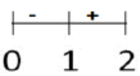
DEFINITE INTEGRALS

Q.1)	$I = \int_0^{\pi} \log(1 + \cos x)\, dx$
Sol.1)	$I = \int_0^{\pi} \log(1+\cos x)\, dx$ .....(1) $I = \int_0^{\pi} \log[1+\cos(\pi - x)]\, dx$ ......(P-IV) $I = \int_0^{\pi} log\,(1-\cos x)\, dx$ ....(2) $(1) + (2)$ $2I = \int_0^{\pi} \log\big((1+\cos x)(1-\cos x)\big)\, dx$ $2I = \int_0^{\pi} \log(1-\cos^2 x)\, dx$ $2I = \int_0^{\pi} \log(\sin^2 x)\, dx$ $2I = 2\int_0^{\pi} \log(\sin x)\, dx$ .....$[\log m^n = n\log m]$ $I = \int_0^{\pi} log\,(\sin x)\, dx$ $I = 2\int_0^{\frac{\pi}{2}} \log(\sin x)\, dx$ ......(P-VI) $\frac{I}{2} = \int_0^{\frac{\pi}{2}} \log(\sin x)\, dx$ ....(3) $\frac{I}{2} = \int_0^{\frac{\pi}{2}} \log\left(\sin\left(\frac{\pi}{2} - x\right)\right) dx$ .....(P-IV) $\frac{I}{2} = \int_0^{\frac{\pi}{2}} \log(\cos x)\, dx$ ....(4) $(3) + (4)$ $I = \int_0^{\frac{\pi}{2}} \log(\sin x . \cos x)\, dx$ $I = \int_0^{\frac{\pi}{2}} log\left(\frac{\sin(2x)}{2}\right) dx$ $I = \int_0^{\frac{\pi}{2}} log\big(\sin(2x)\big) - \log 2\, dx$ $I = \int_0^{\frac{\pi}{2}} log\,(\sin(2x))\, dx - \int_0^{\frac{\pi}{2}} \log 2\, dx$ $I = \int_0^{\frac{\pi}{2}} log\,(\sin(2x))\, dx - log_2(x)_0^{\frac{\pi}{2}}$ $I = \int_0^{\frac{\pi}{2}} log\,(\sin(2x))\, dx - \frac{\pi}{2}\log 2$ $I = I_1 - \frac{\pi}{2}\log 2$ ....(5) Where $I_1 = \int_0^{\frac{\pi}{2}} \log\big(\sin(2x)\big)\, dx$ $Put\ 2x = t$ $when\ x = 0\ ;\ t = 0$ $dx = \frac{dt}{2}$ $x = \frac{\pi}{2}\ ;\ t = \pi$ $\therefore\ I_1 = \frac{1}{2}\int_0^{\pi} \log(\sin t)\, dx$

	$I_1 = \frac{1}{2} \times 2 \int_0^{\frac{\pi}{2}} \log(sin\ t)\, dt$ ....(P-VI) $I_1 = \int_0^{\frac{\pi}{2}} \log(sin\ t)\, dt$ $I_1 = \int_0^{\frac{\pi}{2}} \log(sin\ x)\, dx$ ....(P-I) $I_1 = \frac{\mathrm{I}}{2}$ ....$\{from\ eq. 3\}$ $\therefore eq. (5)$ becomes $I\ = \frac{\mathrm{I}}{2} - \frac{\pi}{2}\log 2$ $\Rightarrow I\ = \frac{\mathrm{I}}{2} = \frac{-\pi}{2}\log 2$ $\Rightarrow \frac{\mathrm{I}}{2} = \frac{-\pi}{2}\log 2$ $I\ = -\pi \log 2$ ans.
Q.2)	$I\ = \int_0^{\pi} \frac{x\ tan\ x}{sec\ x\ .\ cosec\ x} dx$
Sol.2)	$I\ = \int_0^{\pi} \frac{\frac{x\ sin\ x}{cos\ x}}{\frac{1}{cos\ x} \cdot \frac{1}{sin\ x}} dx$ $I\ = \int_0^{\pi} x\ sin^2\ dx$ ....(1) $I\ = \int_0^{\pi} (\pi - x) sin^2 (\pi - x)\, dx$ ....(P-IV) $I\ = \int_0^{\pi} (\pi - x) sin^2 x\, dx$ ....(2) $(1)\ + (2)$ $2\, I\ = \int_0^{\pi} x\ sin^2 x + \pi\ sin^2 x - x\ sin^2 x\, dx$ $2\, I\ = \pi \int_0^{\pi} sin^2 x\, dx$ $2\, I\ = 2\pi \int_0^{\frac{\pi}{2}} sin^2\ dx$ ....(P-VI) $I\ = \pi \int_0^{\frac{\pi}{2}} sin^2 x\, dx$ $I\ = \pi \int_0^{\frac{\pi}{2}} \frac{1 - cos(2x)}{2} dx$ $I\ = \frac{\pi}{2} \left[x - \frac{sin(2x)}{2} \right]_0^{\frac{\pi}{2}}$ $I\ = \frac{\pi}{2} \left[\left(\frac{\pi}{2} - \frac{sin\ \pi}{2} \right) - (0 - 0) \right]$ $I\ = \frac{\pi}{2} \left[\frac{\pi}{2} - 0 \right]$ $I\ = \frac{\pi^2}{4}$ $ans.$
Q.3)	$\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x} + \sqrt{x}} dx$
Sol.3)	$I\ = \int_1^2 \frac{\sqrt{x}}{\sqrt{3-x} + \sqrt{x}} dx$ $\dots\dots\dots(1)$

	$I = \int_1^2 \frac{\sqrt{1+2-x}}{\sqrt{3-(1+2-x)}+\sqrt{(1+2-x)}} dx$ ……… $\left[\int_a^b f(x)dx = \int_a^b f(a+b-x)\, dx\right]$ $I = \int_1^2 \frac{\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}} dx$ ………(2) $(1) + (2)$ $2I = \int_1^2 \frac{\sqrt{x}+\sqrt{3-x}}{\sqrt{3-x}+\sqrt{x}} dx$ $2I = \int_1^2 1\, .dx$ $2I = (x)_1^2$ $2I = 2-1$ $\Rightarrow I = \frac{1}{2}$ ans.
Q.4)	$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\cot x}} dx$
Sol.4)	$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\cot x}} dx$ ……..(1) $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\cot\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}} dx$ …..(P-V) (above) $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\tan x}} dx$ $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\frac{1}{\sqrt{\cot x}}} dx$ $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+1} dx$ ………(2) $(1) + (2)$ $2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}+1}{\sqrt{\cot x}+1} dx$ $2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1\, .dx$ $2I = (x)_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $2I = \frac{\pi}{3}-\frac{\pi}{6}$ $2I = \frac{\pi}{6}$ $I = \frac{\pi}{12}$ ans.
Q.5)	$I = \int_0^1 \frac{\lvert 5x-3\rvert}{(3/5)} dx$
Sol.5)	$I = \int_0^1 \frac{\lvert 5x-3\rvert}{(3/5)} dx$

	$I \;=\; -\int_0^{\frac{3}{5}}(5x-3)\,dx + \int_{\frac{3}{5}}^{1}(5x-3)\,dx$ $I \;=\; -\left[\frac{5x^2}{2} - 3x\right]_0^{\frac{3}{5}} + \left[\frac{5x^2}{2} - 3x\right]_{\frac{3}{5}}^{1}$ $I \;=\; -\left[\frac{5}{2}.\frac{9}{25} - 3\,.\frac{3}{5}\right] + \left[\left(\frac{5}{2} - 3\right) - \left(\frac{5}{2}\,.\frac{9}{25} - 3\,.\frac{3}{5}\right)\right]$ $I \;=\; -\left(\frac{9}{10} - \frac{9}{5}\right) + \left[-\frac{1}{2} - \frac{9}{10} + \frac{9}{5}\right]$ $I \;=\; -\frac{9}{10} + \frac{9}{5} - \frac{1}{2} - \frac{9}{10} + \frac{9}{5}$ $I \;=\; \frac{13}{10} \qquad ans.$.
Q.6)	$I \;=\; \int_{-5}^{5} \begin{matrix}\lvert x-2\rvert \\ (2)\end{matrix}\, dx$
Sol.6)	$I \;=\; \int_{-5}^{5} \begin{matrix}\lvert x-2\rvert \\ (2)\end{matrix}\, dx$ $I \;=\; -\int_5^2(x-2)\,dx + \int_2^5(x-2)\,dx$ $I \;=\; -\left[\frac{x^2}{2} - 2x\right]_{-5}^{2} + \left[\frac{x^2}{2} - 2x\right]_2^5$ $I \;=\; -\left[(2-4) - \left(\frac{25}{2} + 10\right)\right] + \left[\left(\frac{25}{2} - 10\right) - (2-4)\right]$ $I \;=\; -\left[-2 - \frac{45}{2}\right] + \left[\frac{5}{2} + 2\right]$ $I \;=\; 2 + \frac{45}{2} + \frac{5}{2} + 2 = 29 \qquad ans\,.$
Q.7)	$I \;=\; \int_1^5 \begin{matrix}\lvert x-6\rvert \\ (6)\end{matrix}\, dx$
Sol.7)	$I \;=\; \int_1^5 \begin{matrix}\lvert x-6\rvert \\ (6)\end{matrix}\, dx$ $I \;=\; -\int_1^5(x-6)\,dx$ $I \;=\; -\left[\frac{x^2}{2} - 6x\right]_1^5$ $I \;=\; -\left[\left(\frac{25}{2} - 30\right) - \left(\frac{1}{2} - 6\right)\right]$ $I \;=\; -\left[\frac{-35}{2} + \frac{11}{2}\right] = 12 \qquad ans.$
Q.8)	$I \;=\; \int_{-1}^{1} \begin{matrix}e\,\lvert x\rvert \\ (0)\end{matrix}\, dx$
Sol.8)	$I \;=\; \int_{-1}^{0} e^{-x}\,dx + \int_0^1 e^{x}\,dx$ $I \;=\; \left[\frac{e^{-x}}{-1}\right]_{-1}^{0} + [e^{x}]_0^1$

	$I = \left[\frac{1}{-1} - \frac{e^1}{-1}\right] + [e^1 - e^0]$ $I = [-1 + e] + [e - 1]$ $I = 2e - 2 \quad ans.$
Q.9)	$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin\,\lvert x\rvert + cos\lvert x\rvert\,dx$
Sol.9)	$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin\,\lvert x\rvert + cos\lvert x\rvert\,dx$ $I = \int_{-\frac{\pi}{2}}^{0} sin(-x) + cos(-x)\,dx + \int_{0}^{\frac{\pi}{2}} sin(x) + cos(x)\,dx$ $I = \int_{-\pi/2}^{0} -sin\,x + cos\,x\,dx + \int_{0}^{\frac{\pi}{2}} sin\,x + cos\,x\,dx$ $I = [cos\,x + sin\,x]_{-\frac{\pi}{2}}^{0} + [-cos\,x + sin\,x]_{0}^{\frac{\pi}{2}}$ $I = \left[(cos\,0 + sin\,0) - \left(cos\left(-\frac{\pi}{2}\right)\right) + sin\left(-\frac{\pi}{2}\right)\right] + \left[\left(-cos\frac{\pi}{2} + sin\frac{\pi}{2}\right) - (-cos\,0 + sin\,0)\right]$ $I = [(1 + 0) - (0 - 1)] + [(0 + 1) — (-1 + 0)]$ $I = 2 + 2 = 4$ ans.
Q.10)	$I = \int_0^2 \lvert x^2 + 2x - 3\rvert\,dx$
Sol.10)	$I = \int_0^2 \lvert\underset{(-3)}{(x + 3)}\underset{(1)}{(x - 1)}\rvert\,dx$  $\therefore\ I = -\int_0^1 (x^2 + 2x - 3)\,dx + \int_1^2 (x^2 + 2x - 3)$ $I = -\left[\frac{x^2}{3} + x^2 - 3x\right]_0^1 + \left[\frac{x^3}{3} + x^2 - 3x\right]_1^2$ $I = -\left[\left(\frac{1}{3} + 1 - 3\right) - (0)\right] + \left[\left(\frac{8}{3} + 4 - 6\right) - \left(\frac{1}{3} + 1 - 3\right)\right]$ $I = -\left[\frac{-5}{3}\right] + \left[\frac{2}{3} + \frac{5}{3}\right]$ $I = \frac{12}{3} = 4 \quad ans.$