5. Triangles

Exercise 5.1

1. Question

Fill in the blanks with the correct word given in brackets:

(i) All squares are having the same length of sides are……….

[similar, congruent, both congruent and similar]

(ii) All circles having the same radius are ……….

[similar, congruent, both congruent and similar]

(iii) All rhombuses having one angle 90^{0} ………. [similar, congruent]

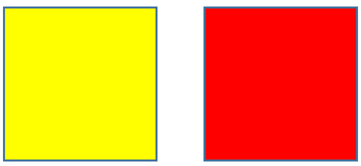
(iv) All photographs of a given building made by the same negative are ………..

[similar, congruent, both congruent and similar]

(v) Two polygons having equal numbers of sides are similar if their corresponding angles are equal and their corresponding sides are ……….

[equal, proportional]

Answer

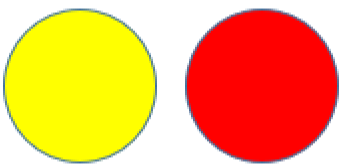
(i) 

Both congruent and similar because

(a) all squares have same shape and size

(b) their corresponding angles are equal

(c) their corresponding sides are proportional

(ii) 

Both congruent and similar because all circles have same shape and size

(iii) Similar because all rhombuses have the same angle, but size can vary.

(iv) Similar because all photographs have the same shape but not necessarily the same size.

(v) Two polygons having equal numbers of sides are similar if their corresponding angles are equal and their corresponding sides are Proportional

2. Question

State which of the following statements are true and which are false:

(i) Two similar figures are congruent.

(ii) All congruent figures are similar.

(iii) All isosceles triangles are similar.

(iv) All right-angled triangles are similar.

(v) All squares are similar.

(vi) All rectangles are similar.

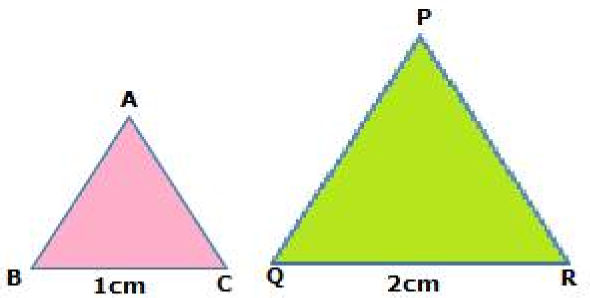
(vii) Two photographs of a person made by the same negative are similar.

(viii) Two photographs of a person one at the age of 5 years and other at the age of 50 years are similar.

Answer

(i) This statement is false because all the congruent figures are similar, but similar figures need not be congruent.

E.g. Two equilateral triangles having sides 1cm and 2cm.



In case of equilateral triangles, all the sides are equal, and all the angles are of 60°.

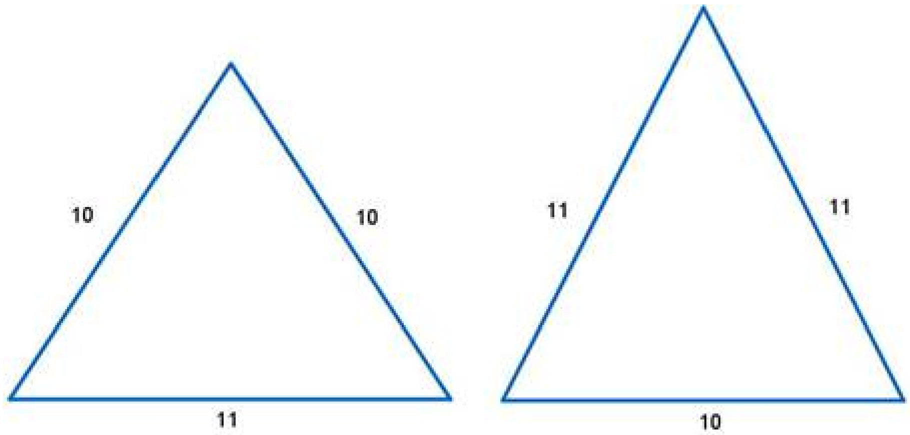
But here, their corresponding angles are equal, but sides of triangle ABC and PQR are not equal in length.

So, they are similar figures but not congruent.

(ii) This statement is true because all congruent figures are similar, but similar figures need not be congruent.

(iii) This statement is false because for two triangles to be similar to the angles in one triangle must have the same values as the angles in the other triangle. The sides must be proportionate.

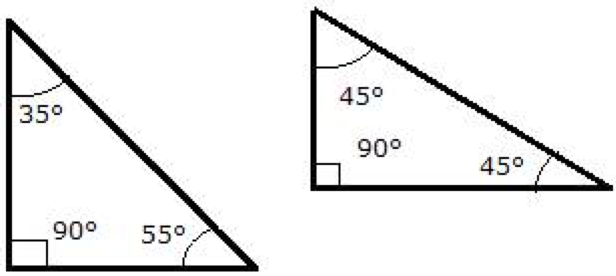
E.g.



These are the two isosceles triangles having two equal sides, but we can see that the sides are not proportionate.

(iv) This statement is false.

Suppose these are two right-angled triangles



Here, both of the triangles are right-angled, but other corresponding two angles are not equal. So, these are not similar figures.

(v) This statement is true because all the angles in a square are right angles and all the sides are equal. Hence, a smaller square can be enlarged to the size of a larger square, and vice-versa is also true.

(vi) This statement is false because similarity preserves the ratio of length. Therefore, two rectangles with a different ratio between their sides cannot be similar.

(vii) This statement is true because photographs are produced by projecting the image from a negative through an enlarger to a photographic paper. The enlarger reproduces the image from the negative but makes it bigger. The images are not identical and are not of the same size, but they are similar.

These two photographs of Sadie are the same shape, but they are not the same size.

(viii) This statement is false because here the photograph of a person is taken at the different ages.

3. Question

Give two examples of:

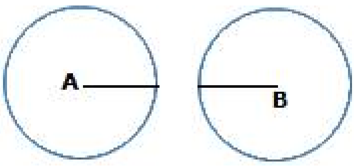
(i) Congruent figures.

(ii) Similar figures which are not congruent.

(iii) Non-similar figures.

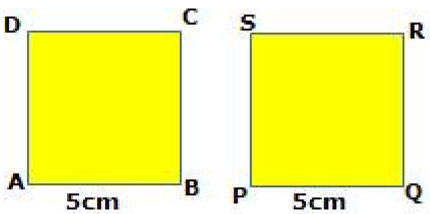
Answer

(i) (a) Two circles having radii 2cm and different centres



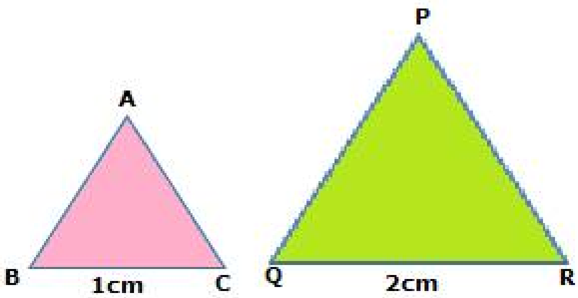
In this, both of them have the same radii, but their centres are different.

(b) Two squares having the same length of side 5cm



We know that in a square all the sides are equal and all angles are of 90°. So, these two squares are congruent.

(ii) (a) Two equilateral triangles having sides 1cm and 2cm.

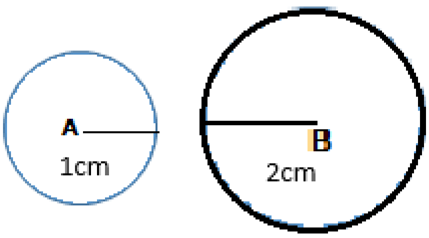


In case of equilateral triangles, all the sides are equal, and all the angles are of 60°.

But here, their corresponding angles are equal, but sides of triangle ABC and PQR are not equal in length.

So, they are similar figures but not congruent.

(b) Two circles having radii 1cm and 2cm



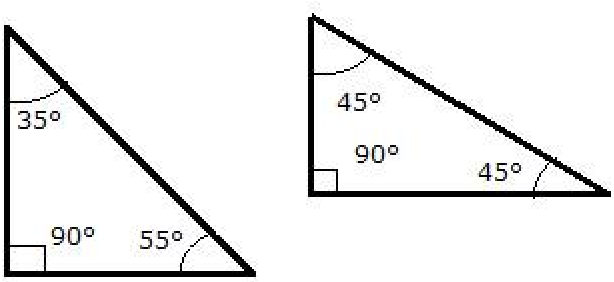
Both of the figures are of circle but they are having different radii. So, they are similar but not congruent.

(iii) (a) A rhombus and a rectangle

In the case of a rhombus, all the sides are equal, and the angles can either be right angles or combination of acute and obtuse angles but in rectangle all angles are equal, and opposite sides are equal.

Hence, a rhombus and a rectangle are non-similar figures.

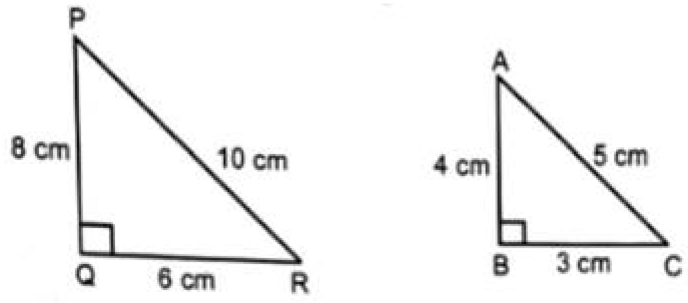
(b)



Here, both of the triangles are right-angled but other two angles are not equal. So, these are not similar figures.

4. Question

State whether the following right-angled triangles are similar or not:



Answer

Two polygons of a same number of sides are similar, if

a) all the corresponding angles are equal.

b) all the corresponding sides are in the same ratio (or proportion)

In case of right-angled triangle PQR and ABC

$$\frac{PQ}{AB} = \frac{8}{4} = 2,$$

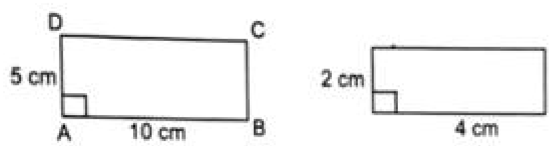
$$\frac{PR}{AC} = \frac{10}{5} = 2$$

and $\frac{QR}{BC} = \frac{6}{3} = 2$

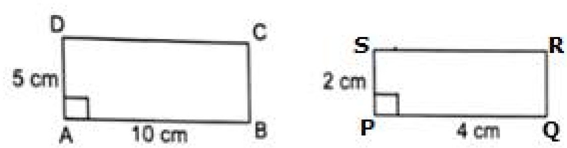
The corresponding sides of a right-angled triangle ABC and PQR are proportional, and their corresponding angles are not equal. Hence, triangles ABC and PQR are not similar.

5. Question

State whether the following rectangles are similar or not.



Answer



Two polygons of the same number of sides are similar, if

a) All the corresponding angles are equal.

b) all the corresponding sides are in the same ratio (or proportion)

In case of rectangles ABCD and PQRS

$\frac{AD}{PS} = \frac{5}{2},$

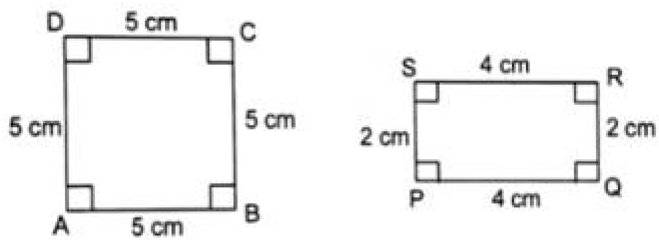
$\frac{AB}{PQ} = \frac{10}{4} = \frac{5}{2}$

And it is given that both are rectangles and we know that, in rectangle all angles are of 90°

The corresponding sides of a rectangle ABCD and PQRS are proportional, and their corresponding angles are equal. Hence, rectangles ABCD and PQRS are similar.

6. Question

State whether the following quadrilaterals are similar or not:



Answer

Two polygons of the same number of sides are similar, if

a) All the corresponding angles are equal.

b) all the corresponding sides are in the same ratio (or proportion)

In case of quadrilaterals ABCD and PQRS

$\frac{AD}{PS} = \frac{5}{2},$

$\frac{AB}{PQ} = \frac{5}{4},$

$\frac{BC}{QR} = \frac{5}{2},$

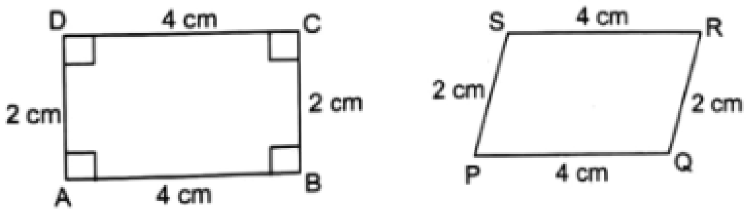
$\frac{CD}{RS} = \frac{5}{4}$

and ∠A=∠B =∠C =∠D =∠P =∠Q =∠R=∠S =90°

The corresponding sides of a quadrilateral ABCD and PQRS are not proportional. Hence, quadrilaterals ABCD and PQRS are not similar.

7 A. Question

State whether the following pair of polygons are similar or not.



Answer

Two polygons of a same number of sides are similar, if

a) all the corresponding angles are equal.

b) all the corresponding sides are in the same ratio (or proportion)

In case of polygons ABCD and PQRS

$\frac{AD}{PS} = \frac{2}{2} = 1$,

$\frac{AB}{PQ} = \frac{4}{4} = 1$,

$\frac{BC}{QR} = \frac{2}{2} = 1$,

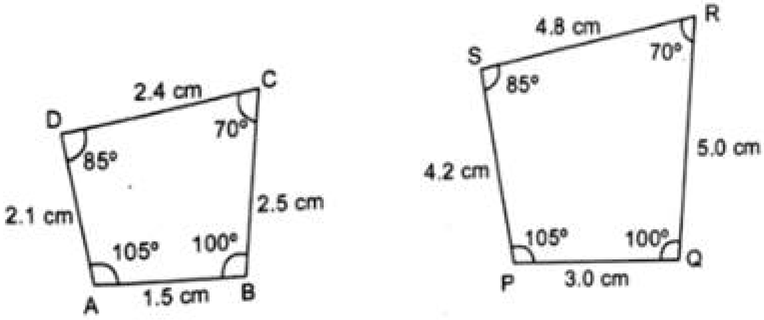
$\frac{CD}{RS} = \frac{4}{4} = 1$

and $\angle A = \angle B = \angle C = \angle D = 90°$ but $\angle P, \angle Q, \angle R, \angle S \neq 90°$

The corresponding sides of a polygon ABCD and PQRS are proportional, but their corresponding angles are not equal. Hence, polygon ABCD and PQRS are not similar.

7 B. Question

State whether the following pair of polygons are similar or not.



Answer

Two polygons of a same number of sides are similar if

a) all the corresponding angles are equal.

b) all the corresponding sides are in the same ratio (or proportion)

In case of polygons ABCD and PQRS

$\frac{AD}{PS} = \frac{2.1}{4.2} = \frac{1}{2},$

$\frac{AB}{PQ} = \frac{1.5}{3.0} = \frac{1}{2},$

$\frac{BC}{QR} = \frac{2.5}{5.0} = \frac{1}{2},$

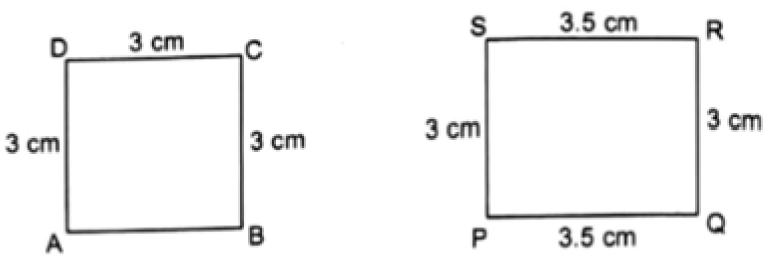
$\frac{CD}{RS} = \frac{2.4}{4.8} = \frac{1}{2}$

and ∠A=∠P =105°, ∠B =∠Q =100°, ∠C =∠R=70°, ∠D=∠S =85°

The corresponding sides of a polygon ABCD and PQRS are proportional, and their corresponding angles are also equal. Hence, polygon ABCD and PQRS are similar.

7 C. Question

State whether the following pair of polygons are similar or not.



Answer

Two polygons of the same number of sides are similar, if

a) All the corresponding angles are equal.

b) all the corresponding sides are in the same ratio (or proportion)

In case of polygons ABCD and PQRS

$$\frac{AD}{PS} = \frac{3}{3} = 1\,,$$

$$\frac{AB}{PQ} = \frac{3}{3.5},$$

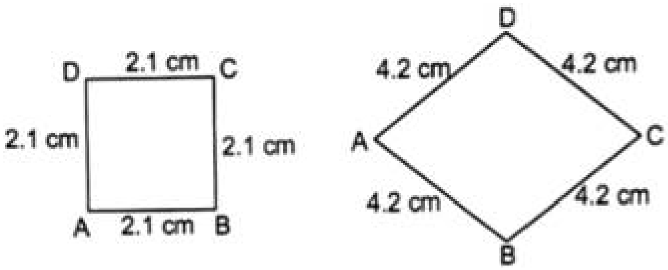
$$\frac{BC}{QR} = \frac{3}{3} = 1,$$

$$\frac{CD}{RS} = \frac{3}{3.5}$$

Clearly, the corresponding sides of a polygon ABCD and PQRS are not proportional. Hence, polygon ABCD and PQRS are not similar.

7 D. Question

State whether the following pair of polygons are similar or not.



Answer

Two polygons of the same number of sides are similar if

a) all the corresponding angles are equal.

b) all the corresponding sides are in the same ratio (or proportion)

In case of polygons □ ABCD and ◊ ABCD

$$\frac{AB}{AB} = \frac{2.1}{4.2} = 2\,,$$

$$\frac{BC}{BC} = \frac{2.1}{4.2} = 2,$$

$$\frac{CD}{CD} = \frac{2.1}{4.2} = 2,$$

$$\frac{DA}{DA} = \frac{2.1}{4.2} = 2$$

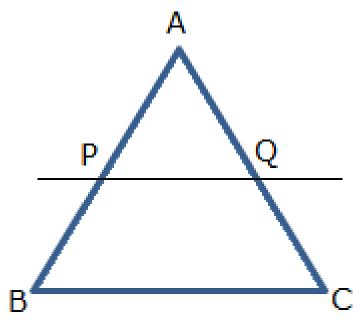
The corresponding sides of a polygon ABCD and ABCD are proportional, but their corresponding angles are not equal as we can see the first figure is of a square (all angles are of 90°) and other is of a rhombus (in rhombus the diagonal meet in the middle at a right angle). Hence, polygon ABCD and ABCD are not similar.

Exercise 5.2

1. Question

In ΔABC, P and Q are two points on AB and AC respectively such that PQ || BC and $\frac{AP}{PB} = \frac{2}{3}$, then find $\frac{AQ}{QC}$.

Answer



Given: PQ || BC

and $\frac{AP}{PB} = \frac{2}{3}$

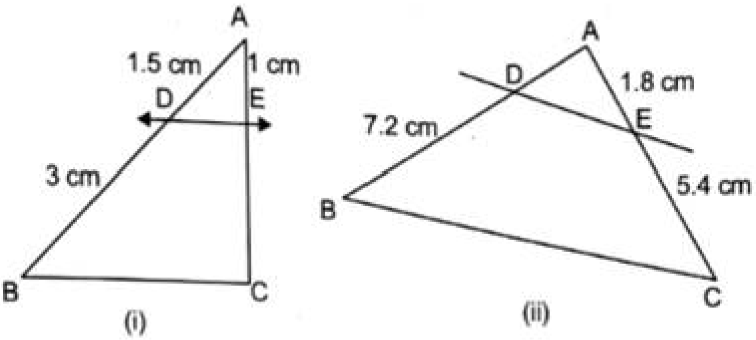
By **Basic Proportionality theorem** which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.

$$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$$

$$\Rightarrow \frac{AP}{PB} = \frac{AQ}{QC} = \frac{2}{3}$$

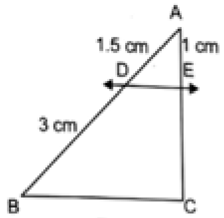
2. Question

In figures (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).



Answer

(i)



Given: DE || BC

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

[by basic proportionality theorem which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.]

$\Rightarrow \frac{1.5}{3} = \frac{1}{EC}$ [given: AD= 1.5cm, DB =3cm & AE =1cm]

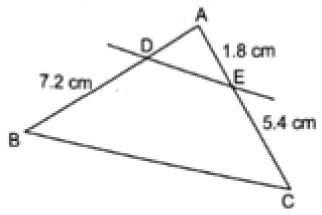
$$\Rightarrow EC = \frac{3}{1.5}$$

$$\Rightarrow EC = \frac{3 \times 10}{15}$$

$$\Rightarrow EC = \frac{30}{15}$$

$\Rightarrow$ EC = 2cm

(ii)



Given: DB = 7.2 cm, AE = 1.8 cm and EC = 5.4 cm

and DE || BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$

[by basic proportionality theorem which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.]

$\Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4}$

$\Rightarrow AD = \frac{7.2 \times 1.8}{5.4}$

$\Rightarrow AD = \frac{72 \times 18}{54 \times 10}$

$\Rightarrow AD = \frac{24}{10}$

⇒ AD = 2.4cm

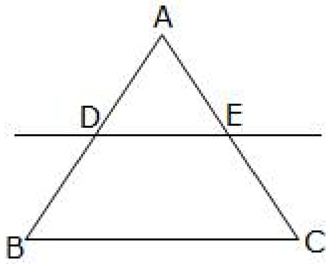
3. Question

In a ΔABC, DE || BC, where D is a point on AB and E is a point on AC, then

(i) $\frac{EC}{DB}$ =……… (ii) $\frac{AD}{AE}$ =………

(iii) $\frac{AB}{DB}$ =……… (iv) $\frac{EC}{DB}$ =………

Answer



(i) Given: DE || BC

Basic Proportionality theorem which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [by basic proportionality theorem]

(ii) **Basic Proportionality theorem** which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.

By basic proportionality theorem, we know that

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{AD}{AB - AD} = \frac{AE}{AC - AE}$$

$$\Rightarrow \frac{AD}{AD\left(\frac{AB}{AD} - 1\right)} = \frac{AE}{AE\left(\frac{AC}{AE} - 1\right)}$$

$$\Rightarrow \frac{AB}{AD} - 1 = \frac{AC}{AE} - 1$$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

$$\Rightarrow \frac{AD}{AE} = \frac{AB}{AC}$$

(iii) From part (i), we know that $\frac{AD}{DB} = \frac{AE}{EC}$

On adding 1 to both the sides, we get

$$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

$$\Rightarrow \frac{AD + DB}{DB} = \frac{AE + EC}{EC}$$

$$\Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$$

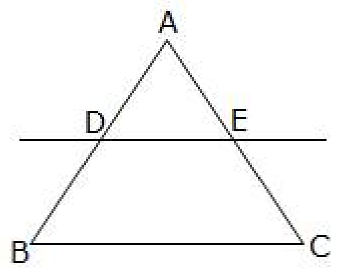
(iv) From part (iii), we have $\frac{AB}{DB} = \frac{AC}{EC}$

$$\Rightarrow \frac{EC}{DB} = \frac{AC}{AB}$$

4. Question

If in Δ ABC, DE || BC and DE cuts sides AB and AC at D and E respectively such that AD: DB = 4: 5, then find AE: EC.

Answer



Given: DE || BC

and $\frac{AD}{DB} = \frac{4}{5}$

To find: AE: EC

Given: DE || BC

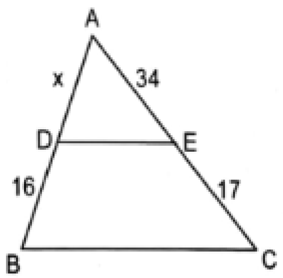
Basic Proportionality theorem which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.

∴ $\frac{AD}{DB} = \frac{AE}{EC}$ [by basic proportionality theorem]

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC} = \frac{4}{5}$$

5. Question

In the adjoining figure, DE || BC. Find x.



Answer

Given: AD = x

DB =16, AE = 34 and EC = 17

Given: DE || BC

Basic Proportionality theorem which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.

So, by basic proportionality theorem

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$

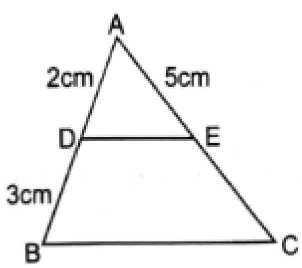
$\Rightarrow \frac{x}{16} = \frac{34}{17}$

$\Rightarrow \frac{x}{16} = 2$

⇒ x = 32

6. Question

In the adjoining figure, AD = 2 cm, DB = 3 cm, AE = 5 cm and DE || BC, then find EC.



Answer

Given: AD = 2cm, DB =3cm, AE = 5cm

and DE || BC

Basic Proportionality theorem which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.

So, by basic proportionality theorem

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

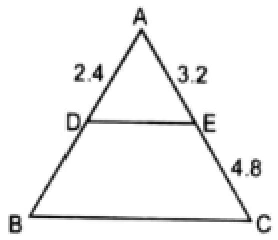
$$\Rightarrow \frac{2}{3} = \frac{5}{EC}$$

$$\Rightarrow EC = \frac{5 \times 3}{2}$$

⇒ EC = 7.5 cm

7. Question

In the adjoining figure, DE || BC, AD = 2.4 cm, AE = 3.2 cm, CE = 4.8 cm, find BD.



Answer

Given: AD = 2.4cm, AE = 3.2cm and EC = 4.8cm

and DE || BC

Basic Proportionality theorem which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.

So, by basic proportionality theorem

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{2.4}{DB} = \frac{3.2}{4.8}$$

$$\Rightarrow \frac{2.4}{DB} = \frac{2}{3}$$

$\Rightarrow \ DB = \frac{2.4 \times 3}{2}$

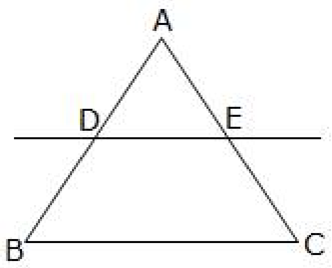
⇒ DB = 3.6 cm

or BD = 3.6 cm

8. Question

If DE has been drawn parallel to side BC of ΔABC cutting AB and AC at points D and E respectively, such that $\frac{AD}{DB} = \frac{3}{4}$, then find the value of $\frac{AE}{EC}$.

Answer



Given: DE || BC

and $\frac{AD}{DB} = \frac{3}{4}$

To find : $\frac{AE}{EC}$

Given: DE || BC

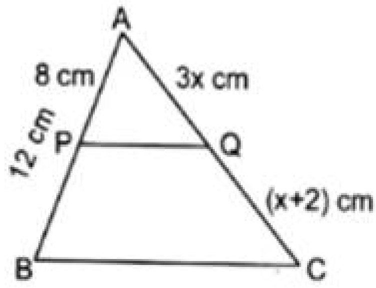
Basic Proportionality theorem which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.

$\therefore \ \frac{AD}{DB} = \frac{AE}{EC}$ [by basic proportionality theorem]

$\Rightarrow \ \frac{AD}{DB} = \frac{AE}{EC} = \frac{3}{4}$

9. Question

In the adjoining figure, P and Q are points on sides AB and AC respectively of mix such that PQ || BC and AP= 8 cm, AB =12 cm, AQ = 3x cm, QC = (x + 2) cm. Find x.



Answer

Given: AP = 8cm , AB =12cm, AQ = (3x)cm and QC = (x+2)cm

and PQ || BC

Basic Proportionality theorem which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.

$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$ [by basic proportionality theorem]

$$\Rightarrow \frac{AP}{AB - AP} = \frac{AQ}{QC}$$

$$\Rightarrow \frac{8}{12 - 8} = \frac{3x}{x + 2}$$

$$\Rightarrow \frac{8}{4} = \frac{3x}{x + 2}$$

$$\Rightarrow 2 = \frac{3x}{x + 2}$$

⇒ 2(x+2) = 3x

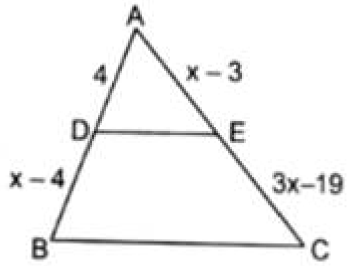
⇒ 2x + 4 = 3x

⇒ 2x – 3x = - 4

⇒ x = 4

10. Question

In the adjoining figure, DE || BC, find x.



Answer

Given: AD = 4, DB =x – 4, AE = x – 3 and EC = 3x – 19

and DE || BC

Basic Proportionality theorem which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.

So, by basic proportionality theorem

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{4}{x-4} = \frac{x-3}{3x-19}$

$\Rightarrow 4(3x - 19) = (x - 4)(x - 3)$

$\Rightarrow 12x - 76 = x^2 - 3x - 4x + 12$

$\Rightarrow 12x - 76 = x^2 - 7x + 12$

$\Rightarrow x^2 - 7x + 12 - 12x + 76 = 0$

$\Rightarrow x^2 - 19x + 88 = 0$

Solving the Quadratic equation by splitting themiddle term, we get,

$\Rightarrow x^2 - 11x - 8x + 88 = 0$

$\Rightarrow x(x - 11) - 8(x - 11) = 0$

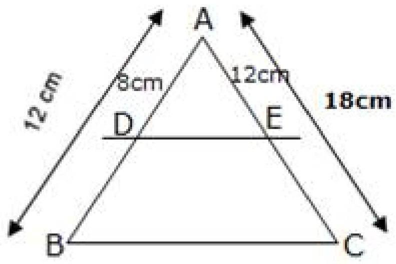
$\Rightarrow (x - 8)(x - 11) = 0$

$\Rightarrow x = 8$ and 11

11. Question

If D and E are points on sides AB and AC respectively of ΔABC and AB = 12 cm, AD = 8 cm, AE = 12 cm, AC =18 cm, then prove that DE || BC.

Answer



Given: AB = 12cm, AD =8cm, AE = 12cm and AC = 18cm

To Prove: DE || BC

In $\triangle$ ABC,

$$\frac{AD}{DB} = \frac{AD}{AB - AD} = \frac{8}{12 - 8} = \frac{8}{4} = 2$$

and $\frac{AE}{EC} = \frac{AE}{AC - EC} = \frac{12}{18 - 12} = \frac{12}{6} = 2$

Thus, $\frac{AD}{DB} = \frac{AE}{EC}$

Basic Proportionality theorem which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.

Hence, DE || BC [by converse of basic proportionality theorem]

Hence, Proved.

12. Question

P and Q are points on sides AB and AC respectively of Δ ABC. For each of the following cases, state whether PQ || BC.

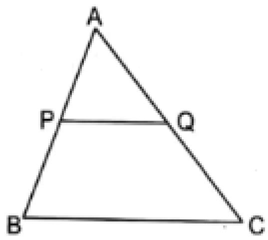
(i) AP= 8 cm, PB = 3 cm, AC = 22 cm and AQ =16 cm.

(ii) AB= 1.28 cm, AC = 2.56 cm, AP= 0.16 cm and AQ = 0.32 cm

(iii) AB = 5 cm, AC =10 cm, AP= 4 cm, AQ = 8 cm.

(iv) AP= 4 cm, PB= 4.5 cm, AQ = 4 cm, QC = 5 cm.

Answer



(i) Given: AP= 8 cm, PB = 3 cm, AC = 22 cm and AQ =16 cm

To find: PQ || BC

In $\triangle$ ABC,

$$\frac{AP}{PB} = \frac{8}{3}$$

$$\text{and} \frac{AQ}{QC} = \frac{16}{AC - AQ} = \frac{16}{22 - 16} = \frac{16}{6} = \frac{8}{3}$$

$$\text{Thus,} \frac{AP}{PB} = \frac{AQ}{QC}$$

<u>Basic Proportionality theorem</u> which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.

Hence, PQ || BC [by converse of basic proportionality theorem]

Hence, Proved.

(ii) Given: AB= 1.28 cm, AC = 2.56 cm, AP= 0.16 cm and AQ = 0.32 cm

To find: PQ || BC

In $\triangle$ ABC,

$$\frac{AP}{PB} = \frac{0.16}{AB - AP} = \frac{0.16}{1.28 - 0.16} = \frac{0.16}{1.12} = \frac{1}{7}$$

$$\text{and} \frac{AQ}{QC} = \frac{16}{AC - AQ} = \frac{0.32}{2.56 - 0.32} = \frac{0.32}{2.24} = \frac{1}{7}$$

$$\text{Thus,} \frac{AP}{PB} = \frac{AQ}{QC}$$

<u>Basic Proportionality theorem</u> which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.

Hence, PQ || BC [by converse of basic proportionality theorem]

Hence, Proved.

(iii) Given: AB = 5 cm, AC =10 cm, AP= 4 cm, AQ = 8 cm

To find: PQ || BC

In $\triangle$ ABC,

$$\frac{AP}{PB} = \frac{4}{AB - AP} = \frac{4}{5-4} = \frac{4}{1} = 4$$

$$\text{and} \frac{AQ}{QC} = \frac{8}{AC - AQ} = \frac{8}{10-8} = \frac{8}{2} = 4$$

$$\text{Thus,} \frac{AP}{PB} = \frac{AQ}{QC}$$

Basic Proportionality theorem which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.

Hence, PQ || BC [by converse of basic proportionality theorem]

Hence, Proved.

(iv) Given: AP= 4 cm, PB= 4.5 cm, AQ = 4 cm, QC = 5 cm

To find: PQ || BC

In $\triangle$ ABC,

$$\frac{AP}{PB} = \frac{4}{4.5} = \frac{4 \times 10}{45} = \frac{8}{9}$$

$$\text{and} \frac{AQ}{QC} = \frac{4}{5}$$

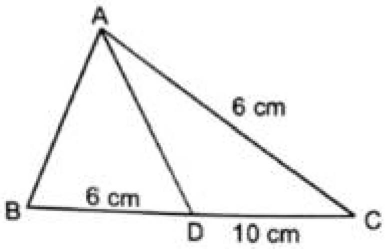
$$\text{Thus,} \frac{AP}{PB} \neq \frac{AQ}{QC}$$

Basic Proportionality theorem which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.

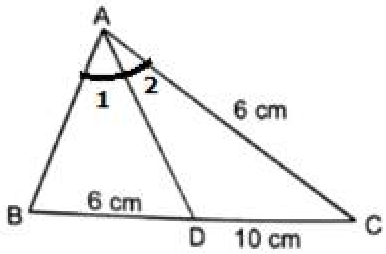
⇒ PQ is not parallel to BC

13. Question

In the adjoining figure, AD is the bisector of ∠BAC. If BC = 10 cm, BD = 6 cm AC = 6 cm, then find AB.



Answer



Given: AD is the bisector of ∠BAC

and by **Angle-Bisector theorem** which states that if a ray bisects an **angle** of a **triangle**, then it divides the opposite side into segments that are proportional to the other two sides.

$$\therefore \frac{BD}{DC} = \frac{AB}{AC}$$

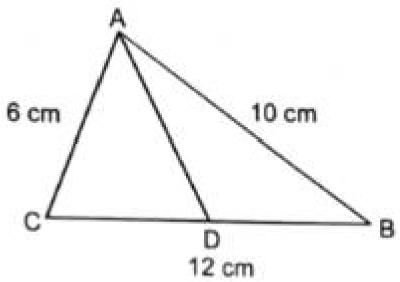
$$\Rightarrow \frac{6}{10} = \frac{AB}{6}$$

$$\Rightarrow AB = \frac{36}{10}$$

⇒ AB = 3.6cm

14. Question

In the adjoining figure, AD is the bisector of ∠BAC. If AB = 10 cm, AC = 6 cm, BC = 12 cm, find BD.



Answer

Given: AD is the bisector of ∠BAC

and by **Angle-Bisector theorem** which states that if a ray bisects an **angle** of a **triangle**, then it divides the opposite side into segments that are proportional to the other two sides.

$$\therefore \frac{CD}{DB} = \frac{AC}{AB}$$

$$\Rightarrow \frac{CD}{BC - CD} = \frac{6}{10}$$

$$\Rightarrow \frac{CD}{CD\left(\frac{BC}{CD} - 1\right)} = \frac{6}{10}$$

$$\Rightarrow \frac{BC}{CD} - 1 = \frac{10}{6}$$

$$\Rightarrow \frac{BC}{CD} = \frac{10}{6} + 1$$

$$\Rightarrow \frac{12}{CD} = \frac{10 + 6}{6}$$

$$\Rightarrow \frac{12}{CD} = \frac{16}{6}$$

$$\Rightarrow CD = \frac{12 \times 6}{16}$$

$$\Rightarrow CD = \frac{9}{2} = 4.5\text{cm}$$

And BC – CD = DB

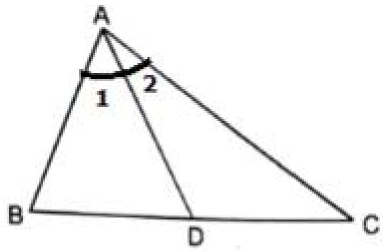
⇒ 12 – 4.5 = DB

⇒ DB = 7.5cm

15. Question

In EABC, AD is the bisector of ∠A. If AB = 3.5 cm, AC = 4.2 cm, DC = 2.4 cm. Find BD.

Answer



Given: AD is the bisector of $\angle A$

and by **Angle-Bisector theorem** which states that if a ray bisects an **angle** of a **triangle**, then it divides the opposite side into segments that are proportional to the other two sides.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{BD}{2.4} = \frac{3.5}{4.2}$

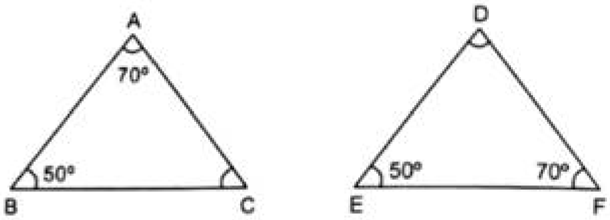
$\Rightarrow BD = \frac{3.5 \times 2.4}{4.2}$

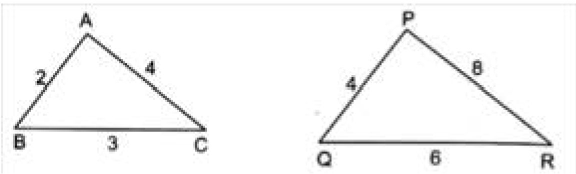
$\Rightarrow$ BD = 2cm

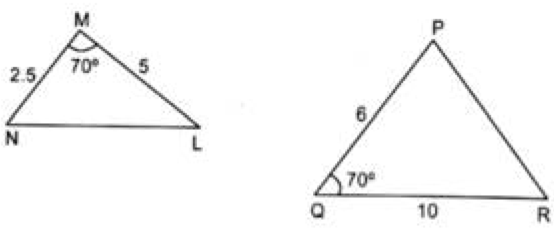
Exercise 5.3

1. Question

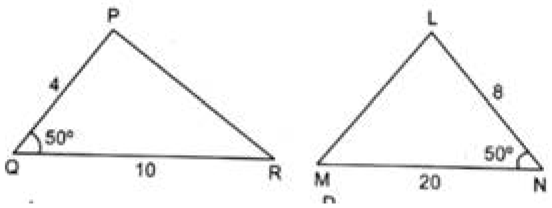
State which of the following pairs of triangles are similar. Write the similarity criterion used and write the pairs of similar triangles in symbolic form (all lengths of sides are in cm).

(i) 

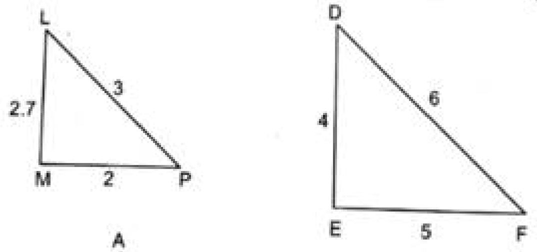
(ii) 

(iii) 

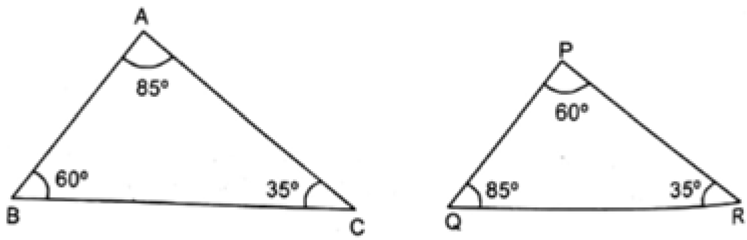
(iv)



(v)



(vi)



(vii)

Answer

(i) In △ ABC,

∠A = 70° and ∠B = 50°

And we know that, sum of the angles = 180°

⇒∠A + ∠B +∠C = 180°

⇒70° + 50° +∠C = 180°

⇒ 120° +∠C = 180°

⇒ ∠C = 60°

And In △ DEF

∠F = 70° and ∠E = 50°

And we know that, sum of the angles = 180°

⇒∠D + ∠E +∠F = 180°

$\Rightarrow \angle D + 50° + 70° = 180°$

$\Rightarrow \angle D = 60°$

Yes, $\triangle ABC \sim \triangle DEF$ [by AAA similarity criterion]

(ii) In $\triangle ABC$ and $\triangle PQR$

Here, $\frac{AB}{PQ} = \frac{2}{4} = \frac{1}{2}, \frac{BC}{QR} = \frac{3}{6} = \frac{1}{2}, \frac{AC}{PR} = \frac{4}{8} = \frac{1}{2}$

As, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

So, $\triangle ABC \sim \triangle PQR$ [by SSS similarity criterion]

(iii) In $\triangle MNL$ and $\triangle PQR$

$\angle NML = \angle PQR = 70°$

$$\frac{MN}{PQ} = \frac{2.5}{6} = \frac{25}{6 \times 10} = \frac{5}{6 \times 2} = \frac{5}{12}$$

and $\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$

$$\Rightarrow \frac{MN}{PQ} \neq \frac{ML}{QR}$$

No, the two triangles are not similar.

(iv) In $\triangle PQR$ and $\triangle LMN$

$\angle PQR = \angle LNM = 50°$

$$\frac{PQ}{LN} = \frac{4}{8} = \frac{1}{2}$$

and $\frac{QR}{MN} = \frac{10}{20} = \frac{1}{2}$

$$\Rightarrow \frac{PQ}{LN} = \frac{QR}{MN}$$

$\therefore \triangle PQR \sim \triangle LMN$ [by SAS similarity criterion]

(v) In $\triangle LMP$ and $\triangle DEF$

Here, $\frac{LM}{DE} = \frac{2.7}{4} = \frac{1}{2}, \frac{LP}{DF} = \frac{3}{6} = \frac{1}{2}, \frac{MP}{EF} = \frac{2}{5}$

As $\frac{AB}{PQ} \neq \frac{BC}{QR} \neq \frac{AC}{PR}$

So, no two triangles are not similar

(vi) In $\triangle$ ABC and $\triangle$ PQR

∠A = ∠Q = 85°

∠B = ∠P = 60°

and ∠C =∠R = 35°

So, $\triangle$ PQR ~ $\triangle$ LMN [by AAA similarity]

(vii) In $\triangle$ ABC and $\triangle$ PQR

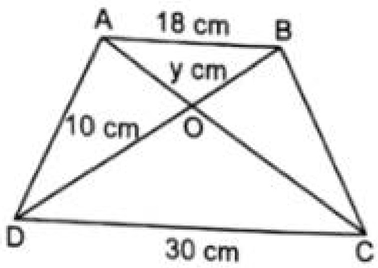
Here, $\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}, \frac{BC}{PR} = \frac{2.5}{5} = \frac{1}{2}, \frac{AC}{PQ} = \frac{3}{6} = \frac{1}{2}$

As, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{AC}{PQ}$

So, $\triangle$ ABC ~ $\triangle$ PQR [by SSS similarity criterion]

2. Question

If diagonals AC and BD of trapezium ABCD with AB || CD intersect each other at 0 and AB= 18 cm, DC = 30 cm, OB =y cm, OD= 10 cm, find y.



Answer

Given: ABCD is a trapezium with AB || CD

and diagonals AB and CD intersecting at O

To find: y

Firstly, we prove that $\triangle$ OAB ~ $\triangle$ ODC

Let $\triangle$ OAB and $\triangle$ ODC

∠AOB = ∠COD [vertically opposite angles]

∠OBA = ∠ODC [∵AB || CD with BD as transversal.

alternate angles are equal]

∠OAB = ∠OCD [∵AB || CD with BD as transversal.

alternate angles are equal]

∴ △ OAB ~ △ ODC [by AAA similarity]

Since triangles are similar. Hence corresponding sides are proportional.

$$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD} = \frac{AB}{DC}$$

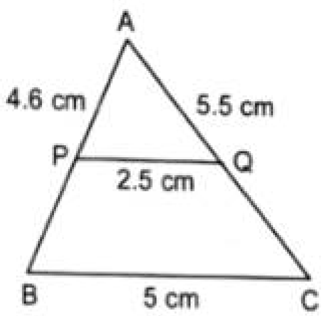
$$\Rightarrow \frac{OB}{OD} = \frac{AB}{DC}$$

$$\Rightarrow \frac{y}{10} = \frac{18}{30}$$

⇒ y = 6cm

3. Question

In the given figure BC = 5 cm, AC = 5.5 cm and AB= 4.6 cm. P and Q are points on AB and AC respectively such that PQ || BC. If PQ = 2.5 cm, find other sides of ΔAPQ.



Answer

Given: PQ || BC

To find: AP and AQ

Since, PQ || BC, AB is transversal, then,

△ APQ = △ ABC [by corresponding angles]

Since, PQ || BC, AC is transversal, then,

△ APQ = △ ABC [by corresponding angles]

In △ APQ and △ ABC

∠APQ = ∠ABC

$\angle AQP = \angle ACB$

$\therefore \triangle APQ \cong \triangle ABC$ [by AAA similarity]

Since, the corresponding sides of similar triangles are proportional

$$\therefore \frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC}$$

$$\Rightarrow \frac{AP}{AB} = \frac{PQ}{BC}$$

$$\Rightarrow \frac{AP}{4.6} = \frac{2.5}{5}$$

$$\Rightarrow AP = \frac{2.5 \times 4.6}{5}$$

$\Rightarrow AP = 2.3$

Now, taking $\frac{PQ}{BC} = \frac{AQ}{AC}$

$$\Rightarrow \frac{2.5}{5} = \frac{AQ}{5.5}$$

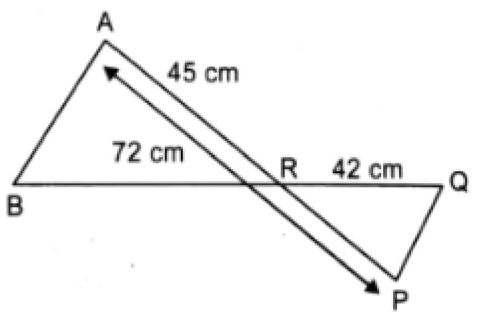
$$\Rightarrow AQ = \frac{2.5 \times 5.5}{5}$$

$\Rightarrow AQ = 2.75$

Therefore, AP = 2.3cm and AQ = 2.75cm

4. Question

In the given figure $\Delta ABR \sim \triangle PQR$, if PQ = 30 cm, AR = 45 cm, AP = 72 cm and QR = 42 cm, find PR and BR.



Answer

Given: $\triangle ABR \sim \triangle PQR$

As, $\triangle$ ABR and $\triangle$ PQR are similar

$$\therefore \frac{AR}{PR} = \frac{BR}{QR} = \frac{AB}{QP}$$

$$\Rightarrow \frac{45}{AP - AR} = \frac{BR}{42} = \frac{AB}{30}$$

$$\Rightarrow \frac{45}{72 - 45} = \frac{BR}{42}$$

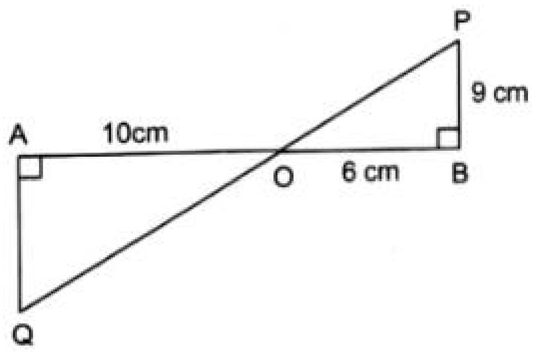
$$\Rightarrow \frac{45}{27} = \frac{BR}{42}$$

⇒BR = 70cm

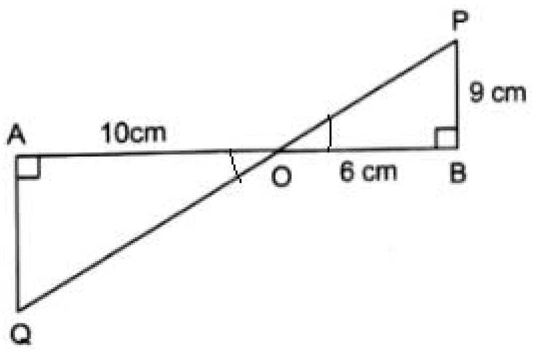
and PR = AP – AR = 72 – 45 = 27cm

5. Question

In the given figure, QA and PB are perpendiculars to AB. If AO = 10 cm, BO = 6 cm and PB = 9 cm, find AQ.



Answer



Let us take $\triangle$ OAQ and $\triangle$ OBP

∠AOQ = ∠BOP (vertically opposite angles)

∠OAQ = ∠OBP (each 90°)

$\therefore$ $\triangle$ OAQ ~ $\triangle$ OBP (by AA similarity criterion)

Given: AO = 10 cm, BO = 6 cm and PB = 9 cm

As, $\triangle$ OAQ ~ $\triangle$ OBP

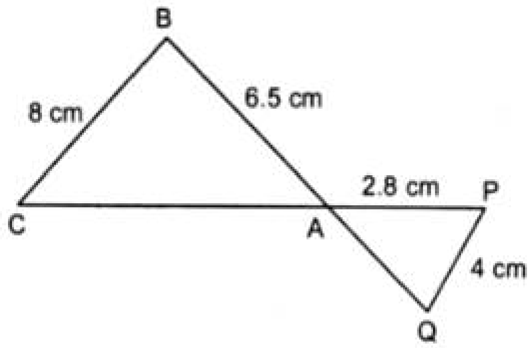
$$\therefore \frac{AO}{BO} = \frac{AQ}{BP}$$

$$\Rightarrow \frac{10}{6} = \frac{AQ}{9}$$

⇒AQ = 15cm

6. Question

In the given figure ΔACB ~ ΔAPQ, if BC = 8 cm, PQ = 4 cm, BA = 6.5 cm, AP= 2.8 cm, Find CA and AQ.



Answer

Given: $\triangle$ ACB ~ $\triangle$ APQ

As, $\triangle$ ACB and $\triangle$ APQ are similar

$$\therefore \frac{CA}{AP} = \frac{BA}{AQ} = \frac{CB}{QP}$$

$$\Rightarrow \frac{CA}{2.8} = \frac{6.5}{AQ} = \frac{8}{4}$$

$$\Rightarrow \frac{CA}{2.8} = \frac{6.5}{AQ} = 2$$

Taking $\frac{CA}{2.8} = 2$

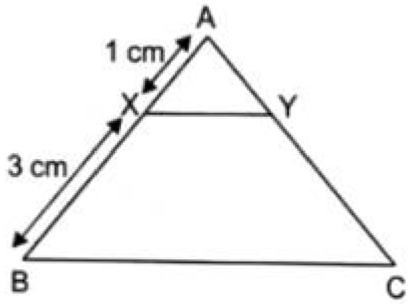
⇒CA = 5.6cm

Now, taking $\frac{6.5}{AQ} = 2$

⇒AQ =3.25cm

7. Question

In the given figure, XY || BC. Find the length of XY, given BC = 6 cm.



Answer

Given: XY || BC

To find: XY

Since, XY || BC, AB is transversal, then,

$\triangle$ AXY = $\triangle$ ABC [by corresponding angles]

Since, XY || BC, AC is transversal, then,

$\triangle$ AYX = $\triangle$ ABC [by corresponding angles]

In $\triangle$ AXY and $\triangle$ ABC

$\angle AXY = \angle ABC$

$\angle AYX = \angle ACB$

$\therefore$ $\triangle$ AXY ≅ $\triangle$ ABC [by AA similarity]

Since, triangles are similar, hence corresponding sides will be proportional

$$\therefore \frac{AX}{AB} = \frac{XY}{BC} = \frac{AY}{AC}$$

$$\Rightarrow \frac{AX}{AB} = \frac{XY}{BC}$$

$$\Rightarrow \frac{1}{AX + XB} = \frac{XY}{6}$$

$$\Rightarrow \frac{1}{1 + 3} = \frac{XY}{6}$$

$$\Rightarrow \frac{1}{4} = \frac{XY}{6}$$

$\Rightarrow XY = \frac{6}{4}$

$\Rightarrow XY = 1.5$

Therefore, XY= 1.5cm

8. Question

The perimeters of two similar triangles, ABC and PQR ($\Delta ABC \sim \Delta PQR$) are respectively 72 cm and 48 cm. If PQ = 20 cm, find AB.

Answer

Given: $\Delta ABC \sim \Delta PQR$, PQ =20cm

And perimeter of Δ ABC and Δ PQR are 72cm and 48cm respectively.

As, $\Delta ABC \sim \Delta PQR$

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ (corresponding sides are proportional)

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AB + BC + AC}{PQ + QR + PR}$$

$$\Rightarrow \frac{AB + BC + AC}{PQ + QR + PR} = \frac{AB}{PQ}$$

$$\Rightarrow \frac{\text{Perimeter of ABC}}{\text{Perimeter of PQR}} = \frac{AB}{PQ}$$

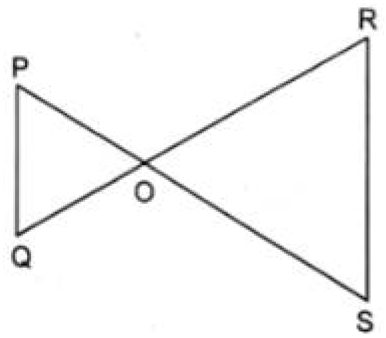
$$\Rightarrow \frac{72}{48} = \frac{AB}{20}$$

$$\Rightarrow AB = \frac{72 \times 20}{48}$$

$\Rightarrow AB = 30cm$

9. Question

In the given figure, if PQ || RS, prove that $\Delta POQ \sim \Delta SOR$.



Answer

Given: PQ || RS

To Prove: △ POQ ~ △ SOR

Let us take △ POQ and △ SOR

∠OPQ = ∠OSR (as PQ || RS, Alternate angles)

∠POQ = ∠ROS (vertically opposite angles)

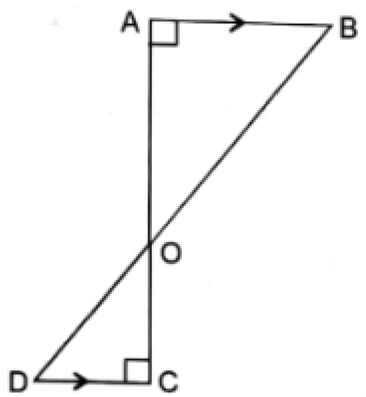
∠OQP = ∠ORS (as PQ || RS, Alternate angles)

∴ △ POQ ~ △ SOR (by AAA similarity criterion)

Hence Proved

10. Question

In the given figure, if ∠A = ∠C, then prove that Δ AOB ~ Δ COD



Answer

Given: ∠A = ∠C

To Prove: △ AOB ~ △ COD

Let us take △ AOB and △ COD

∠A = ∠C (given)

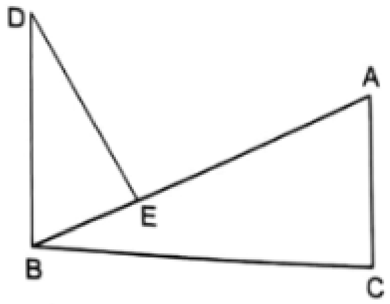
∠AOB = ∠COD (vertically opposite angles)

∴ △ AOB ~ △ COD (by AA similarity criterion)

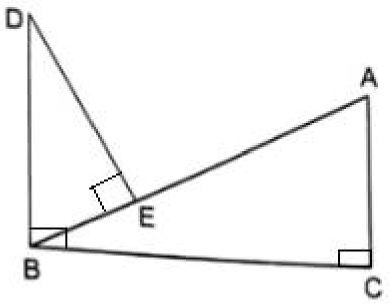
Hence Proved

11. Question

In the given figure DB ⊥ BC, DE ⊥ AB and AC ⊥ BC, prove that ΔBDE ~ΔABC.



Answer



We have, DB ⊥ BC and AC ⊥ BC

∠B + ∠C = 90° + 90°

⇒ ∠B + ∠C = 180°

∴ BD || AC

⇒∠EBD = ∠CAB (alternate angles)

Let us take △ BDE and △ ABC

∠BED = ∠ACB (each 90°)

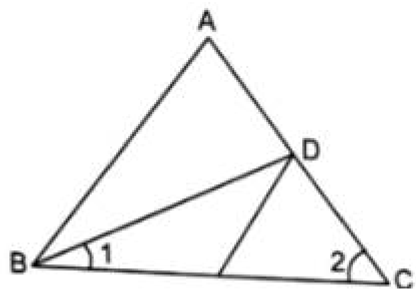
∠EBD = ∠CAB (alternate angles)

∴ △ BDE ~ △ ABC (by AA similarity criterion)

Hence Proved

12. Question

In the given figure, $\angle 1 = \angle 2$ and $\frac{AC}{BD} = \frac{CB}{CE}$, prove that $\Delta ACB \sim \Delta DCE$



Answer

We have, $\frac{AC}{BD} = \frac{CB}{CE}$

$\Rightarrow \frac{AC}{CB} = \frac{BD}{CE}$

$\Rightarrow \frac{AC}{CB} = \frac{CD}{CE}$ ($\because$, BD = DC as $\angle 1 = \angle 2$) ...(i)

Also, $\angle 1 = \angle 2$

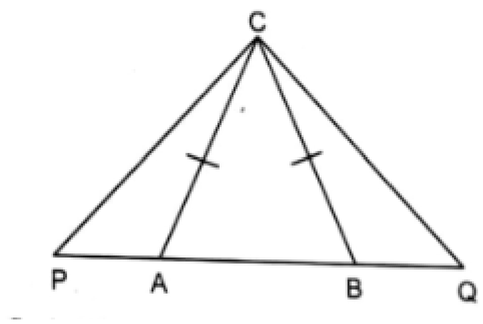
i.e. $\angle DBC = \angle ACB$

$\therefore \triangle ACB \sim \triangle DCE$ (by SAS similarity criterion)

Hence Proved

13. Question

In an isosceles ΔABC with AC = BC, the base AB is produced both ways to P and Q such that AP x BQ = AC^2. Prove that : $\Delta ACP \sim \Delta BQC$



Answer

Given ABC is an isosceles triangle and AC = BC

$\because$ AC = BC

$\Rightarrow \angle CAB = \angle CBA$

$\Rightarrow 180° - \angle CAB = 180° - \angle CBA$

$\Rightarrow \angle CAP = \angle CBQ$

Also, AP x BQ = AC^2

$$\Rightarrow \frac{AP}{AC} = \frac{AC}{BQ}$$

$\Rightarrow \frac{AP}{AC} = \frac{BC}{BQ}$ ($\because$ AC =BC)

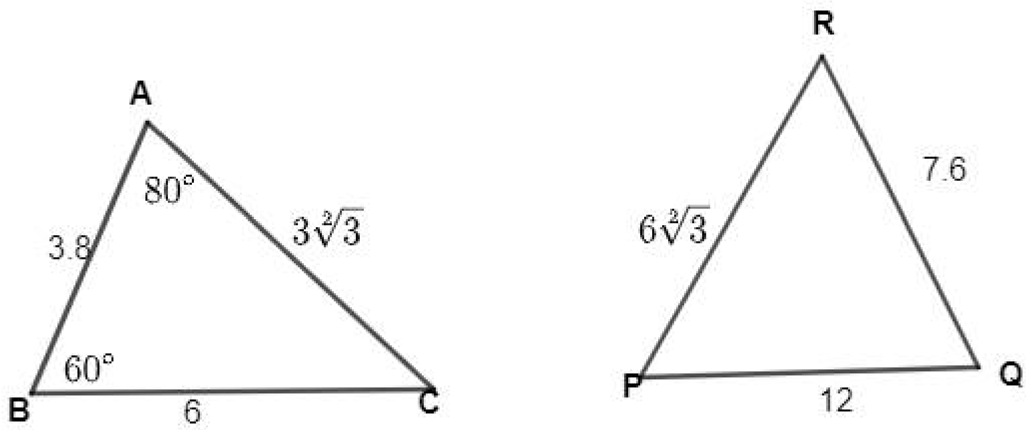
Thus, by SAS similarity, we get

$\triangle ACP \sim \triangle BQC$

Hence Proved

14. Question

In the given figure, find $\angle P$.



Answer

From the figure,

$$\frac{AB}{RQ} = \frac{3.8}{7.6} = \frac{1}{2}$$

$$\frac{BC}{PQ} = \frac{6}{12} = \frac{1}{2}$$

$$\frac{AC}{PR} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$$

Hence, $\frac{AB}{RQ} = \frac{BC}{PQ} = \frac{AC}{PR} = \frac{1}{2}$

Now it can be seen that both the triangles are similar as the corresponding sides are propotional.

From the figure we can see that,

∠ P = ∠ C

From ΔABC,

∠ A + ∠ B + ∠ C = 180°

60° + 80° + ∠ C = 180°

∠ C = 180° – 140°

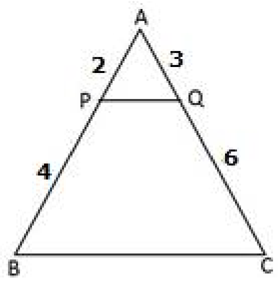
∠ C = 40°

Hence, ∠ P = 40°

15. Question

P and Q are points on the sides AB and AC respectively of a ΔABC. If AP = 2 cm, PB = 4 cm, AQ = 3 cm and QC = 6 cm, show that BC = 3 PQ.

Answer



Here, $\frac{AP}{PB} = \frac{2}{4} = \frac{1}{2}$

and $\frac{AQ}{QC} = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow \frac{AP}{PB} = \frac{AQ}{QC}$

∴ PQ || BC [by converse of basic proportionality theorem]

Now, take △ APQ and △ ABC

∠APQ = ∠ABC (corresponding angles)

∠AQP = ∠ACB (corresponding angles)

∴ △ APQ ~ △ ABC (by AA similarity criterion)

Since, triangles are similar, hence corresponding sides will be proportional

$$\therefore \frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC}$$

$$\Rightarrow \frac{2}{6} = \frac{PQ}{BC} = \frac{3}{9}$$

$$\Rightarrow \frac{2}{6} = \frac{PQ}{BC}$$

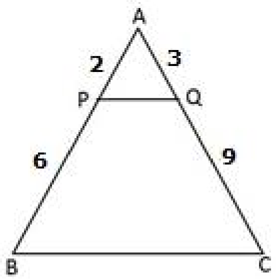
⇒BC = 3PQ

Hence Proved

16. Question

P and Q are respectively the points on the sides AB and AC of a ΔABC. If AP = 2 cm, PB = 6 cm, AQ = 3 cm and QC = 9, Prove that BC = 4PQ.

Answer



Here, $\frac{AP}{PB} = \frac{2}{6} = \frac{1}{3}$

and $\frac{AQ}{QC} = \frac{3}{9} = \frac{1}{3}$

$$\Rightarrow \frac{AP}{PB} = \frac{AQ}{QC}$$

∴ PQ || BC [by the converse of basic proportionality theorem]

Now, take △ APQ and △ ABC

∠APQ = ∠ABC (corresponding angles)

∠AQP = ∠ACB (corresponding angles)

∴ △ APQ ~ △ ABC (by AA similarity criterion)

Since, triangles are similar, hence corresponding sides will be proportional

$\therefore \frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC}$

$\Rightarrow \frac{2}{8} = \frac{PQ}{BC} = \frac{3}{12}$

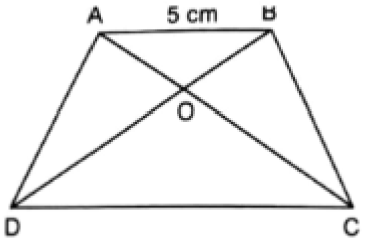
$\Rightarrow \frac{2}{8} = \frac{PQ}{BC}$

$\Rightarrow BC = 4PQ$

Hence Proved

17. Question

In the given figure, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and AB = 5 cm. Find the value of DC.



Answer

In $\triangle$ AOB and $\triangle$ COD,

$\angle$ AOB = $\angle$COD (Vertically opposite angles)

$\frac{AO}{OC} = \frac{BO}{OD}$ (given)

Therefore according to SAS similarity criterion,

$\therefore \triangle AOB \sim \triangle COD$

Since, triangles are similar, hence corresponding sides will be proportional

$\therefore \frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{DC}$

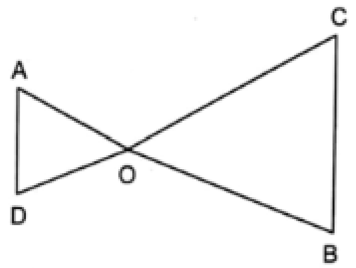
$\Rightarrow \frac{1}{2} = \frac{1}{2} = \frac{5}{DC}$

$\Rightarrow \frac{1}{2} = \frac{5}{DC}$

$\Rightarrow$ DC = 10cm

18. Question

In the given figure, OA .OB = OC.OD, show that: ∠A = ∠C and ∠B = ∠D.



Answer

Given: OA × OB = OC × OD

To Prove: ∠A = ∠C and ∠B = ∠D

Now, OA .OB = OC.OD

$\Rightarrow \frac{OA}{OC} = \frac{OD}{OB}$ ...(i)

In △AOD and △COB

$\frac{OA}{OC} = \frac{OD}{OB}$ (from (i))

∠AOD = ∠COB (vertically opposite angles)

∴ △AOD ~ △COB (by SAS similarity criterion)

We know that if two triangles are similar then their corresponding angles are equal.

⇒ ∠A = ∠C and ∠B = ∠D

Hence Proved

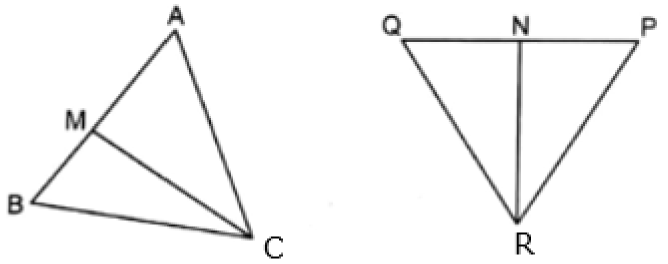
19. Question

In the given figure, CM and RN are respectively the medians of ΔABC and ΔPQR. If ΔABC ~ ΔPQR, prove that:

(i) ΔAMC ~ ΔPNR

(ii) $\frac{CM}{RN} = \frac{AB}{PQ}$

(iii) ΔCMB ~ ΔRNQ



Answer

Given: CM is the median of △ABC and RN is the median of △PQR

Also, △ ABC ~ △ PQR

To Prove: (i) △ AMC ~ △ PNR

CM is median of △ ABC

So, $AM = MB = \frac{1}{2}AB$ ...(1)

Similarly, RN is the median of △ PQR

So, $PN = QN = \frac{1}{2}PQ$ ...(2)

Given △ ABC ~ △ PQR

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

(corresponding sides of similar triangle are proportional)

$$\frac{AB}{PQ} = \frac{CA}{RP}$$

$\frac{2AM}{2PN} = \frac{CA}{RP}$ (from (1) and (2))

$\Rightarrow \frac{AM}{PN} = \frac{CA}{RP}$ ...(3)

Also, since △ ABC ~ △ PQR

∠A = ∠P ...(4)

(corresponding angles of similar triangles are equal)

In △ AMC and △ PNR

∠A = ∠P (from (4))

$\frac{AM}{PN} = \frac{CA}{RP}$ (from (3))

$\therefore \triangle AMC \sim \triangle PNR$ (by SAS similarity)

Hence Proved

(ii)To Prove: $\frac{CM}{RN} = \frac{AB}{PQ}$

In part (i), we proved that $\triangle AMC \sim \triangle PNR$

So, $\frac{CM}{RN} = \frac{AC}{PR} = \frac{AM}{PN}$

(corresponding sides of a similar triangle are proportional)

Therefore, $\frac{CM}{RN} = \frac{AM}{PN}$

$$\frac{CM}{RN} = \frac{2AM}{2PN}$$

$$\frac{CM}{RN} = \frac{AB}{PQ}$$

Hence Proved

(iii) $\triangle CMB \sim \triangle RNQ$

Given $\triangle ABC \sim \triangle PQR$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

(corresponding sides of similar triangle are proportional)

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$\frac{2BM}{2QN} = \frac{BC}{QR}$ (from (1) and (2))

$\Rightarrow \frac{2M}{QN} = \frac{BC}{QR}$ ...(5)

Also, since $\triangle ABC \sim \triangle PQR$

$\angle B = \angle Q$ ...(6)

(corresponding angles of similar triangles are equal)

In $\triangle CMB$ and $\triangle RNQ$

$\angle B = \angle Q$ (from (6))

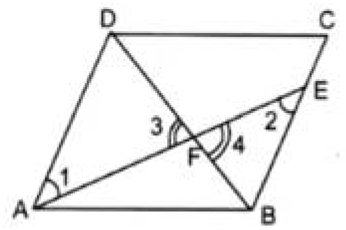
$\frac{BM}{QN} = \frac{BC}{QR}$ (from (5))

$\therefore \triangle CMB \sim \triangle RNQ$ (by SAS similarity)

Hence Proved

20. Question

In the adjoining figure, the diagonal BD of a parallelogram ABCD intersects the segment AE at the point F, where E is any point on the side BC. Show that DF x FE = BF x FA.



Answer

Given: ABCD is a parallelogram

To Prove: DF x FE = BF x FA

In $\triangle AFD$ and $\triangle BFE$

$\angle 1 = \angle 2$ (alternate angles)

$\angle 3 = \angle 4$ (vertically opposite angles)

$\therefore \triangle AFD \sim \triangle BFE$ (by AA similarity criterion)

So, $\frac{FB}{FD} = \frac{FE}{FA}$

(corresponding sides of similar triangle are proportional)

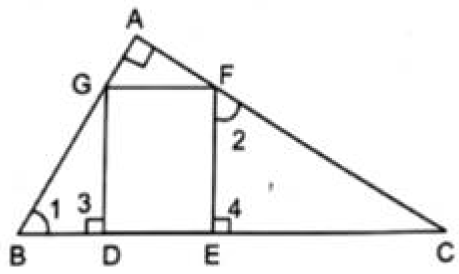
$\Rightarrow \frac{BF}{DF} = \frac{FE}{FA}$

$\Rightarrow$ DF x FE = BF x FA

Hence Proved

21. Question

In the given figure, DEFG is a square and $\angle BAC$ is a right angle. Show that DE^2 = BD x EC.



Answer

Given: DEFG is a square and ∠BAC = 90°

To Prove: $DE^2 = BD \times EC$.

In △ AGF and △ DBG

∠GAF = ∠BDG [each 90°]

∠AGF = ∠DBG

[corresponding angles because GF|| BC and AB is the transversal]

∴ △ AFG ~ △ DBG [by AA Similarity Criterion] ...(1)

In △ AGF and △ EFC

∠GAF = ∠CEF [each 90°]

∠AFG = ∠ECF

[corresponding angles because GF|| BC and AC is the transversal]

∴ △ AGF ~ △ EFC [by AA Similarity Criterion] ...(2)

From equation (1) and (2), we have

△ DBG ~ △ EFC

Since, the triangle is similar. Hence corresponding sides are proportional

$$\Rightarrow \frac{BD}{EF} = \frac{DG}{EC}$$

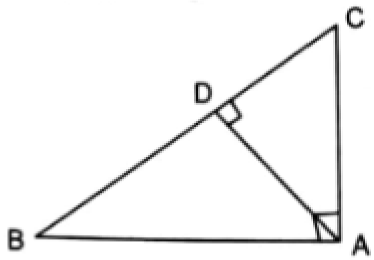
$$\Rightarrow \frac{BD}{DE} = \frac{DE}{EC}$$ [∵DEFG is a square]

$\Rightarrow DE^2 = BD \times EC$

Hence Proved

22. Question

In the given figure, ABD is a right angled triangle being right angled at A and AD ⊥ BC. Show that:



(i) AB^2= BC.BD

(ii) AC^2 = BC. DC

(iii) AB. AC. = BC. AD

Answer

(i) In ΔDAB and ΔACB

∠ADB = ∠CAB [each 90°]

∠DAB = ∠CAB [common angle]

∴ $\triangle$ DAB ~ $\triangle$ ACB [by AA similarity]

Since the triangles are similar, hence corresponding sides are in proportional.

$$\Rightarrow \frac{AB}{DB} = \frac{BC}{AB}$$

⇒ AB^2= BC×BD

(ii) In $\triangle$ ACB and $\triangle$ DAC

∠CAB = ∠ADC [each 90°]

∠CAB = ∠CAD [common angle]

∴ $\triangle$ ACB ~ $\triangle$ DAC [by AA similarity]

Since the triangles are similar, hence corresponding sides are in proportional.

$$\Rightarrow \frac{DC}{AC} = \frac{AC}{BC}$$

⇒ AC^2 = BC. DC

(iii) In part (i) we proved that $\triangle$ DAB ~ $\triangle$ ACB

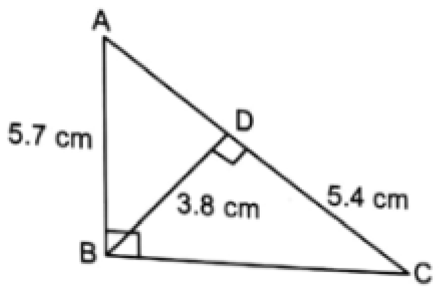
$\Rightarrow \frac{AB}{AD} = \frac{BC}{AC}$

⇒ AB × AC = BC × AD

Hence Proved

23. Question

In the given figure, ∠ABC = 90° and BD ⊥ AC. If AB = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm, find BC.



Answer

Given: ∠ABC = 90° and BD ⊥ AC

and AB = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm

To find: BC

Firstly, we have to show that △ ABC ~ △ BDC

Let △ ABC and △ BDC

∠ABC = ∠BDC [each 90°]

∠ACB = ∠BCD [common angle]

∴ △ ABC ~ △ BDC [by AA similarity criterion]

Since, triangles are similar, hence corresponding sides are proportional.

$\Rightarrow \frac{AB}{BC} = \frac{BD}{DC}$

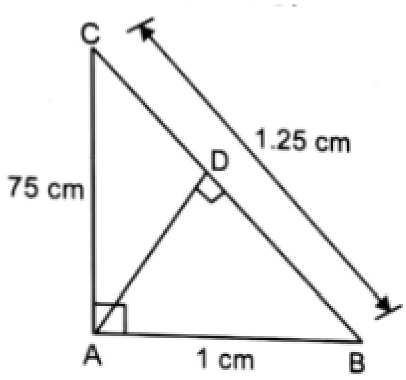
$\Rightarrow \frac{5.7}{BC} = \frac{3.8}{5.4}$

$\Rightarrow BC = \frac{5.7 \times 5.4}{3.8}$

⇒ BC = 8.1cm

24. Question

In the given figure, ∠CAB =90° and AD ⊥ BC. Show that ΔBDA ~ ΔBAC. If AC = 75 cm, AB = 1 cm and BC = 1.25 cm, find AD.



Answer

Given: ∠CAB =90° and AD ⊥ BC

and AC = 75 cm, AB = 1 cm and BC = 1.25 cm

Now, In △ADB and △CAB

∠ADB = ∠CAB [each 90°]

∠ABD = ∠CBA [common angle]

∴ △ADB ~ △CAB [by AA similarity]

Since the triangles are similar, hence corresponding sides are in proportional.

$$\Rightarrow \frac{AC}{BC} = \frac{AD}{AB}$$

$$\Rightarrow \frac{75}{1.25} = \frac{AD}{1}$$

⇒ AD = 60cm

Exercise 5.4

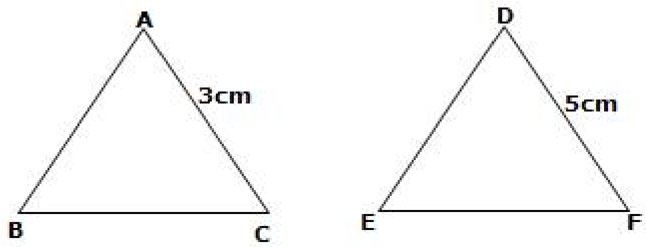
1. Question

In two similar triangles ABC and DEF, AC = 3 cm and DF = 5 cm. Find the ratio of the areas of the two triangles.

Answer

Given: ΔABC ~ ΔDEF and AC = 3 cm and DF = 5 cm

To find: Areas of the two triangles



We know that, the ratio of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(AC)^2}{(DF)^2}$

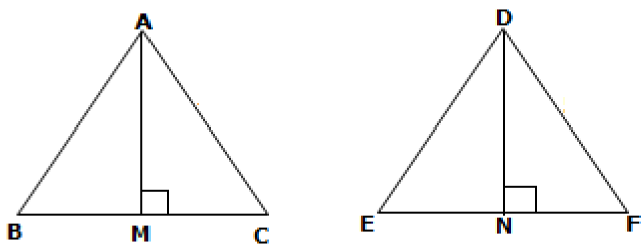
$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(3)^2}{(5)^2}$

$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{9}{25}$

2. Question

The corresponding altitudes of two similar triangles are 6 cm and 9 cm respectively. Find the ratio of their areas.

Answer



Given: AM = 6cm and DN = 9cm

Here, ΔABC and ΔDEF are similar triangles

We know that, in similar triangles, corresponding angles are in the same ratio.

$\Rightarrow \angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$ ……(i)

In Δ ABM and Δ DEN

$\angle B = \angle E$ [from (i)]

and $\angle M = \angle N$ [each 90°]

$\therefore \Delta ABC \sim \Delta DEF$ [by AA similarity]

So, $\frac{AM}{DN} = \frac{AB}{DE} = \frac{BM}{EN}$ ……(ii)

We know that, the ratio of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(AB)^2}{(DE)^2}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(AM)^2}{(DN)^2} \text{ [from (ii)]}$$

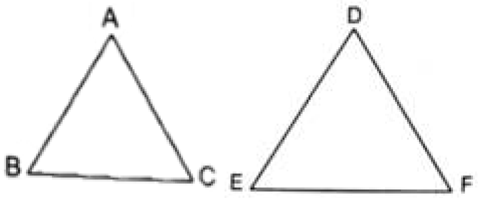
$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(6)^2}{(9)^2}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{36}{81}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{4}{9}$$

3. Question

In the given figure, ΔABC and ΔDEF are similar, BC = 3cm, EF = 4 cm and area of ΔABC = 54 sq cm. Determine the area of ΔDEF.



Answer

Given: ΔABC ~ ΔEF, BC = 3cm, EF = 4 cm

and area of ΔABC = 54 sq cm

We know that, the ratio of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(BC)^2}{(EF)^2}$$

$$\Rightarrow \frac{54}{ar(\Delta DEF)} = \frac{(3)^2}{(4)^2} \text{ [given]}$$

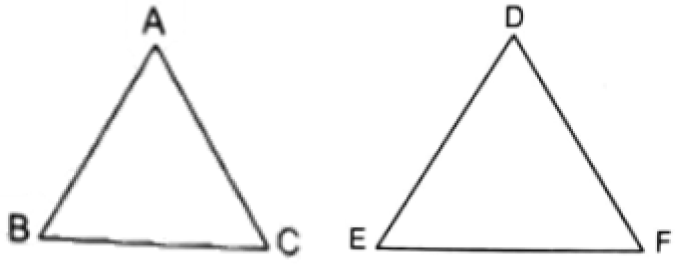
$$\Rightarrow \frac{54}{ar(\Delta DEF)} = \frac{9}{16}$$

$$\Rightarrow ar(\Delta DEF) = \frac{54 \times 16}{9}$$

⇒ ar (ΔDEF) = $96cm^2$

4. Question

If ΔABC ~ ΔDEF, AB =10 cm, area (ΔABC) = 20 sq. cm, area (ΔDEF) = 45 sq. cm. Determine DE.



Answer

Given: △ ABC ~ △ DEF, AB = 10cm,

and area of △ ABC = 20 sq cm , area of △ DEF = 45 sq cm

We know that, the ratio of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(AB)^2}{(DE)^2}$

$\Rightarrow \frac{20}{45} = \frac{(10)^2}{(DE)^2}$ [given]

$\Rightarrow \frac{20}{45} = \frac{100}{(DE)^2}$

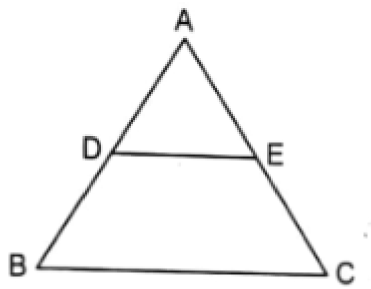
$\Rightarrow (DE)^2 = \frac{100 \times 45}{20}$

$\Rightarrow (DE)^2 = 5 \times 45$

$\Rightarrow$ DE = 15cm

5. Question

In ΔABC ~ ΔADE and DE|| BC. If DE = 3cm, BC = 6 cm and area (ΔADE) =15 sq. cm, find the area of ΔABC.



Answer

Given: ΔABC ~ ΔADE

DE = 3cm, BC = 6 cm and area (ΔADE) =15 sq. cm

We know that, the ratio of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\therefore \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{(DE)^2}{(BC)^2}$

$\Rightarrow \frac{15}{ar(\Delta ABC)} = \frac{(3)^2}{(6)^2}$ [given]

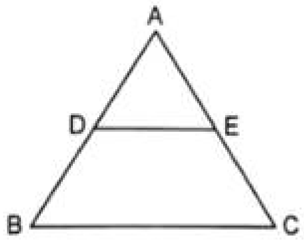
$\Rightarrow \frac{15}{ar(\Delta ABC)} = \frac{9}{36}$

$\Rightarrow ar(\Delta ABC) = \frac{15 \times 36}{9}$

⇒ ar (ΔABC) = $60cm^2$

6. Question

In the figure DE || BC. If DE = 4 cm, BC = 8 cm and area (ΔADE) = 25 sq. cm, find the area of ΔABC.



Answer

Given: DE || BC

DE = 4cm, BC = 8cm and area (Δ ADE) =25 sq. cm

In Δ ABC and Δ ADE

∠B = ∠D [∵ DE || BC and AB is transversal,

Corresponding angles are equal]

∠C = ∠E [∵ DE || BC and AC is transversal,

Corresponding angles are equal]

∠BAC =∠DAE [common angle]

∴ Δ ABC ~ Δ ADE [by AAA similarity]

Now, we know that, the ratio of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\therefore \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{(DE)^2}{(BC)^2}$

$\Rightarrow \frac{25}{ar(\Delta ABC)} = \frac{(4)^2}{(8)^2}$ [given]

$\Rightarrow \frac{25}{ar(\Delta ABC)} = \frac{16}{64}$

$\Rightarrow ar(\Delta ABC) = \frac{25 \times 64}{16}$

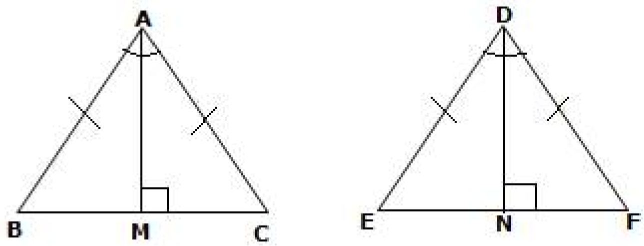
$\Rightarrow ar(\Delta ABC) = 25 \times 4$

$\Rightarrow ar(\Delta ABC) = 100cm^2$

7. Question

Two isosceles triangles have equal vertical angles and their areas are in the ratio 16 : 25. find the ratio of their corresponding heights.

Answer



Let Δ ABC and Δ DEF are two isosceles triangles with AB =AC and DE = DF and ∠A = ∠D

Now, let AM and DN are their respective altitudes or heights.

Let Δ ABC and Δ DEF

$\frac{AB}{DE} = \frac{AC}{DF}$

∠A = ∠D [given]

$\therefore \Delta ABC \sim \Delta DEF$ [by SAS similarity]

We know that, in similar triangles, corresponding angles are in the same ratio.

⇒∠B = ∠E and ∠C = ∠F ……(i)

In Δ ABM and Δ DEN

∠B = ∠E [from (i)]

and ∠M = ∠N [each 90°]

$\therefore \triangle ABC \sim \triangle DEF$ [by AA similarity]

So, $\frac{AM}{DN} = \frac{AB}{DE} = \frac{BM}{EN}$ ......(ii)

We know that, the ratio of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(AB)^2}{(DE)^2}$

$\Rightarrow \frac{16}{25} = \frac{(AM)^2}{(DN)^2}$ [from (ii)]

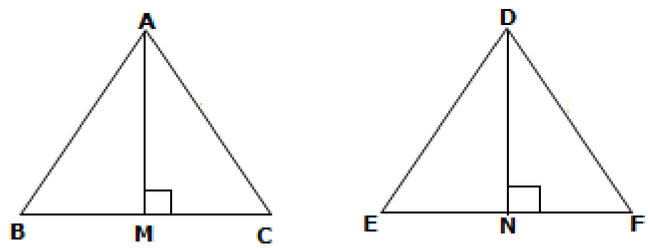
$\Rightarrow \frac{(4)^2}{(5)^2} = \frac{(AM)^2}{(DN)^2}$

$\Rightarrow \frac{AM}{DN} = \frac{4}{5}$

8. Question

The areas of two similar triangles are 100 cm^2 and 49 cm^2, respectively. If the altitude of the bigger triangle is 5 cm, find the corresponding altitude of the other.

Answer



Given: Let $\Delta ABC = 100cm^2$ and $\Delta DEF = 49cm^2$

Let AM = 5cm

Here, ΔABC and ΔDEF are similar triangles

We know that, in similar triangles, corresponding angles are in the same ratio.

$\Rightarrow \angle B = \angle E$ and $\angle C = \angle F$ ...(i)

In $\triangle$ ABM and $\triangle$ DEN

$\angle B = \angle E$ [from (i)]

and $\angle M = \angle N$ [each 90°]

$\therefore \triangle ABC \sim \triangle DEF$ [by AA similarity]

So, $\frac{AM}{DN} = \frac{AB}{DE} = \frac{BM}{EN}$ ......(ii)

We know that, the ratio of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(AB)^2}{(DE)^2}$

$\Rightarrow \frac{100}{49} = \frac{5^2}{(DN)^2}$ [from (ii)]

$\Rightarrow \frac{100}{49} = \frac{25}{(DN)^2}$

$\Rightarrow (DN)^2 = \frac{25 \times 49}{100}$

$\Rightarrow (DN)^2 = \frac{49}{4}$

$\Rightarrow DN = \frac{7}{2}$

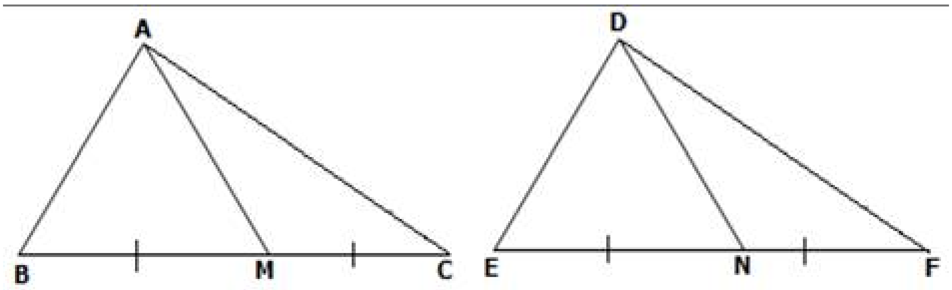
$\Rightarrow$DN = 3.5cm

The height of the other altitude is 3.5cm

9. Question

The areas of two similar triangles are 100 cm^2 and 64cm^2 respectively. If a median of the smaller triangle is 5.6 cm, find the corresponding median of the other.

Answer



Let Δ ABC and Δ DEF are two similar triangles such that ar (Δ ABC) =100cm^2 and ar (Δ DEF) = 64cm^2

Also, let AM and DN are medians of Δ ABC and Δ DEF respectively.

Now in Δ ABC and Δ DEF

$\angle B = \angle E$ [$\because \Delta ABC \sim \Delta DEF$]

and $\frac{AB}{DE} = \frac{BM}{EN}$ $\left[\because \frac{AB}{DE} = \frac{BC}{EF} \Rightarrow \frac{AB}{DE} = \frac{2BM}{2EN}\right]$

∴ $\triangle$ ABC ~ $\triangle$ DEF [by SAS similarity]

$\Rightarrow \frac{AB}{DE} = \frac{AM}{DN}$ ...(i)

Now, as $\triangle$ ABC ~ $\triangle$ DEF

We know that, the ratio of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{(AB)^2}{(DE)^2}$

$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{(AM)^2}{(DN)^2}$ [from (i)]

$\Rightarrow \frac{100}{64} = \frac{(AM)^2}{(5.6)^2}$

$\Rightarrow (AM)^2 = \frac{100 \times 5.6 \times 5.6}{64}$

$\Rightarrow (AM)^2 = \frac{100 \times 56 \times 56}{64 \times 10 \times 10}$

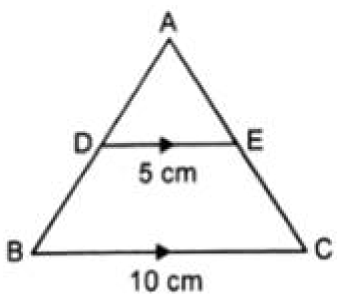
$\Rightarrow (AM)^2 = 7 \times 7$

$\Rightarrow$AM = 7cm

Hence, the length of the other median is 7cm.

10. Question

In the given figure, DE || BC. If DE = 5 cm, BC =10 cm and ar(ΔADE) = 20 cm^2, find the area of ΔABC.



Answer

Given: DE || BC

DE = 5cm, BC = 10cm and area ($\triangle$ ADE) =20 sq. cm

In $\triangle$ ABC and $\triangle$ ADE

$\angle B = \angle D$ [∵ DE || BC and AB is transversal,

Corresponding angles are equal]

$\angle C = \angle E$ [∵ DE || BC and AB is transversal,

Corresponding angles are equal]

$\angle BAC = \angle DAE$ [common angle]

∴ $\triangle ABC \sim \triangle ADE$ [by AAA similarity]

Now, we know that the ratio of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\therefore \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{(DE)^2}{(BC)^2}$

$\Rightarrow \frac{20}{ar(\Delta ABC)} = \frac{(5)^2}{(10)^2}$ [given]

$\Rightarrow \frac{20}{ar(\Delta ABC)} = \frac{25}{100}$

$\Rightarrow ar(\Delta ABC) = \frac{20 \times 100}{25}$

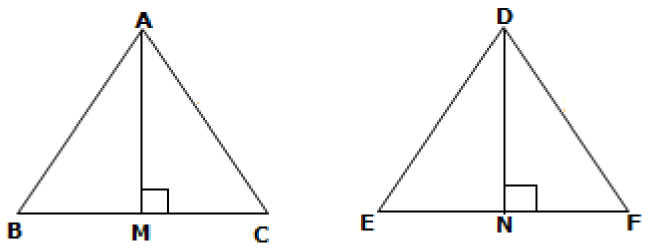
$\Rightarrow ar(\triangle ABC) = 20 \times 4$

$\Rightarrow ar(\triangle ABC) = 80cm^2$

11. Question

The areas of two similar triangles are 81 cm^2 and 49 cm^2 respectively. If the altitude of the first triangle is 6.3 cm, find the corresponding altitude of the other.

Answer



Given: Let $\triangle ABC = 81cm^2$ and $\triangle DEF = 49cm^2$

Let AM = 6.3cm

Here, $\triangle ABC$ and $\triangle DEF$ are similar triangles

We know that, in similar triangles, corresponding angles are in the same ratio.

$\Rightarrow \angle B = \angle E$ and $\angle C = \angle F$ ...(i)

In $\triangle$ ABM and $\triangle$ DEN

$\angle B = \angle E$ [from (i)]

and $\angle M = \angle N$ [each 90°]

$\therefore \triangle ABC \sim \triangle DEF$ [by AA similarity]

So, $\frac{AM}{DN} = \frac{AB}{DE} = \frac{BM}{EN}$ ...(ii)

We know that, the ratio of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(AB)^2}{(DE)^2}$

$\Rightarrow \frac{81}{49} = \frac{(6.3)^2}{(DN)^2}$ [from (ii)]

$\Rightarrow \frac{81}{49} = \frac{6.3 \times 6.3}{(DN)^2}$

$\Rightarrow (DN)^2 = \frac{6.3 \times 6.3 \times 49}{81}$

$\Rightarrow (DN)^2 = \frac{63 \times 63 \times 49}{81 \times 10 \times 10}$

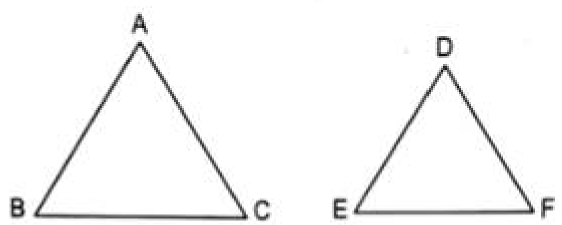
$\Rightarrow (DN)^2 = \frac{7 \times 7 \times 49}{100}$

$\Rightarrow DN = 4.9cm$

Height of the other altitude is 4.9cm

12. Question

In the given figure, ΔABC ~ ΔDEF. If AB = 2DE and area of ΔABC is 56 sq. cm, find the area of ΔDEF.



Answer

Given: ΔABC ~ ΔDEF and AB =2DE

And area of Δ ABC is 56 sq. cm

We know that, the ratio of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(AB)^2}{(DE)^2}$

$\Rightarrow \frac{56}{ar(\Delta DEF)} = \frac{(2DE)^2}{(DE)^2}$ [given]

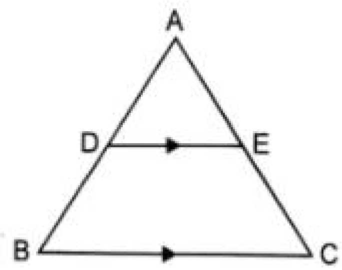
$\Rightarrow \frac{56}{ar(\Delta DEF)} = \frac{4(DE)^2}{(DE)^2}$

$\Rightarrow ar(\Delta DEF) = \frac{56}{4}$

⇒ ar(ΔDEF) = 14sq cm

13. Question

In the given figure, DE || BC and DE : BC = 4 : 5. Calculate the ratio of the areas of ΔADE and the trapezium ΔCEDB.



Answer

It is given that DE || BC and DE : BC = 4 : 5

Let Δ ADE and ΔABC

∠ADE = ∠ABC [corresponding angles]

∠AED = ∠ACB [corresponding angles]

∴ Δ ADE ~ ΔABC [by AA similarity]

We know that the ratio of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{(BC)^2}{(DE)^2}$

Subtracting 1 from both the sides, we get

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta ADE)} - 1 = \frac{(5)^2}{(4)^2} - 1 \text{ [given]}$$

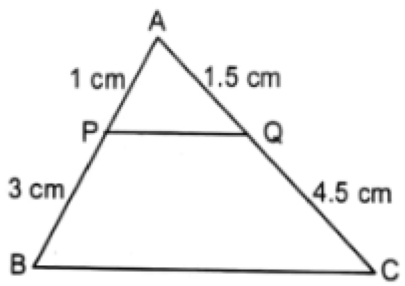
$$\Rightarrow \frac{ar(\Delta ABC) - ar(\Delta ADE)}{ar(\Delta ADE)} = \frac{25-16}{16}$$

$$\Rightarrow \frac{ar(CEDB)}{ar(\Delta ADE)} = \frac{9}{16}$$

$$\Rightarrow \frac{ar(\Delta ADE)}{ar(CEDB)} = \frac{16}{9}$$

14. Question

ABC is a triangle, and PQ is a straight line meeting AB in P and AC in Q. If AP= 1 cm, 1 BP = 3 cm, AQ = 1.5 cm, CQ = 4.5 cm. Prove that the area of Δ APQ = 1/16 (area of ΔABC).



Answer

Given: AP= 1 cm, 1 BP= 3 cm, AQ = 1.5 cm, CQ = 4.5 cm

Here, $\frac{AP}{PB} = \frac{1}{3}$ and $\frac{AQ}{QC} == \frac{1.5}{4.5} = \frac{1}{3}$

⇒PQ || BC [by converse of basic proportionality theorem]

In Δ ABC and Δ APQ

∠B = ∠P [∵ PQ || BC and AB is transversal,

Corresponding angles are equal]

∠C = ∠Q [∵ PQ || BC and AC is transversal,

Corresponding angles are equal]

∠BAC =∠PAQ [common angle]

∴ Δ ABC ~ Δ APQ [by AAA similarity]

Now, we know that, the ratio of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\therefore \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{(AP)^2}{(AB)^2}$$

$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{(1)^2}{(1+3)^2}$ [given]

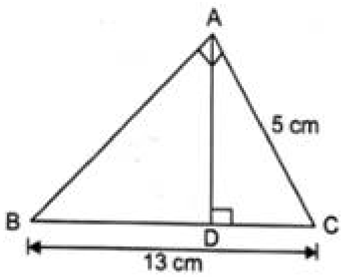
$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{1}{16}$

$\Rightarrow ar(\Delta APQ) = \frac{1}{16} ar(\Delta ABC)$

Hence Proved

15. Question

ΔABC is right angled at A and AD ⊥ BC. If BC = 13 cm and AC = 5 cm, find the ratio of the areas of ΔABC and ΔADC.



Answer

Given: AD ⊥ BC

and BC = 13 cm and AC = 5 cm

Let △ ABC and △ ADC

∠A = ∠D [each 90°]

∠C = ∠C [common angle]

∴ △ ABC ~ △ ADC [by AA similarity]

We know that, the ratio of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta ADC)} = \frac{(BC)^2}{(AC)^2}$

$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta ADC)} = \frac{(13)^2}{(5)^2}$

$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta ADC)} = \frac{169}{25}$

Exercise 5.5

1. Question

Sides of some triangles are given below. Determine which of them are right triangles

(i) 8 cm, 15 cm, 17 cm

(ii) (2a —1) cm, $2\sqrt{2a}$ cm and (2a + 1) cm

(iii) 7 cm, 24 cm, 25 cm

(iv) 1.4 cm, 4.8 cm, 5 cm

Answer

(i) Using Pythagoras theorem, i.e. if the square of the hypotenuse is equal to the sum of the other two sides. Then, the given triangle is a right angled triangle, otherwise not.

Here, $(8)^2 + (15)^2 = 64 + 225 = 289 = (17)^2$

∴ given sides 8cm, 15cm and 17cm make a right angled triangle.

(ii) Using Pythagoras theorem, i.e. if the square of the hypotenuse is equal to the sum of the other two sides. Then, the given triangle is a right angled triangle, otherwise not.

Here, $(2a - 1)^2 + (2\sqrt{(2a)})^2$

$\Rightarrow 4a^2 + 1 - 4a + 8a$

$\Rightarrow 4a^2 + 1 + 4a$

$= (2a + 1)^2$

∴ given sides (2a —1) cm, $2\sqrt{2a}$ cm and (2a + 1) cm make a right angled triangle.

(iii) Using Pythagoras theorem, i.e. if the square of the hypotenuse is equal to the sum of the other two sides. Then, the given triangle is a right angled triangle, otherwise not.

Here, $(7)^2 + (24)^2 = 49 + 576 = 625 = (25)^2$

∴ given sides 7cm, 24cm and 25cm make a right angled triangle.

(iv) Using Pythagoras theorem, i.e. if the square of the hypotenuse is equal to the sum of the other two sides. Then, the given triangle is a right angled triangle, otherwise not.

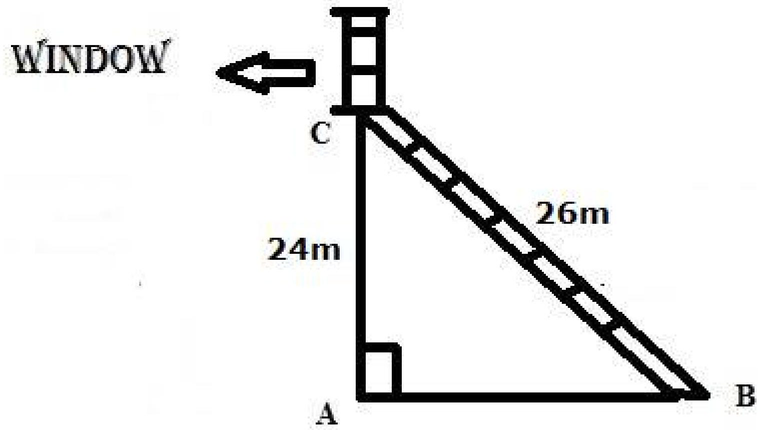
Here, $(1.4)^2 + (4.8)^2 = 1.96 + 23.04 = 25 = (5)^2$

∴ given sides 1.4cm, 4.8cm and 5cm make a right angled triangle.

2. Question

A ladder 26 m long reaches a window 24 m above the ground. Find the distance of the foot of the ladder from the base of the wall.

Answer



Let AC be the position of a window from the ground and BC be the ladder, then the height of the window, AC =24m and length of the ladder, BC = 26m

Let AB = x m be the distance of the foot of the ladder from the base of the wall.

In ΔCAB, using Pythagoras Theorm,

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (AC)^2 + (AB)^2 = (BC)^2$

$\Rightarrow (24)^2 + (AB)^2 = (26)^2$

$\Rightarrow (AB)^2 = (26)^2 - (24)^2$

$\Rightarrow (AB)^2 = (26 - 24)(26+24)$

$[\because (a^2 - b^2)=(a+b)(a - b)]$

$\Rightarrow (AB)^2 = (2)(50)$

$\Rightarrow (AB)^2 = 100$

$\Rightarrow AB = \pm 10$

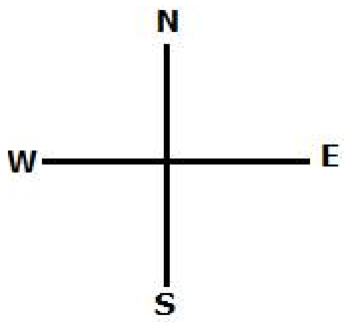
$\Rightarrow AB = 10$ [taking positive square root]

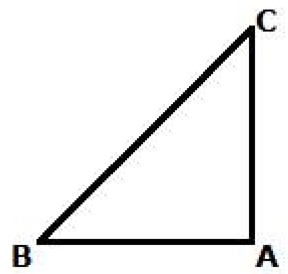
Hence, the distance of the foot of the ladder from base of the wall is 10m

3. Question

A man goes 15 m due west and then 8 m due north. How far is he from the starting point?

Answer





Let AB = 15m and AC = 8m

In ΔCAB, using Pythagoras Theorm,

(Perpendicular)^2 + (Base)^2 = (Hypotenuse)^2

⇒ $(AC)^2 + (AB)^2 = (BC)^2$

⇒ $(8)^2 + (15)^2 = (BC)^2$

⇒ $(BC)^2 = 64 + 225$

⇒ $(BC)^2 = 289$

⇒ BC = ±17

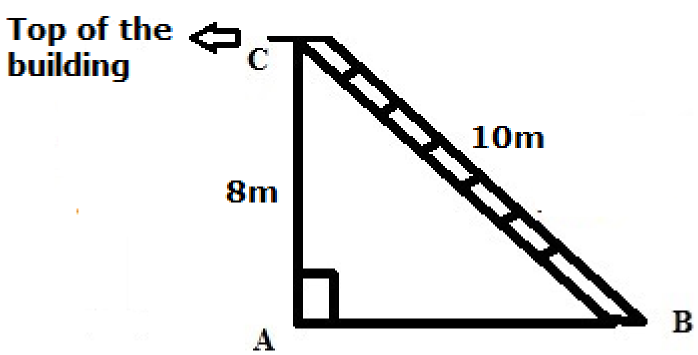
⇒ BC = 17 [taking positive square root]

Hence, the man is 17m far from the starting point.

4. Question

A ladder 10 m long just reaches the top of a building 8 m high from the ground. Find the distance of the foot of the ladder from the building.

Answer



Let AC be the top of the building from the ground and BC be the ladder, then the height of the building, AC = 8m and length of the ladder, BC = 10m

Let AB = x m be the distance of the foot of the ladder from the building.

In ΔCAB, using Pythagoras Theorm,

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (AC)^2 + (AB)^2 = (BC)^2$

$\Rightarrow (8)^2 + (AB)^2 = (10)^2$

$\Rightarrow (AB)^2 = (10)^2 - (8)^2$

$\Rightarrow (AB)^2 = (10 - 8)(10+8)$

$[\because (a^2 - b^2)=(a+b)(a - b)]$

$\Rightarrow (AB)^2 = (2)(18)$

$\Rightarrow (AB)^2 = 36$

$\Rightarrow AB = \pm 6$

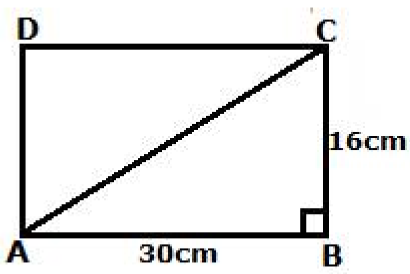
$\Rightarrow AB = 6$ [taking positive square root]

Hence, the distance of the foot of the ladder from building is 6m

5. Question

Find the length of a diagonal of a rectangle whose adjacent sides are 30 cm and 16 cm.

Answer



Let ABCD be a rectangle and AB and BC are the adjacent sides of length 30cm and 16cm respectively.

Let AC be the diagonal.

In ΔCBA, using Pythagoras Theorm,

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (30)^2 + (16)^2 = (AC)^2$

$\Rightarrow (AC)^2 = 900 + 256$

$\Rightarrow (AC)^2 = 1156$

$\Rightarrow AB = \pm 34$

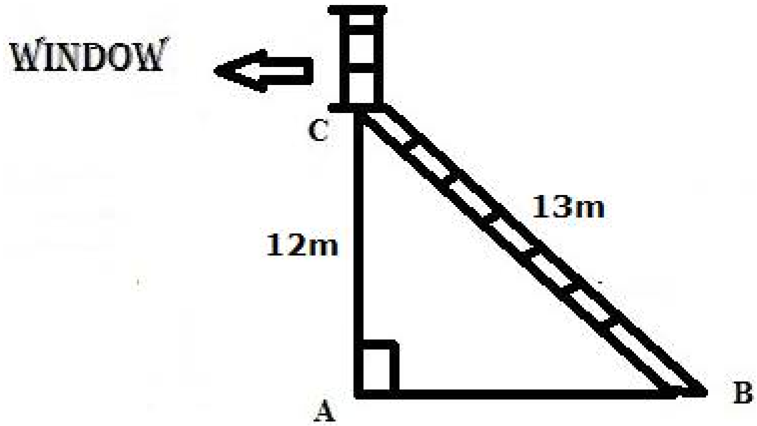
$\Rightarrow AB = 34$ [taking positive square root]

Hence, the length of a diagonal of a rectangle is 34cm

6. Question

A 13 m-long ladder reaches a window of a building 12 m above the ground. Determine the distance of the foot of the ladder from the building.

Answer



Let AC be the position of a window from the ground and BC be the ladder, then the height of the window, AC =12m and length of the ladder, BC = 13m

Let AB = x m be the distance of the foot of the ladder from the base of the wall.

In ΔCAB, using Pythagoras Theorem,

(Perpendicular)^2 + (Base)^2 = (Hypotenuse)^2

$\Rightarrow (AC)^2 + (AB)^2 = (BC)^2$

$\Rightarrow (12)^2 + (AB)^2 = (13)^2$

$\Rightarrow (AB)^2 = (13)^2 - (12)^2$

$\Rightarrow (AB)^2 = (13 - 12)(13+12)$

[$\because (a^2 - b^2)=(a+b)(a - b)$]

⇒ $(AB)^2 = (1)(25)$

⇒ $(AB)^2 = 25$

⇒ AB = ±5

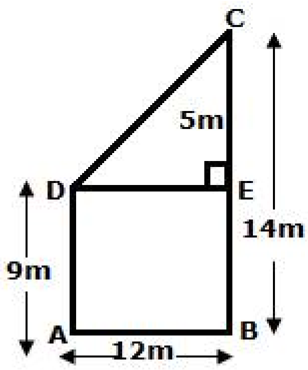
⇒ AB = 5 [taking positive square root]

Hence, the distance of the foot of the ladder from base of the wall is 5m

7. Question

Two vertical poles of height 9 m and 14 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Answer



Let BC and AD be the two poles of height 14m and 9m respectively. Again, let CD be the distance between tops of the poles.

Then, CE = BC – AD = 14 – 9 = 5m [∵AD =BE]

Also, AB =12m

In ΔCED, using Pythagoras theorem, we get

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

⇒ $(CE)^2 + (DE)^2 = (CD)^2$

⇒ $(5)^2 + (12)^2 = (CD)^2$

⇒ $(CD)^2 = 25 + 144$

⇒ $(CD)^2 = 169$

⇒ CD = $\sqrt{169}$

⇒ CD = ±13

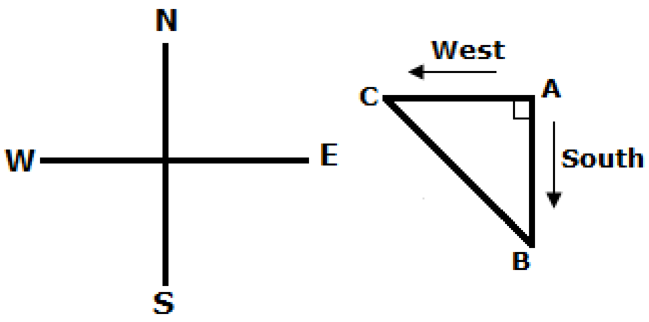
⇒ CD = 13 [taking positive square root]

Hence, the distance between the tops of the poles is 13m

8. Question

A man goes 10 m due south and then 24 m due west. How far is he from the starting point?

Answer



Let AB = 10m and AC = 24m

In ΔCAB, using Pythagoras Theorem,

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (AC)^2 + (AB)^2 = (BC)^2$

$\Rightarrow (24)^2 + (10)^2 = (BC)^2$

$\Rightarrow (BC)^2 = 576 + 100$

$\Rightarrow (BC)^2 = 676$

$\Rightarrow BC = \pm 26$

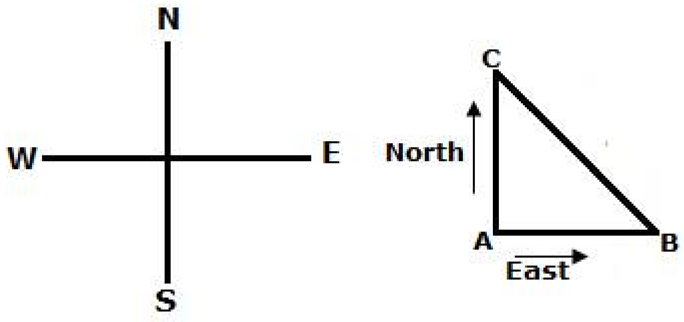
$\Rightarrow BC = 26$ [taking positive square root]

Hence, the man is 26m far from the starting point.

9. Question

A man goes 80 m due east and then 150 m due north. How far is he from the starting point?

Answer



Let AB = 80m and AC = 150m

In ΔCAB, using Pythagoras Theorem,

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (AC)^2 + (AB)^2 = (BC)^2$

$\Rightarrow (150)^2 + (80)^2 = (BC)^2$

$\Rightarrow (BC)^2 = 22500 + 6400$

$\Rightarrow (BC)^2 = 28900$

$\Rightarrow BC = \pm 170$

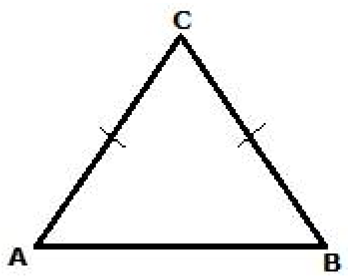
$\Rightarrow BC = 170$ [taking positive square root]

Hence, the man is 170 m far from the starting point.

10. Question

ΔABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ΔABC is a right triangle.

Answer



Given an isosceles triangle ABC with AC = BC, and $AB^2 = 2AC^2$

To Prove: ΔABC is a right triangle

Proof: $AB^2 = 2AC^2$ (given)

$\Rightarrow AB^2 = AC^2 + AC^2$

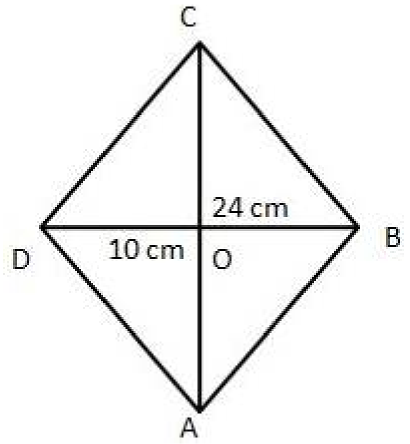
⇒ $AB^2 = AC^2 + BC^2$ [∵AC =BC]

⇒ ΔABC is a right triangle right angled at C.

11. Question

Find the length of each side of a rhombus whose diagonals are 24 cm and 10 cm long.

Answer



Let ABCD be a rhombus where AC = 10cm and BD =24cm

Let AC and BD intersect each other at O.

Now, we know that diagonals of rhombus bisect each other at 90°

Thus, we have

$AO = \frac{1}{2} \times AC \Rightarrow \frac{1}{2} \times 10 = 5cm$

$BO = \frac{1}{2} \times BD = \frac{1}{2} \times 24 = 12cm$

Since, AOB is a right angled triangle

So, by Pythagoras theorem, we have

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

⇒ $(AO)^2 + (BO)^2 = (AB)^2$

⇒ $(5)^2 + (12)^2 = (AB)^2$

⇒ $(AB)^2 = 25 + 144$

⇒ $(AB)^2 = 169$

⇒ $AB = \sqrt{169}$

$\Rightarrow$ AB = ±13

$\Rightarrow$ AB = 13 [taking positive square root]

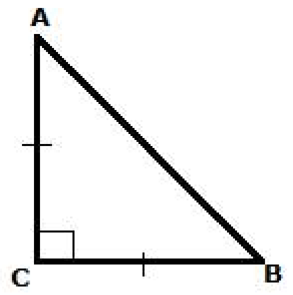
Hence, AB = 13cm

Thus, length of each side of rhombus is 13cm

12. Question

ΔABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Answer



Given: ABC is an isosceles triangle right angled at C.

Let AC = BC

In ΔACB, using Pythagoras theorem, we have

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (AC)^2 + (BC)^2 = (AB)^2$

$\Rightarrow (AC)^2 + (AC)^2 = (AB)^2$

[$\because$ABC is an isosceles triangle, AC =BC]

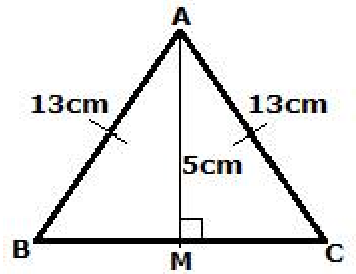
$\Rightarrow 2(AC)^2 = (AB)^2$

Hence Proved

13. Question

ΔABC is an isosceles triangle with AB = AC = 13 cm. The length of altitude from A on BC is 5 cm. Find BC.

Answer



Given: $\triangle$ ABC is an isosceles triangle with AB = AC = 13 cm

Suppose the altitude from A on Bc meets BC at M.

∴ M is the midpoint of BC. AM = 5 cm

In ΔAMB, using Pythagoras theorem, we have

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (AM)^2 + (BM)^2 = (AB)^2$

$\Rightarrow (5)^2 + (BM)^2 = (13)^2$

$\Rightarrow (BM)^2 = (13)^2 - (5)^2$

$\Rightarrow (BM)^2 = (13 - 5)(13+5)$

$[\because (a^2 - b^2) = (a + b)(a - b)]$

$\Rightarrow (BM)^2 = (8)(18)$

$\Rightarrow (BM)^2 = 144$

$\Rightarrow BM = \pm 12$

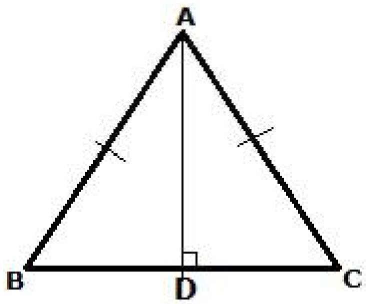
⇒ BM = 12 [taking positive square root]

∴ BC = 2BM or 2MC = 2×12 = 24cm

14. Question

In an equilateral triangle ABC, AD is drawn perpendicular to BC, meeting BC in D. Prove that $AD^2 = 3BD^2$.

Answer



Given: ABC is an equilateral triangle

∴ AB = AC = BC

and AD ⊥ BC

Now, In ΔADB, using Pythagoras theorem, we have

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

⇒ $(AD)^2 + (BD)^2 = (AB)^2$

⇒ $(AD)^2 + (BD)^2 = (BC)^2$ [∵ AB = BC]

⇒ $(AD)^2 + (BD)^2 = (2BD)^2$ [as AD⊥BC]

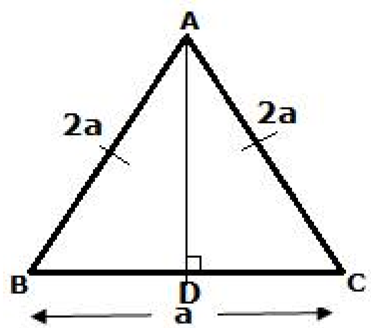
⇒ $(AD)^2 + (BD)^2 = 4BD^2$

⇒ $AD^2 = 3BD^2$

15. Question

Find the length of altitude AD of an isosceles Δ ABC in which AB = AC = 2a units and BC = a units.

Answer



Given: ABC is an isosceles triangle

∴ AB = AC = 2a and BC = a

and AD is the altitude on BC. Therefore, BC = 2BD

Now, In ΔADB, using Pythagoras theorem, we have

(Perpendicular)^2 + (Base)^2 = (Hypotenuse)^2

⇒ $(AD)^2 + (BD)^2 = (AB)^2$

$$\Rightarrow\ (AD)^2 + \left(\frac{a}{2}\right)^2 = (2a)^2$$

$$\Rightarrow\ (AD)^2 = (2a)^2 - \left(\frac{a}{2}\right)^2$$

$$\Rightarrow\ (AD)^2 = 4a^2 - \frac{a^2}{4}$$

$$\Rightarrow\ (AD)^2 = \frac{16a^2 - a^2}{4}$$

$$\Rightarrow\ (AD)^2 = \frac{15a^2}{4}$$

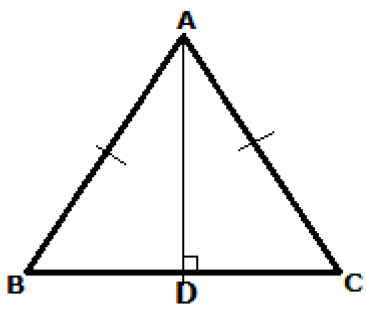
$$\Rightarrow\ AD = \sqrt{\frac{15a^2}{4}}$$

$\Rightarrow\ AD = \frac{\sqrt{15}}{2}a$ [taking positive square root]

16. Question

Δ ABC is an equilateral triangle of side 2a units. Find each of its altitudes.

Answer



Given: ABC is an equilateral triangle

∴ AB = AC = BC = 2a

And let AD is an altitude on BC. Therefore, $BD = \frac{1}{2} \times BC = a$

Now, In ΔADB, using Pythagoras theorem, we have

(Perpendicular)^2 + (Base)^2 = (Hypotenuse)^2

⇒ $(AD)^2 + (BD)^2 = (AB)^2$

$\Rightarrow (AD)^2 + (a)^2 = (2a)^2$

$\Rightarrow (AD)^2 = 4a^2 - a^2$

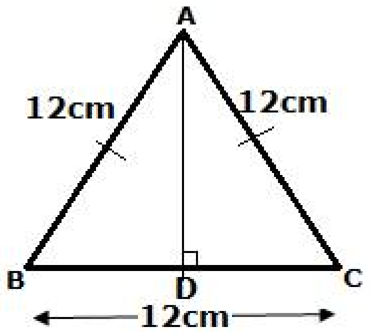
$\Rightarrow (AD)^2 = 3a^2$

$\Rightarrow AD = a\sqrt{3}$ units

17. Question

Find the height of an equilateral triangle of side 12 cm.

Answer



Given: ABC is an equilateral triangle

∴ AB = AC = BC = 12cm

And let AD is an altitude on BC. Therefore, $BD = \frac{1}{2} \times BC = 6cm$

Now, In ΔADB, using Pythagoras theorem, we have

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (AD)^2 + (BD)^2 = (AB)^2$

$\Rightarrow (AD)^2 + (6)^2 = (12)^2$

$\Rightarrow (AD)^2 = 144 - 36$

$\Rightarrow (AD)^2 = 108$

$\Rightarrow AD = \sqrt{108}$

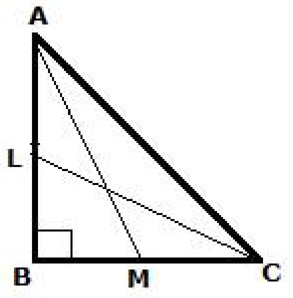
$\Rightarrow AD = 6\sqrt{3}$

Hence, the height of an equilateral triangle is $6\sqrt{3}$ cm

18. Question

L and M are the mid-points of AB and BC respectively of ΔABC, right-angled at B. Prove that $4LC^2 = AB^2 + 4BC^2$

Answer



Given: ABC is a right triangle right angled at B

and L and M are the mid-points of AB and BC respectively.

⇒ AL = LB and BM = MC

In ΔLBC, using Pythagoras theorem we have,

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (LB)^2 + (BC)^2 = (LC)^2$

$\Rightarrow \left(\frac{AB}{2}\right)^2 + (BC)^2 = (LC)^2$

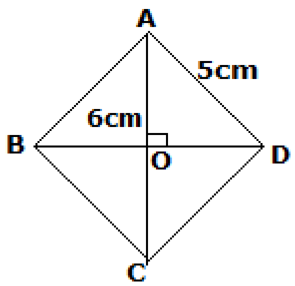
$\Rightarrow (AB)^2 + 4(BC)^2 = 4(LC)^2$

Hence Proved

19. Question

Find the length of the second diagonal of a rhombus, whose side is 5 cm and one of the diagonals is 6 cm.

Answer



Let ABCD be a rhombus having AD = 5cm and AC = 6cm

Now, we know that diagonals of rhombus bisect each other at 90°

Thus, we have

$AO = \frac{1}{2} \times AC \Rightarrow \frac{1}{2} \times 6 = 3cm$

Since, AOD is a right angled triangle

So, by Pythagoras theorem, we have

$(Perpendicular)^2 + (Base)^2 = (Hypotenuse)^2$

$\Rightarrow (AO)^2 + (BO)^2 = (AD)^2$

$\Rightarrow (3)^2 + (BO)^2 = (5)^2$

$\Rightarrow (BO)^2 = 25 - 9$

$\Rightarrow (BO)^2 = 16$

$\Rightarrow BO = \sqrt{16}$

$\Rightarrow BO = \pm 4$

$\Rightarrow BO = 4$ [taking positive square root]

Hence, BO = 4cm

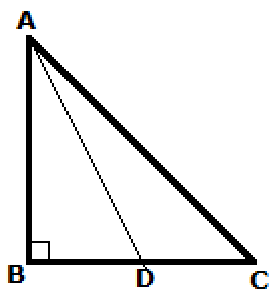
$\Rightarrow BC = 2BO = 2 \times 4 = 8cm$

Thus, length of each side of rhombus is 13cm.

20. Question

In ΔABC, $\angle B = 90°$ and D is the midpoint of BC. Prove that $AC^2 = AD^2 + 3CD^2$.

Answer



Given: $\angle B = 90°$ and D is the midpoint of BC .i.e. BD = DC

To Prove: $AC^2 = AD^2 + 3CD^2$

In ΔABC, using Pythagoras theorem we have,

$(Perpendicular)^2 + (Base)^2 = (Hypotenuse)^2$

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (AB)^2 + (2CD)^2 = (AC)^2$

$\Rightarrow (AB)^2 + 4(CD)^2 = (AC)^2$

$\Rightarrow (AD^2 - BD^2) + 4(CD^2) = AC^2$

[$\because$ In right triangle ΔABD, $AD^2 = AB^2 + BD^2$]

$\Rightarrow AD^2 - BD^2 + 4CD^2 = AC^2$

$\Rightarrow AD^2 - CD^2 + 4CD^2 = AC^2$

[$\because$ D is the midpoint of BC, BD = DC]

$\Rightarrow AD^2 + 3CD^2 = AC^2$

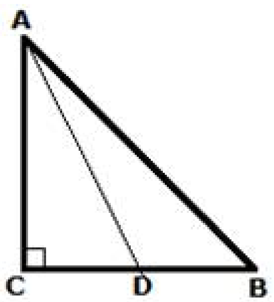
or $AC^2 = AD^2 + 3CD^2$

Hence Proved

21. Question

In ΔABC, ∠C = 90° and D is the midpoint of BC. Prove that $AB^2 = 4AD^2 - 3AC^2$.

Answer



Given: ∠C = 90° and D is the midpoint of BC .i.e. BC = 2CD

To Prove: $AB^2 = 4AD^2 - 3AC^2$

In ΔABC, using Pythagoras theorem we have,

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (AC)^2 + (BC)^2 = (AB)^2$

$\Rightarrow (AC)^2 + (2CD)^2 = (AB)^2$

$\Rightarrow (AC)^2 + 4(CD)^2 = (AB)^2$

$\Rightarrow (AC)^2 + 4(AD^2 - AC^2) = AB^2$

[∵ In right triangle ΔACD, $AD^2 = AC^2 + CD^2$]

$\Rightarrow AC^2 + 4AD^2 - 4AC^2 = AB^2$

$\Rightarrow 4AD^2 - 3AC^2 = AB^2$

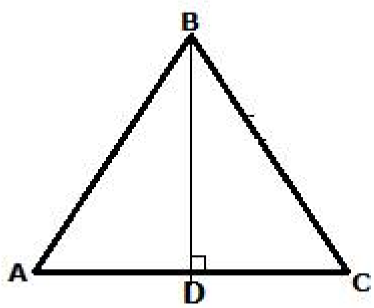
or $AB^2 = 4AD^2 - 3AC^2$

Hence Proved

22. Question

In an isosceles ΔABC, AB = AC and BD ⊥ AC. Prove that $BD^2 - CD^2 = 2CD\ AD$.

Answer



Given: AB = AC and BD ⊥ AC

To Prove: $BD^2 - CD^2 = 2CD \times AD$

In ΔBDC, using Pythagoras theorem we have,

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (BD)^2 + (CD)^2 = (BC)^2$ ...(i)

In ΔBDA, using Pythagoras theorem we have,

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (BD)^2 + (AD)^2 = (AB)^2$

$\Rightarrow (BD)^2 + (AD)^2 = (AC)^2$ [∵ AB = AC]

Multiply this eq. by 2, we get

$\Rightarrow 2(BD)^2 + 2(AD)^2 = 2(AC)^2$ ...(ii)

Subtracting Eq. (ii) from (i), we get

$\Rightarrow CD^2 - BD^2 = BC^2 - 2\,AC^2 + 2\,AD^2$

$= BC^2 - 2\,(AD + CD)^2 + 2\,AD^2$

$= BC^2 - 2\,CD^2 - 4\,AD \times CD$

$= BD^2 + CD^2 - 2\,CD^2 - 4\,AD \times CD$

$= BD^2 - CD^2 - 4\,AD \times CD$

$\Rightarrow CD^2 - BD^2 - BD^2 + CD^2 = -4AD \times CD$

$\Rightarrow -2(BD^2 - CD^2) = -4AD \times CD$

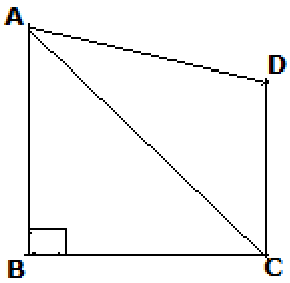
$\Rightarrow BD^2 - CD^2 = 2CD \times AD$

Hence Proved

23. Question

In a quadrilateral, ΔBCD, ∠B = 90°. If $AD^2 = AB^2 + BC^2 + CD^2$, prove that ∠ACD = 90°.

Answer



Given: ABCD is a quadrilateral and ∠B = 90°

and $AD^2 = AB^2 + BC^2 + CD^2$

To Prove: ∠ACD = 90°

In right triangle ΔABC, using Pythagoras theorem, we have

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$ ...(i)

Given: $AD^2 = AB^2 + BC^2 + CD^2$

$\Rightarrow AD^2 = AC^2 + CD^2$ [from (i)]

In ΔACD

$AD^2 = AC^2 + CD^2$

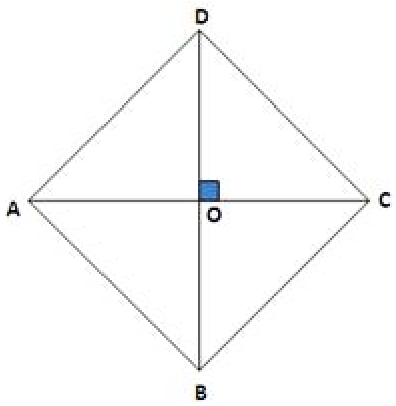
∴ ∠ACD = 90° [converse of Pythagoras theorem]

Hence Proved

24. Question

In a rhombus ABCD, prove that: $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Answer



In rhombus ABCD, AB = BC = CD = DA

We know that diagonals bisect each other at 90°

And $OA = OC = \frac{1}{2} \times AC$, $OB = OD = \frac{1}{2} \times BC$

Consider right triangle AOB

(Perpendicular)^2 + (Base)^2 = (Hypotenuse)^2

$\Rightarrow (OA)^2 + (OB)^2 = (AB)^2$

$\Rightarrow \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 = AB^2$

$\Rightarrow AC^2 + BD^2 = 4AB^2$

$\Rightarrow AC^2 + BD^2 = AB^2 + AB^2 + AB^2 + AB^2$

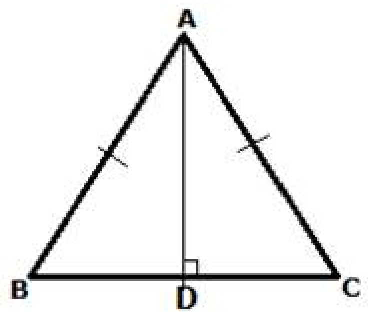
$\Rightarrow AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$

Hence Proved

25. Question

In an equilateral triangle ABC, AD is the altitude drawn from A on side BC. Prove that $3AB^2 = 4AD^2$.

Answer



Given: ABC is an equilateral triangle

and AD is the altitude on side BC

To Prove: $3AB^2 = 4AD^2$

In right triangle ΔADB, using Pythagoras theorem

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow AD^2 + BD^2 = AB^2$

$\Rightarrow AD^2 + \left(\frac{BC}{2}\right)^2 = AB^2$

$\Rightarrow 4AD^2 + BC^2 = 4AB^2$

$\Rightarrow 4AD^2 = 4AB^2 - BC^2$

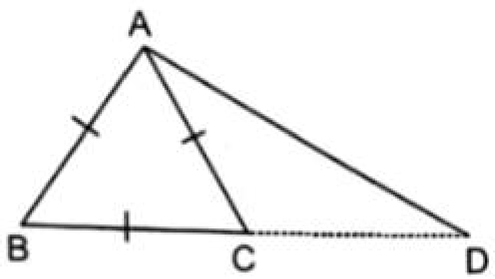
$\Rightarrow 4AD^2 = 4AB^2 - AB^2$ [∵ABC is an equilateral triangle]

$\Rightarrow 4AD^2 = 3AB^2$

Hence Proved

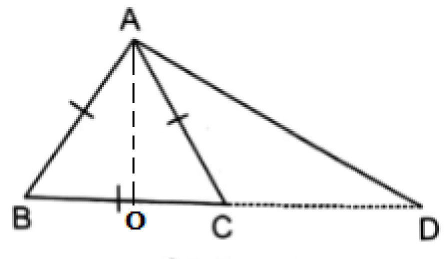
26. Question

In ΔABC, AB = AC. Side BC is produced to D. Prove that $(AD^2 - AC^2) = BD \cdot CD$



Answer

Construction: Draw an altitude from A on BC and named it O.



Given: ABC is an isosceles triangle with AB = AC

To Prove: $AD^2 — AC^2 = BD \times CD$

In right triangle ΔAOD, using Pythagoras theorem, we have

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow AO^2 + OD^2 = AD^2$ ...(i)

Now, in right triangle ΔAOB, using Pythagoras theorem, we have

$\Rightarrow AO^2 + BO^2 = AB^2$ ...(ii)

Subtracting eq (ii) from (i), we get

$AD^2 - AB^2 = AO^2 + OD^2 - AO^2 - BO^2$

$\Rightarrow AD^2 - AB^2 = OD^2 - BO^2$

$\Rightarrow AD^2 - AB^2 = (OD + BO)(OD - OB)$

[$\because (a^2 - b^2) = (a + b)(a - b)$]

$\Rightarrow AD^2 - AB^2 = (BD)(OD - OC)$ [$\because OB = OC$]

$\Rightarrow AD^2 - AB^2 = (BD)(CD)$

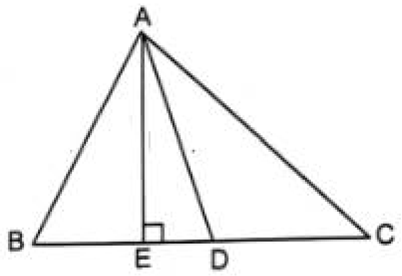
$\Rightarrow AD^2 - AC^2 = (BD)(CD)$ [$\because AB = AC$]

Hence Proved

27. Question

In ΔABC, D is the mid-point of BC and AE ⊥ BC . If AC > AB, show that $AB^2 = AD^2 — BC .DE + 1/4\ BC^2$

Answer



Given: In $\triangle$ ABC, D is the mid-point of BC and AE $\perp$ BC

and AC > AB

In right triangle ΔAEB, using Pythagoras theorem, we have

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (AE)^2 + (BE)^2 = (AB)^2$

$\Rightarrow (AE)^2 + (BD - ED)^2 = (AB)^2$

$\Rightarrow (AE)^2 + (ED)^2 + (BD)^2 - 2\,(ED)(BD) = (AB)^2$

[$\because (a - b)^2 = a^2 + b^2 - 2ab$]

$\Rightarrow (AE^2 + ED^2) + (BD)^2 - 2\,(ED)(BD) = (AB)^2$

$\Rightarrow (AD)^2 + (BD)^2 - 2\,(ED)(BD) = (AB)^2$

[$\because$ In right angled ΔAED, $AE^2 + ED^2 = AD^2$]

$$\Rightarrow (AD)^2 + \left(\frac{BC}{2}\right)^2 - 2ED\left(\frac{BC}{2}\right) = (AB)^2$$

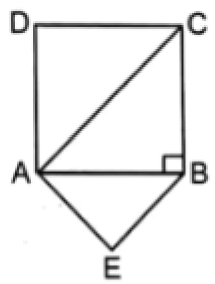
[$\because$ D is the midpoint of BC, so 2DC = BC]

$$\Rightarrow AB^2 = AD^2 - BC \times ED + \frac{BC^2}{4}$$

Hence Proved

28. Question

ABC is an isosceles triangle, right angled at B. Similar triangles ACD and ABE are constructed on sides AC and AB. Find the ratio between the areas of ΔABE and ΔACD.



Answer

Given ΔABC is an isosceles triangle in which ∠B is right angled i.e. 90°

⇒ AB = BC

In right angled ΔABC, by Pythagoras theorem, we have

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

⇒ $(AB)^2 + (BC)^2 = (AC)^2$

⇒ $(AB)^2 + (AB)^2 = (AC)^2$

[∵ABC is an isosceles triangle, AB =BC]

⇒ $2(AB)^2 = (AC)^2$

⇒ $(AC)^2 = 2(AB)^2$ ...(i)

It is also given that ΔABE ~ ΔADC

And we also know that, the ratio of similar triangles is equal to the ratio of their corresponding sides.

∴ $\frac{ar\,(\Delta ABE)}{ar(\Delta ADC)} = \frac{AB^2}{AC^2}$

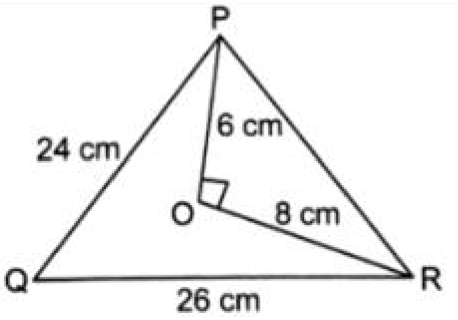
⇒ $\frac{ar\,(\Delta ABE)}{ar(\Delta ADC)} = \frac{AB^2}{2AB^2}$ [from (i)]

⇒ $\frac{ar\,(\Delta ABE)}{ar(\Delta ADC)} = \frac{1}{2}$

∴ ar(ΔABE) : ar(ΔADC) = 1 : 2

29. Question

In the given figure, 0 is a point inside a ∠PQR such that ∠POR = 90°, OP = 6 cm and OR= 8 cm. If PQ = 24 cm and QR = 26 cm, prove that ΔPQR is right angled. P



Answer

Given: ∠POR = 90°, OP = 6 cm and OR= 8 cm

and PQ = 24 cm and QR = 26 cm

To Prove: Δ PQR is right angled at P

In ΔPOR, using Pythagoras theorem, we get

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (PO)^2 + (OR)^2 = (PR)^2$

$\Rightarrow (6)^2 + (8)^2 = (PR)^2$

$\Rightarrow 36 + 64 = (PR)^2$

$\Rightarrow (PR)^2 = 100$

$\Rightarrow PR = \sqrt{100}$

⇒ PR = 10 [taking positive square root]

In ΔPQR

Using Pythagoras theorem, i.e. if the square of the hypotenuse is equal to the sum of the other two sides. Then, the given triangle is a right angled triangle, otherwise not.

Here, $(PR)^2 + (PQ)^2$

$\Rightarrow (10)^2 + (24)^2$

= 100 + 576

= 676

$= (26)^2 = (QR)^2$

∴ given sides 10cm, 24cm and 26cm make a right triangle right angled at P.

Hence Proved

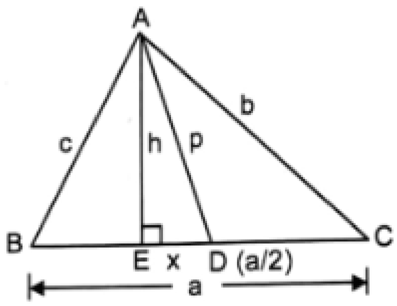
30. Question

In the given figure, D is the mid-point of side BC and AE ⊥ BC. If BC = a, AC = b, AB= c, ED = x, AD =p and AE = h, prove that

(i) $b^2 = p^2 + ax + a^2/4$

(ii)$(b^2+c^2)=2p^2 + 1/2\ a^2$

(iii) $(b^2 - c^2) = 2ax$



Answer

Given: D is the mid-point of side BC and AE ⊥ BC

and BC = a, AC = b, AB= c, ED = x, AD =p and AE = h

To Prove: (i) $b^2 = p^2 + ax + \frac{a^2}{4}$

or $AC^2 = AD^2 + BC \times ED + \frac{BC^2}{4}$

Proof: In right triangle ΔAEC, using Pythagoras theorem, we have

(Perpendicular)^2 + (Base)^2 = (Hypotenuse)^2

⇒ $(AE)^2 + (EC)^2 = (AC)^2$

⇒ $(AE)^2 + (ED + DC)^2 = (AC)^2$

⇒ $(AE)^2 + (ED)^2 + (DC)^2 + 2\ (ED)(DC) = (AC)^2$

[∵ $(a + b)^2 = a^2 + b^2 + 2ab$]

⇒ $(AE^2 + ED^2) + (DC)^2 + 2\ (ED)(DC) = (AC)^2$

⇒ $(AD)^2 + (DC)^2 + 2\ (ED)(DC) = (AC)^2$

[∵ In right angled ΔAED, $AE^2 + ED^2 = AD^2$]

$$\Rightarrow\ (AD)^2 + \left(\frac{BC}{2}\right)^2 + 2ED\left(\frac{BC}{2}\right) = (AC)^2$$

[∵ D is the midpoint of BC, so 2DC = BC]

$$\Rightarrow AC^2 = AD^2 + BC \times ED + \frac{BC^2}{4} \text{ ...(i)}$$

$$\Rightarrow b^2 = p^2 + ax + \frac{a^2}{4}$$

To Prove: (ii) $b^2 + c^2 = 2p^2 + \frac{a^2}{2}$

or $AC^2 + AB^2 = 2AD^2 + \frac{BC^2}{2}$

Proof: In right triangle ΔAEB, using Pythagoras theorem, we have

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (AE)^2 + (BE)^2 = (AB)^2$

$\Rightarrow (AE)^2 + (BD - ED)^2 = (AB)^2$

$\Rightarrow (AE)^2 + (ED)^2 + (BD)^2 - 2\,(ED)(BD) = (AB)^2$

[∵ $(a - b)^2 = a^2 + b^2 - 2ab$]

$\Rightarrow (AE^2 + ED^2) + (BD)^2 - 2\,(ED)(BD) = (AB)^2$

$\Rightarrow (AD)^2 + (BD)^2 - 2\,(ED)(BD) = (AB)^2$

[∵ In right angled ΔAED, $AE^2 + ED^2 = AD^2$]

$$\Rightarrow (AD)^2 + \left(\frac{BC}{2}\right)^2 - 2ED\left(\frac{BC}{2}\right) = (AB)^2$$

[∵ D is the midpoint of BC, so 2DC = BC]

$$\Rightarrow AB^2 = AD^2 - BC \times ED + \frac{BC^2}{4} \text{ ...(ii)}$$

On adding eq. (i) and (ii), we get

$$AC^2 + AB^2 = AD^2 + BC \times ED + \frac{BC^2}{4} + AD^2 - BC \times ED + \frac{BC^2}{4}$$

$$\Rightarrow AC^2 + AB^2 = AD^2 + \frac{BC^2}{2}$$

$$\Rightarrow b^2 + c^2 = 2p^2 + \frac{a^2}{2}$$

To Prove: (iii) $(b^2 - c^2) = 2ax$

or $(AC)^2 - (AB)^2 = 2\,(BC)(ED)$

Proof: Subtracting Eq. (ii) from (i), we get

$$\Rightarrow\ AC^2 - AB^2 = AD^2 + BC \times ED + \frac{BC^2}{4} - AD^2 + BC \times ED - \frac{BC^2}{4}$$

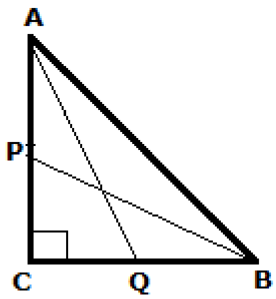
$\Rightarrow (AC)^2 - (AB)^2 = 2\,(BC)(ED)$

Hence Proved

31. Question

P and Q are the mid–points of the sides CA and CB respectively of ΔABC right angled at C. Prove that $4(AQ^2 + BP^2) = 5AB^2$

Answer



Given: $\triangle$ ABC ia right triangle right angled at C

P and Q are the mid–points of the sides CA and CB respectively.

$\Rightarrow$ AP = PC and CQ = QB

In $\triangle$ ACB, using Pythagoras Theorem, we have

(Perpendicular)^2 + (Base)^2 = (Hypotenuse)^2

$\Rightarrow (AC)^2 + (BC)^2 = (AB)^2$ ...(i)

Now, In $\triangle$ ACQ, using Pythagoras Theorem, we have

(Perpendicular)^2 + (Base)^2 = (Hypotenuse)^2

$\Rightarrow (AC)^2 + (CQ)^2 = (AQ)^2$

$$\Rightarrow\ (AC)^2 + \left(\frac{BC}{2}\right)^2 = (AQ)^2$$

$\Rightarrow 4(AC)^2 + (BC)^2 = 4(AQ)^2$

$\Rightarrow (BC)^2 = 4(AQ)^2 - 4(AC)^2$ ...(ii)

Now, In $\triangle$ PCB, using Pythagoras Theorem, we have

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (PC)^2 + (BC)^2 = (BP)^2$

$\Rightarrow \left(\frac{AC}{2}\right)^2 + (BC)^2 = (BP)^2$

$\Rightarrow (AC)^2 + 4(BC)^2 = 4(BP)^2$

$\Rightarrow (AC)^2 = 4(BP)^2 - 4(BC)^2$ ...(ii)

Putting the value of $(AC)^2$ and $(BC)^2$ in eq. (i), we get

$4(BP)^2 - 4(BC)^2 + 4(AQ)^2 - 4(AC)^2 = (AB)^2$

$\Rightarrow 4(BP^2 + AQ^2) - 4(BC^2 + AC^2) = (AB)^2$

$\Rightarrow 4(BP^2 + AQ^2) - 4(AB^2) = (AB)^2$ [from eq(i)]

$\Rightarrow 4(BP^2 + AQ^2) = 5(AB)^2$

Hence Proved