

## 11. Circles

### Exercise 11.1

#### 1. Question

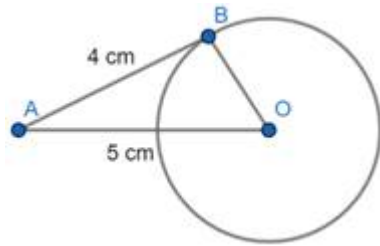
The length of a tangent from a point A at a distance 5cm from the centre of a circle is 4cm. Find the radius of the circle.

#### Answer

Let the centre of circle be O so that  $AO = 5 \text{ cm}$

Tangent is AB whose length is 4 cm

OB is radius as shown



Now we know that radius is perpendicular to point of contact

Hence OB is perpendicular to AB

Hence  $\angle ABO = 90^\circ$

Consider  $\triangle ABO$

Using Pythagoras theorem

$$\Rightarrow AB^2 + OB^2 = AO^2$$

$$\Rightarrow 4^2 + OB^2 = 5^2$$

$$\Rightarrow 16 + OB^2 = 25$$

$$\Rightarrow OB^2 = 25 - 16$$

$$\Rightarrow OB^2 = 9$$

$$\Rightarrow OB = \pm 3$$

As length cannot be negative

$$\Rightarrow OB = 3 \text{ cm}$$

Hence length of radius is 3 cm

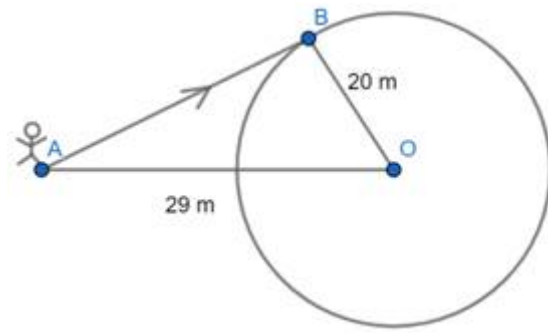
## 2. Question

Rajesh is 29 m away from the centre of a circular flower bed. Find the distance he has to cover to reach the flower bed along the tangential path if the radius of the flower bed is 20m.

### Answer

Let the centre of circular flower bed be O and radius OB = 20 m

Let Rajesh is at point A, and he has to travel tangential path to reach flower bed which is AB as shown



Now we know that radius is perpendicular to point of contact

Hence OB is perpendicular to AB

$$\text{Hence } \angle ABO = 90^\circ$$

Consider  $\triangle ABO$

Using Pythagoras theorem

$$\Rightarrow AB^2 + OB^2 = AO^2$$

$$\Rightarrow AB^2 + 20^2 = 29^2$$

$$\Rightarrow AB^2 = 29^2 - 20^2$$

$$\text{Using } a^2 - b^2 = (a + b)(a - b)$$

$$\Rightarrow AB^2 = (29 - 20)(29 + 20)$$

$$\Rightarrow AB^2 = 9 \times 49$$

$$\Rightarrow AB = \sqrt{9 \times 49}$$

$$\Rightarrow AB = \pm (3 \times 7)$$

$$\Rightarrow AB = \pm 21$$

As length cannot be negative

$$\Rightarrow AB = 21 \text{ m}$$

Hence Rajesh has to cover 21 m to reach the flower bed along the tangential path.

### 3. Question

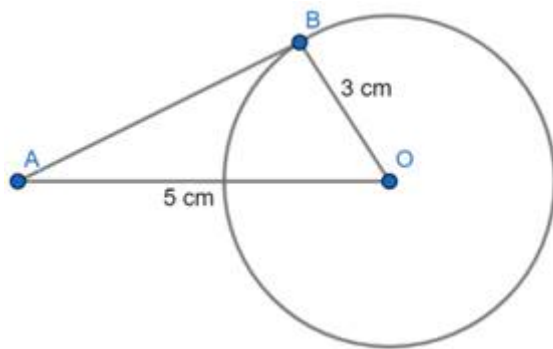
Find the length of the tangent drawn from a point, whose distance from the centre of a circle is 5 cm, and the radius of the circle is 3 cm.

#### Answer

Let A be the point at distance of 5 cm from the centre as  $AO = 5 \text{ cm}$

AB is the tangent at point B as shown

OB is the radius which is 3 cm



Now we know that radius is perpendicular to point of contact

Hence OB is perpendicular to AB

Hence  $\angle ABO = 90^\circ$

Consider  $\triangle ABO$

Using Pythagoras theorem

$$\Rightarrow AB^2 + OB^2 = AO^2$$

$$\Rightarrow AB^2 + 3^2 = 5^2$$

$$\Rightarrow AB^2 + 9 = 25$$

$$\Rightarrow AB^2 = 25 - 9$$

$$\Rightarrow AB^2 = 16$$

$$\Rightarrow AB = \pm 4$$

As length cannot be negative

$$\Rightarrow AB = 4 \text{ cm}$$

Hence length of tangent is 4 cm

#### 4. Question

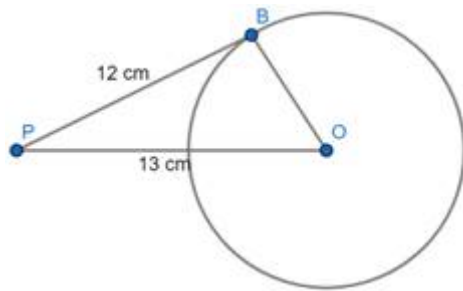
A point P is 13 cm from the centre of the circle. The length of the tangent drawn from P to the circle is 12 cm. Find the radius of the circle.

#### Answer

Let the centre of circle be O so that PO = 13 cm

Tangent is PB whose length is 12 cm

OB is radius as shown



Now we know that radius is perpendicular to point of contact

Hence OB is perpendicular to PB

$$\text{Hence } \angle PBO = 90^\circ$$

Consider  $\triangle PBO$

Using Pythagoras theorem

$$\Rightarrow PB^2 + OB^2 = PO^2$$

$$\Rightarrow 12^2 + OB^2 = 13^2$$

$$\Rightarrow OB^2 = 13^2 - 12^2$$

$$\Rightarrow OB^2 = 169 - 144$$

$$\Rightarrow OB^2 = 25$$

$$\Rightarrow OB = \pm 5$$

As length cannot be negative

$$\Rightarrow OB = 5 \text{ cm}$$

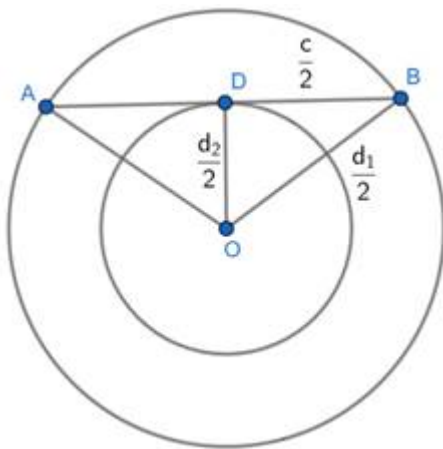
Hence length of radius is 5 cm

### 5. Question

If  $d_1, d_2$  ( $d_2 < d_1$ ) are the diameters of two concentric circles and chord of one circle of length  $C$  is tangent to other circle, then prove that  $d_2^2 = C^2 + d_1^2$ .

### Answer

Let the two concentric circles have the centre  $O$  and let  $AB$  be the chord of an outer circle whose length is  $D$  and which is also tangent to the inner circle at point  $D$  as shown



The diameters are given as  $d_1$  and  $d_2$  hence the radius will be  $\frac{d_1}{2}$  and  $\frac{d_2}{2}$

In  $\triangle OAB$

$$\Rightarrow OA = OB \text{ ...radius of the outer circle}$$

Hence  $\triangle OAB$  is an isosceles triangle

As radius is perpendicular to tangent  $OC$  is perpendicular to  $AB$

$OC$  is altitude from the apex, and in an isosceles triangle, the altitude is also the median

$$\text{Hence } AD = DB = \frac{C}{2}$$

Consider  $\triangle ODB$

$$\Rightarrow \angle ODB = 90^\circ \text{ ...radius perpendicular to tangent}$$

Using Pythagoras theorem

$$\Rightarrow OD^2 + BD^2 = OB^2$$

$$\Rightarrow \frac{d_2^2}{2^2} + \frac{C^2}{2^2} = \frac{d_1^2}{2^2}$$

Multiply the whole by  $2^2$

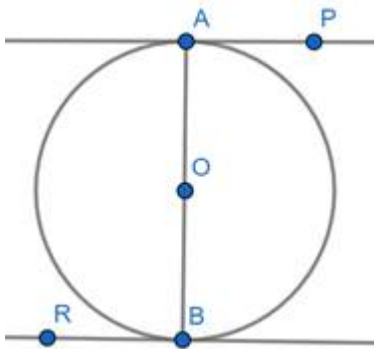
$$\Rightarrow d_2^2 + C^2 = d_1^2$$

Hence proved

## 6. Question

Prove that the line segment joining the point of contact of two parallel tangents to a circle is a diameter of the circle.

**Answer**



Let lines AP and BR are parallel tangents to circle having centre O

We have to prove that AB is the diameter

To prove AB as diameter, we have to prove that AB passes through O which means that points A, O and B are on the same line or collinear

OA is perpendicular to PA at A because the line from the centre is perpendicular to the tangent at the point of contact

PA || RB

Hence OA is also perpendicular to RB

$\Rightarrow$  OA perpendicular to PA and RB ...(i)

Similarly, OB is perpendicular to RB at B because the line from the centre is perpendicular to the tangent at the point of contact

PA || RB

Hence OB is also perpendicular to PA

$\Rightarrow$  OB perpendicular to PA and RB ...(ii)

From (i) and (ii) we can say that OA and OB can be same line or parallel lines, but we have a common point O which implies that OA and OB are same lines

Hence A, O, B lies on the same line, i.e. A, O and B are collinear

Thus AB passes through O

Hence AB is the diameter

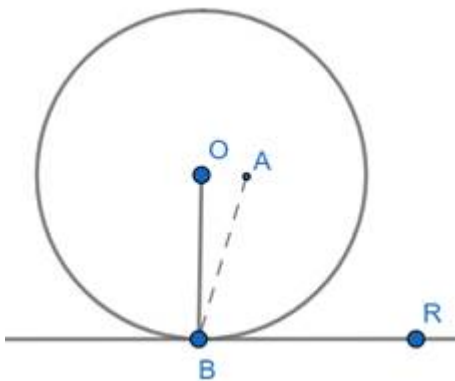
Hence, the line segment joining the point of contact of two parallel tangents to a circle is a diameter of the circle.

## 7. Question

Prove that the perpendicular at the point of contact of the tangent to a circle passes through the centre.

### Answer

Let there be a circle with centre O and BR as tangent with the point of contact as B



Let AB be the line perpendicular to BR

$$\Rightarrow \angle ABR = 90^\circ \dots(i)$$

As OB is the radius of the circle and we know that radius is perpendicular to the tangent at the point of contact

OB is perpendicular to BR

$$\Rightarrow \angle OBR = 90^\circ \dots(ii)$$

Equation (i) and (ii) implies that

$$\Rightarrow \angle ABR = \angle OBR$$

This is only possible iff A and O lie on the same line or A and O are the same points

Case 1: Suppose A and O are on the same line

If A and O are on the same line, then the perpendicular AB to tangent BR has passed through the centre

Case 2: suppose A and O are the same points

As O itself is the centre of the circle, and A and O are the same points hence the perpendicular to the tangent at the point of contact passes through the circle

In any scenario, the line has to pass through the centre.

Hence, the perpendicular at the point of contact of the tangent to a circle passes through the centre

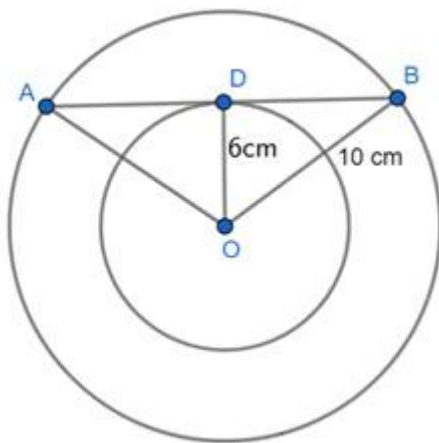
### 8. Question

Two concentric circles are of radii 10 cm, and 6 cm Find the length of the chord of the larger circle which touches the smaller circle.

#### Answer

Let the two concentric circles have the centre O and let AB be the chord of an outer circle whose length is D and which will also be tangent to the inner circle at point D because it is given that the chord touches the inner circle.

The radius of inner circle  $OD = 6$  cm and the radius of outer circle  $OB = 10$  cm



In  $\triangle OAB$

$\Rightarrow OA = OB$  ...radius of outer circle

Hence  $\triangle OAB$  is isosceles triangle

As radius is perpendicular to tangent  $OC$  is perpendicular to  $AB$

$OC$  is altitude from apex and in isosceles triangle the altitude is also the median

Hence  $AD = DB$

Hence  $AB = 2DB$

Consider  $\triangle ODB$

$\Rightarrow \angle ODB = 90^\circ$  ...radius perpendicular to tangent

Using Pythagoras theorem

$$\Rightarrow OD^2 + BD^2 = OB^2$$

$$\Rightarrow 6^2 + BD^2 = 10^2$$

$$\Rightarrow 36 + BD^2 = 100$$

$$\Rightarrow BD^2 = 100 - 36$$

$$\Rightarrow BD^2 = 64$$

$$\Rightarrow BD = \pm 8$$

As length cannot be negative

$$\Rightarrow BD = 8 \text{ cm}$$

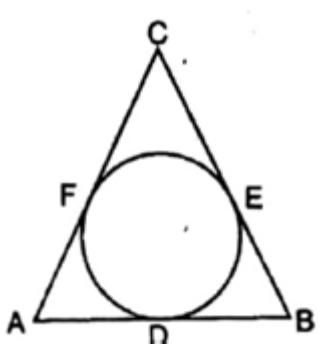
$$\Rightarrow AB = 2 \times 8 \text{ ...since } AB = 2BD$$

$$\Rightarrow AB = 16 \text{ cm}$$

### 9. Question

(i) A circle is inscribed in a  $\triangle ABC$  having sides BC, CA and AB 16 cm, 20 cm and 24 cm respectively as shown in the figure Find AD, BE and CF.

(ii) If  $AF=4\text{cm}$ ,  $BE=3\text{cm}$ ,  $AC=11\text{cm}$ , then find BC.



### Answer

i) Tangents drawn from external point are equal

AD and AF are tangents from point A

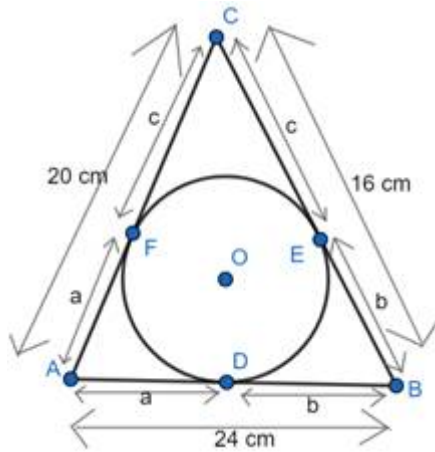
$$\Rightarrow AD = AF = a$$

BF and BE are tangents from point B

$$\Rightarrow BD = BE = b$$

CD and CE are tangents from point C

$$\Rightarrow CF = CE = c$$



From figure

We have  $AC = AF + FC$

$$\Rightarrow 20 = a + c \dots(i)$$

Also,  $AB = AD + DB$

$$\Rightarrow 24 = a + b \dots(ii)$$

And  $CB = CE + EB$

$$\Rightarrow 16 = c + b \dots(iii)$$

Add (i), (ii) and (iii)

$$\Rightarrow 20 + 24 + 16 = a + c + a + b + c + b$$

$$\Rightarrow 60 = 2(a + b + c)$$

$$\Rightarrow a + b + c = 30 \dots(iv)$$

Substitute (i) in (iv)

$$\Rightarrow 20 + b = 30$$

$$\Rightarrow b = 10$$

Substitute (ii) in (iv)

$$\Rightarrow 24 + c = 30$$

$$\Rightarrow c = 6$$

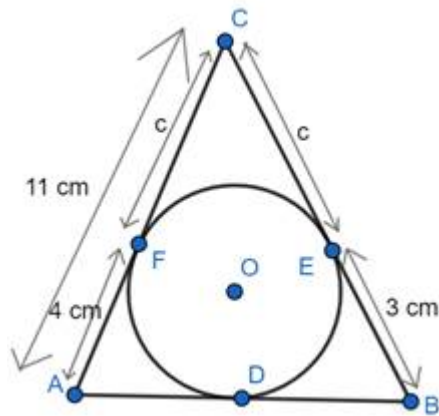
Substitute (iii) in (iv)

$$\Rightarrow 16 + a = 30$$

$$\Rightarrow a = 14$$

Hence  $AD = a = 14$  cm,  $BE = b = 10$  cm and  $CF = c = 6$  cm

ii)



Tangents drawn from external point are equal

CF and CE are tangents from point C

$$\Rightarrow CF = CE = c$$

From figure

$$AC = AF + FC$$

$$\Rightarrow 11 = 4 + c$$

$$\Rightarrow c = 7 \text{ cm}$$

$$\text{Hence } EC = c = 7 \text{ cm}$$

$$\text{We have } BC = BE + EC$$

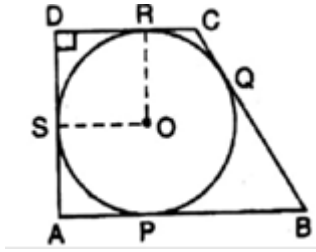
$$\Rightarrow BC = 3 + 7$$

$$\Rightarrow BC = 10 \text{ cm}$$

Hence BC is 10 cm

### 10. Question

In the given figure, ABCD is a quadrilateral in which  $\angle D = 90^\circ$ . A circle C (O,r) touches the sides AB, BC, CD and DA at P,Q,R,S respectively, If  $BC = 38$  cm,  $CD = 25$  cm and  $BP = 27$  cm, find the value of r.



### Answer

$r$  is the radius which is  $OR = r$

Consider quadrilateral DROS

$$\Rightarrow \angle RDS = 90^\circ \dots \text{given}$$

$$\Rightarrow \angle DRO = 90^\circ \dots \text{radius is perpendicular to the tangent}$$

$$\Rightarrow DR = DS \dots \text{tangents drawn from the same point are equal}$$

As the adjacent angles are  $90^\circ$  and adjacent sides are same hence DROS is a square

$$\text{Hence } OR = DR = r \dots (i)$$

As tangents drawn from the same point are equal

BQ and BP are tangents drawn from B

$$\Rightarrow BQ = BP \Rightarrow BQ = 27 \text{ cm} \dots \text{BP is 27 cm given}$$

From figure

$$\Rightarrow BC = BQ + QC$$

$$\Rightarrow 38 = 27 + QC \dots \text{BC is 38 cm given}$$

$$\Rightarrow QC = 11 \text{ cm}$$

CQ and CR are tangents drawn from C

$$\Rightarrow CQ = CR \dots \text{tangents from same point}$$

$$\Rightarrow CR = 11 \text{ cm}$$

Again from figure

$$\Rightarrow CD = CR + RD$$

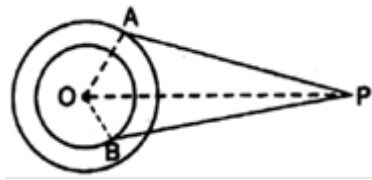
$$\Rightarrow 25 = 11 + r \dots \text{CD is 25 given and } RD = r \text{ from (i)}$$

$$\Rightarrow r = 14 \text{ cm}$$

Hence  $r$  radius is 14 cm

### 11. Question

In the given figure, O is the centre of two concentric circles of radii 4 cm and 6cm respectively. PA and PB are tangents to the outer and inner circle respectively. If PA=10cm, find the length of PB up to one place of decimal.



### Answer

Consider  $\triangle POA$

OA = 6 cm ...radius of the outer circle

PA = 10 cm ...given

$\angle OAP = 90^\circ$  ...radius is perpendicular to the tangent

Hence  $\triangle POA$  is right-angled triangle

Using Pythagoras  $\Rightarrow OA^2 + AP^2 = OP^2$

$$\Rightarrow 6^2 + 10^2 = OP^2$$

$$\Rightarrow 36 + 100 = OP^2$$

$$\Rightarrow OP^2 = 136 \dots(i)$$

Consider  $\triangle PBO$

OB = 4 cm ...radius of inner circle

$\angle OBP = 90^\circ$  ...radius is perpendicular to the tangent

Hence  $\triangle POB$  is right-angled triangle

Using Pythagoras  $\Rightarrow OB^2 + BP^2 = OP^2$

Using (i)

$$\Rightarrow 4^2 + BP^2 = 136$$

$$\Rightarrow 16 + BP^2 = 136$$

$$\Rightarrow BP^2 = 120$$

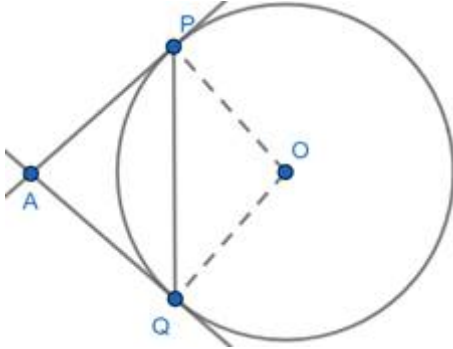
$$\Rightarrow BP = 10.9 \text{ cm}$$

Hence length of PB is 10.9 cm

## 12. Question

Show that the tangents at the extremities of any chord of a circle make equal angles with the chord.

**Answer**



Let the circle with centre O and chord PQ with tangents from point A as AP and AQ as shown

We have to prove that  $\angle APQ = \angle AQP$

Consider  $\triangle OPQ$

$\Rightarrow OP = OQ$  ...radius

Hence  $\triangle OPQ$  is an isosceles triangle

$\Rightarrow \angle OPQ = \angle OQP$  ...base angles of isosceles triangle ...(a)

As radius OP is perpendicular to tangent AP at point of contact P

$\Rightarrow \angle APO = 90^\circ$

From figure  $\angle APO = \angle APQ + \angle OPQ$

$\Rightarrow 90^\circ = \angle APQ + \angle OPQ$

$\Rightarrow \angle APQ = 90^\circ - \angle OPQ$  ...(i)

As radius OQ is perpendicular to tangent AQ at point of contact Q

$\Rightarrow \angle AQO = 90^\circ$

From figure  $\angle AQO = \angle AQP + \angle OQP$

$\Rightarrow 90^\circ = \angle AQP + \angle OQP$

$\Rightarrow \angle AQP = 90^\circ - \angle OQP$

Using (a)

$\Rightarrow \angle AQP = 90^\circ - \angle OPQ$  ...(ii)

Using (i) and (ii), we can say that

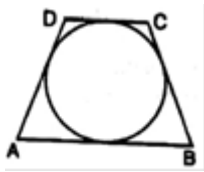
$$\Rightarrow \angle APQ = \angle AQP$$

Hence proved

Hence, the tangents at the extremities of any chord of a circle make equal angles with the chord

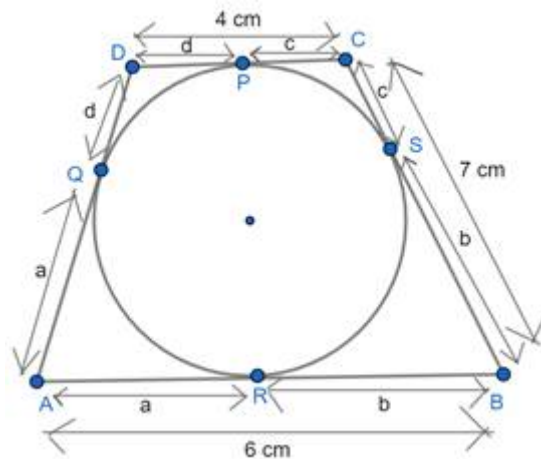
### 13. Question

In the given figure, a circle touches all the four sides of a quadrilateral ABCD whose three sides are  $AB = 6\text{ cm}$ ,  $BC = 7\text{ cm}$ , and  $CD = 4\text{ cm}$ . Find AD.



### Answer

Mark the touching points as P, Q, R and S as shown



As tangents from a point are of equal length we have

$$AQ = AR = a$$

$$BR = BS = b$$

$$CP = CS = c$$

$$DP = DQ = d$$

From figure

$$\Rightarrow BC = BS + SC$$

$$\Rightarrow 7 = b + c$$

$$\Rightarrow b = 7 - c \dots BC \text{ is } 7 \text{ cm given } \dots(i)$$

Also,

$$\Rightarrow DC = DP + PC$$

$$\Rightarrow 4 = d + c$$

$$\Rightarrow c = 4 - d \dots DC \text{ is 4 cm given } \dots(ii)$$

And

$$\Rightarrow AB = AR + RB$$

$$\Rightarrow 6 = a + b \dots AB \text{ is 6 cm given}$$

$$\Rightarrow a = 6 - b$$

Using (i)

$$\Rightarrow a = 6 - (7 - c)$$

Using (ii)

$$\Rightarrow a = 6 - (7 - (4 - d))$$

$$\Rightarrow a = 6 - (7 - 4 + d)$$

$$\Rightarrow a = 6 - 7 + 4 - d$$

$$\Rightarrow a + d = 3$$

$$\Rightarrow AQ + QD = 3 \dots \text{since } AQ = a \text{ and } QD = d$$

From figure  $AQ + QD = AD$

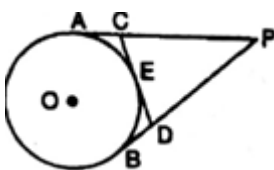
$$\Rightarrow AD = 3 \text{ cm}$$

Hence AD is 3 cm

#### 14. Question

(i) From an external point P, tangents PA and PB are drawn to a circle with centre O. If CD is the tangent to the circle at the point E and  $PA = 14 \text{ cm}$ , find the perimeter of  $\triangle PCD$ .

(ii) If  $PA = 11 \text{ cm}$ ,  $PD = 7 \text{ cm}$ , then  $DE = ?$



**Answer**

i) From P we have tangents PA and PB

Hence  $PA = PB$  ...tangents from same point are equal ...(a)

Point C is on PA

From C we have tangents CA and CE

$\Rightarrow CA = CE$  ...tangents from same point are equal ...(i)

Point D is on PB

From D we have two tangents DE and DB

$\Rightarrow DE = DB$  ... tangents from same point are equal ...(ii)

Consider  $\triangle PCD$

$\Rightarrow$  perimeter of  $\triangle PCD = PC + CD + PD$

From figure  $CD = CE + ED$

$\Rightarrow$  perimeter of  $\triangle PCD = PC + CE + ED + PD$

Using (i) and (ii)

$\Rightarrow$  perimeter of  $\triangle PCD = PC + CA + DB + PD$

From figure we have

$PC + CA = PA$  and  $DB + PD = PB$

$\Rightarrow$  perimeter of  $\triangle PCD = PA + PB$

Using (a)

$\Rightarrow$  perimeter of  $\triangle PCD = PA + PA$

$\Rightarrow$  perimeter of  $\triangle PCD = 2(PA)$

PA is 14 cm given

$\Rightarrow$  perimeter of  $\triangle PCD = 2 \times 14$

$\Rightarrow$  perimeter of  $\triangle PCD = 28$  cm

ii)  $PA = 11$  cm ...given

using (a)

$PB = 11$  cm

From figure

$$\Rightarrow PB = PD + DB$$

Using (ii)

$$\Rightarrow PB = PD + DE$$

$$\Rightarrow 11 = 7 + DE \text{ ...PD is 7 cm given}$$

$$\Rightarrow DE = 5 \text{ cm}$$

Hence  $DE = 5 \text{ cm}$

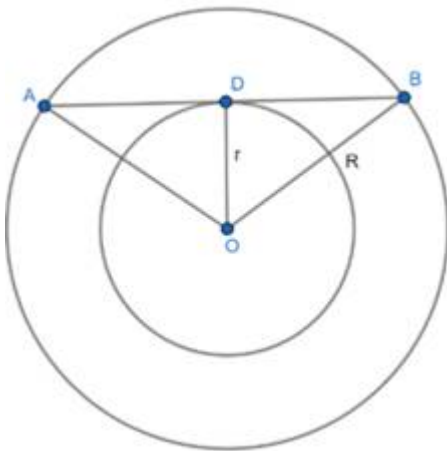
### 15. Question

In two concentric circles, prove that all chords of the outer circle which touch the inner arc of equal length.

### Answer

Let  $O$  be the centre of concentric circles with radius ' $r$ ' and ' $R$ ' ( $R > r$ ) and  $AB$  be the chord which touches the inner circle at point  $D$

We have to prove that  $AB$  has a fixed length



Consider  $\triangle OAB$

$$OA = OB \text{ ...radius}$$

Hence  $\triangle OAB$  is an isosceles triangle

Radius  $OD$  is perpendicular to tangent  $AB$  at the point of contact  $D$

Hence  $OD$  is the altitude, and we know that the altitude from the apex of the isosceles triangle is also the median

$$\Rightarrow AD = BD \text{ ...(a)}$$

Now consider  $\triangle ODB$

$$\Rightarrow \angle ODB = 90^\circ \text{ ...radius is perpendicular to tangent}$$

Using Pythagoras

$$\Rightarrow OD^2 + BD^2 = OB^2$$

The radius are  $OB = R$  and  $OD = r$

$$\Rightarrow r^2 + BD^2 = R^2$$

$$\Rightarrow BD^2 = R^2 - r^2$$

$$\Rightarrow BD = \sqrt{(R^2 - r^2)} \dots(i)$$

From figure

$$\Rightarrow AB = AD + BD$$

Using (a)

$$\Rightarrow AB = BD + BD$$

$$\Rightarrow AB = 2BD$$

Using (i)

$$\Rightarrow AB = 2\sqrt{(R^2 - r^2)}$$

Here observe that  $AB$  only depends on  $R$  and  $r$  which are fixed radius of inner circle and outer circle.

And as the radius of both the circle will not change however one may draw the chord the radius will always be fixed.

And hence  $AB$  won't change  $AB$  is fixed length

Hence proved

Hence, in two concentric circles, all chords of the outer circle which touch the inner circle are of equal length.

## **Exercise 11.2**

### **1. Question**

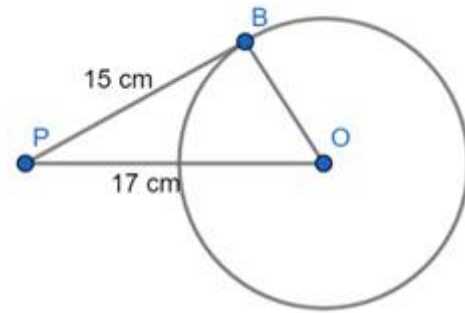
From a point  $P$ , the length of the tangent to a circle is 15 cm, and the distance of  $P$  from the centre of the circle is 17cm. Then what is the radius of the circle?

**Answer**

Let the centre of circle be  $O$  so that  $PO = 17$  cm

Tangent is  $PB$  whose length is 15 cm

OB is radius as shown



Now we know that radius is perpendicular to point of contact

Hence OB is perpendicular to PB

Hence  $\angle PBO = 90^\circ$

Consider  $\Delta PBO$

Using Pythagoras theorem

$$\Rightarrow PB^2 + OB^2 = PO^2$$

$$\Rightarrow 15^2 + OB^2 = 17^2$$

$$\Rightarrow OB^2 = 17^2 - 15^2$$

$$\Rightarrow OB^2 = 289 - 225$$

$$\Rightarrow OB^2 = 64$$

$$\Rightarrow OB = \pm 8$$

As length cannot be negative

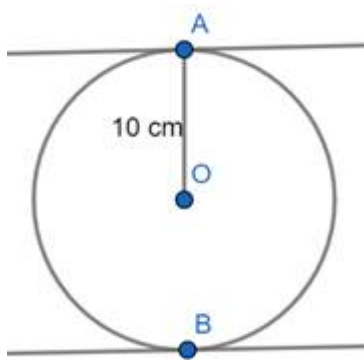
$$\Rightarrow OB = 8 \text{ cm}$$

Hence length of radius is 8 cm

## 2. Question

What is the distance between two parallel tangents of a circle of radius 10cm?

**Answer**



O is the centre of circle and tangents from point A and B are parallel

We know that the line joining point of contacts of two parallel tangents (here AB) passes through the centre

And as a line from the centre is perpendicular to tangent, hence that line(AB) will be the distance between parallel tangents

AB passes through centre O hence AB is also the diameter of the circle

Hence the distance between the two parallel tangents will be the diameter of the circle

Radius is given 10 cm

Hence diameter of circle =  $2 \times \text{radius}$

Hence  $AB = 2 \times 10$

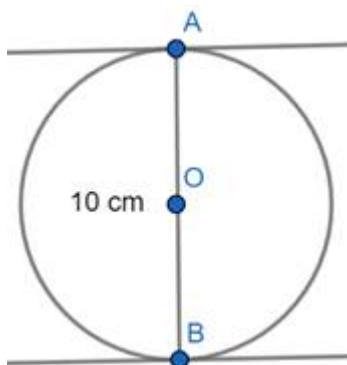
$\Rightarrow AB = 20 \text{ cm}$

Hence distance between parallel tangents is 20 cm

### 3. Question

If the distance between two parallel tangents of a circle is 10cm, what is the radius of the circle?

**Answer**



O is the centre of circle and tangents from point A and B are parallel

We know that the line joining point of contacts of two parallel tangents (here AB) passes through the centre

And as a line from the centre is perpendicular to tangent, hence that line(AB) will be the distance between parallel tangents

AB passes through centre O hence AB is also the diameter of the circle

Hence the distance between the two parallel tangents will be the diameter of the circle

The distance AB = 10 cm in diameter of the circle

Hence radius will be half of the diameter which is 5 cm

#### 4. Question

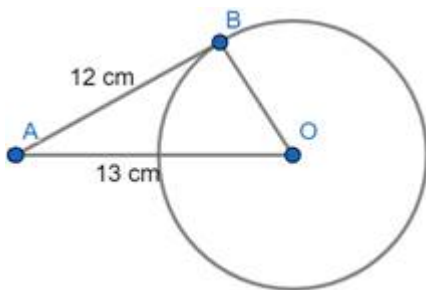
The length of the tangent from a point A at a distance of 13cm from the centre of the circle is 12cm. What is the radius of the circle?

#### Answer

Let the centre of circle be O so that AO = 13 cm

Tangent is AB whose length is 12 cm

OB is radius as shown



Now we know that radius is perpendicular to point of contact

Hence OB is perpendicular to AB

Hence  $\angle ABO = 90^\circ$

Consider  $\Delta ABO$

Using Pythagoras theorem

$$\Rightarrow AB^2 + OB^2 = AO^2$$

$$\Rightarrow 12^2 + OB^2 = 13^2$$

$$\Rightarrow OB^2 = 13^2 - 12^2$$

$$\Rightarrow OB^2 = 169 - 144$$

$$\Rightarrow OB^2 = 25$$

$$\Rightarrow OB = \pm 5$$

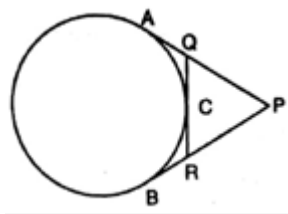
As length cannot be negative

$$\Rightarrow OB = 5 \text{ cm}$$

Hence length of radius is 5 cm

### 5 A. Question

In the given figure if  $PA=20$  cm, what is the perimeter of  $\Delta PQR$ .



### Answer

From P we have tangents PA and PB

Hence  $PA = PB$  ...tangents from same point are equal ...(a)

Point Q is on PA

From Q we have tangents QA and QC

$$\Rightarrow QA = QC \text{ ...tangents from same point are equal ...}(i)$$

Point R is on PB

From R we have two tangents RC and RB

$$\Rightarrow RC = RB \text{ ... tangents from same point are equal ...}(ii)$$

Consider  $\Delta PQR$

$$\Rightarrow \text{perimeter of } \Delta PQR = PQ + QR + PR$$

$$\text{From figure } QR = QC + CR$$

$$\Rightarrow \text{perimeter of } \Delta PQR = PQ + QC + CR + PR$$

Using (i) and (ii)

$$\Rightarrow \text{perimeter of } \Delta PQR = PQ + QA + RB + PR$$

From figure we have

$$PQ + QA = PA \text{ and } RB + PR = PB$$

$$\Rightarrow \text{perimeter of } \Delta PQR = PA + PB$$

Using (a)

$$\Rightarrow \text{perimeter of } \Delta PQR = PA + PA$$

$$\Rightarrow \text{perimeter of } \Delta PQR = 2(PA)$$

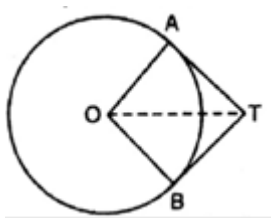
PA is 20 cm given

$$\Rightarrow \text{perimeter of } \Delta PQR = 2 \times 20$$

$$\Rightarrow \text{perimeter of } \Delta PQR = 40 \text{ cm}$$

### 5 B. Question

In the given figure if  $\angle ATO = 40^\circ$ , find  $\angle AOB$ .



### Answer

$$\angle ATO = 40^\circ \text{ ...given}$$

From T we have two tangents TA and TB

We know that if we join point T and centre of circle O then the line TO divides the angle between tangents

$$\Rightarrow \angle ATO = \angle OTB = 40^\circ \text{ ...(i)}$$

$$\angle OAT = \angle OBT = 90^\circ \text{ ...radius is perpendicular to tangent ...(ii)}$$

Consider quadrilateral OATB

$$\Rightarrow \angle OAT + \angle ATB + \angle TBO + \angle AOB = 360^\circ \text{ ...sum of angles of quadrilateral}$$

$$\text{From figure } \angle ATB = \angle ATO + \angle OTB$$

$$\Rightarrow \angle OAT + \angle ATO + \angle OTB + \angle TBO + \angle AOB = 360^\circ$$

Using (i) and (ii)

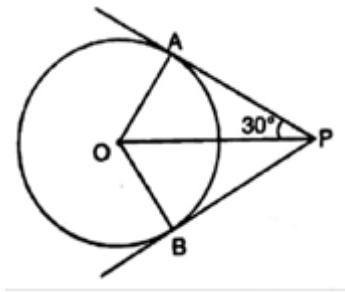
$$\Rightarrow 90^\circ + 40^\circ + 40^\circ + 90^\circ + \angle AOB = 360^\circ$$

$$\Rightarrow 260^\circ + \angle AOB = 360^\circ$$

$$\Rightarrow \angle AOB = 100^\circ$$

## 6. Question

in the figure PA and PB are tangents to the circle. If  $\angle APO = 30^\circ$ , find  $\angle AOB$



## Answer

$$\angle APO = 30^\circ \text{ ...given}$$

From P we have two tangents PA and PB

We know that if we join point P and centre of circle O then the line PO divides the angle between tangents

$$\Rightarrow \angle APO = \angle OPB = 30^\circ \text{ ...(i)}$$

$$\angle OAP = \angle OBP = 90^\circ \text{ ...radius is perpendicular to tangent ...(ii)}$$

Consider quadrilateral OAPB

$$\Rightarrow \angle OAP + \angle APB + \angle PBO + \angle AOB = 360^\circ \text{ ...sum of angles of quadrilateral}$$

$$\text{From figure } \angle APB = \angle APO + \angle OPB$$

$$\Rightarrow \angle OAP + \angle APO + \angle OPB + \angle PBO + \angle AOB = 360^\circ$$

Using (i) and (ii)

$$\Rightarrow 90^\circ + 30^\circ + 30^\circ + 90^\circ + \angle AOB = 360^\circ$$

$$\Rightarrow 240^\circ + \angle AOB = 360^\circ$$

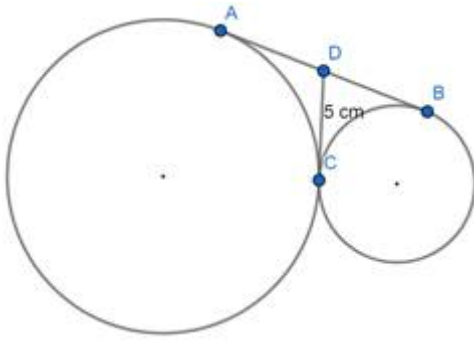
$$\Rightarrow \angle AOB = 120^\circ$$

Hence  $\angle AOB$  is  $120^\circ$

## 7. Question

AB and CD are two common tangents of two circles which touch each other at C. If D lies on AB and  $CD = 5$  cm, then what is the length of AB.

## Answer



DC and AB are tangents given to both circle

Point D is on AB which means DA and DB are also tangents to both circle

Now from point D, we have two tangents to bigger circle which are DA and DC

$\Rightarrow DA = DC$  ...tangents from a point to a circle are equal

$\Rightarrow DA = 5 \text{ cm}$  ...DC is 5 cm given...(i)

Also from point D, we have two tangents to smaller circle which are DB and DC

$\Rightarrow DB = DC$  ...tangents from a point to a circle are equal

$\Rightarrow DB = 5 \text{ cm}$  ...DC is 5 cm given...(ii)

Now as point D is on AB from the figure we can say that  $DA + DB = AB$

$\Rightarrow AB = DA + DB$

Using (i) and (ii)

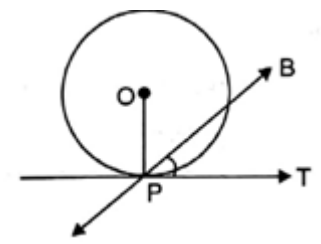
$\Rightarrow AB = 5 + 5$

$\Rightarrow AB = 10 \text{ cm}$

Hence the length of tangent AB is 10 cm

### 8. Question

In the given figure,  $\angle BPT = 50^\circ$ . What is the measure of  $\angle OPB$ ?



**Answer**

PT is tangent to circle and OP is radius

$\Rightarrow \angle OPT = 90^\circ$  ...radius is perpendicular to tangent

From figure

$\Rightarrow \angle OPT = \angle OPB + \angle BPT$

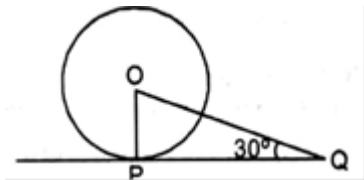
$\Rightarrow 90^\circ = \angle OPB + 50^\circ$  ... $\angle BPT$  is  $50^\circ$  given

$\Rightarrow \angle OPB = 40^\circ$

Hence  $\angle OPB$  is  $40^\circ$

### 9. Question

In the given figure, a measure of  $\angle POQ$  is.....



### Answer

PQ is tangent to circle and OP is radius

$\Rightarrow \angle OPQ = 90^\circ$  ...radius is perpendicular to tangent

$\Rightarrow \angle PQO = 30^\circ$  ...given

Consider  $\triangle OPQ$

$\Rightarrow \angle OPQ + \angle PQO + \angle POQ = 180^\circ$  ...sum of angles of triangle

$\Rightarrow 90^\circ + 30^\circ + \angle POQ = 180^\circ$

$\Rightarrow 120^\circ + \angle POQ = 180^\circ$

$\Rightarrow \angle POQ = 60^\circ$

Hence  $\angle POQ$  is  $60^\circ$

### 10. Question

If all sides of a parallelogram touch a circle, then that parallelogram is....

### Answer

Consider ABCD as a parallelogram touching the circle at points P, Q, R and S as shown

As ABCD is a parallelogram opposites sides are equal

$$\Rightarrow AB = CD \dots(a)$$

$$\Rightarrow AD = BC \dots(b)$$

AP and AS are tangents from point A  $\Rightarrow AP = AS$  ...tangents from point to a circle are equal...(i)

BP and BQ are tangents from point B  $\Rightarrow BP = BQ$  ...tangents from point to a circle are equal...(ii)

CQ and CR are tangents from point C  $\Rightarrow CR = CQ$  ...tangents from point to a circle are equal...(iii)

DR and DS are tangents from point D  $\Rightarrow DR = DS$  ...tangents from point to a circle are equal...(iv)

Add equation (i) + (ii) + (iii) + (iv)

$$\Rightarrow AP + BP + CR + DR = AS + DS + BQ + CQ$$

From figure  $AP + BP = AB$ ,  $CR + DR = CD$ ,  $AS + DS = AD$  and  $BQ + CQ = BC$

$$\Rightarrow AB + CD = AD + BC$$

Using (a) and (b)

$$\Rightarrow AB + AB = AD + AD$$

$$\Rightarrow 2AB = 2AD$$

$$\Rightarrow AB = AD$$

AB and AD are adjacent sides of parallelogram which are equal hence parallelogram ABCD is a rhombus

Hence if all sides of a parallelogram touch a circle then that parallelogram is a rhombus

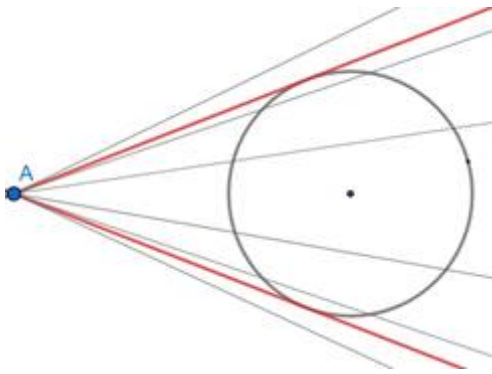
## 11. Question

From an external point..... tangents can be drawn to a circle.

### Answer

We can see that only 2 tangents can be drawn from an external point (here A) to a circle

Every other line either don't intersect the circle or intersects at two points hence there can be only 2 tangents from external point to a circle



### 12. Question

From an external point P two tangents PA and PB have been drawn. If PA = 6cm, then what is the length of PB.

### Answer

From P we have two tangents PA and PB

The tangents from an external point to a circle are equal

Hence  $PA = PB$

PA = 6 cm given

Hence PB is also 6 cm

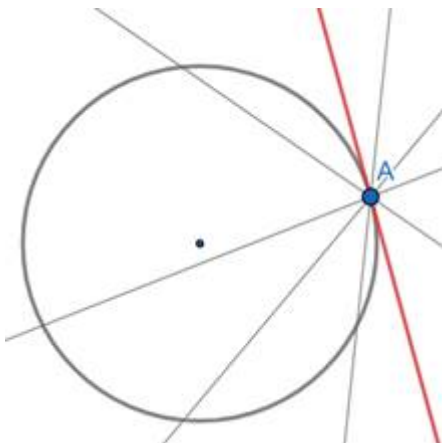
### 13. Question

How many tangents can be drawn at a point on a circle ?

### Answer

Only one tangent can be drawn from a point on the circle

Every other line will intersect the circle at two points which won't be a tangent

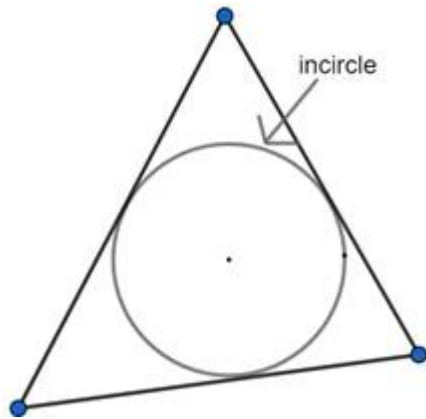


### 14. Question

What is the name of the circle touching the three sides of a triangle internally?

**Answer**

Touching three sides internally of a triangle which means the circle is inside the triangle hence it is called incircle of triangle



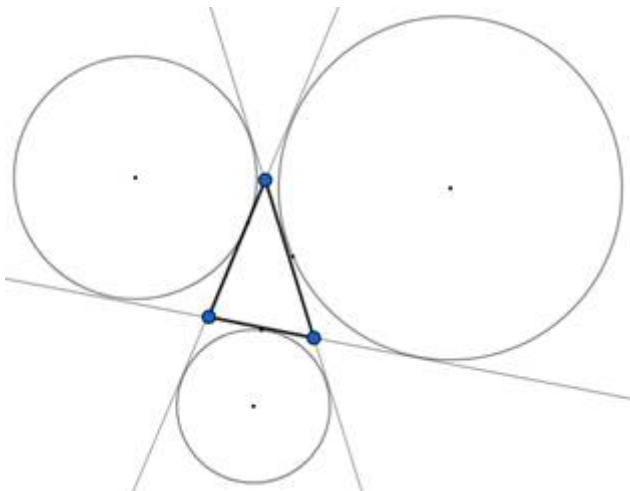
**15. Question**

How many excircles can be drawn to a triangle?

**Answer**

A circle is lying outside the triangle having one side of the triangle as tangent and also other two sides as tangent when they are extended.

There are three sides of a triangle hence there can be three excircles.



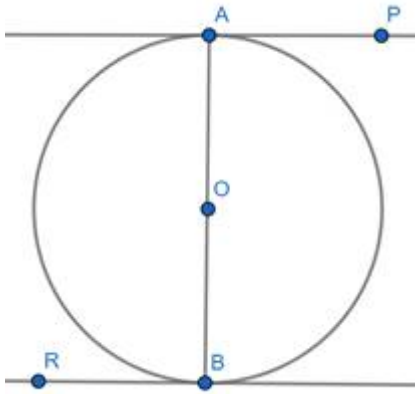
**16. Question**

What is the relation between the tangents at the extremities of a diameter of a circle?

**Answer**

The tangents at the extremities of diameter are parallel to each other

Proof:



Consider a circle with centre O and diameter AB having tangents as PA and RB as shown

$\angle OAP = 90^\circ$  ...radius is perpendicular to the tangent at the point of contact

$\angle RBO = 90^\circ$  ... radius is perpendicular to the tangent at the point of contact

$\Rightarrow \angle OAP = \angle RBO$

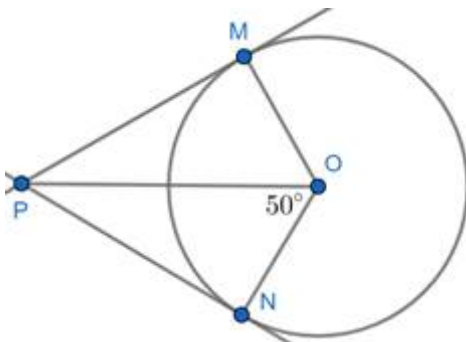
$\angle OAP$  and  $\angle RBO$  are alternate angles between lines PA and RB having transversal as AB

As alternate angles are equal lines, PA and RB are parallel

### 17. Question

O is the centre of a circle. From an external point, P two tangents PM and PN have been drawn which touch the circle at M and N. If  $\angle PON = 50^\circ$ , then find the value of  $\angle MPN$ .

**Answer**



From P we have two tangents PM and PN

We know that if we join point P and centre of circle O then the line PO divides the angle between tangents

$\Rightarrow \angle MPO = \angle NPO$  ...(a)

Consider  $\triangle PNO$

$$\Rightarrow \angle PON = 50^\circ \text{ ...given}$$

As radius  $ON$  is perpendicular to tangent  $PN$

$$\Rightarrow \angle PNO = 90^\circ$$

Now

$$\Rightarrow \angle PON + \angle PNO + \angle NPO = 180^\circ \text{ ...sum of angles of triangle}$$

$$\Rightarrow 50^\circ + 90^\circ + \angle NPO = 180^\circ$$

$$\Rightarrow 140^\circ + \angle NPO = 180^\circ$$

$$\Rightarrow \angle NPO = 40^\circ \text{ ...(i)}$$

From figure

$$\Rightarrow \angle MPN = \angle MPO + \angle NPO$$

Using (a)

$$\Rightarrow \angle MPN = \angle NPO + \angle NPO$$

$$\Rightarrow \angle MPN = 2\angle NPO$$

Using (i)

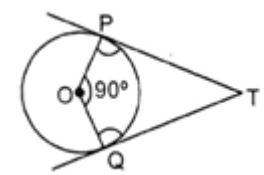
$$\Rightarrow \angle MPN = 2 \times 40^\circ$$

$$\Rightarrow \angle MPN = 80^\circ$$

Hence  $\angle MPN$  is  $80^\circ$

### 18 A. Question

In the given figure, two radii  $OP$  and  $OQ$  of a circle are mutually perpendicular. What is the degree measure of the angle between tangents drawn to the circle at  $P$  and  $Q$ ?



### Answer

a) we have to find  $\angle PTQ$  which is the angle between the tangents  $TP$  and  $TQ$

$\angle OPT = \angle OQT = 90^\circ$  ...radius is perpendicular to tangent at point of contact

$$\angle POQ = 90^\circ \text{ ...given}$$

Consider quadrilateral POQT

$$\Rightarrow \angle OPT + \angle OQT + \angle POQ + \angle PTQ = 360^\circ \text{ ...sum of angles of quadrilateral}$$

$$\Rightarrow 90^\circ + 90^\circ + 90^\circ + \angle PTQ = 360^\circ$$

$$\Rightarrow 270^\circ + \angle PTQ = 360^\circ$$

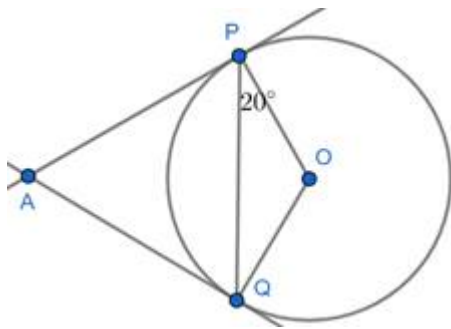
$$\Rightarrow \angle PTQ = 90^\circ$$

Hence angle between tangents is  $90^\circ$

### 18 B. Question

Centre of the circle is O and AP, and AQ is tangent of the circle. If  $\angle OPQ = 20^\circ$ , then what is the value of  $\angle PAQ$ ?

**Answer**



$$\angle OPQ = 20^\circ \text{ ...given}$$

Radius OP is perpendicular to tangent PA at point of contact A

$$\Rightarrow \angle APO = 90^\circ$$

$$\text{From figure } \angle APO = \angle APQ + \angle OPQ$$

$$\Rightarrow 90^\circ = \angle APQ + 20^\circ$$

$$\Rightarrow \angle APQ = 70^\circ \text{ ...(a)}$$

Consider  $\triangle APQ$

AP and AQ are tangents to circle from A

Tangents from a point to a circle are equal

$$\Rightarrow AP = AQ \text{ hence } \triangle APQ \text{ is a isosceles triangle}$$

$$\Rightarrow \angle APQ = \angle AQP$$

Using (a)

$$\Rightarrow \angle AQP = 70^\circ$$

Now

$$\Rightarrow \angle APQ + \angle AQP + \angle PAQ = 180^\circ \text{ ...sum of angles of triangle}$$

$$\Rightarrow 70^\circ + 70^\circ + \angle PAQ = 180^\circ$$

$$\Rightarrow 140^\circ + \angle PAQ = 180^\circ$$

$$\Rightarrow \angle PAQ = 40^\circ$$

Hence  $\angle PAQ$  is  $40^\circ$