

Determinants

Class 12th

Q.1)	Show that $\begin{vmatrix} 1 & 1 & 1 \\ mc_1 & m+1c_1 & m+2c_1 \\ mc_2 & m+1c_2 & m+2c_2 \end{vmatrix} = 1$
Sol.1)	We have $\begin{vmatrix} 1 & 1 & 1 \\ mc_1 & m+1c_1 & m+2c_1 \\ mc_2 & m+1c_2 & m+2c_2 \end{vmatrix}$ $= \begin{vmatrix} 1 & 1 & 1 \\ m & m+1 & m+2 \\ \frac{m(m-1)}{2} & \frac{(m+1)m}{m} & \frac{(m+2)(m+1)}{2} \end{vmatrix} \dots \left\{ \begin{array}{l} nc_1 = n \\ nc_2 = \frac{n(n-1)}{2} \end{array} \right\}$ taking $\left(\frac{1}{2}\right)$ common from R_3 $= \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ m & m+1 & m+2 \\ m^2 - m & m^2 + m & m^2 + 3m + 2 \end{vmatrix}$ $c_2 \rightarrow c_2 - c_1$ and $c_3 \rightarrow c_3 - c_2$ $= \frac{1}{2} \begin{vmatrix} 1 & 0 & 0 \\ m & 1 & 2 \\ m^2 - m & 2m & 4m + 2 \end{vmatrix}$ expanding along R_1 $= \frac{1}{2} [4m + 2 - 4m]$ $= \frac{2}{2} = 1 = \text{RHS} \quad \text{ans.}$
Q.2)	Show $\begin{vmatrix} (b+c)^2 & ba & ca \\ ab & (c+a)^2 & cb \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$
Sol.2)	We have $\begin{vmatrix} (b+c)^2 & ba & ca \\ ab & (c+a)^2 & cb \\ ac & bc & (a+b)^2 \end{vmatrix}$ $= R_1 \rightarrow aR_1, R_2 \rightarrow bR_2, R_3 \rightarrow cR_3$ $= \frac{1}{abc} \begin{vmatrix} a(b+c)^2 & ba^2 & ca^2 \\ ab^2 & b(c+a)^2 & cb^2 \\ ac^2 & bc^2 & c(a+b)^2 \end{vmatrix}$ taking a, b, c common from c_1, c_2 and c_3 resp. $= \frac{1}{abc} \begin{vmatrix} a(b+c)^2 & ba^2 & ca^2 \\ ab^2 & b(c+a)^2 & cb^2 \\ ac^2 & bc^2 & c(a+b)^2 \end{vmatrix}$ $c_1 \rightarrow c_1 - c_3$ and $c_2 \rightarrow c_2 - c_3$ $= \begin{vmatrix} (b+c+a)(b+c-a) & 0 & a^2 \\ 0 & (c+a+b)(c-a-b) & b^2 \\ (c+a+b)(c-a-b) & (c+a+b)(c-a-b) & (a+b)^2 \end{vmatrix}$ taking $(a+b+c)$ common from C_1 & C_2 both

	$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ c-a-b & c-a-b & (a+b)^2 \end{vmatrix}$ $R_3 \rightarrow R_3(R_1 + R_2)$ $= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ -2b & -2a & 2ab \end{vmatrix}$ $c_1 \rightarrow ac_1 \text{ and } c_2 \rightarrow bc_2$ $= \frac{(a+b+c)^2}{ab} \begin{vmatrix} ab+ac-a^2 & 0 & a^2 \\ 0 & bc+ab-b^2 & b^2 \\ -2ab & -2ab & 2ab \end{vmatrix}$ $c_1 \rightarrow c_1 + c_3 \text{ and } c_2 \rightarrow c_2 + c_3$ $= \frac{(a+b+c)^2}{ab} \begin{vmatrix} ab+ac & a^2 & a^2 \\ b^2 & bc+ab & b^2 \\ 0 & 0 & 2ab \end{vmatrix}$ taking a, b and 2ab common from R_1, R_2 and R_3 resp. $= \frac{(a+b+c)^2}{ab} \cdot ab(2ab) \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ 0 & 0 & 1 \end{vmatrix}$ expanding $= 2ab(a+b+c)^2[(b+c)(c+a) - a(b) + a(0)]$ $= 2ab(a+b+c)^2(bc+ab+c^2+ac-ab)$ $= 2ab(a+b+c)^2 \cdot c(b+c+a)$ $= 2abc(a+b+c)^3 = \text{RHS} \quad \text{ans.}$
Q.3)	Show that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$
Sol.3)	We have $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$ Main step $c_1 \rightarrow c_1 - bc_3$ and $c_2 \rightarrow ac_3$ $= \begin{vmatrix} 1+a^2-b^2+2b^2 & 2ab-2ab & -2b \\ 2ab-2ab & 1-a^2+b^2+2a^2 & 2a \\ 2b-b+a^2b+b^3 & -2a+a-a^3-ab^2 & 1-a^2-b^2 \end{vmatrix}$ $= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix}$ taking $(1+a^2+b^2)$ common from c_1 and c_2 $= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$ expanding $= (1+a^2+b^2)^2[1[1-a^2-b^2+2a^2] - 2b(-b)]$ $= (1+a^2+b^2)^2[1-a^2-b^2+2a^2+2b^2]$ $= (1+a^2+b^2)^2(1+a^2+b^2)$ $= (1+a^2+b^2)^3 = \text{RHS} \quad \text{ans.}$

Solving System of Linear Equations (Matrix Method)

Q.4) Solve the equations using matrix method $x + 2y + z = 7$; $x + 3z = 11$; $2x - 3y = 1$

Sol.4) The given equation are

$$x + 2y + z = 7$$

$$x + 0y + 3z = 11$$

$$2x - 3y + 0z = 1$$

these equation can be written in matrix form

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$$

(or) $Ax = B$

$$\Rightarrow x = A^{-1}B$$

$$\text{Where } A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix} ; B = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix} \text{ \& } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Now

$$|A| = 1(0 + 9) - 2(0 - 6) + 1(-3 - 0) = 9 + 12 - 3$$

$$|A| = 18 \neq 0$$

$\therefore$ system is consistent and unique solution

Cofactors

$$c_{11} = 9 ; c_{12} = -6 ; c_{14} = -3$$

$$c_{21} = -3 ; c_{22} = -2 ; c_{23} = 7$$

$$c_{31} = 6 ; c_{32} = -2 ; c_{33} = -2$$

$$\text{Now } Adj(A) = \begin{bmatrix} 9 & 6 & -3 \\ -3 & -2 & 7 \\ 6 & -2 & -2 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot Adj(A)$$

$$A^{-1} = \frac{1}{18} \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$$

We have $x = A^{-1}B$

$$x = \frac{1}{18} \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix} \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$$

$$\Rightarrow x = \frac{1}{18} \begin{bmatrix} 63 - 33 + 6 \\ 42 - 22 - 2 \\ -21 + 77 - 2 \end{bmatrix}$$

$$\Rightarrow x = \frac{1}{18} \begin{bmatrix} 36 \\ 18 \\ 54 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$\therefore x = 2, y = 1, z = 3$ is the required solution ans.

Q.5) Solve the equations

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10 ; \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10 ; \frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$$

Sol.5) The given equations are

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$$

$$\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$$

These equations can be written in matrix form

$$\begin{bmatrix} 2 & -3 & 3 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$$

$$(or) AX = B$$

$$\Rightarrow X = A^{-1}B$$

$$\text{Where } A = \begin{bmatrix} 2 & -3 & 3 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix}; \quad B = \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}; \quad X = \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix}$$

$$|A| = 2(2 + 1) + 3(2 - 3) + 3(-1 - 3) = 6 - 3 - 12 = -9$$

$$|A| = -9 \neq 0 \quad \therefore \text{system is consistent and unique solution}$$

Cofactors

$$c_{11} = 3 \quad c_{12} = 1 \quad c_{13} = -4$$

$$c_{21} = 3 \quad c_{22} = -5 \quad c_{23} = -7$$

$$c_{31} = -6 \quad c_{32} = 1 \quad c_{33} = 5$$

$$\therefore \text{Adj}A = \begin{bmatrix} 3 & 3 & -6 \\ 1 & -5 & 1 \\ -4 & -7 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj}A = -\frac{1}{9} \begin{bmatrix} 3 & 3 & -6 \\ 1 & -5 & 1 \\ -4 & -7 & 5 \end{bmatrix}$$

We have $X = A^{-1}B$

$$X = -\frac{1}{9} \begin{bmatrix} 3 & 3 & -6 \\ 1 & -5 & 1 \\ -4 & -7 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$$

$$X = -\frac{1}{9} \begin{bmatrix} 30 + 30 - 78 \\ 10 - 50 + 13 \\ -40 - 70 + 65 \end{bmatrix}$$

$$\begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = -\frac{1}{9} \begin{bmatrix} -18 \\ -27 \\ -45 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

$$\Rightarrow x = \frac{1}{2}; \quad y = \frac{1}{3} \text{ and } z = \frac{1}{5} \text{ is the required solution} \quad \text{Ans.....}$$

Q.6)

$$\text{Find } A^{-1}, \text{ where } A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}. \text{ Hence solve the system of equations } x +$$

$$2y - 3z = -4,$$

$$2x + 3y + 2z = 2 \text{ and } 3x - 3y - 4z = 11$$

Sol.6)

$$\text{We have, } A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$$

$$|A| = 1(-12 + 6) - 2(-8 - 6) - 3(-6 - 9) = -6 + 28 + 45$$

$$|A| = 67 \neq 0 \quad \therefore (A \text{ is Invertible } | \text{consistent} | \text{ unique solution})$$

Cofactors

$$c_{11} = -6 \quad c_{12} = 14 \quad c_{13} = -15$$

$$c_{21} = 17 \quad c_{22} = 5 \quad c_{23} = 9$$

$$c_{31} = 13 \quad c_{32} = -8 \quad c_{33} = -1$$

$$\therefore \text{Adj}(A) = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$\text{Now } A^{-1} = \frac{1}{|A|} \cdot \text{Adj}A$$

$$A^{-1} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \quad \dots\dots(1)$$

Given equation are

$$x + y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 1$$

These equation can be written in the form

$$\begin{bmatrix} 1 & 1 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$(\text{or}) AX = B \Rightarrow X = A^{-1}B$$

$$\text{Where } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$X = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} \quad \dots\dots\{A^{-1} \text{ from eq. (i)}\}$$

$$X = \frac{1}{67} \begin{bmatrix} 24 + 34 + 143 \\ -56 + 10 - 88 \\ 60 + 18 - 11 \end{bmatrix} = \frac{1}{67} \begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$\therefore x = 3, y = -2, z = 1$ is the required solution ans.

Q.7)

$$\text{If } A = \begin{bmatrix} 1 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}. \text{ Find } A^{-1} \text{ and hence solve the equation } x + 2y + z = 4; -x +$$

$$y + z = 0 \text{ and } x - 3y + z = 2.$$

Sol.7)

$$\text{We have } A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

Do yourself

$$|A| = 10 \neq 0 \therefore (A \text{ is invertible})$$

$$\text{Adj}A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj}A$$

$$A^{-1} = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \quad \dots\dots(1)$$

Given equation are

$$x + 2y + z = 4$$

$$-x + y + z = 0$$

$$x - 3y + z = 2$$

$\rightarrow$ the matrix of above equation is clearly the transpose of given matrix A

$\therefore$ these equations can be written in the form

$$A'X = B \quad \text{where } B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\Rightarrow X = (A')^{-1}B$$

$$\Rightarrow X = (A^{-1})^{-1}B \quad \dots\dots\dots\{\text{By prop. } (A^{-1})^{-1} = (A^{-1})^1\}$$

$$\Rightarrow X = \frac{1}{10} \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{10} \begin{bmatrix} 16+2 \\ 8-4 \\ 8+6 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 18 \\ 4 \\ 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9/5 \\ 2/5 \\ 7/5 \end{bmatrix}$$

$$\Rightarrow x = \frac{9}{5}, y = \frac{2}{5}, z = \frac{7}{5} \text{ is the req. solution} \quad \text{ans..}$$

Q.8) Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and hence (or) use it to solve the equations $x - y + z = 4$; $x - 2y - 2z = 9$; $2x + y + 3z = 1$

Sol.8) Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$

$$CA = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$\Rightarrow CA = 8I$$

Post multiply by A^{-1}

$$\Rightarrow CAA^{-1} = 8IA^{-1}$$

$$\Rightarrow CI = 8A^{-1} \quad \dots\dots\dots\left\{ \begin{array}{l} AA^{-1} = I \\ IA^{-1} = A^{-1} \end{array} \right\}$$

$$\Rightarrow A^{-1} = \frac{1}{8}C = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

Given equation are

$$x - y + z = 4$$

$$x - 2y - 2z = 9$$

$$2x + y + 3z = 1$$

There equation can be in the form

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$(\text{or}) AX = B$$

$$\Rightarrow X = A^{-1}B \quad \text{where } B = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{8} \begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$$\Rightarrow x = 3, y = -2 \text{ and } z = -1 \text{ is the req. solution} \quad \text{ans.}$$

Q.9) $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$ find AB and hence solve the equations

$$x - 2y = 0; 2x + y + 3z = 8; -2y + x = 7$$

Sol.9)

$$AB = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$\Rightarrow AB = 11I$$

Pre by A^{-1}

$$\Rightarrow A^{-1}AB = 11 A^{-1}I$$

$$\Rightarrow IB = 11 A^{-1}$$

$$\Rightarrow B = 11 A^{-1}$$

$$\Rightarrow A^{-1} = \frac{1}{11}B = \frac{1}{11} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$$

Given equations are

$$x - 2y = 10$$

$$2x + y + 3z = 7$$

$$-2y + 0y + z = 7$$

These equations can be written in the form

$$AX = C \Rightarrow X = A^{-1}C$$

$$\text{Where } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } C = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$X = \frac{1}{11} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{11} \begin{bmatrix} 70 + 16 - 42 \\ -20 + 8 - 21 \\ -40 + 16 + 35 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

$\therefore x = 4, y = -3, z = 1$ is the required solution ans.

Q.10) Show that system of equations is consistent and also find the solution

$$2x - y + 3z = 5 ; 3x + 2y - z = 7 ; 4x + 5y - 5z = 9$$

Sol.10) Given equation are

$$2x - y + 3z = 5$$

$$3x + 2y - z = 7$$

$$4x + 5y - 5z = 9$$

given equation can be written in the form

$$AX = B \Rightarrow X = A^{-1}B$$

$$\text{where } A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 4 & 5 & -5 \end{bmatrix} ; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ \& } B = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$$

$$|A| = 0 \text{ \{solution can be infinite many or no solution\}}$$

$$\text{Now } AdjA = \begin{bmatrix} -5 & 10 & -5 \\ 11 & -22 & 11 \\ 7 & -14 & 7 \end{bmatrix}$$

$$\text{Now } (AdjA)B = \begin{bmatrix} -5 & 10 & -5 \\ 11 & -12 & 11 \\ 7 & -14 & 7 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0$$

Since $|A| = 0$ also $(AdjA) \cdot B = 0$

$\therefore$ System is consistent and Infinite many solutions

$\rightarrow$ Put $z = k$ in first two equations, we get

$$2x - y = 5 - 3k \quad \dots (k \in R)$$

$$3x + 2y = 7 + k$$

$$\text{(or)} \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 - 3k \\ 7 + k \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} ; X = \begin{bmatrix} x \\ y \end{bmatrix} ; B = \begin{bmatrix} 5-3k \\ 7+k \end{bmatrix}$$

$$|A| = 7 \text{ and } AdjA = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$X = A^{-1}B$$

$$X = \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 5-3k \\ 7+k \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 17-5k \\ 11k-1 \end{bmatrix}$$

$$\therefore x = \frac{17-5k}{7} ; y = \frac{11k-1}{7} \text{ and } z = k \quad \text{ans.}$$