Determinants
Class 12th

Q.1)	Show $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$
Sol.1)	$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ $C_3 \rightarrow C_3 + C_2$ = $\begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$ taking (a + b + c) common from C_3 = $(a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$ = $(a+b+c) \times 0 = 0$ .....{ ∵ C_1 & C_3 are identical}
Q.2)	Show that $\begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix} = 0$
Sol.2)	let $\Delta = \begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix}$ $c_1 \rightarrow c_1 + c_2 + c_3$ = $\begin{vmatrix} 0 & c-a & a-b \\ 0 & a-b & b-c \\ 0 & b-c & c-a \end{vmatrix}$ = 0 ....{ all elements of C_1 are z easily}
Q.3)	Show that $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+S) \\ \sin\beta & \cos\beta & \sin(\beta+S) \\ \sin\gamma & \cos\gamma & \sin(\gamma+S) \end{vmatrix} = 0$
Sol.3)	$\Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & \sin\alpha.\cos S + \cos\alpha.\sin S \\ \sin\beta & \cos\beta & \sin\beta.\cos S + \cos\beta.\sin S \\ \sin\gamma & \cos\gamma & \sin\gamma.\cos S + \cos\gamma.\sin S \end{vmatrix}$ Applying sum property in C_3 = $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin\alpha.\cos S \\ \sin\beta & \cos\beta & \sin\beta.\cos S \\ \sin\gamma & \cos\gamma & \sin\gamma.\cos S \end{vmatrix} + \begin{vmatrix} \sin\alpha & \cos\alpha & \cos\alpha.\sin S \\ \sin\beta & \cos\beta & \cos\beta.\sin S \\ \sin\gamma & \cos\gamma & \cos\gamma.\sin S \end{vmatrix}$ = $\cos S \begin{vmatrix} \sin\alpha & \cos\alpha & \sin\alpha \\ \sin\beta & \cos\beta & \sin\beta \\ \sin\gamma & \cos\gamma & \sin\gamma \end{vmatrix} + \sin S \begin{vmatrix} \sin\alpha & \cos\alpha & \cos\alpha \\ \sin\beta & \cos\beta & \cos\beta \\ \sin\gamma & \cos\gamma & \cos\gamma \end{vmatrix}$ = $\cos S(0) + \sin S(0) = 0$ ans.

Q.4)	If a, b, c are in A.P find value of $\begin{vmatrix} 2y+45y+78y+a \\ 3y+56y+89y+b \\ 4y+67y+910y+c \end{vmatrix}$
Sol.4)	let $\Delta=\begin{vmatrix} 2y+45y+78y+a \\ 3y+56y+89y+b \\ 4y+67y+910y+c \end{vmatrix}$ given a, b, c are in A.P $a+c=2\text{b}$ $\therefore$ $R_1 \rightarrow R_1 + R_3$ $=\begin{vmatrix} 6y+1012y+1618y+a+c \\ 3y+56y+89y+b \\ 4y+67y+910y+c \end{vmatrix}$ $=\begin{vmatrix} 6y+1012y+1618y+2\text{b} \\ 3y+56y+89y+b \\ 4y+67y+910y+c \end{vmatrix}$ .....{$\because$ a + c = 2b} taking 2 common from R_1 $= 2\begin{vmatrix} 3y+56y+89y+b \\ 3y+56y+89y+b \\ 4y+67y+910y+c \end{vmatrix}$ clearly R_1 and R_2 are identical $\therefore 2\times 0=0$ ans.
Q.5)	Show that $\begin{vmatrix} (a^x+a^{-x})^2(a^x-a^{-x})^2 1 \\ (a^y+a^{-y})^2(a^y-a^{-y})^2 1 \\ (a^z+a^{-z})^2(a^z-a^{-z})^2 1 \end{vmatrix}=0$
Sol.5)	$c_1 \rightarrow c_1 - c_2$ $=\begin{vmatrix} (a^x+a^{-x})^2-(a^x-a^{-x})^2(a^x-a^{-x})^2 1 \\ (a^y+a^{-y})^2-(a^y-a^{-y})^2(a^y-a^{-y})^2 1 \\ (a^z+a^{-z})^2-(a^z-a^{-z})^2(a^z-a^{-z})^2 1 \end{vmatrix}$ $=\begin{vmatrix} 4(a^x-a^{-x})^2 1 \\ 4(a^y-a^{-y})^2 1 \\ 4(a^z-a^{-z})^2 1 \end{vmatrix}$ .....$\{(a+b)^2-(a-b)^2=4\text{abhere4ab}=4\text{a}^x.a^{-x}-$ $4\}$ $= 4\begin{vmatrix} 1(a^x-a^{-x})^2 1 \\ 1(a^y-a^{-y})^2 1 \\ 1(a^z-a^{-z})^2 1 \end{vmatrix}$ $= 4\times 0=0$ .....$\{C_1 \& C_3 are identical\}$
Q.6)	Show that $\begin{vmatrix} 4115 \\ 7979 \\ 2953 \end{vmatrix}=0$
Sol.6)	let $\Delta=\begin{vmatrix} 4115 \\ 7979 \\ 2953 \end{vmatrix}$

	$c_2 \to c_2 + 8c_3$ = $\begin{vmatrix} 41415 \\ 79799 \\ 29293 \end{vmatrix} = 0$ .....$\{C_1 \& C_3 are identical\}$
Q.7)	Show $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & a+b \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$
Sol.7)	$R_1 \to aR_1, R_2 \to bR_2 \text{and} R_3 \to cR_3$ = $\frac{1}{abc}\begin{vmatrix} ab^2c^2 & abc & ab+ac \\ bc^2a^2 & abc & bc+ab \\ ca^2b^2 & abc & ca+bc \end{vmatrix}$ taking abc common from $C_1$and C_2 = $\frac{1}{abc}.(abc)(abc)\begin{vmatrix} bc & 1 & ab+ac \\ ca & 1 & bc+ab \\ ab & 1 & ca+bc \end{vmatrix}$ $c_3 \to c_3 + c_1$ = $abc\begin{vmatrix} bc 1 ab+bc+ca \\ ca 1 ab+bc+ca \\ ab 1 ab+bc+ca \end{vmatrix}$ taking $(ab+bc+ca)$common from C_3 = $abc(ab+bc+ca)\begin{vmatrix} bc & 1 & 1 \\ ca & 1 & 1 \\ ab & 1 & 1 \end{vmatrix}$ = $abc(ab+bc+ca)(0) = 0$ .... $\{C_1 \& C_3 are identical\}$
Q.8)	Show that $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$
Sol.8)	let $\Delta = \begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$ Applying sum property inC_3 = $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ $R_1 \to aR_1, R_2 \to bR_2, R_3 \to cR_3$ = $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^3 \end{vmatrix} - \frac{1}{abc}\begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$ taking abc common fromC_3 in 2nd Det. = $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \frac{abc}{abc}\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$ $c_2 \leftrightarrow c_3$

	= $\begin{vmatrix}1 & a & a^2\\1 & b & b^2\\1 & c & c^2\end{vmatrix} + \begin{vmatrix}a & 1 & a^2\\b & 1 & b^2\\c & 1 & b^2\end{vmatrix}$ again $c_1 \leftrightarrow c_2$ = $\begin{vmatrix}1 & a & a^2\\1 & b & b^2\\1 & c & c^2\end{vmatrix} - \begin{vmatrix}1 & a & a^2\\1 & b & b^2\\1 & c & c^2\end{vmatrix}$ = 0 ans.
Q.9)	If a , b , c are the P^{th}, q^{th}andr^{th}terms of G.P then show that $\begin{vmatrix}\log a & p & 1\\\log b & q & 1\\\log c & r & 1\end{vmatrix} = 0$
Sol.9)	We know n^{th} term of G.P : $a_n = ar^{n-1}$ here let $A \rightarrow I^{st}$term and R common ratio $\therefore a_p = a = AR^{p-1}; a_q = b = AR^{q-1}; a_r = c = AR^{r-1}$ taking log on both sides $\log a = \log(AR^{p-1}); \log b = \log(AR^{q-1}); \log c = \log(AR^{r-1})$ $\Rightarrow \log a = \log A + (p-1)\log R$ $\log b = \log A + (q-1)\log R$ $\log c = \log A + (r-1)\log R$ Now let$\Delta = \begin{vmatrix}\log a & p & 1\\\log b & q & 1\\\log c & r & 1\end{vmatrix}$ putting values of $\log a, \log b$and $\log c$ = $\begin{vmatrix}\log A + (p-1)\log R & p & 1\\\log A + (q-1)\log R & q & 1\\\log A + (r-1)\log R & r & 1\end{vmatrix}$ Applying sum property in c_1 = $\begin{vmatrix}\log A & p & 1\\\log A & q & 1\\\log A & r & 1\end{vmatrix} + \begin{vmatrix}(p-1)\log R & p & 1\\(q-1)\log R & q & 1\\(r-1)\log R & r & 1\end{vmatrix}$ = $\log A\begin{vmatrix}1 & p & 1\\1 & q & 1\\1 & r & 1\end{vmatrix} + \log R\begin{vmatrix}p-1 & p & 1\\q-1 & q & 1\\r-1 & r & 1\end{vmatrix}$ $c_1 \rightarrow c_1 + c_3$ = $\log A\begin{vmatrix}1 & p & 1\\1 & q & 1\\1 & r & 1\end{vmatrix} + \log R\begin{vmatrix}p & p & 1\\q & q & 1\\r & r & 1\end{vmatrix}$ = $\log A \times (0) + \log R \times (0)$ = $0 + 0 = 0$ ans.
Q.10)	Show $\begin{vmatrix}a & b & c\\x & y & z\\p & q & r\end{vmatrix} = \begin{vmatrix}y & b & q\\x & a & p\\z & c & r\end{vmatrix}$

Sol.10)	let $\Delta = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$ $= \begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix}$ .....$\{\because \mid A \mid = \mid A' \mid\}$ $c_1 \leftrightarrow c_2$ $= -\begin{vmatrix} x & a & p \\ y & b & q \\ z & c & r \end{vmatrix}$ $R_2 \leftrightarrow R_1$ $= (-)(-)\begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$ $= \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$ = RHS Proved