30^\circ) = \frac{2}{\sqrt{3}}$$

Now putting the values;

$$= 2 \times \left(\frac{1}{2}\right)^2 \times (\sqrt{3}) - \left(3 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{2}{\sqrt{3}}\right)^2\right)$$

$$= 2 \times \frac{1}{4} \times (\sqrt{3}) - \left(3 \times \frac{1}{4} \times \frac{4}{3} \right)$$

$$= \frac{\sqrt{3}}{2} - 1$$

$$= \frac{\sqrt{3} - 2}{2}$$

4 D. Question

Evaluate the following :

$$\tan 60^\circ \cdot \operatorname{cosec}^2 45^\circ + \sec^2 60^\circ \cdot \tan 45^\circ$$

Answer

We know that

$$\tan (60^\circ) = \sqrt{3}$$

$$\operatorname{cosec} (45^\circ) = \sqrt{2}$$

$$\sec (60^\circ) = 2$$

$$\tan(45^\circ) = 1$$

Now putting the values;

$$= (\sqrt{3}) \times (\sqrt{2})^2 + (2)^2 \times (1)$$

$$= 2\sqrt{3} + 4$$

$$= 2(\sqrt{3} + 2)$$

4 E. Question

Evaluate the following :

$$\tan 30^\circ \cdot \sec 45^\circ + \tan 60^\circ \cdot \sin 30^\circ$$

Answer

We know that

$$\tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\sec (45^\circ) = \sqrt{2}$$

$$\tan (60^\circ) = \sqrt{3}$$

$$\sec(30^\circ) = \frac{2}{\sqrt{3}}$$

Now putting the values, we get

$$= \frac{1}{\sqrt{3}} \times \sqrt{2} + \sqrt{3} \times \frac{2}{\sqrt{3}}$$

$$= \frac{\sqrt{2}}{\sqrt{3}} + 2$$

$$= 2 + \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 2 + \frac{\sqrt{6}}{3}$$

4 F. Question

Evaluate the following :

$$\cos 30^\circ \cdot \cos 45^\circ - \sin 30^\circ \cdot \sin 45^\circ$$

Answer

We know that

$$\cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\sin(30^\circ) = \frac{1}{2}$$

$$\sin(45^\circ) = \frac{1}{\sqrt{2}}$$

Now putting the values, we get

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

Multiplying and dividing by $(\sqrt{2})$, we get

$$= \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}(\sqrt{3}-1)}{4}$$

4 G. Question

Evaluate the following :

$$\frac{4}{3} \tan^2 30^\circ + \sin^2 60^\circ - 3 \cos^2 60^\circ + \frac{3}{4} \tan^2 60^\circ - 2 \tan^2 45^\circ$$

Answer

We know that

$$\tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(60^\circ) = \frac{1}{2}$$

$$\tan(60^\circ) = \sqrt{3}$$

$$\tan(45^\circ) = 1$$

Now putting the values;

$$= \left(\frac{4}{3} \times \left(\frac{1}{\sqrt{3}} \right)^2 \right) + \left[\left(\frac{\sqrt{3}}{2} \right)^2 \right] - \left[3 \times \left(\frac{1}{2} \right)^2 \right] + \left[\frac{3}{4} (\sqrt{3})^2 \right] - [2 \times (1)^2]$$

$$= \left[\frac{4}{3} \times \frac{1}{3} \right] + \left[\frac{3}{4} \right] - \left[3 \times \frac{1}{4} \right] + \left[\frac{3}{4} \times 3 \right] - [2 \times (1)]$$

$$= \frac{4}{9} + \frac{3}{4} - \frac{3}{4} + \frac{9}{4} - 2$$

$$= \frac{16 + 27 - 27 + 81 - 72}{36}$$

$$= \frac{25}{36}$$

4 H. Question

Evaluate the following :

$$\frac{\tan^2 60^\circ + 4\cos^2 45^\circ + 3\sec^2 30^\circ + 5\cos^2 90^\circ}{\operatorname{cosec} 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$$

Answer

We know that

$$\tan (60^\circ) = \sqrt{3}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\sec (30^\circ) = \frac{2}{\sqrt{3}}$$

$$\cos(90^\circ) = 0$$

$$\operatorname{cosec} (30^\circ) = 2$$

$$\sec (60^\circ) = 2$$

$$\cot (30^\circ) = \sqrt{3}$$

Now putting the values, we get

$$= \frac{(\sqrt{3})^2 + \left[4 \times \left(\frac{1}{\sqrt{2}}\right)^2\right] + \left[3 \times \left(\frac{2}{\sqrt{3}}\right)^2\right] + [5 \times (0)^2]}{(2) + (2) - (\sqrt{3})^2}$$

$$= \frac{(3) + \left[4 \times \frac{1}{2}\right] + \left[3 \times \frac{4}{3}\right] + [5 \times 0]}{2 + 2 - 3}$$

$$= \frac{(3) + [2] + [4] + [0]}{2 + 2 - 3}$$

$$= 9$$

4 I. Question

Evaluate the following :

$$\frac{5\sin^2 30^\circ + \cos^2 45^\circ - 4\tan^2 30^\circ}{2\sin 30^\circ \cdot \cos 30^\circ + \tan 45^\circ}$$

Answer

We know that

$$\sin(30^\circ) = \frac{1}{2}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\tan(45^\circ) = 1$$

Now putting the values, we get

$$= \frac{\left[5 \times \left(\frac{1}{2}\right)^2\right] + \left(\frac{1}{\sqrt{2}}\right)^2 - \left[4 \times \left(\frac{1}{\sqrt{3}}\right)^2\right]}{2 \left(\frac{1}{2}\right) \left(\frac{\sqrt{3}}{2}\right) + (1)}$$

$$= \frac{\left(\frac{5}{4}\right) + \left(\frac{1}{2}\right) - \left(\frac{4}{3}\right)}{\left(\frac{\sqrt{3}}{2}\right) + 1}$$

$$= \frac{\left(\frac{15 + 6 - 16}{12}\right)}{\left(\frac{\sqrt{3} + 2}{2}\right)}$$

$$= \frac{5}{12} \times \frac{2}{\sqrt{3} + 1}$$

$$= \frac{5}{6} \times \frac{1}{\sqrt{3} + 2}$$

5 A. Question

Prove the following :

$$\frac{(1 - \cos B)(1 + \cos B)}{(1 - \sin B)(1 + \sin B)} = \frac{1}{3} \text{ When } B = 30^\circ$$

Answer

Solving, L.H.S.

$$= \frac{(1)^2 - (\cos B)^2}{(1)^2 - (\sin B)^2} [(a)^2 - (b)^2 = (a+b)(a-b)]$$

$$= \frac{1 - \cos^2 B}{1 - \sin^2 B}$$

We know that,

$$\cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin(30^\circ) = \frac{1}{2}$$

Putting the values, we get

$$= \frac{1 - \left(\frac{\sqrt{3}}{2}\right)^2}{1 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{1 - \frac{3}{4}}{1 - \frac{1}{4}}$$

$$= \frac{4 - 3}{4 - 1}$$

$$= \frac{1}{3}$$

=R.H.S.

Hence Proved

5 B. Question

Prove the following :

$$\frac{(1 - \cos \alpha)(1 + \cos \alpha)}{(1 - \sin \alpha)(1 + \sin \alpha)} = 3 \text{ When } \alpha = 60^\circ$$

Answer

Solving, L.H.S.

$$= \frac{(1)^2 - (\cos \alpha)^2}{(1)^2 - (\sin \alpha)^2} [(a)^2 - (b)^2 = (a+b)(a-b)]$$

$$= \frac{1 - \cos^2 \alpha}{1 - \sin^2 \alpha}$$

We know that

$$\cos(60^\circ) = \frac{1}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

Putting the values, we get

$$= \frac{1 - \left(\frac{1}{2}\right)^2}{1 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1 - \frac{1}{4}}{1 - \frac{3}{4}}$$

$$= \frac{4 - 1}{4 - 3}$$

$$= 3 = \text{R.H.S.}$$

5 C. Question

Prove the following :

$$\cos(A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B \text{ if } A=B=60^\circ$$

Answer

Solving, L.H.S.

$$= \cos(60^\circ - 60^\circ) [\text{Putting the value } A=B=60^\circ]$$

$$= \cos(0^\circ)$$

$$= 1$$

Solving, R.H.S.

$$= \cos(60^\circ) \times \cos(60^\circ) + \sin(60^\circ) \times \sin(60^\circ) \text{ [Putting the value } A=B=60^\circ]$$

$$= \cos^2(60^\circ) + \sin^2(60^\circ)$$

We know that,

$$\cos(60^\circ) = \frac{1}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= \frac{1+3}{4}$$

$$= 1$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence Proved

5 D. Question

Prove the following :

$$4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) = 2$$

Answer

We know that,

$$\sin(30^\circ) = \frac{1}{2}$$

$$\cos(60^\circ) = \frac{1}{2}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\sin(90^\circ) = 1$$

Now solving, L.H.S.

$$= 4\{(\sin 30^\circ)^2\}^2 + \{(\cos 60^\circ)^2\}^2 - 3[(\cos 45^\circ)^2 - (\sin 90^\circ)^2]$$

Putting the values

$$= 4 \times \left[\left\{ \left(\frac{1}{2} \right)^2 \right\}^2 + \left\{ \left(\frac{1}{2} \right)^2 \right\}^2 \right] - 3 \left[\left(\frac{1}{\sqrt{2}} \right)^2 - 1 \right]$$

$$= 4 \times \left[\left\{ \frac{1}{4} \right\}^2 + \left\{ \frac{1}{4} \right\}^2 \right] - 3 \left[\frac{1}{2} - 1 \right]$$

$$= 4 \times \left[\frac{1}{16} + \frac{1}{16} \right] - 3 \left[-\frac{1}{2} \right]$$

$$= 4 \times \left[\frac{1}{8} \right] - 3 \left[-\frac{1}{2} \right]$$

$$= \left[\frac{1}{2} \right] + \left[\frac{3}{2} \right]$$

$$= \left[\frac{4}{2} \right]$$

$$= 2 = \text{R.H.S.}$$

Hence Proved

5 E. Question

Prove the following :

$$\sin 90^\circ = 2 \sin 45^\circ \cdot \cos 45^\circ$$

Answer

We know that,

$$\sin(90^\circ) = 1$$

$$\sin(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

Taking LHS = $\sin 90^\circ = 1$

Now, taking RHS

$$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$= 2 \times \frac{1}{2}$$

$$= 1$$

= R.H.S.

Hence Proved

5 F. Question

Prove the following :

$$\cos 60^\circ = 2\cos^2 30^\circ - 1 = 1 - 2\sin^2 30^\circ$$

Answer

We know that,

$$\cos(60^\circ) = \frac{1}{2}$$

$$\cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin(30^\circ) = \frac{1}{2}$$

$$\text{Taking LHS} = \cos 60^\circ = \frac{1}{2}$$

Now, solving RHS = $2\cos^2 30^\circ - 1$, we get

$$= 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1$$

$$= 2 \times \frac{3}{4} - 1$$

$$= \frac{3}{2} - 1$$

$$= \frac{3}{2} - 1$$

$$= \frac{1}{2}$$

= RHS

Now taking RHS = $1 - 2\sin^2 30^\circ$

$$= 1 - 2\left(\frac{1}{2}\right)^2$$

$$= 1 - \frac{1}{2}$$

$$= \frac{2 - 1}{2}$$

$$= \frac{1}{2}$$

= RHS

Hence, proved.

5 G. Question

Prove the following :

$$\cos 90^\circ = 1 - 2 \sin^2 45^\circ = 2 \cos^2 45^\circ - 1$$

Answer

We know that

$$\cos(90^\circ) = 0$$

$$\sin(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

taking LHS = $\cos 90^\circ = 0$

Now solving RHS $1 - 2\sin^2 45^\circ$

$$= 1 - 2 \left(\frac{1}{\sqrt{2}} \right)^2$$

$$= 1 - 2 \times \frac{1}{2}$$

$$= 1 - 1$$

$$= 0$$

$$= \text{RHS}$$

Now, solving $\text{RHS} = 2\cos^2 45^\circ - 1$, we get

$$= 1 - 2 \times \left(\frac{1}{\sqrt{2}} \right)^2$$

$$= 1 - 2 \times \frac{1}{2}$$

$$= 1 - 1$$

$$= 0$$

Hence, proved.

5 H. Question

Prove the following :

$$\sin 30^\circ \cdot \cos 60^\circ + \cos 30^\circ \cdot \sin 60^\circ = \sin 90^\circ$$

Answer

We know that

$$\sin(30^\circ) = \frac{1}{2}$$

$$\cos(60^\circ) = \frac{1}{2}$$

$$\cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin(90^\circ) = 1$$

Taking LHS =

$$= \left[\left(\frac{1}{2} \right) \times \left(\frac{1}{2} \right) \right] + \left[\left(\frac{\sqrt{3}}{2} \right) \times \left(\frac{\sqrt{3}}{2} \right) \right]$$

$$= \left[\left(\frac{1}{4} \right) \right] + \left[\left(\frac{3}{4} \right) \right]$$

$$= \left[\frac{1+3}{4} \right]$$

$$= 1$$

$$\text{Now, RHS} = \sin 90^\circ = 1$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence, proved.

5 I. Question

Prove the following :

$$\cos 60^\circ \cdot \cos 30^\circ - \sin 60^\circ \cdot \sin 30^\circ = \cos 90^\circ$$

Answer

We know that

$$\cos(60^\circ) = \frac{1}{2}$$

$$\cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin(30^\circ) = \frac{1}{2}$$

$$\cos(90^\circ) = 0$$

Taking LHS

$$= \left[\left(\frac{1}{2} \right) \left(\frac{\sqrt{3}}{2} \right) \right] - \left[\left(\frac{\sqrt{3}}{2} \right) \left(\frac{1}{2} \right) \right]$$

$$= \left[\left(\frac{\sqrt{3}}{4} \right) \right] - \left[\left(\frac{\sqrt{3}}{4} \right) \right]$$

$$= 0$$

$$\text{Now, RHS} = \cos 90^\circ = 0$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence, proved.

5 J. Question

Prove the following :

$$\cos 60^\circ = \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ}$$

Answer

We know that,

$$\cos(60^\circ) = \frac{1}{2}$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\text{Taking LHS} = \cos 60^\circ = \frac{1}{2}$$

Now, solving RHS

$$= \frac{1 - \left(\frac{1}{\sqrt{3}} \right)^2}{1 + \left(\frac{1}{\sqrt{3}} \right)^2}$$

$$= \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}}$$

$$\begin{aligned}
&= \frac{\frac{3-1}{3}}{\frac{3+1}{3}} \\
&= \frac{2}{4} \\
&= \frac{1}{2}
\end{aligned}$$

$\therefore \text{L.H.S.} = \text{R.H.S.}$

Hence, proved.

5 K. Question

Prove the following :

$$\frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \cdot \tan 30^\circ} = \tan 30^\circ$$

Answer

We know that

$$\tan(60^\circ) = \sqrt{3}$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}}$$

Taking LHS

$$\begin{aligned}
&= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + (\sqrt{3}) \times \left(\frac{1}{\sqrt{3}}\right)} \\
&= \frac{\frac{3-1}{\sqrt{3}}}{1+1} \\
&= \frac{\frac{2}{\sqrt{3}}}{2} \\
&= \frac{1}{\sqrt{3}}
\end{aligned}$$

$$\text{Now, RHS} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

∴ L.H.S. = R.H.S.

Hence, proved.

5 L. Question

Prove the following :

$$\frac{1 - \tan 30^\circ}{1 + \tan 30^\circ} = \frac{1 - \sin 60^\circ}{\cos 60^\circ}$$

Answer

$$\tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(60^\circ) = \frac{1}{2}$$

Taking LHS

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3} - 1}{\sqrt{3}}}{\frac{\sqrt{3} + 1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

Multiplying and Dividing, LHS by $(\sqrt{3} - 1)$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$= \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - (1)^2} [(a)^2 - (b)^2 = (a+b)(a-b)]$$

$$= \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{3 + 1 - 2\sqrt{3}}{3 - 1}$$

$$= \frac{4 - 2\sqrt{3}}{2}$$

Multiplying and Dividing, LHS by 2

$$= 2 - \sqrt{3}$$

Now, RHS

$$= \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= \frac{\frac{2 - \sqrt{3}}{2}}{\frac{1}{2}}$$

$$= 2 - \sqrt{3}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence, proved.

5 M. Question

Prove the following :

$$\frac{\sin 60^\circ + \cos 30^\circ}{\sin 30^\circ + \cos 60^\circ + 1} = \cos 30^\circ$$

Answer

We know that

$$\sin(30^\circ) = \frac{1}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(60^\circ) = \frac{1}{2}$$

Taking LHS

$$= \frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{\frac{1}{2} + \frac{1}{2} + 1}$$

$$= \frac{2 \times \frac{\sqrt{3}}{2}}{\frac{1+1+2}{2}}$$

$$= \frac{\sqrt{3}}{2}$$

$$\text{Now, RHS} = \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$\therefore$ LHS = RHS

Hence Proved

5 N. Question

Prove the following :

$$\sin 60^\circ = 2 \sin 30^\circ \cdot \cos 30^\circ = \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ}$$

Answer

We know that

$$\sin(30^\circ) = \frac{1}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\text{Taking LHS} = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\text{Now, solving RHS} = 2 \sin 30^\circ \cos 30^\circ$$

$$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{2}$$

$$= \text{LHS}$$

$$\text{Now, RHS} = \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ}$$

$$= \frac{2 \left(\frac{1}{\sqrt{3}} \right)}{1 + \left(\frac{1}{\sqrt{3}} \right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4}$$

$$= \frac{\sqrt{3}}{2}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved

6 A. Question

If $A=60^\circ$ and $B = 30^\circ$, verify that :

$$\cos (A+B) = \cos A \cos B - \sin A \sin B$$

Answer

Given: $A=60^\circ$ and $B=30^\circ$

Now, LHS = $\cos (A+B)$

$$\Rightarrow \cos (60^\circ + 30^\circ)$$

$$\Rightarrow \cos (90^\circ)$$

$$\Rightarrow 0 [\because \cos 90^\circ = 0]$$

Now, RHS = $\cos A \cos B - \sin A \sin B$

$$\Rightarrow \cos(60^\circ) \cos(30^\circ) - \sin(60^\circ) \sin(30^\circ)$$

$$\Rightarrow \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$$\Rightarrow 0$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence Proved

6 B. Question

If $A=60^\circ$ and $B=30^\circ$, verify that :

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$

Answer

Given: $A=60^\circ$ and $B=30^\circ$

Now, LHS = $\sin (A-B)$

$$\Rightarrow \sin (60^\circ - 30^\circ)$$

$$\Rightarrow \sin (30^\circ)$$

$$\Rightarrow \left(\frac{1}{2}\right)$$

Now, RHS = $\sin A \cos B - \cos A \sin B$

$$\Rightarrow \sin(60^\circ) \cos(30^\circ) - \cos(60^\circ) \sin(30^\circ)$$

$$\Rightarrow \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$\Rightarrow \frac{3}{4} - \frac{1}{4}$$

$$\Rightarrow \left(\frac{1}{2}\right)$$

$\therefore \text{LHS} = \text{RHS}$

Hence Proved

6 C. Question

If $A=60^\circ$ and $B = 30^\circ$, verify that :

$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Answer

Given: $A=60^\circ$ and $B = 30^\circ$

Now, $\text{LHS} = \tan (A-B)$

$$\Rightarrow \tan (60^\circ - 30^\circ)$$

$$\Rightarrow \tan (30^\circ)$$

$$\Rightarrow \left(\frac{1}{\sqrt{3}}\right)$$

$$\text{Now, RHS} = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$$

$$\Rightarrow \frac{\sqrt{3} - \left(\frac{1}{\sqrt{3}}\right)}{1 + (\sqrt{3})\left(\frac{1}{\sqrt{3}}\right)}$$

$$\Rightarrow \frac{\frac{3 - 1}{\sqrt{3}}}{1 + 1}$$

$$\Rightarrow \left(\frac{1}{\sqrt{3}}\right)$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence Proved

7 A. Question

If $A = 30^\circ$, verify that :

$$\sin 2A = 2 \sin A \cos A$$

Answer

Given: $A = 30^\circ$

$$\text{Now, LHS} = \sin 2(30^\circ)$$

$$\Rightarrow \sin 60^\circ$$

$$\Rightarrow \frac{\sqrt{3}}{2}$$

$$\text{Now, RHS} = 2 \sin A \cos A$$

$$\Rightarrow 2 \sin (30^\circ) \cos (30^\circ)$$

$$\Rightarrow 2 \left(\frac{1}{2} \right) \left(\frac{\sqrt{3}}{2} \right)$$

$$\Rightarrow \frac{\sqrt{3}}{2}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence Proved

7 B. Question

If $A = 30^\circ$, verify that :

$$\cos 2A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1$$

Answer

Given: $A = 30^\circ$

$$\text{Now, LHS} = \cos 2(30^\circ)$$

$$\Rightarrow \cos 60^\circ$$

$$\Rightarrow \frac{1}{2}$$

$$\text{Now, RHS} = 1 - 2\sin^2 A$$

$$\Rightarrow 1 - 2\sin^2 (30^\circ)$$

$$\Rightarrow 1 - 2\left(\frac{1}{2}\right)^2$$

$$\Rightarrow 1 - 2\left(\frac{1}{4}\right)$$

$$\Rightarrow \frac{2 - 1}{2}$$

$$\Rightarrow \frac{1}{2}$$

$$\text{Now, RHS} = 2\cos^2 A - 1$$

$$\Rightarrow 2\cos^2 (30^\circ) - 1$$

$$\Rightarrow 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1$$

$$\Rightarrow 2\left(\frac{3}{4}\right) - 1$$

$$\Rightarrow \frac{3 - 2}{2}$$

$$\Rightarrow \frac{1}{2}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence Proved

8 A. Question

If $\theta = 30^\circ$, verify that :

$$\sin 3\theta = 3 \sin\theta - 4 \sin^3\theta$$

Answer

Given: $\theta = 30^\circ$

$$\text{Now, LHS} = \sin 3(30^\circ)$$

$$\Rightarrow \sin 90^\circ$$

$$= 1$$

$$\text{Now, RHS} = 3 \sin \theta - 4 \sin^3 \theta$$

$$\Rightarrow 3 \sin (30^\circ) - 4 \sin^3 (30^\circ)$$

$$\Rightarrow 3 \left(\frac{1}{2} \right) - 4 \left(\frac{1}{2} \right)^3$$

$$\Rightarrow \frac{3}{2} - \frac{1}{2}$$

$$= 1$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence Proved

8 B. Question

If $\theta = 30^\circ$, verify that :

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

Answer

$$\text{Given: } \theta = 30^\circ$$

$$\text{Now, LHS} = \cos 3(30^\circ)$$

$$\Rightarrow \cos 90^\circ$$

$$= 0$$

$$\text{Now, RHS} = 4 \cos^3 \theta - 3 \cos \theta$$

$$\Rightarrow 4 \cos^3 (30^\circ) - 3 \cos (30^\circ)$$

$$\Rightarrow 4 \left(\frac{\sqrt{3}}{2} \right)^3 - 3 \left(\frac{\sqrt{3}}{2} \right)$$

$$\Rightarrow \left(\frac{3\sqrt{3}}{2} \right) - \left(\frac{3\sqrt{3}}{2} \right)$$

$$= 0$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence Proved

9. Question

If $\sin(A + B) = 1$ and $\cos(A - B) = \frac{\sqrt{3}}{2}$, then find A and B.

Answer

Given : $\sin(A+B) = 1$

$$\Rightarrow \sin(A+B) = \sin(90^\circ) [\because \sin(90^\circ) = 1]$$

On equating both the sides, we get

$$A + B = 90^\circ \dots(1)$$

$$\text{And } \cos(A - B) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos(A - B) = \cos(30^\circ) [\because \cos(30^\circ) = \frac{\sqrt{3}}{2}]$$

On equating both the sides, we get

$$A - B = 30^\circ \dots(2)$$

On Adding Eq. (1) and (2), we get

$$2A = 120^\circ$$

$$\Rightarrow A = 60^\circ$$

Now, Putting the value of A in Eq.(1), we get

$$60^\circ + B = 90^\circ$$

$$\Rightarrow B = 30^\circ$$

Hence, $A = 60^\circ$ and $B = 30^\circ$

10. Question

If $\sin(A + B) = 1$ and $\cos(A - B) = 1$, find A and B.

Answer

Given : $\sin(A+B) = 1$

$$\Rightarrow \sin(A+B) = \sin(90^\circ) [\because \sin(90^\circ) = 1]$$

On equating both the sides, we get

$$A + B = 90^\circ \dots(1)$$

$$\text{And } \cos(A - B) = 1$$

$$\Rightarrow \cos(A - B) = \cos(0^\circ) [\because \cos(0^\circ) = 1]$$

On equating both the sides, we get

$$A - B = 0^\circ \dots(2)$$

On Adding Eq. (1) and (2), we get

$$2A = 90^\circ$$

$$\Rightarrow A = 45^\circ$$

Now, Putting the value of A in Eq.(1), we get

$$45^\circ + B = 90^\circ$$

$$\Rightarrow B = 45^\circ$$

Hence, $A = 45^\circ$ and $B = 45^\circ$

11. Question

If $\sin(A + B) = \cos(A - B) = \frac{\sqrt{3}}{2}$, find A and B.

Answer

$$\text{Given : } \sin(A + B) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin(A + B) = \sin(60^\circ) [\because \sin(60^\circ) = \frac{\sqrt{3}}{2}]$$

On equating both the sides, we get

$$A + B = 60^\circ \dots(1)$$

$$\text{And } \cos(A - B) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos(A - B) = \cos(30^\circ) [\because \cos(30^\circ) = \frac{\sqrt{3}}{2}]$$

On equating both the sides, we get

$$A - B = 30^\circ \dots(2)$$

On Adding Eq. (1) and (2), we get

$$2A = 90^\circ$$

$$\Rightarrow A = 45^\circ$$

Now, Putting the value of A in Eq.(1), we get

$$45^\circ + B = 60^\circ$$

$$\Rightarrow B = 15^\circ$$

Hence, $A = 45^\circ$ and $B = 15^\circ$

12. Question

If $\sin(A - B) = 1/2$, $\cos(A + B) = 1/2$; $0^\circ < A+B < 90^\circ$; $A > B$, find A and B.

Answer

$$\text{Given : } \sin(A - B) = \frac{1}{2}$$

$$\Rightarrow \sin(A-B) = \sin(30^\circ) \left[\because \sin(30^\circ) = \frac{1}{2} \right]$$

On equating both the sides, we get

$$A - B = 30^\circ \dots(1)$$

$$\text{And } \cos(A + B) = \frac{1}{2}$$

$$\Rightarrow \cos(A + B) = \cos(60^\circ) \left[\because \cos(60^\circ) = \frac{1}{2} \right]$$

On equating both the sides, we get

$$A + B = 60^\circ \dots(2)$$

On Adding Eq. (1) and (2), we get

$$2A = 90^\circ$$

$$\Rightarrow A = 45^\circ$$

Now, Putting the value of A in Eq.(2), we get

$$45^\circ + B = 60^\circ$$

$$\Rightarrow B = 15^\circ$$

Hence, $A = 45^\circ$ and $B = 15^\circ$

13 A. Question

Show by an example that

$$\cos A - \cos B \neq \cos (A - B)$$

Answer

Let $A = 60^\circ$ and $B = 30^\circ$, then

$$\text{L.H.S.} = \cos A - \cos B = \cos 60^\circ - \cos 30^\circ = \frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2}$$

$$\text{R. H. S.} = \cos (A - B) = \cos (60^\circ - 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\therefore \text{L.H.S.} \neq \text{R.H.S}$$

13 B. Question

Show by an example that

$$\cos C + \cos D \neq \cos (C + D)$$

Answer

Let $C = 60^\circ$ and $D = 30^\circ$, then

$$\text{L.H.S.} = \cos C + \cos D = \cos 60^\circ + \cos 30^\circ$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1 + \sqrt{3}}{2}$$

$$\text{R. H. S.} = \cos (C+D) = \cos (60^\circ + 30^\circ) = \cos 90^\circ = 0$$

$$\therefore \text{L.H.S.} \neq \text{R.H.S}$$

13 C. Question

Show by an example that

$$\sin A + \sin B \neq \sin (A + B)$$

Answer

Let $A = 60^\circ$ and $B = 30^\circ$, then

$$\text{L.H.S.} = \sin A + \sin B = \sin 60^\circ + \sin 30^\circ$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3} + 1}{2}$$

$$\text{R. H. S.} = \sin (A + B) = \sin (60^\circ + 30^\circ) = \sin 90^\circ = 1$$

$\therefore \text{L.H.S.} \neq \text{R.H.S}$

13 D. Question

Show by an example that

$$\sin A - \sin B \neq \sin (A - B)$$

Answer

Let $A = 60^\circ$ and $B = 30^\circ$, then

$$\text{L.H.S.} = \sin A - \sin B = \sin 60^\circ - \sin 30^\circ$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3} - 1}{2}$$

$$\text{R. H. S.} = \sin (A - B) = \sin (60^\circ - 30^\circ) = \sin 30^\circ$$

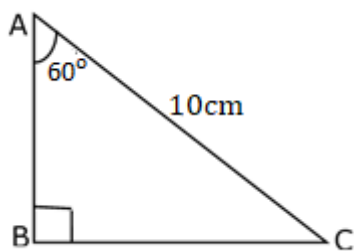
$$= \frac{1}{2}$$

$\therefore \text{L.H.S.} \neq \text{R.H.S}$

14. Question

In a right $\triangle ABC$ hypotenuse $AC = 10$ cm and $\angle A = 60^\circ$, then find the length of the remaining sides.

Answer



Given: $\angle A = 60^\circ$ and $AC = 10$ cm

$$\text{Now, } \sin 60^\circ = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{BC}{10}$$

$$\text{Now, we know that } \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{BC}{10}$$

$$\Rightarrow BC = 5\sqrt{3} \text{ cm}$$

In right angled ΔABC , we have

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2 \text{ [by using Pythagoras theorem]}$$

$$\Rightarrow (AB)^2 + (5\sqrt{3})^2 = (10)^2$$

$$\Rightarrow (AB)^2 + (25 \times 3) = 100$$

$$\Rightarrow (AB)^2 + 75 = 100$$

$$\Rightarrow (AB)^2 = 100 - 75$$

$$\Rightarrow (AB)^2 = 25$$

$$\Rightarrow AB = \sqrt{25}$$

$$\Rightarrow AB = \pm 5$$

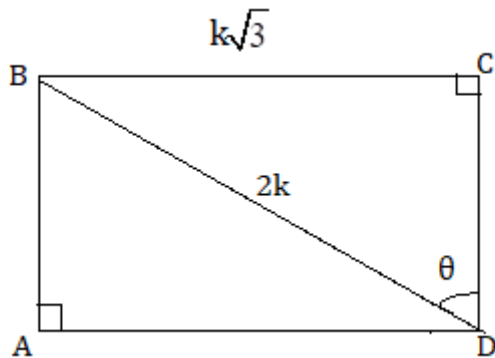
$$\Rightarrow AB = 5 \text{ cm [taking positive square root since, side cannot be negative]}$$

$$\therefore \text{Length of the side } AB = 5 \text{ cm and } BC = 5\sqrt{3} \text{ cm}$$

15. Question

In a rectangle ABCD, $BD : BC = 2 : \sqrt{3}$, then find $\angle BDC$ in degrees.

Answer



$$\text{Given } BD : BC = 2 : \sqrt{3}$$

We have to find the $\angle BDC$

We know that,

$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\Rightarrow \sin \theta = \frac{k\sqrt{3}}{2k}$$

$$\Rightarrow \sin \theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin \theta = \sin 60^\circ$$

$$\Rightarrow \theta = 60^\circ$$

Exercise 4.3

1. Question

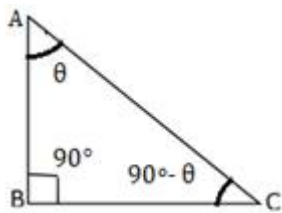
Express the following as trigonometric ratio of complementary angle of θ .

(i) $\cos \theta$ (ii) $\sec \theta$

(iii) $\cot \theta$ (iv) $\operatorname{cosec} \theta$

(v) $\tan \theta$

Answer



(i) We know that

$$\cos \theta = \frac{\text{base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \sin(90^\circ - \theta)$$

$$\Rightarrow \cos \theta = \sin(90^\circ - \theta)$$

(ii) We know that

$$\sec \theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \operatorname{cosec}(90^\circ - \theta)$$

(iii) We know that

$$\cot \theta = \frac{\text{base}}{\text{perpendicular}} = \frac{AB}{BC} = \tan(90^\circ - \theta)$$

(iv) We know that

$$\operatorname{cosec} \theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{BC} = \sec(90^\circ - \theta)$$

(v) We know that

$$\tan \theta = \frac{\text{perpendicular}}{\text{base}} = \frac{BC}{AB} = \cot(90^\circ - \theta)$$

2. Question

Express the following as trigonometric ratio of complementary angle of $90^\circ - \theta$.

(i) $\tan (90^\circ - \theta)$

(ii) $\cos (90^\circ - \theta)$

Answer

(i) We know that,

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\Rightarrow \tan (90^\circ - \theta) = \frac{\sin(90^\circ - \theta)}{\cos(90^\circ - \theta)}$$

$$\Rightarrow \tan (90^\circ - \theta) = \frac{\sin 90^\circ \cos \theta - \cos 90^\circ \sin \theta}{\cos 90^\circ \cos \theta + \sin 90^\circ \sin \theta} [\because \sin 90^\circ = 1 \text{ and } \cos 90^\circ = 0]$$

$$\Rightarrow \tan(90^\circ - \theta) = \frac{\cos \theta}{\sin \theta}$$

$$\Rightarrow \tan (90^\circ - \theta) = \cot \theta$$

(ii) We know that.

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\Rightarrow \cos (90^\circ - \theta) = \cos 90^\circ \cos \theta + \sin 90^\circ \sin \theta$$

$$\Rightarrow \cos (90^\circ - \theta) = (0) \cos \theta + (1) \sin \theta$$

$$\Rightarrow \cos (90^\circ - \theta) = \sin \theta$$

3. Question

Fill up the blanks by an angle between 0° and 90° :

(i) $\sin 70^\circ = \cos(\dots)$ (ii) $\sin 35^\circ = \cos(\dots)$

(iii) $\cos 48^\circ = \sin (\dots)$ (iv) $\cos 70^\circ = \sin (\dots)$

(v) $\cos 50^\circ = \sin (\dots)$ (vi) $\sec 32^\circ = \operatorname{cosec}(\dots)$

Answer

(i) We know that

$$\sin \theta = \cos (90^\circ - \theta)$$

$$\text{Here, } \theta = 70^\circ$$

$$\Rightarrow \sin 70^\circ = \cos(90^\circ - 70^\circ)$$

$$\Rightarrow \sin 70^\circ = \cos 20^\circ$$

(ii) We know that

$$\sin \theta = \cos (90^\circ - \theta)$$

$$\text{Here, } \theta = 35^\circ$$

$$\Rightarrow \sin 35^\circ = \cos(90^\circ - 35^\circ)$$

$$\Rightarrow \sin 35^\circ = \cos 55^\circ$$

(iii) $\cos \theta = \sin (90^\circ - \theta)$

$$\text{Here, } \theta = 48^\circ$$

$$\Rightarrow \cos 48^\circ = \sin (90^\circ - 48^\circ)$$

$$\Rightarrow \cos 48^\circ = \sin 42^\circ$$

(iv) $\cos \theta = \sin (90^\circ - \theta)$

$$\text{Here, } \theta = 70^\circ$$

$$\Rightarrow \cos 70^\circ = \sin (90^\circ - 70^\circ)$$

$$\Rightarrow \cos 70^\circ = \sin 20^\circ$$

(v) $\cos \theta = \sin (90^\circ - \theta)$

$$\text{Here, } \theta = 50^\circ$$

$$\Rightarrow \cos 50^\circ = \sin (90^\circ - 50^\circ)$$

$$\Rightarrow \cos 50^\circ = \sin 40^\circ$$

(vi) $\sec \theta = \operatorname{cosec} (90^\circ - \theta)$

$$\text{Here, } \theta = 32^\circ$$

$$\Rightarrow \sec 32^\circ = \operatorname{cosec}(90^\circ - 32^\circ)$$

$$\Rightarrow \sec 32^\circ = \operatorname{cosec} 58^\circ$$

4. Question

If $A+B=90^\circ$, then fill up the blanks with suitable trigonometric ratio of complementary angle of A or B.

(i) $\sin A = \dots$ (ii) $\cos B = \dots$

(iii) $\sec A = \dots$ (iv) $\tan B = \dots$

(v) $\operatorname{cosec} B = \dots$ (vi) $\cot A = \dots$

Answer

(i) Here, $A+B = 90^\circ$

$$\Rightarrow A = 90^\circ - B$$

Multiplying both sides by Sin, we get

$$\sin A = \sin (90^\circ - B)$$

$$\Rightarrow \sin A = \cos B \quad [\because \cos \theta = \sin (90^\circ - \theta)]$$

(ii) Here, $A+B = 90^\circ$

$$\Rightarrow B = 90^\circ - A$$

Multiplying both sides by cos, we get

$$\cos B = \cos (90^\circ - A)$$

$$\Rightarrow \cos B = \sin A \quad [\because \sin \theta = \cos (90^\circ - \theta)]$$

(iii) Here, $A+B = 90^\circ$

$$\Rightarrow A = 90^\circ - B$$

Multiplying both sides by sec, we get

$$\sec A = \sec (90^\circ - B)$$

$$\Rightarrow \sec A = \operatorname{cosec} B \quad [\because \operatorname{cosec} \theta = \sec (90^\circ - \theta)]$$

(iv) Here, $A+B = 90^\circ$

$$\Rightarrow B = 90^\circ - A$$

Multiplying both sides by tan, we get

$$\tan B = \tan (90^\circ - A)$$

$$\Rightarrow \tan B = \cot A \quad [\because \cot \theta = \tan (90^\circ - \theta)]$$

(v) Here, $A+B = 90^\circ$

$$\Rightarrow B = 90^\circ - A$$

Multiplying both sides by cosec, we get

$$\text{Cosec } B = \text{cosec } (90^\circ - A)$$

$$\Rightarrow \text{cosec } B = \sec A \quad [\because \sec \theta = \text{cosec } (90^\circ - \theta)]$$

$$\text{(vi) Here, } A+B = 90^\circ$$

$$\Rightarrow A = 90^\circ - B$$

Multiplying both sides by Sin, we get

$$\cot A = \cot (90^\circ - B)$$

$$\Rightarrow \cot A = \tan B \quad [\because \tan \theta = \cot (90^\circ - \theta)]$$

5 A. Question

If $\sin 37^\circ = a$, then express $\cos 53^\circ$ in terms of a .

Answer

$$\text{Given } \sin 37^\circ = a$$

$$\text{We know that } \sin \theta = \cos (90^\circ - \theta)$$

$$\text{Here, } \theta = 37^\circ$$

$$\Rightarrow \cos (90^\circ - 37^\circ) = a$$

$$\Rightarrow \cos 53^\circ = a$$

5 B. Question

If $\cos 47^\circ = a$, then express $\sin 43^\circ$ in terms of a .

Answer

$$\text{Given } \cos 47^\circ = a$$

$$\text{We know that } \cos \theta = \sin (90^\circ - \theta)$$

$$\text{Here, } \theta = 47^\circ$$

$$\Rightarrow \sin (90^\circ - 47^\circ) = a$$

$$\Rightarrow \sin 43^\circ = a$$

5 C. Question

If $\sin 52^\circ = a$, then express $\sin 38^\circ$ in terms of a .

Answer

Given $\sin 52^\circ = a$

We know that $\sin \theta = \cos (90^\circ - \theta)$

Here, $\theta = 52^\circ$

$$\Rightarrow \cos (90^\circ - 52^\circ) = a$$

$$\Rightarrow \cos 38^\circ = a$$

5 D. Question

If $\sin 56^\circ = x$, then express $\sin 34^\circ$ in terms of x .

Answer

Given $\sin 56^\circ = x$

We know that $\sin \theta = \cos (90^\circ - \theta)$

Here, $\theta = 56^\circ$

$$\Rightarrow \cos (90^\circ - 56^\circ) = x$$

$$\Rightarrow \cos 34^\circ = x$$

6. Question

Find the value of

(i) $\frac{\cos 59^\circ}{\sin 31^\circ}$ (ii) $\frac{\cos 53^\circ}{\sin 37^\circ}$

(iii) $\frac{\sin 20^\circ}{\cos 70^\circ}$ (iv) $\frac{\sqrt{2} \sin 22^\circ}{\cos 68^\circ}$

(v) $\frac{\sin 10^\circ}{\cos 80^\circ}$ (vi) $\frac{\sin 27^\circ}{\cos 63^\circ}$

(vii) $\frac{\sqrt{3} \cos 65^\circ}{\sin 25^\circ}$ (viii) $\frac{\cos 29^\circ}{\sin 61^\circ}$

(ix) $\sin 54^\circ - \cos 36^\circ$ (x) $\frac{\tan 80^\circ}{\cot 10^\circ}$

(xi) $\operatorname{cosec} 31^\circ - \sec 59^\circ$ (xii) $\frac{\sin 18^\circ}{\cos 72^\circ}$

$$(xiii) \frac{\tan 65^\circ}{\cot 25^\circ}$$

Answer

$$(i) \frac{\cos 59^\circ}{\sin 31^\circ} = \frac{\sin(90^\circ - 59^\circ)}{\sin 31^\circ} = \frac{\sin 31^\circ}{\sin 31^\circ} = 1 \quad [\because \cos \theta = \sin (90^\circ - \theta)]$$

$$(ii) \frac{\cos 53^\circ}{\sin 37^\circ} = \frac{\sin(90^\circ - 53^\circ)}{\sin 37^\circ} = \frac{\sin 37^\circ}{\sin 37^\circ} = 1 \quad [\because \cos \theta = \sin (90^\circ - \theta)]$$

$$(iii) \frac{\sin 20^\circ}{\cos 70^\circ} = \frac{\cos(90^\circ - 20^\circ)}{\cos 70^\circ} = \frac{\cos 70^\circ}{\cos 70^\circ} = 1 \quad [\because \sin \theta = \cos (90^\circ - \theta)]$$

$$(iv) \frac{\sqrt{2} \sin 22^\circ}{\cos 68^\circ} = \frac{\sqrt{2} \cos(90^\circ - 22^\circ)}{\cos 68^\circ} = \frac{\sqrt{2} \cos 68^\circ}{\cos 68^\circ} = \sqrt{2} \quad [\because \sin \theta = \cos (90^\circ - \theta)]$$

$$(v) \frac{\sin 10^\circ}{\cos 80^\circ} = \frac{\cos(90^\circ - 10^\circ)}{\cos 80^\circ} = \frac{\cos 80^\circ}{\cos 80^\circ} = 1 \quad [\because \sin \theta = \cos (90^\circ - \theta)]$$

$$(vi) \frac{\sin 27^\circ}{\cos 63^\circ} = \frac{\cos(90^\circ - 27^\circ)}{\cos 63^\circ} = \frac{\cos 63^\circ}{\cos 63^\circ} = 1 \quad [\because \sin \theta = \cos (90^\circ - \theta)]$$

$$(vii) \frac{\sqrt{3} \cos 65^\circ}{\sin 25^\circ} = \frac{\sqrt{3} \sin(90^\circ - 65^\circ)}{\sin 25^\circ} = \frac{\sin 25^\circ}{\sin 25^\circ} = \sqrt{3} \quad [\because \cos \theta = \sin (90^\circ - \theta)]$$

$$(viii) \frac{\cos 29^\circ}{\sin 61^\circ} = \frac{\sin(90^\circ - 29^\circ)}{\sin 61^\circ} = \frac{\sin 61^\circ}{\sin 61^\circ} = 1 \quad [\because \cos \theta = \sin (90^\circ - \theta)]$$

$$(ix) \sin 54^\circ - \sin(90^\circ - 36^\circ) \quad [\because \cos \theta = \sin (90^\circ - \theta)]$$

$$\Rightarrow \sin 54^\circ - \sin 54^\circ$$

$$\Rightarrow 0$$

$$(x) \frac{\tan 80^\circ}{\cot 10^\circ} = \frac{\cot(90^\circ - 80^\circ)}{\cot 10^\circ} = \frac{\cot 10^\circ}{\cot 10^\circ} = 1 \quad [\because \tan \theta = \cot (90^\circ - \theta)]$$

$$(xi) \operatorname{cosec} 31^\circ - \operatorname{cosec}(90^\circ - 59^\circ) \quad [\because \sec \theta = \operatorname{cosec} (90^\circ - \theta)]$$

$$\Rightarrow \operatorname{cosec} 31^\circ - \operatorname{cosec} 31^\circ$$

$$\Rightarrow 0$$

$$(xii) \frac{\sin 18^\circ}{\cos 72^\circ} = \frac{\cos(90^\circ - 18^\circ)}{\cos 72^\circ} = \frac{\cos 72^\circ}{\cos 72^\circ} = 1 \quad [\because \sin \theta = \cos (90^\circ - \theta)]$$

$$(xiii) \frac{\tan 65^\circ}{\cot 25^\circ} = \frac{\cot(90^\circ - 65^\circ)}{\cot 25^\circ} = \frac{\cot 25^\circ}{\cot 25^\circ} = 1 \quad [\because \tan \theta = \cot (90^\circ - \theta)]$$

7. Question

Fill up the blanks :

$$(i) \text{ If } \sin 50^\circ = 0.7660, \text{ then } \cos 40^\circ = \dots\dots$$

(ii) If $\cos 44^\circ = 0.7193$, then $\sin 46^\circ = \dots$

(iii) $\sin 50^\circ + \cos 40^\circ = 2 \sin (\dots)$

(iv) Value of $\frac{\sin 70^\circ}{\cos 20^\circ}$ is

Answer

(i) Given: $\sin 50^\circ = 0.7660$

We know that

$$\sin \theta = \cos (90^\circ - \theta)$$

$$\Rightarrow \cos (90^\circ - 50^\circ) = 0.7660$$

$$\Rightarrow \cos 40^\circ = 0.7660$$

(ii) Given: $\cos 44^\circ = 0.7193$

We know that,

$$\cos \theta = \sin (90^\circ - \theta)$$

$$\Rightarrow \sin (90^\circ - 44^\circ) = 0.7193$$

$$\Rightarrow \sin 46^\circ = 0.7193$$

(iii) LHS = $\sin 50^\circ + \cos 40^\circ$

$$\Rightarrow \sin 50^\circ + \sin (90^\circ - 40^\circ) [\because \cos \theta = \sin (90^\circ - \theta)]$$

$$\Rightarrow \sin 50^\circ + \sin 50^\circ$$

$$\Rightarrow 2\sin 50^\circ$$

$$(iii) \frac{\sin 70^\circ}{\cos 20^\circ} = \frac{\cos(90^\circ - 70^\circ)}{\cos 20^\circ} = \frac{\cos 20^\circ}{\cos 20^\circ} = 1 [\because \sin \theta = \cos (90^\circ - \theta)]$$

8 A. Question

If $A + B = 90^\circ$, then express $\cos B$ in terms of simplest trigonometric ratio of A .

Answer

Given: $A + B = 90^\circ$

$$\Rightarrow B = 90^\circ - A$$

Multiplying both side by $\cos$, we get

$$= \cos B = \cos (90^\circ - A)$$

$$\Rightarrow \cos B = \sin A [\because \sin \theta = \cos (90^\circ - \theta)]$$

8 B. Question

If $X + Y = 90^\circ$, then express $\cos X$ in terms of simplest trigonometric ratio of Y .

Answer

$$\text{Given: } X + Y = 90^\circ$$

$$\Rightarrow X = 90^\circ - Y$$

Multiplying both side by $\cos$, we get

$$= \cos X = \cos (90^\circ - Y)$$

$$\Rightarrow \cos X = \sin Y [\because \sin \theta = \cos (90^\circ - \theta)]$$

9 A. Question

If $A + B = 90^\circ$, $\sin A = a$, $\sin B = b$, then prove that

$$(a) \ a^2 + b^2 = 1$$

$$(b) \ \tan A = \frac{a}{b}$$

Answer

$$(a) \ \text{LHS} = a^2 + b^2$$

$$= (\sin A)^2 + (\sin B)^2$$

$$= \sin^2 A + \sin^2 B$$

$$= \sin^2 A + \sin^2 (90^\circ - A) [\because \sin \theta = \cos (90^\circ - \theta)]$$

$$= \sin^2 A + \cos^2 A$$

$$= 1 [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= \text{RHS}$$

Hence Proved

$$(b) \ \text{LHS} = \tan A$$

$$\text{Now, taking RHS} = \frac{a}{b}$$

$$\Rightarrow \frac{\sin A}{\sin B}$$

$$\Rightarrow \frac{\sin A}{\sin(90^\circ - A)} \text{ \{given, } A + B = 90^\circ\}}$$

$$\Rightarrow \frac{\sin A}{\cos A} [\because \cos \theta = \sin (90^\circ - \theta)]$$

$$\Rightarrow \tan A$$

$$= \text{LHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence Proved

9 B. Question

Show that $\sin(50^\circ + \theta) - \cos(40^\circ - \theta) = 0$.

Answer

$$\text{LHS} = \sin(50^\circ + \theta) - \cos(40^\circ - \theta)$$

We know that,

$$\sin A = \cos(90^\circ - A)$$

$$\text{Here, } A = 50^\circ + \theta$$

$$\Rightarrow \cos\{90^\circ - (50^\circ + \theta)\} - \cos(40^\circ - \theta)$$

$$\Rightarrow \cos(40^\circ - \theta) - \cos(40^\circ - \theta)$$

$$= 0 = \text{RHS}$$

Hence Proved

10. Question

$$\text{Prove that } \frac{\cos \theta}{\sin(90^\circ - \theta)} + \frac{\sin \theta}{\cos(90^\circ - \theta)} = 2$$

Answer

Taking LHS,

$$\frac{\cos \theta}{\sin(90^\circ - \theta)} + \frac{\sin \theta}{\cos(90^\circ - \theta)} [\because \cos \theta = \sin(90^\circ - \theta) \text{ and } \sin \theta = \cos(90^\circ - \theta)]$$

$$\Rightarrow \frac{\cos \theta}{\cos \theta} + \frac{\sin \theta}{\sin \theta}$$

$$\Rightarrow 1 + 1$$

$$= 2 = \text{RHS}$$

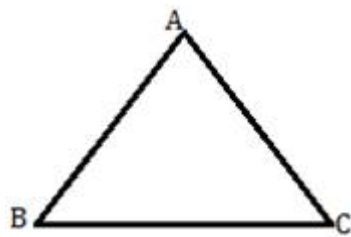
Hence Proved

11 A. Question

In a ΔABC prove that

$$\sin \frac{B+C}{2} = \cos \frac{A}{2}$$

Answer



In ΔABC ,

Sum of angles of a triangle = 180°

$$A + B + C = 180^\circ$$

$$\Rightarrow B + C = 180^\circ - A$$

Multiplying both sides by $\frac{1}{2}$

$$= \frac{B+C}{2} = \frac{180^\circ - A}{2}$$

$$= \frac{B+C}{2} = \frac{180^\circ}{2} - \frac{A}{2}$$

$$= \frac{B+C}{2} = 90^\circ - \frac{A}{2} \dots (1)$$

Taking LHS

$$\sin \frac{B+C}{2}$$

$$= \sin \left(90^\circ - \frac{A}{2} \right) \text{ (from eq (1))}$$

$$= \cos \frac{A}{2} [\because \sin (90^\circ - \theta) = \cos \theta]$$

=RHS

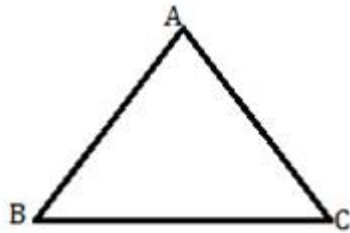
Hence Proved

11 B. Question

In a ΔABC prove that

$$\tan \frac{B+C}{2} = \cot \frac{A}{2}$$

Answer



In ΔABC ,

Sum of angles of a triangle = 180°

$$A + B + C = 180^\circ$$

$$\Rightarrow B + C = 180^\circ - A$$

Multiplying both sides by $\frac{1}{2}$

$$= \frac{B+C}{2} = \frac{180^\circ - A}{2}$$

$$= \frac{B+C}{2} = \frac{180^\circ}{2} - \frac{A}{2}$$

$$= \frac{B+C}{2} = 90^\circ - \frac{A}{2} \dots (2)$$

Taking LHS

$$\tan \frac{B+C}{2}$$

$$= \tan \left(90^\circ - \frac{A}{2} \right) \text{ (from eq (2))}$$

$$= \cot \frac{A}{2} [\because \tan (90^\circ - \theta) = \cot \theta]$$

=RHS

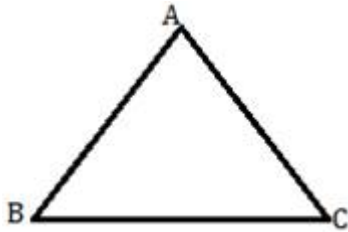
Hence Proved

11 C. Question

In a ΔABC prove that

$$\cos \frac{A+B}{2} = \sin \frac{C}{2}$$

Answer



In ΔABC ,

Sum of angles of a triangle = 180°

$$A + B + C = 180^\circ$$

$$\Rightarrow A + B = 180^\circ - C$$

Multiplying both sides by $\frac{1}{2}$

$$= \frac{A+B}{2} = \frac{180^\circ - C}{2}$$

$$= \frac{A+B}{2} = \frac{180^\circ}{2} - \frac{C}{2}$$

$$= \frac{A+B}{2} = 90^\circ - \frac{C}{2} \dots (3)$$

Taking LHS

$$\cos \frac{A+B}{2}$$

$$= \cos \left(90^\circ - \frac{C}{2} \right) \text{ (from eq (3))}$$

$$= \sin \frac{C}{2} [\because \cos (90^\circ - \theta) = \sin \theta]$$

=RHS

Hence Proved

12 A. Question

If $\sin 3A = \cos(A - 26^\circ)$, where $3A$ is an acute angle, find the value of A .

Answer

$$\sin 3A = \cos (A - 26^\circ) \dots(i)$$

We know that

$$\sin \theta = \cos (90^\circ - \theta)$$

So, Eq. (i) become

$$\cos (90^\circ - 3A) = \cos (A - 26^\circ)$$

On Equating both the sides, we get

$$90^\circ - 3A = A - 26^\circ$$

$$\Rightarrow -3A - A = -26^\circ - 90^\circ$$

$$\Rightarrow -4A = -116^\circ$$

$$\Rightarrow A = 29^\circ$$

12 B. Question

Find θ if $\cos(2\theta + 54^\circ) = \sin \theta$, where $(2\theta + 54^\circ)$ is an acute angle.

Answer

$$\cos(2\theta + 54^\circ) = \sin \theta \dots(i)$$

We know that

$$\sin \theta = \cos (90^\circ - \theta)$$

So, Eq. (i) become

$$\cos(2\theta + 54^\circ) = \cos(90^\circ - \theta)$$

On Equating both the sides, we get

$$2\theta + 54^\circ = 90^\circ - \theta$$

$$\Rightarrow 2\theta + \theta = 90^\circ - 54^\circ$$

$$\Rightarrow 3\theta = 36^\circ$$

$$\Rightarrow \theta = 12^\circ$$

12 C. Question

If $\tan 3\theta = \cot(\theta + 18^\circ)$, where 3θ and $\theta + 18^\circ$ are acute angles, find the value of θ .

Answer

$$\tan 3\theta = \cot(\theta + 18^\circ) \dots(i)$$

We know that

$$\tan \theta = \cot(90^\circ - \theta)$$

So, Eq. (i) become

$$\cot(90^\circ - 3\theta) = \cot(\theta + 18^\circ)$$

On Equating both the sides, we get

$$90^\circ - 3\theta = \theta + 18^\circ$$

$$\Rightarrow -3\theta - \theta = 18^\circ - 90^\circ$$

$$\Rightarrow -4\theta = -72^\circ$$

$$\Rightarrow \theta = 18^\circ$$

12 D. Question

If $\sec 5\theta = \operatorname{cosec}(\theta - 36^\circ)$, where 5θ is an acute angle, find the value of θ .

Answer

$$\sec 5\theta = \operatorname{cosec}(\theta - 36^\circ) \dots(i)$$

We know that

$$\sec \theta = \operatorname{cosec}(90^\circ - \theta)$$

So, Eq. (i) become

$$\operatorname{Cosec}(90^\circ - 5\theta) = \operatorname{cosec}(\theta - 36^\circ)$$

On Equating both the sides, we get

$$90^\circ - 5\theta = \theta - 36^\circ$$

$$\Rightarrow -5\theta - \theta = -36^\circ - 90^\circ$$

$$\Rightarrow -6\theta = -126^\circ$$

$$\Rightarrow \theta = 21^\circ$$

13. Question

Prove that :

$$\sin 70^\circ \cdot \sec 20^\circ = 1$$

Answer

Taking LHS

$$\sin 70^\circ \sec 20^\circ$$

$$\Rightarrow \sin 70^\circ \times \frac{1}{\cos 20^\circ}$$

$$\Rightarrow \sin 70^\circ \times \frac{1}{\sin(90^\circ - 20^\circ)} [\because \cos \theta = \sin(90^\circ - \theta)]$$

$$\Rightarrow \frac{\sin 70^\circ}{\sin 70^\circ}$$

$$= 1 = \text{RHS}$$

Hence Proved

14. Question

Prove that :

$$\sin(90^\circ - \theta) \tan \theta = \sin \theta$$

Answer

Taking LHS

$$\sin(90^\circ - \theta) \tan \theta [\because \cos \theta = \sin(90^\circ - \theta)]$$

$$\Rightarrow \cos \theta \tan \theta$$

$$\Rightarrow \cos \theta \times \frac{\sin \theta}{\cos \theta} [\because \tan \theta = \frac{\sin \theta}{\cos \theta}]$$

$$= \sin \theta = \text{RHS}$$

Hence Proved

15. Question

Prove that :

$$\tan 63^\circ \cdot \tan 27^\circ = 1$$

Answer

Taking LHS

$$\tan 63^\circ \tan 27^\circ$$

$$\Rightarrow \tan 63^\circ \cot (90^\circ - 27^\circ) [\because \tan \theta = \cot (90^\circ - \theta)]$$

$$\Rightarrow \tan 63^\circ \cot 63^\circ$$

$$\Rightarrow \tan 63^\circ \times \frac{1}{\tan 63^\circ} \left[\because \tan \theta = \frac{1}{\cot \theta} \right]$$

$$= 1 = \text{RHS}$$

Hence Proved

16. Question

Prove that :

$$\frac{\sin(90^\circ - \theta) \sin \theta}{\tan \theta} - 1 = -\sin^2 \theta$$

Answer

Taking LHS

$$= \frac{\sin(90^\circ - \theta) \sin \theta}{\tan \theta} - 1$$

$$= \frac{\cos \theta \sin \theta}{\frac{\sin \theta}{\cos \theta}} - 1$$

$$= \frac{\cos \theta \sin \theta \times \cos \theta}{\sin \theta} - 1$$

$$= \cos^2 \theta - 1$$

$$= -\sin^2 \theta [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \text{RHS}$$

Hence Proved

17. Question

Prove that :

$$\sin 55^\circ \cdot \cos 48^\circ = \cos 35^\circ \cdot \sin 42^\circ$$

Answer

$$\text{Taking LHS} = \sin 55^\circ \cos 48^\circ$$

We know that

$$\cos \theta = \sin (90^\circ - \theta)$$

$$\text{Here, } \theta = 48^\circ$$

$$\Rightarrow \sin 55^\circ \sin (90^\circ - 48^\circ)$$

$$\Rightarrow \sin 55^\circ \sin 42^\circ$$

We also know that

$$\sin \theta = \cos (90^\circ - \theta)$$

$$\text{Here, } \theta = 55^\circ$$

$$\Rightarrow \cos (90^\circ - 55^\circ) \sin 42^\circ$$

$$\Rightarrow \cos 35^\circ \sin 42^\circ = \text{RHS}$$

Hence Proved

18. Question

Prove that :

$$\sin^2 25^\circ + \sin^2 65^\circ = \cos^2 63^\circ + \cos^2 39^\circ$$

Answer

$$\text{Taking LHS} = \sin^2 25^\circ + \sin^2 65^\circ$$

We know that

$$\sin \theta = \cos (90^\circ - \theta)$$

$$\text{Here, } \theta = 25^\circ$$

$$\Rightarrow \cos^2 (90^\circ - 25^\circ) + \sin^2 65^\circ$$

$$\Rightarrow \cos^2 65^\circ + \sin^2 65^\circ$$

$$= 1 [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$\text{Now, RHS} = \cos^2 63^\circ + \cos^2 39^\circ$$

We know that

$$\cos \theta = \sin (90^\circ - \theta)$$

$$\text{Here, } \theta = 39^\circ$$

$$\Rightarrow \cos^2 63^\circ + \sin^2 (90^\circ - 39^\circ)$$

$$\Rightarrow \cos^2 63^\circ + \sin^2 63^\circ$$

$$= 1 [\because \cos^2 \theta + \sin^2 \theta = 1]$$

LHS = RHS

Hence Proved

19. Question

Prove that :

$$\sin 54^\circ + \cos 67^\circ = \sin 23^\circ + \cos 36^\circ$$

Answer

$$\text{Taking LHS} = \sin 54^\circ + \cos 67^\circ$$

We know that

$$\cos \theta = \sin (90^\circ - \theta)$$

$$\text{Here, } \theta = 67^\circ$$

$$\Rightarrow \sin 54^\circ + \sin (90^\circ - 67^\circ)$$

$$\Rightarrow \sin 54^\circ + \sin 23^\circ$$

We also know that

$$\sin \theta = \cos (90^\circ - \theta)$$

$$\text{Here, } \theta = 54^\circ$$

$$\Rightarrow \cos (90^\circ - 54^\circ) + \sin 23^\circ$$

$$\Rightarrow \cos 36^\circ + \sin 23^\circ = \text{RHS}$$

Hence Proved

20. Question

Prove that :

$$\cos 27^\circ + \sin 51^\circ = \sin 63^\circ + \cos 39^\circ$$

Answer

$$\text{Taking LHS} = \cos 27^\circ + \sin 51^\circ$$

We know that

$$\cos \theta = \sin (90^\circ - \theta)$$

Here, $\theta = 27^\circ$

$$\Rightarrow \sin (90^\circ - 27^\circ) + \sin 51^\circ$$

$$\Rightarrow \sin 63^\circ + \sin 51^\circ$$

We also know that

$$\sin \theta = \cos (90^\circ - \theta)$$

Here, $\theta = 51^\circ$

$$\Rightarrow \sin 63^\circ + \cos (90^\circ - 51^\circ)$$

$$\Rightarrow \sin 63^\circ + \cos 39^\circ = \text{RHS}$$

Hence Proved

21. Question

Prove that :

$$\sin^2 40^\circ + \sin^2 50^\circ = 1$$

Answer

$$\text{Taking LHS} = \sin^2 40^\circ + \sin^2 50^\circ$$

$$\Rightarrow \cos^2 (90^\circ - 40^\circ) + \sin^2 50^\circ [\because \sin \theta = \cos (90^\circ - \theta)]$$

$$\Rightarrow \cos^2 50^\circ + \sin^2 50^\circ$$

$$= 1 = \text{RHS} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

Hence Proved

22. Question

Prove that :

$$\sin^2 29^\circ + \sin^2 61^\circ = 1$$

Answer

$$\text{Taking LHS} = \sin^2 29^\circ + \sin^2 61^\circ$$

$$\Rightarrow \cos^2 (90^\circ - 29^\circ) + \sin^2 61^\circ [\because \sin \theta = \cos (90^\circ - \theta)]$$

$$\Rightarrow \cos^2 61^\circ + \sin^2 61^\circ$$

$$= 1 = \text{RHS} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

Hence Proved

23. Question

Prove that :

$$\sin \theta .\cos (90^{\circ} - \theta) + \cos \theta \sin (90^{\circ} - \theta).$$

Answer

$$\text{Taking LHS} = \sin \theta \cos (90^{\circ} - \theta) + \cos \theta \sin (90^{\circ} - \theta)$$

$$\Rightarrow \sin \theta \times \sin \theta + \cos \theta \times \cos \theta [\because \sin \theta = \cos (90^{\circ} - \theta) \text{ and } \cos \theta = \sin (90^{\circ} - \theta)]$$

$$\Rightarrow \cos^2 \theta + \sin^2 \theta [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= 1 = \text{RHS}$$

Hence Proved

24. Question

Prove that :

$$\cos \theta . \cos(90^{\circ} - \theta) + \sin \theta \sin (90^{\circ} - \theta) = 0$$

Answer

$$\text{Taking LHS} = \cos \theta \cos (90^{\circ} - \theta) + \sin \theta \sin (90^{\circ} - \theta)$$

$$\Rightarrow \cos \theta \times \sin \theta - \sin \theta \times \cos \theta [\because \sin \theta = \cos (90^{\circ} - \theta) \text{ and } \cos \theta = \sin (90^{\circ} - \theta)]$$

$$= 0 = \text{RHS}$$

Hence Proved

25. Question

Prove that :

$$\sin 42^{\circ} . \cos 48^{\circ} + \cos 42^{\circ} . \sin 48^{\circ} = 1$$

Answer

Taking LHS

$$= \sin 42^{\circ} \cos 48^{\circ} + \cos 42^{\circ} \sin 48^{\circ}$$

$$= \cos (90^{\circ} - 42^{\circ}) \cos 48^{\circ} + \sin (90^{\circ} - 42^{\circ}) \sin 48^{\circ}$$

$$[\because \sin \theta = \cos (90^{\circ} - \theta) \text{ and } \cos \theta = \sin (90^{\circ} - \theta)]$$

$$= \cos 48^{\circ} \cos 48^{\circ} + \sin 48^{\circ} \sin 48^{\circ}$$

$$= \cos^2 48^\circ + \sin^2 48^\circ$$

$$= 1 [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \text{LHS} = \text{RHS}$$

Hence Proved

26. Question

Prove that :

$$\frac{\cos 20^\circ}{\sin 70^\circ} + \frac{\cos \theta}{\sin(90^\circ - \theta)} = 2$$

Answer

Taking LHS

$$= \frac{\cos 20^\circ}{\sin 70^\circ} + \frac{\cos \theta}{\sin(90^\circ - \theta)}$$

$$= \frac{\cos 20^\circ}{\cos(90^\circ - 70^\circ)} + \frac{\cos \theta}{\cos \theta} [\because \sin \theta = \cos(90^\circ - \theta) \text{ and } \cos \theta = \sin(90^\circ - \theta)]$$

$$= \frac{\cos 20^\circ}{\cos 20^\circ} + 1$$

$$= 1 + 1$$

$$= 2 = \text{RHS}$$

Hence Proved

27. Question

Prove that :

$$\tan 27^\circ \tan 45^\circ \tan 63^\circ$$

Answer

Taking LHS

$$= \tan 27^\circ \tan 45^\circ \tan 63^\circ$$

$$= \tan(90^\circ - 27^\circ) \tan 45^\circ \tan 63^\circ [\because \tan \theta = \cot(90^\circ - \theta)]$$

$$= \cot 63^\circ \tan 45^\circ \tan 63^\circ \left[\because \tan \theta = \frac{1}{\cot \theta} \right]$$

$$= \frac{1}{\tan 63^\circ} \times \tan 45^\circ \times \tan 63^\circ$$

$$= \tan 45^\circ [\because \tan 45^\circ = 1]$$

$$= 1 = \text{RHS}$$

Hence Proved

28. Question

Prove that :

$$\tan 9^\circ \cdot \tan 27^\circ \cdot \tan 45^\circ \cdot \tan 63^\circ \cdot \tan 81^\circ = 1$$

Answer

Taking LHS

$$= \tan 9^\circ \tan 27^\circ \tan 45^\circ \tan 63^\circ \tan 81^\circ$$

$$= \cot(90^\circ - 9^\circ) \tan(90^\circ - 27^\circ) \tan 45^\circ \tan 63^\circ \tan 81^\circ [\because \tan \theta = \cot(90^\circ - \theta)]$$

$$= \cot 81^\circ \cot 63^\circ \tan 45^\circ \tan 63^\circ \tan 81^\circ \left[\because \tan \theta = \frac{1}{\cot \theta} \right]$$

$$= \frac{1}{\tan 81^\circ} \times \frac{1}{\tan 63^\circ} \times \tan 45^\circ \times \tan 63^\circ \times \tan 81^\circ$$

$$= \tan 45^\circ [\because \tan 45^\circ = 1]$$

$$= 1 = \text{RHS}$$

Hence Proved

29. Question

Prove that :

$$\sin 9^\circ \cdot \sin 27^\circ \cdot \sin 63^\circ \cdot \sin 81^\circ$$

$$= \cos 9^\circ \cdot \cos 27^\circ \cdot \cos 63^\circ \cdot \cos 81^\circ$$

Answer

Taking LHS

$$= \sin 9^\circ \sin 27^\circ \sin 63^\circ \sin 81^\circ$$

$$= \cos(90^\circ - 9^\circ) \cos(90^\circ - 27^\circ) \cos(90^\circ - 63^\circ) \cos(90^\circ - 81^\circ)$$

$$= \cos 81^\circ \cos 63^\circ \cos 27^\circ \cos 9^\circ$$

$$\text{Or } \cos 9^\circ \cos 27^\circ \cos 63^\circ \cos 81^\circ = \text{RHS}$$

Hence Proved

30 A. Question

Prove that :

$$\tan 7^\circ \cdot \tan 23^\circ \cdot \tan 60^\circ \cdot \tan 67^\circ \cdot \tan 83^\circ = \sqrt{3}$$

Answer

Taking LHS

$$= \tan 7^\circ \tan 23^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ$$

$$= \cot(90^\circ - 7^\circ) \tan(90^\circ - 23^\circ) \tan 60^\circ \tan 67^\circ \tan 83^\circ [\because \tan \theta = \cot(90^\circ - \theta)]$$

$$= \cot 83^\circ \cot 67^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ \left[\because \tan \theta = \frac{1}{\cot \theta} \right]$$

$$= \frac{1}{\tan 83^\circ} \times \frac{1}{\tan 67^\circ} \times \tan 60^\circ \times \tan 67^\circ \times \tan 83^\circ$$

$$= \tan 60^\circ [\because \tan 60^\circ = \sqrt{3}]$$

$$= \sqrt{3} = \text{RHS}$$

30 B. Question

Prove that :

$$\tan 15^\circ \tan 25^\circ \tan 60^\circ \tan 65^\circ \tan 75^\circ = \sqrt{3}$$

Answer

Taking LHS

$$= \tan 15^\circ \tan 25^\circ \tan 60^\circ \tan 65^\circ \tan 75^\circ$$

$$= \cot(90^\circ - 15^\circ) \tan(90^\circ - 25^\circ) \tan 60^\circ \tan 65^\circ \tan 75^\circ [\because \tan \theta = \cot(90^\circ - \theta)]$$

$$= \cot 75^\circ \cot 65^\circ \tan 60^\circ \tan 65^\circ \tan 75^\circ \left[\because \tan \theta = \frac{1}{\cot \theta} \right]$$

$$= \frac{1}{\tan 75^\circ} \times \frac{1}{\tan 65^\circ} \times \tan 60^\circ \times \tan 65^\circ \times \tan 75^\circ$$

$$= \tan 60^\circ [\because \tan 60^\circ = \sqrt{3}]$$

$$= \sqrt{3} = \text{RHS}$$

31. Question

Find the value off the following:

$$\frac{\sin 50^\circ}{\cos 40^\circ} + \frac{\operatorname{cosec} 40^\circ}{\sec 50^\circ} - 4 \cos 50^\circ \cdot \operatorname{cosec} 40^\circ$$

Answer

$$\frac{\sin 50^\circ}{\cos 40^\circ} + \frac{\operatorname{cosec} 40^\circ}{\sec 50^\circ} - 4 \cos 50^\circ \operatorname{cosec} 40^\circ$$

$$= \frac{\sin 50^\circ}{\sin(90^\circ - 40^\circ)} + \frac{\operatorname{cosec} 40^\circ}{\operatorname{cosec}(90^\circ - 50^\circ)} - 4 \sin(90^\circ - 50^\circ) \operatorname{cosec} 40^\circ$$

$$[\because \cos \theta = \sin(90^\circ - \theta) \text{ and } \sec \theta = \operatorname{cosec}(90^\circ - \theta)]$$

$$= \frac{\sin 50^\circ}{\sin 50^\circ} + \frac{\operatorname{cosec} 40^\circ}{\operatorname{cosec} 40^\circ} - 4 \sin 40^\circ \times \frac{1}{\sin 40^\circ} \left[\because \sin \theta = \frac{1}{\operatorname{cosec} \theta} \right]$$

$$= 1 + 1 - 4$$

$$= -2$$

32. Question

Find the value off the following:

$$\frac{\cos^2 20^\circ + \cos^2 70^\circ}{\sin^2 59^\circ + \sin^2 31^\circ} + \sin 35^\circ \cdot \sec 55^\circ$$

Answer

$$\frac{\cos^2 20^\circ + \cos^2 70^\circ}{\sin^2 59^\circ + \sin^2 31^\circ} + \cos(90^\circ - 35^\circ) \sec 55^\circ$$

$$= \frac{\cos^2 20^\circ + \sin^2(90^\circ - 70^\circ)}{\sin^2(59^\circ) + \cos^2(90^\circ - 31^\circ)} + \cos 55^\circ \sec 55^\circ$$

$$[\because \cos \theta = \sin(90^\circ - \theta) \text{ and } \sec \theta = \operatorname{cosec}(90^\circ - \theta)]$$

$$= \frac{\cos^2 20^\circ + \sin^2 20^\circ}{\sin^2(59^\circ) + \cos^2 59^\circ} + \cos 55^\circ \times \frac{1}{\cos 55^\circ} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= 1 + 1$$

$$= 2$$

33. Question

Find the value off the following:

$$\frac{\tan 50^\circ + \sec 50^\circ}{\cot 40^\circ + \operatorname{cosec} 40^\circ} + \cos 40^\circ \cdot \operatorname{cosec} 50^\circ$$

Answer

$$\frac{\tan 50^\circ + \sec 50^\circ}{\cot 40^\circ + \operatorname{cosec} 40^\circ} + \cos 40^\circ \operatorname{cosec} 50^\circ$$

$$= \frac{\cot(90^\circ - 50^\circ) + \operatorname{cosec}(90^\circ - 50^\circ)}{\cot 40^\circ + \operatorname{cosec} 40^\circ} + \sin(90^\circ - 40^\circ) \operatorname{cosec} 50^\circ$$

$$[\because \tan \theta = \cot(90^\circ - \theta), \sec \theta = \operatorname{cosec}(90^\circ - \theta) \text{ and } \cos \theta = \sin(90^\circ - \theta)]$$

$$= \frac{\cot 40^\circ + \operatorname{cosec} 40^\circ}{\cot 40^\circ + \operatorname{cosec} 40^\circ} + \sin 50^\circ \operatorname{cosec} 50^\circ$$

$$= 1 + \sin 50^\circ \times \frac{1}{\sin 50^\circ} \because \sin \theta = \frac{1}{\operatorname{cosec} \theta}$$

$$= 1 + 1$$

$$= 2$$

34. Question

Find the value off the following:

$$\operatorname{cosec}(65^\circ + \theta) - \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \cot(35^\circ + \theta)$$

Answer

$$\operatorname{cosec}(65^\circ + \theta) - \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \cot(35^\circ + \theta)$$

$$= \sec\{90^\circ - (65^\circ + \theta)\} - \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \tan\{90^\circ - (35^\circ + \theta)\}$$

$$[\because \operatorname{cosec} \theta = \sec(90^\circ - \theta) \text{ and } \cot \theta = \tan(90^\circ - \theta)]$$

$$= \sec(90^\circ - 65^\circ - \theta) - \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \tan(90^\circ - 35^\circ - \theta)$$

$$= \sec(25^\circ - \theta) - \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \tan(55^\circ - \theta)$$

$$= 0$$

35. Question

Find the value off the following:

$$\frac{\cos 35^\circ}{\sin 55^\circ} + \frac{\sin 11^\circ}{\cos 79^\circ} - \cos 28^\circ \cdot \operatorname{cosec} 62^\circ$$

Answer

$$\frac{\cos 35^\circ}{\sin 55^\circ} + \frac{\sin 11^\circ}{\cos 79^\circ} - \cos 28^\circ \operatorname{cosec} 62^\circ$$

$$\begin{aligned}
&= \frac{\sin(90^\circ - 35^\circ)}{\sin 55^\circ} + \frac{\sin 11^\circ}{\sin(90^\circ - 79^\circ)} - \sin(90^\circ - 28^\circ) \operatorname{cosec} 62^\circ \\
&= \frac{\sin 55^\circ}{\sin 55^\circ} + \frac{\sin 11^\circ}{\sin 11^\circ} - \sin 62^\circ \operatorname{cosec} 62^\circ \\
&= 1 + 1 - \sin 62^\circ \times \frac{1}{\sin 62^\circ} \\
&= 1 + 1 - 1 \\
&= 1
\end{aligned}$$

36. Question

Find the value of the following:

$$\frac{\cos^2 20^\circ + \cos^2 70^\circ}{\sin^2 59^\circ + \sin^2 31^\circ}$$

Answer

$$\begin{aligned}
&\frac{\cos^2 20^\circ + \cos^2 70^\circ}{\sin^2 59^\circ + \sin^2 31^\circ} \\
&= \frac{\cos^2 20^\circ + \sin^2(90^\circ - 70^\circ)}{\sin^2(59^\circ) + \cos^2(90^\circ - 31^\circ)}
\end{aligned}$$

$[\because \cos \theta = \sin(90^\circ - \theta) \text{ and } \sec \theta = \operatorname{cosec}(90^\circ - \theta)]$

$$= \frac{\cos^2 20^\circ + \sin^2 20^\circ}{\sin^2(59^\circ) + \cos^2 59^\circ}$$

$$= 1$$

37. Question

Find the value of the following:

$$\operatorname{cosec}(65^\circ + \theta) - \sec(25^\circ - \theta)$$

Answer

$$\begin{aligned}
&\operatorname{cosec}(65^\circ + \theta) - \sec(25^\circ - \theta) \\
&= \sec\{90^\circ - (65^\circ + \theta)\} - \sec(25^\circ - \theta) \\
&[\because \operatorname{cosec} \theta = \sec(90^\circ - \theta)] \\
&= \sec(90^\circ - 65^\circ - \theta) - \sec(25^\circ - \theta)
\end{aligned}$$

$$= \sec (25^\circ - \theta) - \sec (25^\circ - \theta)$$

$$= 0$$

38. Question

Find the value off the following:

$$\cos (60^\circ + \theta) - \sin (30^\circ - \theta)$$

Answer

$$\cos (60^\circ + \theta) - \sin (30^\circ - \theta)$$

$$= \sin \{90^\circ - (60^\circ + \theta)\} - \sin (30^\circ - \theta) [\because \cos \theta = \sin (90^\circ - \theta)]$$

$$= \sin (90^\circ - 60^\circ - \theta) - \sin (30^\circ - \theta)$$

$$= \sin (30^\circ - \theta) - \sin (30^\circ - \theta)$$

$$= 0$$

39. Question

Find the value off the following:

$$\sec 70^\circ \cdot \sin 20^\circ - \cos 20^\circ \cdot \operatorname{cosec} 70^\circ$$

Answer

$$\sec 70^\circ \sin 20^\circ - \cos 20^\circ \operatorname{cosec} 70^\circ$$

$$= \operatorname{cosec} (90^\circ - 70^\circ) \cos (90^\circ - 20^\circ) - \cos 20^\circ \operatorname{cosec} 70^\circ$$

$$[\because \sec \theta = \operatorname{cosec} (90^\circ - \theta) \text{ and } \sin \theta = \cos (90^\circ - \theta)]$$

$$= \operatorname{cosec} 70^\circ \cos 20^\circ - \cos 20^\circ \operatorname{cosec} 70^\circ$$

$$= 0$$

40. Question

Find the value off the following:

$$(\sin 72^\circ + \cos 18^\circ)(\sin 72^\circ - \cos 18^\circ)$$

Answer

$$(\sin 72^\circ + \cos 18^\circ)(\sin 72^\circ - \cos 18^\circ)$$

$$\text{Using the identity , } (a-b)(a+b) = a^2 - b^2$$

$$= (\sin 72^\circ)^2 - (\cos 18^\circ)^2$$

$$\begin{aligned}
&= \{\cos(90^\circ - 72^\circ)\}^2 - (\cos 18^\circ)^2 [\because \sin \theta = \cos (90^\circ - \theta)] \\
&= (\cos 18^\circ)^2 - (\cos 18^\circ)^2 \\
&= 0
\end{aligned}$$

41. Question

Find the value off the following:

$$\left(\frac{\sin 35^\circ}{\cos 55^\circ}\right)^2 + \left(\frac{\cos 55^\circ}{\sin 35^\circ}\right) - 2 \cos 60^\circ$$

Answer

$$\begin{aligned}
&\left(\frac{\sin 35^\circ}{\cos 55^\circ}\right)^2 + \left(\frac{\cos 55^\circ}{\sin 35^\circ}\right) - 2 \cos 60^\circ \\
&= \left(\frac{\sin 35^\circ}{\sin(90^\circ - 55^\circ)}\right)^2 + \left(\frac{\sin(90^\circ - 55^\circ)}{\sin 35^\circ}\right) - 2 \cos 60^\circ [\because \cos \theta = \sin (90^\circ - \theta)] \\
&= \left(\frac{\sin 35^\circ}{\sin 35^\circ}\right)^2 + \left(\frac{\sin 35^\circ}{\sin 35^\circ}\right) - 2 \left(\frac{1}{2}\right) \\
&= 1 + 1 - 1 \\
&= 1
\end{aligned}$$

42. Question

Find the value off the following:

$$\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \cdot \operatorname{cosec} 31^\circ$$

Answer

$$\begin{aligned}
&\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ \\
&= \frac{\sin(90^\circ - 80^\circ)}{\sin 10^\circ} + \sin(90^\circ - 59^\circ) \operatorname{cosec} 31^\circ \\
&= \frac{\sin 10^\circ}{\sin 10^\circ} + \sin 31^\circ \operatorname{cosec} 31^\circ \\
&= 1 + \sin 31^\circ \times \frac{1}{\sin 31^\circ} \\
&= 1 + 1
\end{aligned}$$

$$= 2$$

43. Question

Find the value off the following:

$$(\sin 50^\circ + \theta) - \cos (40^\circ - \theta) + \tan 1^\circ \cdot \tan 10^\circ \tan 20^\circ \cdot \tan 70^\circ \cdot \tan 80^\circ \cdot \tan 89^\circ$$

Answer

$$\begin{aligned} & (\sin 50^\circ + \theta) - \cos (40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 20^\circ \tan 70^\circ \tan 80^\circ \tan 89^\circ \\ &= \cos \{90^\circ - (50^\circ + \theta)\} - \cos (40^\circ - \theta) + \cot(90^\circ - 1^\circ) \tan (90^\circ - 10^\circ) \cot(90^\circ - 20^\circ) \\ & \quad \tan 70^\circ \tan 80^\circ \tan 89^\circ [\because \sin \theta = \cos (90^\circ - \theta) \text{ \& } \tan \theta = \cot (90^\circ - \theta)] \\ &= \cos (40^\circ - \theta) - \cos (40^\circ - \theta) + \cot 89^\circ \cot 80^\circ \cot 70^\circ \tan 70^\circ \tan 80^\circ \tan 89^\circ \\ &= \frac{1}{\tan 89^\circ} \times \frac{1}{\tan 80^\circ} \times \frac{1}{\tan 70^\circ} \times \tan 70^\circ \tan 80^\circ \tan 89^\circ \left[\because \cot \theta = \frac{1}{\tan \theta} \right] \\ &= 1 \end{aligned}$$

44. Question

Find the value off the following:

$$\sec^2 10^\circ - \cot^2 80^\circ + \frac{\sin 15^\circ \cos 75^\circ + \cos 15^\circ \cdot \sin 75^\circ}{\cos \theta \sin(90^\circ - \theta) + \sin \theta \cos(90^\circ - \theta)}$$

Answer

$$\begin{aligned} & \sec^2 10^\circ - \cot^2 80^\circ + \frac{\sin 15^\circ \cos 75^\circ + \cos 15^\circ \sin 75^\circ}{\cos \theta \sin(90^\circ - \theta) + \sin \theta \cos(90^\circ - \theta)} \\ &= \sec^2 10^\circ - \tan^2 (90^\circ - 80^\circ) \\ & \quad + \frac{\cos (90^\circ - 15^\circ) \cos 75^\circ + \sin(90^\circ - 15^\circ) \sin 75^\circ}{\cos \theta \sin(90^\circ - \theta) + \sin \theta \cos(90^\circ - \theta)} \\ & [\because \cot \theta = \tan (90^\circ - \theta), \cos \theta = \sin (90^\circ - \theta) \text{ and } \sin \theta = \cos (90^\circ - \theta)] \\ &= \sec^2 10^\circ - \tan^2 10^\circ + \frac{\cos 75^\circ \cos 75^\circ + \sin 75^\circ \sin 75^\circ}{\cos \theta \cos(\theta) + \sin \theta \sin \theta} \\ & [\because 1 + \tan^2 \theta = \sec^2 \theta \text{ and } \cos^2 \theta + \sin^2 \theta = 1] \\ &= 1 + \frac{\cos^2 75^\circ + \sin^2 75^\circ}{\cos^2 \theta + \sin^2 \theta} \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

45. Question

Find the value off the following:

$$\cos(40^\circ + \theta) - \sin(50^\circ - \theta) + \frac{\cos^2 40^\circ + \cos^2 50^\circ}{\sin^2 40^\circ + \sin^2 50^\circ}$$

Answer

$$\begin{aligned} & \cos(40^\circ + \theta) - \sin(50^\circ - \theta) + \frac{\cos^2 40^\circ + \cos^2 50^\circ}{\sin^2 40^\circ + \sin^2 50^\circ} \\ &= \sin\{90^\circ - (40^\circ + \theta)\} - \sin(50^\circ - \theta) + \frac{\cos^2 40^\circ + \sin^2(90^\circ - 50^\circ)}{\sin^2 40^\circ + \cos^2(90^\circ - 50^\circ)} \end{aligned}$$

$$[\because \sin \theta = \cos(90^\circ - \theta) \text{ and } \cos \theta = \sin(90^\circ - \theta)]$$

$$= \sin(50^\circ - \theta) - \sin(50^\circ - \theta) + \frac{\cos^2 40^\circ + \sin^2(40^\circ)}{\sin^2 40^\circ + \cos^2(40^\circ)}$$

$$= 0 + 1$$

$$= 1$$

46. Question

Find the value off the following:

$$\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 55^\circ, \operatorname{cosec} 35^\circ}{\tan 5^\circ \cdot \tan 25^\circ \cdot \tan 45^\circ \tan 65^\circ \tan 85^\circ}$$

Answer

$$\begin{aligned} & \frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 55^\circ \operatorname{cosec} 35^\circ}{\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ} \\ &= \left(\frac{\sin(90^\circ - 70^\circ)}{\sin 20^\circ} \right) + \frac{\sin(90^\circ - 55^\circ) \operatorname{cosec} 35^\circ}{\cot(90^\circ - 5^\circ) \cot(90^\circ - 25^\circ) \tan 45^\circ \tan 65^\circ \tan 85^\circ} \end{aligned}$$

$$[\because \cos \theta = \sin(90^\circ - \theta) \text{ and } \tan \theta = \cot(90^\circ - \theta)]$$

$$= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\sin 35^\circ \operatorname{cosec} 35^\circ}{\cot 85^\circ \cot 65^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ}$$

$$= 1 + \frac{\sin 35^\circ \times \frac{1}{\sin 35^\circ}}{\frac{1}{\tan 85^\circ} \times \frac{1}{\tan 65^\circ} \times \tan 45^\circ \times \tan 65^\circ \times \tan 85^\circ}$$

$$= 1 + 1 [\because \tan 45^\circ = 1]$$

$$= 2$$

47. Question

Find the value off the following:

$$\left(\frac{\sin 27^\circ}{\cos 63^\circ} \right)^2 + \left(\frac{\cos 63^\circ}{\sin 27^\circ} \right)^2$$

Answer

$$\begin{aligned} & \left(\frac{\sin 27^\circ}{\cos 63^\circ} \right)^2 + \left(\frac{\cos 63^\circ}{\sin 27^\circ} \right)^2 \\ &= \left(\frac{\sin 27^\circ}{\sin(90^\circ - 63^\circ)} \right)^2 + \left(\frac{\sin(90^\circ - 63^\circ)}{\sin 27^\circ} \right)^2 [\because \cos \theta = \sin(90^\circ - \theta)] \\ &= \left(\frac{\sin 27^\circ}{\sin 27^\circ} \right)^2 + \left(\frac{\sin 27^\circ}{\sin 27^\circ} \right)^2 \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

48 A. Question

Evaluate the following

$$\frac{3 \sin 5^\circ}{\cos 85^\circ} + \frac{2 \cos 33^\circ}{\sin 57^\circ}$$

Answer

$$\begin{aligned} & \frac{3 \sin 5^\circ}{\cos 85^\circ} + \frac{2 \cos 33^\circ}{\sin 57^\circ} \\ &= \frac{3 \sin 5^\circ}{\sin(90^\circ - 85^\circ)} + \frac{2 \sin(90^\circ - 33^\circ)}{\sin 57^\circ} [\because \cos \theta = \sin(90^\circ - \theta)] \\ &= \frac{3 \sin 5^\circ}{\sin 5^\circ} + \frac{2 \sin 57^\circ}{\sin 57^\circ} \\ &= 3 + 2 \\ &= 5 \end{aligned}$$

48 B. Question

Evaluate the following

$$\frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2$$

Answer

$$\begin{aligned} & \frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2 \\ &= \frac{\cot 54^\circ}{\cot(90^\circ - 36^\circ)} + \frac{\cot(90^\circ - 20^\circ)}{\cot 70^\circ} - 2 \quad [\because \tan \theta = \cot(90^\circ - \theta)] \\ &= \frac{\cot 54^\circ}{\cot 54^\circ} + \frac{\cot 70^\circ}{\cot 70^\circ} - 2 \\ &= 1 + 1 - 2 \\ &= 0 \end{aligned}$$

48 C. Question

Evaluate the following

$$\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ$$

Answer

$$\begin{aligned} & \frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ \\ &= \frac{\cos 80^\circ}{\cos(90^\circ - 10^\circ)} + \sin(90^\circ - 59^\circ) \operatorname{cosec} 31^\circ \\ & \quad [\because \cos \theta = \sin(90^\circ - \theta) \text{ and } \sec \theta = \operatorname{cosec}(90^\circ - \theta)] \\ &= \frac{\cos 80^\circ}{\cos 80^\circ} + \sin 31^\circ \operatorname{cosec} 31^\circ \\ &= 1 + \sin 31^\circ \times \frac{1}{\sin 31^\circ} \quad \because \sin \theta = \frac{1}{\operatorname{cosec} \theta} \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

48 D. Question

Evaluate the following

$$\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$$

Answer

We know that

$$\begin{aligned}
 \cos \theta &= \sin (90^\circ - \theta) \\
 &= \sin (90^\circ - 38^\circ) \sin (90^\circ - 52^\circ) - \sin 38^\circ \sin 52^\circ \\
 &= \sin 52^\circ \sin 38^\circ - \sin 38^\circ \sin 52^\circ \\
 &= 0
 \end{aligned}$$

48 E. Question

Evaluate the following

$$\sec 41^\circ \sin 49^\circ + \cos 49^\circ \operatorname{cosec} 41^\circ$$

Answer

We know that

$$\begin{aligned}
 \sec \theta &= \operatorname{cosec} (90^\circ - \theta) \text{ and } \cos \theta = \sin (90^\circ - \theta) \\
 \operatorname{Cosec} (90^\circ - 41^\circ) \sin 49^\circ + \sin (90^\circ - 49^\circ) \operatorname{cosec} 41^\circ \\
 &= \operatorname{cosec} 49^\circ \sin 49^\circ + \sin 41^\circ \operatorname{cosec} 41^\circ \\
 &= \frac{1}{\sin 49^\circ} \times \sin 49^\circ + \sin 41^\circ \times \frac{1}{\sin 41^\circ} \left[\because \sin \theta = \frac{1}{\operatorname{cosec} \theta} \right] \\
 &= 1 + 1 \\
 &= 2
 \end{aligned}$$

Exercise 4.4**1. Question**

Fill in the blanks

(i) $\sin^2 \theta \operatorname{cosec}^2 \theta = \dots\dots\dots$

(ii) $1 + \tan^2 \theta = \dots\dots\dots$

(iii) Reciprocal $\sin \theta \cdot \cot \theta = \dots\dots\dots$

(iv) $1 - \dots\dots\dots = \cos^2 \theta$

(v) $\tan A = \frac{\dots\dots\dots}{\cos A}$

$$(vi) \dots = \frac{\cos A}{\sin A}$$

(vii) $\cos \theta$ is reciprocal of

(viii) Reciprocal of $\sin \theta$ is.....

(ix) Value of $\sin \theta$ in terms of $\cos \theta$ is

(x) Value of $\cos \theta$ in terms of $\sin \theta$ is

Answer

(i) Given: $\sin^2 \theta \operatorname{cosec}^2 \theta$

$$\Rightarrow \sin^2 \theta \times \frac{1}{\sin^2 \theta} \left[\because \sin \theta = \frac{1}{\csc \theta} \right]$$

$$= 1$$

(ii) Given: $1 + \tan^2 \theta$

$$= 1 + \left(\frac{\sin \theta}{\cos \theta} \right)^2 \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$= 1 + \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$= \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} \left[\because \cos^2 \theta + \sin^2 \theta = 1 \right]$$

$$= \frac{1}{\cos^2 \theta}$$

$$= \sec^2 \theta \left[\because \cos \theta = \frac{1}{\sec \theta} \right]$$

(iii) Given : $\sin \theta \cot \theta$

Firstly, we simplify the given trigonometry

$$= \sin \theta \times \frac{\cos \theta}{\sin \theta} \left[\because \cot \theta = \frac{\cos \theta}{\sin \theta} \right]$$

$$= \cos \theta$$

Now, the reciprocal of $\cos \theta$ is

$$= \frac{1}{\cos \theta}$$

$$= \sec \theta \left[\because \cos \theta = \frac{1}{\sec \theta} \right]$$

(iv) Given: $1 - x = \cos^2 \theta$

Subtracting 1 to both the sides, we get

$$1 - x - 1 = \cos^2 \theta - 1$$

$$\Rightarrow -x = -\sin^2 \theta$$

$$\Rightarrow x = \sin^2 \theta$$

(v) $\tan A = \frac{\sin A}{\cos A}$

(vi) $\cot A = \frac{\cos A}{\sin A}$

(vii) $\cos \theta = \frac{1}{\sec \theta}$

(viii) $\sin \theta = \frac{1}{\csc \theta}$

(ix) We know that

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$$

$$\Rightarrow \sin \theta = \sqrt{1 - \cos^2 \theta}$$

(x) We know that

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$$

$$\Rightarrow \cos \theta = \sqrt{1 - \sin^2 \theta}$$

2. Question

If $\sin \theta = p$ and $\cos \theta = q$, what is the relation between p and q ?

Answer

We know that,

$$\cos^2 \theta + \sin^2 \theta = 1 \dots (i)$$

Given : $\sin \theta = p$ and $\cos \theta = q$

Putting the values of $\sin \theta$ and $\cos \theta$ in eq. (i), we get

$$(q)^2 + (p)^2 = 1$$

$$\Rightarrow p^2 + q^2 = 1$$

3. Question

If $\cos A = x$, express $\sin A$ in terms of x

Answer

We know that

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$$

$$\Rightarrow \sin \theta = \sqrt{1 - \cos^2 \theta}$$

And Given that $\cos \theta = x$

$$\Rightarrow \sin \theta = \sqrt{1 - x^2}$$

4. Question

If $x \cos \theta = 1$ and $y \sin \theta = 1$ find the value of $\tan \theta$.

Answer

Given $x \cos \theta = 1$ and $y \sin \theta = 1$

$$\Rightarrow \cos \theta = \frac{1}{x} \text{ and } \sin \theta = \frac{1}{y}$$

Now, we know that

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

Putting the value of $\sin \theta$ and $\cos \theta$, we get

$$\tan \theta = \frac{\frac{1}{y}}{\frac{1}{x}}$$

$$\Rightarrow \tan \theta = \frac{x}{y}$$

5. Question

If $\cos 40^\circ = p$, then write the value of $\sin 40^\circ$ in terms of p .

Answer

We know that

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\Rightarrow \cos^2 40^\circ + \sin^2 40^\circ = 1$$

$$\Rightarrow \sin^2 40^\circ = 1 - \cos^2 40^\circ$$

$$\Rightarrow \sin 40^\circ = \sqrt{(1 - \cos^2 40^\circ)}$$

And Given that $\cos 40^\circ = p$

$$\Rightarrow \sin 40^\circ = \sqrt{(1 - p^2)}$$

6. Question

If $\sin 77^\circ = x$, then write the value of $\cos 77^\circ$ in terms of x .

Answer

We know that

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\Rightarrow \cos^2 77^\circ + \sin^2 77^\circ = 1$$

$$\Rightarrow \cos^2 77^\circ = 1 - \sin^2 77^\circ$$

$$\Rightarrow \cos 77^\circ = \sqrt{(1 - \sin^2 77^\circ)}$$

And Given that $\sin 77^\circ = x$

$$\Rightarrow \cos 77^\circ = \sqrt{(1 - x^2)}$$

7. Question

If $\cos 55^\circ = x^2$, then write the value of $\sin 55^\circ$ in terms of x .

Answer

We know that

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\Rightarrow \cos^2 55^\circ + \sin^2 55^\circ = 1$$

$$\Rightarrow \sin^2 55^\circ = 1 - \cos^2 55^\circ$$

$$\Rightarrow \sin 55^\circ = \sqrt{(1 - \cos^2 55^\circ)}$$

And Given that $\cos 55^\circ = x^2$

$$\Rightarrow \sin 55^\circ = \sqrt{1 - (x^2)^2}$$

$$\Rightarrow \sin 55^\circ = \sqrt{1 - x^4}$$

8. Question

If, $\sin 50^\circ = \alpha$ then write the value of $\cos 50^\circ$ in terms of α .

Answer

We know that

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\Rightarrow \cos^2 50^\circ + \sin^2 50^\circ = 1$$

$$\Rightarrow \cos^2 50^\circ = 1 - \sin^2 50^\circ$$

$$\Rightarrow \cos 50^\circ = \sqrt{1 - \sin^2 50^\circ}$$

And Given that $\sin 50^\circ = a$

$$\Rightarrow \cos 50^\circ = \sqrt{1 - a^2}$$

9. Question

If $x \cos A = 1$ and $\tan A = y$, then what is the value of $x^2 - y^2$.

Answer

Given $x \cos A = 1$ and $\tan A = y$

$$\Rightarrow x = \frac{1}{\cos A} \text{ and } \frac{\sin A}{\cos A} = y \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

To find: $x^2 - y^2$

Putting the values of x and y , we get

$$\left(\frac{1}{\cos A} \right)^2 - \left(\frac{\sin A}{\cos A} \right)^2$$

$$= \frac{1}{\cos^2 A} - \frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{1 - \sin^2 A}{\cos^2 A}$$

$$= \frac{\cos^2 A}{\cos^2 A} \left[\because \cos^2 \theta + \sin^2 \theta = 1 \right]$$

$$= 1$$

10. Question

Prove the followings identities:

$$(1 - \sin \theta)(1 + \sin \theta) = \cos^2 \theta$$

Answer

Taking LHS = $(1 - \sin \theta)(1 + \sin \theta)$

Using identity , $(a + b)(a - b) = (a^2 - b^2)$, we get

$$= (1)^2 - (\sin \theta)^2$$

$$= 1 - \sin^2 \theta$$

$$= \cos^2 \theta [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \text{RHS}$$

Hence Proved

11. Question

Prove the followings identities:

$$(1 + \cos \theta)(1 - \cos \theta) = \sin^2 \theta$$

Answer

Taking LHS = $(1 - \cos \theta)(1 + \cos \theta)$

Using identity , $(a + b)(a - b) = (a^2 - b^2)$, we get

$$= (1)^2 - (\cos \theta)^2$$

$$= 1 - \cos^2 \theta$$

$$= \sin^2 \theta [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \text{RHS}$$

Hence Proved

12. Question

Prove the followings identities:

$$\frac{(1 - \cos \theta)(1 + \cos \theta)}{(1 - \sin \theta)(1 + \sin \theta)} = \tan^2 \theta$$

Answer

$$\begin{aligned} \text{Taking LHS } & \frac{(1 - \cos \theta)(1 + \cos \theta)}{(1 - \sin \theta)(1 + \sin \theta)} \\ &= \frac{(1)^2 - (\cos \theta)^2}{(1)^2 - (\sin \theta)^2} \text{ [Using identity , (a + b) (a - b) = (a}^2 - \text{b}^2)] \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} [\because \cos^2 \theta + \sin^2 \theta = 1] \\ &= \tan^2 \theta \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right] \\ &= \text{RHS} \end{aligned}$$

Hence Proved

13. Question

Prove the followings identities:

$$\frac{1}{\sec \theta + \tan \theta} = \sec \theta - \tan \theta$$

Answer

$$\begin{aligned} \text{Taking LHS } &= \frac{1}{\sec \theta + \tan \theta} \\ \text{Multiplying and divide by the conjugate of } &\sec \theta + \tan \theta \\ &= \frac{1}{\sec \theta + \tan \theta} \times \frac{\sec \theta - \tan \theta}{\sec \theta - \tan \theta} \\ &= \frac{\sec \theta - \tan \theta}{\sec^2 \theta - \tan^2 \theta} \text{ [Using identity , (a + b) (a - b) = (a}^2 - \text{b}^2)] \\ &= \sec \theta - \tan \theta [\because 1 + \tan^2 \theta = \sec^2 \theta] \\ &= \text{RHS} \end{aligned}$$

Hence Proved

14 A. Question

Prove the following identities :

$$\sin \theta \cdot \cot \theta = \cos \theta$$

Answer

Taking LHS = $\sin \theta \cot \theta$

$$= \sin \theta \times \frac{\cos \theta}{\sin \theta} \left[\because \cot \theta = \frac{\cos \theta}{\sin \theta} \right]$$

$$= \cos \theta$$

$$= \text{RHS}$$

Hence Proved

14 B. Question

Prove the following identities :

$$\sin^2 \theta (1 + \cot^2 \theta) = 1$$

Answer

Taking LHS = $\sin^2 \theta (1 + \cot^2 \theta)$

$$= \sin^2 \theta (\operatorname{cosec}^2 \theta) \left[\because \cot^2 \theta + 1 = \operatorname{cosec}^2 \theta \right]$$

$$= \sin^2 \theta \times \frac{1}{\sin^2 \theta} \left[\because \sin \theta = \frac{1}{\operatorname{csc} \theta} \right]$$

$$= 1$$

$$= \text{RHS}$$

Hence Proved

14 C. Question

Prove the following identities :

$$\cos^2 A (\tan^2 A + 1) = 1$$

Answer

Taking LHS = $\cos^2 A (\tan^2 A + 1)$

$$= \cos^2 \theta (\sec^2 \theta) \left[\because 1 + \tan^2 \theta = \sec^2 \theta \right]$$

$$= \cos^2 \theta \times \frac{1}{\cos^2 \theta} \left[\because \cos \theta = \frac{1}{\sec \theta} \right]$$

$$= 1$$

$$= \text{RHS}$$

Hence Proved

14 D. Question

Prove the following identities :

$$\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$$

Answer

$$\text{Taking LHS} = \tan^4\theta + \tan^2\theta$$

$$= (\tan^2\theta)^2 + \tan^2\theta$$

$$= (\sec^2\theta - 1)^2 + (\sec^2\theta - 1) [\because 1 + \tan^2\theta = \sec^2\theta]$$

$$= \sec^4\theta + 1 - 2\sec^2\theta + \sec^2\theta - 1 [\because (a - b)^2 = (a^2 + b^2 - 2ab)]$$

$$= \sec^4\theta - \sec^2\theta$$

$$= \text{RHS}$$

Hence Proved

14 E. Question

Prove the following identities :

$$\frac{(1 + \tan^2\theta)\sin^2\theta}{\tan\theta} = \tan\theta$$

Answer

$$\text{Taking LHS} = \frac{(1 + \tan^2\theta)\sin^2\theta}{\tan\theta}$$

$$= \frac{(\sec^2\theta)\sin^2\theta}{\frac{\sin\theta}{\cos\theta}} [\because 1 + \tan^2\theta = \sec^2\theta]$$

$$= \frac{1 \times \sin\theta \times \cos\theta}{\cos^2\theta} \left[\because \cos\theta = \frac{1}{\sec\theta} \right]$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$

$$= \text{RHS}$$

Hence Proved

14 F. Question

Prove the following identities :

$$\frac{\sin^2 \theta}{\cos^2 \theta} + 1 = \frac{\tan^2 \theta}{\sin^2 \theta}$$

Answer

$$\text{Taking LHS} = \frac{\sin^2 \theta}{\cos^2 \theta} + 1$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \frac{1}{\cos^2 \theta}$$

$$= \sec^2 \theta \left[\because \cos \theta = \frac{1}{\sec \theta} \right]$$

$$\text{Now, RHS} = \frac{\tan^2 \theta}{\sin^2 \theta}$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$= \frac{\sin^2 \theta}{\sin^2 \theta \times \cos^2 \theta}$$

$$= \frac{1}{\cos^2 \theta}$$

$$= \sec^2 \theta \left[\because \cos \theta = \frac{1}{\sec \theta} \right]$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence Proved

14 G. Question

Prove the following identities :

$$\frac{3 - 4\sin^2 \theta}{\cos^2 \theta} = 3 - \tan^2 \theta$$

Answer

$$\text{Taking LHS} = \frac{3 - 4\sin^2 \theta}{\cos^2 \theta}$$

$$= \frac{3}{\cos^2 \theta} - \frac{4 \sin^2 \theta}{\cos^2 \theta}$$

$$= 3 \sec^2 \theta - 4 \tan^2 \theta$$

We know that,

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$= 3(1 + \tan^2 \theta) - 4 \tan^2 \theta$$

$$= 3 + 3 \tan^2 \theta - 4 \tan^2 \theta$$

$$= 3 - \tan^2 \theta$$

=RHS

Hence Proved

14 H. Question

Prove the following identities :

$$(1 + \tan^2 \theta) \cos \theta \cdot \sin \theta = \tan \theta$$

Answer

$$\text{Taking LHS} = (1 + \tan^2 \theta) \cos \theta \sin \theta$$

$$= (\sec^2 \theta) \cos \theta \sin \theta \left[\because 1 + \tan^2 \theta = \sec^2 \theta \right]$$

$$= \frac{1}{\cos^2 \theta} \times \cos \theta \times \sin \theta \left[\because \cos \theta = \frac{1}{\sec \theta} \right]$$

$$= \frac{\sin \theta}{\cos \theta}$$

$$= \tan \theta \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

=RHS

Hence Proved

14 I. Question

Prove the following identities :

$$\sin^2 \theta - \cos^2 \phi = \sin^2 \phi - \cos^2 \theta$$

Answer

Taking LHS = $\sin^2 \theta - \cos^2 \varphi$

$$= (1 - \cos^2 \theta) - (1 - \sin^2 \varphi) [\because \cos^2 \theta + \sin^2 \theta = 1] \text{ \& } [\because \cos^2 \varphi + \sin^2 \varphi = 1]$$

$$= 1 - \cos^2 \theta - 1 + \sin^2 \varphi$$

$$= \sin^2 \varphi - \cos^2 \theta$$

=RHS

Hence Proved

14 J. Question

Prove the following identities :

$$\frac{1 - \tan^2 \theta}{\cot^2 \theta - 1} = \tan^2 \theta$$

Answer

$$\text{Taking LHS} = \frac{1 - \tan^2 \theta}{\cot^2 \theta - 1}$$

$$= \frac{1 - \frac{\sin^2 \theta}{\cos^2 \theta}}{\frac{\cos^2 \theta}{\sin^2 \theta} - 1}$$

$$= \frac{\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}}{\frac{\cos^2 \theta - \sin^2 \theta}{\sin^2 \theta}}$$

$$= \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta} \times \frac{\sin^2 \theta}{\cos^2 \theta - \sin^2 \theta}$$

$$= \tan^2 \theta \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

=RHS

Hence Proved

15 A. Question

Prove the following identities :

$$(1 - \cos \theta)(1 + \cos \theta)(1 + \cot^2 \theta) = 1$$

Answer

$$\text{Taking LHS} = (1 - \cos\theta)(1 + \cos\theta)(1 + \cot^2 \theta)$$

Using identity, $(a + b)(a - b) = (a^2 - b^2)$ in first two terms, we get

$$= (1)^2 - (\cos\theta)^2 (\operatorname{cosec}^2 \theta) [\because \cot^2 \theta + 1 = \operatorname{cosec}^2 \theta]$$

$$= (1 - \cos^2 \theta) (\operatorname{cosec}^2 \theta)$$

$$= (\sin^2 \theta) (\operatorname{cosec}^2 \theta) [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \sin^2 \theta \times \frac{1}{\sin^2 \theta} \left[\because \sin \theta = \frac{1}{\csc \theta} \right]$$

$$= 1$$

$$= \text{RHS}$$

Hence Proved

15 B. Question

Prove the following identities :

$$\frac{(1 + \sin \theta)^2 + (1 - \sin \theta)^2}{2 \cos^2 \theta} = \frac{1 + \sin^2 \theta}{1 - \sin^2 \theta}$$

Answer

$$\begin{aligned} \text{Taking LHS} &= \frac{(1 + \sin \theta)^2 + (1 - \sin \theta)^2}{2 \cos^2 \theta} \\ &= \frac{1 + \sin^2 \theta + 2 \sin \theta + 1 + \sin^2 \theta - 2 \sin \theta}{2 \cos^2 \theta} \end{aligned}$$

$$[\because (a + b)^2 = (a^2 + b^2 + 2ab) \text{ and } (a - b)^2 = (a^2 + b^2 - 2ab)]$$

$$= \frac{2 + 2 \sin^2 \theta}{2 \cos^2 \theta}$$

$$= \frac{2(1 + \sin^2 \theta)}{2(1 - \sin^2 \theta)} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \frac{1 + \sin^2 \theta}{1 - \sin^2 \theta}$$

$$= \text{RHS}$$

Hence Proved

15 C. Question

Prove the following identities :

$$\frac{\cos^2 \theta (1 - \cos \theta)}{\sin^2 \theta (1 - \sin \theta)} = \frac{1 + \sin \theta}{1 + \cos \theta}$$

Answer

$$\text{Taking LHS} = \frac{\cos^2 \theta (1 - \cos \theta)}{\sin^2 \theta (1 - \sin \theta)}$$

Multiplying and divide by the conjugate of $(1 - \sin \theta)$, we get

$$\begin{aligned} &= \frac{\cos^2 \theta (1 - \cos \theta)}{\sin^2 \theta (1 - \sin \theta)} \times \frac{(1 + \sin \theta)}{(1 + \sin \theta)} \\ &= \frac{\cos^2 \theta (1 - \cos \theta) (1 + \sin \theta)}{\sin^2 \theta [(1)^2 - (\sin \theta)^2]} \\ &= \frac{\cos^2 \theta (1 - \cos \theta) (1 + \sin \theta)}{\sin^2 \theta \times \cos^2 \theta} \\ &= \frac{(1 - \cos \theta) (1 + \sin \theta)}{\sin^2 \theta} \end{aligned}$$

Now, multiply and divide by conjugate of $1 - \cos \theta$, we get

$$\begin{aligned} &= \frac{(1 - \cos \theta) (1 + \sin \theta)}{\sin^2 \theta} \times \frac{(1 + \cos \theta)}{(1 + \cos \theta)} \\ &= \frac{(1^2 - \cos^2 \theta) (1 + \sin \theta)}{\sin^2 \theta (1 + \cos \theta)} \\ &= \frac{\sin^2 \theta (1 + \sin \theta)}{\sin^2 \theta (1 + \cos \theta)} \\ &= \frac{1 + \sin \theta}{1 + \cos \theta} \end{aligned}$$

=RHS

Hence Proved

15 D. Question

Prove the following identities :

$$(\sin \theta - \cos \theta)^2 = 1 - 2 \sin \theta \cdot \cos \theta$$

Answer

Taking LHS = $(\sin \theta - \cos \theta)^2$

Using the identity, $(a - b)^2 = (a^2 + b^2 - 2ab)$

$$= \sin^2 \theta + \cos^2 \theta - 2\sin \theta \cos \theta$$

$$= 1 - 2\sin \theta \cos \theta [\because \cos^2 \theta + \sin^2 \theta = 1]$$

=RHS

Hence Proved

15 E. Question

Prove the following identities :

$$(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$$

Answer

Taking LHS = $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2$

Using the identity, $(a + b)^2 = (a^2 + b^2 + 2ab)$ and $(a - b)^2 = (a^2 + b^2 - 2ab)$

$$= \sin^2 \theta + \cos^2 \theta + 2\sin \theta \cos \theta + \sin^2 \theta + \cos^2 \theta - 2\sin \theta \cos \theta$$

$$= 1 + 1 [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= 2$$

=RHS

Hence Proved

15 F. Question

Prove the following identities :

$$(\sin \theta + b \cos \theta)^2 + (a \cos \theta - b \sin \theta)^2 = a^2 + b^2$$

Answer

Taking LHS = $(\sin \theta + b \cos \theta)^2 + (a \cos \theta - b \sin \theta)^2$

Using the identity, $(a + b)^2 = (a^2 + b^2 + 2ab)$ and $(a - b)^2 = (a^2 + b^2 - 2ab)$

$$= a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta + a^2 \cos^2 \theta + b^2 \sin^2 \theta - 2ab \sin \theta \cos \theta$$

$$= a^2 \sin^2 \theta + a^2 \cos^2 \theta + b^2 \sin^2 \theta + b^2 \cos^2 \theta$$

$$= a^2 (\sin^2 \theta + \cos^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta)$$

$$= a^2 + b^2 [\because \cos^2 \theta + \sin^2 \theta = 1]$$

=RHS

Hence Proved

15 G. Question

Prove the following identities :

$$\cos^4 A + \sin^4 A + 2 \sin^2 A \cos^2 A = 1$$

Answer

$$\text{Taking LHS} = \cos^4 A + \sin^4 A + 2 \sin^2 A \cos^2 A$$

$$\text{Using the identity, } (a + b)^2 = (a^2 + b^2 + 2ab)$$

$$\text{Here, } a = \cos^2 A \text{ and } b = \sin^2 A$$

$$= (\cos^2 A + \sin^2 A) [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= 1$$

15 H. Question

Prove the following identities :

$$\sin^4 A - \cos^4 A = 2 \sin^2 A - 1 = 1 - 2 \cos^2 A = \sin^2 A - \cos^2 A$$

Answer

Given:

$$\sin^4 A - \cos^4 A = 2 \sin^2 A - 1 = 1 - 2 \cos^2 A = \sin^2 A - \cos^2 A$$

I
II
III
IV

Taking I term

$$= \sin^4 A - \cos^4 A \rightarrow \text{I term}$$

$$= (\sin^2 A)^2 - (\cos^2 A)^2$$

$$= (\sin^2 A - \cos^2 A)(\sin^2 A + \cos^2 A)$$

$$[\because (a^2 - b^2) = (a + b)(a - b)]$$

$$= (\sin^2 A - \cos^2 A)(1) [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= (\sin^2 A - \cos^2 A) \dots(i) \rightarrow \text{IV term}$$

From Eq. (i)

$$= \{\sin^2 A - (1 - \sin^2 A)\} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \sin^2 A - 1 + \sin^2 A$$

$$= 2 \sin^2 A - 1 \rightarrow \text{II term}$$

Again, From Eq. (i)

$$= \{(1 - \cos^2 A) - \cos^2 A\} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= 1 - 2 \cos^2 A \rightarrow \text{III term}$$

Hence, I = II = III = IV

Hence Proved

15 I. Question

Prove the following identities :

$$\cos^4 \theta - \sin^4 \theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1$$

Answer

Given:

$$\begin{array}{ccccc} \cos^4 \theta - \sin^4 \theta & = & \cos^2 \theta - \sin^2 \theta & = & 2 \cos^2 \theta - 1 \\ \text{I} & & \text{II} & & \text{III} \end{array}$$

Taking I term

$$= \cos^4 \theta - \sin^4 \theta \rightarrow \text{I term}$$

$$= (\cos^2 \theta)^2 - (\sin^2 \theta)^2$$

$$= (\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta)$$

$$[\because (a^2 - b^2) = (a + b)(a - b)]$$

$$= (\cos^2 \theta - \sin^2 \theta)(1) [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= (\cos^2 \theta - \sin^2 \theta) \dots(i) \rightarrow \text{II term}$$

From Eq. (i)

$$= \{\cos^2 \theta - (1 - \cos^2 \theta)\} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= 2 \cos^2 \theta - 1 \rightarrow \text{III term}$$

Hence, I = II = III

Hence Proved

15 J. Question

Prove the following identities :

$$2 \cos^2 \theta - \cos^4 \theta + \sin^4 \theta = 1$$

Answer

$$\text{Taking LHS} = 2 \cos^2 \theta - \cos^4 \theta + \sin^4 \theta$$

$$= 2 \cos^2 \theta - (\cos^4 \theta - \sin^4 \theta)$$

$$= 2 \cos^2 \theta - [(\cos^2 \theta)^2 - (\sin^2 \theta)^2]$$

$$\text{Using identity, } (a^2 - b^2) = (a + b)(a - b)$$

$$= 2 \cos^2 \theta - [(\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta)]$$

$$= 2 \cos^2 \theta - [(\cos^2 \theta - \sin^2 \theta)(1)] \quad [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= 2 \cos^2 \theta - \cos^2 \theta + \sin^2 \theta$$

$$= \cos^2 \theta + \sin^2 \theta$$

$$= 1 \quad [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \text{RHS}$$

Hence Proved

15 K. Question

Prove the following identities :

$$1 - 2 \cos^2 \theta + \cos^4 \theta = \sin^4 \theta$$

Answer

$$\text{Taking LHS} = 1 - 2 \cos^2 \theta + \cos^4 \theta$$

We know that,

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$= 1 - 2 \cos^2 \theta + (\cos^2 \theta)^2$$

$$\begin{aligned}
&= 1 - 2 \cos^2 \theta + (1 - \sin^2 \theta)^2 \\
&= 1 - 2 \cos^2 \theta + 1 + \sin^4 \theta - 2\sin^2 \theta \\
&= 2 - 2(\cos^2 \theta + \sin^2 \theta) + \sin^4 \theta \\
&= 2 - 2(1) + \sin^4 \theta \\
&= \sin^4 \theta \\
&= \text{RHS}
\end{aligned}$$

Hence Proved

15 L. Question

Prove the following identities :

$$1 - 2 \sin^2 \theta + \sin^4 \theta = \cos^4 \theta$$

Answer

$$\text{Taking LHS} = 1 - 2 \sin^2 \theta + \sin^4 \theta$$

We know that,

$$\begin{aligned}
&\cos^2 \theta + \sin^2 \theta = 1 \\
&= 1 - 2 \sin^2 \theta + (\sin^2 \theta)^2 \\
&= 1 - 2 \sin^2 \theta + (1 - \cos^2 \theta)^2 \\
&= 1 - 2 \sin^2 \theta + 1 + \cos^4 \theta - 2\cos^2 \theta \\
&= 2 - 2(\cos^2 \theta + \sin^2 \theta) + \cos^4 \theta \\
&= 2 - 2(1) + \cos^4 \theta \\
&= \cos^4 \theta \\
&= \text{RHS}
\end{aligned}$$

Hence Proved

16 A. Question

Prove that the following identities :

$$\sec^2 \theta + \operatorname{cosec}^2 \theta = \sec^2 \theta \cdot \operatorname{cosec}^2 \theta$$

Answer

Taking LHS = $\sec^2 \theta + \operatorname{cosec}^2 \theta$

$$= \frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta} \because \left[\cos \theta = \frac{1}{\sec \theta} \right] \text{ and } \left[\sin \theta = \frac{1}{\csc \theta} \right]$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta \sin^2 \theta} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \frac{1}{\cos^2 \theta \sin^2 \theta}$$

$$= \sec^2 \theta \times \operatorname{cosec}^2 \theta \because \left[\cos \theta = \frac{1}{\sec \theta} \right] \text{ and } \left[\sin \theta = \frac{1}{\csc \theta} \right]$$

=RHS

Hence Proved

16 B. Question

Prove that the following identities :

$$\frac{\cos^2 \theta}{\sin \theta} + \sin \theta = \operatorname{cosec} \theta$$

Answer

$$\text{Taking LHS} = \frac{\cos^2 \theta}{\sin \theta} + \sin \theta$$

$$= \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta}$$

$$= \frac{1}{\sin \theta} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \operatorname{cosec} \theta$$

=RHS

Hence Proved

16 C. Question

Prove that the following identities :

$$\cot \theta + \tan \theta = \operatorname{cosec} \theta \cdot \sec \theta$$

Answer

$$\text{Taking LHS} = \cot \theta + \tan \theta$$

$$= \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \left[\because \cot \theta = \frac{\cos \theta}{\sin \theta} \right] \text{ and } \left[\tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$= \frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta \sin \theta} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \frac{1}{\cos \theta \sin \theta}$$

$$= \operatorname{cosec} \theta \sec \theta \because \left[\cos \theta = \frac{1}{\sec \theta} \right] \text{ and } \left[\sin \theta = \frac{1}{\csc \theta} \right]$$

=RHS

Hence Proved

17. Question

Prove that the following identities :

$$\frac{1 - \sin \theta}{1 + \sin \theta} = \left(\frac{1 - \sin \theta}{\cos \theta} \right)^2$$

Answer

$$\text{Taking LHS} = \frac{1 - \sin \theta}{1 + \sin \theta}$$

Multiplying and divide by the conjugate of $1 + \sin \theta$, we get

$$= \frac{1 - \sin \theta}{1 + \sin \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta}$$

$$= \frac{(1 - \sin \theta)^2}{(1)^2 - (\sin \theta)^2}$$

$$[\because (a - b)(a - b) = (a - b)^2 \text{ and } (a + b)(a - b) = (a^2 - b^2)]$$

$$= \frac{(1 - \sin \theta)^2}{1 - \sin^2 \theta}$$

$$= \frac{(1 - \sin \theta)^2}{\cos^2 \theta} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \left(\frac{1 - \sin \theta}{\cos \theta} \right)^2$$

=RHS

Hence Proved

18. Question

Prove that the following identities :

$$\frac{1 - \cos \theta}{1 + \cos \theta} = \left(\frac{1 - \cos \theta}{\sin \theta} \right)^2$$

Answer

$$\text{Taking LHS} = \frac{1 - \cos \theta}{1 + \cos \theta}$$

Multiplying and divide by the conjugate of $1 + \cos \theta$, we get

$$= \frac{1 - \cos \theta}{1 + \cos \theta} \times \frac{1 - \cos \theta}{1 - \cos \theta}$$

$$= \frac{(1 - \cos \theta)^2}{(1)^2 - (\cos \theta)^2}$$

$$[\because (a - b)(a - b) = (a - b)^2 \text{ and } (a + b)(a - b) = (a^2 - b^2)]$$

$$= \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}$$

$$= \frac{(1 - \cos \theta)^2}{\sin^2 \theta} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \left(\frac{1 - \cos \theta}{\sin \theta} \right)^2$$

=RHS

Hence Proved

19. Question

Prove that the following identities :

$$\frac{1 - \cos \theta}{1 + \cos \theta} = \left(\frac{1 - \cos \theta}{\sin \theta} \right)^2$$

Answer

$$\text{Taking LHS} = \frac{1 - \cos \theta}{1 + \cos \theta}$$

Multiplying and divide by the conjugate of $1 + \cos \theta$, we get

$$= \frac{1 - \cos \theta}{1 + \cos \theta} \times \frac{1 - \cos \theta}{1 - \cos \theta}$$

$$= \frac{(1 - \cos \theta)^2}{(1)^2 - (\cos \theta)^2}$$

$$[\because (a - b)(a - b) = (a - b)^2 \text{ and } (a + b)(a - b) = (a^2 - b^2)]$$

$$= \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}$$

$$= \frac{(1 - \cos \theta)^2}{\sin^2 \theta} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \left(\frac{1 - \cos \theta}{\sin \theta} \right)^2$$

=RHS

Hence Proved

20. Question

Prove that the following identities :

$$\frac{\cos \theta}{1 + \sin \theta} = \frac{1 - \sin \theta}{\cos \theta}$$

Answer

$$\text{Taking LHS} = \frac{\cos \theta}{1 + \sin \theta}$$

Multiplying and divide by the conjugate of $1 + \sin \theta$, we get

$$= \frac{\cos \theta}{1 + \sin \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta}$$

$$= \frac{\cos \theta (1 - \sin \theta)}{(1)^2 - (\sin \theta)^2} [\because (a + b)(a - b) = (a^2 - b^2)]$$

$$= \frac{\cos \theta (1 - \sin \theta)}{1 - \sin^2 \theta}$$

$$= \frac{\cos \theta (1 - \sin \theta)}{\cos^2 \theta} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \frac{1 - \sin \theta}{\cos \theta}$$

=RHS

Hence Proved

21. Question

Prove that the following identities :

$$(\sin^8\theta - \cos^8\theta) = (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta \cos^2\theta)$$

Answer

Taking LHS

$$= \sin^8\theta - \cos^8\theta$$

$$= (\sin^4\theta)^2 - (\cos^4\theta)^2$$

$$= (\sin^4\theta - \cos^4\theta)(\sin^4\theta + \cos^4\theta)$$

$$[\because (a^2 - b^2) = (a + b)(a - b)]$$

$$= \{(\sin^2\theta)^2 - (\cos^2\theta)^2\}\{(\sin^2\theta)^2 + (\cos^2\theta)^2\}$$

$$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)[(\sin^2\theta + \cos^2\theta) - 2\sin^2\theta \cos^2\theta]$$

$$[\because (a^2 + b^2) = (a + b)^2 - 2ab]$$

$$= (1)[\sin^2\theta - \cos^2\theta][(1) - 2\sin^2\theta \cos^2\theta]$$

$$= (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta \cos^2\theta)$$

=RHS

Hence Proved

22. Question

Prove that the following identities :

$$2(\sin^6\theta - \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + (\sin^2\theta + \cos^2\theta)$$

Answer

Taking LHS

$$= 2(\sin^6\theta - \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + (\sin^2\theta + \cos^2\theta)$$

$$= 2[(\sin^2\theta)^3 - (\cos^2\theta)^3] - 3[(\sin^2\theta)^2 + (\cos^2\theta)^2] + 1 [\because \cos^2\theta + \sin^2\theta = 1]$$

Now, we use these identities, $(a^3 - b^3) = (a + b)^3 - 3ab(a + b)$ and $(a^2 + b^2) = (a + b)^2 - 2ab$

$$= 2[(\sin^2 \theta + \cos^2 \theta)^3 - 3\sin^2 \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta)] - 3[(\sin^2 \theta + \cos^2 \theta) - 2\sin^2 \theta \cos^2 \theta] + 1$$

$$= 2[(1) - 3\sin^2 \theta \cos^2 \theta (1)] - 3[(1) - 2\sin^2 \theta \cos^2 \theta] + 1 [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= 2(1 - 3\sin^2 \theta \cos^2 \theta) - 3 + 6\sin^2 \theta \cos^2 \theta + 1$$

$$= 2 - 6\sin^2 \theta \cos^2 \theta - 3 + 6\sin^2 \theta \cos^2 \theta$$

$$= 0$$

$$= \text{RHS}$$

Hence Proved

23. Question

Prove the following identities

$$\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$$

Answer

$$\text{Taking LHS} = \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A}$$

$$= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin A}{1 - \frac{\cos A}{\sin A}} \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta} \right]$$

$$= \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}} + \frac{\sin A}{\frac{\sin A - \cos A}{\sin A}}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

Using the identity, $(a^2 - b^2) = (a + b)(a - b)$

$$= \frac{(\cos A - \sin A)(\cos A + \sin A)}{(\cos A - \sin A)}$$

$$= \sin A + \cos A$$

$$= \text{RHS}$$

Hence Proved

24. Question

Prove the following identities

$$\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{2}{\sin \theta}$$

Answer

$$\begin{aligned}\text{Taking LHS} &= \frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} \\&= \frac{\sin^2 \theta + (1 + \cos \theta)^2}{(1 + \cos \theta)(\sin \theta)} \\&= \frac{\sin^2 \theta + 1 + \cos^2 \theta + 2 \cos \theta}{(1 + \cos \theta)(\sin \theta)} \\&= \frac{2 + 2 \cos \theta}{(1 + \cos \theta)(\sin \theta)} [\because \cos^2 \theta + \sin^2 \theta = 1] \\&= \frac{2(1 + \cos \theta)}{(1 + \cos \theta)(\sin \theta)} \\&= \frac{2}{\sin \theta}\end{aligned}$$

=RHS

Hence Proved

25. Question

Prove the following identities

$$\frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta} = 2 \sec^2 \theta$$

Answer

$$\begin{aligned}\text{Taking LHS} &= \frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta} \\&= \frac{1 - \sin \theta + 1 + \sin \theta}{(1 + \sin \theta)(1 - \sin \theta)}\end{aligned}$$

Using the identity, $(a^2 - b^2) = (a + b)(a - b)$

$$= \frac{2}{(1)^2 - (\sin \theta)^2}$$

$$\begin{aligned}
&= \frac{2}{1 - \sin^2 \theta} \\
&= \frac{2}{\cos^2 \theta} [\because \cos^2 \theta + \sin^2 \theta = 1] \\
&= 2 \sec^2 \theta \\
&= \text{RHS}
\end{aligned}$$

Hence Proved

26. Question

Prove the following identities

$$\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} = 2 \sec \theta$$

Answer

$$\begin{aligned}
\text{Taking LHS} &= \frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} \\
&= \frac{(1 + \sin \theta)^2 + \cos^2 \theta}{(1 + \sin \theta)(\cos \theta)} \\
&= \frac{1 + \sin^2 \theta + 2 \sin \theta + \cos^2 \theta}{(\cos \theta)(1 + \sin \theta)} \\
&= \frac{2 + 2 \sin \theta}{(\cos \theta)(1 + \sin \theta)} [\because \cos^2 \theta + \sin^2 \theta = 1] \\
&= \frac{2(1 + \sin \theta)}{(\cos \theta)(1 + \sin \theta)} \\
&= \frac{2}{\cos \theta} \\
&= 2 \sec \theta \\
&= \text{RHS}
\end{aligned}$$

Hence Proved

27. Question

Prove the following identities

$$\frac{\cos \theta}{1 - \sin \theta} + \frac{\cos \theta}{1 + \sin \theta} = \frac{2}{\cos \theta}$$

Answer

$$\begin{aligned}\text{Taking LHS} &= \frac{\cos \theta}{1 - \sin \theta} + \frac{\cos \theta}{1 + \sin \theta} \\&= \frac{\cos \theta (1 + \sin \theta) + \cos \theta (1 - \sin \theta)}{(1 - \sin \theta) (1 + \sin \theta)} \\&= \frac{\cos \theta + \cos \theta \sin \theta + \cos \theta - \cos \theta \sin \theta}{(1 - \sin \theta) (1 + \sin \theta)} \\&= \frac{2 \cos \theta}{(1 - \sin \theta) (1 + \sin \theta)}\end{aligned}$$

Using the identity, $(a^2 - b^2) = (a + b) (a - b)$

$$\begin{aligned}&= \frac{2 \cos \theta}{(1)^2 - (\sin \theta)^2} \\&= \frac{2 \cos \theta}{1 - \sin^2 \theta} \\&= \frac{2 \cos \theta}{\cos^2 \theta} [\because \cos^2 \theta + \sin^2 \theta = 1] \\&= \frac{2}{\cos \theta}\end{aligned}$$

=RHS

Hence Proved

28. Question

Prove the following identities

$$\frac{1}{1 + \cos \theta} + \frac{1}{1 - \cos \theta} = \frac{2}{\sin^2 \theta}$$

Answer

$$\begin{aligned}\text{Taking LHS} &= \frac{1}{1 + \cos \theta} + \frac{1}{1 - \cos \theta} \\&= \frac{1 - \cos \theta + 1 + \cos \theta}{(1 + \cos \theta) (1 - \cos \theta)}\end{aligned}$$

Using the identity, $(a^2 - b^2) = (a + b) (a - b)$

$$= \frac{2}{(1)^2 - (\cos \theta)^2}$$

$$= \frac{2}{1 - \cos^2 \theta}$$

$$= \frac{2}{\sin^2 \theta} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

=RHS

Hence Proved

29. Question

Prove the following identities

$$\frac{1}{1 - \sin \theta} - \frac{1}{1 + \sin \theta} = \frac{2 \tan \theta}{\cos \theta}$$

Answer

$$\text{Taking LHS} = \frac{1}{1 - \sin \theta} - \frac{1}{1 + \sin \theta}$$

$$= \frac{1 + \sin \theta - 1 + \sin \theta}{(1 + \sin \theta)(1 - \sin \theta)}$$

Using the identity, $(a^2 - b^2) = (a + b)(a - b)$

$$= \frac{2 \sin \theta}{(1)^2 - (\sin \theta)^2}$$

$$= \frac{2 \sin \theta}{1 - \sin^2 \theta}$$

$$= \frac{2 \sin \theta}{\cos^2 \theta} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \frac{2 \tan \theta}{\cos \theta} \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

=RHS

Hence Proved

30. Question

Prove the following identities

$$\cot^2 \theta - \cos^2 \theta = \cot^2 \theta \cdot \cos^2 \theta$$

Answer

$$\text{Taking LHS} = \cot^2 \theta - \cos^2 \theta$$

$$\begin{aligned}
&= \frac{\cos^2 \theta}{\sin^2 \theta} - \cos^2 \theta \left[\because \cot \theta = \frac{\cos \theta}{\sin \theta} \right] \\
&= \frac{\cos^2 \theta - \sin^2 \theta \cos^2 \theta}{\sin^2 \theta} \\
&= \frac{\cos^2 \theta (1 - \sin^2 \theta)}{\sin^2 \theta} [\because \cos^2 \theta + \sin^2 \theta = 1] \\
&= \frac{\cos^2 \theta \cos^2 \theta}{\sin^2 \theta} \\
&= \cot^2 \theta \cos^2 \theta \left[\because \cot \theta = \frac{\cos \theta}{\sin \theta} \right]
\end{aligned}$$

=RHS

Hence Proved

31. Question

Prove the following identities

$$\tan^2 \varphi - \sin^2 \varphi - \tan^2 \varphi \cdot \sin^2 \varphi = 0$$

Answer

Taking LHS = $\tan^2 \varphi - \sin^2 \varphi - \tan^2 \varphi \sin^2 \varphi$

$$\begin{aligned}
&= \frac{\sin^2 \varphi}{\cos^2 \varphi} - \sin^2 \varphi - \frac{\sin^2 \varphi}{\cos^2 \varphi} \sin^2 \varphi \\
&= \frac{\sin^2 \varphi - \sin^2 \varphi \cos^2 \varphi - \sin^4 \varphi}{\cos^2 \varphi} \\
&= \frac{\sin^2 \varphi (1 - \cos^2 \varphi - \sin^2 \varphi)}{\cos^2 \varphi} \\
&= \frac{\sin^2 \varphi \{1 - (\cos^2 \varphi + \sin^2 \varphi)\}}{\cos^2 \varphi} [\because \cos^2 \varphi + \sin^2 \varphi = 1] \\
&= \frac{\sin^2 \varphi \{1 - 1\}}{\cos^2 \varphi}
\end{aligned}$$

= 0

=RHS

Hence Proved

32. Question

Prove the following identities

$$\tan^2 \phi + \cot^2 \phi + 2 = \sec^2 \phi \cdot \operatorname{cosec}^2 \phi$$

Answer

$$\text{Taking LHS} = \tan^2 \phi + \cot^2 \phi + 2$$

$$= \frac{\sin^2 \phi}{\cos^2 \phi} + \frac{\cos^2 \phi}{\sin^2 \phi} + 2$$

$$= \frac{\sin^4 \phi + \cos^4 \phi + 2\sin^2 \phi \cos^2 \phi}{\cos^2 \phi \sin^2 \phi} [\because (a+b)^2 = (a^2 + b^2 + 2ab)]$$

$$= \frac{(\sin^2 \phi + \cos^2 \phi)^2}{\cos^2 \phi \sin^2 \phi} [\because \cos^2 \phi + \sin^2 \phi = 1]$$

$$= \frac{1}{\cos^2 \phi \sin^2 \phi}$$

$$= \sec^2 \phi \operatorname{cosec}^2 \phi \left[\because \cos \theta = \frac{1}{\sec \theta} \text{ and } \sin \theta = \frac{1}{\csc \theta} \right]$$

=RHS

Hence Proved

33. Question

Prove the following identities

$$\frac{\operatorname{cosec} \theta + \cot \theta - 1}{\cot \theta - \operatorname{cosec} \theta + 1} = \frac{1 + \cos \theta}{\sin \theta}$$

Answer

$$\text{Taking LHS} = \frac{\operatorname{cosec} \theta + \cot \theta - 1}{\cot \theta - \operatorname{cosec} \theta + 1}$$

$$= \frac{(\cot \theta + \operatorname{cosec} \theta) - (\operatorname{cosec}^2 \theta - \cot^2 \theta)}{\cot \theta - \operatorname{cosec} \theta + 1} [\because \cot^2 \theta - \operatorname{cosec}^2 \theta = 1]$$

$$= \frac{(\cot \theta + \operatorname{cosec} \theta) - \{(\operatorname{cosec} \theta + \cot \theta)(\operatorname{cosec} \theta - \cot \theta)\}}{\cot \theta - \operatorname{cosec} \theta + 1}$$

$$= \frac{(\cot \theta + \operatorname{cosec} \theta)(1 - \operatorname{cosec} \theta + \cot \theta)}{\cot \theta - \operatorname{cosec} \theta + 1}$$

$$= \cot \theta + \operatorname{cosec} \theta$$

$$= \frac{\cos \theta}{\sin \theta} + \frac{1}{\sin \theta} \left[\because \cot \theta = \frac{\cos \theta}{\sin \theta} \text{ and } \sin \theta = \frac{1}{\csc \theta} \right]$$

$$= \frac{1 + \cos \theta}{\sin \theta}$$

=RHS

Hence Proved

34. Question

Prove the following identities

$$\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \tan \theta + \cot \theta$$

Answer

$$\text{Taking LHS} = \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta}$$

$$= \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}} \left[\because \cot \theta = \frac{\cos \theta}{\sin \theta} \text{ and } \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$= \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}}$$

$$= \frac{\sin \theta}{\cos \theta} \times \frac{\sin \theta}{\sin \theta - \cos \theta} + \frac{\cos \theta}{\sin \theta} \times \frac{\cos \theta}{\cos \theta - \sin \theta}$$

$$= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} + \frac{\cos^2 \theta}{\sin \theta (\cos \theta - \sin \theta)}$$

$$= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} + \frac{\cos^2 \theta}{\sin \theta \{-(\sin \theta - \cos \theta)\}}$$

$$= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} - \frac{\cos^2 \theta}{\sin \theta (\sin \theta - \cos \theta)}$$

$$= \frac{\sin^3 \theta - \cos^3 \theta}{(\cos \theta \sin \theta) (\sin \theta - \cos \theta)}$$

$$= \frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)}{(\cos \theta \sin \theta)(\sin \theta - \cos \theta)} \left[\because (a^3 - b^3) = (a - b)(a^2 + b^2 + ab) \right]$$

$$= \frac{1 + \sin \theta \cos \theta}{\cos \theta \sin \theta}$$

$$= \frac{1}{\cos \theta \sin \theta} + \frac{\sin \theta \cos \theta}{\cos \theta \sin \theta}$$

$$= \tan \theta \cot \theta + 1$$

=RHS

Hence Proved

35. Question

Prove the following identities

$$\frac{1 - \cos \theta}{1 + \cos \theta} = (\cot \theta - \operatorname{cosec} \theta)^2$$

Answer

$$\text{Taking LHS} = \frac{1 - \cos \theta}{1 + \cos \theta}$$

Multiplying and divide by the conjugate of $1 + \cos \theta$, we get

$$= \frac{1 - \cos \theta}{1 + \cos \theta} \times \frac{1 - \cos \theta}{1 - \cos \theta}$$

$$= \frac{(1 - \cos \theta)^2}{(1)^2 - (\cos \theta)^2}$$

$$[\because (a - b)(a + b) = (a - b)^2 \text{ and } (a + b)(a - b) = (a^2 - b^2)]$$

$$= \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}$$

$$= \frac{(1 - \cos \theta)^2}{\sin^2 \theta} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \left(\frac{1 - \cos \theta}{\sin \theta} \right)^2$$

$$= \left(\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2$$

$$= (\operatorname{cosec} \theta - \cot \theta)^2$$

$$= \{ -(\cot \theta - \operatorname{cosec} \theta) \}^2$$

$$= (\cot \theta - \operatorname{cosec} \theta)^2$$

=RHS

Hence Proved

36. Question

Prove the following identities

$$\sqrt{\frac{1+\cos \theta}{1-\cos \theta}} = \frac{1+\cos \theta}{\sin \theta}$$

Answer

$$\text{Taking LHS} = \sqrt{\frac{1+\cos \theta}{1-\cos \theta}}$$

$$= \sqrt{\frac{1+\cos \theta}{1-\cos \theta} \times \frac{1+\cos \theta}{1+\cos \theta}} \text{ [multiplying and divide by conjugate of } 1-\cos \theta]$$

$$= \sqrt{\frac{(1+\cos \theta)^2}{(1)^2 - (\cos \theta)^2}}$$

$$= \sqrt{\frac{(1+\cos \theta)^2}{(1-\cos^2 \theta)}}$$

$$= \sqrt{\frac{(1+\cos \theta)^2}{\sin^2 \theta}} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \frac{1+\cos \theta}{\sin \theta}$$

=RHS

Hence Proved

37. Question

Prove the following identities

$$\sqrt{\frac{1+\cos \theta}{1-\cos \theta}} = \frac{\sin \theta}{1-\cos \theta}$$

Answer

$$\text{Taking LHS} = \sqrt{\frac{1+\cos \theta}{1-\cos \theta}}$$

$$= \sqrt{\frac{1+\cos \theta}{1-\cos \theta} \times \frac{1+\cos \theta}{1+\cos \theta}} \text{ [multiplying and divide by conjugate of } 1-\cos \theta]$$

$$= \sqrt{\frac{(1+\cos \theta)^2}{(1)^2 - (\cos \theta)^2}}$$

$$\begin{aligned}
&= \sqrt{\frac{(1 + \cos \theta)^2}{(1 - \cos^2 \theta)}} \\
&= \sqrt{\frac{(1 + \cos \theta)^2}{\sin^2 \theta}} [\because \cos^2 \theta + \sin^2 \theta = 1] \\
&= \frac{1 + \cos \theta}{\sin \theta}
\end{aligned}$$

Multiply and divide by conjugate of $1 + \cos \theta$, we get

$$\begin{aligned}
&= \frac{1 + \cos \theta}{\sin \theta} \times \frac{1 - \cos \theta}{1 - \cos \theta} \\
&= \frac{1 - \cos^2 \theta}{\sin \theta \times (1 - \cos \theta)} \\
&= \frac{\sin^2 \theta}{\sin \theta \times (1 - \cos \theta)} \\
&= \frac{\sin \theta}{1 - \cos \theta}
\end{aligned}$$

=RHS

Hence Proved

38. Question

Prove the following identities

$$\sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} = \sec \theta - \tan \theta$$

Answer

$$\begin{aligned}
\text{Taking LHS} &= \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} \\
&= \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta}} [\text{multiplying and divide by conjugate of } 1 + \sin \theta] \\
&= \sqrt{\frac{(1 - \sin \theta)^2}{(1)^2 - (\sin \theta)^2}}
\end{aligned}$$

$$\begin{aligned}
&= \sqrt{\frac{(1 - \sin \theta)^2}{(1 - \sin^2 \theta)}} \\
&= \sqrt{\frac{(1 - \sin \theta)^2}{\cos^2 \theta}} [\because \cos^2 \theta + \sin^2 \theta = 1] \\
&= \frac{1 - \sin \theta}{\cos \theta} \\
&= \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \\
&= \sec \theta - \tan \theta \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cos \theta = \frac{1}{\sec \theta} \right]
\end{aligned}$$

=RHS

Hence Proved

39. Question

Prove the following identities

$$\sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} = \frac{\cos \theta}{1 + \sin \theta}$$

Answer

$$\begin{aligned}
\text{Taking LHS} &= \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} \\
&= \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta} \times \frac{1 + \sin \theta}{1 + \sin \theta}} [\text{multiplying and divide by of } 1 + \sin \theta] \\
&= \sqrt{\frac{(1)^2 - (\sin \theta)^2}{(1 + \sin \theta)^2}} \\
&= \sqrt{\frac{1 - \sin^2 \theta}{(1 + \sin \theta)^2}} \\
&= \sqrt{\frac{\cos^2 \theta}{(1 + \sin \theta)^2}} [\because \cos^2 \theta + \sin^2 \theta = 1] \\
&= \frac{\cos \theta}{1 + \sin \theta}
\end{aligned}$$

=RHS

Hence Proved

40. Question

Prove the following identities

$$\sqrt{\frac{1+\sin \theta}{1-\sin \theta}} + \sqrt{\frac{1-\sin \theta}{1+\sin \theta}} = 2 \sec \theta$$

Answer

$$\text{Taking LHS} = \sqrt{\frac{1+\sin \theta}{1-\sin \theta}} + \sqrt{\frac{1-\sin \theta}{1+\sin \theta}}$$

[multiplying and divide by conjugate of $1-\sin \theta$ in 1st term and $1+\sin \theta$ in 2nd term]

$$= \sqrt{\frac{1+\sin \theta}{1-\sin \theta} \times \frac{1+\sin \theta}{1+\sin \theta}} + \sqrt{\frac{1-\sin \theta}{1+\sin \theta} \times \frac{1-\sin \theta}{1-\sin \theta}}$$

$$= \sqrt{\frac{(1+\sin \theta)^2}{(1)^2 - (\sin \theta)^2}} + \sqrt{\frac{(1-\sin \theta)^2}{(1)^2 - (\sin \theta)^2}}$$

$$= \sqrt{\frac{(1+\sin \theta)^2}{(1-\sin^2 \theta)}} + \sqrt{\frac{(1-\sin \theta)^2}{(1-\sin^2 \theta)}}$$

$$= \sqrt{\frac{(1+\sin \theta)^2}{\cos^2 \theta}} + \sqrt{\frac{(1-\sin \theta)^2}{\cos^2 \theta}} [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \frac{1+\sin \theta}{\cos \theta} + \frac{1-\sin \theta}{\cos \theta}$$

$$= \frac{1+\sin \theta + 1-\sin \theta}{\cos \theta}$$

$$= \frac{2}{\cos \theta}$$

$$= 2 \sec \theta$$

=RHS

Hence Proved

41. Question

If $\sec \theta + \tan \theta = m$ and $\sec \theta - \tan \theta = n$, then prove that $\sqrt{mn} = 1$

Answer

Given : $\sec \theta + \tan \theta = m$ and $\sec \theta - \tan \theta = n$

To Prove : $\sqrt{mn} = 1$

Taking LHS = $\sqrt{mn}$

Putting the value of m and n, we get

$$= \sqrt{(\sec \theta + \tan \theta)(\sec \theta - \tan \theta)}$$

Using the identity, $(a + b)(a - b) = (a^2 - b^2)$

$$= \sqrt{\sec^2 \theta - \tan^2 \theta}$$

$$= \sqrt{1} [\because 1 + \tan^2 \theta = \sec^2 \theta]$$

$$= \pm 1$$

$$= \text{RHS}$$

Hence Proved

42. Question

If $\cos \theta + \sin \theta = 1$, then prove that $\cos \theta - \sin \theta = \pm 1$.

Answer

Given: $\cos \theta + \sin \theta = 1$

On squaring both the sides, we get

$$(\cos \theta + \sin \theta)^2 = (1)^2$$

$$\Rightarrow \cos^2 \theta + \sin^2 \theta + 2\sin \theta \cos \theta = 1$$

$$\Rightarrow \cos^2 \theta + \sin^2 \theta = \cos^2 \theta + \sin^2 \theta - 2\sin \theta \cos \theta$$

$$[\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$\Rightarrow \cos^2 \theta + \sin^2 \theta = (\cos \theta - \sin \theta)^2$$

$$[\because (a - b)^2 = (a^2 + b^2 - 2ab)]$$

$$\Rightarrow 1 = (\cos \theta - \sin \theta)^2$$

$$\Rightarrow (\cos \theta - \sin \theta) = \pm 1$$

Hence Proved

43. Question

If $\sin\theta + \sin^2\theta = 1$, then prove that $\cos^2\theta + \cos^4\theta = 1$

Answer

$$\text{Given : } \sin\theta + \sin^2\theta = 1$$

$$\Rightarrow \sin\theta = 1 - \sin^2\theta$$

$$\text{Taking LHS} = \cos^2\theta + \cos^4\theta$$

$$= \cos^2\theta + (\cos^2\theta)^2$$

$$= (1 - \sin^2\theta) + (1 - \sin^2\theta)^2 \dots(i)$$

Putting $\sin\theta = 1 - \sin^2\theta$ in Eq. (i), we get

$$= \sin\theta + (\sin\theta)^2$$

$$= \sin\theta + \sin^2\theta$$

$$= 1 \text{ [Given: } \sin\theta + \sin^2\theta = 1]$$

$$= \text{RHS}$$

Hence Proved

44. Question

If $\tan\theta + \sec\theta = x$, show that $\sin\theta = \frac{x^2 - 1}{x^2 + 1}$

Answer

$$\text{To show : } \sin\theta = \frac{x^2 - 1}{x^2 + 1}$$

$$\text{Taking RHS} = \frac{x^2 - 1}{x^2 + 1}$$

$$\text{Given } \tan\theta + \sec\theta = x$$

$$= \frac{(\tan\theta + \sec\theta)^2 - 1}{(\tan\theta + \sec\theta)^2 + 1}$$

$$= \frac{\tan^2\theta + \sec^2\theta + 2\tan\theta\sec\theta - 1}{\tan^2\theta + \sec^2\theta + 2\tan\theta\sec\theta + 1}$$

$$= \frac{\tan^2\theta + \sec^2\theta + 2\tan\theta\sec\theta}{\sec^2\theta - 1 + \sec^2\theta + 2\tan\theta\sec\theta + 1} [\because 1 + \tan^2\theta = \sec^2\theta]$$

$$\begin{aligned}
&= \frac{2\tan^2\theta + 2\tan\theta\sec\theta}{2\sec^2\theta + 2\tan\theta\sec\theta} \\
&= \frac{\frac{\sin^2\theta}{\cos^2\theta} + \frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta}}{\frac{1}{\cos^2\theta} + \frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta}} \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \sec\theta = \frac{1}{\cos\theta} \right] \\
&= \frac{\sin^2\theta + \sin\theta}{1 + \sin\theta} \\
&= \frac{\sin\theta(\sin\theta + 1)}{1 + \sin\theta}
\end{aligned}$$

$$= \sin\theta$$

$$= \text{LHS}$$

Hence Proved

45. Question

If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, then show $q(p^2 - 1) = 2p$

Answer

Given: $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$

To show $q(p^2 - 1) = 2p$

Taking LHS = $q(p^2 - 1)$

Putting the value of $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, we get

$$\begin{aligned}
&= (\sec\theta + \operatorname{cosec}\theta) \{ (\sin\theta + \cos\theta)^2 - 1 \} \\
&= (\sec\theta + \operatorname{cosec}\theta) \{ (\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta) - 1 \} \\
&[\because (a+b)^2 = (a^2 + b^2 + 2ab)] \\
&= (\sec\theta + \operatorname{cosec}\theta) (1 + 2\sin\theta\cos\theta - 1) \\
&= (\sec\theta + \operatorname{cosec}\theta) (2\sin\theta\cos\theta) \\
&= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta} \right) \times (2\sin\theta\cos\theta) \left[\because \sec\theta = \frac{1}{\cos\theta} \text{ and } \operatorname{cosec}\theta = \frac{1}{\sin\theta} \right] \\
&= \frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta} \times 2\sin\theta\cos\theta \\
&= 2(\sin\theta + \cos\theta)
\end{aligned}$$

$$= 2p \text{ [given } \sin \theta + \cos \theta = p \text{]}$$

=RHS

Hence Proved

46. Question

If $x \cos \theta = a$ and $y = a \tan \theta$, then prove that $x^2 - y^2 = a^2$

Answer

Given: $x \cos \theta = a$ and $y = a \tan \theta$

$$\Rightarrow x = \frac{a}{\cos \theta} \text{ and } y = a \tan \theta$$

To Prove : $x^2 - y^2 = a^2$

Taking LHS = $x^2 - y^2$

Putting the values of x and y , we get

$$\begin{aligned} &= \left(\frac{a}{\cos \theta} \right)^2 - (a \tan \theta)^2 \\ &= \frac{a^2}{\cos^2 \theta} - a^2 \tan^2 \theta \\ &= \frac{a^2}{\cos^2 \theta} - a^2 \frac{\sin^2 \theta}{\cos^2 \theta} \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right] \\ &= \frac{a^2 - a^2 \sin^2 \theta}{\cos^2 \theta} \\ &= \frac{a^2 (1 - \sin^2 \theta)}{\cos^2 \theta} \\ &= \frac{a^2 \cos^2 \theta}{\cos^2 \theta} \left[\because \cos^2 \theta + \sin^2 \theta = 1 \right] \\ &= a^2 \end{aligned}$$

= RHS

Hence Proved

47. Question

If $x = r \cos \alpha \sin \beta$, $y = r \sin \alpha \sin \beta$ and $z = r \cos \alpha$ then prove that $x^2 + y^2 + z^2 = r^2$.

Answer

Taking LHS = $x^2 + y^2 + z^2$

Putting the values of x, y and z , we get

$$=(r \cos \alpha \sin \beta)^2 + (r \sin \alpha \sin \beta)^2 + (r \cos \alpha)^2$$

$$= r^2 \cos^2 \alpha \sin^2 \beta + r^2 \sin^2 \alpha \sin^2 \beta + r^2 \cos^2 \alpha$$

Taking common $r^2 \sin^2 \alpha$, we get

$$= r^2 \sin^2 \alpha (\cos^2 \beta + \sin^2 \beta) + r^2 \cos^2 \alpha$$

$$= r^2 \sin^2 \alpha + r^2 \cos^2 \alpha [\because \cos^2 \beta + \sin^2 \beta = 1]$$

$$= r^2 (\sin^2 \alpha + \cos^2 \alpha)$$

$$= r^2 [\because \cos^2 \alpha + \sin^2 \alpha = 1]$$

$$= \text{RHS}$$

Hence Proved

48. Question

If $\sec \theta - \tan \theta = x$, then prove that

$$(i) \cos \theta = \frac{2x}{1+x^2}$$

$$(ii) \sin \theta = \frac{1-x^2}{1+x^2}$$

Answer

(i) Given $\sec \theta - \tan \theta = x$

$$\text{Taking RHS} = \frac{2x}{1+x^2}$$

Putting the value of x, we get

$$= \frac{2(\sec \theta - \tan \theta)}{1 + (\sec \theta - \tan \theta)^2}$$

$$= \frac{2(\sec \theta - \tan \theta)}{1 + \sec^2 \theta + \tan^2 \theta - 2\sec \theta \tan \theta}$$

$$= \frac{2(\sec \theta - \tan \theta)}{\sec^2 \theta + \sec^2 \theta - 2\sec \theta \tan \theta} [\because 1 + \tan^2 \theta = \sec^2 \theta]$$

$$= \frac{2(\sec \theta - \tan \theta)}{2 \sec \theta (\sec \theta - \tan \theta)}$$

$$= \frac{1}{\sec \theta}$$

$$= \cos \theta \left[\because \cos \theta = \frac{1}{\sec \theta} \right]$$

=RHS

Hence Proved

(ii) Given $\sec \theta - \tan \theta = x$

$$\text{Taking RHS} = \frac{1-x^2}{1+x^2}$$

Putting the value of x, we get

$$= \frac{1 - (\sec \theta - \tan \theta)^2}{1 + (\sec \theta - \tan \theta)^2}$$

$$= \frac{1 - \sec^2 \theta - \tan^2 \theta + 2 \sec \theta \tan \theta}{1 + \sec^2 \theta + \tan^2 \theta - 2 \sec \theta \tan \theta}$$

$$= \frac{-\tan^2 \theta - \tan^2 \theta + 2 \sec \theta \tan \theta}{\sec^2 \theta + \sec^2 \theta - 2 \sec \theta \tan \theta} \left[\because 1 + \tan^2 \theta = \sec^2 \theta \right]$$

$$= \frac{2 \tan \theta (\sec \theta - \tan \theta)}{2 \sec \theta (\sec \theta - \tan \theta)}$$

$$= \frac{\sin \theta}{\cos \theta} \times \cos \theta$$

$$= \sin \theta \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

=RHS

Hence Proved

49. Question

If $a \cos \theta + b \sin \theta = c$, then prove that $a \sin \theta - b \cos \theta = \pm \sqrt{a^2 + b^2 - c^2}$

Answer

Let

$$(a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \cos \theta \sin \theta + a^2 \sin^2 \theta$$

$$+ b^2 \cos^2 \theta - 2ab \cos \theta \sin \theta$$

$$\Rightarrow c^2 + (a \sin \theta - b \cos \theta)^2 = a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\cos^2 \theta + \sin^2 \theta)$$

$$\Rightarrow c^2 + (a \sin \theta - b \cos \theta)^2 = a^2 + b^2$$

$$\Rightarrow (a \sin \theta - b \cos \theta)^2 = a^2 + b^2 - c^2$$

$$\Rightarrow (a \sin \theta - b \cos \theta) = \pm \sqrt{a^2 + b^2 - c^2}$$

50. Question

If $1 + \sin^2 \theta = 3 \sin \theta \cdot \cos \theta$, then prove that $\tan \theta = 1$ or $1/2$.

Answer

$$\text{Given: } 1 + \sin^2 \theta = 3 \sin \theta \cos \theta$$

Divide by $\cos^2 \theta$ to both the sides, we get

$$\Rightarrow \frac{1}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{3 \sin \theta \cos \theta}{\cos^2 \theta}$$

$$\Rightarrow \sec^2 \theta + \tan^2 \theta = 3 \tan \theta$$

$$\Rightarrow 1 + \tan^2 \theta + \tan^2 \theta = 3 \tan \theta$$

$$\Rightarrow 2 \tan^2 \theta - 3 \tan \theta + 1 = 0$$

$$\text{Let } \tan \theta = x$$

$$\Rightarrow 2x^2 - 3x + 1 = 0$$

$$\Rightarrow 2x^2 - 2x - x + 1 = 0$$

$$\Rightarrow 2x(x - 1) - 1(x - 1) = 0$$

$$\Rightarrow (2x - 1)(x - 1) = 0$$

Putting each of the factor = 0, we get

$$\Rightarrow x = 1 \text{ or } \frac{1}{2}$$

And above, we let $\tan \theta = x$

$$\Rightarrow \tan \theta = 1 \text{ or } \frac{1}{2}$$

Hence Proved

51. Question

If $a \cos \theta - b \sin \theta = x$ and $a \sin \theta + b \cos \theta = y$ that $a^2 + b^2 = x^2 + y^2$.

Answer

Taking RHS $= x^2 + y^2$

Putting the values of x and y, we get

$$\begin{aligned} & (a \cos \theta - b \sin \theta)^2 + (a \sin \theta + b \cos \theta)^2 \\ &= a^2 \cos^2 \theta + b^2 \sin^2 \theta - 2ab \cos \theta \sin \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \cos \theta \sin \theta \\ &= a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\cos^2 \theta + \sin^2 \theta) \\ &= a^2 + b^2 [\because \cos^2 \theta + \sin^2 \theta = 1] \\ &= \text{RHS} \end{aligned}$$

Hence Proved

52. Question

If $x = a \sec \theta + b \tan \theta$ and $y = a \tan \theta + b \sec \theta$, then prove that $x^2 - y^2 = a^2 - b^2$.

Answer

Taking LHS $= x^2 - y^2$

Putting the values of x and y, we get

$$\begin{aligned} & (a \sec \theta + b \tan \theta)^2 - (a \tan \theta + b \sec \theta)^2 \\ &= a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta - a^2 \tan^2 \theta - b^2 \sec^2 \theta - 2ab \sec \theta \tan \theta \\ &= a^2 (\sec^2 \theta - \tan^2 \theta) - b^2 (\sec^2 \theta - \tan^2 \theta) \\ &= a^2 - b^2 [\because 1 + \tan^2 \theta = \sec^2 \theta] \\ &= \text{RHS} \end{aligned}$$

Hence Proved

53. Question

If $(a^2 - b^2) \sin \theta + 2ab \cos \theta = a^2 + b^2$, then prove that $\tan \theta = \frac{a^2 - b^2}{2ab}$.

Answer

Taking $(a^2 - b^2) \sin \theta + 2ab \cos \theta = a^2 + b^2$

We know that $\sin \theta = \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$ and $\cos \theta = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$

Then, substituting the above values in the given equation, we get

$$= a^2 - b^2 \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} + 2ab \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} = a^2 + b^2$$

Now, substituting $t = \tan \frac{\theta}{2}$, we have

$$a^2 - b^2 \frac{2t}{1 + t^2} + 2ab \frac{1 - t^2}{1 + t^2} = a^2 + b^2$$

$$\Rightarrow (a^2 - b^2)2t - 2ab(1 - t^2) = (a^2 + b^2)(1 + t^2)$$

Simplify, we get

$$(a^2 + 2ab + b^2)t^2 - 2(a^2 - b^2)t + (a^2 - 2ab + b^2) = 0$$

$$\Rightarrow (a+b)^2 t^2 - 2(a^2 - b^2)t + (a - b)^2 = 0$$

$$\Rightarrow (a+b)^2 t^2 - 2(a - b)(a+b)t + (a - b)^2 = 0$$

$$\Rightarrow [(a+b)t - (a - b)]^2 = 0 \quad [\because (a - b)^2 = (a^2 + b^2 - 2ab)]$$

$$\Rightarrow [(a+b)t - (a - b)] = 0$$

$$\Rightarrow (a+b)t = (a - b)$$

$$\Rightarrow t = \frac{a - b}{a + b}$$

$$\Rightarrow \tan \frac{\theta}{2} = \frac{a - b}{a + b}$$

We know that, $\frac{2t}{1 - t^2} = \tan \theta$, where $t = \tan \frac{\theta}{2}$

$$\Rightarrow \tan \theta = \frac{2 \left(\frac{a - b}{a + b} \right)}{1 - \left(\frac{a - b}{a + b} \right)^2}$$

$$\Rightarrow \tan \theta = \frac{2(a+b)(a-b)}{(a+b)^2 - (a-b)^2}$$

$$\Rightarrow \tan \theta = \frac{a^2 - b^2}{2ab}$$

Hence Proved