DEFINITE INTEGRALS

Q.1)	$I = \int_{-1}^{2} \lvert x^3 - x \rvert \, dx$
Sol.1)	$I = \int_{-1}^{2} \underset{(0)\ (-1)\ (1)}{\lvert x\,(x+1)(x-1) \rvert} \, dx$ $\therefore\ I = +\int_{-1}^{0}(x^3 - x)\,dx - \int_{0}^{1}(x^3 - x)\,dx + \int_{1}^{2}(x^3 - x)\,dx$ $I = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} - \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$ $I = \left[(0) - \left(\frac{1}{4} - \frac{1}{2}\right)\right] - \left[\left(\frac{1}{4} - \frac{1}{2}\right) - (0)\right] + \left[(4-2) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$ $I = \frac{11}{4}$ $ans.$
Q.2)	$I = \int_{1}^{4} \underset{(3)}{\lvert x-3 \rvert} + \underset{(1)}{\lvert x-1 \rvert} + \underset{(2)}{\lvert x-2 \rvert} \, dx$
Sol.2)	$I = \int_{1}^{4} \underset{(1)}{\lvert x-1 \rvert} + \underset{(2)}{\lvert x-2 \rvert} + \underset{(3)}{\lvert x-3 \rvert} \, dx$ $I = \int_{1}^{2}(x-1) - (x-2) - (x-3)\,dx + \int_{2}^{3}(x-1) + (x-2) - (x-3)\,dx +$ $\int_{3}^{4}(x-1) + (x-2) + (x-3)\,dx$ $I = \int_{1}^{2}(-x+4)\,dx + \int_{2}^{3} x\,dx + \int_{3}^{4}(3x-6)\,dx$ $I = \left[\frac{-x^2}{2} + 4x\right]_{1}^{2} + \left[\frac{x^2}{2}\right]_{2}^{3} + \left[\frac{3x^2}{2} - 6x\right]_{3}^{4}$ $I = \left[(-2+8) - \left(\frac{-1}{2} + 4\right)\right] + \left[\frac{9}{2} - 2\right] + \left[(24-24) - \left(\frac{27}{2} - 18\right)\right]$ $I = 6 - \frac{7}{2} + \frac{5}{2} + \frac{9}{2}$ $I = \frac{19}{2}$ $ans.$
Q.3)	$I = \int_{0}^{2\pi} \lvert \sin x \rvert \, dx$
Sol.3)	$I = \int_{0}^{\pi} \sin x\,dx - \int_{\pi}^{2\pi} \sin x\,dx$ $I = [-\cos x]_{0}^{\pi} - [-\cos x]_{\pi}^{2\pi}$ $I = -[\cos x]_{0}^{\pi} + [\cos x]_{\pi}^{2\pi}$ $I = -[\cos \pi - \cos 0] + [\cos 2\pi - \cos \pi]$ $I = -[-1-1] + [0-(-1)]$ $I = 2 + 2$ $I = 4$ $ans.$
Q.4)	$I = \int_{0}^{\frac{\pi}{2}} \lvert \cos(2x) \rvert \, dx$
Sol.4)	$I = \int_{0}^{\frac{\pi}{2}} \lvert \cos(2x) \rvert \, dx$

	Critical $point \quad 2x = \frac{\pi}{2} \qquad x = \frac{\pi}{4}$ $\therefore \quad I = \int_0^{\frac{\pi}{4}} cos(2x)\,dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} cos(2x)\,dx$ $\left\{\begin{matrix} \because \ 0 < x < \pi/4 & \because \ \pi/4 < x < \pi/2 \\ 0 < 2x < \pi/2 & \pi/2 < 2x < \pi \end{matrix}\right\}$ $1st\ quad\ (+ve) \quad 2nd\ quad\ (-ve)$ $I = \frac{1}{2}[sin(2x)]_0^{\pi/4} - \frac{1}{2}[sin(2x)]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $I = \frac{1}{2}\left[\left(sin\frac{\pi}{2} - sin\,0\right)\right] - \frac{1}{2}\left[sin\,\pi - sin\frac{\pi}{2}\right]$ $I = \frac{1}{2}[1-0] - \frac{1}{2}[0-1]$ $I = \frac{1}{2} + \frac{1}{2}$ $I = 1 \quad ans.$
Q.5)	$I = \int_0^3 [x]\,dx \ \ (greatest\ integer\ function)$
Sol.5)	$I = \int_0^1 [x]\,dx + \int_1^2 [x]\,dx + \int_2^3 [x]\,dx$ $I = \int_0^1 (0)\,dx + \int_1^2 (1)\,dx + \int_2^3 (2)\,dx$ $I = 0 + (x)_1^2 + (2x)_2^3$ $I = (2-1) + (6-4)$ $I = 3 \quad ans\ .$
Q.6)	$I = \int_0^2 [x^2]\,dx$
Sol.6)	$I = \int_0^2 [x^2]\,dx$ $I = \int_0^1 [x^2]\,dx + \int_1^{\sqrt{2}} [x^2]\,dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2] + \int_{\sqrt{3}}^2 [x^2]\,dx$ $I = \int_0^1 (0)\,dx + \int_1^{\sqrt{2}} (1)\,dx + \int_{\sqrt{2}}^{\sqrt{3}} (2)\,dx + \int_{\sqrt{3}}^2 (3)\,dx$ $I = 0 + (x)_1^{\sqrt{2}} + (2x)_{\sqrt{2}}^{\sqrt{3}} + (3x)_{\sqrt{3}}^2$ $I = (\sqrt{2}-1) + (2\sqrt{3} - 2\sqrt{2}) + (6 - 3\sqrt{3})$ $I = 5 - \sqrt{2} - \sqrt{3} \quad Ans$ ...
Q.7)	$I = \int_0^{1.5} [x^2]\,dx$
Sol.7)	$I = \int_0^{1.5} [x^2]\,dx$ $I = \int_0^1 [x^2]\,dx + \int_1^{\sqrt{2}} [x^2]\,dx + \int_{\sqrt{2}}^{1.5} [x^2]\,dx$ $I = \int_0^1 0\,dx + \int_1^{\sqrt{2}} (1)\,dx + \int_{\sqrt{2}}^{1.5} (2)\,dx$ $I = 0 + (x)_1^{\sqrt{2}} + (2x)_{\sqrt{2}}^{1.5}$

	$I = (\sqrt{2}-1)+(3-2\sqrt{2})$ $I = 2-\sqrt{2} \quad ans.$
Q.8)	$I = \int_{-1}^{\frac{3}{2}} \lvert x\, sin(\pi\, x)\rvert\, dx$
Sol.8)	Case – I $-1 < x < 0 \quad ; \quad -\pi < \pi\, x < 0$ $\lvert x\, sin(\pi\, x)\rvert = x\, sin(\pi\, x)$ Case$-II$ $1 < x < 1 \quad ; \quad 0 < \pi\, x < \pi$ $\lvert x\, sin\,(\pi\, x)\rvert = x\, sin(\pi\, x)$ Case$-III$ $1 < x < \frac{3}{2} \quad ; \quad \pi < \pi\, x < \frac{3\pi}{2}$ $\lvert x\, sin(\pi\, x)\rvert = -x\, sin(\pi\, x)$ $\therefore \quad I = \int_{-1}^{1} x\, sin(\pi\, x)\, dx - \int_{1}^{\frac{3}{2}} x\, sin(\pi\, x)\, dx$ let $I_1 = \int x\, sin\,(\pi\, x)\, dx$ $I_1 = \left[\frac{-x\, cos(\pi\, x)}{\pi} - \int (1)\frac{(-cos\, \pi\, x)}{\pi} dx\right]$ $I1 = \frac{-x\, cos(\pi\, x)}{\pi} + \frac{1}{\pi}\int cos(\pi\, x)\, dx$ $I_1 = \frac{-x\, cos(\pi\, x)}{\pi} + \frac{sin(\pi\, x)}{\pi^2}$ $\therefore\ I = \left[\frac{-x\, cos(\pi\, x)}{\pi} + \frac{sin(\pi\, x)}{\pi^2}\right]_{-1}^{1} - \left[\frac{-x\, cos\,(\pi\, x)}{\pi} + \frac{sin(\pi\, x)}{\pi^2}\right]_{1}^{3/2}$ $I = \left[\left(\frac{-cos\,(\pi)}{\pi} + \frac{sin\,(\pi)}{\pi^2}\right) - \left(\frac{cos(-\pi)}{\pi} + \frac{sin(-\pi)}{\pi^2}\right)\right] - \left[\left(\frac{-3}{2}\,\frac{cos\left(\frac{3\pi}{2}\right)}{\pi} + \frac{sin\left(\frac{3\pi}{2}\right)}{\pi^2}\right) - \left(\frac{-cos(\pi)}{\pi} + \frac{sin(\pi)}{\pi^2}\right)\right]$ $I = \left[\left(\frac{1}{\pi}+0\right) - \left(\frac{-1}{\pi}+0\right)\right] - \left[\left(0-\frac{1}{\pi^2}\right) - \left(\frac{+1}{\pi}+0\right)\right] \quad \ldots.. \begin{Bmatrix} cos(\pi) = -1 \quad ; \quad cos(-\pi) = -1 \\ sin(-\pi) = 0 \quad ; \quad sin\left(\frac{3\pi}{2}\right) = -1 \\ cos(3\pi/2) = 0 \end{Bmatrix}$ $I = \frac{1}{\pi} + \frac{1}{\pi} + \frac{1}{\pi^2} + \frac{1}{\pi}$ $\therefore \quad I = \frac{3}{\pi} + \frac{1}{\pi^2}$ ans.
Q.9)	$I = \int_{1/e}^{e} \lvert \log x \rvert\, dx$
Sol.9)	$I = \int_{1/e}^{e} \lvert \log x \rvert\, dx$ Critical $x = 1$ since $\log 1 = 0$ $\therefore\ I = -\int_{1/e}^{1} \log x\, dx + \int_{1}^{e} \log x\, dx$ let $I_1 = \int \log x\, dx$ $= \int \log x\,.1\, dx$

	$= \int \log x \,.\, x - \int \frac{1}{x} . x \, dx$ $I_1 = x \log x - x$ $\therefore \; I = -[x \log x - x]_{1/e}^{1} + [x \log x - x]_1^e$ $= -\left[(\log 1 - 1) - \left(\frac{1}{e}\log\left(\frac{1}{e}\right) - \frac{1}{e}\right)\right] + [(e \log e - e) - (\log 1 - 1)]$ $= -\left[(0-1) - \left(\frac{-1}{e} - \frac{1}{e}\right)\right] + [(e-e) - (0-1)]$ ......$\left\{\because \; \log = 1 \;,\; \log\left(\frac{1}{e}\right) = -1\right\}$ $= 1 - \frac{2}{e} + 1$ $= 2 - \frac{2}{e}$ ans.
Q.10)	$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^7 \, x \, dx$
Sol.10)	$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^7 \, x \, dx$ Here $f(x) = sin^7 x$ $f(-x) = sin^7(-x) = (-\sin x)^7 = -f(x)$ $\therefore \; f(-x) = -f(x)$ $f(x)$ → odd function $\therefore I = 0$ Ans. ......$\left\{\int_{-a}^{a} f(x)dx = 0 \;\; when \, f(x) is \, odd \, function\right\}$